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THE RÔLE OF PRICE DISPERSION

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I. *Introductory*

There has hitherto been no positive theory of price index that can prove to measure the macro-economic price-niveau; the only one set of rigid theories recognized being connected with the micro-economic niveau of cost-of-living prices. L. R. Klein has once tried to introduce the macro-economic conditions into the price-index theory,[1] but his attempt was limited to clarify those conditions which make a price-index the medium factor between the micro-economic and macro-economic models and could ever give no positive theory as such of how an index formula can show a macro-economic price-niveau.

Theories of cost-of-living index, the only one set of theories recognized hitherto, are based on the economic theory of consumer's preference and given on its behalf the name of economic "functional approach" as R. Frisch did it. The functional approach uses, on principle, the so-called "aggregative forms" as the index formulae, so that it can afford the definition of "equivalent index". And it does not need to mention here that aggregative forms of price index make no direct use of price-relatives, which are the direct component factors in case of "relative forms" of index formulae.

It is owing to the above-said context that little attention has been paid

[1] L. R. Klein, Macro-economics and the Theory of Rational Behavior, *Econometrica*, Vol. 14, No. 2, 1946.

to price-relatives in the theoretical discussion of price index numbers. The rôle of price-relatives was abandoned or neglected at the same time when the so-called "atomistic approach" proved to be deficient to give an economic-meaningful index, as the approach was going to use the distribution of price-relatives and deduce unduly therefrom an average level of price variation, that is the required answer of true price-niveau. Thus, the behavior of price-relatives distribution, or the price dispersion, seems ever to have been in exile from the theoretical context of price indices as if it committed an original sin with its accomplice, the atomistic approach as such.

It is the aim of this article to recover the rôle of price-dispersion from its exile and put it in a right position in the theory and practice of measurement of price-niveau. According to me, the price-dispersion must play an important rôle in the formation of unambiguous level of prices, contrary to what has been observed in the popular view of price-index theory. If this view of mine is of any worth, the practice of compiling price indices as well as those theoretical discussions which bear any relation to price-niveau, e. g. the current discussion on the relative price-level between industrial and agricultural sectors and so on, will be influenced to the extent.

II. *Price-dispersion and the theory of price-indices*

Price dispersion, or a behavior of price-relatives distribution, has been reminded of in one context according to the popular view of price-indices theory: in the context of the necessity to renew the base period of index numbers. As Mitchell duly pointed out[2], price dispersion has a strong tendency to increase as the distance from the base-period becomes large. Generally speaking, price-relatives show a fairly concentrated distribution in those periods near enough to the base, while this distribution becomes the more decentralized and dispersed, the more are they off the base. It is on this mere fact that Mitchell and other theoretists of price indices recommend the proper renewal of base-period, and we must ascertain here the reason of this recommendation as done from a somewhat practical point of view on one hand and from the theoretical point of view, on the other hand, which belongs right to the atomistic approach.

Why, then, must we renew the base-period before the price-dispersion grows large enough? It is because the increase of dispersion makes the average value unreliable as well as unstable, they say. We can easily find from under this sort of view the atomistic reasoning that seeks in the average price-relatives the true position of price-niveau change; which is no-

[2] W. C. Mitchell, The Making and Using of Index Numbers, *Bulletin No. 656 of the Bureau of Labour Statistics, U.S., 1938.*

thing but the cause of the failure the atomistic approaches do suffer.

I am, too, for this side; namely, I think it due in accordance with Mitchell to pay good attentions to the growing price-dispersion and to renew the base in due time. It is, however, on a reasoning different from Mitchell's and the atomistic view-point that I am for this side. Theoretically speaking, I must cling to the functional approach in the price-indices theory and yet I must, contrary to the common view, put much stress upon the fact of price-dispersion, because the factor of price-dispersion should, I suppose, be introduced into the very center of the functional approach of price indices.

As is generally known, the functional approach of price-indices defines the true index of prices as a ratio of an equivalent total expenditure in a current period to that in the base-period and explains the equivalency on the ground of one and the same indifference surface of utility, this surface being touched by two different price planes respectively in the base and current period. But very little attentions are paid to the existence of an efficient domain within which alone the system of indifference surface is available, so that, it seems, the two price-planes can be unlimitedly different each other in their angle, namely, the two positions of relative price in the base and current period are allowed to be quite different. Sometimes the efficient domain of indifference surface, it is true, has been referred to, but the reference has had little or no conscious relation to the relative-price changes.

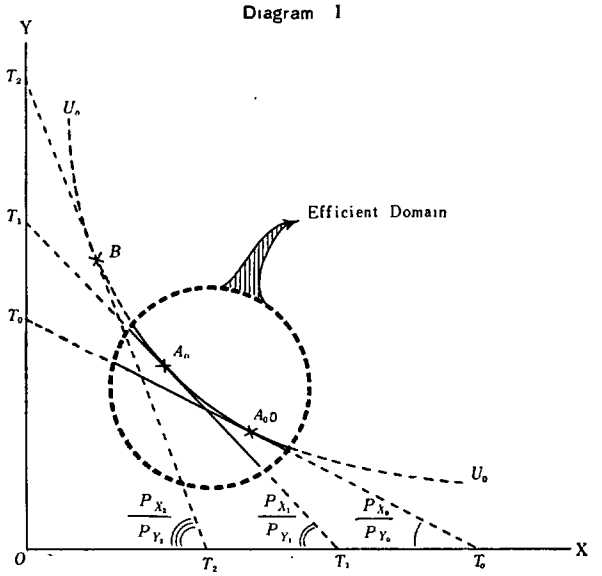
Since an indifference surface indicates the possibility of continuous change of the rate of substitution among goods to be purchased by a consumer, there must be a limitation to the availability of the surface and the two touching points of price-plane to indifference surface must be within the efficient domain. And if the domain is not so much wide—and I think this is quite plausible—, the efficient angle-change of the price-plane should not also be so large that the price-relatives between two periods, if expressed in a distribution form, show rather large a dispersion. This is the point of argument here—to point out the limitation of available surface and, what is more important, to combine the limitation with the price dispersion. Without the reminiscence of this relation the functional approach of price-indices would seem to be conclusive under any degree of price-relatives dispersion.

I will not go farther and say with the atomistic view that the price dispersion and its center-value, the mean, can alone explain the position of price-niveau change. But I would dare say with much stress that the price-dispersion gives to the central part of price-indices theory a necessary condition, —a condition fundamental for the efficient formation of price-niveau.

The situation above stated is explained in **Diagram I** in a two dimensional case. The existence of an efficient domain of indifference curve

(U_0) is shown by a circle, within which the continuous part of the curve can give the efficient touches (A_0 and A_0') to the price-lines (T_0 T_0 and T_1 T_1) and without which the touch (B) has no realistic meaning. Therefore the price-angle T_0 can move efficiently to the angle T_1, but a change to the angle T_2 (probably large change) has no meaning for the proof-making

Diagram 1



of (say) the limits theory of index numbers. These changes of price-angles between commodities X, Y $\left(\frac{p_{X0}}{p_{Y0}}\right.$ to $\frac{p_{X1}}{p_{Y1}}$, or to $\left.\frac{p_{X2}}{p_{Y2}}\right)$, if considered among much more commodities, can be transformed into price-dispersions in those distributions of price-relatives between two periods $\left(\frac{p_{X1}}{p_{X0}}, \frac{p_{Y1}}{p_{Y0}}, \cdots\cdots\right)$. And those price-dispersions which are formed by the touches within the efficient domain can not be so large in degree; those without the domain can grow with no bound. The existence of the efficient domain, therefore, makes an important condition for the functional approach of price indices.

III. *Price-dispersion in the course of price-changes*

The view pointed above may be nothing new, as it is what the functional approach should originally assume implicitly. It may be that the writer has only to make it explicit and give it some metrical expressions.

Logically speaking, the first necessary step may be to make clear the demarcation of the efficient domain of the indifference surface. But this

will lead us to a tedius work, probably more tedious than the work R. Frisch was dragged into when he tried to measure marginal utility of a commodity.[3] So, it will be a rough but economical step if, by simply assuming the existence of the domain, we measure directly price-dispersions shown in some periods of price-stability and interpret other periods with larger price-dispersion as periods when most purchases (or trades) are made beyond and without the efficient domain; in other words, periods when the formation of price-niveau is ambiguous.

The arguments above shown are due only for the micro-economic levels of cost-of-living prices. As for the macro-economic levels of, say, wholesale prices the above-stated theorem of efficient domain is of no direct use. But even apart from the foundation of indifference surfaces and efficient domain thereof, the observation of price dispersion has its own significance and is accompanied by various technical problems. There may, perhaps, loom some prospect of developing a new theoretical foundation of macro-economic price-indices on the basis of dispersion or distribution analysis.

Now let us consider some aspects of price-dispersion analysis, using wholesale price data in Japan for example. The tool of analysis here, for the time being, is the "coefficient of variation" as a measure of relative dispersion This coefficient is convenient for the observation of price dispersion in periods of straight prices change. For in periods when prices are rising in trend, that is, price relatives move upward in average M, the standard deviation σ of these price relatives is apt to increase, so that the coefficient of variation c.v., or σ/M, can keep rather constant a value if the degree of price-dispersion in those periods is moderate enough to show an unambiguous rise of price-niveau. In periods of price fall, it matters just the contrary situation. Situations are, however, somewhat complicated in most actual cases: rises and falls of prices usually appear alternately and in various lengths and degrees, so that price dispersions expressed in σ's do not necessarily show a parallel move with average prices M's. There may happen very often price-falls with ever increasing dispersions, resulting in a series of ever growing coefficients of variation. It is in such cases, I suppose, that the use of this coefficient takes a full effect, because the price situations then are such that there can never exist a unique, unambiguous price-niveau in its proper sense; some of the falling prices, for example, may be caused by a kind or so of conspicuous price-strategies, other prices being kept relatively high by the inertia lasting since the previous periods.

One more point must be added here in connection with the appraisal of this coefficient. One and the same price-situation can be interpreted to have different values of this coefficient according as which period is selected as basis to form these price-relatives. The price-relatives of a certain

[3] Ragnar Frisch, *New Methods of Measuring Marginal Utility*, 1932.

month (t) to the previous month (t−1) give usually a very small value of dispersion both in terms of standard deviation and coefficient of variation, and the price-relatives of the same month to the (t−n)th month are expected to give larger values of dispersion if the trend is rising. But there often arise those cases where the dispersion for (t−n)th month happens to be smaller than that for (t−m)th, (m<n). The fact is that the rising trend includes some falling periods in the first half, of which the bottom lies at (t−m)th month, and more rapidly rising periods follow in the latter half, to the effect that the whole trend runs upwards. In these cases the price-niveau for the t-th month must not be appraised simply to be more ambiguous on the basis of (t−m)th month than on the more remote basis of (t−n)th month. For the price-structure may have entirely changed from (t−n)th month to t-th, which details it is that we must put under further investigations in such a complicated case.

I selected, for examining the above-stated arguments by actual data, series of Tokyo Wholesale Price Index (Revised) published by the Bank of Japan. As this new series (1952=100) are composed of as many as 403 items, I took only two groups of item, textile goods (98 items) and materials for building and construction (37 items). The prices of the first group (textiles) showed a downward trend, while those of the second (materials for building) gave an upward trend, for the periods 1952 to 1954. My tentative intention is to find the degree of ambiguity of price-niveau w. r. t. each of these groups. September of 1954 was selected as the t-th month, and price relatives were calculated on the basis of August of the same year (t−1th month), the average of 1953 and the average of 1952 (namely the base-period of this index). A frequency distribution of each set of these price-relatives was formed by using as frequencies, not the numbers of items, but the weights belonging to each class of the price-relative classifications. M's, σ's and c v. 's calculated from these distributions are as follows:

Table I.

	Group I (Textiles)			Group II (Materials for Building)		
	M	σ	c.v.	M	σ	c.v.
Sept. 1954 / Aug. 1954	99.8	4.2	0.042	101.0	1.2	0.011
〃 / 1953	90.9	12.2	0.135	106.1	5.9	0.056
〃 / 1952	88.8	11.0	0.126	131.2	24.5	0.187
1953 / 1952	98.5	11.7	0.118	123.2	16.3	0.132

* This line is added for reference.
(Distribution data for this table are shown in Appendix of this article.)

The M's column, which shows but slightly different values from those calculated directly from the group index published by the Bank because of the formation process of frequency distribution, indicates clearly the respective direction of average-price trend w.r.t. these two groups.

On inspection, Group II shows almost parallel changes in M and σ, resulting in a comparatively small change in c.v., as was expected in case of upward trend. Group I, however, contains something to be explained. That is, while the averages of price-relatives (M) become smaller as the basis of price-relatives goes farther, the coefficient of variation for the 1953 basis is rather higher than that for the 1952 basis and show the highest among the three cases. There must be something abnormal somewhere during these periods considered. The fact is that average prices of textile goods, though they show a downward trend throughout the whole periods, had periods of upward change in 1953 and subsequent periods of steep fall in 1954, so that a balancing effect came to force, leaving the c.v. for 1952 basis unexpectedly small (0.126). To endorse this, we have only to inspect the c.v. in the reference line (values for 1953 relative to 1952) which is as low as 0.118 compared with 0.135 for Sept.1954/1952.

All this tells us the use of coefficient of variation must not be on a single definite base (as 1952), but in a series of gradually shifting bases as shown in Table I. In other words, the degree of price dispersion for a period must be interpreted as variant according to the basis considered. Large values of c.v., if there appears any among these variant degrees, are worth while further investigations.

What is then the limit of size of c.v., within which a unique and unambiguous price-niveau could be seen formed? Table I shows 0.011 as the smallest and 0.187 as the largest. The niveau may be said the more clearly-formed, the smaller is the c.v., but we can not give at this stage any theoretical limit towards large values. Foughly speaking from the formal point of view, 0.2 may be taken as too large. It may also be said to be not yet so large. The distinct limit, however, could be found only empirically through a more extensive system of observations in connection with chronological studies.

IV. *Conclusion and some of its applications*

I have above suggested the rôle of price dispersion in forming a distinct price-niveau. The idea that the efficient domain of indifference surfaces has a close relation to price dispersion is, I suppose, what has hitherto been neglected in the theoretical as well as practical approach of price-index and price-niveau. But economy of calculating labour on my side could not make me attain at any significant result to discern whether a period has

or has not a distinct, unambiguous price-niveau. What was attained at is nothing but some informations about price dispersion by means of the coefficient of variation.

Even at this extremely incomplete stage of study, however, some applications of the view are not impossible. Two ideas of application can be mentioned here, one practical and the other rather theoretical.

The first is concerned with the practical treatment of price index data. No published price-indices have ever informed us a measure of dispersion with them. It may be that no such a practice has been necessary in countries and periods with fair stability of prices. However, in a country as post-war Japan where constant instability of economy prevails, all the price indices published formally by authorized statistical agencies had better inform to the public at the same time with their release of current index some data of price dispersion as an important "red sign" to warn the possible ambiguity of the calculated change of price-niveau. This suggestion may add the importance all the more when we think of the fact most of price-indices are not founded on a meaningful theory of price-niveaus.

The second application, of a more theoretical tint, is concerning the recent situation of economic theory where the factor of price, especially relative price, came to reobtain economic theoreticians' interest long after Keynes neglected the factor of price in his system. For instance in the theory of economic growth, relative prices between agricultural and non-agricultural sectors are put under lime-light. In such a case relative prices between sectors is usually treated as if a distinct price-niveau were established in each of the sectors under consideration. If this assumption of an unambiguous formation of price-niveau be removed, the whole structure, at least the model-building of economic growth, would suffer a considerable revision, I think. The rôle of price dispersion will be found not small in these context.

—Written in Dec., 1954—

APPENDIX

The distribution of price-relatives used in the calculations of M, σ and c.v. in III of this article are as follows :—

Group I (Textile goods)

(1) Sept.1954/Aug.1954

x	f	x	f
88	1.6	101	16.4
89	—	102	4.9
90	1.0	103	8.4
91	—	104	0.5

(2) Sept. 1954/Average of 1953

x	f
65	1.6
75	16.2
85	33.5
95	27.9

92	3.3	105	1.7
93	3.6	106	0.2
94	0.3	107	0.4
95	1.1	108	0.1
96	2.7	109	—
97	8.9	110	0.7
98	5.8	111	4.9
99	8.4		
100	25.1		100.0

105	16.3
115	0.7
125	3.7
135	0.1
	100.0

(3) Sept. 1954/Average of 1952

x	f
65	3.0
75	15.9
85	39.5
95	27.9
105	10.5
115	1.9
125	1.3
	100.0

(4) 1953/1952

x	f
65	3.3
75	1.5
85	15.2
95	31.9
105	35.2
115	10.9
125	2.0
	100.0

Group II (Materials for building and construction)

(1) Sept. 1954/Aug. 1954

x	f
94	0.5
98	8.8
100	49.0
101	20.8
102	10.3
103	8.1
105	2.5
	100.0

(2) Sept. 1954/Average of 1953

x	f
92.5	4.3
97.5	14.4
102.5	20.2
107.5	27.9
112.5	33.2
	100.0

(3) Sept. 1954/Average of 1952

x	f
85	4.5
95	17.6
105	0.8
115	6.0
125	7.6
135	18.3
145	13.8
155	31.4
	100.0

(4) 1953/1952

f
2.6
9.0
16.5
11.8
19.5
17.5
23.1
—
100.0