A test of serial independence of deviations from cointegrating relations

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December 2005

Abstract

In this paper, we develop a method of testing whether deviations from a cointegration relationship are serially independent. Because whether deviations are serially independent is an important issue in the study of economics, the test offers benefits to practitioners.

Keywords: Cointegration; Serial independence

JEL classification: C12; C32

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1 INTRODUCTION

The problem of cointegration, i.e. situations in time-series regression analysis where deviations from a long-run relationship follow a stationary process, has attracted much research interest in recent years. Cointegration techniques aiming to investigate for the presence of cointegrating vectors and to estimate their values have come to be widely used by econometricians. However, researchers applying cointegration methods need to pay attention not only to the cointegration relationship itself but in some cases also to the deviations from the relationship. That is to say, in some empirical applications of cointegration methods, the serial independence of the errors needs to be examined. For example, when examining the unbiasedness hypothesis in foreign exchange markets, the cointegrating regression might be specified as follows:

$$s_t = \beta_0 + \beta_1 f_{t-1} + u_t, \tag{1}$$

where s_t and f_t are the natural logarithms of the spot rate and the forward rate, respectively. For the unbiasedness hypothesis to hold, β_0 must be equal to zero and β_1 equal to one. In addition, u_t must be serially uncorrelated. The intuition underlying these requirements, in the words of Brenner and Kroner (1995, 33) is that 'if all relevant information is immediately impounded into asset prices, then on average, the forward rate should equal the realized spot rate, and there should be no information left in the residuals to help predict future spot rates'. However, as Zivot (1998) notes, many practitioners fail to address the question of serial correlation.

In this paper, using Johansen's vector error correction (VEC) model, we develop a method of testing whether deviations from a cointegration relationship are serially independent. The proposed test statistic is easy to calculate and asymptotically χ^2-distributed as shown in Section 2. In order to evaluate the test, we provide Monte Carlo comparisons of the proposed test statistic and the test suggested by Kellard, Newbold, and Rayner (2001), hereafter KNR, for the size distortions and the power of the tests. KNR (2001) argued that if Schwartz's Bayesian information criterion (SBIC) fit ARMA(0,0) to the estimated residual u_t obtained from Johansen's VEC model, then it can then be concluded that u_t is serially uncorrelated. The Monte Carlo study reported in Section 3 indicates that in terms of power, the proposed test is superior. Section 4 contains a small empirical application of the test. Section 5 concludes.

2 TESTING FOR SERIAL INDEPENDENCE

Consider an m-vector process for the cointegrating relationship generated by Johansen's VEC model,

$$\Delta x_t = \alpha\beta' x_{t-1} + \sum_{j=1}^{p-1} \Gamma_j \Delta x_{t-j} + \Theta D_t + \varepsilon_t, \tag{2}$$

where x_t is an $(m \times 1)$ vector of variables integrated of order one, Γ_i is an $(m \times m)$ matrix, Δ is the first difference operator and ε_t is distributed $IIN(0, \Omega)$. The deterministic terms D_t contain regressors that we consider non-stochastic. Supposing the system (2) is cointegrated with rank r and $0 < r < m$, α is a full rank $(m \times r)$ matrix and β is a full rank $(m \times r)$ matrix consisting of r cointegrating vectors such that $u_t = \beta' x_t$ is stationary.

Our interest is in whether deviations from cointegration relations are serially uncorrelated, i.e. whether u_t is serially uncorrelated. Obviously, whether errors from cointegration relationships are serially uncorrelated is independent of the deterministic terms D_t; in what follows, we therefore drop these terms for convenience. We state the following proposition, giving the testable condition for the null hypothesis of serial independence of the deviations:

Proposition 1. *Let x_t be generated by (2) with cointegration rank $0 < r < m$. Then, $\beta' x_t$ is serially uncorrelated if and only if*

$$\beta'[\alpha \ \Gamma_1 \ \cdots \ \Gamma_2] = [-I \ 0 \ \cdots \ 0] \tag{3}$$

Proof: We solve (2) for $\beta' x_t$. Multiplying equation (2) by β' and collecting terms, we obtain

$$\beta' x_t = (I - (I + \beta'\alpha)L)^{-1}[\sum_{j=1}^{p-1} \beta' \Gamma_j C(L) L^j + \beta']\varepsilon_t \tag{4}$$

where L is the lag operator and $C(L)$ is the lag polynomial such that $\Delta x_t = C(L)\varepsilon_t$. According to Johansen (1995), Δx_t is expressed as the vector moving average process, $\Delta x_t = C(L)\varepsilon_t$. Equation (4) represents the time series structure of $u_t = \beta' x_t$ itself. Substituting (3) in (4), we obtain

$$\beta' x_t = \beta' \varepsilon_t.$$

This means that if (3) holds, then deviations from cointegration relations are serially uncorrelated.

Furthermore, equation (4) establishes that the restriction that makes all the coefficients of ε_i for $i = 1, 2, \cdots$ zero is only (3). $\square$

In what follows, we take (3) as the testable condition of the null of serial independence of the forecast errors. That is, we consider the following testing problem:

$$H_0: \ \beta'[\alpha \ \Gamma_1 \ \cdots \ \Gamma_2] = [-I \ 0 \ \cdots \ 0] \text{ versus } H_1 : \text{not } H_0. \tag{5}$$

For notational convenience we denote $[\alpha \ \Gamma_1 \ \cdots \ \Gamma_2]$ and $[-I \ 0 \ \cdots \ 0]$ as θ and I_0, respectively.

The Wald test is exploited to handle problem (5). Specifically,

$$W = T\{vec(\hat{\beta}'\hat{\theta}) - vec(I_0)\}'(\hat{\Sigma})^{-1}\{vec(\hat{\beta}'\hat{\theta}) - vec(I_0)\}, \tag{6}$$

where $\Sigma = \beta'\Omega\beta \otimes \Sigma_{XX}^{-1}$, $\Sigma_{XX} = Var[(\beta' x_{t-1}); \Delta x_{t-1}; \cdots ; \Delta x_{t-p+1})]$, T denotes the sample size, "ˆ" denotes the ML estimator, and $vec(\cdot)$ is the row-stacking operator.

Next, we derive the limiting distribution of (6).

Proposition 2.

$$W \xrightarrow{d} \chi^2_{r(r+(p-1)m)}$$

Proof: Johansen's (1995) Theorem 13.5 states

$$\sqrt{T}\{vec(\hat{\theta}) - vec(\theta)\} \xrightarrow{d} N(0, \Omega \otimes \Sigma_{XX}^{-1}).$$

Hence, we obtain

$$\begin{aligned}\sqrt{T}\{vec(\hat{\beta}'\hat{\theta}) - vec(\beta'\theta)\} &= \sqrt{T}vec(\beta'\hat{\theta}) - \sqrt{T}vec(\beta'\theta) + \sqrt{T}(vec(\hat{\beta}'\hat{\theta}) - vec(\beta'\hat{\theta})) \\ &\xrightarrow{d} N(\beta'\Omega\beta \otimes \Sigma_{XX}^{-1}) + 0. \end{aligned} \tag{7}$$

We derive the desired result by using (7). □

3 MONTE CARLO SIMULATION

In this section, we conduct a Monte Carlo simulation to investigate the finite-sample properties of the test statistic. The numerical performance of (6) is compared with that of the alternative method of testing serial independence proposed by KNR (2001).

In our experiment, the following data generating process is employed:

$$\Delta x_t = \begin{bmatrix} 0.75 \\ -0.5 \end{bmatrix} \begin{bmatrix} -1.0 & 0.5 \end{bmatrix} x_{t-1} + \Gamma_1 \Delta x_{t-1} + \varepsilon_t,$$

where ε_t is i.i.d.$N(0, I_2)$. To test the null hypothesis of serial independence, we let

$$\Gamma_1 = \begin{bmatrix} 0.35 & -0.35 \\ 0.7 & -0.7 \end{bmatrix}.$$

Under the alternative, we let

$$\Gamma_1 = \begin{bmatrix} 0.35 & -0.35 \\ 0.7 & -0.7 \end{bmatrix} + \delta \begin{bmatrix} 0 & 0.05 \\ -0.05 & 0 \end{bmatrix}$$

where $\delta = 1, 2$. The sample sizes are $T = 100, 200, 400$, and the nominal size of our test is 5%.

In order to evaluate the performance of the method used by KNR (2001), we count the number of times SBIC does not fit ARMA(0,0) to $\hat{\beta}' x_t$, where $\hat{\beta}'$ is the maximum likelihood estimator, based on 10,000 replications. Comparing this count with the rejection frequency of the test is an appropriate way in which the two methods can be compared.

Table 1: The numerical performance of the test and SBIC model selection

	size		power($\delta = 1$)		power($\delta = 2$)	
	W	SBIC	W	SBIC	W	SBIC
$T = 100$	6.7	10.6	16.0	12.9	44.0	20.7
$T = 200$	6.4	6.6	23.8	9.5	72.2	24.6
$T = 400$	5.3	4.1	42.5	8.7	96.3	43.9

Note: Our test is denoted by "W" and the method used by KNR (2001) is denoted by "SBIC".

Table 1 shows the results of the experiment. The results of the experiment suggest that the size distortions of our test are moderate. The power property of our test contrasts with the corresponding part of SBIC model selection. Under the alternative, the proportion of correct selections by SBIC is considerably smaller than the empirical power of our test. Seen in this light, our test performs considerably better than SBIC model selection.

4 AN EMPIRICAL EXAMPLE

In this section, we provide an empirical example to demonstrate the proposed test. As mentioned in the introduction, the unbiasedness hypothesis with regard to foreign exchange markets requires the error from the cointegration relationship to be white noise. Specifically, it is required that u_t in equation (1) is serially independent.

We consider a bivariate VEC model,

$$\Delta y_t = \alpha \beta' y_{t-1} + \sum_{j=1}^{p-1} \Gamma_j \Delta y_{t-j} + \Theta D_t + \varepsilon_t, \tag{8}$$

with $y_t = [s_t, f_{t-1}]'$, where s_t and f_t are the natural logarithms of the dollar-yen spot rate and the one month dollar-yen forward rate, respectively. Monthly data for the dollar-yen spot rate and the one month forward rate were taken from DataStream. The sample period for which monthly data are available is 1984:6 and to 2004:6, giving a sample size of $T = 241$. $\beta' y_{t-1}$ in (8) represents cointegration relationship (1) with an appropriate specification of the deterministic terms, D_t.

Table 2: Testing the unbiasedness hypothesis in foreign exchange market

(A) Test for unit root of exchange rates

	ADF test
spot	-1.532
forward	-1.374

(B) Estimated lag length of VAR by SBIC: 4

(C) Estimated equation: $\Delta x_t = \alpha \beta^{+\prime} x_{t-1}^{+} + \sum_{j=1}^{3} \Gamma_j \Delta x_{t-j} + \varepsilon_t$,
where $x_{t-1}^{+} = [1, x_{t-1}']'$, $\beta^{+} = [\beta_0, \beta']'$.

(D) Test for the cointegration rank

H_0	$r = 0$	$r \leq 1$
Trace	25.190^a	11.893^b

(E) Standardized cointegrating vector $\hat{\beta}^{+\prime}$

$\hat{\beta}_0 = 0.008 \quad \hat{\beta}_1 = 1.000 \quad \hat{\beta}_2 = -1.002$

$(s_t = -0.008 + 1.002 f_{t-1} + u_t, \quad u_t \sim I(0))$

(F) Test statistic for the null of $\beta_0 = 0$: 0.035

(G) Test statistic for the null of $\beta_1 = -\beta_2$: 0.067

(H) Selected model for u_t by SBIC: ARMA(0,0)

(I) Test statistics for $\beta' \theta = I_0$: 19.045^a

Note: Critical values for the Trace test are obtained from Osterwald-Lenum (1992). [a] and [b] indicate rejection of the hypothesis at the 1% and 5% critical level, respectively.

Applying the Augmented Dickey-Fuller test, it appears from (A) in Table 2 that s_t and f_t have a unit root. Next, we apply Johansen's test with specifications (B) and (C). (D) shows the results of the test for cointegration. We conclude that the rank of cointegration r is one because the null of $r = 0$ at the 1% significance level is rejected while the null of $r \leq 1$ is not rejected at the 1% significance level.

(F) and (G) report likelihood ratio test statistics based on Johansen's (1995) Theorem 11.3 and Theorem 7.2, respectively. These statistics are asymptotically χ^2-distributed. In this empirical example, the number of degrees of freedom is one. (F) and (G) show that the stationary combination $s_t = f_{t-1} + u_t$ holds. The next test is to check for serial correlation in u_t. As shown in (H), SBIC selects ARMA(0,0) for the estimated residual u_t, i.e., u_t is found to be uncorrelated. Therefore, the unbiasedness hypothesis is accepted. On the other hand, our test rejects the null of independence at the 1% significance level as shown in (I). The result appears to be consistent with our Monte Carlo experiment. Section 3 indicated that while our test had reasonable empirical power, SBIC tended to select ARMA(0,0) or failed to detect dependence of u_t in the Monte Carlo experiment. While the method proposed by KNR (2001) would have led one to accept the unbiasedness hypothesis, the proposed test suggests that u_t is in fact serially correlated and the unbiasedness hypothesis should be rejected.

5 CONCLUSION

In this paper, we proposed a method to test the null of serial independence of a deviation from a cointegration relation. Whether deviations are serially independent is an important issue in the study of economics and finance, and the test proposed here offers various benefits: The test statistic is easy to calculate and is asymptotically χ^2-distributed. Moreover, our test performs well in the mean in that its size distortion is moderate and it showed reasonably high power in the Monte Carlo experiment. The empirical example examining the unbiasedness hypothesis with regard to foreign exchange rates yielded results corroborating that our test has high power.

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