A variant of nonconsequentialism and its characterization

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Abstract

In this paper we investigate a choice behavior that recognizes the intrinsic value of opportunities for choice. *Nonconsequentialism* is a choice attitude toward outcomes and opportunities and prioritizes opportunities for choice rather than outcomes. It is possible to define various types of nonconsequentialism in terms of criteria for ranking opportunity sets. We consider the *leximax* ordering over opportunity sets, which takes into account both the quality and quantity of elements in opportunity sets. This paper presents an axiomatic characterization of a nonconsequentialist *extended preference* whose opportunity evaluation is based on the leximax ordering.

Key words: nonconsequentialism; consequence; opportunity; extended preference; leximax

JEL classification: D00, D60, D63

1 Introduction

This paper explores the wider conceptual frameworks of welfare economics. Welfare economics has attended rather strictly to welfaristic features of consequences in assessing a social state and an economic policy. In this paper, however, we take into account nonconsequential features in welfare analysis. In particular, we investigate a choice behavior which recognizes the intrinsic value of opportunities for choice, since choosing itself can be seen as a valuable functioning. Sen (1999), for example, explains that "having an x when there is no alternative may be sensibly distinguished from choosing x when substantial alternatives exist. Fasting is not the same thing as being forced to

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starve. Having the option of eating makes fasting what it is, to wit, choosing not to eat when one could have eaten (Sen 1999, p.76)." Hence, choosing x from the opportunity set A, which contains some elements other than x, is better than choosing x from the singleton set $\{x\}$.

In a recent paper, Suzumura and Xu (2001) develop an extended framework to express the importance of opportunities for choice, where individuals have *extended preference orderings* over the pairs of outcomes and opportunity sets from which choices are made. [1] Extended preference orderings are effective in analyzing the following situations.

Suppose that an individual faces a choice situation of a political party and have two one-party rules, one is dictatorship by the Left and the other is dictatorship by the Right. Formally, they are defined by

$$\{(x, \{x\}), (z, \{z\})\},$$

where x stands for the Left and z stands for the Right. In this situation, suppose that he prefers the Right to the Left, that is, $(z, \{z\})$ is preferred to $(x, \{x\})$. Now, let us suppose that the political system transforms from dictatorship into democracy and the Center (y) forms against the Left. The situation changes as follows:

$$\{(x, \{x, y\}), (z, \{z\})\}.$$

In this situation he may choose $(x, \{x, y\})$ over $(z, \{z\})$ since democracy is preferred to dictatorship, and then he compensates the poorness of consequences by the richness of opportunities. Thus, extended preference orderings can express such a substitution of welfare evaluation.

Within the extended framework Suzumura and Xu (2001) provide two concepts, *consequentialism* and *nonconsequentialism*, and characterize them ax-

[1] See Baharad and Nitzan (2000) and Gravel (1994, 1998) for alternative analyses of extended preference orderings. They mainly focus on the inconsistency between extended preference orderings on the pairs of outcomes and opportunity sets and orderings on outcomes, and seem to be based on more traditional rational choice theory. Namely, given an opportunity set, each individual chooses the optimal outcome with respect to his own preference on outcomes from the opportunity set. On the other hand, our approach is close to Sen's (1985) view of rational choice. That is, "considerations of rational choice must introduce other aspects of a person's choice, e.g., values other than pursuing one's own well-being. A serious consideration of what a person should choose has to take fuller note of the resulting state of affairs [and in non-consequentialist approaches, of other things as well $\cdots$ (Sen 1985, p.66)]." Indeed, Suzumura and Xu (2001) also stress that Gravel's (1994, 1998) approach is quite different from theirs. We will subsequently give an example which appropriately expresses our approach.

iomatically. This paper focuses especially on nonconsequentialism, which prioritizes opportunities for choice rather than outcomes. Moreover, we consider both the richness of opportunities and the quality of outcomes.

The criteria for ranking opportunity sets determine to a large extent the extended preference orderings of nonconsequentialism. The question posed here concerns the criteria that should be adopted for ranking opportunity sets within the nonconsequentialist framework. One criterion adopted in conventional welfare economics is the indirect-utility ordering of opportunity sets. That is, only the maximal possible utility achievable from an opportunity set matters in establishing the ranking of opportunity sets. The indirect-utility ordering regards the value of opportunities as instrumental; however, the literature has shown the inadequacy of drawing exclusively on achieved utilities and ignoring considerations of freedom in ranking opportunity sets. [2]

Pattanaik and Xu (1990) offer an alternative criterion: Opportunity sets are ranked by the degree of freedom based on the number of elements in each set. According to this cardinality-based ordering, opportunity set A offers at least as much freedom as opportunity set B if and only if A has at least as many elements as B. Suzumura and Xu (2001) also adopt this cardinality-based ordering in introducing the concepts of consequentialism and nonconsequentialism.

We examine whether the cardinality-based ordering, as in Suzumura and Xu (2001), is appropriate within the extended framework. Let us consider the following example: An individual working for a railroad company wants to improve working conditions. There exist three conceivable alternatives: protesting alone (x), not protesting (y), and joining a strike (z). We suppose that if he protests alone, he may lose his job, whereas striking will be the most effective in improving working conditions. Therefore, he prefers z to y, and y to x in terms of his own well-being. Then consider the following situation:

$$\{(y, \{x, y\}), (y, \{y, z\})\}.$$

In the first circumstance, he is not able to join a strike (he may have a non-permanent contract), and chooses not to protest for his own well-being; on the other hand, in the second circumstance, he could join a strike, but he chooses not to protest because he predicts that if a strike takes place, the lives of a great many of passengers will be disrupted. [3]

[2] See Bossert et al. (1994), Pattanaik and Xu (1990, 2000), Puppe (1995, 1996) and Sen (1988, 1991) for discussions of freedom of choice. See also Barberà et al. (2003) for surveys of the literature on the ranking of opportunity sets.

[3] A consideration of choosing a non-optimal outcome in terms of one's own well-being is not insignificant since "(t)here is an enormous difference between choosing

Then it seems plausible that $(y, \{y, z\})$ is better than $(y, \{x, y\})$, since whereas in $(y, \{x, y\})$ he does not have recourse to a strike and $(y, \{x, y\})$ leaves no potential for improvement in terms of his well-being, in $(y, \{y, z\})$ he does not join a strike but $(y, \{y, z\})$ could improve his well-being if he joined a strike. Individuals who care about both the richness of opportunities and the quality of outcomes should take into account the quality of elements in opportunity sets. In this situation $\{y, z\}$ offers a richer opportunity than $\{x, y\}$ since z is preferred to y, and y to x for his own well-being. However, if one adopts the cardinality-based ordering in ranking opportunity sets, the decision maker would be indifferent between $(y, \{x, y\})$ and $(y, \{y, z\})$, since each opportunity set contains the same number of elements and y is chosen from each set. In the subsequent section, we will discuss a relevant axiom that springs out of the above criticism.

In order to propose an extended preference ordering which plausibly prefers $(y, \{y, z\})$ to $(y, \{x, y\})$, we should consider a ranking of opportunity sets that takes into account not only the degree of freedom based on the number of elements, but also the quality of opportunity sets in terms of individual's indirect utility. In that case, the issue of ranking opportunity sets is to discuss how to combine the indirect-utility ordering with the cardinality-based ordering. An approach to this issue is to make a lexicographic comparison between two criteria. One of the two criteria takes priority, and there exist two possible lexicographic relations: the *cardinality-first lexicographic* relation and the *indirect-utility-first lexicographic* relation.[4] We adopt the *leximax* ordering over opportunity sets (Bossert et al. 1994).[5] If the best element in opportunity set A is better than that of opportunity set B, the leximax ordering coincides with the indirect-utility ordering in the comparison of A and B. However, unlike the indirect-utility ordering, the leximax ordering takes into account the number of elements in a given opportunity set. This is because if an opportunity set is a strict subset of another opportunity set and the best elements in each set are indifferent, then the leximax ordering coincides with the cardinality-based ordering rather than the indirect-utility ordering.

This paper introduces an extended preference ordering of nonconsequentialism whose opportunity evaluation is based not on the cardinality-based ordering, but on the leximax ordering. Moreover, we provide an axiomatic characterization of this proposed extended preference ordering, and apply these axioms in

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tea or coffee according to one's taste (and concern for personal well-being), and choosing to join, or not to join, a strike, taking note, *inter alia*, of obligations to others; or working hard or giving to charity out of sympathy or commitment (Sen 1985, pp.18-19)."

[4] See Bossert et al. (1994) and Dutta and Sen (1996) for details.

[5] In this paper, opportunity sets are finite. Therefore, the leximax ordering is defined for finite sets, as established in Bossert et al. (1994). See Arlegi et al. (2005) for the infinite case.

a comparison of two nonconsequentialist extended preference orderings, one introduced here and the other proposed in Suzumura and Xu (2001).

In section 2 we present basic notation and definitions. Section 3 shows several axioms used in this paper and discusses their implications. Section 4 contains the main results, including the axiomatic characterization of a nonconsequentialist extended preference ordering. Section 5 offers a brief conclusion.

2 Notation and definitions

Let $\mathbb{R}$ ($\mathbb{R}_{++}$) be the set of all (all positive) real numbers. $\mathbb{R}^m$ is the m-fold Cartesian product of $\mathbb{R}$. $\mathbf{0}^m$ is the origin of $\mathbb{R}^m$.

Let X be the set of all social states; we assume that $3 \leq |X| = m < \infty$. The elements of X are denoted by $x, y, z, \ldots$, and they are called *outcomes*. Let K be the set of all non-empty subsets of X. The elements of K are denoted by $A, B, C, \ldots$, and they are called *opportunity sets*. $X \times K$ denotes the Cartesian product of X and K. The elements of $X \times K$ are denoted by $(x, A), (y, B), (z, C), \ldots$, and they are called *extended alternatives*. In particular, if an extended alternative is the pair of an outcome and a singleton set, e.g. $(x, \{x\})$, we call it a *singleton pair alternative*. Let $\Omega = \{(x, A) | A \in K \textit{ and } x \in A\}$. That is, Ω contains all (x, A) such that $x \in A$ whenever $(x, A) \in \Omega$. The interpretation of $(x, A) \in \Omega$ is that the outcome x is chosen from the opportunity set A.

Let R be an extended preference ordering over Ω, which is *reflexive*, *complete* and *transitive*. The asymmetric and symmetric parts of R are denoted by P and I, respectively. For any $(x, A), (y, B) \in \Omega$, $(x, A)R(y, B)$ implies that choosing x from A is at least as good as choosing y from B. Without loss of generality, we assume that the elements of $A \in K$ are ordered in decreasing preference according to R over singleton pair alternatives. That is, we have $(a_1, \{a_1\})R(a_2, \{a_2\})R \cdots R(a_r, \{a_r\})$ for all $A = \{a_1, \ldots, a_r\} \in K$ with $r := |A| \leq m$.

3 Axioms

In this section, we discuss axioms used in our characterization. We also mention axioms introduced by Suzumura and Xu (2001). We first present a list of axioms and then discuss their implications.

Independence (IND). For all $(x, A), (y, B) \in \Omega$ and all $z \in X \backslash (A \cup B)$,

$(x, A)R(y, B) \Leftrightarrow (x, A \cup \{z\})R(y, B \cup \{z\})$.

Strong Independence (SIND). For all $(x, A), (y, B) \in \Omega$ and all $z, w \in X \backslash (A \cup B)$ such that $(z, \{z\})I(w, \{w\})$, $(x, A)R(y, B) \Leftrightarrow (x, A \cup \{z\})R(y, B \cup \{w\})$.

Simple Indifference (SI). For all $x \in X$ and all $y, z \in X \backslash \{x\}$, $(x, \{x, y\})I(x, \{x, z\})$.

Natural Indifference (NI). For all $x \in X$ and all $y, z \in X \backslash \{x\}$ such that $(y, \{y\})I(z, \{z\})$, $(x, \{x, y\})I(x, \{x, z\})$.

Monotonicity (MON). For all $(x, A), (x, B) \in \Omega$, $B \subseteq A \Rightarrow (x, A)R(x, B)$.

Simple Preference for Opportunities (SPO). For all $x, y \in X$, $x \neq y$, $(y, \{x, y\})P(x, \{x\})$.

Simple Extension (SE). For all $x, y \in X$, $(x, \{x, y\})R(y, \{x, y\}) \Leftrightarrow (x, \{x\})R(y, \{y\})$.

Before offering the last axiom, we present a definition. Given R defined over Ω, for all $A, B \in K$, A is *equivalent to B with respect to R* if and only if there exists a bijective mapping σ from A to B such that for all $a \in A$, $(a, \{a\})I(\sigma(a), \{\sigma(a)\})$. If there exists no such bijective mapping, A is *not equivalent to B with respect to R*.

Robustness of Preference (RP). Given R defined over Ω, for all $(x, A), (y, B) \in \Omega$ such that A is not equivalent to B with respect to R, and for all $z \in X \backslash (A \cup B)$, if $(x, A)P(y, B)$, $(a, \{a\})P(z, \{z\})$ for all $a \in A$, and $(b, \{b\})R(z, \{z\})$ for all $b \in B$, then $(x, A)P(y, B \cup \{z\})$.

Axioms (IND), (SI), (MON) and (SPO) are discussed by Suzumura and Xu (2001).[6] (IND) requires that for all opportunity sets A and B, when an outcome z is added to or eliminated from both A and B, extended preference orderings after transformation are consistent with original ones, independent of the property of the outcome z. The stronger version of (IND), that is, (SIND) requires that whenever $(z, \{z\})I(w, \{w\})$, after the elements z and w are added to or eliminated from A and B respectively, the transformation does not affect original extended preference ordering.

(SI) requires that choosing x from the two opportunity sets consisting of two

[6] See the concluding remarks in Suzumura and Xu (2001) for the discussion of (MON).

outcomes be regarded as indifferent no matter what outcomes are added to x. As stated in the introduction, (SI) is not plausible if individuals care about both the richness of opportunities and the quality of outcomes. In that case, if $(z, \{z\})$ is better than $(x, \{x\})$, and $(x, \{x\})$ is better than $(y, \{y\})$, then it is plausible that $(x, \{x, z\})$ is better than $(x, \{x, y\})$. Therefore, we do not use (SI) in our characterization result. Instead of (SI), we adopt a weak condition of (SI), that is (NI). In addition to the same situation as (SI), (NI) assumes that $(y, \{y\})I(z, \{z\})$.

(MON) seems plausible in this context. It requires that choosing x from the opportunity set A be at least as good as choosing the same x from the opportunity set B which is a subset of A.

(SPO) represents the desire for opportunities for choice, stipulating that the choice from the opportunity set containing two elements is always better than the choice from the singleton set, where the element in the singleton set could be chosen from the opportunity set containing two elements.

(SE) requires that the extended preference ordering over two singleton pair alternatives $(x, \{x\})$ and $(y, \{y\})$ be ranked in the same way as the extended preference ordering over extended alternatives $(x, \{x, y\})$ and $(y, \{x, y\})$. The gist of (SE) is that when an individual faces the alternative of x or y, he should choose the better outcome.

(RP) corresponds to a condition, *Robustness of Strict Preference*, used by Bossert et al. (1994), but it is independent of Suzumura and Xu's (2001) *robustness* axiom. To make sure of the intuitive content of (RP), let us consider the example in the introduction, where $X = \{$protesting alone (x), not protesting (y), joining a strike $(z)\}$. We first consider the following situation:

$$\{(y, \{y\}), (y, \{y, z\})\}.$$

In the first circumstance, an individual have no means to protest; on the other hand, in the second circumstance, he could join a strike, but he chooses not to protest. Note that for any extended preference ordering R, $\{y\}$ is not equivalent to $\{y, z\}$ with respect to R. We suppose that $(y, \{y, z\})$ is ranked higher than $(y, \{y\})$ with respect to R. (RP) requires that even if a worse outcome x is added to the set $\{y\}$, $(y, \{y, z\})$ is still ranked higher than $(y, \{x, y\})$. Indeed, this conclusion sounds plausible as discussed in the introduction. (RP) implies that increasing the cardinality of the opportunity set does not always improve the situation in that the strict preference cannot be undone by adding worse outcomes to an extended alternative (y, B) that is worse than (x, A).

The following results establish certain consequences and relationships between some of the above axioms, which will be useful to prove our characterization result. See also the next section for an independence result. The first result extends the property of (SPO) to the case where opportunity sets contain more than two elements.

Lemma 1. If R satisfies (IND) and (SPO), then for all $(x, A) \in \Omega$ and all $y \in X \backslash A$, $(y, A \cup \{y\}) P(x, A)$.

Proof. Suppose that R satisfies (IND) and (SPO). Suppose that there exist some $(x, A) \in \Omega$ and some $y \in X \backslash A$ such that $(x, A) R(y, A \cup \{y\})$. By (IND), for all $z \in A \backslash \{x\}$, we have $(x, A \backslash \{z\}) R(y, A \backslash \{z\} \cup \{y\})$. It follows from the repeated use of (IND) that $(x, \{x\}) R(y, \{x, y\})$. However, by (SPO), we must have $(y, \{x, y\}) P(x, \{x\})$, which leads to a contradiction. Thus, we have $(y, A \cup \{y\}) P(x, A)$ for all $(x, A) \in \Omega$ and all $y \in X \backslash A$. ■

The next result implies that if R satisfies (IND) and (SE), the ordering over two extended alternatives which have the same opportunity set is, in fact, judged on their outcomes.

Lemma 2. If R satisfies (IND) and (SE), then for all $(x, A), (y, A) \in \Omega$, $(x, A) R(y, A) \Leftrightarrow (x, \{x\}) R(y, \{y\})$.

Proof. Suppose that R satisfies (IND) and (SE). By (SE), for all $x, y \in X$, we have $(x, \{x, y\}) R(y, \{x, y\}) \Leftrightarrow (x, \{x\}) R(y, \{y\})$. From (IND), for all $z \in A \backslash \{x, y\}$, we obtain $(x, \{x, y\} \cup \{z\}) R(y, \{x, y\} \cup \{z\}) \Leftrightarrow (x, \{x, y\}) R(y, \{x, y\})$. Therefore, we have $(x, \{x, y\} \cup \{z\}) R(y, \{x, y\} \cup \{z\}) \Leftrightarrow (x, \{x\}) R(y, \{y\})$. It follows from the repeated use of (IND) that $(x, A) R(y, A) \Leftrightarrow (x, \{x\}) R(y, \{y\})$. ■

The following two results represent relationships between some of the above axioms.

Lemma 3. R satisfies (IND) and (NI) if and only if it satisfies (SIND).

Proof. If R satisfies (SIND), then it is easy to check that R also satisfies (IND) and (NI). Now, suppose that R satisfies (IND) and (NI). First, by (NI), for all $x \in X$, and all $z, w \in X \backslash \{x\}$ such that $(z, \{z\}) I(w, \{w\})$, we have $(x, \{x, z\}) I(x, \{x, w\})$. From (IND), for all $h \in X \backslash \{x, z, w\}$, we can get $(x, \{x, z\} \cup \{h\}) I(x, \{x, w\} \cup \{h\})$. By the repeated use of (IND), we obtain $(x, A \cup \{z\}) I(x, A \cup \{w\})$. Next, suppose $(x, A) R(y, B)$. From (IND), for all $w \in X \backslash (A \cup B)$, we have $(x, A \cup \{w\}) R(y, B \cup \{w\})$. Therefore, if $(z, \{z\}) I(w, \{w\})$, then we can obtain $(x, A \cup \{z\}) R(y, B \cup \{w\})$ from the transitivity of R. If we suppose $(x, A \cup \{z\}) R(y, B \cup \{w\})$, then we have $(x, A) R(y, B)$ by the similar argument. That is, (SIND) holds. ■

Lemma 4. If R satisfies (IND), (SPO) and (SE), then it also satisfies (MON).

Proof. Suppose that R satisfies (IND), (SPO) and (SE). First, we show that we have the following:

$$\text{For all } (x, A) \in \Omega \text{ and all } y \in X \backslash A,\ (x, A \cup \{y\})R(x, A). \tag{1}$$

Suppose that for some $(x, A) \in \Omega$ and some $y \in X \backslash A$, $(x, A)P(x, A \cup \{y\})$. From (IND), for all $z \in A \backslash \{x\}$, we have $(x, A \backslash \{z\})P(x, A \backslash \{z\} \cup \{y\})$. By the repeated use of (IND), we can obtain the following:

$$(x, \{x\})P(x, \{x, y\}). \tag{2}$$

Then we can distinguish three cases: (a)$(x, \{x\})P(y, \{y\})$, (b)$(y, \{y\})P(x, \{x\})$ and (c)$(x, \{x\})I(y, \{y\})$.

Case (a): By (SE), we have $(x, \{x, y\})P(y, \{x, y\})$. It follows from (2) and the transitivity of R that $(x, \{x\})P(y, \{x, y\})$, which contradicts to (SPO).

Case (b): Given (2) and the assumption in case (b), we have $(y, \{y\})P(x, \{x, y\})$ by the transitivity of R, which leads to a contradiction to (SPO).

Case (c): (SE) implies $(x, \{x, y\})I(y, \{x, y\})$. From (2) and the transitivity of R, we have $(x, \{x\})P(y, \{x, y\})$, which is a contradiction to (SPO).

Thus, we must have (1) since R is complete. It clearly follows from (1) and the transitivity of R that R satisfies (MON). ■

4 A characterization result

In this section, we define and characterize a nonconsequentialist extended preference ordering. We first present a choice behavior toward extended alternatives. Let $\succsim_\Theta$ be a complete ordering over K such that, for all $A, B \in K$, $A \succsim_\Theta B$ implies that A contains as much opportunity as B according to $\succsim_\Theta$. $\succ_\Theta$ and $\sim_\Theta$ are the asymmetric and symmetric parts of $\succsim_\Theta$ respectively. We can consider $\succsim_\Theta$ as a measure of the richness of opportunities.

R is said to be *opportunity-first consequence-second* (OFCS) if, for all (x, A), $(y, B) \in \Omega$, $A \succ_\Theta B \Rightarrow (x, A)P(y, B)$ and $A \sim_\Theta B \Rightarrow [(x, A)R(y, B) \Leftrightarrow (x, \{x\})R(y, \{y\})]$. This attitude toward extended alternatives is the generalization of *strong nonconsequentialism*, which is provided by Suzumura and Xu (2001).[7] By this generalization, we can discuss the extended framework beyond a special measure. According to OFCS, two extended alternatives, (x, A)

[7] Strong nonconsequentialism is equivalent to *Cardinality-OFCS* defined below. See also Suzumura and Xu (2004) for the generalized measure of opportunities.

and (y, B), are ranked exclusively in terms of a measure $\succsim_\Theta$ between A and B, and if it is indifferent between A and B, (x, A) and (y, B) are judged on the extended preference ordering over $(x, \{x\})$ and $(y, \{y\})$. We now define a choice attitude toward the extended alternatives characterized in this paper and follow with remarks.

Given R defined over Ω, let $u : X \to \mathbb{R}_{++}$ be such that, for all $x, y \in X$, $u(x) \geq u(y) \Leftrightarrow (x, \{x\})R(y, \{y\})$. For $A = \{a_1, \ldots, a_r\} \in K$ with $r := |A| \leq m$, let

$$v(A) := (u(a_1), \ldots, u(a_r), \mathbf{0}^{m-r}).$$

Note that we assume $(a_1, \{a_1\})R(a_2, \{a_2\})R \cdots R(a_r, \{a_r\})$. Let $\geq_l$ be the lexicographic ordering on $\mathbb{R}^m$. That is, $\geq_l = \{(x, y) \in \mathbb{R}^m \times \mathbb{R}^m | \exists j$ such that $x_j > y_j$ and $\forall i < j\ x_i = y_i$, or $\forall i\ x_i = y_i\}$. The *leximax* ordering $\succsim_L$ over K is defined by

$$\forall A, B \in K, \qquad A \succsim_L B :\Leftrightarrow v(A) \geq_l v(B).$$

The asymmetric and symmetric parts of $\succsim_L$ are denoted by $\succ_L$ and $\sim_L$, respectively. We are now ready to identify *Leximax-OFCS*.

Leximax-OFCS. R is said to be *Leximax-OFCS* if, for all $(x, A), (y, B) \in \Omega$, $A \succ_L B \Rightarrow (x, A)P(y, B)$ and $A \sim_L B \Rightarrow [(x, A)R(y, B) \Leftrightarrow (x, \{x\})R(y, \{y\})]$.

Thus, Leximax-OFCS is a nonconsequentialist extended preference ordering with the leximax ordering over opportunity sets. The leximax ordering over K makes a lexicographic comparison between elements in the opportunity sets in decreasing preference order. If this comparison is not decisive, the role of the number of elements becomes crucial in ranking opportunity sets. If two opportunity sets contain the same number of elements, then the two sets are indifferent according to the leximax ordering. Within the extended framework, the correspondence to the utility function is derived from an extended preference ordering over singleton pair alternatives consisting of each element in a given opportunity set.

Suzumura and Xu (2001) propose the OFCS whose opportunity set evaluation is the cardinality-based ordering. The formal definition is as follows:

Cardinality-OFCS. R is said to be *Cardinality-OFCS* if, for all $(x, A), (y, B) \in \Omega$, $|A| > |B| \Rightarrow (x, A)P(y, B)$ and $|A| = |B| \Rightarrow [(x, A)R(y, B) \Leftrightarrow (x, \{x\})R$ $(y, \{y\})]$.

To exemplify extended preference orderings for Leximax-OFCS and Cardinality-OFCS, let us consider the example discussed in the introduction, where $X = \{$protesting alone (x), not protesting (y), joining a strike $(z)\}$.

Example 1. Let R_1 be a Leximax-OFCS's extended preference ordering and

R_2 be a Cardinality-OFCS's extended preference ordering. We assume that $(z, \{z\})P_i(y, \{y\})P_i(x, \{x\})$ for $i \in \{1, 2\}$. Then two extended orderings R_1 and R_2 are as follows:

$(z, X)P_1(y, X)P_1(x, X)P_1(z, \{y, z\})P_1(y, \{y, z\})P_1(z, \{x, z\})P_1(x, \{x, z\})P_1(z, \{z\})$
$P_1(y, \{x, y\})P_1(x, \{x, y\})P_1(y, \{y\})P_1(x, \{x\})$.

$(z, X)P_2(y, X)P_2(x, X)P_2(z, \{x, z\})I_2(z, \{y, z\})P_2(y, \{x, y\})I_2(y, \{y, z\})P_2(x, \{x, y\})$
$I_2(x, \{x, z\})P_2(z, \{z\})P_2(y, \{y\})P_2(x, \{x\})$.

As noted in the introduction, it is plausible that an individual who cares about both the richness of opportunities and the quality of outcomes takes into account the quality of elements in opportunity sets. For this reason, $(y, \{y, z\})$ is better than $(y, \{x, y\})$. Notice that whereas R_1 appropriately grasps this situation, the decision maker is indifferent in terms of R_2 between $(y, \{x, y\})$ and $(y, \{y, z\})$.

Our main result is the following theorem, which characterizes Leximax-OFCS axiomatically.

Theorem 1. R satisfies (IND), (NI), (SPO), (SE) and (RP) if and only if it is Leximax-OFCS.

Proof. If R is Leximax-OFCS, then it clearly satisfies (IND), (NI), (SPO), (SE) and (RP). Therefore we have only to prove that if R satisfies (IND), (NI), (SPO), (SE) and (RP), then, for all $(x, A), (y, B) \in \Omega$, $A \succ_L B \Rightarrow (x, A)P(y, B)$ and $A \sim_L B \Rightarrow [(x, A)R(y, B) \Leftrightarrow (x, \{x\})R(y, \{y\})]$.

At first, we show that if $A \succ_L B$, then $(x, A)P(y, B)$. Let $A \succ_L B$. Let $A = \{a_1, \dots, a_r\}$, $B = \{b_1, \dots, b_s\}$ with $r, s \leq m$. There are two possible cases:

(a) $r > s$, and for all $\lambda \in \{1, \dots, s\}$, $(a_\lambda, \{a_\lambda\})I(b_\lambda, \{b_\lambda\})$;

(b) $\exists k \in \{1, \dots, min\{r, s\}\}$ such that for all $\lambda \in \{1, \dots, k-1\}$, $(a_\lambda, \{a_\lambda\})I$ $(b_\lambda, \{b_\lambda\})$ and $(a_k, \{a_k\})P(b_k, \{b_k\})$.

In case (a), given $(a_1, \{a_1\})I(b_1, \{b_1\})$, it follows from (SPO) and the transitivity of R that $(a_r, \{a_1, a_r\})P\ (b_1, \{b_1\})$. If $|B| > 1$, by the assumption in case (a), for all $z \in B \backslash \{b_1\}$, there exists $\hat{z} \in A \backslash \{a_1, a_r\}$ such that $(z, \{z\})I(\hat{z}, \{\hat{z}\})$. Since R satisfies (SIND) by lemma 3, we have $(a_r, \{a_1, a_r\} \cup \{\hat{z}\})P(b_1, \{b_1, z\})$. By the repeated use of (SIND), we can obtain $(a_r, \{a_1, \dots, a_s\} \cup \{a_r\})P(b_1, B)$. Since R satisfies (MON) by lemma 4, $\{a_1, \dots, a_s\} \cup \{a_r\} \subseteq A$ implies $(a_r, A)R$ $(a_r, \{a_1, \dots, a_s\} \cup \{a_r\})$. $(a_r, A)P(b_1, B)$ follows from the transitivity of R. By lemma 2, we have $(x, A)R(a_r, A)$ for all $x \in A$, and $(b_1, B)R(y, B)$ for all $y \in B$. Then it follows from the transitivity of R that $(x, A)P(y, B)$.

In case (b), we distinguish two subcases: $k = 1$ and $k > 1$.

Subcase $k = 1$: We can get $(a_1, \{a_1\})P(b_1, \{b_1\})$ by the assumption in case (b). If $|B| > 1$, (RP) implies $(a_1, \{a_1\})P(b_1, B)$. If $|A| > 1$, we have $\{a_1\} \subseteq A \backslash \{a_r\}$. From (MON), we can obtain $(a_1, A\backslash\{a_r\})R(a_1, \{a_1\})$. By lemma 1, we have $(a_r, A)P(a_1, A\backslash\{a_r\})$. It follows from the transitivity of R that $(a_r, A)P(b_1, B)$. From lemma 2 and the transitivity of R, we obtain $(x, A)P(y, B)$ by an argument similar to case (a).

Subcase $k > 1$: Given $(a_1, \{a_1\})I(b_1, \{b_1\})$, we have $(a_k, \{a_1, a_k\})P(b_1, \{b_1\})$ from (SPO) and the transitivity of R. By the repeated use of (SIND), we have $(a_k, \{a_1, \ldots, a_k\})P(b_1, \{b_1, \ldots, b_{k-1}\})$. (RP) implies $(a_k, \{a_1, \ldots, a_k\})P(b_1, B)$. If $r > k$, we have $\{a_1, \ldots, a_k\} \subseteq A\backslash\{a_r\}$. $(a_k, A\backslash\{a_r\})R(a_k, \{a_1, \ldots, a_k\})$ follows from (MON). By Lemma 1, we obtain $(a_r, A)P(a_k, A\backslash\{a_r\})$. It follows from the transitivity of R that $(a_r, A)P(b_1, B)$. From lemma 2 and the transitivity of R, we can get $(x, A)P(y, B)$, which is analogous to case (a).

Next, we show that if $A \sim_L B$, then for all $(x, A), (y, B) \in \Omega$, $[(x, A)R(y, B) \Leftrightarrow (x, \{x\})R(y, \{y\})]$. Note that $A \sim_L B$ implies $v(A) = v(B)$.

We first prove that $(x, A)P(y, B) \Leftrightarrow (x, \{x\})P(y, \{y\})$. Suppose $(x, \{x\})P(y, \{y\})$. By $v(A) = v(B)$, for $x \in A$, there exists $\hat{x} \in B$ such that $(x, \{x\})I(\hat{x}, \{\hat{x}\})$. On the other hand, for $y \in B$, there exists $\hat{y} \in A$ such that $(y, \{y\})I(\hat{y}, \{\hat{y}\})$. From (IND), we have $(x, \{x, \hat{y}\})I(\hat{x}, \{\hat{x}, \hat{y}\})$ and $(y, \{\hat{x}, y\})I(\hat{y}, \{\hat{x}, \hat{y}\})$. It follows from (SE) that $(\hat{x}, \{\hat{x}, \hat{y}\})P(\hat{y}, \{\hat{x}, \hat{y}\}) \Leftrightarrow (\hat{x}, \{\hat{x}\})P(\hat{y}, \{\hat{y}\})$. By the transitivity of R, we have $(x, \{x, \hat{y}\})P(y, \{\hat{x}, y\})$. Since for all $z \in A\backslash\{x, \hat{y}\}$, there exists $\hat{z} \in B\backslash\{y, \hat{x}\}$ such that $(z, \{z\})I(\hat{z}, \{\hat{z}\})$, we can get $(x, \{x, \hat{y}\} \cup \{z\})P(y, \{\hat{x}, y\} \cup \{\hat{z}\})$ by (SIND). From the repeated use of (SIND), we have $(x, A)P(y, B)$. We can show the converse by a similar argument.

Secondly, we prove that $(x, A)I(y, B) \Leftrightarrow (x, \{x\})I(y, \{y\})$. Suppose $(x, \{x\})I(y, \{y\})$. Notice that $v(A) = v(B)$, and we have $(x, A)I(y, B)$ by the repeated use of (SIND). The converse also follows from a similar argument. ■

The next result proves the independence of the axioms used in Theorem 1.

Theorem 2. Axioms (IND), (NI), (SPO), (SE) and (RP) are independent.

Proof. Five examples are used to prove the theorem. In the examples, we will show that the corresponding orderings satisfy all axioms but one. Let $X = \{x, y, z\}$.

(a) $\neg$(IND): Consider R_1 defined by $(x, X)I_1(y, X)I_1(z, X)P_1(x, \{x, y\})I_1(y, \{x, y\})$ $P_1(x, \{x, z\})P_1(y, \{y, z\})P_1(z, \{x, z\})I_1(z, \{y, z\})P_1(x, \{x\})I_1(y, \{y\})P_1(z, \{z\})$. R_1 satisfies (NI), (SPO), (SE) and (RP), but it violates (IND) since $(y, \{y, z\})$

$P_1(z, \{y, z\})$ and $(y, X)I_1(z, X)$.

(b) $\neg$(NI): Consider R_2 defined by $(x, X)P_2(y, X)I_2(z, X)P_2(x, \{x, y\})P_2(x, \{x, z\})$ $P_2(y, \{x, y\})I_2(z, \{x, z\})P_2(x, \{x\})P_2(y, \{y, z\})I_2(z, \{y, z\})P_2(y, \{y\})I_2(z, \{z\})$. R_2 satisfies (IND), (SPO), (SE) and (RP), but it violates (NI) since $(y, \{y\})I_2(z, \{z\})$ and $(x, \{x, y\})P_2(x, \{x, z\})$.

(c) $\neg$(SPO): Consider R_3 defined by $(x, X)I_3(y, X)I_3(z, X)I_3(x, \{x, y\})I_3(y, \{x, y\})$ $I_3(x, \{x, z\})I_3(y, \{y, z\})I_3(z, \{x, z\})I_3(z, \{y, z\})I_3(x, \{x\})I_3(y, \{y\})I_3(z, \{z\})$. R_3 satisfies (IND), (NI), (SE) and (RP), but it violates (SPO) since $(y, \{x, y\})I_3(x, \{x\})$.

(d) $\neg$(SE): Consider R_4 defined by $(x, X)I_4(y, X)I_4(z, X)P_4(x, \{x, y\})I_4(y, \{x, y\})$ $P_4(x, \{x, z\})I_4(y, \{y, z\})I_4(z, \{x, z\})I_4(z, \{y, z\})P_4(x, \{x\})I_4(y, \{y\})P_4(z, \{z\})$. R_4 satisfies (IND), (NI), (SPO) and (RP), but it violates (SE) since $(y, \{y\})P_4(z, \{z\})$ and $(y, \{y, z\})I_4(z, \{y, z\})$.

(e) $\neg$(RP): Consider R_5 defined by $(x, X)P_5(y, X)I_5(z, X)P_5(x, \{x, y\})I_5(x, \{x, z\})$ $P_5(y, \{x, y\})I_5(y, \{y, z\})I_5(z, \{x, z\})I_5(z, \{y, z\})P_5(x, \{x\})P_5(y, \{y\})I_5(z, \{z\})$. R_5 satisfies (IND), (NI), (SPO) and (SE), but it violates (RP) since $(x, \{x\})P_5(y, \{y\})$, $(x, \{x\})P_5(z, \{z\})$, $(y, \{y\})I_5(z, \{z\})$ and $(y, \{y, z\})P_5(x, \{x\})$. ■

5 Concluding remarks

We investigated a nonconsequentialist extended preference ordering over the pairs of outcomes and opportunity sets from which outcomes are chosen. It is possible to characterize various types of noncosequentialist extended preference ordering in terms of criteria of ranking opportunity sets. In the case where an individual cares about both the quality of outcomes and the richness of opportunities, it is appropriate to consider not only the number of elements in opportunity sets but also the quality of opportunity sets in terms of indirect-utility. We adopted the leximax ordering which is a ranking of opportunity sets that combines the indirect-utility ordering with the cardinality-based ordering, and applied it to the nonconsequentilaism framework. Moreover, we can compare axioms used in characterization results of Leximax-OFCS and Cardinality-OFCS. The following theorem is proposed by Suzumura and Xu (2001) to characterize Cardinality-OFCS.

Theorem 3 (Suzumura and Xu 2001, Theorem 5.2). R satisfies (IND), (SI) and (SPO) if and only if it is Cardinality-OFCS.

Axioms which characterize the above two OFCS are marked with $\oplus$ in Table

1. Axioms which are consistent (resp. inconsistent) with the OFCS get a + (resp. $\times$) mark.

[Insert Table 1]

(IND) and (SPO) are used in both characterization results. (SI) clearly implies (NI). Whereas (SI) is used in the characterization of Cardinality-OFCS, Leximax-OFCS does not satisfy (SI). Leximax-OFCS uses (NI) instead of (SI) in the characterization. (SI) takes into consideration the quantity of opportunity sets, yet (NI) considers the quality of opportunity sets as well as the quantity. (SE) is satisfied by both Leximax-OFCS and Cardinality-OFCS. (RP) is used in the characterization of Leximax-OFCS but it is inconsistent with Cardinality-OFCS. It is possible to identify (RP) as an axiom that takes the quality of opportunity sets into consideration. Note that (MON), defined in the previous section, is satisfied by Leximax-OFCS as well as by Cardinality-OFCS. Despite these observations, note that the only source of discrepancy lies in the axioms considering either quality or quantity of opportunity sets.

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Table 1

	Leximax-OFCS	Cardinality-OFCS
IND	$\oplus$	$\oplus$
SI	$\times$	$\oplus$
NI	$\oplus$	$+$
SPO	$\oplus$	$\oplus$
SE	$\oplus$	$+$
RP	$\oplus$	$\times$

(IND): Independence
(SI): Simple Indifference
(NI): Natural Indifference
(SPO): Simple Preference for Opportunities
(SE): Simple Extension
(RP): Robustness of Preference
$\oplus$ stands for that the axiom is used for the characterization.
$+$ stands for that the axiom is consistent with the OFCS.
$\times$ stands for that the axiom is inconsistent with the OFCS.