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Privately informed parties and policy divergence

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Abstract

This paper presents a Downsian model of political competition in which parties have incomplete but richer information than voters on policy effects. Each party can observe a private signal of the policy effects, while voters cannot. In this setting, voters infer the policy effects from the party platforms. In this political game with private information, we show that there exist weak perfect Bayesian equilibria (WPBEs) at which the parties play different strategies, and thus, announce different platforms even when their signals coincide. This result is in contrast with the conclusion of the Median Voter Theorem in the classical Downsian model. Our equilibrium analysis suggests similarity between the set of WPBEs in this model and the set of uniformly perfect equilibria of Harsanyi and Selten (1988) in the model with completely informed parties which we studied in a previous paper (Kikuchi, 2010).

1 Introduction

Elections often involve uncertainty about the effects of policy alternatives. Which policy works the best depends not only on parties' motivations and voters' preferences but also on unobservable external conditions. In such situations, parties can take advantage of expertise offered by private think tanks or government officials, whereas most voters only have publicly accessible information such as that provided by the mass media. In this paper, we present a model of political competition in which parties have incomplete but richer information than voters on policy effects.

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Our earlier paper constructed a model of political competition in which two office-seeking parties (that is, "Downsian" parties[1]) have complete information on policy effects, while voters are only endowed with a common prior distribution (Kikuchi, 2010). In this setting, voters infer the policy effects from the party platforms.

In this political game with asymmetric information, we showed that there exist weak perfect Bayesian equilibria (WPBEs) at which the party platforms diverge with positive probability. We refined WPBEs, showing that there exist uniformly perfect equilibria with policy divergence.[2]

On the one hand, these results are in contrast with the conclusion of the Median Voter Theorem in the classical Downsian model, which states that under some natural assumptions, two office-seeking parties will announce the same platform. On the other hand, our results have some consistency with empirical studies of two-party politics. For example, Fiorina (2006) and McCarty et al. (2008) report that in recent decades, the positions of the Democratic and the Republican parties in the United States have polarized rather than converged.

The present paper extends the previous model to the case where the parties can only observe private signals of the policy effects, while voters only have a common prior distribution. An interpretation of the model is that parties have different sources of policy-relevant information which are unavailable to voters. For example, the Democratic and the Republican parties in the United States rely on different think tanks for policy research.

Even in this alternative setting, party platforms may transmit useful information on the policy effects to voters. For example, suppose that the diplomatic relations with a foreign country X is at issue. The number w of weapons of mass destruction that X possesses affects voters' preferences over policy alternatives. Each party I receives a private signal $\hat{w}_I$ that serves as an estimate of w. Suppose that each party I's strategy is characterized by a cutpoint α_I such that it announces a hard-line policy against X if $\hat{w}_I > \alpha_I$, while it announces a soft-line policy if $\hat{w}_I < \alpha_I$. Thus, if, say, $\alpha_1 < \alpha_2$ and parties 1 and 2 announce hard-line and soft-line policies, respectively, then voters will think that w is likely to be somewhere between α_1 and α_2.

In this political game with private information, we show that there exist WPBEs at which the parties play different strategies, and thus, the parties announce different platforms even when their signals coincide. This result is in marked contrast with the conclusion of the Median Voter Theorem.

[1]In this paper, the qualifier "Downsian" indicates the assumption made by Downs (1957) that parties or candidates are solely motivated to win office.

[2]The concept of uniform perfectness is introduced by Harsanyi and Selten (1988).

Our equilibrium analysis suggests similarity between the set of WPBEs in this model and the set of uniformly perfect equilibria in the model with completely informed parties. We thus expect that the number of WPBEs is much smaller in the present model than in the previous model. In Section 4, we provide an example for which this conjecture is true at least for a class of WPBEs with simple strategies of the parties.

There are other papers that study political competition between privately informed parties. Among them, Heidhues and Lagerlöf (2003) present a model closest to ours. The only difference between their and our settings is that they assume a binary signal space, while we assume a continuous signal space. Yet, an interesting difference exists between the equilibria of the two models. In their model, the only pure-strategy equilibria that satisfy a condition called "symmetric voting" are the completely pooling equilibria at which the two parties choose the same fixed policy irrespective of their signals. Hence, the parties' platforms always converge and, moreover, reveal no information on policy effects to voters. By contrast, in our model, there exist pure-strategy equilibria satisfying symmetric voting at which the parties choose different policies even when they observe the same signal. Moreover, at the equilibria, one party's platform reveals some information on policy effects to voters.

Banks (1990) studies incomplete information about candidates' "types," where the type of a candidate represents his ideal policy. Each candidate can select any platform, while the policy that he actually implements if he wins the election is fixed at his type. In his model, there exist universally divine equilibria of Banks and Sobel (1990) at which the candidates play different pure strategies if and only if the costs of lying (that is, announcing platforms distant from the candidates' types) are sufficiently low. In our model, parties play different strategies, although they are assumed to commit to their platforms.

The rest of this paper is organized as follows. In Section 2, we construct the model of political competition with privately informed parties. In Section 3, we study WPBEs in this game. In Section 4, we provide an example for which we can derive the set of WPBEs with simple strategies of the parties. Section 5 concludes.

2 The model

2.1 Information structure

There are two political parties, 1 and 2, and a fixed number of voters. A majority voting determines one party as the winner. Before the election, the parties simultaneously announce their platform policies. The parties can choose either policy L or policy R. The winning party implements its platform.

Each voter i's preference between the two policies depends on his type $t_i \in [-1,1] = X$ and the state $\theta \in X$. We assume that there exists a unique median type $t_m \in (-1,1)$. In each state θ, voter i's preference is represented by the utility difference between policies L and R, which we denote by $u(t_i,\theta)$. In state θ, he prefers policy L if $u(t_i,\theta) > 0$, and prefers policy R if $u(t_i,\theta) < 0$.

The state is unobservable for the parties and voters. We denote by P the prior distribution function of the state on interval X. Let Θ be a random variable which describes the state.

Each party receives a private signal of the state before its platform choice. Party 1 and party 2's signals are values s and t, respectively, in interval X. They cannot observe each other's signal. Let S and T be random variables that describe party 1's signal and party 2's signal, respectively. We assume that given $\Theta = \theta$, the random variables S and T are distributed identically and independently according to a distribution function $Q(\cdot \mid \theta)$ on X.

Voters observe the parties' platforms before voting. This is equivalent to that they observe the *event* $\Lambda \in \{\emptyset, 1, 2, 12\}$ which represents the set of the parties that announce policy L. Events can be classified according to whether the parties' platforms converge or diverge. Events $\emptyset$ and 12 are called *convergence events*, while events 1 and 2 are called *divergence events*.

We assume the following:

Assumption 1. *Function $u(t,\cdot)$ is continuous and decreasing for all type t. Moreover, $u(t,\theta) < 0 < u(t,\theta')$ if and only if $\theta' < t < \theta$.*

Assumption 2. *P has a density p with support X. $Q(\cdot \mid \theta)$ has a density $q(\cdot \mid \theta)$ with support X for all state θ.*

Assumption 3. *q has the strict monotone likelihood ratio property (SMLRP). That is, $s > s'$, $\theta > \theta'$ implies $q(s \mid \theta)q(s' \mid \theta') > q(s' \mid \theta)q(s \mid \theta')$.*

Assumption 1 says that every voter's type represents his utility threshold of the state in the sense that he prefers policy L to policy R if and only if his type exceeds the state. Assumption 2 says that the distribution functions P and $Q(\cdot \mid \theta)$, $\theta \in X$, have densities with full support. Assumption 3 means roughly that a higher signal indicates a higher state, and is stronger than the assumption that if $\theta' < \theta$, then $Q(\cdot \mid \theta)$ first-order stochastically dominates $Q(\cdot \mid \theta')$.

We denote by $\bar{u}_m$ the prior mean of the median voters' utility:

$$\bar{u}_m = E(u(t_m, \Theta)).$$

2.2 Strategies

Each party seeks to maximize the probability of being elected.

Party 1's strategy is a Borel function $f : X \to [0,1]$ that assigns to each signal s the probability that it announces policy L given signal s. Party 2's strategy is a Borel function $g : X \to [0,1]$ that assigns to each signal t the probability that it announces policy L given signal t.

In every event, each voter votes for a party if its platform gives him a higher expected utility; if he is indifferent between the two parties' platforms, he votes for each party with probability 1/2. This implies that in every convergence event, all voters vote randomly.

Each voter i's strategy is a function $h_i : \{1,2\} \to [0,1]$. It assigns to each divergence event Λ the probability that in event Λ, he votes for the party Λ, that is, the party with platform L. In this definition, we restrict the domain of h_i to divergence events, since we have fixed voters' local strategies in convergence events. We denote by $h = (h_i)$ the strategy profile of voters, and denote by $h(\Lambda) = (h_i(\Lambda))$ the profile of voting probabilities in event Λ.

We focus on equilibrium strategies of the parties characterized by cutpoints. Parties 1's strategy f and party 2's strategy g are said to have cutpoints if there exist $x, y \in X$ such that

$$f(s) = \begin{cases} 1 & \text{if } s < x \\ 0 & \text{if } s > x \end{cases}, \quad g(t) = \begin{cases} 1 & \text{if } t < y \\ 0 & \text{if } t > y \end{cases}.$$

We call the strategy with cutpoint x and the strategy with cutpoint y simply *strategy x* and *strategy y*, respectively.

3 Political equilibrium

In this section, we study the pure strategy weak perfect Bayesian equilibria in which the parties play cutpoint strategies. We denote by x and y the respective strategies of parties 1 and 2. If the strategy profile (x, y, h) is a weak perfect Bayesian equilibrium for some strategy profile h of voters, then (x, y) is simply called a *weak perfect Bayesian equilibrium (WPBE)*.

Remark. *The strategy pairs $(-1,-1)$ and $(1,1)$, that is, the strategy pairs in which both parties always choose the same fixed policy, are WPBEs.*

To observe this, consider the strategy pair $(-1,-1)$. Given this strategy pair, the only reachable event is the convergence event $\emptyset$, that is, the event that both parties choose policy R. In event $\emptyset$, both parties win with probability 1/2.

Suppose that in every divergence event, all voters share a belief that the state is so large that the median voters prefer policy R to policy L. Then, in any state, no party cannot gain a majority by changing its platform to policy L. The same argument applies for the strategy pair $(1,1)$. $\square$

We denote by $\pi(h,\Lambda)$ the probability that in the divergence event Λ, policy L defeats policy R, given a strategy profile h of voters. If h is a profile of voters' best responses to (x,y), then for every divergence event Λ reachable given (x,y), we have

$$\pi(h,\Lambda) = \begin{cases} 1 & \text{if } E(u(t_m,\Theta)\,|\,x,y,\Lambda) > 0 \\ \frac{1}{2} & \text{if } E(u(t_m,\Theta)\,|\,x,y,\Lambda) = 0 \\ 0 & \text{if } E(u(t_m,\Theta)\,|\,x,y,\Lambda) < 0 \end{cases}, \tag{1}$$

where $E(\cdot\,|\,x,y,\Lambda)$ denotes the expectation given the strategy pair (x,y) and event Λ. In the second line of (1), we use the standard assumption that each party wins with probability 1/2 if the election results in a tie.

By Assumptions 1-3, if $E(u(t_m,\Theta)\,|\,x,y,\Lambda)$ exists, it is continuously decreasing in x and y (See Milgrom, 2001).

The conditional distribution function of T given $S=s$ is defined by

$$\Phi(t\,|\,s) = \frac{\int_X Q(t\,|\,\theta)q(s\,|\,\theta)p(\theta)\,d\theta}{\int_X q(s\,|\,\theta)p(\theta)\,d\theta}.$$

Assumption 3 implies that $\Phi(t\,|\,\cdot)$ is decreasing for all t.

We denote by $\Pi_1(L\,|\,y,h,s)$ and $\Pi_1(R\,|\,y,h,s)$ the respective winning probabilities of party 1 when it announces policy L and when it announces policy R, given its signal s, party 2's strategy y, and a profile h of voters' strategies. These probabilities are defined by

$$\begin{aligned} \Pi_1(L\,|\,y,h,s) &= \tfrac{1}{2}\Phi(y\,|\,s) + \pi(h,1)(1-\Phi(y\,|\,s)), \\ \Pi_1(R\,|\,y,h,s) &= \tfrac{1}{2}(1-\Phi(y\,|\,s)) + (1-\pi(h,1))\Phi(y\,|\,s). \end{aligned} \tag{2}$$

Party 2's winning probabilities $\Pi_2(L\,|\,x,h,t)$ and $\Pi_2(R\,|\,x,h,t)$ are similarly defined.

Lemma 1 says that (x,y) is a WPBE if and only if the median voters are indifferent between policies L and R in every divergence event reachable given (x,y).

Lemma 1. *Suppose that Assumptions 1-3 hold.*

(i) (x,y) *with* $x,y \in (-1,1)$ *is a WPBE if and only if* $E(u(t_m,\Theta)\,|\,x,y,\Lambda) = 0$ *for* $\Lambda = 1,2$.

(ii) $(1,y)$ *with* $y<1$ *is a WPBE if and only if* $E(u(t_m,\Theta)\,|\,x,y,1)=0$*.* $(-1,y)$ *with* $y>-1$ *is a WPBE if and only if* $E(u(t_m,\Theta)\,|\,x,y,2)=0$*.*

Proof. See Appendix. □

In both parts (i) and (ii) of Lemma 1, the sufficiency is easy to show. Suppose that the median voters are indifferent between policies L and R in every divergence event Λ reachable given (x,y). Let h be a profile of voters' best responses to (x,y). Then, by (1), $\pi(h,\Lambda)=1/2$ for every reachable divergence event Λ. Suppose moreover that in every unreachable divergence event, voters have the same belief on the state (and thus behave in the same manner) as in a reachable divergence event. Then, $\pi(h,\Lambda)=1/2$ for every unreachable divergence event Λ. Therefore, no party can increase its winning probability by deviation given any signal.[3]

We provide an intuition of the necessity only for part (ii). Consider a strategy pair $(1,y)$ with $y\in(-1,1)$. The reachable events are events 1 and $\emptyset$. We show that $\pi(h,1)=1/2$ for any profile h of voters' best responses, which is equivalent to that the median voters are indifferent between policies L and R in event 1. Suppose that $\pi(h,1)=0$. Then, party 2 should deviate to strategy 1 so that it wins irrespective of the state. Now, suppose that $\pi(h,1)=1$. Then, ex ante, party 2 wins with probability less than 1/2. Party 2 can increase the winning probability by choosing strategy 1 so that the election results in a tie in all states. Therefore, $\pi(h,1)=1/2$ if $(1,y)$ is a WPBE.

Proposition 1 says that under an additional distributional assumption, there exists a WPBE (x,y) with $x=y\in(-1,1)$, and such a WPBE is unique.

Assumption 4. $\int_X q(s\,|\,\theta)\,dP(\theta)>0$ *for* $s=-1$ *and* $s=1$*. Moreover,* $E(u(t_m,\Theta)\,|\,S=-1)>0>E(u(t_m,\Theta)\,|\,S=1)$*.*

Assumption 4 guarantees that there are a private signal indicating the median voters' preference for policy L and a private signal indicating their preference for policy R.

Proposition 1. *Suppose that Assumptions 1-4 hold. Then, there exists a unique* $z\in(-1,1)$ *such that* $(x,y)=(z,z)$ *is a WPBE.*

Proof. Fix $z\in(-1,1)$. By Lemma 1, (z,z) is a WPBE if and only if

$$E(u(t_m,\Theta)\,|\,z,z,\Lambda)=\frac{\int_X u(t_m,\theta)Q(z\,|\,\theta)(1-Q(z\,|\,\theta))\,dP(\theta)}{\int_X Q(z\,|\,\theta)(1-Q(z\,|\,\theta))\,dP(\theta)}=0 \tag{3}$$

[3] Thus, a strategy pair (x,y) satisfying the sufficient condition in either part of Lemma 1 is a WPBE which satisfies "symmetric voting" of Heidhues and Lagerlöf (2003) with the profile h of voters' strategies constructed above, where a WPBE (x,y,h) is said to satisfy symmetric voting if $h_i(1)=h_i(2)$ for every voter i.

for $\Lambda = 1,2$, which is a single equation for z. Recall that $E(u(t_m,\Theta)\,|\,z,z,\Lambda)$ is continuously decreasing in z. Note that both $\lim_{z\to-1} E(u(t_m,\Theta)\,|\,z,z,\Lambda)$ and $\lim_{z\to 1} E(u(t_m,\Theta)\,|\,z,z,\Lambda)$ are limits of an indeterminate form. Thus, by l'Hopital's rule,

$$\begin{aligned}\lim_{z\to-1} E(u(t_m,\Theta)\,|\,z,z,\Lambda) &= \frac{\int_X u(t_m,\theta)\lim_{z\to-1}(1-2Q(z\,|\,\theta))q(z\,|\,\theta)\,dP(\theta)}{\int_X \lim_{z\to-1}(1-2Q(z\,|\,\theta))q(z\,|\,\theta)\,dP(\theta)} \\ &= E(u(t_m,\Theta)\,|\,S=-1).\end{aligned}$$

Similarly, we have $\lim_{z\to 1} E(u(t_m,\Theta)\,|\,z,z,\Lambda) = E(u(t_m,\Theta)\,|\,S=1)$. Therefore, by Assumption 4, there uniquely exists $z \in (-1,1)$ satisfying (3). □

Proposition 2 says that generically, there exists a WPBE at which one party chooses a fixed policy regardless of its private signal, while the other party plays a strategy that is responsive to its signal. Moreover, such an equilibrium is unique up to the labeling of parties.

Proposition 2. *Suppose that Assumptions 1-4 hold.*

(i) Suppose $\bar{u}_m < 0$. Then, there exists a unique $y \in (-1,1)$ such that $(-1,y)$ is a WPBE. There exists no WPBE of the form $(1,y)$, $y < 1$.

(ii) Suppose $\bar{u}_m > 0$. Then, there exists a unique $y \in (-1,1)$ such that $(1,y)$ is a WPBE. There exists no WPBE of the form $(-1,y)$, $y > -1$.

Proof. Part (i). Fix $y > -1$. By Lemma 1, $(-1,y)$ is a WPBE if and only if

$$E(u(t_m,\Theta)\,|\,-1,y,2) = \frac{\int_X u(t_m,\theta)Q(y\,|\,\theta)\,dP(\theta)}{\int_X Q(y\,|\,\theta)\,dP(\theta)} = 0. \tag{4}$$

Since $E(u(t_m,\Theta)\,|\,-1,y,2)$ is continuously decreasing in y, the same argument as in the proof of Proposition 1 yields

$$\lim_{y\to-1} E(u(t_m,\Theta)\,|\,-1,y,2) = E(u(t_m,\Theta)\,|\,S=-1) > 0,$$

where the last inequality is due to Assumption 4. Since $E(u(t_m,\Theta)\,|\,-1,1,2) = \bar{u}_m < 0$, there exists a unique $y \in (-1,1)$ satisfying (4).

If $y < 1$, then $E(u(t_m,\Theta)\,|\,1,y,1) \le E(u(t_m,\Theta)\,|\,1,-1,1) = \bar{u}_m < 0$. Thus, there exists no WPBE of the form $(1,y)$, $y < 1$.

The proof of part (ii) is similar. □

In this section, we have shown that there exist five WPBEs which have one of the following forms: (a) the parties choose the same interior cutpoint; (b) one party chooses an interior cutpoint, while the other chooses a corner cutpoint; (c) the parties choose the same corner cutpoint.

It is conceivable that the set of WPBEs with cutpoint strategies is finite, since WPBEs (x,y) with interior cutpoints are solutions to two equations in part (i) of Lemma 1 and the number of WPBEs with corner cutpoints (WPBEs of type (b) or (c)) is finite. This conjecture implies a marked difference from the model with completely informed parties, in which there exists a continuum of WPBEs with cutpoint strategies (Kikuchi, 2010).

In Section 4, we provide an example in which the five WPBEs of types (a)-(c) indeed constitute the set of WPBEs with cutpoint strategies.

4 An example

In this example, we assume that u is defined by $u(t,\theta) = t - \theta$, and Θ is uniformly distributed on interval X, that is, $p(\theta) = 1/2$ for all $\theta \in X$. We also assume that given $\Theta = \theta$, S and T are identically and independently distributed with the density function $q(\cdot\,|\,\theta)$ defined by

$$q(s\,|\,\theta) = \begin{cases} \frac{1}{2}(1-\theta) & \text{if } s \leq 0, \\ \frac{1}{2}(1+\theta) & \text{if } s > 0. \end{cases}$$

Then, Assumptions 1 and 2 are satisfied. Function q satisfies the monotone likelihood ratio property with weak inequalities, but it violates SMLRP (Assumption 3). Assumption 4 is satisfied if and only if $t_m \in (-1/3, 1/3)$, since $E(\Theta\,|\,S=-1) = -1/3$ and $E(\Theta\,|\,S=1) = 1/3$.

The conditional expectations of the state in the divergence events 1 and 2 given (x,y) are as follows.

$$E(\Theta\,|\,x,y,1) = \begin{cases} \frac{y}{1-2y} & \text{if } x,y \leq 0, \\ \frac{x+y}{1+2(x-y)-xy} & \text{if } y < 0 \leq x, \\ \frac{x}{1+2x} & \text{if } x,y > 0, \\ 0 & \text{if } x < 0 \leq y, \end{cases}$$

$$E(\Theta\,|\,x,y,2) = \begin{cases} \frac{x}{1-2x} & \text{if } x,y \leq 0, \\ 0 & \text{if } y < 0 \leq x, \\ \frac{y}{1+2y} & \text{if } x,y > 0, \\ \frac{x+y}{1+2(y-x)-xy} & \text{if } x < 0 \leq y. \end{cases}$$

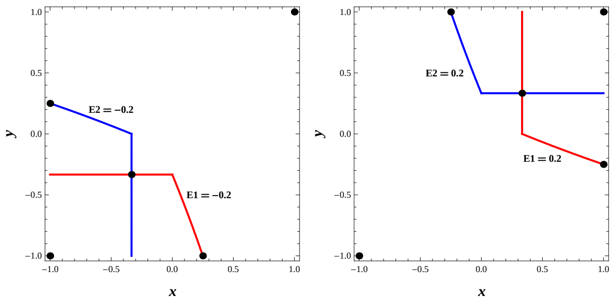

Figure 1: WPBEs in the example

Let W denote the set of WPBEs (x,y) with $x \geq y$ in this example. By simple calculations using Lemma 1, we have

$$W = \begin{cases} \left\{(-1,-1),\,(1,1),\,\left(\frac{t_m}{1+2t_m},\frac{t_m}{1+2t_m}\right),\,\left(\frac{1+3t_m}{1-3t_m},-1\right)\right\} & \text{if } t_m \in [-\frac{1}{3},0], \\ \left\{(-1,-1),\,(1,1),\,\left(\frac{t_m}{1-2t_m},\frac{t_m}{1-2t_m}\right),\,\left(1,-\frac{1-3t_m}{1+3t_m}\right)\right\} & \text{if } t_m \in [0,\frac{1}{3}], \\ \{(-1,-1),\,(1,1)\} & \text{otherwise.} \end{cases}$$

Thus, counting the WPBE (x,y) with $y > x$, we conclude that the five WPBEs of types (a)-(c) constitute the set of WPBEs with cutpoint strategies if Assumption 4 holds and if $u_m \neq 0$, that is, if $t_m \in (-1/3,0)$ or $t_m \in (0,1/3)$.[4]

Figure 1 illustrates the set of WPBEs with cutpoint strategies in this example for the cases where $t_m = -0.2$ (the left graph) and $t_m = 0.2$ (the right graph). WPBEs are represented by dots. The curves labeled "$E1 = t_m$" and "$E2 = t_m$" represent the sets of (x,y) satisfying the equations $E(u(t_m,\Theta)\,|\,x,y,1) = 0$ and $E(u(t_m,\Theta)\,|\,x,y,2) = 0$, respectively.

In this example, there exist significantly fewer WPBEs with cupoint strategies than in the model with completely informed parties. Moreover, the set of WPBEs with cupoint strategies in this example has some similarity with the set of uniformly perfect equilibria (in the sense of Harsanyi and Selten (1988)) with cutpoint strategies (UPECs) in the model with completely informed parties.

Figure 2 illustrates the sets of WPBEs and UPECs in the model with completely informed parties for the cases where $t_m = -0.2$ (the left graph) and $t_m = 0.2$ (the right graph). Except for the complete information of the parties, the same setting as the present example is assumed. The union of the two line segments represents the set of WPBEs with cutpoint strategies. The three dots represent the UPECs.

[4]Note that in this example, the sign of t_m equals the sign of $\bar{u}_m = E(t_m - \Theta)$.

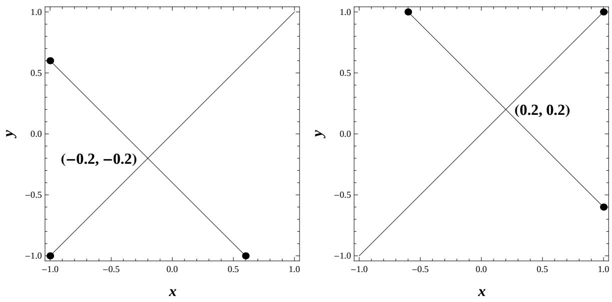

Figure 2: WPBEs and UPECs in the example when parties have complete information

5 Conclusion

In this paper, we have constructed a Downsian model of political competition in which parties can observe private signals of policy effects, while voters only have a prior distribution. In this model, we have shown that there exist WPBEs at which the parties play different strategies, and hence, choose different policies even when their signals coincide. This result is in marked contrast with the Median Voter Theorem in the classical Downsian model.

We have derived equilibrium conditions which suggest that this model has only a finite number of WPBEs with cutpoint strategies, whereas the model with completely informed parties has a continuum of WPBEs (Kikuchi, 2010). In an example, we have shown that there exist exactly five WPBEs with cutpoint strategies. Moreover, three of these equilibria locate at points close to the three uniformly perfect equilibria with cutpoint strategies (UPECs) in the model with completely informed parties.

An important next step in this research would be to refine the WPBEs. For this purpose, the trembling-hand perfection will again be useful. We expect that the set of UPECs in the present model comprises the three WPBEs close to the UPECs in the model with completely informed parties.

The model in this paper would also be instrumental in refining WPBEs in the model with completely informed parties. We may consider the limits of WPBEs when the precisions of the signals go to infinity as equilibria that are stable against incomplete information of the parties. Our analysis indicates that the set of the limit WPBEs with cutpoint strategies will resemble, but not coincide with the set of UPECs.[5] These remaining problems are open to future research efforts.

[5] For example, if $\bar{u}_m < 0$, then in the model with completely informed parties, the strategy pair $(1,1)$ is not a UPEC, whereas in the model with private information, it is a WPBE irrespective of the distribution functions P and Q.

6 Appendix: proof of Lemma 1

Proof. Part (i). Let (x,y) be a strategy pair with $x,y \in (-1,1)$. Let h be the profile of voters' best responses to (x,y). Since both events 1 and 2 are reachable given (x,y), $\pi(h,\Lambda)$ is determined by (1) for $\Lambda = 1,2$. Hence, our goal is to show that the strategy pair (x,y) is a WPBE if and only if

$$\pi(h,\Lambda) = 1/2 \text{ for } \Lambda = 1,2. \tag{5}$$

We first show that if the strategy pair (x,y) is a WPBE, then it satisfies (5). By (2), policy L is optimal for party 1 given signal $s' < x$ if and only if

$$\Phi(y\,|\,s')\{\pi(h,2) - \pi(h,1)\} \geq \tfrac{1}{2} - \pi(h,1). \tag{6}$$

Similarly, policy R is optimal for party 1 given signal $s > x$ if and only if

$$\Phi(y\,|\,s)\{\pi(h,2) - \pi(h,1)\} \leq \tfrac{1}{2} - \pi(h,1). \tag{7}$$

By Assumption 3, $\Phi(y\,|\,s) < \Phi(y\,|\,s')$ for $s', s \in X$ with $s' < x < s$. Thus, (6) and (7) imply that

$$\pi(h,1) \leq \pi(h,2). \tag{8}$$

If (8) holds with equality, then by (6) and (7), $\pi(h,1) = \pi(h,2) = 1/2$. Thus, it remains to show the inequality opposite to (8).

To do this, observe that party 2's respective winning probabilities given signal t when it chooses policy L and when it chooses policy R are as follows.

$$\Pi_2(L\,|\,x,h,t) = \tfrac{1}{2}\Phi(x\,|\,t) + \pi(h,2)(1 - \Phi(x\,|\,t)),$$
$$\Pi_2(R\,|\,x,h,t) = \tfrac{1}{2}(1 - \Phi(x\,|\,t)) + \pi(h,1)\Phi(x\,|\,t).$$

Policy L is optimal for party 2 given signal $t' < y$ if and only if

$$\Phi(x\,|\,t')\{\pi(h,1) - \pi(h,2)\} \geq \tfrac{1}{2} - \pi(h,2). \tag{9}$$

Policy R is optimal for party 2 given signal $t > y$ if and only if

$$\Phi(x\,|\,t)\{\pi(h,1) - \pi(h,2)\} \leq \tfrac{1}{2} - \pi(h,2). \tag{10}$$

The same argument as the preceding paragraph yields that (9) and (10) imply

$$\pi(h,1) \geq \pi(h,2),$$

which is the desired inequality.

The converse is also true since (5) implies that conditions (6), (7), (9), and (10) hold with equalities.

Part (ii). Consider a strategy pair $(1,y)$ with $y<1$. Given this strategy pair, the only reachable divergence event is event 1. Thus, we have to prove that the strategy pair is a WPBE if and only if (5) holds for $\Lambda=1$.

Suppose first that $y\in(-1,1)$. For $(1,y)$ to be a WPBE, conditions (6), (9), and (10) are necessary and sufficient, where $\pi(h,2)$ is interpreted as the winning probability given some beliefs of voters. The last two conditions are equivalent to (5) for $\Lambda=1$ since $Q(1\,|\,t')=Q(1\,|\,t)=1$. Conversely, suppose that (5) holds for $\Lambda=1$. Suppose, moreover, that h is a strategy profile of voters in which they vote optimally according to the common belief that in event Λ, the median voters are indifferent between policies L and R. Then, (6) holds with equality.

For the strategy pair $(1,-1)$ to be a WPBE, (6) and (10) are necessary and sufficient. Since $Q(-1\,|\,s')=0$ and $Q(1\,|\,t)=1$, these conditions are equivalent to $\pi(h,1)=1/2$. The proof for strategy pairs $(-1,y)$, $y>-1$, is similar. □

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