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Will a Growth Miracle Reduce Debt in Japan?*

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Japan has the highest debt to GDP ratio among the developed nations. In addition, the population is projected to age rapidly over the next few decades, which will significantly increase the ratio of government expenditures to GDP. In this paper, we explore the effect of economic growth driven by total factor productivity on Japanese debt in the face of higher future social security expenditures. Our main finding is that a decade of fast growth of total factor productivity, at an average of 6% per year, may help Japan eliminate its debt. This result suggests that the policy makers may well focus on growth-inducing policies such as lower distorting taxation and structural reforms that incentivize the entrepreneurial activity and innovation to drive growth.
JEL Classification Codes: E00, H20, H50

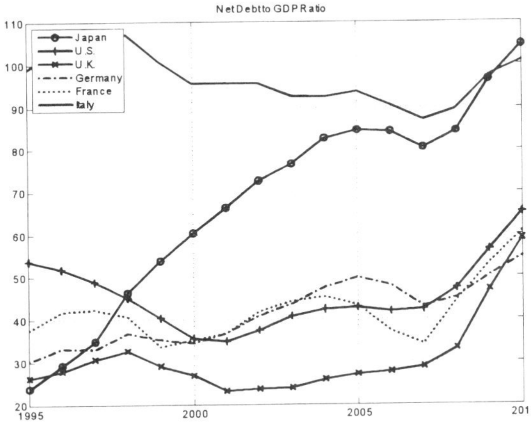
1. Introduction

Among the developed countries facing severe demographic and fiscal problems, Japan is projected to be affected the worst. The population is aging very rapidly, inducing large increases in the government's social security related pension, health and long term care expenditures. In addition, Japan has already overtaken Italy as the nation with the largest debt to output ratio on a net basis among the developed countries. Figure (1) shows the ratio of net government debt to GDP in a subset of OECD countries. The fiscal response to the lost decade has pushed this ratio from less than 20% in early 1990s to 104.6% projected for 2010.

In our earlier work, we used the standard growth model to produce short term forecasts on the Japanese economy when the consumption tax rate is raised from 5% to 15%, taking as given the projected increases in government expenditures[1]. In particular, we explored whether this 10-percentage point increase in the consumption tax by itself is sufficient to deliver persistent primary surpluses that can be used to reduce the debt to GNP ratio in Japan. Our finding, unfortunately, was not affirmative. Even under optimistic scenarios, the government must raise other taxes or cut spending significantly if the objective of policy is to reduce the debt to GNP ratio significantly.

In this paper, we extend our earlier work by studying how exogenous growth can influence the set of options available to the Japanese fiscal authority. Although there is no consensus yet on a theory of total factor productivity (hereafter TFP), it is well known that certain economic institutions and policies can help produce faster growth. For example, those associated with higher accumulation of human capital, increase in competition, openness, more incentive to innovate are considered to be important. We take the future paths of increases in government purchases of goods and services and transfer payments as given according to the estimates of Fukawa and Sato (2009). Since they estimate the ratios of these expenditure items to GNP assuming a rate of growth of real GNP close to 2%, we back out the paths of real government expenditures under the same assumption but we fix these items in our computations using different rates of growth of TFP and hence real GNP. Our model traces out the equilibrium responses of macroeconomic indicators such as consumption, investment, and output, as well as the endogenously calculated primary balance for the government. Since the primary balance is affected by the endogenous changes in the tax base, it reacts differently to rates of TFP growth. We assume that the Japanese economy experiences a given rate of growth of TFP for the next 10 years and we add up the successive budget balances generated by the optimal response of the Japanese eco-

Figure 1. Net Debt to GDP Ratio Among Developed Countries



nomy to the assumed TFP growth rate from 2010 through 2019. By varying the rate of growth of TFP, we then tabulate how far 'fast growth' can go in terms of reducing the debt to GNP ratio in Japan over the next 10 years.

Our model does not explicitly include government debt. The main reason is that inclusion would require us to introduce an additional feature or institution to yield the well-known rate-of-return dominance of private capital over government indebtedness. Instead, we model the primary balance and budget balance. We then add the annual flows of budget balances to the existing stock of debt as of the end of 1980 in order to compare with the data from 1981 to 2009 and to introduce our model's forecasts on the stock of debt over the next 40 years. Note that we are taking the current stock of debt as given and beyond the scope of this model. However, the additions and subtractions in the form of budget balances are equilibrium quantities produced by our model.

We find that a decade of fast growth, at an average of more than 6% per year, may reduce Japanese debt to zero by 2050. While the debt amounts to about 100% of GDP in 2010, a sufficiently higher TFP growth increases the tax revenue, improving the government budget balance. Otherwise, the increase in government expense dominates the development of budget balance, accumulating the debt further than the current level. To give the range of our forecast, we conduct two sensitivity analysis. First, we simulate the model under an assumption that consumption tax is permanently raised from 5% to 15% in 2011. Since tax revenues immediately go up, the debt is eliminated in a shorter period of time and with a smaller productivity growth rate. Second, we consider a case in which the interest rate on debt is equal to the return to private capital. This assumption of more costly interest payments will now add further strain to the fiscal situation. Because higher TFP growth rate causes higher return to the private capital, reduction of debt is partially offset by the increasing interest payments. In this case, at least 8% of TFP growth is needed to eliminate the debt.

The paper is organized as follows. Section 2 describes the model. Section 3 contains the measurement and calibration used in the paper. Our numerical results are reported in Section 4. Section 5 concludes.

2. The Standard Neoclassical Growth Model

This paper explores the quantitative effects of economic growth on the government debt in Japan taking as given projected increases in government expenditures due to the aging of the population. In order to study these effects, a model of the Japanese economy is needed. Following Hayashi and Prescott (2002), Chen, İmrohoroğlu, and İmrohoroğlu (2006), and İmrohoroğlu and Sudo (2010), and we use the standard growth model to measure how faster TFP growth affects the Japanese government's indebtedness. The model is an infinite horizon, complete markets framework that has been successfully used to address a variety of economic issues concerning the Japanese economy. In this model, a representative household chooses its decision rules for consumption and saving, taking as given factor prices and government policy. A stand-in firm maximizes its profits, setting factor prices equal to their marginal productivities. There is a government that finances exogenously given government purchases, transfer payments, and interest payments on government debt by taxing factor incomes and consumption, or by issuing new one-period bonds. The engine of growth in the model is exogenously growing TFP. Agents in this environment take into account future policy and prices and maximize their objective functions. Therefore, the model can be a useful device to measure the responsiveness of the private sector to future demographic changes and fiscal policy experiments. Below, we present our model in detail[2].

2.1 Household's Problem

Time is discrete, starting from period 0. There is a representative household with N_t working-age members at date t, facing the following problem in a complete markets environment:

$$\max \sum_{t=0}^{\infty} \beta^t N_t \log c_t$$

subject to

$$(1+\tau_{c,t})C_t + K_{t+1} \leq [1+(1-\tau_{k,t})(r_t-\delta_t)]K_t + (1-\tau_{h,t})w_t H_t + TR_t - N_t\tau_t + N_t\pi_t^p,$$

where $c_t=C_t/N_t$ is consumption per household member, $h_t=H_t/N_t$ is the fraction of hours worked per member of the household, β is the subjective discount factor, H_t is total hours worked by all working-age members of the household, $\tau_{h,t}$ and $\tau_{k,t}$ are tax rates on labor and capital income, respectively, $\tau_{c,t}$ is the consumption tax rate, τ_t is a per-capita lump-sum indirect tax distinct from the consumption tax, w_t is the real wage, TR_t is aggregate government transfers, π_t^p is the per-member primary balance, r_t is the rental rate of capital, and δ_t is the time-t depreciation rate[3]. Beginning of period t assets are denoted by K_t. Population growth is given by the change in the size of the household, which evolves over time exogenously at the rate $n_t=N_t/N_{t-1}$. We assume that the representative household receives the interest earnings on the government debt I_t.

It should be noted that the tax rates are assumed to be proportional, although there is some progressivity in the actual Japanese tax system. Since we do not conduct income tax reform experiments in this paper, we believe that this abstraction is reasonable. As we show later on, our model's tax revenues closely mirror actual government revenues.

2.2 Firm's Problem

There is a stand-in firm with access to a constant returns to scale Cobb-Douglas production function given by

$$Y_t = A_t K_t^{\theta} H_t^{1-\theta},$$

where θ is the income share of capital and A_t is total factor productivity, which grows exogenously at the rate $\gamma_t=A_t/A_{t-1}$. Aggregate capital stock follows the law of motion

$$K_{t+1} = (1-\delta_t)K_t + X_t, \tag{1}$$

where X_t is gross investment at period t.

The representative firm maximizes its profits by choosing capital and labor, taking factor prices as given. This produces the usual equilibrium conditions that equate factor prices to their marginal productivities.

2.3 Government Budget

The government faces exogenously given streams of government purchases G_t, transfer payments TR_t, and interest payments to holders of its debt I_t. These can be financed by taxing consumption, income from labor and capital, or by raising new debt. In this paper we do not explicitly model government debt. The main reason is that modeling debt requires a way of introducing rate-of-return dominance of private capital over government debt as we observe in the data over several decades and across many countries[4]. Instead, we focus on the additions to existing debt by carefully modeling the government's flow budget constraint. Denoting the (per-capita) budget balance by π_t^b and the primary balance by π_t^p, we specify the government budgets as follows.

$$G_t + TR_t + I_t = \tau_{h,t} w_t H_t + \tau_{k,t}(r_t-\delta_t)K_t + \tau_{c,t}C_t + N_t\tau_t - N_t\pi_t^b \tag{2}$$

$$G_t + TR_t = \tau_{h,t} w_t H_t + \tau_{k,t}(r_t-\delta_t)K_t + \tau_{c,t}C_t + N_t\tau_t - N_t\pi_t^p. \tag{3}$$

We can now implicitly calculate the government debt in the beginning of period t, B_t^g, by the cumulative sum of π_t^b,

$$B_t^g = B_1^g + \sum_{s=1}^{t-1} N_s\pi_s^b,$$

where B_1^g is the government debt at the initial period.

In this paper we take interest payments on government debt exogenous. In order to study the effects of higher interest costs on the economy, we also consider a case in which the interest rate on government debt is equal to the rate of return on private capital. This would presumably portray a 'worse case' scenario.

It should be emphasized that we do not have a theory as to household's holding of government debt. There is no consensus in the literature on the optimal size of government debt primarily because there is no agreement on a theory of debt. For this reason, we concentrate on the effects of its financing on the economy as well as the effects of growth on the size of debt. In this sense, debt is endogenous in our model as we determine its level by accumulating budget deficits that are endogenously determined by the interaction of demographics, policy, and private sector behavior. Note that the projected increases in G_t and TR_t will proxy for the impact of the demographic transition in Japan. Government's fiscal policy will be represented with the assumed paths of the expenditure items and the tax rates. Finally, the private sector will optimally respond to changes in this environment by adjusting its consumption-saving behavior, and the general equilibrium effects will show up as the wage rate and rate of return to capital adjust accordingly.

2.4 Competitive Equilibrium

It is useful to define a competitive equilibrium of our model so that it can guide our computational method. For a government fiscal policy $\{G_t, TR_t, I_t, \tau_{h,t}, \tau_{k,t}, \tau_{c,t}, \tau_t\}_{t=0}^{\infty}$, a competitive equilibrium consists of an allocation $\{C_t, X_t, H_t, K_{t+1}, Y_t\}_{t=0}^{\infty}$, a budget balance π_t^b, a primary balance π_t^p, and factor prices $\{w_t, r_t\}$ such that

- the allocation solves household's problem,
- the allocation solves the firm's profit maximization problem with factor prices given by: $w_t=(1-\theta)A_tK_t^{\theta}H_t^{-\theta}$, and $r_t=\theta A_tK_t^{\theta-1}H_t^{1-\theta}$,
- the government budget is satisfied,
- the goods market clears: $C_t+X_t+G_t=Y_t$.

2.5 Equilibrium Conditions

We can combine the equilibrium conditions of the model in two equations below:

$$\frac{(1+\tau_{c,t+1})C_{t+1}}{N_{t+1}} = \frac{(1+\tau_{c,t})C_t}{N_t}\beta\{1+(1-\tau_{k,t+1})[\theta A_{t+1}K_{t+1}^{\theta-1}H_{t+1}^{1-\theta}-\delta_{t+1}]\}, \tag{4}$$

$$K_{t+1} = (1-\delta_t)K_t+A_tK_t^{\theta}H_t^{1-\theta}-C_t-G_t. \tag{5}$$

Our approach is to start from given initial conditions and then compute an equilibrium transition path towards a balanced growth path at which per capita aggregate variables grow at the rate $g_t=\gamma_t^{1/(1-\theta)}$ For a variable z_t, detrending is done by applying $\tilde{z}_t=z_t/\left[A_t^{\frac{1}{1-\theta}}N_t\right]$. Using this change of variables to (4) and (5), we obtain equations

$$\tilde{c}_{t+1} = \frac{(1+\tau_{c,t})}{(1+\tau_{c,t+1})}\frac{\tilde{c}_t}{g_{t+1}}\beta\{1+(1-\tau_{k,t+1})[\theta x_{t+1}^{\theta-1}-\delta_{t+1}]\},$$

$$\tilde{k}_{t+1} = \frac{1}{g_{t+1}n_{t+1}}[1-\delta_t)+(1-\psi_t)x_t^{\theta-1}]\tilde{k}_t-\tilde{c}_t,$$

where ψ_t is the ratio of government purchases to output, G_t/Y_t, and x_t is detrended capitallabor ratio, $(K_t/H_t)/A_t^{\frac{1}{1-\theta}}$

The steady-state conditions are obtained by setting $\tilde{z}_t=\tilde{z}$ for all t:

$$1 = \frac{1}{g}\beta\{1+(1-\tilde{\tau}_k)[\theta x^{\theta-1}-\tilde{\delta}]\}$$

$$\tilde{k} = \frac{1}{gn}[(1-\tilde{\delta})+(1-\tilde{\psi})x^{\theta-1}]\tilde{k}-\tilde{c}.$$

These two equations deliver the steady-state values of detrended capital and consumption where $\tilde{\delta}$, $\tilde{\tau}_h$ and $\tilde{\tau}_k$ are the steady-state depreciation rate, labor income tax rate and capital income tax rate, respectively.

3. Measurement and Calibration

The next step is to align the macroeconomic and government accounts in the model with those in the data. Then we can calibrate the model properly so that it represents aggregate behavior in Japan and we can use it to shed some light on how aging and future economic growth affects the macroeconomy and government debt.

3.1 Adjustments to National Accounts

Our measurement and calibration approach follows Hayashi and Prescott (2002) and views the capital stock in Japan as the sum of domestic private capital and foreign capital owned by the Japanese household. In standard growth theory, government consumption and investment are expensed. In calculating output in the model, which we take it as real GNP, we use

$$C_t+X_t+G_t = Y_t,$$

where C_t is "Private Final Consumption Expenditure", X_t is the sum of "Private Gross Capital Formation" , "Change in Inventories," "Net Exports," and "Net Factor Payments from Abroad," and G_t is "Government Final Consumption Expenditure" and "Capital Formation" minus "depreciation of government capital." Our output is then the sum of the above three components.

Note that we include foreign capital as part of Japanese capital and also include the income flow from this capital in our measure of output. Although this is not quite the open economy representation of the economy

Table 1. Calibration in the Steady State

β	0.97	Subjective Discount Factor
θ	0.377	Output Share of Capital
δ	0.08	Depreciation Rate
$g-1$	0.02	Growth Rate of TFP factor
$n-1$	0.0	Population Growth Rate
τ_k	0.398	Capital Income Tax Rate
τ_h	0.298	Labor Income Tax Rate
G/Y	0.25	Ratio of Government Purchases to GNP

which would shut down the general equilibrium effects coming from the response of factor prices to changes in quantities, we believe that we do represent the income flows that arise as part of interacting with the rest of the world.

3.2 Adjustment to Government Accounts

In this subsection we describe how we align government accounts in the model and the data. We use data from the 93 SNA national accounts.

3.2.1 Consumption Tax Revenue

- Consumption tax revenue:

(Model)	(Data)
$\tau_{c,t}C_t$	"Value added taxes (VAT)"

3.2.2 Factor Income Tax Revenue

- Factor income tax revenue:

(Model)	(Data)
$\tau_{h,t}(1-\theta)Y_t$	"direct tax on households" +"social security tax (gross)"
$\tau_{k,t}(\theta Y_t-\delta_t K_t)$	"direct tax on nonfinancials" +"direct tax on financials"

3.2.3 Budget Balance

- Budget balance:

$$\tau_{c,t}C_t+\tau_{h,t}(1-\theta)Y_t+\tau_{k,t}(\theta Y_t-\delta_t K_t)$$
$$+N_t\tau_t-TR_t-I_t-G_t.$$

We need to specify the last four items. Indirect tax revenue other than consumption tax, $N_t\tau_t$, is calculated as "Import Duties" plus "Others" plus "Other Taxes on Production" minus "Subsidies (payable)" plus "Capital Transfers (receivable)" minus "Capital Transfers (payable)."

- Transfer payments, TR_t, are calculated as "Social Benefits other than Social Transfers in Kind (payable)" plus "Other Current Transfers (payable)" minus "Other Current Transfers (receivable)."
- Interest payments on government debt, I_t, are calculated as Property Income (payable)" minus"Property Income (receivable)."
- Government purchases of goods and services, G_t, are calculated as "Final Consumption Expenditure" plus "Gross Fixed Capital Formation" minus "Consumption of Fixed Capital."

3.3 Calibration of the Model

We take 1981 as the starting point for our analysis, primarily because this is the first year when national account series with a consistent set of definitions are available. The last period for which we have data for all of the variables is 2008. Therefore, the model will take observed inputs as given for the 1981-2008 period, and some values for 2009, and will make assumptions about the values of these exogenous inputs for 2009 and beyond. We assume that the economy will reach a steady state far into the future. As a result, we have a two-point boundary problem, starting with given initial conditions in 1981, and ending at a steady state far into the future. We use a shooting algorithm to calculate an equilibrium transition path that connects these two boundary points. Since the steady state is reached far into the future, our assumptions about that steady state will have minimal effect, if any, on the immediate future along the transition path.

We present our calibration choices in more detail in the next subsections.

3.3.1 Constant Parameters and Steady-State Calibration

The calibrated steady state values of the parameters are given in Table 1. These are constructed following Hayashi and Prescott (2002), and equal to those used in our earlier work, İmrohoroğlu and Sudo (2010).

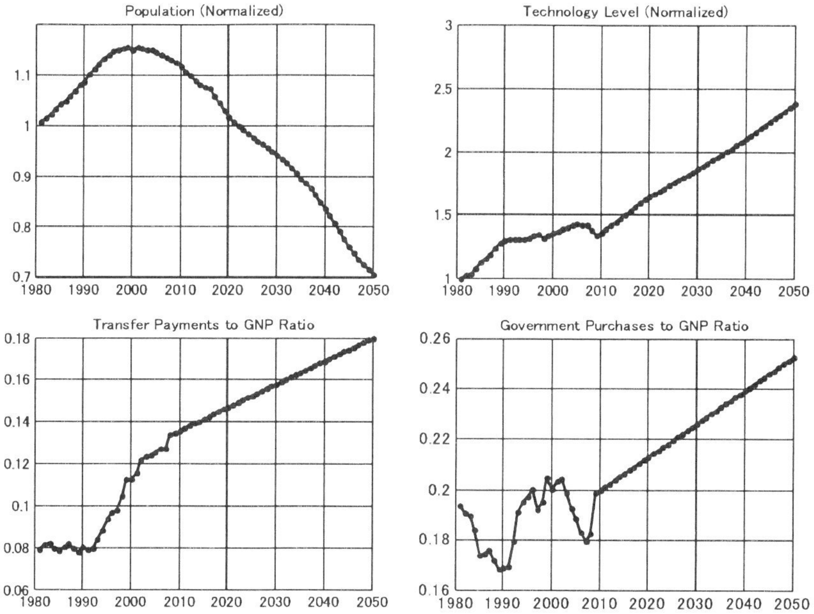
3.3.2 Inputs for 1981-2008 and beyond

There are two reasons why we start from given initial conditions in 1981. First, we obtain similar results for years between 2010 and 2040 if we start from an earlier year such as 1961 or a later year such as 1990. Second, national accounts are available in a consistent manner only from 1981. From 1981 until 2008, we use the observed values for the following exogenous variables: $\{G_t/Y_t, TR_t/Y_t, I_t/Y_t, \tau_{h,t}, \tau_{k,t}, \tau_{c,t}, \tau_t, \delta_t, \gamma_t, N_t, n_t, H_t\}_{t=1981}^{2008}$.

Below we provide our assumptions for various exogenous variables.

- $\{G_t/Y_t, TR_t/Y_t\}_{t=1981}^{2008}$: In order to represent the projected burden of an aging Japan and

Figure 2. Population, TFP, and Government Expenditures



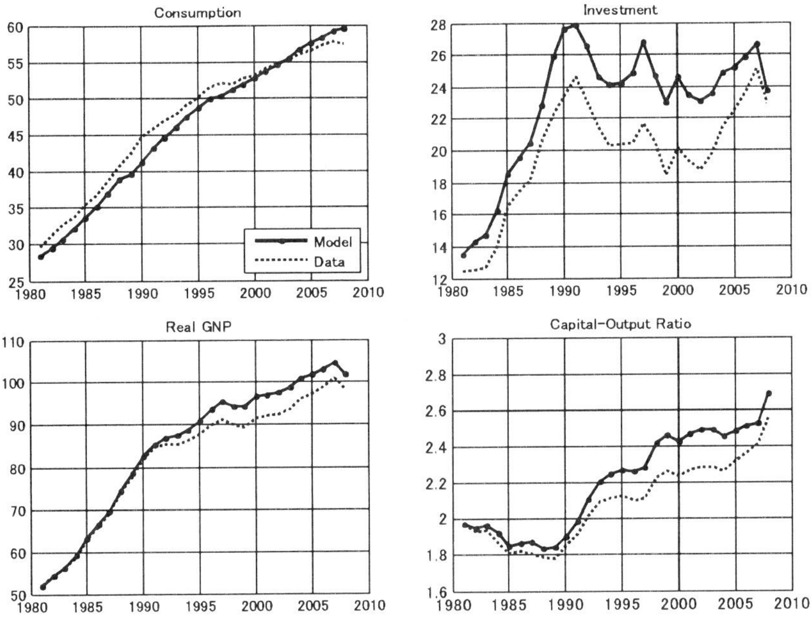
the increases in social security and health expenditures on the government budget, we follow our earlier paper İmrohoroğlu and Sudo (2010). For 2009, the data on G_t/Y_t and TR_t/Y_t for 2009 are approximated from the publicly available but preliminary data[5]. For G_t/Y_t in 2009 and beyond, we set different time paths for each component of G_t, depending on its characteristics. The ratios of "Gross Fixed Capital Formation," "Individual Consumption Expenditure," and, "Transfers of Individual Non-Market Goods and Services" to GNP are assumed to linearly increase to their respective sample averages from 1999 to 2008 in year 2050 and to remain constant at these 2050 levels forever. "Social Transfers in Kind, payable," is assumed to linearly converge to 12% of GNP in year 2050, following the projections of Fukawa and Sato (2009), and to stay constant onward. Hence, G_t/Y_t is assumed to converge from 20% in 2009 to 25% by 2050. We assume that TR_t/Y_t increases linearly from 13.5% in 2009 to 18% in year 2050 where we again rely on the estimates of Fukawa and Sato (2009). These projections introduce the fiscal burden due to the expected aging of the Japanese population into our simple growth model. Figure (2) describes the benchmark paths of government expenditures, together with those of population and TFP, including the assumptions about their out-of-sample values.

- $\{I_t/Y_t\}_{t=1981}^{2008}$: The ratio of "Property Income (payable)" minus "Property Income (receivable)" to GNP is assumed to increase with the average growth rate of sample 1999 to 2008 in year 2050 and to remain constant at these 2050 levels forever.

- $\{\delta_t, \gamma_t, N_t, H_t\}_{t=1981}^{2008}$: For the last three exogenous variables, 2009 values are available and used in the simulations. For the rate of depreciation of capital, δ_t, we set it equal to the value in 2008 for 2009 and onward. We extend N_t from 2010 to 2050 based on the medium-fertility and medium-mortality population projections made by the National Institute of Population and Social Security Research, and assume that population remains unchanged after 2050. We set H_t equal to the average from 1999 to 2009 for year 2010 and onward. Our TFP is calculated as $A_t = Y_t/K_t^{\theta}(H_t)^{1-\theta}$. The growth rate of TFP, $\gamma_t = A_t/A_{t-1}$ is a key exogenous variable that influences the growth of the tax base and therefore the size of the additional consumption tax needed to attain a primary surplus. Our calculations will allow TFP growth rates $\gamma_t - 1 \in \{0, 0.02, 0.04, 0.06, 0.08\}$.

- $\{\tau_{h,t}, \tau_{k,t}\}_{t=1981}^{2006}$: The labor income tax rate series is an updated version of that calculated by Mendoza, Razin, and Tesar (1994). They use national accounts and government revenue statistics for large industrial countries to compute annual time series of effective tax rates on factor incomes. The last year for which this tax data set is updated is 2006, and we assume that $\tau_{h,2006} = 0.298$ for all years after 2006.

Figure 3.　National Accounts: Model and Data



The capital income tax rate is constructed according to the methodology in Hayashi and Prescott (2002). The last year for which we can construct this tax rate is 2008 and we assume that $\tau_{k,2008}=0.398$ remains unchanged forever. This way, we can trace any changes in the model's accounts to our assumptions on the consumption tax rate, government expenditures and TFP growth rates.

- $\{\tau_{c,t}\}_{t=1981}^{2008}$ The consumption tax in the model is assumed to rise from zero to 3% in 1989, and to 5% in 1997. In the data, there are taxes that are typically classified as consumption taxes such as import and excise taxes that existed before 1989. In the model and data, we classify these as non-consumption (lump-sum) taxes so that we can concentrate on the more recent and targeted consumption taxes[6].

4. Numerical Findings

This section presents the main numerical results. First, the simulations of the benchmark model under the assumption of continued and projected fiscal policy and a 5% consumption tax are displayed and then two sensitivity analyses are conducted to explore the effects of a higher consumption tax rate and of higher costs of interest payments.

To explore the effect of productivity growth on the primary balance and government debt between now and 2050, we simulate our model for periods 2010 and onward, under a different set of assumptions about productivity growth rate from 2010 through 2019. We choose the simulation where $\{\gamma_t\}_{t=2010}^{2019}=1.02$ as the benchmark. In this case, the model delivers projected output growth rates from 2010 to 2019 equal to 1. 016 on average, which is close to the calculation made by Fukawa and Sato (2009). We then recover the sequences of the levels of exogenous government variables in the case of $\{\gamma_t\}_{t=2010}^{2019}=1.02$, that are denoted by $\{\widehat{G}_t, \widehat{TR}_t, \hat{I}_t, \hat{\tau}_t\}_{t=2010}^{\infty}$. based on the sequence of model-generated output. In the simulations where $\{\gamma_t\}_{t=2010}^{2019}\neq 1.02$ we feed through $\{\widehat{G}_t, \widehat{TR}_t, \hat{I}_t, \hat{\tau}_t\}_{t=1981}^{\infty}$ into the model instead of $\{G_t/Y_t, TR_t/Y_t, I_t/Y_t, \tau_t/Y_t\}_{t=1981}^{\infty}$. Under this setting, the set of the ratio of government variables to output $\{G_t/Y_t, TR_t/Y_t, I_t/Y_t, \tau_t/Y_t\}_{t=1981}^{\infty}$ is affected by the development of output and no longer exogenously determined.

4.1　Benchmark Results: Growth and Debt

Figure (3) displays consumption, investment, output and the capital-output ratio in the data and in the benchmark model where the consumption tax is assumed to remain at 5% forever. The model comes close to replicating the actual behavior of the Japanese economy between 1981 and 2009. The performance of the model is not as good in the more recent time period, primarily because the model does not do as good a job replicating investment behavior[7].

Figure (4) depicts government accounts and indicates how close the model comes to replicating the government revenue and debt quantities in the data. As before, the performance is not perfect, especially in the late 1990s

Figure 4. Government Accounts: Model and Data

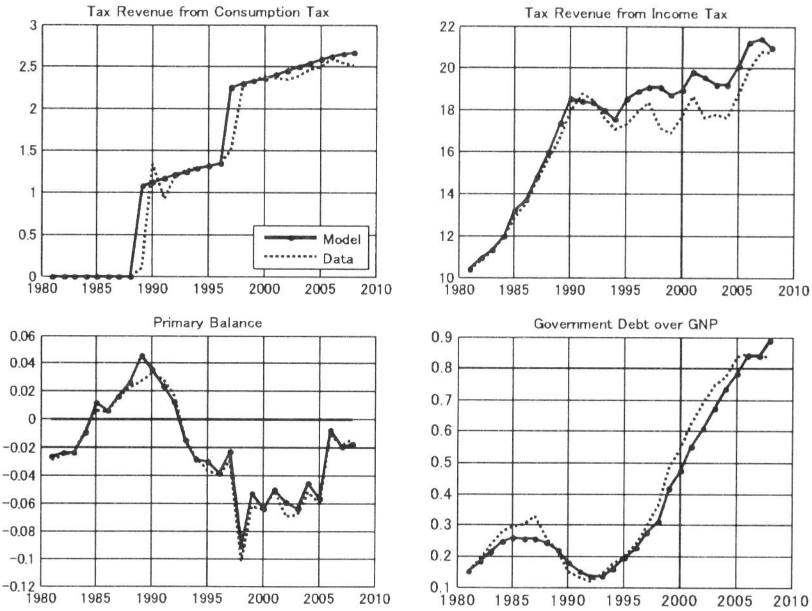
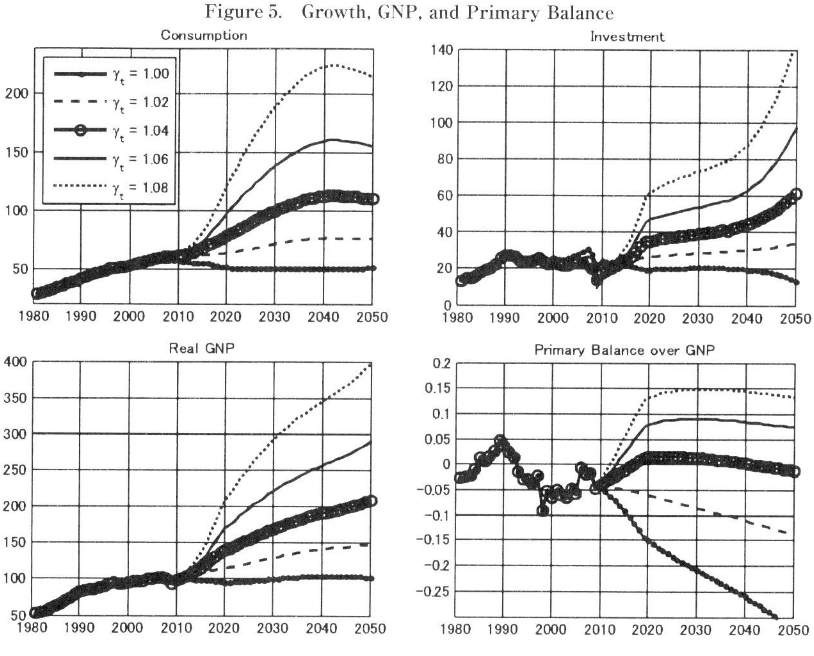


Figure 5. Growth, GNP, and Primary Balance



and early 2000s but the model does a remarkable job of capturing the time path of primary balance. As a result, starting from the actual debt to GNP ratio in 1981, our 'built-up' debt to GNP ratio in the last frame of Figure (4) is very close to the actual debt to GNP ratio in the data. Our simple growth model seems to do a good job matching with the aggregate characteristics of the Japanese economy.

In Figure (5), we conduct deterministic simulations with different growth assumptions. In particular, we present five alternative equilibrium transition paths. In each transition path, individuals take the projected increases in government purchases and expenditures into account, as well as the factor prices and the exogenous rate of TFP growth. In several earlier papers, such as Hayashi and Prescott (2002) and Chen, İmrohoroğ lu, and İmrohoroğ lu (2006) among others, deterministic simulations produce very similar, and sometimes nearly identical, time series compared to stochastic simulations. For this reason, we only present perfect foresight simulations in this paper.

In this section, our simulations assume that the rate of growth TFP for the next decade (between 2010 and 2019) will be such that the rate of growth of real GNP will be close to one of five alternative values depicted above[8]. According to Figure (5), a TFP growth rate at 2% will deliver a typical path for real GNP (and other indicators) that are consistent with past averages and future expected growth. Primary balance, on the other hand, will continue to worsen despite a 2% TFP growth rate. The reason for the worsening of primary balance is the expected increases in the ratios of government expenditures to GNP. Of course,

Figure 6. Growth, Government Expenditures, and Debt

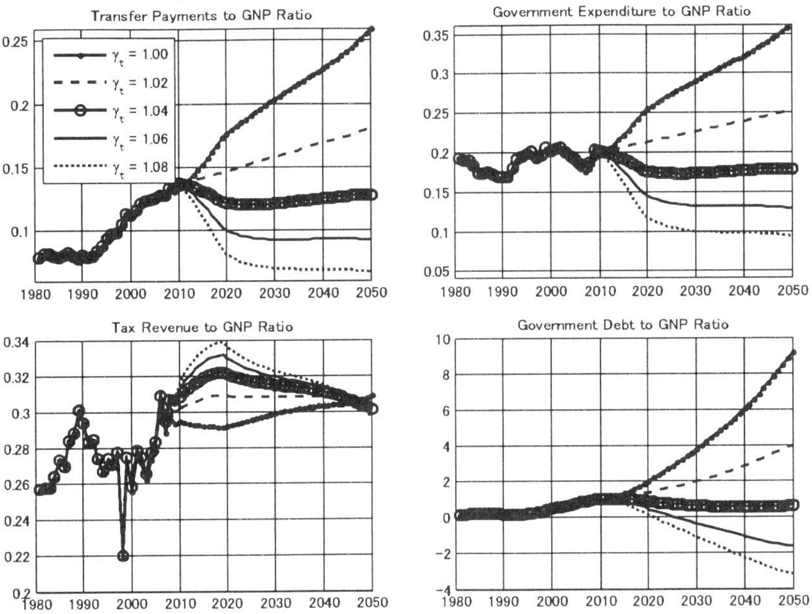


Table 2. Y_t/Y_{2010} : GNP Under Different TFP Growth Assumptions $\{\gamma_t\}_{t=2010}^{2019}$

	γ_t=1.00	γ_t=1.02	γ_t=1.04	γ_t=1.06	γ_t=1.08
2010	100	100	100	100	100
2020	95	116	141	172	212
2030	99	130	171	226	297
2040	103	142	193	261	351
2050	101	149	212	296	406

Table 3. B_t^g/Y_t : Debt to GNP Ratio Under Different TFP Growth Assumptions $\{\gamma_t\}_{t=2010}^{2019}$

	γ_t=1.00	γ_t=1.02	γ_t=1.04	γ_t=1.06	γ_t=1.08
2010	0.97	0.99	1.00	1.00	1.01
2020	2.03	1.40	0.88	0.46	0.41
2030	3.79	2.00	0.65	—	—
2040	6.03	2.85	0.57	—	—
2050	9.21	4.00	0.63	—	—

zero growth over the net decade, in other words, another lost decade, will push the primary balance into uncharted territory, into deficits of about 15-20% of GNP, with similarly alarming predictions of the government's debt to GNP ratio. A growth miracle, on the other hand, such as a 6% or 8% TFP growth rate, will yield persistently positive primary balance over the four decades. What will be the impact of growth on the level of debt? Figure (6) below provides the answer.

Note that Fukawa and Sato (2009) make an assumption of about 2% growth of GNP when they calculate their projections for future ratios of G_t/Y_t and TR_t/Y_t. As a result, when economic growth is better than 2% annually for the next decade, this serves to 'lower' the projected ratio of government expenditures to GNP. In other words, faster economic growth acts to help deliver a 'reduced' fiscal burden in addition to the increased tax base to help pay for expenditures. Growth is truly a gift that keeps on giving.

According to Figure (6), a TFP growth rate of 4% will likely maintain the debt to GNP ratio at its current level of about 100%. In other words, if TFP grows at 4% for the next 10 years, it will neutralize the fiscal burden from the future expenditure increases due to the aging of the population. Anything less, and the debt to GNP ratio rises. If there is zero growth, it could get very high[9)]. A growth miracle like 8% TFP growth for 10 years will actually eliminate all debt and allow the government to build a small public fund.

The tables below summarizes the simulation outcome. Table 2 displays the level of GNP in specific year with respect to that in 2010, and Table 3 display debt to GNP ratio under the five scenarios. The sign "—" implies that the debt to GNP is smaller than zero, indicating that debt is eliminated. Even under 4% growth rate, the debt never reaches zero although the size of debt to ratio shrinks compared with that in 2010. In contrast, when growth rate is either 6% or 8%, the debt is eliminated by 2030.

4.2 Sensitivity Analysis

4.2.1 Higher Consumption Tax

In this subsection, we allow the government to permanently raise the consumption tax from 5% to 15% in 2011. Does the new and higher consumption tax help reduce the government's debt? The answer is yes in general, but the magnitude depends on the realized growth rate of output over the next 10 years. Following figures show the projection of macro variables from 2010 up to 2050 when consumption tax is raised from 5% to

Figure 7. 15% Consumption Tax Rate and the Economy

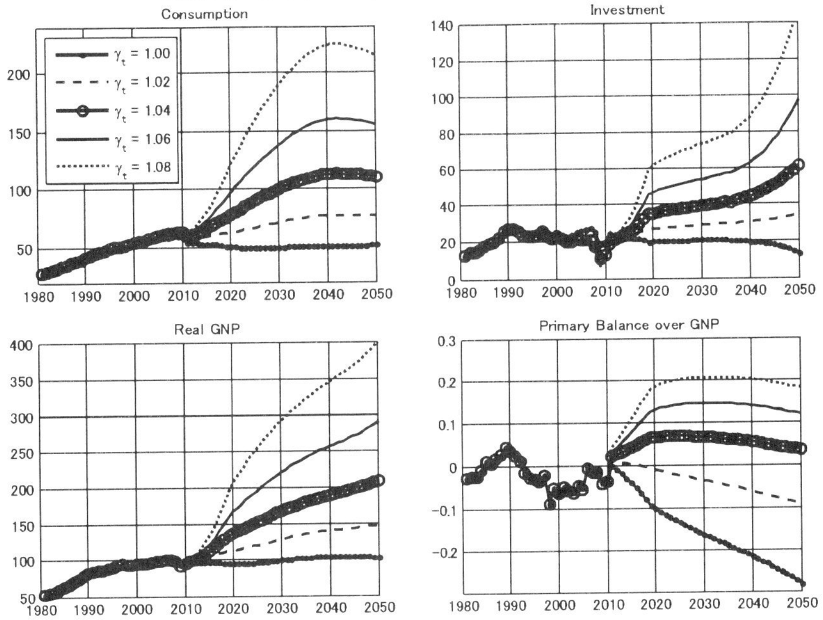
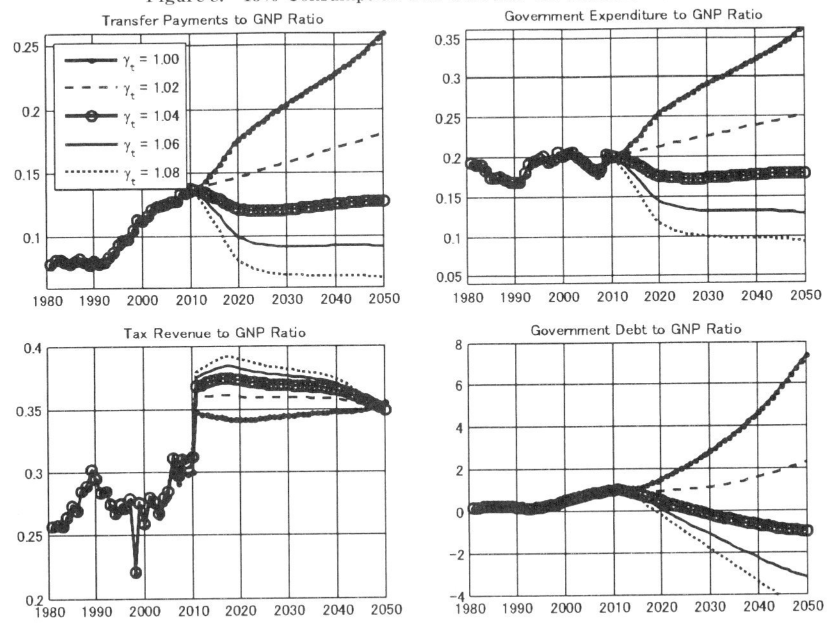


Figure 8. 15% Consumption Tax Rate and Government Debt



15% in 2011.

Figure (7) shows that the higher consumption tax rate delivers a positive primary balance sooner than the previous case. Furthermore, the magnitude and persistency of the primary surplus both improve significantly. As a result, we can expect that a relatively smaller rate of TFP growth, combined with the higher consumption tax rate, might now be able to relieve the fiscal pressure. The immediate impact of the tax rate increase is seen in the path of tax revenue to GNP ratio in Figure (8). In all cases, the tax revenue jumps up in 2011. Consequently, even a 4% growth rate may reduce the debt to GNP ratio immediately and eliminate it altogether eventually. A growth rate of only 2%, however, would still lead to a much higher debt to GNP ratio by 2050.

Table 4. B_t^g/Y_t : Debt to GNP Ratio Under Different TFP Growth Assumptions $\{\gamma_t\}_{t=2010}^{2019}$

	γ_t=1.00	γ_t=1.02	γ_t=1.04	γ_t=1.06	γ_t=1.08
2010	0.96	0.98	0.99	0.99	1.00
2020	1.52	0.93	0.45	0.08	—
2030	2.83	1.11	—	—	—
2040	4.66	1.55	—	—	—
2050	7.36	2.30	—	—	—

The table 4 below summarizes the simulation outcome. With consumption tax increase, debt shrinks quicker. Consequently, relatively slower TFP growth rate is sufficient to bring the debt to zero. It is also notable, however, that with 0% and 2% growth rate, debt to GNP ratio never reaches zero despite of the consumption tax increase.

4. 2. 2 Higher Interest Costs of Debt

Although our model is silent about the determination of I_t, from the perspective of projection accuracy, it is useful to conduct

Figure 9. Costly Interest Payments and the Economy

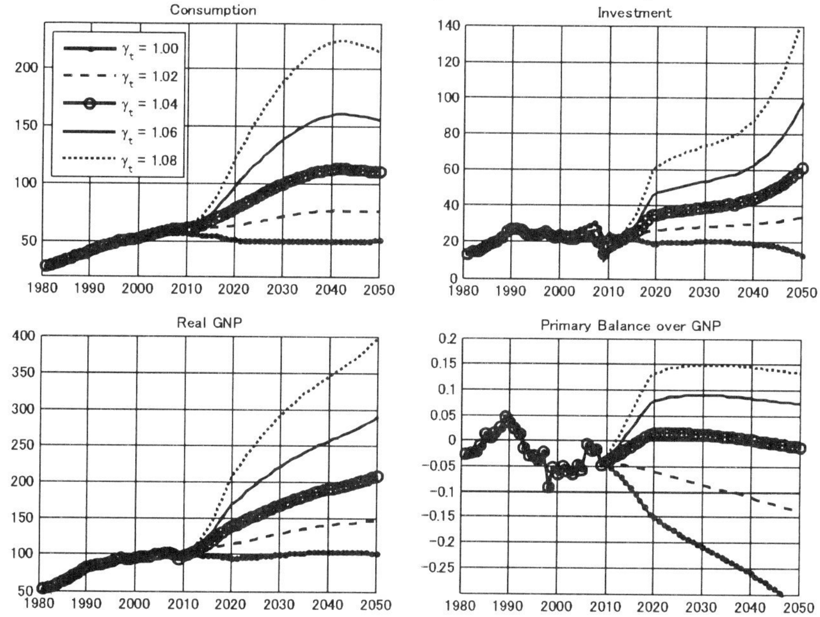
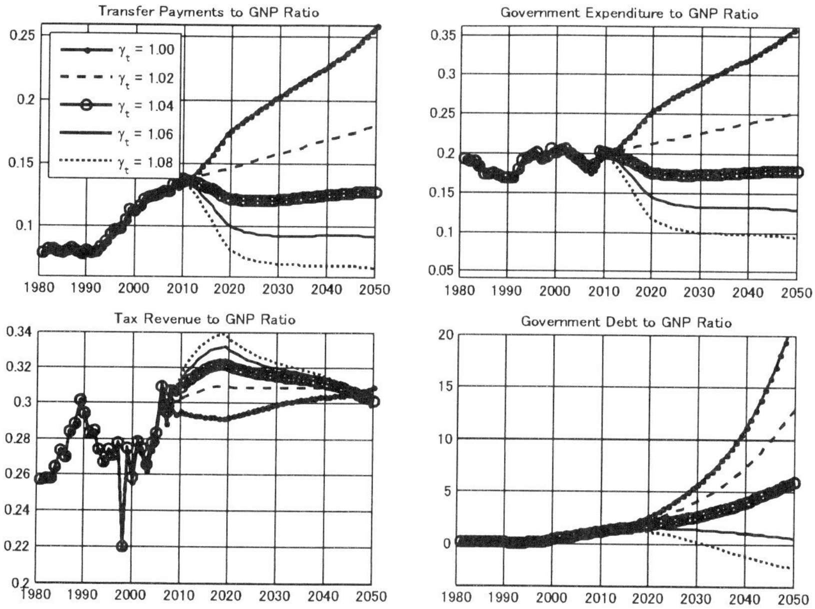


Figure 10. Costly Interest Payments and Government Debt



simulations under an alternative assumption where I_t is endogenously determined. Here, we construct the sequence of I_t so that it reflects the level of current government debt B_t^g and the real return from holding the private capital $(1-\tau_{k,t})(r_t-\delta_t)$. We assume that I_t evolves according to the following equation.

$$I_t = B_t^g(1-\tau_{k,t})(r_t-\delta_t) \quad \text{if } B_t^g > 0,$$
$$I_t = 0 \quad \text{if } B_t^g \leq 0. \tag{6}$$

As we discuss above, the return from holding government bond is often lower than the return from private capital. In this sense, this setting delivers the simulation results under a worse case scenario. Notice that a higher TFP growth rate causes higher interest payments, deteriorating the government budget balance although it broadens the tax base. In addition, I_t becomes larger as debt increases. Consequently, long lasting budget deficits may worsen the budget balance itself through the accumulation of debt.

Figures (9) and (10) show the projection of macro variables from 2010 up to 2050. When interest payments faced by the Japanese government are high, even a 6% growth performance is insufficient to curtail the downward trend in the primary balance. Despite the fiscal relief for a few years in the case of 6% growth, fiscal situation deteriorates eventually. In the case of more costly interest payments, a better economic performance is needed.

Figure (10) shows that 4% growth will end up rasing debt to GNP ratio to just under 600% by 2050. Even 6% growth rate is not sufficiently fast to eliminate the debt by year 2050. In addition, debt to GNP ratio almost diverge with 0% and 2% growth. In these scenarios, debt accumulates quicker and it

Table 5. B_t^g/Y_t : Debt to GNP Ratio Under Different TFP Growth Assumptions $\{\gamma_t\}_{t=2010}^{2019}$

	$\gamma_t=1.00$	$\gamma_t=1.02$	$\gamma_t=1.04$	$\gamma_t=1.06$	$\gamma_t=1.08$
2010	1.14	1.20	1.24	1.26	1.28
2020	2.66	2.21	1.82	1.47	1.16
2030	5.64	4.08	2.65	1.37	0.24
2040	11.16	7.36	3.98	1.06	—
2050	22.32	12.86	5.88	0.59	—

further deteriorates government deficit, leading to higher debt.

Table 5 below summarizes the simulation outcome. With costly interest payments, since the government's interest payments rise with the size of the debt and the return to private capital, debt accumulates faster compared with the previous cases. For example, under the TFP growth rate of 4%, the debt to GNP ratio is 5.88 in 2050 while it is 0.63 in the benchmark case. Consequently, relatively faster growth rate of TFP, in particular that faster than 6%, is needed to bring the debt to zero.

5. Concluding Remarks

Japan currently faces serious fiscal challenges that can be summarized in the unprecedentedly high current government debt. In this paper, we explore the impact of productivity growth rate on the Japanese government accounts using a standard growth model. The model is a general equilibrium model with complete markets and perfect foresight. A representative household and a stand-in firm take factor prices, demographics, and fiscal policy as given, and maximize their objective functions with respect to their budget constraints. The government finances its exogenous spending, including interest payments, with taxes on factor incomes and consumption.

Based on the calibration to the Japanese economy employed in İmrohoroğlu and Sudo (2010), and the future forecasts of government expenditure and social transfer reported by Fukawa and Sato (2009), our model generates the predicted time path of primary balance and debt from 2010 to 2050 under various assumptions as to the future growth rate of TFP. Our quantitative exercise suggests that current debt is eliminated in the following few decades only if a new Japanese miracle, a productivity growth rate far faster than 4% per year, is realized again in the next decade. Otherwise, the budget balance is never achieved and debt increases over time. To check the sensitivity of our benchmark simulation, we conduct two alternative experiments. First, we simulate the model under an assumption that consumption tax is permanently raised from 5% to 15% in 2011. Since tax revenues immediately jump up in 2011, the debt is eliminated within a shorter horizon and with a smaller TFP growth rate. Second, we consider a case where interest payments increase with the return to the private capital. Because faster TFP growth rate leads to higher return to the private capital, reduction of debt is partially offset by a rise in the interest costs. In this case, the Japanese economy needs a growth rate of TFP at least faster than 6% to eliminate the debt.

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Notes

1) See Doi, Ihori, and Mitsui (2006), and Broda and Weinstein (2004) for the other approach of analyzing Japanese fiscal deficit problem.

2) Chen, İmrohoroğlu, and İmrohoroğlu (2007) and Braun, Ikeda, and Joines (2009) develop overlapping generations models with incomplete markets to study the Japanese economy. By construction, these models deliver richer implications by disaggregating the economy into cohorts and different income and wealth groups. However, their aggregate predictions on the main macro variables seem to be consistent with those from the standard model with infinite horizon and complete markets.

3) Lower case letter will refer to per-capita items and upper case letters will be used to denote economy-wide aggregate quantities.

4) There are several modeling approaches that would allow for two assets, capital and debt, such that one would dominate the other in rate of return. One possible way would be to introduce a banking sector that has a positive value added by intermediating between borrowers and lenders. Another way would be to attribute some direct utility to holding government bonds that would deliver a lower rate of return to bonds since the agent derives direct utility from them due to an unspecified reason. Yes another approach would be to introduce uncertainty into the model and allow different assets to possess different distributional properties, serving differing risk preferences.

In this paper, government debt is exogenous and implicitly computed by accumulating budget deficits. Although none holds debt, interest payments are received by Japanese individuals and these payments impose a financing burden on the government. Introducing debt endogenously is an important extension which is beyond the scope of this paper.

5) See İmrohoroğlu and Sudo (2010) for the construction of G_t in 2009.

6) Note that some categories of entities and goods may be exempt from taxes. Since the tax rates faced by the representative agent in the model are calculated from different sources, they will not produce model accounts that come close to the observed government accounts. As a result, an adjustment is necessary so that the tax revenues in the model and those in the data are reasonably aligned. For each time period t, we multiply the tax rate on consumption by a correction factor of 0.9, and the tax rates on labor and capital income are multiplied by constants 0.8 and 0.85, respectively. Note that this is only a level adjustment and aims to align the government accounts in the model and the data.

7) This observation may stem from that our one sector model abstracts from the change in productivity that is peculiar to the investment goods producing sector. See Braun and Shioji (2007) and Hirose and Kurozumi (2010) where the role of investment specific technology in explaining Japanese economic fluctuations is discussed.

8) Note that real GNP growth rates are not equal to TFP growth rates. Along the balanced growth path, we have $g=\gamma^{1/(1-\tau)}$. However, along the transition, this relation does not hold. In this benchmark simulation, setting the TFP growth rate to 0%, 2%, 4%, 6%, and 8% from 2010 to 2019 delivers an average real GNP growth rate of -0.2%, 1.6%, 3.5%, 5.5%, and 7.5%, respectively.

9) Note that we rule out Ponzi schemes by imposing a steady state in the far distant future. However, debt to GNP ratios of 500-600% are unprecedented and our simple model cannot be a good measuring device to study the effects on the economy once indebtedness reaches these levels.

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