Discussion Paper Series A No.615

One Million Miles to Go: Taking the Axiomatic Road to Defining Exploitation

Roberto Veneziani
(School of Economics and Finance,
Queen Mary University of London)
and
Naoki Yoshihara
(Institute of Economic Research, Hitotsubashi University)

August, 2014

Institute of Economic Research
Hitotsubashi University
Kunitachi, Tokyo, 186-8603 Japan

One million miles to go: taking the axiomatic road to defining exploitation*

Roberto Veneziani† Naoki Yoshihara‡

August 7, 2014

Abstract

This paper analyses the Marxian theory of exploitation. A general axiomatic approach is developed which is appropriate to study the concept of exploitation - what it is and how it should be captured empirically. Two properties are presented that capture some fundamental Marxian insights. It is shown that, contrary to the received view, there exists a nonempty class of definitions of exploitation that preserve the relation between exploitation and profits - called Profit-Exploitation Correspondence Principle - in general economies with heterogeneous agents, complex class structures, and production technologies with heterogeneous labour inputs. However, among the main approaches, only the so-called 'New Interpretation' satisfies the Profit-Exploitation Correspondence Principle in general.

JEL: B51.

Keywords: Exploitation, profits, axiomatic analysis.

*We thank Gary Dymski, Don Katzner, Gil Skillman, Peter Skott, and participants in the Analytical Political Economy workshop and the American Economic Association conference (Philadelphia) for useful comments and suggestions. We worked on this paper while visiting the University of Massachusetts, Amherst: their support and hospitality is gratefully acknowledged. The usual disclaimer applies.

†(Corresponding author) School of Economics and Finance, Queen Mary University of London, Mile End Road, London E1 4NS, United Kingdom (r.veneziani@qmul.ac.uk)

‡The Institute of Economic Research, Hitotsubashi University, Naka 2-1, Kunitachi, Tokyo 186-0004, Japan (yosihara@ier.hit-u.ac.jp)

1 Introduction

After a wave of intense research and debates in the 1970s and 1980s, the Marxian theory of exploitation has moved to the sidelines of academic economics. The notion of exploitation has never had much appeal for mainstream economists. But heterodox scholars, in the main, also seem sceptical about the possibility of developing a logically coherent and empirically meaningful concept of exploitation. Two broad (and related) sets of objections have been moved against the Marxian theory of exploitation.

First, it is unclear what Marxian exploitation actually *is*. At a general level, it can be defined as the (forced) extraction of surplus labour, or as a power relation leading to systematic differences between the labour contributed (in some relevant sense) by workers in productive activities and the labour that they receive (in some relevant sense) in return. As intuitive as these general formulations are, outside of very stylised two-class economies with linear technologies and one type of homogeneous labour, the notions of surplus labour, or of the labour contributed and received by workers have no obvious interpretation. Different definitions of exploitation can be, and have in fact been proposed that rely on different interpretations of these concepts (see, for example, Okishio [27]; Morishima [25]; Roemer [31]; Flaschel [6, 7]; Duménil [3]; Foley [10]). Actually, as Steedman [37] famously argued, it is not even clear that these concepts - and, a fortiori, Marxian exploitation - *can* be coherently defined due to the conceptual problems inherent in the notion of labour embodied.

Second, it is unclear what the concept of exploitation *does*. The conceptual problems, anomalies and counterexamples identified in the literature have led to increasingly complex and often counterfactual definitions that have progressively lost intuitive appeal, and also their applicability to actual capitalist economies, calling into question the usefulness of the notion of exploitation. At a methodological level, this is often attributed to the mathematical turn taken by the debate on exploitation in the 1970s, which, according to critics, has shifted the focus from the broader conceptual issues to relatively minor technical details.

This paper defends the relevance of Marxian exploitation while adopting a formal approach to exploitation theory. First, a definition is proposed, which is conceptually related to the 'New Interpretation' (Duménil [3]; Foley [10]; Duménil, Foley, and Lévy [4]; see also Mohun [23]). According to this definition, an agent is exploited (resp., an exploiter) if and only if the labour

she contributes is greater (resp., lower) than the share of aggregate social labour that she receives via her income. This approach defines exploitation as a feature of the competitive allocation of social labour rather than as the result of productive inefficiencies, or labour market imperfections. Unlike the received definitions, it has a clear empirical content, for it is firmly anchored to the actual data of the economy. Further, we show that it preserves several widely shared positive and normative intuitions concerning the notion of exploitation, including the existence of a robust relation between exploitation and profits. This is an important and surprising result.

At a general, theoretical level, it is often argued that the concept of exploitation is central in Marxian economics because it links the visible, epiphenomenal characteristics of capitalist economies (as revealed by monetary variables) with deeper, structural forces and mechanisms. Whether profits are indissolubly linked to exploitation is important, in this perspective, both if exploitation is meant to uncover the fundamental forces underlying the dynamics of accumulation in capitalist economies; and if it is an essentially normative criterion to evaluate, and indict, capitalism. For, given private ownership of productive assets, one should expect profits to be a counterpart of the transfer of social surplus and social labour from workers to capitalists, and one of the causes of inequalities in well-being freedom.

The existence of a relation between exploitation and profits has been famously proved by Okishio [28] in linear economies with homogeneous labour. Given its relevance the result has been dubbed the *Fundamental Marxian Theorem* (henceforth, FMT), and has sparked a substantial literature.[1] Whereas the result was successfully extended to Leontief economies with heterogeneous labour (see, e.g., Fujimori [12]; Krause [14]; Bowles and Gintis [1]), Steedman [37] proved that the FMT does not hold in von Neumann economies with joint production. Morishima [25] and Roemer [31] proposed two alternative definitions of exploitation that meet Steedman's critique. Nonetheless, these results are far from conclusive: Yoshihara and Veneziani [46] have shown that neither Morishima's [25] nor Roemer's [31] definition preserves the FMT in economies with a convex technology and homogeneous labour. Further, the robustness of the FMT in more general economies with heterogeneous skills and labour inputs is an open question.

[1] The literature is too vast for a comprehensive list of references. In addition to the classic contributions cited in the text, it is worth mentioning the more recent detailed discussions by Fleurbaey [9]; Mohun [22]; Flaschel [7]; Yoshihara and Veneziani [46].

As explained in section 4 below, our analysis bears only a broad conceptual relation with the FMT literature. Yet, in the light of the debates on the FMT, it is remarkable that, if the New Interpretation is adopted, then it is possible to establish a robust relation between exploitation and profits in general economic environments with heterogeneous labour. Indeed, the New Interpretation is the *only* definition - among the main ones in the literature - with this property.[2]

The second contribution of the paper is methodological. A general axiomatic approach is developed, whereby the desirable characteristics of a definition of exploitation are stated using the language of formal logic. As this is one of the first applications of axiomatic analysis to Marxian and classical economics,[3] the next subsection addresses some methodological issues related to the use of mathematics in economics, placing our approach in the context of recent debates. The aim is not to provide a general methodological discussion, but rather to explain why the axiomatic approach adopted in this paper is both appropriate and insightful in the analysis of the concept of exploitation. The uninterested reader may safely skip to section 2.

1.1 Taking the axiomatic road

This paper addresses the issue of the appropriate definition of exploitation and measure of exploitative relations in capitalist economies. The motivation of our analysis is not the lack, but the *wealth* of plausible candidates: as noted above, many different definitions can be, and have in fact been proposed, which incorporate different positive and normative intuitions.

The fundamental question is how to choose among all of the existing and the conceivable definitions. Thus far, the debate has largely been reactive: new definitions have often emerged as the product of a process of adjustment of the theory to various anomalies and counterexamples identified in the literature. We adopt a different approach. Rather than proposing another definition, and comparing it with the existing alternatives, we develop a general axiomatic framework to analyse what exploitation is, and how it should be measured. The axiomatic method is used to rigorously and explicitly state

[2]This paper significantly generalises Veneziani and Yoshihara [41], which focuses on a smaller set of economies with homogeneous labour inputs and on a specific equilibrium notion.

[3]Notable exceptions include Yoshihara [44], Veneziani and Yoshihara [41], and Flaschel et al. [8].

the normative and positive foundations of the notion of exploitation.

The starting point of our analysis is the acknowledgment that the concept of exploitation has a quantitative dimension. This is not to say that exploitation can or should be *reduced* to a quantitative phenomenon. Purely distributive approaches that reduce exploitation to an inequality in productive assets (such as Roemer's [31] seminal *property relations* approach), for example, are ultimately unsatisfactory. For exploitative social relations arguably involve some form of power, force, or coercion, which need not be clearly measurable.[4]

Yet the concept of exploitation also has an inherently quantitative dimension, such that it is meaningful in principle to say that "advanced economies have become significantly more exploitative over the past four decades", or that "exploitation is worse in country A than in country B". Exploitation diagnoses the process through which "certain inequalities in incomes are generated by inequalities in rights and powers over productive resources: the inequalities occur, in part at least, through the ways in which the exploiters, by virtue of their exclusionary rights and powers over resources, are able to appropriate labour effort of the exploited" (Wright [43], p.1563). The analytical focus of this paper is precisely the quantitative dimension of exploitation and the most appropriate way of capturing this aspect of exploitative social relations.

It may be objected that the possibility of measuring a certain phenomenon does not imply that formal tools should be used to study it.[5] Critical realism, for example, has long argued against the mainstream insistence that mathematics or the logical deductivist method be used always and everywhere in economics (Lawson [16]). According to critical realists, social reality is most plausibly construed as an open, structured, dynamic and internally related system and mathematical-deductivist methods are inappropriate in the causal-explanatory analysis of open systems (Lawson [16, 18]).

A thorough discussion of the role of formal tools in social theorising goes beyond the scope of this paper. But it is worth explaining why the main objections against the use of mathematics in economics do not apply here, and the axiomatic method is both appropriate and insightful for our analysis.

[4] A critical discussion of Roemer's approach, and of the role of power in exploitation theory is in Veneziani [39, 40]. See also the insightful analysis in Skillman [35] and Vrousalis [42].

[5] Conversely, the fact that some variables are not measurable does not rule out *a priori* the use of formal tools and concepts (Katzner [13]).

This paper examines the category of "exploitation", in order to understand what it means, and how it can be captured empirically. Our theoretical effort can be conceived of as philosophical underlabouring a central concept of Marxian economics and we use the axiomatic method in order to identify the desirable properties of a definition (and measure) of exploitation. The analysis is *not* based on any assumptions - whether explicit or implicit - about the nature of social reality as a closed system, and about the pervasiveness of strict regularities and constant conjunctions of events.

Ours is an investigation in *scientific ontology* and in this respect it is similar in nature to Sen's [33] analysis of the concepts of functionings and capabilities (see Martins [19]).[6] Our axiomatic framework does not incorporate any hypotheses concerning causal laws (or even tendencies) within a predictionist perspective (Lawson [16], p.60). Rather, it aims to clarify the social category of exploitation and to capture exploitative relations *a posteriori*, focusing on the state of the economy at a given point in time. It is therefore conceptually analogous to the approach used to identify appropriate measures of poverty or inequality (e.g., Foster [11]), labour productivity (Flaschel et al [8]), or labour content (Yoshihara and Veneziani [48]).

The axioms are thus *abstract* in that they incorporate relevant philosophical views about the nature and the positive and normative foundations of Marxian exploitation, and about the properties of an appropriate definition of (one aspect of) exploitative social relations. But they are also *empirically oriented*, in that they focus on observable, well-defined magnitudes, and do not rely on "claims that are believed to be false of our world, and of any really possible counterfactual world" (Lawson [17], p.766).

The axiomatic framework also precisely identifies the domain and scope of the analysis by focusing on a set of economic agents and on a class of economic environments and allocations. But this by no means entails a reductionist perspective. On the one hand, no strong *a priori* restrictions are imposed on agent behaviour or on the institutional mechanisms (market-based or otherwise) regulating economic interactions. The axioms incorporate no assumptions concerning individuals' ontology (including their selfish or ethical motivations, the origin of their preferences, or the notion of individual rationality), the nature of social interactions, and so on. On the other hand,

[6]Interestingly, this is not the only similarity with Sen's theory: at the substantive level, the definition of exploitation that emerges from our analysis is conceptually related to Sen's theory of *well-being freedom*. For a discussion see Veneziani and Yoshihara [41].

although the axioms focus on the exploitation status of *individual agents at a given point in time*, they do not entail a commitment to atomism, to a static view of social reality, or to the existence of a fixed, given unit of analysis. The axiomatic system can be modified to incorporate aggregate, dynamic, or relational properties, depending on the object of analysis.

To be sure, the intuitive appeal of a definition is fundamental, and a certain definition should provide the right answers in situations in which we feel that intuition is a reliable guide. The axiomatic method is not "a substitute for intuition ... but instead ... a way of articulating [the intuitions that hold in specific situations] into operationally useful conditions pertaining to an entire class of cases" (Thomson [38], p.356). Indeed, as shown by the often surprising impossibility results obtained in social choice theory, or by the very difficulty in providing a definition of exploitation that preserves key Marxian insights, intuition alone can be insufficient. When delineating the properties of the appropriate definition of exploitation, "Informal insights, important as they are, cannot replace the formal investigations that are needed to examine the congruity and cogency of combinations of values and of apparently plausible demands" (Sen [34], p.353).

Of course, the axiomatic method does not necessarily lead to univocal conclusions: one can reject any of the axioms below, and propose a new one, possibly leading to an alternative definition of exploitation. This indeterminacy is not a property of axiomatic analysis *per se*: it is inherent in all social theorising and it simply reflects different positive and normative intuitions.[7] If anything, the axiomatic method has the advantage of making such intuitions explicit, thus forcefully directing research and debate to the foundational issues concerning the nature and measurement of exploitation.

This is an important point. Unlike in much of the mainstream, in this paper the axiomatic method is not used in a purely instrumental way, for example in order to generate predictions that match empirical data, and regardless of the actual relevance of the axioms (Lawson [16, 18]). The content of the axioms is central to our analysis, and the relevance and meaningfulness of the conclusions ultimately depends on it.[8] As noted by Foster, in his

[7] As Sen ([32], p.187) noted in his insightful discussion of the labour theory of value, "since several motivations underlying the labour theory of value can be distinguished, it is not really surprising that different conventions for calculating aggregate labour magnitudes would exist and also appear natural in different contexts. The source of these ambiguities ... [rests] in the basic multiplicity of motivation underlying the labour theory."

[8] In this respect, the axiomatic approach adopted in this paper is fundamentally differ-

thoughtful discussion of inequality and poverty measures, "The relevance of an axiomatic result depends entirely on the acceptability or usefulness of its constituent properties" (Foster [11], p.367).

2 Economic states

The aim is to analyse exploitation without imposing any significant restrictions on agents' behaviour, market structure, and so on. Therefore we keep the description of the economic environment to a bare minimum. Consider an economy with N agents, n commodities, and T types of labour. Let $\mathcal{N}$ be the set of agents with generic element ν and let $\mathcal{T}$ be the set of types of labour with generic element τ.

Technology is described by a production set P with elements - activities - of the form $\alpha = (-\alpha_l, -\underline{\alpha}, \overline{\alpha})$ where $\alpha_l \equiv (\alpha_{l\tau})_{\tau\in\mathcal{T}} \in \mathbb{R}^T_+$ is a profile of labour inputs used in production and measured in hours; $\underline{\alpha} \in \mathbb{R}^n_+$ are the inputs of the produced goods; and $\overline{\alpha} \in \mathbb{R}^n_+$ are the outputs of the n goods.[9] By measuring labour inputs in terms of time, this description of the technology includes standard economies as a special case.[10]

The net output vector arising from α is denoted as $\widehat{\alpha} \equiv \overline{\alpha} - \underline{\alpha}$ and the set of efficient activities in P is denoted as ∂P.[11] In the rest of the paper, we assume that: the technology displays constant returns to scale; firms can decide not to activate any process; and the production of any output requires *some* labour and *some* capital.[12] These restrictions are rather mild and standard in heterodox (and even mainstream) approaches.

Each agent $\nu \in \mathcal{N}$ is endowed with a (possibly zero) vector of productive assets, $\omega^\nu \in \mathbb{R}^n_+$, and a nonempty set of types of labour $\mathcal{T}^\nu \subseteq \mathcal{T}$ that can

ent, for example, from Gérard Debreu's classic *Theory of value*.

[9] In principle, all variables should be dated. However, because we analyse exploitative relations in a given economy *at a given point in time*, we drop the time subscript for notational convenience.

[10] Consider, for example, a production technology with homogeneous labour inputs but heterogeneous skills among agents. The set $\mathcal{T}$ of types of labour can be interpreted as the qualitatively homogeneous but quantitatively different labour skills. Then this technology can be represented by a set $P \subseteq \mathbb{R}^T_- \times \mathbb{R}^n_- \times \mathbb{R}^n_+$ with $T = N$ types of heterogeneous labour, where each component of $\alpha_l \in \mathbb{R}^N_+$ is measured in the time unit, thus incorporating skills in the description of the technology.

[11] Formally: $\partial P \equiv \{\alpha \in P \mid \nexists \alpha' \in P \text{ such that } \alpha'_i > \alpha_i \text{ for all } i\}$.

[12] These assumptions on P are stated formally in appendix A.1.

be used in production. The total amount of *time* that can be used either productively (possibly in different types of labour) or in leisure activities, is normalised to one. Let $C \subseteq \mathbb{R}^n_+ \times [0,1]^T$ be the set of all conceivable choices of each agent with generic element (c^ν, λ^ν), where $c^\nu \in \mathbb{R}^n_+$ is ν's consumption vector and $\lambda^\nu = (\lambda^\nu_1, ..., \lambda^\nu_T)$ describes the amount of time of each type of labour spent by ν in productive activities, where $\lambda^\nu_\tau \in [0,1]$ for all $\tau \in \mathcal{T}^\nu$, $\lambda^\nu_\tau = 0$ for all $\tau \in \mathcal{T} \backslash \mathcal{T}^\nu$, and $\sum_{\tau \in \mathcal{T}^\nu} \lambda^\nu_\tau \leqq 1$.

Let (p, w) be the $1 \times (n+T)$ vector describing the (positive) prices of the n commodities and the (nonnegative) wages of the T types of labour. Let $\pi^{\max} = \max_{\alpha \in P} \frac{p\widehat{\alpha} - w\alpha_l}{p\underline{\alpha}}$ be the maximum profit rate that can be obtained at prices (p, w), and let $P^\pi(p, w)$ be the set of production processes that yield the maximum profit rate at (p, w).[13]

Our analysis does not depend on any specific assumptions on individual households' or firms' behaviour, or on the institutional framework in which agents interact. However, both the key axioms and the main results apply only to a class of economic scenarios which may, or may not, turn out to be true ex post. Yet they are sufficiently general and theoretically relevant to warrant investigation and to cover a large set of possible cases.

Let $\mathbf{0}$ be the null vector. An *economic environment* is a set of agents, $\mathcal{N}$; a production set, P; a consumption space, C; a profile of agents' sets of types of labour, $(\mathcal{T}^\nu)_{\nu \in \mathcal{N}}$; and a profile of agents' endowments of productive assets, $(\omega^\nu)_{\nu \in \mathcal{N}}$. Given an economic environment, an *economic outcome* is a price vector, (p, w), a profile of consumption and labour decisions $(c^\nu, \lambda^\nu)_{\nu \in \mathcal{N}}$, and a profile of production activities operated by agents $(\alpha^\nu)_{\nu \in \mathcal{N}}$, with aggregate production activity, $\alpha^{p,w} = \sum_{\nu \in \mathcal{N}} \alpha^\nu$, such that:[14] (i) $\widehat{\alpha}^{p,w} \geqq \sum_{\nu \in \mathcal{N}} c^\nu$; (ii) $p\widehat{\alpha}^{p,w} - w\alpha^{p,w}_l \geqq 0$; (iii) $\alpha^{p,w} \in P^\pi(p, w)$ with $\alpha^{p,w}_l = \sum_{\nu \in \mathcal{N}} \lambda^\nu$; and (iv) $pc^\nu = \pi^{\max} p\underline{\alpha}^\nu + w\lambda^\nu$ for all $\nu \in \mathcal{N}$. A pair of an economic environment and an economic outcome is an *economic state* (henceforth, ES).

The concept of ES is very general, with no substantive restrictions on behaviour, technology or institutions. Conditions (i) and (ii) are hardly demanding: they simply state that aggregate net output can at least provide for consumption, and that *aggregate* profits are nonnegative, respectively. Condition (iii) is only slightly more restrictive in that it postulates that producers activate profit-rate-maximising processes. This is consistent with Marx's

[13] Formally, $P^\pi(p, w) = \left\{ \alpha \in P \mid \pi^{\max} = \frac{p\widehat{\alpha} - w\alpha_l}{p\underline{\alpha}} \right\}$.

[14] For all vectors $x, y \in \mathbb{R}^n$, $x \geqq y$ if and only if $x_i \geqq y_i$ $(i = 1, \ldots, n)$; $x \geq y$ if and only if $x \geqq y$ and $x \neq y$; $x > y$ if and only if $x_i > y_i$ $(i = 1, \ldots, n)$.

([21], ch.10) analysis of capitalist behaviour and is common in heterodox approaches (including Sraffian price theory). It is also theoretically justified because it is desirable to obtain a definition of exploitation that can capture exploitative social relations under optimal capitalist behaviour. In Marxian theory, exploitation is the product of capitalist social relations, rather than mistakes, technical inefficiencies, or market imperfections.[15] Condition (iv) postulates that individual expenditure is subject to the budget constraint. This yields no significant loss of generality and it does not rule out savings, as the bundle c^ν is not necessarily restricted to consumption goods and services. Further, it is theoretically justified because in Marxian theory, exploitation is primarily the product of capitalist social relations, and in particular of the wage relation, rather than credit markets or individual life-cycle decisions.

The notion of ES does not incorporate any of the standard features of mainstream models such as utility functions, selfish optimising (or even boundedly rational) consumers, and differentiable production functions. The production set is sufficiently general to allow for the existence of a differentiable production function, but it does *not* postulate it and nothing depends on neoclassical assumptions of any sort. Nor does the definition of ES rely on the concept of equilibrium. According to Lawson [18], a focus on equilibrium is a requirement of mathematical models proceeding in deductive mode to specify causal influences. But our inquiry is of a different, ontological nature and so we do not need any notion of equilibrium or closure.

Perhaps more importantly, the concept of ES is not a description of the functioning of the economy: how it is structured, what agents do or how they choose, and so on. Rather, it acts formally as a domain condition: it defines the scope and boundaries of the analysis. Social scientific theorising is always context-specific, and the definition of ES makes the context explicit. It is important to note, however, that such boundaries are rather wide indeed (and significantly wider than in the literature), for conditions (i)-(iv) can obtain in a range of economies and under many different assumptions concerning, e.g., institutions and behaviour.[16] Further, although it is an open question whether our conclusions hold even more generally, the key insights of the

[15] Note that the condition $\alpha_l^{p,w} = \sum_{\nu \in \mathcal{N}} \lambda^\nu$ does not imply any form of labour market equilibrium or optimising behaviour. It simply states that the labour used in production is performed by the agents in the economy.

[16] The set of economic states is nonempty. It contains, for example, generalisations of Roemer's [30, 31] economies as special cases (see the Addendum).

paper are robust to several changes in the definition of ES.[17]

3 Defining Marxian exploitation

In Marxian theory, exploitation can be seen in two related but slightly different ways. One can focus on the difference between the labour performed by workers and the labour socially necessary to produce the goods they consume. Or, one can conceptualise exploitative social relations as characterised by systematic differences between the labour that agents 'contribute' to the economy and the labour they 'receive'. As soon as one moves away from the simplest linear economies, none of these concepts is clearly defined and, as already noted, various definitions have been proposed which reflect different views concerning the concept of exploitation.

Most (though not all) of the previous debates have revolved around the appropriate definition of socially necessary labour time, or of the labour received by agents. In order to focus on this issue, contributors have analysed economies with homogeneous labour, in which the labour performed by workers, or the labour that they contribute to the economy is uncontroversial: it corresponds to the labour time spent in productive activities. In more general economies, however, these concepts are not trivial.

In this section, we extend some of the main definitions to economies with heterogeneous labour. The key step is to acknowledge that although the notion of labour performed, or contributed to the economy is theoretically important, the differences between alternative approaches lie elsewhere. Therefore we adopt a common definition of the labour performed, or contributed by agents, measured by the value of the labour time spent in production.

This approach is consistent with the classical economists' view on how to convert different types of labour into a single unit, whereby "the different kinds of labour are to be aggregated via the (gold) money wage rates" (Kurz and Salvadori [15], p.324). According to Smith, for example,

> "It is often difficult to ascertain the proportion between two different quantities of labour. The time spent in two different

[17] For example, all of our results continue to hold if condition (iv) is weakened to require that expenditure does *not exceed* income, and condition (i) is weakened to require that *aggregate* net output is at least as high as the consumption of any *individual* agent ($\widehat{\alpha}^{p,w} \geqq c^{\nu}$ for each $\nu \in \mathcal{N}$), or, indeed, that aggregate net output be strictly positive.

> sorts of work will not always alone determine this proportion. The different degrees of hardship endured, and of ingenuity exercised, must likewise be taken into account. There may be more labour in an hour's hard work, than in two hours easy business; or in an hour's application to a trade which it cost ten years labour to learn, than in a month's industry, at an ordinary and obvious employment. But it is not easy to find any accurate measure either of hardship or ingenuity. In exchanging, indeed, the different productions of different sorts of labour for one another, some allowance is commonly made for both. It is adjusted, however, not by any accurate measure, but by the higgling and bargaining of the market, according to that sort of rough equality which, though not exact, is sufficient for carrying on the business of common life" (Smith [36], ch.V, pp.34-35).

One can similarly interpret Ricardo's ([29], ch.I, section II, p.11) arguments that "The estimation in which different quantities of labour are held, comes soon to be adjusted in the market with sufficient precision for all practical purposes, and depend much on the comparative skill of the labourer, and intensity of the labour performed". Despite some debates on the concept of "abstract labour", our approach is also consistent with Marx's ([20], pp.51-2) views on the conversion of complex labour into simple labour.

Indeed, although exploitation theorists often do not provide a complete extension of their definitions to economies with heterogeneous labour, in the main they do endorse (albeit sometimes implicitly) the classical economists' view on the use of wages for the reduction of different types of labour to a single unit (see, e.g., Morishima [24], ch.14; Duménil et al. [4]).[18,19]

Consider first Morishima's [24, 25] classic definition. For any ES and any bundle $c \in \mathbb{R}^n_+$, let $l.v.(c;w)$ denote the minimum amount of labour necessary to produce c as net output.[20] According to Morishima, the amount of labour received by agent ν, who consumes c^ν, is $l.v.(c^\nu;w)$. Therefore:

Definition 1: Consider any ES. Agent $\nu \in \mathcal{N}$, who supplies λ^ν and consumes c^ν, is *exploited* if and only if $w\lambda^\nu > l.v.(c^\nu;w)$ and *an exploiter* if and

[18] It is also worth noting that Yoshihara and Veneziani [48] have proved that in economies with heterogeneous labour the *wage-additive measure* is the unique measure of labour content that satisfies a small set of theoretically robust and intuitive axioms.

[19] For a different approach see Okishio [26, 28]; Fujimori [12]; Krause [14].

[20] Formally: $l.v.(c;w) \equiv \min\{w\alpha_l \mid \alpha \in P \;\&\; \widehat{\alpha} \geqq c\}$.

only if $w\lambda^{\nu} < l.v.\,(c^{\nu}; w)$.

Definition 1 has some desirable characteristics, according to Morishima ([25], pp.616-618): the notion of exploitation is well-defined because $l.v.\,(c; w)$ is unique, well-defined and nonnegative.[21] Furthermore, although wages are necessary to convert different types of labour into a single unit, exploitation status can be determined prior to and independent of the prices of commodities, as in classical Marxian theory.

According to Roemer [30, 31], however, Definition 1 is conceptually flawed because it identifies exploitation status based on production techniques that may never be used by profit-maximising capitalists, and because the labour received by agents should be defined incorporating information about the prices of produced commodities, also. Like Morishima [25], Roemer [31] focuses on the bundle c^{ν} actually consumed by agents but argues that its labour content, denoted as $l.v.\,(c^{\nu}; p, w)$, is given by the minimum amount of labour necessary to produce it as net output using *profit-rate-maximising production processes*,[22] for only the latter are activated by capitalists. Then, according to Roemer, exploitation status is determined as follows:

Definition 2: Consider any ES. Agent $\nu \in \mathcal{N}$, who supplies λ^{ν} and consumes c^{ν}, is *exploited* if and only if $w\lambda^{\nu} > l.v.\,(c^{\nu}; p, w)$ and *an exploiter* if and only if $w\lambda^{\nu} < l.v.\,(c^{\nu}; p, w)$.

Although they preserve some important Marxian insights, Definitions 1 and 2 have been criticised because exploitation status depends on counterfactual amounts of labour content (see, e.g., Flaschel [6, 7]). For the production activities yielding $l.v.\,(c^{\nu}; w)$ or $l.v.\,(c^{\nu}; p, w)$ may be different from those actually used in the economy. According to critics, this use of counterfactuals is theoretically undesirable and makes exploitation an empirically vacuous notion, since the computation of $l.v.\,(c^{\nu}; w)$ and $l.v.\,(c^{\nu}; p, w)$ requires information that is normally unavailable, including, in Morishima's own words, "information about *all* the available techniques of production, actually chosen *or potentially usable*" ([25], p.617, italics added).[23]

An alternative approach has been recently proposed by Yoshihara and Veneziani ([45, 44, 41]). Consider any ES. For any $c \in \mathbb{R}^{n}_{+}$ such that

[21] This follows from assumptions **A0**∼**A2** in Appendix A.1 (see Roemer [30], Proposition 2.1). The same holds for $l.v.\,(c; p, w)$ below.

[22] Formally: for any $c \in \mathbb{R}^{n}_{+}$, $l.v.\,(c; p, w) \equiv \min\{w\alpha_l \mid \alpha \in P^{\pi}(p, w)\ \&\ \widehat{\alpha} \geqq c\}$.

[23] See Yoshihara and Veneziani [46] for a detailed discussion.

$pc \leqq p\widehat{\alpha}^{p,w}$, the *labour content of* c is defined as $l.v.(c;p,w,\alpha^{p,w}) \equiv \tau^c w\alpha_l^{p,w}$, where $\tau^c \in [0,1]$ is such that $\tau^c p\widehat{\alpha}^{p,w} = pc$.[24] Thus, the labour content of aggregate net output, $\widehat{\alpha}^{p,w}$, is equal to the value of total social labour, $\alpha_l^{p,w}$, and the labour contained in any bundle c (whose value does not exceed national income) is equal to the fraction τ^c of social labour necessary to produce a fraction of aggregate net output, $\tau^c\widehat{\alpha}^{p,w}$, that has the same value as c. Then:

Definition 3: Consider any ES. Agent $\nu \in \mathcal{N}$, who supplies λ^ν and consumes c^ν is *exploited* if and only if $w\lambda^\nu > l.v.(c^\nu;p,w,\alpha^{p,w})$ and an *exploiter* if and only if $w\lambda^\nu < l.v.(c^\nu;p,w,\alpha^{p,w})$.

Definition 3 is conceptually related to the 'New Interpretation' of Duménil [3] and Foley [10]. In fact, for any agent $\nu \in \mathcal{N}$, τ^{c^ν} represents ν's share of national income, and so $\tau^{c^\nu} w\alpha_l^{p,w}$ is the share of (the value of) social labour that ν receives by earning the income exactly necessary to buy c^ν. As in Roemer's [31] approach (and unlike in Morishima's), exploitation status can be identified only if goods' prices are known, but social relations play a more central role, because the definition of exploitation requires knowledge of the social reproduction point, and it is related to the production and distribution of national income and social labour. Indeed, unlike Definitions 1 and 2, Definition 3 depends exclusively on empirically observable magnitudes. Nonetheless, the New Interpretation has been criticised because, unlike Definitions 1 and 2, the agents' actual consumption choices are only indirectly relevant to determine exploitation status, and unlike Definition 1, exploitation status depends on information about commodity prices.

This brief (and admittedly partial) survey shows that there are many possible approaches - incorporating different positive and normative intuitions - to explain what exploitation is and how it should be captured. The question then is how to adjudicate between them. Some preliminary answers are provided in the next section.

4 Axiomatising Marxian exploitation

In this section, we discuss two axioms incorporating some key properties that a definition of exploitation, and measure of exploitative relations, should

[24] If $p\widehat{\alpha}^{p,w} = 0$, we set $\tau^c = 0$ by definition.

satisfy and analyse their implications.

To be specific, for any ES, let $W_+ \equiv \{\nu \in \mathcal{N} \mid \omega^\nu = \mathbf{0} \ \& \ w\lambda^\nu > 0\}$: W_+ is the set of agents with no initial endowments who supply some (productive) labour at a given ES. Our axiomatic analysis focuses on the exploitation status of the agents in W_+. Theoretically, the set W_+ is of focal interest from a Marxian perspective even in societies with a complex class structure: if *any* agents are exploited, propertyless agents who supply wage labour should be among them. Formally, as argued below, focusing on a strict subset of the set of agents makes the axiomatic restrictions weak and undemanding.

The first axiom is a domain condition capturing some minimal intuitions that represent the core of Marxian exploitation theory and that are shared by all of the main approaches.

Labour Exploitation with Heterogeneous Labour (LEH): *Consider any ES. Given any definition of exploitation, the set of exploited agents $N^{ted} \subseteq \mathcal{N}$ has the following property. There exists a profile $(c_e^1, ..., c_e^{|W_+|})$ such that, for any $\nu \in W_+$, $c_e^\nu \in \mathbb{R}^n_+$, $pc_e^\nu = w\lambda^\nu$, and for some $\alpha^{c_e^\nu} \in \partial P$ with $\widehat{\alpha}^{c_e^\nu} \geqq c_e^\nu$ and $\widehat{\alpha}^{c_e^\nu} \not> c_e^\nu$:*

$$\nu \in N^{ted} \text{ if and only if } w\alpha_l^{c_e^\nu} < w\lambda^\nu.$$

In order to interpret **LEH**, recall that the exploitation status of agent ν is determined by the difference between the amount of labour that ν 'contributes' to the economy, and the amount she 'receives'. As argued in section 3, in the main approaches consistent with the classical economists' view, the former quantity is given by the value of the labour supplied by ν, $w\lambda^\nu$. But there are many possible views concerning the latter quantity. As a domain condition, **LEH** provides some minimal, key restrictions on the definition of the amount of labour that a theoretically relevant subset of agents receives.[25]

To be specific, **LEH** requires that the exploitation status of each propertyless worker $\nu \in W_+$ be determined by identifying a nonnegative vector c_e^ν - call it an *exploitation reference bundle* (hereafter, ERB) - whose labour content - the amount of labour that ν receives - is the labour necessary to produce the ERB efficiently as net output, valued at current wages, $w\alpha_l^{c_e^\nu}$. If

[25] **LEH** only applies to labour-based definitions of exploitation. It is not relevant, for example, for Roemer's [31] property-relations definition or for entitlement-based theories (Ferguson [5]). Related axioms are analysed in Yoshihara and Veneziani [45], Yoshihara [44] and Veneziani and Yoshihara [41].

ν supplies $w\lambda^\nu$, and $w\lambda^\nu$ is more than $w\alpha_l^{c_e^\nu}$, then ν is regarded as contributing more labour than ν receives. According to **LEH**, all such agents should be considered exploited, i.e. members of N^{ted}.

The ERB must have two properties. First, it must be (*just*) *affordable*, at prices p, by a propertyless worker $\nu \in W_+$, who supplies λ^ν units of labour at wages w ($pc_e^\nu = w\lambda^\nu$). This embodies the idea that the amount of labour that $\nu \in W_+$ receives depends on her income, or more precisely, it is determined by some reference bundle that ν can purchase. In standard approaches, the ERB is the bundle actually chosen by the agent. **LEH** is weaker in that it only requires that the ERB be *potentially* chosen.

Second, the ERB must be *technically feasible* with an efficient production process ($\alpha^{c_e^\nu} \in \partial P$ with $\widehat{\alpha}^{c_e^\nu} \geqq c_e^\nu$). This embodies the intuition that the amount of labour received by an agent is related to production conditions. More precisely, **LEH** states that the ERB be technologically feasible as net output, and its labour content is the amount of labour socially necessary to produce it. Observe that the axiom requires that the amount of labour associated with each ERB be uniquely determined with reference to production conditions, but it does not specify how such amount should be chosen. There may be in principle many (efficient) ways of producing the ERB, c_e^ν, and thus of determining its labour content $w\alpha_l^{c_e^\nu}$.[26]

LEH imposes extremely weak and theoretically reasonable restrictions on the appropriate definition of exploitation, and *all* of the main approaches, including Definitions 1-3 above, satisfy it.[27] **LEH** does *not* provide comprehensive conditions for the determination of exploitation status: it only focuses on the strict subset of agents who own no physical assets and supply some labour in return for a wage; it is silent on the exploitation status of all other agents; and it imposes no restrictions on the set of exploiters. More generally, **LEH** does not incorporate any assumptions on individual behaviour or on the structure of economic interactions. Nor is it based upon any (classical or neoclassical) equilibrium notion. It does not capture any causal mechanisms and embodies no assumptions on the nature of individuals or of social reality. It captures the properties of the concept of exploitation in a

[26] We note in passing that **LEH** allows the ERB to be variable and a function of (p, w). Further, once c_e^ν is identified, the existence of $\alpha^{c_e^\nu}$ is guaranteed by assumptions **A0** and **A2** in Appendix A.1.

[27] For a proof of this claim, see the Addendum. It can also be proved that the definition of exploitation proposed in Flaschel [6, 7] - suitably extended to economies with heterogeneous labour - also satisfies **LEH**.

purely a posteriori - rather than predictionist - perspective, by focusing on the data emerging from economic processes at a given point in time.

Because **LEH** is a domain condition that captures some aspects of Marxian exploitation theory shared by all of the main approaches, further restrictions must be imposed in order to discriminate among alternative definitions. A key tenet of Marxian theory is the idea that in capitalist economies profits are closely related to the existence of exploitation. Given private ownership of productive assets, profits should be a counterpart of the transfer of social surplus and social labour from asset-poor agents to wealthy ones, and a general correspondence should exist between positive profits and the exploitation of at least the poorest segments of the working class. This is formalised in the next axiom.

Profit-Exploitation Correspondence Principle (PECP): *Given any ES such that* $W_+ \neq \varnothing$,

$$\left[p\widehat{\alpha}^{p,w} - w\alpha_l^{p,w} > 0 \text{ if and only if } N^{ted} \supseteq W_+\right].$$

In other words, at any ES, aggregate profits are strictly positive if and only if propertyless workers are exploited. This incorporates a key intuition of Marxian exploitation theory at the centre of the debates on the FMT. Yet, **PECP** is both conceptually and formally distinct from the FMT.

Conceptually, the FMT is conceived of as a causal statement: it is meant to prove that profits emerge from (are caused by) the exploitation of workers. Thus in the standard literature it is a *result* that may or may not hold in certain economies under a given definition of exploitation. **PECP** is instead conceived of as a fundamental, definitional property of Marxian exploitation theory. It is a statement about what exploitation is, or what intuitions it incorporates, and so it is formulated without specifying *any* definition: the appropriate definition should be such that propertyless workers are exploited if and only if profits are positive.

Formally, unlike the FMT, **PECP** establishes a connection between the existence of *aggregate* profits and the exploitation status of a *subset* of the set of agents, namely those who have 'nothing to lose but their chains', rather than the aggregate rate of exploitation. Thus, **PECP** allows for the possibility that propertyless workers are a *strict* subset of the set of exploited agents N^{ted}, and when profits are zero it does *not* require that there be no

exploitation in the economy, but only that *some* propertyless agents are not exploited. Moreover, like **LEH**, **PECP** focuses only on propertyless agents who supply *some* wage labour and so it imposes no constraints on the definition of exploitation whenever $W_+ = \varnothing$. This restriction is theoretically appropriate, because the exploitation status of agents who do not engage in any economic activities is unclear.

PECP establishes a rather weak link between exploitation and profits.[28] It is therefore surprising that, in conjunction with **LEH**, it characterises a family of definitions of exploitation.[29]

Theorem 1: *For any definition of exploitation satisfying* **LEH**, *the following two statements are equivalent at any ES*:
(1) **PECP** *holds under this definition*;
(2) *if* $\pi^{\max} > 0$, *then for all* $\nu \in W_+$ *with* $w\alpha_l^{c_e^\nu} > 0$, *there is* $\alpha_\pi^\nu \in \partial P$ *such that* $\widehat{\alpha}_\pi^\nu \in \mathbb{R}_+^n$, $p\widehat{\alpha}_\pi^\nu > w\alpha_{\pi l}^\nu = w\lambda^\nu$ *and* $(\alpha_{\pi l}^\nu, \underline{\alpha}_\pi^\nu, \overline{\alpha}_\pi^\nu) \geqq \eta^\nu \left(\alpha_l^{c_e^\nu}, \underline{\alpha}^{c_e^\nu}, \overline{\alpha}^{c_e^\nu}\right)$ *for some* $\eta^\nu > 1$.

Theorem 1 is mainly a technical result: it provides a condition that can be used to check whether a given definition satisfies **PECP**. As such, it does not identify a unique definition of exploitation that meets **PECP**, but rather a class of definitions satisfying condition (2). Yet it has relevant implications for the main received approaches. For there are economies in which condition (2) never holds, if Definitions 1 and 2 are adopted. In contrast, Definition 3 satisfies condition (2), and thus the **PECP**, in general.[30]

Corollary 1: *There exist ES's such that neither* Definition 1 *nor* Definition 2 *satisfies* **PECP**. *Instead,* Definition 3 *satisfies* **PECP** *at any ES.*

5 Conclusions

This paper explores a novel axiomatic approach to Marxian exploitation theory. Two properties - a domain axiom and the **Profit-Exploitation Correspondence Principle** - are analysed, which incorporate some widely shared intuitions concerning the normative and positive foundations of the concept

[28] Oberve also that **PECP** is silent on the set of exploiters.

[29] The proofs of Theorem 1 and Corollary 1 below are in Appendix A.2.

[30] It can also be proved that if a definition of exploitation satisfies **LEH** and **PECP**, then no agent in W_+ is exploited whenever profits are zero.

of exploitation. Contrary to the received view, a nonempty class of definitions of exploitation is characterised, which preserve the relation between profits and the exploitation of propertyless workers in general economies with a complex class structure, heterogeneous agents, complex technologies with heterogeneous labour inputs, general market structures, and so on. Interestingly, however, among the main approaches, only the 'New Interpretation' is shown to satisfy **PECP** in general. Given the theoretical relevance of **PECP** in Marxian theory, this provides strong support for Definition 3 above.

To be sure, the relation between exploitation and profits is only one - albeit important - aspect of Marxian theory and the results in this paper do not exhaust the analysis of the concept of exploitation. Yet, they do show the potential of the axiomatic method and in closing the paper we briefly mention some lines for further research.

First of all, the concept of exploitation is meant to be a diagnostic of the characteristics of the social structure, both in its power dimension and in its inequality dimension. In this paper, we have focused on the latter. It would be important to extend our analysis to incorporate power, and coercive social relations into the axiomatic framework.

Further, Theorem 1 does not identify a single definition that meets **PECP**, but rather a *class* of definitions. It would be interesting to analyse whether a unique definition can be characterised by imposing further properties, and if so, whether such definition is indeed the 'New Interpretation'.

This is an open question, but two points are worth making that suggest that the key insights of the paper are indeed robust. First, in the standard Okishio-Morishima approach, the existence of exploitation is just a numerical representation of the existence of surplus products. Thus, the FMT establishes the equivalence between positive profits and the productiveness of the economy measured in terms of the labour numéraire. Yet, Bowles and Gintis [2] and Roemer [31] have proved that a similar result holds when productiveness is measured in terms of any other good, thus raising doubts on the significance of the relation between exploitation and profits. Yoshihara and Veneziani [47] have proved that this is not true if the 'New Interpretation' is adopted: no equivalence between profits and exploitation holds if another commodity is used to define exploitation.

Second, in Marxian theory, the social positions identified by the notion of exploitation are *internally related*: 'they are what they are ... by virtue of the relation to other in which they stand' (Lawson [16], p. 17). The existence of an exploiter is inextricably linked to the existence of exploited agents and

'you cannot have the one without the other' (ibid.). Perhaps surprisingly, Yoshihara and Veneziani [45] have proved that the 'New Interpretation' is the only conceivable definition that possesses this relational property.

In summary, this paper provides a general theoretical framework to analyse the notion of exploitation. The results derived shed light on some important (and vexed) issues. More importantly, they show that it is fruitful to take the axiomatic road to exploitation theory, even though there are still a million miles to go.

A Formal analysis

A.1 Assumption on technology

The following assumptions on P hold throughout the paper.

Assumption 0 (A0). P is a closed convex cone in $\mathbb{R}^{2n+T}$ and $\mathbf{0} \in P$.

Assumption 1 (A1). For all $\alpha \in P$, if $\overline{\alpha} \geq \mathbf{0}$ then $\alpha_l \geq \mathbf{0}$ and $\underline{\alpha} \geq \mathbf{0}$.

Assumption 2 (A2). For all $c \in \mathbb{R}^n_+$, there is a $\alpha \in P$ such that $\widehat{\alpha} \geqq c$.

A.2 Proofs

Proof of Theorem 1: Consider any ES. If $W_+ = \varnothing$, the equivalence is immediately established, for both **PECP** and condition (2) are vacuously satisfied. Therefore, in the rest of the proof, suppose that $W_+ \neq \varnothing$.

(2)⇒(1): Suppose that if $\pi^{\max} > 0$, then for each $\nu \in W_+$ with $w\alpha_l^{c_e^\nu} > 0$, there exists $\alpha_\pi^\nu \in \partial P$ such that $p\widehat{\alpha}_\pi^\nu > w\alpha_{\pi l}^\nu = w\lambda^\nu$ and $(\alpha_{\pi l}^\nu, \underline{\alpha}_\pi^\nu, \overline{\alpha}_\pi^\nu) \geqq \eta^\nu \left(\alpha_l^{c_e^\nu}, \underline{\alpha}^{c_e^\nu}, \overline{\alpha}^{c_e^\nu}\right)$ for some $\eta^\nu > 1$.

Let $p\widehat{\alpha}^{p,w} - w\alpha_l^{p,w} = 0$. Then by the definition of ES, $\pi^{\max} = 0$ and condition (2) is vacuously satisfied. By **LEH**, for each $\nu \in W_+$, $c_e^\nu \in \mathbb{R}^n_+$, $pc_e^\nu = w\lambda^\nu > 0$ and $\alpha^{c_e^\nu} \in \partial P$ with $\widehat{\alpha}^{c_e^\nu} \geqq c_e^\nu$. Therefore, noting that $p\widehat{\alpha}^{c_e^\nu} \geqq pc_e^\nu = w\lambda^\nu > 0$, $\pi^{\max} = 0$ implies that $w\alpha_l^{c_e^\nu} \geqq w\lambda^\nu$. Hence, by **LEH**, $\nu \notin N^{ted}$ holds for all $\nu \in W_+$.

Let $p\widehat{\alpha}^{p,w} - w\alpha_l^{p,w} > 0$ so that $\pi^{\max} > 0$. Consider any $\nu \in W_+$. If $w\alpha_l^{c_e^\nu} = 0$, then by definition $w\alpha_l^{c_e^\nu} < w\lambda^\nu$. If $w\alpha_l^{c_e^\nu} > 0$, then by condition (2), $\alpha_\pi^\nu \in \partial P$ with $w\alpha_{\pi l}^\nu = w\lambda^\nu > 0$ and $\eta^\nu > 1$ together imply $w\alpha_l^{c_e^\nu} < w\lambda^\nu$. Thus, by **LEH**, $\nu \in N^{ted}$ holds for any $\nu \in W_+$.

In summary, (2) implies that **PECP** holds under any definition of exploitation satisfying **LEH**.

(1)⇒(2): Suppose that $p\widehat{\alpha}^{p,w} - w\alpha_l^{p,w} > 0 \Leftrightarrow N^{ted} \supseteq W_+$.

Suppose that $\pi^{\max} > 0$. By the definition of ES, $p\widehat{\alpha}^{p,w} - w\alpha_l^{p,w} > 0$ holds, and by **LEH** and **PECP**, for each $\nu \in W_+$, there exist $c_e^\nu \in \mathbb{R}_+^n$ and $\alpha^{c_e^\nu} \in \partial P$ with $\widehat{\alpha}^{c_e^\nu} \geqq c_e^\nu$ and $\widehat{\alpha}^{c_e^\nu} \not> c_e^\nu$ such that $pc_e^\nu = w\lambda^\nu > 0$ and $w\alpha_l^{c_e^\nu} < w\lambda^\nu$. Consider $\nu \in W_+$ such that $w\alpha_l^{c_e^\nu} > 0$. Then let $\eta^\nu \in \mathbb{R}_+$ be such that $\eta^\nu w\alpha_l^{c_e^\nu} = w\lambda^\nu$ and let $\alpha_\pi^\nu \equiv \eta^\nu \alpha^{c_e^\nu}$. Since $\alpha^{c_e^\nu} \in \partial P$ with $\widehat{\alpha}^{c_e^\nu} \geqq c_e^\nu \geqq \mathbf{0}$, then $\widehat{\alpha}_\pi^\nu \in \mathbb{R}_+^n$ and, by **A0**, $\alpha_\pi^\nu \in \partial P$. Moreover, by construction, $(\alpha_{\pi l}^\nu, \underline{\alpha}_\pi^\nu, \overline{\alpha}_\pi^\nu) \geqq \eta^\nu \left(\alpha_l^{c_e^\nu}, \underline{\alpha}^{c_e^\nu}, \overline{\alpha}^{c_e^\nu}\right)$ for some $\eta^\nu > 1$. Finally, $p\widehat{\alpha}_\pi^\nu \geqq \eta^\nu p\widehat{\alpha}^{c_e^\nu} > p\widehat{\alpha}^{c_e^\nu} \geqq pc_e^\nu = w\lambda^\nu = w\alpha_{\pi l}^\nu$ holds.

In summary, if **PECP** holds, then (2) holds under any definition of exploitation satisfying **LEH**. ■

Proof of Corollary 1: For a proof that Definitions 1 and 2 do not satisfy **PECP**, see Veneziani and Yoshihara [41], which proves the result in a subset of the economic environments considered here.

We prove that Definition 3 satisfies condition (2) of Theorem 1. Consider any ES with $W_+ \neq \varnothing$.

Suppose $\pi^{\max} > 0$. By the definition of ES, this implies $p\widehat{\alpha}^{p,w} - w\alpha_l^{p,w} > 0$; and since $w\lambda^\nu > 0$ for all $\nu \in W_+$ then $w\alpha_l^{p,w} \geqq \sum_{\nu \in W_+} w\lambda^\nu > 0$. Then, for all $\nu \in W_+$, let $\alpha_\pi^\nu = \frac{w\lambda^\nu}{w\alpha_l^{p,w}} \alpha^{p,w}$: $\alpha^{p,w} \in \partial P$ holds by condition (iii) of ES and so by **A0**, $\alpha_\pi^\nu \in \partial P$. Moreover $\widehat{\alpha}_\pi^\nu = \frac{w\lambda^\nu}{w\alpha_l^{p,w}} \widehat{\alpha}^{p,w}$, and so $\widehat{\alpha}_\pi^\nu \in \mathbb{R}_+^n$, and since $p\widehat{\alpha}^{p,w} - w\alpha_l^{p,w} > 0$, it follows that $p\widehat{\alpha}_\pi^\nu > w\alpha_{\pi l}^\nu = w\lambda^\nu$.

Finally, under Definition 3, $\alpha^{c_e^\nu} = \tau^{c^\nu} \alpha^{p,w}$ holds, where $\tau^{c^\nu} = \frac{pc^\nu}{p\widehat{\alpha}^{p,w}}$ for all $\nu \in W_+$. Hence, $(\alpha_{\pi l}^\nu, \underline{\alpha}_\pi^\nu, \overline{\alpha}_\pi^\nu) \geqq \eta^\nu \left(\alpha_l^{c_e^\nu}, \underline{\alpha}^{c_e^\nu}, \overline{\alpha}^{c_e^\nu}\right)$ for some $\eta^\nu > 1$ if and only if $\frac{w\lambda^\nu}{w\alpha_l^{p,w}} (\alpha_l^{p,w}, \underline{\alpha}^{p.w}, \overline{\alpha}^{p,w}) \geqq \eta^\nu \frac{pc^\nu}{p\widehat{\alpha}^{p,w}} (\alpha_l^{p,w}, \underline{\alpha}^{p.w}, \overline{\alpha}^{p,w})$ for some $\eta^\nu > 1$, and the latter inequality holds for all $\nu \in W_+$ whenever $p\widehat{\alpha}^{p,w} - w\alpha_l^{p,w} > 0$, since $pc^\nu = w\lambda^\nu$ for all $\nu \in W_+$ by condition (iv) of ES.

In summary, condition (2) of Theorem 1 holds at any ES. ■

References

[1] Bowles, S., Gintis, H., 1977. The Marxian Theory of Value and Heterogeneous Labour: A Critique and Reformulation. *Cambridge Journal of*

Economics 1, 173-192.

[2] Bowles, S., Gintis, H., 1981. Structure and Practice in the Labor Theory of Value. *Review of Radical Political Economics* 12, 1-26.

[3] Duménil, G., 1980. De la Valeur aux Prix de Production. Economica, Paris.

[4] Duménil, G., Foley, D.K., Lévy, D., 2009. A Note on the Formal Treatment of Exploitation in a Model with Heterogeneous Labor. *Metroeconomica* 60, 560-567.

[5] Ferguson, B., 2014. Two Paradoxes of Exploitation. Mimeo, LSE and Bayreuth.

[6] Flaschel, P., 1983. Actual Labor Values in a General Model of Production. *Econometrica* 51, 435-454.

[7] Flaschel, P., 2010. *Topics in Classical Micro- and Macroeconomics.* Springer, New York.

[8] Flaschel, P., Franke, R., Veneziani, R., 2013. Labor Productivity and the Law of Decreasing Labor Content. *Cambridge Journal of Economics* 37, 379-402.

[9] Fleurbaey, M., 1996. *Théories Économiques de la Justice.* Economica, Paris.

[10] Foley, D.K., 1982. The Value of Money, the Value of labour Power, and the Marxian Transformation Problem. *Review of Radical Political Economics* 14, 37-47.

[11] Foster, J.E., 1994. Normative Measurement: Is Theory Relevant? *American Economic Review* 84, 365-370.

[12] Fujimori, Y., 1982. *Modern Analysis of Value Theory.* Springer-Verlag, Berlin.

[13] Katzner, D.W., 2009. *Analysis without Measurement.* Cambridge University Press, Cambridge.

[14] Krause, U., 1982. Heterogeneous Labour and the Fundamental Marxian Theorem. *Review of Economic Studies* 48, 173-178.

[15] Kurz, H.D., Salvadori, N., 1995. *Theory of Production: A long-Period Analysis*. Cambridge University Press, Cambridge.

[16] Lawson, T., 2003. *Reorienting Economics*. Routledge, London.

[17] Lawson, T., 2009. The current economic crisis: its nature and the course of academic economics. *Cambridge Journal of Economics* 33, 759-777.

[18] Lawson, T., 2013. What is this 'school' called neoclassical economics? *Cambridge Journal of Economics* 37, 947-983.

[19] Martins, N., 2006. Capabilities as causal powers. *Cambridge Journal of Economics* 30, 671-685.

[20] Marx, K., 1954 [1867]. *Capital*, Vol.I. London, Lawrence & Wishart.

[21] Marx, K., 1959 [1894]. *Capital*, Vol.III. London, Lawrence & Wishart.

[22] Mohun, S., 2003. On the TSSI and the Exploitation Theory of Profit. *Capital and Class* 81, 85-102.

[23] Mohun, S., 2004. The Labour Theory of Value as Foundation for Empirical Investigations. *Metroeconomica* 55, 65-95.

[24] Morishima, M., 1973. *Marx's Economics*. Cambridge University Press, Cambridge.

[25] Morishima, M., 1974. Marx in the Light of Modern Economic Theory. *Econometrica* 42, 611-632.

[26] Okishio, N., 1957. *Theory of Reproduction* (in Japanese). Sobun Publisher.

[27] Okishio, N., 1963. A mathematical note on Marxian Theorems. *Weltwirtschaftliches Archiv* 91, 287-299.

[28] Okishio, N., 1965. *Fundamental Theory of Capitalist Economy* (in Japanese). Sobun Publisher.

[29] Ricardo, D., 2004 [1819]. *The Principles of Political Economy and Taxation*. Dover Publication.

[30] Roemer, J.E., 1981. *Analytical foundations of Marxian economic theory*. Harvard University Press, Cambridge, MA.

[31] Roemer, J.E., 1982. *A General Theory of Exploitation and Class*. Harvard University Press, Cambridge, MA.

[32] Sen, A.K., 1978. On the labour theory of value: some methodological issues. *Cambridge Journal of Economics* 2, 175-190.

[33] Sen, A.K., 1985a. *Commodities and Capabilities*. North-Holland, Amsterdam.

[34] Sen, A.K., 1999. The Possibility of Social Choice. *American Economic Review* 89, 349-378.

[35] Skillman, G.L., 1995. *Ne Hic Saltaveris*: the Marxian Theory of Exploitation after Roemer. *Economics and Philosophy* 11, 309-331.

[36] Smith, A., 2000 [1776]. *The Wealth of Nations*. Modern Library, New York.

[37] Steedman, I., 1977. *Marx after Sraffa*. New Left Books, London.

[38] Thomson, W., 2001. On the axiomatic method and its recent applications to game theory and resource allocation. *Social Choice and Welfare* 18, 327-386.

[39] Veneziani, R., 2007. Exploitation and Time. *Journal of Economic Theory* 132, 189-207.

[40] Veneziani, R., 2013. Exploitation, Inequality, and Power. *Journal of Theoretical Politics* 25, 526-545.

[41] Veneziani, R., Yoshihara, N., 2014. Exploitation in economies with heterogeneous preferences, skills and assets: An axiomatic approach. *Journal of Theoretical Politics*, forthcoming.

[42] Vrousalis, N., 2013. Exploitation, Vulnerability, and Social Domination. *Philosophy & Public Affairs* 41, 131-157.

[43] Wright, E. O., 2000. Class, Exploitation, and Economic Rents: Reflections on Sorensen's "Sounder Basis". *The American Journal of Sociology* 105, 1559-71.

[44] Yoshihara, N., 2010. Class and Exploitation in General Convex Cone Economies. *Journal of Economic Behavior & Organization* 75, 281-296.

[45] Yoshihara, N., Veneziani, R., 2009. Exploitation as the Unequal Exchange of Labour: An Axiomatic Approach. WP N.655, Queen Mary University of London.

[46] Yoshihara, N., Veneziani, R., 2012. Profits and Exploitation: A Reappraisal. *Advances in Mathematical Economics* 16, 85-109.

[47] Yoshihara, N., Veneziani, R., 2013. Exploitation of Labour and Exploitation of Commodities: a 'New Interpretation'. *Review of Radical Political Economics* 45, 517-524.

[48] Yoshihara, N., Veneziani, R., 2014. The Measurement of Labour Content: A General Approach. DP 2013-5, University of Massachusetts, Amherst.

1 Proof that Definitions 1, 2 and 3 satisfy LEH

For Definition 1, for all $\nu \in W_+$, $c_e^\nu \equiv c^\nu$ and $\alpha^{c_e^\nu} \in \arg\min\{w\alpha_l \mid \alpha \in P \;\&\; \widehat{\alpha} \geqq c_e^\nu\}$, so that $w\alpha_l^{c_e^\nu} = l.v.(c_e^\nu; w)$.

For Definition 2, for all $\nu \in W_+$, $c_e^\nu \equiv c^\nu$ and $\alpha^{c_e^\nu} \in \arg\min\{w\alpha_l \mid \alpha \in P^\pi(p,w) \;\&\; \widehat{\alpha} \geqq c_e^\nu\}$, so that $w\alpha_l^{c_e^\nu} = l.v.(c_e^\nu; p, w)$.

For Definition 3, for all $\nu \in W_+$, let $\tau^{c^\nu} = \frac{pc^\nu}{p\widehat{\alpha}^{p,w}}$, if $p\widehat{\alpha}^{p,w} > 0$ and $\tau^{c^\nu} = 0$, otherwise. Then, $c_e^\nu \equiv \tau^{c^\nu}\widehat{\alpha}^{p,w}$ and $\alpha^{c_e^\nu} \equiv \tau^{c^\nu}\alpha^{p,w}$, so that $w\alpha_l^{c_e^\nu} = \tau^{c^\nu} w\alpha_l^{p,w}$.

2 An example of an economic state

Consider an economic environment as described in the paper. In the economy, agents produce, consume, and trade labour. On the production side, they can either sell their labour-power, or hire others to work on their capital, or they can be self-employed and work on their own assets. Then, $\alpha^\nu = (-\alpha_l^\nu, -\underline{\alpha}^\nu, \overline{\alpha}^\nu) \in P$ is the production process operated by ν as a self-employed producer, with her own capital; $\beta^\nu = \left(-\beta_l^\nu, -\underline{\beta}^\nu, \overline{\beta}^\nu\right) \in P$ is the production process that ν operates by hiring labour β_l^ν; γ^ν is ν's vector of labour supply. Thus, let $\lambda_\tau^\nu = \alpha_{\tau l}^\nu + \gamma_\tau^\nu$ be the total amount of labour time of each type $\tau \in \mathcal{T}^\nu$ of labour expended by ν.

Agent ν's choices can be represented by a function $u^\nu : C \to \mathbb{R}_+$, which is strictly increasing in consumption commodities and weakly decreasing in each type of labour. For the sake of simplicity, and with no loss of generality, the consumption space is assumed to be sufficiently large for consumption goods.

Given (p, w), each agent ν is assumed to choose a plan $(\alpha^\nu, \beta^\nu, \gamma^\nu, c^\nu)$ to maximise her welfare subject to the constraint that (1) net income is sufficient for consumption plans; (2) wealth is sufficient to purchase the inputs necessary for production plans; (3) production plans are technically feasible; and (4) the consumption and labour vector are in the feasible set. Formally, each agent ν solves:

$$MP^\nu : \max_{(\alpha^\nu, \beta^\nu, \gamma^\nu, c^\nu)} u^\nu(c^\nu, \lambda^\nu)$$

subject to

$$\begin{aligned}
[p(\overline{\alpha}^\nu - \underline{\alpha}^\nu)] + \left[p\left(\overline{\beta}^\nu - \underline{\beta}^\nu\right) - w\beta_l^\nu\right] + [w\gamma^\nu] &\geqq pc^\nu, && (1)\\
p\left(\underline{\alpha}^\nu + \underline{\beta}^\nu\right) &\leqq p\omega^\nu, && (2)\\
\alpha^\nu, \beta^\nu &\in P, && (3)\\
(c^\nu, \lambda^\nu) &\in C. && (4)
\end{aligned}$$

Let $E\left\langle P, \mathcal{N}, \mathcal{T}, C, (u^\nu)_{\nu\in\mathcal{N}}, (\mathcal{T}^\nu)_{\nu\in\mathcal{N}}, (\omega^\nu)_{\nu\in\mathcal{N}}\right\rangle$, or as a shorthand notation E, denote the economy with technology P, agents $\mathcal{N}$, types of heterogeneous labour $\mathcal{T}$, consumption space C, utility functions $(u^\nu)_{\nu\in\mathcal{N}}$, labour endowments $(\mathcal{T}^\nu)_{\nu\in\mathcal{N}}$, and commodity endowments $(\omega^\nu)_{\nu\in\mathcal{N}}$. Let the set of all such economies be denoted by $\mathcal{E}$. Note that this is a subset of the set of economic environments analysed in the paper.

Following Roemer (1982), the equilibrium concept can be defined.

Definition 1: A *reproducible solution* (RS) for $E \in \mathcal{E}$ is a price vector (p, w) and an associated profile of actions $(\alpha^\nu, \beta^\nu, \gamma^\nu, c^\nu)_{\nu\in\mathcal{N}}$ such that:

(i) $(\alpha^\nu, \beta^\nu, \gamma^\nu, c^\nu)$ solves MP^ν for all ν (optimality);
(ii) $\sum_{\nu\in\mathcal{N}} \left(\widehat{\alpha}^\nu + \widehat{\beta}^\nu\right) \geqq \sum_{\nu\in\mathcal{N}} c^\nu$ (reproducibility);
(iii) $\sum_{\nu\in\mathcal{N}} \left(\underline{\alpha}^\nu + \underline{\beta}^\nu\right) \leqq \sum_{\nu\in\mathcal{N}} \omega^\nu$ (feasibility);
(iv) $\sum_{\nu\in\mathcal{N}} \beta_l^\nu = \sum_{\nu\in\mathcal{N}} \gamma^\nu$ (labour market equilibrium).

In other words, at a reproducible solution (i) every agent optimises; (iii) there are enough resources for production plans in aggregate; and (iv) the labour market clears. Condition (ii) states that aggregate net outputs should at least suffice for aggregate consumption. This is a reproducibility condition because it is equivalent to requiring that the vector of social endowments does not decrease component-wise. For (ii) is equivalent to $\sum_{\nu\in\mathcal{N}} \left[\omega^\nu - \left(\underline{\alpha}^\nu + \underline{\beta}^\nu\right) + \left(\overline{\alpha}^\nu + \overline{\beta}^\nu - c^\nu\right)\right] \geqq \sum_{\nu\in\mathcal{N}} \omega^\nu$, which states that aggregate stocks at the beginning of next period should not be smaller than aggregate stocks at the beginning of the current period.

By the assumptions on u^ν, it immediately follows that all prices must be strictly positive and wages must be nonnegative, at any non-trivial reproducible solution - i.e. at any equilibrium where *some* production process is activated. The existence of a RS in such economies can be shown as in the standard proofs of competitive equilibrium developed in the general equilibrium theory.

Recall that $\pi^{\max} = \max_{\alpha\in P} \frac{p\widehat{\alpha} - w\alpha_l}{p\underline{\alpha}}$ and $P^\pi(p,w) = \left\{\alpha \in P \mid \pi^{\max} = \frac{p\widehat{\alpha} - w\alpha_l}{p\underline{\alpha}}\right\}$. It is straightforward to prove that any RS where at least some production process is activated has the following properties.

Lemma 1: *Let (p,w) be a RS for $E \in \mathcal{E}$ such that $\sum_{\nu\in\mathcal{N}} c^\nu \geq \mathbf{0}$. Then, (i) $p\widehat{\alpha}^{p,w} - w\alpha_l^{p,w} \geqq 0$ for some $\alpha^{p,w} \in P\setminus\{\mathbf{0}\}$ with $\widehat{\alpha}^{p,w} \geq \sum_{\nu\in\mathcal{N}} c^\nu$ and $\alpha_l^{p,w} = \sum_{\nu\in\mathcal{N}} \lambda^\nu$, (ii) $\alpha^\nu + \beta^\nu \in P^\pi(p,w)$ for all $\nu \in \mathcal{N}$ with $p\omega^\nu > 0$, and (iii) $pc^\nu = \pi^{\max} p\left(\underline{\alpha}^\nu + \underline{\beta}^\nu\right) + w\lambda^\nu$ for all $\nu \in \mathcal{N}$.*

Therefore, any economy $E \in \mathcal{E}$ and any associated RS represent an economic state as defined in the paper.