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PRICE DISCRIMINATION THROUGH GROUP BUYING*

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Abstract

This paper argues that when consumers are heterogeneous in group-buying costs, a monopolist seller may practice price discrimination through inducing certain consumers to participate in group buying. In contrast to the standard model, the optimal quantity/quality level for low valuation consumers without group buying is further distorted downward, whereas the levels for other consumers are socially optimal. Inducing group buying is more favorable when the proportion of high valuation consumers is higher, or the valuation differential is larger. We also discuss two extensions: one allowing for consumers' arbitrage behavior and the other one allowing for more potential group buying consumers.

Keywords: group buying; nonlinear pricing; price discrimination
JEL Classification Codes: D42, D82, L11, L12

I. *Introduction*

In recent years, various types of group buying have become a widespread mechanism in business-to-consumer transactions.[1] Particularly, a new type of buyer-initiated group buying,

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[1] For example, for the traditional web-based group buying sites that first emerged in the US in the late 1990s, operators of these websites function as the intermediaries between buyers and sellers, and they seek to aggregate distinct buyers by announcing a price schedule conditional on the total number of customer orders. Some well-known group-buying websites of this type are Mercata.com, Mobshop.com and LetsBuyIt.com; see Anand and Aron (2003),

called *team purchase* or *tuangou*, is prevalent in China and other Asian countries.[2] Team purchases are generally initiated by a number of buyers who have the same intent to buy a certain product. To this end, buyers may solicit more other buyers to joint them via online forums, blogs, or word of mouth, and then they form a team to make purchases (Tang, 2008; Wang et al., 2011). Team purchases are different from other types of group buying in that only specific buyers, such as the members of certain online forums, are eligible for the discount price, whereas other buyers are unable to form a team due to higher search costs or transaction costs. While team purchases seem detrimental to sellers, one may observe that some sellers choose to actively react to such group-buying behavior by offering additional group-buying options rather than preventing consumers from making group purchases. For example, GOME, one of the major electronics retailers in China, used to launch a 'tuangou day project', according to which some selected stores are only open to group-buying consumers on certain dates (Yen and Huang, 2009).

The aforementioned phenomenon raises a natural question about sellers' rationale for providing consumers with a group-guying option. More specifically, this paper intends to inquire the following questions: when consumers differ in their abilities to engage in group buying, how does a seller react to such consumers' group-buying behavior? What is the seller's optimal pricing strategy given this consumers' group-buying behavior? Is such consumer's group-buying behavior detrimental or favorable to the seller? To address and answer the aforementioned questions, we construct a stylized model that incorporates the features of buyer-initiated group buying.

Our model considers that consumers are able to make an individual purchase or self-organize a group purchase at the expense of paying a group-buying cost, and they are heterogeneous along two dimensions: valuations for the good and group-buying costs.[3] For tractability, we consider a binary setting where consumers' type in each dimension takes two possible values. From this perspective, we enrich the standard model of second-degree price discrimination in a two-type setting by incorporating the heterogeneity in consumers' group-buying costs. Notably, the optimal price schedule can be replicated by a menu of bundles (quantity-price or quality-price combinations) intended for each type of consumers, and consumers choose between packages with different quantities of the same good or between goods with different qualities.

We start with a scenario wherein only low-valuation consumers may engage in group buying. In this scenario, we investigate the impact of group buying on the design of price schedule and on the seller's profit. In contrast to the canonical nonlinear-pricing model in a two-type setting, we demonstrate that the optimal quantity/quality level intended for low-valuation consumers with a high group-buying cost is further distorted downward, whereas the quantity/quality levels intended for other consumers are socially optimal. We also show that all low-valuation consumers obtain zero surplus, whereas high-valuation consumers obtain a

and Tang (2008). Although this new business model is innovative, most group-buying sites failed their services in recent years; see Flynn (2001) and Tang (2008). Another recent example is the Groupon-type group buying, wherein numerous third-party operators offer in-advance sales on certain items for a short time period, and each consumer only needs to decide whether to accept the offer.

[2]Tuangou is the direct translation of team purchases from Chinese; see Tang (2008) for more details.

[3] This group-buying cost may be justified by the fact that although most buyer-initiated group buying is organized online, no intermediaries are involved in the process of transaction; see Yen and Huang (2009).

positive surplus. However, this surplus of high-valuation consumers is lower than its counterpart in the optimal menu without group buying.

Because a fraction of low-valuation consumers can choose to participate in group buying, the presence of these group-buying consumers alleviates the tension between efficiency consideration and rent extraction. Hence, the seller raises the quantity/quality level intended for consumers with a low group-buying cost. This induces them to participate in group buying without intensifying the incentive problem of high-valuation consumers. Moreover, the seller distorts the quantity/quality level intended for low-valuation consumers with a high group-buying cost, thereby reducing the information rent of high-valuation consumers. We show that inducing group buying is more favorable when the proportion of high-valuation consumers is higher, the valuation differential is larger, or the group-buying cost is lower.

We further show that while inducing group buying is not always profitable, the seller can always gain a larger profit from consumers with a high group-buying cost. This is because the presence of group-buying (low-cost) consumers allows the seller to design a menu of bundles specifically intended for high-cost consumers, thereby implementing more sophisticated segmentation between high-valuation and low-valuation consumers. Particularly, when the cost to participate in group buying is lower than the surplus distortion caused by information asymmetry, inducing group buying always ameliorates the seller's profitability.

We also discuss two variants of the model. First, we consider a scenario wherein consumers have access to a more general form of group buying that allows for consumers' arbitrage behavior. In this scenario, because consumers are allowed to buy any fraction of any bundles, such consumers' arbitrage behavior certainly impairs the seller's ability to discriminate between various buyers, and it leads to a detrimental effect on the seller's profit. Although a complete characterization of analytical results is intractable, we use an example to graphically and numerically illustrate that group buying may still be favorable to the seller. Second, we consider another scenario that allows all more potential consumers to engage in group buying. Similarly, we argue that the profit-enhancing result can be generalized to this more general framework.

In the context of this paper, group buying leads to several conflicting forces for the seller's profitability. On the one hand, the heterogeneity in group-buying costs allows the seller to distinguish between consumers with various costs through their selections of individual purchases or group purchases; hence, group buying improves the seller's ability of price discrimination, and it is favorable to the seller. On the other hand, group-buying costs aggravate consumers' participation constraints and lower consumers' willingness to pay; consumers' arbitrage behavior in the form of group buying limits the seller's ability to price discriminate between consumers with various valuations. Therefore, group buying may be detrimental to the seller.

This paper accentuates the interplay between the conflicting forces caused by group buying. The stylized results of this paper may justify the seller's reaction to consumers' group-buying behavior, and provide a rationale for providing consumers with group-buying options. While a number of existing studies have offered justifications for the impact of group buying on sellers' profitability from various viewpoints,[4] we complement their views by providing a different rationale as well as analyzing both the beneficial and detrimental impact caused by

[4] For example, see Anand and Aron (2003), Chen and Zhang (2012), Edelman et al. (2011), and Jing and Xie (2011).

group buying.

The remainder of this paper is organized as follows. Section II reviews the related literature. Section III delineates the model that features consumers' heterogeneity in valuations and group-buying costs. Section IV characterizes the seller's optimal menu of bundles, and compares the seller's profits. Section V discusses other variants of the model. Finally, concluding remarks are presented in Section VI, and proofs of the propositions are relegated to Appendix.

II. *Related Literature*

This paper is related to a stream of growing literature on group buying. A number of papers have argued that group buying may serve as an advantageous marketing strategy from the seller's perspective. Anand and Aron (2003) investigate the performance of group-buying pricing schedule under demand uncertainty, and they demonstrate that group-buying pricing may outperform the conventional posted-price mechanism. Jing and Xie (2011) model group buying as a selling mechanism to motivate information sharing among informed and less-informed consumers. They show that group-buying mechanisms may be more profitable than other selling strategies such as individual-selling strategies and referred rewards programs. Edelman et al. (2011) study the impact of online discount vouchers on seller's profitability, and they model two channels through which a discount voucher may benefit the seller: price discrimination and advertising.[5] Chen and Zhang (2012) study the impact of group-buying discounts[6] on a monopolist seller's profitability when the seller faces demand uncertainty. They characterize the sufficient conditions under which group buying dominates separate selling. While we also study the impact of group buying on the monopolist seller's profitability, market segmentation of consumers with various group-buying costs is the driving force that boosts the seller's profit.

This paper is also relevant to a number of papers that investigate the impact of consumers' arbitrage on a monopolist's pricing schedule. Building on the standard non-linear pricing model in a two-type setting, Alger (1999) characterizes the optimal pricing schedule when consumers have access to partial arbitrage in the form of multiple and/or joint purchases. With consumers' joint purchases, she confirms that the optimal pricing schedule exhibits a clear pattern without quantity discounts and demonstrates that consumers' arbitrage is always detrimental to the monopolist seller. McManus (2001) considers the optimal two-part pricing when consumers may collude in making purchases. He shows that a monopolist seller may benefit from the cooperation between consumers with various demand intensities because the seller can set a higher fixed fee equal to the sum of consumers' surpluses. Jeon and Menicucci (2005) study the optimal second-degree price discrimination when various types of buyers may form coalitions in an asymmetric-information environment.[7] They characterize the optimal selling mechanism

[5] Although they don't model online discount vouchers as a quantity discount offered for group buying, price discrimination and advertising can be interpreted as the driving forces that enhance the profitability of group-buying promotions.

[6] This selling practice is termed "interpersonal bundling" by the authors.

[7] This setting is in contrast to Alger (1999), who considers that consumer coalitions are formed under complete information about each other's type, and only consumers with the same type can form coalitions.

and show that a seller may prevent arbitrage among buyers and achieve the same profit as in the absence of buyer coalitions. In this paper, we investigate the impact of group-buying behavior on a monopolist's pricing schedule given that consumers are heterogeneous in both valuations and group-buying costs. We accentuate the interplay between the conflicting forces caused by group buying, based on which the presence of group buying may actually lead to a higher or lower seller's profit.

Another stream of literature has investigated the impact of buyer groups on competition among oligopolist sellers. Marvel and Yang (2008) show that the formation of buyer groups enables sellers to compete by means of nonlinear tariffs, and this may intensify price competition among sellers and lower all sellers' profit. Dana Jr (2012) argues that buyer groups may leverage the commitment to purchase exclusively from one seller to amplify the intensity of competition among rival sellers. Following this argument, Chen and Li (2013) further elaborate on the conditions under which an exclusive purchase commitment is optimal for buyer groups, and they argue this commitment is more likely to be favorable to buyer groups when sellers are more differentiated. Collectively, in this line of research the introduction of buyer groups gives rise to fierce competition among oligopoly sellers, and therefore it is always detrimental to sellers. By contrast, we consider a monopoly model with the aim of claiming that the formation of buyer groups may lead to a higher profit for a monopolist seller.

III. *Model*

A monopolist seller sells a good produced at a constant marginal cost $c>0$ to a continuum of consumers whose preferences are unobservable to the seller. Consumers have heterogeneous tastes characterized by a taste parameter θ. A type θ consumer derives net surplus

$$u(\theta,q,t)=\theta v(q)-t$$

from buying a quantity-price bundle (q,t), where q,t stand for the quantity purchased and the price paid to the seller respectively. The function $v(.)$ is a twice continuously differentiable and strictly concave function with $v(0)=0, v'>0$, and $v''<0$. The taste parameter θ, which is privately known to consumers only, takes two possible values: $\underline{\theta}$ and $\bar{\theta}$, where $\underline{\theta}<\bar{\theta}$. Let λ $(1-\lambda)$ denote the prior probability that a consumer is of type $\underline{\theta}$ $(\bar{\theta})$. Consumers of type $\bar{\theta}$ and $\underline{\theta}$ are referred to as high-valuation and low-valuation consumers respectively. The total population of consumers is normalized to one.

Consumers' group-buying behavior. In contrast to the traditional literature on nonlinear pricing, we consider that consumers have access to a group-buying option. This option allows consumers to self-organize a group purchase at the expense of paying an additional group-buying cost. Following Alger (1999), we assume that only consumers of the same valuation can make group purchases.[8]

[8] However, the group-buying option by Alger (1999) is more general: any number of same-type consumers may make a group purchase, and each consumer can consume any fraction of any bundle the seller offers. In contrast, we consider that a fixed number of same-type consumers have an option to buy a certain bundle jointly. We shall consider a more general form of group buying in Section V. We show that even though consumers have access to a more

Consumers are heterogeneous in their group-buying costs, which are privately known to consumers only. The group-buying cost takes two possible values: a low cost a_l and a high cost a_h, where a_h is sufficiently high such that only consumers with a low group-buying cost are able to engage in group buying. Low-valuation and high-valuation consumers incur a low group-buying cost a_l with a probability α and β respectively. Practically, the group-buying cost can possibly appear in form of transaction costs or search costs.

Quality discrimination. The aforementioned quantity-discrimination model can be transformed into a quality-discrimination one by simply relabeling variables.[9] Hence, a bundle (q,t) offered by a seller can be interpreted as either a quantity-price or quality-price package. With regard to group buying, both quantity and quality interpretations are relevant. For quantity interpretations, we may think that a group of n same-type consumers purchase a bundle jointly, and each consumer pays a price t/n for q/n units of the good.

For quality interpretations, in contrast, we may think that a group of n same-type consumers purchase a bundle jointly, and each consumer pays a price t/n for a fractional $1/n$ ownership of the good with quality q. Notably, this interpretation applies to the goods that are not consumed at all times. To exemplify this thought, imagine that consumers living in the same neighborhood can choose to share a good jointly, such as household hardware (lawn mowers, snow-plows, etc.) and sports facilities (swimming pools, tennis courts, gymnasiums, etc.); alternatively, consider that consumers can buy a vacation timeshare to obtain a fractional right to use a vacation property for a certain time interval.

While our model is relevant for both quantity and quality interpretation, the following analysis is limited to the quantity interpretation for the sake of brevity. Nonetheless, all the main results also apply to the quality interpretation.

IV. *Analysis*

In this section, we start with a benchmark without group buying, and then we proceed to characterize the optimal menu of bundles when low-valuation consumers are able to engage in group buying. We aim to characterize the optimal menu of quantity-price bundles intended for various types of consumers, and examine the conditions under which inducing group buying is favorable to the seller.

1. The Benchmark

The benchmark is a special case wherein all consumers incur a high group-buying cost,

efficient arbitrage technology, the monopolist seller may still benefit from consumers' group-buying behavior.

[9] The transformation of the quantity model into the quality model is shown in section 3.3.3 in Tirole (1988). Let consumers have preferences $u(\theta,s,p)=\theta s-p$, where s stands for the quality of the good and p is the price paid. The cost of producing one unit of the good with quality s, denoted by $c(s)$, is increasing and convex. Let $q\equiv c(s)$ denote the cost of quality s, and $s=v(q)\equiv c^{-1}(q)$ is the quality obtained for cost q, which is an increasing and concave function. Thus, the consumer's preference can be expressed as:

$$u(\theta,s,p)=\theta s-p=\theta v(q)-p(v(q))=\theta v(q)-t,$$

which is equivalent to the setting in the quantity model.

i.e., $\alpha = \beta = 0$, and therefore group buying is restrained. Note that this benchmark is identical to the standard model of second-degree price discrimination in a two-type setting. While this is a well-known model in textbooks,[10] what follows will briefly summarize the main properties of this benchmark in order to accentuate the features of our results.

Typically, a monopolist seller's problem is to look for the optimal price schedule that maximizes his expected profit. By the revelation principle, we can confine our search for the optimal price schedule to a menu of bundles (quantity-price packages), $\{(\bar{q},\bar{t}),(\underline{q},\underline{t})\}$, such that in equilibrium high-valuation consumers choose the bundle $(\bar{q},\bar{t})$, and low-valuation consumers choose the other bundle $(\underline{q},\underline{t})$. Hence, the seller's expected profit is expressed as follows:

$$\lambda(\underline{t} - c\underline{q}) + (1-\lambda)(\bar{t} - c\bar{q}).$$

The relevant incentive-compatibility (IC) and individual rationality (IR) constraints are given by:

$$\text{(IC) } \bar{\theta}v(\bar{q}) - \bar{t} \geq \bar{\theta}v(\underline{q}) - \underline{t}, \underline{\theta}v(\underline{q}) - \underline{t} \geq \underline{\theta}v(\bar{q}) - \bar{t},$$

$$\text{(IR) } \bar{\theta}v(\bar{q}) - \bar{t} \geq 0, \underline{\theta}v(\underline{q}) - \underline{t} \geq 0.$$

For ease of exposition, let $\bar{q}^*$ and $\underline{q}^*$ denote the first-best quantity level for high-valuation and low-valuation consumers respectively; that is, $\bar{\theta}v'(\bar{q}^*) = \underline{\theta}v'(\underline{q}^*) = c$. With regard to the menu of optimal bundles $\{(\bar{q}^b,\bar{t}^b),(\underline{q}^b,\underline{t}^b)\}$ in this benchmark, what follows summarizes the salient features documented in the literature. First, high-valuation consumers obtain an efficient quantity level, so the quantity level intended for high-valuation consumers is $\bar{q}^b = \bar{q}^*$. Second, the quantity level intended for low-valuation consumers, $\underline{q}^b$, is defined by the following expression:

$$\left[\underline{\theta} - \frac{1-\lambda}{\lambda}(\bar{\theta} - \underline{\theta})\right] v'(\underline{q}^b) = c,$$

and the concavity of the consumer's utility function $v(q)$ immediately implies $\underline{q}^b < \underline{q}^*$. Finally, the incentive constraint for high-valuation consumers and the participation constraint for low-valuation consumers are binding at the optimum. Accordingly, the prices $\underline{t}^b$ and $\bar{t}^b$ are given by the following expressions

$$\underline{t}^b = \underline{\theta}v(\underline{q}^b), \bar{t}^b = \bar{\theta}v(\bar{q}^*) - (\bar{\theta} - \underline{\theta})v(\underline{q}^b).$$

This indicates that low-valuation consumers obtain zero surplus, whereas high-valuation consumers obtain a positive surplus equal to $(\bar{\theta} - \underline{\theta})v(\underline{q}^b)$.

As previously mentioned, the above properties of optimal bundles can be directly

[10] See, for example, chapter 2 in Laffont and Martimort (2009), chapter 2 in Salanié (1997), or chapter 3 in Tirole (1988).

transformed into quality interpretations. In this case, the monopolist seller discriminates between consumers with various quality tastes by offering a menu of quality-price bundles, and the main features of the optimal bundles are alike: high-valuation consumers choose the socially optimal quality and obtain a positive surplus, whereas low-valuation consumers choose a suboptimal quality and obtain zero surplus.

2. When Only Low-valuation Consumers May Engage in Group Buying

We now consider that low-valuation consumers incur a low grouping-buying cost with a probability $\alpha > 0$, whereas high-valuation consumers all incur a high grouping-buying cost, i.e., $\beta = 0$. While this is a restrictive case of the general model, it allows us to crystallize the primary message of this paper: consumers' group buying behavior may boost the seller's profit. A justification for this assumption is as follows: as suggested by Alger (1999), high-valuation consumers usually obtain higher income than low-valuation consumers, and they incur higher opportunity costs of spending time in group-buying activities than low-valuation consumers; hence, high-valuation consumers are less involved in the discussion on online forums, blogs, etc., and less likely to self-organize group buying.[11] In section V, we shall relax this assumption ($\beta = 0$) and discuss the generalization of the results obtained in this section.

Because only low-valuation consumers may engage in group buying in this context, the group-buying option is relevant to the design of price schedule only if the seller targets all consumers. Hence, to avoid trivial scenarios, we impose the following assumption that ensures it is never optimal for the seller to abandon certain consumers: $\left[\underline{\theta} - \frac{(1-\lambda)}{\lambda(1-\alpha)}(\bar{\theta} - \underline{\theta})\right] v'(0) > c$.[12] In this context, the seller faces three various consumer types: (i) high-valuation, high-cost consumers (referred to as type H consumers) (ii) low-valuation, high-cost consumers (type LH consumers), and (iii) low-valuation, low-cost consumers (type LL consumers). When the seller intends to induce group buying, by the revelation principle we can confine the search for the optimal price schedule to a menu of bundles $\{(\bar{q}, \bar{t}), (\underline{q}, \underline{t}), (n\tilde{q}, n\tilde{t})\}$ that induces consumers to truthfully reveal their types. Hence, a feasible menu must satisfy three sets of incentive-compatibility constraints as follows.

First, the incentive-compatibility constraints for type H consumers (IC_h) are given by

$$\bar{\theta} v(\bar{q}) - \bar{t} \geq \bar{\theta} v(\underline{q}) - \underline{t}, \tag{IC_{hl}}$$

$$\bar{\theta} v(\bar{q}) - \bar{t} \geq \bar{\theta} v(n\tilde{q}) - n\tilde{t}, \tag{IC_{hn}}$$

$$\bar{\theta} v(\bar{q}) - \bar{t} \geq \bar{\theta} v(\tilde{q}) - \tilde{t} - a_h, \tag{IC_{hg}}$$

$$\bar{\theta} v(\bar{q}) - \bar{t} \geq \bar{\theta} v\left(\frac{\bar{q}}{n}\right) - \frac{\bar{t}}{n} - a_h, \tag{$IC_{hh'}$}$$

[11] We would like to thank an anonymous referee for providing this justification.

[12] Alternatively, we may assume the proportion of low-valuation consumers is sufficiently large such that $\lambda > (\bar{\theta} - \underline{\theta})/(\bar{\theta} - \alpha\underline{\theta})$ and $v'(0)$ is sufficiently greater than zero.

$$\bar{\theta}v(\bar{q})-\bar{t}\geq\bar{\theta}v(\frac{\underline{q}}{n})-\frac{\underline{t}}{n}-a_h. \qquad (IC_{hl'})$$

The inequalities (IC_{hl}), (IC_{hn}) ensure that type H consumers prefer the bundle $(\bar{q},\bar{t})$ to other bundles: $(\underline{q},\underline{t})$ and $(n\tilde{q},n\tilde{t})$. The inequalities (IC_{hg}), $(IC_{hh'})$, and $(IC_{hl'})$ guarantee that type H consumers will not participate in group buying.

Second, the incentive-compatibility constraints for type LH consumers are formulated similarly as follows:

$$\underline{\theta}v(\underline{q})-\underline{t}\geq\underline{\theta}v(\bar{q})-\bar{t}, \qquad (IC_{lh})$$

$$\underline{\theta}v(\underline{q})-\underline{t}\geq\underline{\theta}v(n\tilde{q})-n\tilde{t}, \qquad (IC_{ln})$$

$$\underline{\theta}v(\underline{q})-\underline{t}\geq\underline{\theta}v(\tilde{q})-\tilde{t}-a_h, \qquad (IC_{lg})$$

$$\underline{\theta}v(\underline{q})-\underline{t}\geq\underline{\theta}v(\frac{\bar{q}}{n})-\frac{\bar{t}}{n}-a_h, \qquad (IC_{lh'})$$

$$\underline{\theta}v(\underline{q})-\underline{t}\geq\underline{\theta}v(\frac{\underline{q}}{n})-\frac{\underline{t}}{n}-a_h. \qquad (IC_{ll'})$$

Finally, the incentive compatibility constraints for type LL consumers (IC_g) are given by

$$\underline{\theta}v(\tilde{q})-\tilde{t}-a_l\geq\underline{\theta}v(\bar{q})-\bar{t}, \qquad (IC_{gh})$$

$$\underline{\theta}v(\tilde{q})-\tilde{t}-a_l\geq\underline{\theta}v(\underline{q})-\underline{t}, \qquad (IC_{gl})$$

$$\underline{\theta}v(\tilde{q})-\tilde{t}-a_l\geq\underline{\theta}v(n\tilde{q})-n\tilde{t}, \qquad (IC_{gn})$$

$$\underline{\theta}v(\tilde{q})-\tilde{t}\geq\underline{\theta}v(\frac{\bar{q}}{n})-\frac{\bar{t}}{n}, \qquad (IC_{gh'})$$

$$\underline{\theta}v(\tilde{q})-\tilde{t}\geq\underline{\theta}v(\frac{\underline{q}}{n})-\frac{\underline{t}}{n}, \qquad (IC_{gl'})$$

where (IC_{gh}), (IC_{gl}), and (IC_{gn}) ensure type LL consumers prefer buying the bundle $(n\tilde{q},n\tilde{t})$ jointly rather than buying any other bundle individually, and $(IC_{gh'})$, $(IC_{gl'})$ prevent these consumers from buying other bundles jointly.

Furthermore, the individual rationality constraints (IR) that ensure each consumer type is willing to make a purchase are given by (IR_h), (IR_l), and (IR_g) as follows:

$$\bar{\theta}v(\bar{q})-\bar{t}\geq 0, \qquad (IR_h)$$

$$\underline{\theta}v(\underline{q})-\underline{t}\geq 0, \qquad (IR_l)$$

$$\underline{\theta}v(\tilde{q})-\tilde{t}-a_l\geq 0. \qquad (IR_g)$$

Seller's optimization problem. Having characterized the consumers' behavior, we proceed to formulate the seller's optimization problem:

$$\max_{\underline{q},\underline{t},\tilde{q},\tilde{t},\bar{q},\bar{t}} \lambda(1-\alpha)(\underline{t}-c\underline{q})+\lambda\alpha(\tilde{t}-c\tilde{q})+(1-\lambda)(\bar{t}-c\bar{q}), \tag{1}$$

$$st.\ (IC_h),(IC_l),(IC_g),(IR).$$

To solve the above seller's optimization problem, we can ignore the incentive constraints (IC_{hg}), $(IC_{hh'})$, $(IC_{hl'})$, (IC_{lg}), $(IC_{lh'})$, and $(IC_{ll'})$ because the group-buying cost a_h is sufficiently large, and consumers with a high group-buying cost never participate in group buying. We further consider a relaxed problem wherein the seller maximizes his expected profit with only a subset of constraints, and then verify the other constraints ex post.[13] Solving this optimization problem, we obtain the menu of optimal bundles intended for each consumer type. We thereby characterize this menu of bundles in the following proposition.

Proposition 1. *When the seller intends to induce group buying, the optimal menu of bundles, denoted by* $\{(\underline{q}^g,\underline{t}^g),(\bar{q}^g,\bar{t}^g),(n\tilde{q}^g,n\tilde{t}^g)\}$, *is characterized as follows.*

(i) The quantity (quality) level of each bundle is determined by the following equations:

$\underline{q}^g$	$\bar{q}^g$	$\tilde{q}^g$
$\left[\underline{\theta}-\frac{(1-\lambda)}{\lambda(1-\alpha)}(\bar{\theta}-\underline{\theta})\right]v'(\underline{q}^g)=c$	$\bar{\theta}v'(\bar{q}^g)=c$	$\underline{\theta}v'(\tilde{q}^g)=c$

(ii) The price of each bundle is given by

$\underline{t}^g$	$\bar{t}^g$	$\tilde{t}^g$
$\underline{t}^g=\underline{\theta}v(\underline{q}^g)$	$\bar{t}^g=\bar{\theta}v(\bar{q}^*)-(\bar{\theta}-\underline{\theta})v(\underline{q}^g)$	$\tilde{t}^g=\underline{\theta}v(\tilde{q}^g)-a_l$

(iii) There exists $n^*>0$ *that ensures the optimal menu stated above is always feasible for* $n\geq n^*$.

Proof. See Appendix.

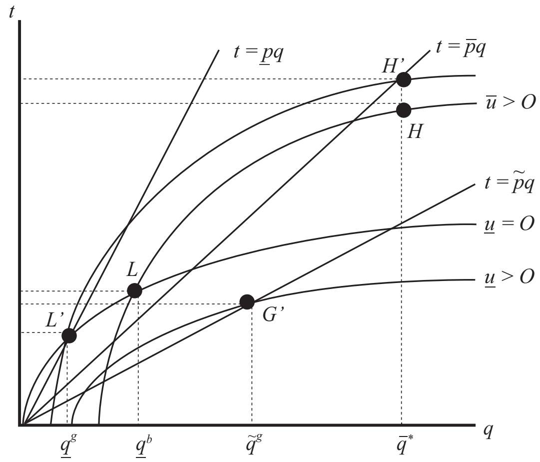
The optimal menu of bundles for each consumer type is illustrated in Figure 1, where H,L,G are the bundles intended for type H, type LH, and type LL consumers, respectively. This proposition shows that the quantity level intended for high-valuation consumers ($\bar{q}^g$) is socially optimal no matter whether group buying is present or not. For the quantity level intended for low-valuation consumers, comparing the results of this model and the benchmark leads to the following relationship among various quantity levels:

$$\underline{q}^g<\underline{q}^b<\tilde{q}^g=\underline{q}^*. \tag{2}$$

This condition shows that the quantity level intended for low-valuation consumers with a low

[13] We start with a relaxed problem where only the incentive constraints (IC_{hl}), (IC_{lh}), (IC_{gh}), (IC_{gl}) and the individual rationality constraints (IR_h), (IR_l), (IR_g) are considered. This is a common approach to solve the multi-dimensional screening problem; see, e.g., Armstrong and Rochet (1999).

FIGURE 1. THE OPTIMAL MENU OF BUNDLES



group-buying cost ($\tilde{q}^g$) is socially optimal; this is because high-valuation (type *H*) consumers never participate in group buying, and hence the tension between rent extraction and efficiency is absent. By contrast, the quantity level intended for high-cost consumers ($\underline{q}^g$) is further distorted downward in comparison with the benchmark ($\underline{q}^b$). This distortion is to further alleviate the incentive problem of high-valuation consumers, and allow the seller to extract a higher surplus from them. Furthermore, direct observations yield the following results: $\frac{\partial \underline{q}^g}{\partial \lambda} > 0$, $\frac{\partial \underline{q}^g}{\partial \alpha} < 0$, showing that the distortion of quantity level shrinks as the proportion of the intended consumer type increases.

When the number of buyers to form group buying (n) is sufficiently large such that$n \geq n^*$, all consumers are unwilling to choose the bundle ($n\tilde{q}, n\tilde{t}$) individually; hence, the incentive constraints (IC_{hn}), (IC_{ln}), and (IC_{gn}) are satisfied. Furthermore, this also ensures that low-cost (type *LL*) consumers will not self-organize a group purchase for the bundles intended for other types of consumers; hence, the incentive constraints ($IC_{gh'}$), ($IC_{gl'}$) are also satisfied.

We show that low-valuation (type *LH* and type *LL)* consumers obtain zero surplus, whereas high-valuation (type *H*) consumers obtain a positive surplus; this is reminiscent of the standard result in the two-type monopoly pricing problem. However, when type *LL* consumers are induced to participate in group buying, the quantity level for type *LH* consumers is further distorted downward, thereby leading to a lower information rent for high-valuation consumers.

In sum, consumers' heterogeneity in group-buying costs allows the seller to discriminate between consumers with various group-buying costs. In comparison with the benchmark, the seller raises the quantity level intended for low-cost (type *LL*) consumers to induce them to participate in group buying, whereas she lowers the quantity level intended for high-cost (type *LH)* consumers to alleviate the incentive problem of high-valuation (type *H*) consumers, thereby allowing the seller to charge a higher price to them.

3. Whether to Induce Group Buying

Thus far we have characterized the menu of optimal bundles by taking into account consumers' group-buying behavior. Naturally, a further question is to inquire whether such group-buying behavior is favorable to the seller and whether the seller should induce it. Hence, we shall proceed to compare the seller's expected profits in the scenario with and without group buying.

Let π_b^* and π_g^* denote the seller's profit in the benchmark and in group-buying model respectively; hence, we have

$$\pi_b^* = \lambda(\underline{t}^b - c\underline{q}^b) + (1-\lambda)(\bar{t}^b - c\bar{q}^b),$$

$$\pi_g^* = \lambda(1-\alpha)(\underline{t}^g - c\underline{q}^g) + \lambda\alpha(\tilde{t}^g - c\tilde{q}^g) + (1-\lambda)(\bar{t}^g - c\bar{q}^g).$$

Comparing the seller's profits in these two scenarios, we obtain a necessary and sufficient condition under which the seller always induces group buying. The following proposition documents this condition.

Proposition 2. *The seller induces group buying if and only if*

$$\lambda(1-\alpha)[(\underline{\theta}v(\underline{q}^g) - c\underline{q}^g) - (\underline{\theta}v(\underline{q}^b) - c\underline{q}^b)] + \lambda\alpha[(\underline{\theta}v(\underline{q}^*) - c\underline{q}^*) - (\underline{\theta}v(\underline{q}^b) - c\underline{q}^b) - a_l]$$
$$+ (1-\lambda)(\bar{\theta} - \underline{\theta})(v(\underline{q}^b) - v(\underline{q}^g)) > 0, \tag{3}$$

where $\underline{q}^*$, $\underline{q}^b$, and $\underline{q}^g$ are determined by the following equations:

$\underline{q}^*$	$\underline{q}^b$	$\underline{q}^g$
$\underline{\theta}v'(\underline{q}^*) = c$	$\left[\underline{\theta} - \frac{1-\lambda}{\lambda}(\bar{\theta} - \underline{\theta})\right]v'(\underline{q}^b) = c$	$\left[\underline{\theta} - \frac{(1-\lambda)}{\lambda(1-\alpha)}(\bar{\theta} - \underline{\theta})\right]v'(\underline{q}^g) = c$

Proof. See Appendix.

The left hand side of the expression (3) refers to the profit differential between the scenarios with and without inducing group buying, and it comprises three terms explained as follows. The first term,

$$\lambda(1-\alpha)[(\underline{\theta}v(\underline{q}^g) - c\underline{q}^g) - (\underline{\theta}v(\underline{q}^b) - c\underline{q}^b)] < 0, \tag{4}$$

refers to the profit differential generated from type *LH* consumers. Likewise, the second term,

$$\lambda\alpha[(\underline{\theta}v(\underline{q}^*) - c\underline{q}^*) - (\underline{\theta}v(\underline{q}^b) - c\underline{q}^b) - a_l] \gtrless 0, \tag{5}$$

and the third term,

$$(1-\lambda)(\bar{\theta} - \underline{\theta})(v(\underline{q}^b) - v(\underline{q}^g)) > 0, \tag{6}$$

refers to the profit differential generated from type *LL* and type *H* consumers respectively. By the derived property in (2) and the concavity of $v(.)$, (4) is always negative, whereas (6) is always positive. This reveals that when the seller induces group buying, the profit from type *LH* consumers decreases, whereas the profit from type *H* consumers increases. In contrast, (5) is indefinite, and it depends on the group-buying cost a_l. A lower cost a_l leads to a higher seller's profit generated from consumers that participate in group buying. In sum, the seller tends to induce group buying when the proportion of high-valuation consumers is higher, the valuation differential is larger, or the group-buying cost is lower.

We articulate the rationale for the aforementioned results as follows. Because a fraction of low-valuation consumers are able to participate in group buying, the presence of these group-buying consumers alleviates the tension between efficiency consideration and rent extraction. To reduce the information rent of high-valuation consumers, the seller downward distorts the quantity level intended for low-valuation consumers without group buying (type *LH* consumers). Hence, the seller's profit from high-valuation consumers increases, whereas the profit from type *LH* consumers decreases. For type *LL* consumers, the seller raises their quantity to induce them to participate in group buying. However, group-buying costs aggravate their participation constraints, and curtail the price they are willingness to pay.

Collectively, two conflicting forces lead to an ambiguous effect on the seller's profit. On the one hand, the consumers' heterogeneity in group-buying costs provides an additional dimension for the seller to discriminate between consumers with various costs. On the other hand, group-buying costs aggravate the participation constraints, and this effect leads to a lower profit. Under certain circumstances, the profit increment from high-valuation consumers may recoup the profit decrement from other consumers, thereby leading to a higher seller's profit.

While the above analysis shows that the seller's profit does not necessarily increase when inducing group buying, we shall demonstrate that the seller can always obtain a higher profit from consumers with a high group-buying cost.

Proposition 3. *In the optimal price schedule with group buying, the profit from high-cost consumers is larger than that in the optimal schedule without group buying.*

Proof. See Appendix.

As explained previously, among high-cost (type *H* and *LH*) consumers, the seller's profit from high-valuation consumers increases, whereas the profit from low-valuation ones decreases. Proposition 3 further validates that the profit increment from high-valuation consumers dominates the profit decrement from low-valuation ones. That is, group buying leads to a higher profit from those who are unable to participate in group buying. With the presence of consumers' heterogeneity in group-buying costs, the seller can effectively separate low-cost consumers from others by inducing them to participate in group buying. Hence, the seller can provide a menu of bundles specifically intended for high-cost consumers, thereby implementing more sophisticated segmentation between high-valuation and low-valuation consumers.

Proposition 2 and 3 immediately imply that if the group-buying cost of type *LL* is sufficiently small, inducing group buying always leads to a higher seller's profit. We hereby highlight this result as a corollary.

Corollary 1. *If the group-buying cost a_l is less than the surplus distortion under asymmetric information, i.e.,*

$$a_l < (\underline{\theta}v(\underline{q}^*) - c\underline{q}^*) - (\underline{\theta}v(\underline{q}^b) - c\underline{q}^b), \tag{7}$$

then the seller should always induce group buying.

Proof. See Appendix.

As previously mentioned, the quantity level for low-valuation consumers is distorted downward due to the trade-off between rent extraction and efficiency in the standard non-linear pricing problem. This surplus distortion caused by informational asymmetry is represented by $(\underline{\theta}v(\underline{q}^*)-c\underline{q}^*)-(\underline{\theta}v(\underline{q}^b)-c\underline{q}^b)$. In our context, since all high-valuation consumers are unable to participate in group buying, the seller raises the quantity level intended for group-buying (type *LL*) consumers without intensifying the incentive problem of high-valuation consumers. Hence, the seller's profit from group-buying consumers may increase, and the maximum of profit increment is equal to the aforementioned surplus distortion. Meanwhile, the group-buying cost a_l aggravates the participation constraints and offsets the profit increment from group-buying consumers. Put differently, the condition (7) shows that the seller tends to induce group-buying when the group-buying costs only partially counteract the potential profit increment from those consumers.

Example 1. To illustrate the condition provided in Proposition 2, we consider an example in which consumers' utility function is given by $v(q)=q^{\frac{1}{2}}$. In this case, we obtain

$$\underline{q}^*=\frac{\underline{\theta}^2}{4c^2},\ \underline{q}^b=\frac{\left[\underline{\theta}-(1-\lambda)\overline{\theta}\right]^2}{4\lambda^2c^2},\ \underline{q}^g=\frac{\left[(1-\lambda\alpha)\underline{\theta}-(1-\lambda)\overline{\theta}\right]^2}{4\lambda^2c^2(1-\alpha)^2}.$$

It is immediate to check that $\underline{q}^*>\underline{q}^b>\underline{q}^g$. To compare the seller's profit in the scenarios with and without group buying, we calculate the three terms in (6) as follows:

$$\lambda(1-\alpha)\left[(\underline{\theta}v(\underline{q}^g)-c\underline{q}^g)-(\underline{\theta}v(\underline{q}^b)-c\underline{q}^b)\right]=\frac{\alpha(\alpha-2)(1-\lambda)^2(\overline{\theta}-\underline{\theta})^2}{4c\lambda(1-\alpha)}<0,$$

$$\lambda\alpha\left[(\underline{\theta}v(\underline{q}^*)-c\underline{q}^*)-(\underline{\theta}v(\underline{q}^b)-c\underline{q}^b)-a_l\right]=\frac{\alpha(1-\lambda)^2(\overline{\theta}-\underline{\theta})^2}{4c\lambda}-\lambda\alpha a_l\lesseqgtr 0,$$

$$(1-\lambda)(\overline{\theta}-\underline{\theta})(v(\underline{q}^b)-v(\underline{q}^g))=\frac{\alpha(1-\lambda)^2(\overline{\theta}-\underline{\theta})^2}{2c\lambda(1-\alpha)}>0.$$

Adding the above three terms together, we obtain

$$\pi_g^*-\pi_b^*=\frac{\alpha(1-\lambda)^2(\overline{\theta}-\underline{\theta})^2}{4c\lambda(1-\alpha)}-\lambda\alpha a_l.$$

Taking the derivative of $\pi_g^*-\pi_b^*$ with respect to λ and α leads to the following results:

$$\frac{\partial(\pi_g^*-\pi_b^*)}{\partial\lambda}=\frac{-\alpha(1+\lambda)(1-\lambda)(\overline{\theta}-\underline{\theta})^2}{4c\lambda^2(1-\alpha)}-\alpha a_l<0,$$

$$\frac{\partial(\pi_g^*-\pi_b^*)}{\partial\alpha}=\frac{(1-\lambda)^2(\overline{\theta}-\underline{\theta})^2}{4c\lambda(1-\alpha)^2}-\lambda a_l\lesseqgtr 0.$$

Hence, a lower group-buying cost a_l, greater differential in valuations $(\overline{\theta}-\underline{\theta})$, or a larger proportion of high-valuation consumers lead to a higher seller's profit. In particular, if the group-buying cost a_l is small enough, the seller will choose to induce group buying.

4. The Pattern of the Price Schedule

We now turn to the impact of group buying on the pattern of the price schedule. To this end, let $p=t/q$ denote the implicit unit price of the bundle (q,t). In the benchmark, the optimal price schedule has the following features. First, the optimal price schedule yields either quantity premia or quantity discounts; that is, the implicit unit price may increase or decrease in quantities. Second, all consumers pay an average unit price greater than their marginal utility; that is, $\overline{p}^b>\overline{\theta}v'(\overline{q}^b)$ and $\underline{p}^b>\underline{\theta}v'(\underline{q}^b)$. By contrast, when all consumers have access to buying any fraction of any bundle, Alger (1999) obtains a significantly different price pattern: (*i*) the optimal price schedule must be non-decreasing in quantities, $\underline{p}\leq \overline{p}$; (*ii*) all consumers pay an average unit price equal to their marginal utility, $\overline{p}=\overline{\theta}v'(\overline{q})$ and $\underline{p}=\underline{\theta}v'(\underline{q})$. The results of Alger (1999) suggest that when consumers have access to such arbitrage opportunities, the optimal price schedule never yields quantity discounts, and each consumer pays a lower unit price.

In our model, consumers are heterogeneous in both valuations and group-buying costs, and the optimal price schedule exhibits a different price pattern. For ease of exposition, we define $\underline{p}^g$ as the implicit unit price for type *LH* consumers, $\tilde{p}^g$ for type *LL* consumers, and $\overline{p}^g$ for type *H* consumers, respectively. The following proposition summarizes the main results regarding the price pattern in our context.

Proposition 4. *In the optimal price schedule with group buying, (i) there may be quantity premia or quantity discounts, but* $\tilde{p}^g<\underline{p}^g$ *always holds; (ii) consumers pay an average price higher than their marginal utility.*

Proof. See Appendix.

In the benchmark, consumers do not necessarily choose the bundle with a lower unit price. We confirm that to segment two groups of consumers with various group-buying costs, the seller must offer a lower unit price to induce low-cost consumers to participate in group buying. Proposition 4 also indicates that consumers still pay an average unit price greater than their marginal utility, and this is in contrast with the result in Alger (1999). To exemplify this point, we provide an example in what follows.

Example 2. We consider the same example in which the consumer's utility function is given by $v(q)=q^{\frac{1}{2}}$. Hence, the implicit unit price of each bundle is shown as follows:

$$\underline{p}^g=\frac{\underline{\theta}v(\underline{q}^g)}{\underline{q}^g}=\frac{2\lambda c\underline{\theta}(1-\alpha)}{\left[(1-\lambda\alpha)\underline{\theta}-(1-\lambda)\overline{\theta}\right]},\ \tilde{p}^g=\frac{\underline{\theta}v(\tilde{q}^g)-a_l}{\tilde{q}^g}=2c-\frac{4a_lc^2}{\underline{\theta}^2},$$

$$\overline{p}^g=\frac{2c\left\{\overline{\theta}(1-\lambda\alpha)(\overline{\theta}-\underline{\theta})+\underline{\theta}\left[(1-\lambda\alpha)\underline{\theta}-(1-\lambda)\overline{\theta}\right]\right\}}{\lambda\overline{\theta}^2(1-\alpha)}.$$

A simple calculation immediately leads to

$$\underline{p}^g - \tilde{p}^g = \frac{2c(1-\lambda)(\bar{\theta}-\underline{\theta})}{\left[(1-\lambda\alpha)\underline{\theta}-(1-\lambda)\bar{\theta}\right]} + \frac{4a_l c^2}{\underline{\theta}^2} > 0,$$

$$\bar{p}^g - \underline{p}^g = \frac{2c(1-\lambda\alpha)(\bar{\theta}-\underline{\theta})\left[2\bar{\theta}\underline{\theta}(1-\lambda)-(1-\lambda)\bar{\theta}^2-(1-\lambda\alpha)\underline{\theta}^2\right]}{\lambda\bar{\theta}^2(1-\alpha)[(1-\lambda\alpha)\underline{\theta}-(1-\lambda)\bar{\theta}]} \gtrless 0.$$

Hence, the seller charges a lower unit price to consumers with a low group-buying cost, and induces them to participate in group buying. However, there may be quantity premia or discount for consumers who make individual purchases only.

V. *Discussion*

In this section, we shall discuss two extensions of the original model analyzed in Section IV. First, we consider a more general form of group buying that allows for consumers' arbitrage behavior. Second, we consider that both low-valuation and high-valuation consumers may engage in group buying.

1. Consumers' Arbitrage Behavior

We start with the discussion on consumers' arbitrage behavior. This arbitrage behavior refers to that consumers are allowed to buy any fraction of any bundles offered by the seller at the expense of paying an additional group-buying cost. In this context, while the heterogeneity in group-buying costs facilitates the seller's ability to segment consumers with various costs, consumer's arbitrage behavior restrains the seller's ability to discriminate between consumers with various valuations. Therefore, consumer's group-buying behavior leads to a more ambiguous effect on the seller's profit.

Using a simple revealed preference argument, Alger (1999) argues that consumers' arbitrage behavior in the form of group-buying always leads to a lower seller's profit. This result is straightforward because a better arbitrage possibility for buyers is always detrimental to the seller when consumers are heterogeneous in valuations only. In contrast with that, our model considers the heterogeneity in group-buying costs, and allows for the interplay between various conflicting forces caused by group buying. Hence, the impact of group buying on the sellers' profit is ambiguous. In what follows, we shall proceed to examine how these driving forces interact, and show that the seller may still possibly benefit from this general form of group buying.

Following the original model, a proportion α of low-valuation consumers incur a low group-buying cost, but now they are allowed to buy any fraction of any bundle. To accentuate the interplay between aforementioned conflicting forces, the group-buying cost a_l is assumed to be zero. Similar to the previous analysis, we can confine our attention to a menu of feasible bundles $\{(\bar{q},\bar{t}),(\underline{q},\underline{t}),(\tilde{q},\tilde{t})\}$ intended for type H, type LH and type LL consumers, respectively.

Specifically, the feasible bundles must ensure that type LL consumers prefer the bundle $(\tilde{q},\tilde{t})$ to any fraction of any bundles. Hence, the incentive-compatibility constraints for type LL consumers, denoted by (IC_g^*), are as follows:[14]

$$\underline{\theta}v(\tilde{q})-\tilde{t}\geq\underline{\theta}v(\frac{\bar{q}}{k})-\frac{\bar{t}}{k},\quad \forall k\geq 1,$$

$$\underline{\theta}v(\tilde{q})-\tilde{t}\geq\underline{\theta}v(\frac{\underline{q}}{k})-\frac{\underline{t}}{k},\quad \forall k\geq 1,$$

$$\underline{\theta}v(\tilde{q})-\tilde{t}\geq\underline{\theta}v(\frac{n\tilde{q}}{k})-\frac{n\tilde{t}}{k},\quad \forall k\geq 1.$$

In this scenario with consumers' arbitrage behavior, the seller's optimization problem is formulated as follows:

$$\max_{\underline{q},\underline{t},\tilde{q},\tilde{t},\bar{q},\bar{t}} \lambda(1-\alpha)(\underline{t}-c\underline{q})+\lambda\alpha(\tilde{t}-c\tilde{q})+(1-\lambda)(\bar{t}-c\bar{q}),$$

$$st.(IC_h),(IC_l),(IC_g^*),(IR).$$

While it is intractable to analytically solve the above optimization problem and characterize the menu of optimal bundles, we can discuss several important features of this solution. First, the incentive constraints (IC_g^*) imply that the implicit unit price of the bundle $(\tilde{q},\tilde{t})$ intended for type LL consumers must be equal to their marginal utility, i.e., $\tilde{p}=\underline{\theta}v'(\tilde{q})$. By $\underline{\theta}v'(\tilde{q})=\tilde{p}>c$, we can conclude that the quantity level intended for type LL consumers must be distorted downward, i.e., $\tilde{q}<\underline{q}^*$. Second, to prevent the arbitrage behavior of type LL consumers, the implicit unit price $\tilde{p}$ must be lower than that of other bundles; that is, $\tilde{p}\leq\min\left\{\frac{\bar{t}}{\bar{q}},\frac{\underline{t}}{\underline{q}}\right\}$. This condition significantly alters the nature of the optimal bundles characterized in the previous section; particularly, the quantity level intended for high-valuation may not be socially optimal, which is significantly different from the result in other scenarios without consumer arbitrage.

Finally, a simple revealed preference argument indicates that the seller's profit is always lower than that in the model analyzed in Section IV because low-cost consumers have a more efficient way to engage in arbitrage. However, the seller's profit may still be higher than that in the benchmark even though consumer's arbitrage aggravates the incentive problem and limits the seller's ability to discriminate between various consumer types. This is because the heterogeneity in group-buying costs facilitates more sophisticated market segmentation. We hereby summarize the above results in the following proposition.

Proposition 5. *If low-cost consumers can buy any fraction of any bundles, then (i) the quantity (quality) levels for type LH consumers* ($\underline{q}$) *and type LL consumers* ($\tilde{q}$) *must be distorted downward; (ii) the seller's profit may be higher than the profit level in the benchmark.*

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[14] The incentive-compatibility constraints for other types of consumers are the same as those in the model described in Section III.

To illustrate that the seller's profit may increase in comparison with the benchmark, we provide an example as follows.

Example 3. Let the consumer's utility function be given by $v(q)=\frac{1}{2}[1-(1-q)^2]$. We consider a menu of bundles $\{(\underline{q}^m,\underline{t}^m),(\overline{q}^m,\overline{t}^m),(\tilde{q}^m,\tilde{t}^m)\}$ defined as follows:

$$\left[\underline{\theta}-\frac{(1-\lambda)}{\lambda(1-\alpha)}(\overline{\theta}-\underline{\theta})\right]v'(\underline{q}^m)=c,\overline{\theta}v'(\overline{q}^m)=c,\underline{\theta}v'(\tilde{q}^m)=\frac{\overline{t}^m}{\overline{q}^m};$$

$$\underline{t}^m=\underline{\theta}v(\underline{q}^m),\overline{t}^m=\overline{\theta}v(\overline{q}^m)-(\overline{\theta}-\underline{\theta})v(\underline{q}^m),\tilde{t}^m=\tilde{q}^m\frac{\overline{t}^m}{\overline{q}^m}.$$

In this menu, the bundles $(\underline{q}^m,\underline{t}^m),(\overline{q}^m,\overline{t}^m),(\tilde{q}^m,\tilde{t}^m)$ are intended for type LH, type H, and type LL consumers, respectively. The quantity levels exhibit the following properties: $\underline{q}^m=\underline{q}^g$, $\overline{q}^m=\overline{q}^g$, $\tilde{q}^m<\tilde{q}^g$. While this menu of bundles is not necessarily optimal,[15] it is a feasible one that satisfies all the constraints described above: $(IC_h),(IC_l),(IC_g^*)$, and (IR). Consider the following parameters: $\underline{\theta}=1$, $\overline{\theta}=1.55$, $c=0.3$, $\lambda=0.45$, $\alpha=1/30$. We immediately obtain the seller's expected profit in the benchmark, π_b^*, and in the group-buying model, π_g^*, as follows: $\pi_b^*=0.2777, \pi_g^*=0.2780$. Hence, this example shows that inducing group buying leads to a higher profit.

2. When All Consumers May Engage in Group Buying

As characterized in Proposition 1 and 2, when only low-valuation consumers may engage in group buying $(\alpha>0,\beta=0)$, we show that the optimal menu of bundles is $\{(\underline{q}^g,\underline{t}^g),(\overline{q}^g,\overline{t}^g),(n\tilde{q}^g,n\tilde{t}^g)\}$, and we also characterize the conditions under which group buying boosts the seller's profit. In this subsection, we shall further discuss whether this main result still holds when both high-valuation and low-valuation consumers may engage in group buying $(\alpha>0,\beta>0)$.

In this new context, the menu $\{(\underline{q}^g,\underline{t}^g),(\overline{q}^g,\overline{t}^g),(n\tilde{q}^g,n\tilde{t}^g)\}$ characterized in Section IV is not necessarily optimal. However, we can show that the seller's profit under this menu may still be higher than its profit level in the benchmark. Given that the seller proposes the same menu, we argue that high-valuation consumers with a low group-buying cost will either choose the bundle $(\overline{q}^g,\overline{t}^g)$ or $(\tilde{q}^g,\tilde{t}^g)$, but they will never choose the bundle $(\underline{q}^g,\underline{t}^g)$.[16] In the former case where these consumers choose the bundle $(\overline{q}^g,\overline{t}^g)$, the seller's profit remains as π_g^* characterized in Section IV:

$$\pi_g^*=\lambda(1-\alpha)(\underline{t}^g-c\underline{q}^g)+\lambda\alpha(\tilde{t}^g-c\tilde{q}^g)+(1-\lambda)(\overline{t}^g-c\overline{q}^g)$$

[15] In the proposed bundles, the quantity intended for type H consumers is socially optimal, which is not necessarily true for optimal bundles.

[16] By the incentive constraints of the type LL consumers, we obtain:

$$\underline{\theta}v(\tilde{q}^g)-\tilde{t}^g-a_l\geq\underline{\theta}v(\underline{q}^g)-\underline{t}^g \Rightarrow \overline{\theta}v(\tilde{q}^g)-\tilde{t}^g-a_l\geq\overline{\theta}v(\underline{q}^g)-\underline{t}^g.$$

By contrast, in the latter case where these consumers choose the bundle $(\tilde{q}^g, \tilde{t}^g)$, the seller's profit is as follows:

$\tilde{\pi}_g^* = \lambda(1-\alpha)(\underline{t}^g - c\underline{q}^g) + \lambda\alpha(\tilde{t}^g - c\tilde{q}^g) + (1-\lambda)\beta(\tilde{t}^g - c\tilde{q}^g) + (1-\lambda)(1-\beta)(\bar{t}^g - c\bar{q}^g)$. By the fact that $(\bar{t}^g - c\bar{q}^g) > (\tilde{t}^g - c\tilde{q}^g)$, it is straightforward to show that $\tilde{\pi}_g^*$ decreases with β, and it is always lower than π_g^*. Because we have characterized the conditions under which π_g^* dominates π_b^*, we can further argue that $\tilde{\pi}_g^*$ also dominates π_b^* provided that β is sufficiently small.

In sum, while the analysis in Section IV is based on a simplified framework, the discussion in this section validates the robustness of our main results in various extensions of the original model.

VI. *Conclusion*

Motivated by the prevalence of buyer-initiated group buying, this paper aims to investigate the impact of such consumers' group-buying behavior on the pricing schedule and on the profitability of a monopolist seller. Building on the literature on second-degree price discrimination, this paper enriches the traditional nonlinear-pricing model by incorporating the heterogeneity in consumers' group-buying costs. In this context, we highlight several conflicting forces caused by buyer-initiated group buying. First, consumers' heterogeneity in group-buying costs renders an additional dimension for the seller to discriminate between consumers with various costs through their selections of individual purchases or group purchases. Second, group-buying costs aggravate consumers' participation constraints and curtail consumers' willingness to pay. Finally, consumers' arbitrage behavior in the form of group buying restrains the seller's ability to discriminate between various consumer types. The first effect is favorable, whereas the other two effects are detrimental to the seller. The main results of this stylized model hinge on the interplay between these conflicting forces caused by group buying.

We characterize the optimal menu of quantity-price (quality-price) bundles, and elaborate on the conditions under which group buying is favorable to the seller. In contrast to the standard nonlinear-pricing model, the optimal quantity/quality level for low valuation consumers without group buying is further distorted downward, whereas the levels for other consumers are socially optimal. Inducing group buying is more favorable when the proportion of high-valuation consumers is higher, the valuation differential is larger, or the group-buying cost is lower. Our main result is robust in two extensions: one that considers a more general form of group buying allowing for consumers' arbitrage behavior and the other one that allows more potential consumers to engage in group buying.

A number of existing studies have investigated the impact of group buying on the seller's pricing strategies and profitability from various viewpoints. On the one hand, Alger (1999) argues that consumers' arbitrage behavior impedes the seller's ability to discriminate between consumers with various valuations, and group buying is always detrimental to the seller. On the other hand, many studies have provided various rationales for offering group-buying options from sellers' perspective, such as a pricing strategy under demand uncertainty (Arand and Aron, 2003; Chen and Zhang, 2012), a pricing strategy to facilitate price discrimination and advertising (Edelman et al., 2011), and a selling mechanism through social interactions (Jie and Xie, 2011). In contrast to the aforementioned studies, this paper accentuates the interplay between various conflicting effects caused by group buying, and we provide a novel rationale for sellers to react actively to such consumers' group-buying behavior.

This paper certainly has its limitations. For example, we consider the heterogeneity in valuations and group-buying costs in a binary setting for tractability; the optimal menu remains unclear when high-valuation consumers are also able to engage in group buying. The extensions to relax these assumptions in our model can possibly merit more fruitful future research.

Appendix

Proof of Proposition 1.

To solve the seller's optimization problem, we start with a relaxed problem where only the incentive constraints (IC_{hl}), (IC_{lh}), (IC_{gh}), (IC_{gl}) and the individual rationality constraints (IR_h), (IR_l), (IR_g) are considered. In this relaxed problem, we first make the following claim.

Claim 1. *In this relaxed problem, the incentive constraints (IC_{hl}), (IC_{gl}), and the individual rationality constraints (IR_l) and (IR_g) are binding at the optimum, whereas the other constraints are slack.*

The proof of this claim consists of the following steps.

Step 1: we show that (IR_l) is binding at the optimum. Suppose not, and then by (IC_{hl}) we obtain

$$\bar{\theta}v(\bar{q})-\bar{t}\geq\bar{\theta}v(\underline{q})-\underline{t}>\underline{\theta}v(\underline{q})-\underline{t}>0,$$

showing that the seller can further increase his profit by raising the price $\bar{t}$ and $\underline{t}$ by the same small amount ε. A small increment of prices increases the seller's profit, whereas this action neither violates the constraints (IC_{gh}), (IC_{gl}), (IR_h), and (IR_l), nor changes the constraints (IC_{hl}), (IC_{lh}) and (IR_g).

Step 2: we show that (IC_{hl}) is binding at the optimum. Suppose not, so we obtain

$$\bar{\theta}v(\bar{q})-\bar{t}>\bar{\theta}v(\underline{q})-\underline{t}>\underline{\theta}v(\underline{q})-\underline{t}=0.$$

Then the seller can further increase his profit by raising the price $\bar{t}$ by a small amount ε, without changing the price $\underline{t}$ and $\tilde{t}$. This increment of the price would increase the seller's profit without any effect on the other constraints in the relaxed problem.

Step 3: we show that $\bar{q}>\underline{q}$. Simply adding (IC_{hl}) and (IC_{lh}), we obtain

$$(\bar{\theta}-\underline{\theta})\left[v(\bar{q})-v(\underline{q})\right]\geq 0,$$

which ensures that $\bar{q}>\underline{q}$.

Step 4: we show that (IR_g) is binding. Since (IC_{hl}) is binding, we have $\bar{t}=\bar{\theta}v(\bar{q})-(\bar{\theta}-\underline{\theta})v(\underline{q})$. Therefore,

$$\begin{aligned}\underline{\theta}v(\bar{q})-\bar{t}&=\underline{\theta}v(\bar{q})-\bar{\theta}v(\bar{q})+(\bar{\theta}-\underline{\theta})v(\underline{q})\\&=(\bar{\theta}-\underline{\theta})\left[v(\underline{q})-v(\bar{q})\right]<0\end{aligned}$$

Suppose that (IR_g) is not binding, and by (IC_{gh}) and (IC_{gl}) we have

$$\underline{\theta}v(\tilde{q})-\tilde{t}-a_l>0>\underline{\theta}v(\overline{q})-\overline{t}$$

$$\underline{\theta}v(\tilde{q})-\tilde{t}-a_l>\underline{\theta}v(\underline{q})-\underline{t}=0$$

Then the seller can further increase his profit by raising the price $\tilde{t}$, without changing the price $\overline{t}$ and $\underline{t}$.

Step 5: we show that (IC_{lh}), (IC_{gh}) and (IR_h) are slack. Substituting $\underline{t}=\underline{\theta}v(\underline{q})$ and $\overline{t}=\overline{\theta}v(\overline{q})-(\overline{\theta}-\underline{\theta})v(\underline{q})$ into (IC_{lh}), we rewrite (IC_{lh}) as

$$\begin{aligned}&\underline{\theta}v(\underline{q})-\underline{t}\geq\underline{\theta}v(\overline{q})-\overline{t}\\ \Leftrightarrow\ &0\geq\underline{\theta}v(\underline{q})-\overline{\theta}v(\overline{q})+(\overline{\theta}-\underline{\theta})v(\underline{q})\\ \Leftrightarrow\ &0\geq\overline{\theta}(v(\underline{q})-v(\overline{q})),\end{aligned}$$

which certainly holds as $\underline{q}<\overline{q}$ and $v'>0$. In addition, the surplus of high-valuation consumers can be rewritten as

$$\overline{\theta}v(\overline{q})-\overline{t}=(\overline{\theta}-\underline{\theta})v(\underline{q})>0,$$

which implies (IR_h) certainly holds. Lastly, substituting $\tilde{t}=\underline{\theta}v(\tilde{q})-a_l$ and $\overline{t}=\overline{\theta}v(\overline{q})-(\overline{\theta}-\underline{\theta})v(\underline{q})$ into (IC_{gh}), we obtain

$$\begin{aligned}&\underline{\theta}v(\tilde{q})-\tilde{t}-a_l\geq\underline{\theta}v(\overline{q})-\overline{t}\\ \Leftrightarrow\ &0\geq\underline{\theta}v(\overline{q})-\overline{\theta}v(\overline{q})+(\overline{\theta}-\underline{\theta})v(\underline{q})\\ \Leftrightarrow\ &0\geq(\overline{\theta}-\underline{\theta})(v(\underline{q})-v(\overline{q})),\end{aligned}$$

which certainly holds as $\underline{q}<\overline{q}$ and $v'>0$.

Step 1 to 5 complete the proof of claim 1.

Next, we characterize the solution to this relaxed problem. By Lemma 1, the incentive constraints (IC_{hl}), (IC_{gl}) and individual rationality constraints (IR_l), (IR_g) are binding at the optimum; consequently, we have

$$\underline{t}=\underline{\theta}v(\underline{q}), \tag{A.1}$$

$$\overline{t}=\overline{\theta}v(\overline{q})-(\overline{\theta}-\underline{\theta})v(\underline{q}). \tag{A.2}$$

$$\tilde{t}=\underline{\theta}v(\tilde{q})-a_l. \tag{A.3}$$

Substituting (A.1), (A.2), and (A.3) into the expected profit function (1), we rewrite (1) as

$$\lambda(1-\alpha)[\underline{\theta}v(\underline{q})-c\underline{q}]+\lambda\alpha\underline{\theta}v(\tilde{q})-a_l-c\tilde{q}]+(1-\lambda)\left[\overline{\theta}v(\overline{q})-(\overline{\theta}-\underline{\theta})v(\underline{q})-c\overline{q}\right]. \tag{A.4}$$

Let $\{(\underline{q}^g,\underline{t}^g),(n\tilde{q}^g,n\tilde{t}^g),(\bar{q}^g,\bar{t}^g)\}$ denote the profit-maximizing menu in the relaxed problem. Since the profit function (A.4) is concave, taking the derivative of (A.4) with respect to $\underline{q}$, $\tilde{q}$, and $\bar{q}$ respectively leads to the following necessary and sufficient conditions for the optimal bundles:

$$\left[\underline{\theta}-\frac{(1-\lambda)}{\lambda(1-\alpha)}(\bar{\theta}-\underline{\theta})\right]v'(\underline{q}^g)=c, \tag{A.5}$$

$$\underline{\theta}v'(\tilde{q}^g)=c, \tag{A.6}$$

$$\bar{\theta}v'(\bar{q}^g)=c. \tag{A.7}$$

Equations (A.5), (A.6), and (A.7) along with (A.1), (A.2), and (A.3) characterize the optimal bundles in the relaxed problem.

Finally, we make the following claim to complete the proof of this proposition.

Claim 2. *There exists $n^*>0$ such that the solution to the relaxed problem is also the solution to the original problem for $n\geq n^*$.*

To characterize the solution to the original profit-maximizing problem, we must verify other constraints in the original problem: (IC_{hn}), (IC_{ln}), (IC_{gn}), ($IC_{gh'}$), and ($IC_{gl'}$).

First, we show that the incentive constraints (IC_{hn}), (IC_{ln}), and (IC_{gn}) hold for a sufficiently large n; this ensures that no consumer is willing to buy the bundle intended for group purchases, $(n\tilde{q},n\tilde{t})$, individually. Consider the constraint (IC_{hn}), and let Δu denote the the difference between the right-hand side and left-hand side of (IC_{hn}); hence, we have

$$\begin{aligned}\Delta u&=[\bar{\theta}v(n\tilde{q})-n\tilde{t}]-[\bar{\theta}v(\bar{q})-\bar{t}]\\&=[\bar{\theta}v(n\tilde{q})-\bar{\theta}v(\bar{q})]-[n\tilde{t}-\bar{t}]\\&=\int_{\bar{q}}^{n\tilde{q}}\bar{\theta}v'(x)dx-[n\tilde{t}-\bar{t}].\end{aligned}$$

The concavity of $v(.)$ leads to the following expressions:

$$\bar{\theta}v'(x)<\bar{\theta}v'(\bar{q})=c \text{ for all } x\in[\bar{q},n\tilde{q}],$$

and

$$\begin{aligned}\Delta u&=\int_{\bar{q}}^{n\tilde{q}}\bar{\theta}v'(x)dx-[n\tilde{t}-\bar{t}]\\&<\int_{\bar{q}}^{n\tilde{q}}cdx-[n\tilde{t}-\bar{t}]=c[n\tilde{q}-\bar{q}]-[n\tilde{t}-\bar{t}]=[\bar{t}-c\bar{q}]-n[\tilde{t}-c\tilde{q}],\end{aligned}$$

which implies that $\Delta u<0$ for $n>\dfrac{\bar{t}-c\bar{q}}{\tilde{t}-c\tilde{q}}$. That is, ($IC_{hn}$) holds for $n>\dfrac{\bar{t}-c\bar{q}}{\tilde{t}-c\tilde{q}}$. A similar argument applies to (IC_{ln}) and (IC_{gn}). Hence, there exists $n_1^*>0$ such that (IC_{hn}), (IC_{ln}), and (IC_{gn}) hold for $n\geq n_1^*$.

Second, we show that ($IC_{gh'}$) and ($IC_{gl'}$) are slack if n is sufficiently large. Since (IR_g) is binding, the left-hand side of ($IC_{gh'}$) and ($IC_{gl'}$) is $\underline{\theta}v(\tilde{q})-\tilde{t}=a_l>0$. The right-hand side of ($IC_{gh'}$) and ($IC_{gl'}$) approaches to $\underline{\theta}v(0)-0=0$ as n approaches to infinity. Hence, there exists $n_2^*>0$ such that both ($IC_{gh'}$) and ($IC_{gl'}$) hold for $n\geq n_2^*$. Define $n=\min\{n_1^*,n_2^*\}$, and then this completes the proof of claim 2.

Proof of Proposition 2.

The seller's profit in the benchmark model, denoted by π_b^*, and the profit in the group-buying model, denoted by π_g^*, are given as follows:

$$\pi_b^* = \lambda(\underline{t}^b - c\underline{q}^b) + (1-\lambda)(\bar{t}^b - c\bar{q}^*), \tag{A.8}$$

$$\pi_g^* = \lambda(1-\alpha)(\underline{t}^g - c\underline{q}^g) + \lambda\alpha(\tilde{t} - c\tilde{q}^g) + (1-\lambda)(\bar{t}^g - c\bar{q}^*), \tag{A.9}$$

where the prices t in each bundle are given by

$$\underline{t}^b = \underline{\theta}v(\underline{q}^b), \bar{t}^b = \bar{\theta}v(\bar{q}^*) - (\bar{\theta} - \underline{\theta})v(\underline{q}^b), \tag{A.10}$$

$$\underline{t}^g = \underline{\theta}v(\underline{q}^g), \bar{t}^g = \bar{\theta}v(\bar{q}^*) - (\bar{\theta} - \underline{\theta})v(\underline{q}^g), \tilde{t}^g = \underline{\theta}v(\tilde{q}^g) - a_l. \tag{A.11}$$

Substituting (A.10) and (A.11) into (A.8) and (A.9), we immediately obtain

$$\pi_g^* > \pi_b^* \Leftrightarrow \lambda(1-\alpha)\left[(\underline{\theta}v(\underline{q}^g) - c\underline{q}^g) - (\underline{\theta}v(\underline{q}^b) - c\underline{q}^b)\right]$$

$$+\lambda\alpha\left[(\underline{\theta}v(\underline{q}^*) - c\underline{q}^*) - (\underline{\theta}v(\underline{q}^b) - c\underline{q}^b) - a_l\right]$$

$$+(1-\lambda)(\bar{\theta} - \underline{\theta})(v(\underline{q}^b) - v(\underline{q}^g)) > 0.$$

Proof of Proposition 3.

As shown in the proof of Proposition 1, π_g^* is the solution to the following optimization problem:

$$[MP_g] \max_{\{\underline{t},\underline{q},\tilde{t},\tilde{q},\bar{t},\bar{q}\}} \lambda(1-\alpha)(\underline{t} - c\underline{q}) + \lambda\alpha(\tilde{t} - c\tilde{q}) + (1-\lambda)(\bar{t} - c\bar{q}),$$

subject to:

$$\bar{\theta}v(\bar{q}) - \bar{t} = \bar{\theta}v(\underline{q}) - \underline{t}, \tag{A.12}$$

$$\underline{\theta}v(\underline{q}) - \underline{t} = 0, \tag{A.13}$$

$$\underline{\theta}v(\tilde{q}) - \tilde{t} - a_l = 0. \tag{A.14}$$

Since $\{\tilde{t}, \tilde{q}\}$ is related to the constraint (A.14) only, the optimization problem MP_g can be transformed into two separate optimization problems as follows:

$$[MP_{g1}] \quad \max_{\{\underline{t},\underline{q},\bar{t},\bar{q}\}} \lambda(1-\alpha)(\underline{t} - c\underline{q}) + (1-\lambda)(\bar{t} - c\bar{q}),$$

$s.t.$ (A.12), (A.13);

$$[MP_{g2}] \quad \max_{\{\tilde{t},\tilde{q}\}} \lambda\alpha(\tilde{t} - c\tilde{q}),$$

$s.t.$ (A.14).

By contrast, as shown in the standard two-type pricing problem, π_b^* is the solution to the following optimization problem:

$$[MP_b]\ \underset{\{\underline{t},\underline{q},\bar{t},\bar{q}\}}{\mathrm{Max}}\ \lambda(1-\alpha)(\underline{t}-c\underline{q})+\lambda\alpha(\underline{t}-c\underline{q})+(1-\lambda)(\bar{t}-c\bar{q}),$$

s.t. (A.12), (A.13).

Since the constraints in both $[MP_{g1}]$ and $[MP_b]$ are the same, this immediately leads to

$$\lambda(1-\alpha)(\underline{t}^g-c\underline{q}^g)+(1-\lambda)(\bar{t}^g-c\bar{q}^*)\geq\lambda(1-\alpha)(\underline{t}^b-c\underline{q}^b)+(1-\lambda)(\bar{t}^b-c\bar{q}^*),\tag{A.15}$$

where $\{(\underline{q}^g,\underline{t}^g),(\bar{q}^*,\bar{t}^g)\}$ is the solution to $[MP_{g1}]$.

Proof of Corollary 1.

Proposition 2 and 3 immediately imply that inducing group buying boosts the seller's profit if the profit from group-buying (type *LL)* consumers increases; that is,

$$\lambda\alpha(\tilde{t}^g-c\tilde{q}^g)>\lambda\alpha(\underline{t}^b-c\underline{q}^b)\Rightarrow\pi_g^*>\pi_b^*.$$

By (A.10) and (A.11), we obtain

$$\lambda\alpha(\tilde{t}^g-c\tilde{q}^g)>\lambda\alpha(\underline{t}^b-c\underline{q}^b)\ \Leftrightarrow\ a_l<(\underline{\theta}v(\underline{q}^*)-c\underline{q}^*)-(\underline{\theta}v(\underline{q}^g)-c\underline{q}^g).$$

Proof of Proposition 4.

The average unit prices for each bundle are given by the following equations:

$$\underline{p}^g\equiv\frac{\underline{t}^g}{\underline{q}^g}=\frac{\underline{\theta}v(\underline{q}^g)}{\underline{q}^g},\tilde{p}^g\equiv\frac{\tilde{t}^g}{\tilde{q}^g}=\frac{\underline{\theta}v(\tilde{q}^g)-a_l}{\tilde{q}^g},$$

$$\bar{p}^g\equiv\frac{\bar{t}^g}{\bar{q}^g}=\frac{\bar{\theta}v(\bar{q}^*)-(\bar{\theta}-\underline{\theta})v(\underline{q}^g)}{\bar{q}^*}.$$

The concavity of $v(.)$ leads to that

$$\underline{p}^g\equiv\frac{\underline{t}^g}{\underline{q}^g}=\frac{\underline{\theta}v(\underline{q}^g)}{\underline{q}^g}>\underline{\theta}v'(\underline{q}^g).$$

Because the quantities offered to type *H* and type *LL* consumers are socially optimal, we have

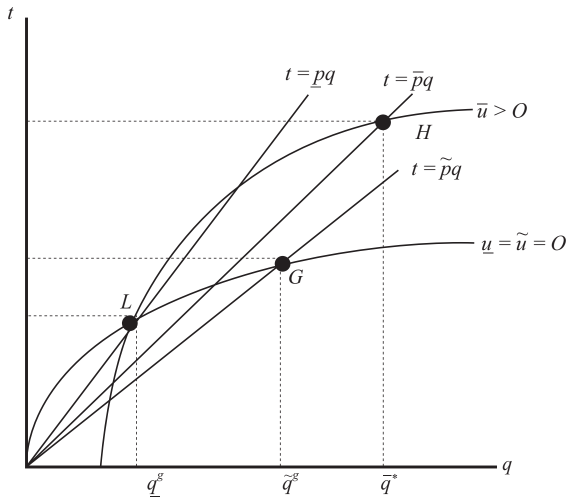
$$\underline{\theta}v'(\tilde{q}^g)=c<\tilde{p}^g,\bar{\theta}v'(\bar{q}^*)=c<\bar{p}^g.$$

Note that $\tilde{p}^g$ is decreasing in the group-buying cost a_l. We first consider that when $a_l=0$, low-valuation consumers obtain zero surplus. Thus, the two bundles offered to low-valuation consumers, $(\underline{q}^g,\underline{t}^g)$ and $(\tilde{q}^g,\tilde{t}^g)$ are on the same indifference curve as shown in Figure 2. By the concavity of $v(.)$ and by $\tilde{q}^g>\underline{q}^g$, the unit price for the bundle $(\tilde{q}^g,\tilde{t}^g)$ must be lower than the unit price for the bundle $(\underline{q}^g,\underline{t}^g)$. For $a_l>0$, the unit price $\tilde{p}^g$ is even lower and less than the price $\underline{p}^g$ for any $a_l\geq0$.

Proof of Proposition 5.

The incentive constraints (IC_g^*) require that type *LL* consumers would not choose any fraction of any

Figure 2. Price Pattern



bundles. If $\tilde{p} > \underline{\theta} v'(\tilde{q})$, then type *LL* consumers would be better off by buying a small fraction of the bundle $(\tilde{q},\tilde{t})$, thereby violating the incentive constraints (IC_g^*). By contrast, if $\tilde{p} < \underline{\theta} v'(\tilde{q})$, the seller can increase his profit by increasing $\tilde{p}$ and $\tilde{q}$ without jeopardizing any constraint; hence, the bundle $(\tilde{q},\tilde{t})$ can not be optimal. By the above arguments, we can conclude that the implicit unit price $\tilde{p}$ must be equal to $\underline{\theta} v'(\tilde{q})$. Furthermore, we can argue that $\underline{\theta} v'(\tilde{q}) = \tilde{p} > c$; otherwise, the seller does not need to induce type *LL* consumers to purchase. By the concavity of $v(.)$, we must have $\tilde{q} < \underline{q}^*$.

The numerical example in the main text shows the existence of menus leading to a higher seller's profit. Hence, this example will suffice to prove that the seller may benefit from group buying in the context with consumers' arbitrage behavior.

References

Alger, I. (1999), "Consumer Strategies Limiting the Monopolist's Power: Multiple and Joint Purchases," *The Rand Journal of Economics* 30, pp.736-757.

Anand, K.S., and R. Aron (2003), "Group Buying on the Web: A Comparison of Price-discovery Mechanisms," *Management Science* 49, pp.1546-1562.

Armstrong, M., and J.-C. Rochet (1999), "Multi-dimensional Screening: A User's Guide," *European Economic Review* 43, pp.959-979.

Chen, Y., and X. Li (2013), "Group Buying Commitment and Sellers' Competitive Advantages," *Journal of Economics and Management Strategy* 22, pp.164-183.

Chen, Y., and T. Zhang (2012), "Group Coupons: Interpersonal Bundling on the Internet," NET Institute Working Paper No.12-09.

Dana Jr, J.D. (2012), "Buyer Groups as Strategic Commitments," *Games and Economic Behavior* 74, pp.470-485.

Edelman, B., S. Jaffe and S. Kominers (2011), "To Groupon or Not to Groupon: The Profitability of Deep Discounts," Working paper, Harvard Business School.

Flynn, L.J. (2001), "MobShop, a Group-Buying Site, Drops Its Consumer Business," *New York Times*.

Jeon, D.S. and D. Menicucci (2005), "Optimal Second-degree Price Discrimination and Arbitrage: On the Role of Asymmetric Information among Buyers," *RAND Journal of Economics* 36, pp.337-360.

Jing, X. and J. Xie (2011), "Group Buying: A New Mechanism for Selling through Social Interactions," *Management Science* 57, pp.1354-1372.

Laffont, J.J. and D. Martimort (2009), *The Theory of Incentives: The Principal-agent Model*, Princeton University Press.

Marvel, H.P. and H. Yang (2008), "Group Purchasing, Nonlinear Tariffs, and Oligopoly," *International Journal of Industrial Organization* 26, pp.1090-1105.

McManus, B. (2001), "Two-Part Pricing with Costly Arbitrage," *Southern Economic Journal* 68, pp.369-386.

Salanié, B. (1997), *The Economics of Contracts: A Primer*, MIT Press.

Tang, C.S. (2008), "United We May Stand," *Wall Street Journal*.

Tirole, J. (1988), *The Theory of Industrial Organization*, MIT press.

Wang, J.J., X.I.N. Zhao and J.J. Li (2011), "Team Purchase: A Case of Consumer Empowerment in China," *Journal of Consumer Affairs* 45, pp.528-538.

Yen, B. and M. Huang (2009), "Tug of War between the Retailers and Consumers: The Phenomenon of Group Purchase in China," The Proceedings of the 9th International Conference on Electronic Business, Macau, November.