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Emissions Trading, Firm Heterogeneity, and Intra-Industry Reallocations in the Long Run

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Abstract: Design of environmental regulation has substantial implications for size distribution and mass of firms within and across industries in the long run. In a general equilibrium model that accounts for endogenous entry and exit of heterogeneous firms, the welfare impacts of emissions trading are analytically decomposed into the effects on economy-wide income, mass of firms, size distribution, markups, and factor prices. Distortionary impacts on size distribution and permit price depend on the conditionality of permit distribution, interactions between changes in entry-exit conditions and in aggregate accounting conditions, the factor intensity of entry, and coverage of non-pollution-intensive sectors in emissions trading.

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1. Introduction

Since Rose-Ackerman (1973), economists have long been concerned with the implications of environmental regulations for the long-run industry dynamics. Conventional wisdom in the earlier literature suggests that emissions tax and (auctioned) emissions trading policies induce efficient allocations in the long run, whereas abatement subsidy and uniform emissions standard policies would distort the entry-exit conditions and induce excessive entry (Spulber, 1985; Baumol, 1988; Polinsky, 1979).

A point of departure for our analysis comes from two observations we make about Spulber’s seminal paper (1985), which showed that a uniform emissions standard would induce excessive entry and thus inefficiency under perfect competition with identical firms. This result occurs because the emissions standard confers firms the right to pollute up to the standard upon entry, which also serves as an entry subsidy. Our first observation is that because Spulber assumes identical firms, excessive entry in his context simply means a larger number (or mass) of firms in the long run equilibrium. When firms are heterogeneous, however, excessive entry could mean either entry of less productive firms, a larger mass of firms, or both. Indeed, an increasing number of empirical studies have substantiated the existence of large and persistent variation in firm-level productivity across firms (e.g., Cabral and Mata, 2003; Eaton, Korum, and Karamarz, 2011). When firms are heterogeneous, environmental regulations might affect different firms differently both at intensive and extensive margins, inducing changes in the size distribution of firms.

Second, the policy-induced effect on the extensive margin alone (i.e., on size distribution and mass of firms) may have a second-order impact on the intensive margin, via its effect on competition in the commodity and factor markets. Since Gibrat’s (1931) seminal work, an extensive body of literature (e.g., Cabral and Mata, 2003; Lucas, 1978; Luttmer, 2007; Simon and Bonini, 1958) has investigated the economic mechanisms underlying the size distribution of firms that is often observed to be stable and approximately Pareto or log-normal. Their motivation comes from the idea that the size distribution alone may have important implications for consumer welfare, industry competition, and anti-trust regulations. Recently, economists (e.g. Melitz, 2003; Eaton et al., 2011) have examined trade-induced variations in the size distribution of firms. In the environmental economics literature, a recent empirical study by Greenstone, List, and Syverson (2012) on U.S. manufacturing firms shows that environmental regulations induced exit of less productive firms, causing the industry to be more concentrated, yet decreased average productivity of firms. Presumably, this occurs because those productive firms that stay in the industry produce substantially less due to the cost of environmental regulations. Such a policy-induced change in the size distribution
of firms creates intra/inter-industry reallocations of firm-level variables. Hence, the overall impacts of environmental regulations on aggregate variables of interest such as permit price, output, and welfare would be determined through intricate interactions between their effects on the extensive margin and on the intensive margin.

This paper proposes a theoretical framework that enables us to disentangle these intricate effects of environmental regulations on the size distribution and mass of firms in a general equilibrium model that accounts for entry and exit of heterogeneous firms. To this end, we focus on the design issues of emissions trading. In first-best settings, a successful ET policy should make the initial distribution of emissions allowances unconditional on all relevant economic decisions by the regulated firms such as emissions, output, or entry. However, conditional allocation rules have often been used in practice in order to protect certain industries or to alleviate pre-existing market distortions. For example, the European Union Emission Trading Scheme (EUETS) has the new entrant and closure provision under which firms lose their permits upon exit (Ellerman and Buchner, 2007). The Waxman-Markey legislation proposed an output-based allocation (OBA) rule where firms receive emissions allowances proportional to their output levels. When permit allocation is conditioned on entry or production, however, firms receive a de facto entry/production subsidy, which may alter firms’ pollution-generating activities both at intensive (i.e., production/abatement) and extensive margins (i.e., entry/exit). In a recent paper, Hahn and Stavins (2011) point out that such a conditional distribution of permits is indeed one of the six ways in which the “independence property” of emissions trading can fail.1

We start with the Melitz-type economy (2003) consisting of a continuum of heterogeneous firms. In the model, the firms make endogenous entry, draw heterogeneous productivity shocks upon entry, and then produce in the monopolistically competitive industry using two inputs, labor and emissions, in a manner analogous to Copeland and Taylor (1994). The model then embeds a suit of conditional allocation rules under the ET policy. As in Melitz (2003) and other related studies, our analysis is restricted to comparison of stationary equilibria, wherein the distribution of all firm-level variables stays constant and firms form perfectly rational expectations about all the industry-level variables (including the price of permits) when making all relevant decisions. The advantage of this approach is its tractability, in particular with respect to the policy-induced effects on both the intensive and extensive margins.

We consider several permit allocation rules, all of which have been applied in practice

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1In the literature, the independence property of emissions trading is defined as the property that the emissions market equilibrium minimizes the total cost of abatement given the emissions cap and the equilibrium allocation of permits is independent of the initial distribution of permits (Hahn and Stavins, 2011).
and received significant attention in previous studies. These are, in the order of increasing latitude of conditionality: (i) auctioning, (ii) grandfathering with a permanent allocation rule (as in the U.S. Acid Rain Program), (iii) grandfathering with an entry/closure provision (as in the European Union Emissions Trading System), and (iv) grandfathering with an output-based allocation (OBA) rule (as discussed in previously proposed U.S. legislations). Considering these schemes one-by-one allows us to disentangle the equilibrium effects of each allocative design. For instance, we demonstrate that while (iii) has a direct impact on firms’ decisions at the extensive margin (i.e., entry and exit), (iv) influences those at both the intensive and extensive margins.

This paper contributes to four areas of research. First, the paper adds to the body of literature that has investigated linkages between environmental regulation and competitiveness of the manufacturing industry (see Jaffe et al., 1996 and Ambec et al., 2013 for extensive reviews). Empirical studies illustrate the impact of U.S. environmental regulation on firms’ output, productivity, and exit decisions in the manufacturing sector (e.g., Greenstone et al., 2012; Ryan, 2012). While these studies find convincing evidence for the causal linkages between environmental regulation and industry performance, the underlying economic mechanisms that induce changes in the size distribution of regulated firms still remain unclear—an aspect that the literature in industrial organization and international trade have found to play a crucial role in determining industry performance. Our paper offers a theoretical foundation to fully explain the mechanisms, and shows that the impact of emissions trading on the size distribution and the average firm profits depends on a number of factors: the conditionality of permit distribution, interactions between changes in entry-exit conditions and in aggregate accounting conditions, the factor intensity of entry, and coverage of non-pollution-intensive sectors in emissions trading.2 The proposed model could be readily extended to other types of environmental regulations such as emissions tax/subsidy and command-and-control policies.

Second, this paper complements a line of studies that incorporate Melitz’ framework in analyzing the impacts of trade liberalization on pollution (Kreickemeier and Richter, Forthcoming) or of environmental regulations on firms’ exports and emissions (Yokoo, 2009; Cui, Lapan, and Moschini, 2012). Kreickemeier and Richter (Forthcoming) assume a constant emissions rate per unit of output. Yokoo (2009) assumes a Copeland-Taylor framework in modeling firms’ variable emissions rates. Cui et al. (2012) also use the Copeland-Taylor framework, but augments it by incorporating firms’ binary technology choice.3 However,

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2We are currently working on a companion paper, empirically investigating the size distribution of firms and its relationship with the distribution of emissions intensities in the Japanese pollution-intensive industries.

3Our model ignores firms’ investments in abatement capital, and hence, firms’ dynamic responses to
none of these studies considers implications of grandfathering schemes, either for empirical implementation or for welfare analysis. As demonstrated in the paper, specific design features of grandfathered emissions trading have substantial implications for both entry-exit and aggregate accounting conditions in the Melitz-type economy, whose impacts can be analytically decomposed into five competing effects on economy-wide income, mass of firms, size distribution, price markup, and factor price. Because many existing emissions markets use grandfathering schemes in practice, these policy-induced differences may have important implications for identification and estimation in empirical studies and, therefore, can motivate future empirical studies. Furthermore, previous studies do not fully explore implications of the emissions cost in either the fixed input of production or entry. Our paper shows that different assumptions on either the fixed input of production or entry yield different theoretical predictions on the productivity cutoffs, which can be of empirical importance.

Third, there is a growing body of literature that has investigated the effects of conditional allocation rules in second-best settings with pre-existing distortions theoretically (Fischer and Fox, 2007; Jensen and Rasmussen, 2000) and empirically (Fowlie, Reguant, and Ryan, 2013). Fischer and Fox (2007) use a computational general equilibrium model to investigate the implications of allocation rules for domestic rebate programs in a static context. They find that an auctioned emissions trading outperforms an OBA rule in terms of social welfare, with a permit price under the auctioned system roughly equal to that under the OBA. Their focus is, however, not on the long-run equilibrium impacts, and hence, they do not address the allocative effects on either the intra-industry firm distribution or the mass of firms in the long run. Fowlie et al. (2013) use a dynamic partial-equilibrium model of an oligopolistic industry to empirically investigate the effects of alternative allocation rules in the U.S. cement industry. They find that dynamically updating permit allocations in proportion to production in the previous period does better than auctioning, for such an allocation rule can mitigate distortions from both emissions leakages and market power in the commodity market. Their approach takes into account firms’ dynamic responses in discrete technology investments to policy designs. Importantly, however, they assume a constant price of permits with a flat permit supply curve in the neighborhood of the cap, assuming that the cement industry is small relative to the overall emissions market. In contrast, ours is a general equilibrium analysis, with a vertical permit supply and an endogenous permit environmental regulations. A discrete technology choice, as in Cui et al., might potentially offer an important channel for future research. However, we have yet to see if such a channel adds substantially to the Melitz-type model of environmental regulations. For example, our model without such a discrete technology choice can still give rise to the same two testable hypotheses of Cui et al. model that “facility productivity is inversely related to emission intensity” and that “export status is negatively correlated with emissions intensity.” Exploring such a channel is left for our future research.
Though much of our paper is organized around a one-sector model, our analysis can be readily extended to multiple sectors where such an assumption is more useful (see Section 9). In this sense, our model is more general in its scope, yet is substantially more tractable than these studies, allowing us to address the size distribution and mass of firms in the long-run equilibrium — a gap in the literature we attempt to fill in.

Fourth, there exists a large body of literature that has investigated the distortionary effects of environmental regulations on entry-exit behavior in a variety of setups (e.g., Carlton and Loury, 1980; Spulber, 1985; Kohn, 1985, 1994; Collinge and Oates, 1982; McKitrick and Collinge, 2000; Pezzey, 2003). However, we are not aware of studies of the entry-exit problem that have explicitly considered heterogeneity of firms. Our analysis suggests that when firms are heterogeneous, there is a subtle and important interaction between the distortion on entry-exit conditions and that on aggregate resource constraints (or equivalently, between the size distribution of firms and the mass of firms) — a pathway that can motivate future empirical and theoretical works.

Our results, however, rest on two qualifying assumptions of the model, which present both advantages and disadvantages over existing studies. The first is the assumption of monopoly competition. Much of the existing literature on the theory of environmental regulations assumes either perfect competition or oligopolistic competition because pollution-intensive industries such as cement, iron and steel, natural gas, and non-ferrous metals have been traditionally perceived as homogeneous-good industries. However, at least some of these industries have increasingly become differentiated-good industries with substantial evidence of intra-industry trade. Dispersion measures of firm size within a sector in the U.S., which “captures the joint effect of the dispersion of firm productivity and the elasticity of substitution” (Helpman, Melitz, and Yeaple, 2004) are 1.48 for stone, minerals, and ceramics, 1.88 for ferrous metals, and 1.49 for non-ferrous metals — these numbers are roughly comparable to some of the well-known monopolistically competitive industries such as textiles (1.84) and apparel (1.57). Furthermore, some of the well-known differentiated-good industries such as chemical are also pollution-intensive. For instance, the organic and inorganic chemical industry accounts for 9.7%, 18.7%, 16.5%, 7.2%, 12.7% and 11.1% of the total emissions from all U.S. manufacturing processes in 1999 for CO, NH₃, NOₓ, PM10, SO₂ and VOC, respectively. These numbers are not small compared to the iron and steel industry, which accounts for 28.6%, 13.9%, 7.3%, 13.7%, 5.6%, and 4.4%, respectively (EPA Sector Notebook, 2005). The Melitz-type economy is known to yield theoretical predictions that are roughly consistent with empirical regularities in manufacturing industries including these pollution-intensive industries (e.g., Helpman et al., 2004; Eaton et al., 2011).

The second qualification is the full-employment assumption. Our model is a general equi-
librium model, and we explicitly use this assumption in deriving the mass of firms and the price of permits (but not the cutoff productivity). Indeed, an important contribution of the paper is this explicit account of the aggregate resource constraints in examining the policy-induced effects. Presumably, however, an introduction of emissions trading would cause reallocation of employment from pollution-intensive industries to less pollution-intensive industries. Hence, the full employment assumption would be more valid in the model incorporating two or more industries with different pollution intensities. Section 9 explores such a model, and shows that initial permit distributions to different sectors have important implications for the equilibrium price of permits as well as inter-industry reallocations of employment, emissions, and firms.

The paper is organized as follows. The next section describes our model environment, with auctioned emissions trading as a benchmark. We first describe the equilibrium properties of the model under the auctioned ET in comparison to no regulation in Section 3. We then examine the equilibrium properties under grandfathering with permanent allocation in Section 4, with entry/closure provision in Section 5, and with output-based allocation in Section 6. We explore welfare implications of our analysis in Section 7. Section 8 discusses alternative assumptions about the cost of emissions in entry. A model with multiple sectors is discussed in Section 9. The last section concludes.

2. The Model Setup

2.A. Regulatory Setup

We first touch on the regulatory environment. Let $Z^s > 0$ be a cap on aggregate emissions, which is assumed exogenous to the model (until section 7) and stay constant for all periods. The only regulatory variable of interest in this paper, therefore, is the allocation rules on the initial distribution of permits. The rules are announced once and for all periods, which firms observe prior to all relevant decisions. This approach is identical to that of Melitz (2003) in his analysis of the impact of international trade. In all cases, a continuum of firms participate in the emissions market with undifferentiated permits, so that the emissions market is perfectly competitive. In this section, we describe our benchmark model for the case of auctioned emissions trading (ET). We then examine the impacts of alternative allocation rules one by one in subsequent sections.

2.B. Demand
Consider an economy characterized by both pollution-intensive production and monopolistic competition (e.g., chemical, iron and steel, and non-ferrous metals). The preferences of a representative consumer are given by the Dixit-Stiglitz CES utility with an additional disutility from aggregate pollution:

$$U = \left[ \int_{\omega \in \Omega} q(\omega)^\rho d\omega \right]^{\frac{1}{\rho}} - L h(Z),$$  \hfill (1)

where $\omega$ is an index of commodities, $\Omega$ the measure of the set of available goods, $L$ is the population size, and $h$ is a convex function of aggregate emissions $Z$. The parameter $\rho$ represents the elasticity of substitution between commodities. We assume that $\rho \in (0, 1)$, i.e., the commodities are substitutes. The standard two-step procedure as in Dixit and Stiglitz (1977) yields the following aggregate price index:

$$P = \left[ \int_{\omega \in \Omega} p(\omega)^{1-\sigma} d\omega \right]^{\frac{1}{1-\sigma}},$$  \hfill (2)

where $\sigma \equiv 1/(1 - \rho) > 1$. Assuming that individual consumers ignore the term $h(Z)$ in making the consumption decision,\(^4\) we obtain the following standard formulas for consumer demand and expenditures:

$$q(\omega) = Q \left[ \frac{p(\omega)}{P} \right]^{-\sigma} \quad \text{and} \quad r(\omega) = R \left[ \frac{p(\omega)}{P} \right]^{1-\sigma},$$  \hfill (3)

where $r(\omega) = p(\omega)q(\omega)$, $Q = \left[ \int_{\omega \in \Omega} q(\omega)^\rho d\omega \right]^{\frac{1}{\rho}}$ is the aggregate quantity index, and $R = PQ$ is the economy’s total expenditure/income.

2.C. Production and Abatement

As in Melitz (2003), each firm is endowed with productivity $\phi \in [0, \infty)$ and employs only one input, labor, which is inelastically supplied at the aggregate level $L^s$. For expositional ease, higher $\phi$ represents higher productivity. Unlike in Melitz (2003), firms discharge pollution as a by-product of production. Firms have access to abatement technologies, which also use labor to reduce emissions. Following Copeland and Taylor (1994), the joint production

\(^4\)This assumption is justified by assuming the representative consumer consists of a continuum of consumers.
function can then be written as:

\[ q = \begin{cases} 
\phi z^{\beta l^{1-\beta}} & \text{if } z < \lambda l, \\
\phi Al & \text{otherwise,}
\end{cases} \quad (4) \]

where \( \lambda > 0 \) is the bound on the substitution possibility between labor and pollution inputs and \( A = \lambda^\beta. \)

The cost function consists of a variable component as well as a fixed overhead component, both of which are assumed to incur the cost of emissions (e.g., a factory or equipment emits a certain amount of pollution irrespective of the amount of output as long as in operation). To avoid undue complexity, the fixed component of production is assumed to have the same emissions intensity as the variable component, as in Bernard, Redding, and Schott (2007). Under these assumptions, firm’s cost minimization with respect to both variable and fixed inputs yields the following cost function:

\[ c(q) = \left[ \frac{q}{\phi} + f \right] \tau^\beta w^{1-\beta}, \quad (5) \]

where \( \tau \) is the price of emissions permits and \( w \) is the unit cost of labor, which we normalize to equal 1.

Given the cost function and input prices, the firm maximizes its profit:

\[ \max p^r(q)q - c(q), \quad (6) \]

where \( p^r(\cdot) \) is a residual demand curve given by (3). Maximizing (6) along with (5) yields

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5 Because output must be bounded above for a given level of labor input, the substitution possibility between labor and pollution must be bounded by some \( \lambda > 0 \). When \( \tau \) is zero (no regulation) or sufficiently low, firms would attempt to substitute more pollution for labor, eventually reaching the maximum substitution possibility. See Copeland and Taylor (1994) for a detailed discussion on this production function.

6 The assumption on the emissions intensities is not innocuous, and has important implications for our results. In our companion paper, we consider a model without a fixed cost of emissions. We shall revisit this issue in Section 8.

7 To be more precise, firm’s cost-minimization along with (4) would yields the following cost function:

\[ c(q) = \left[ \frac{q}{\phi} + f \right] \frac{\tau^\beta w^{1-\beta}}{\beta^\beta (1 - \beta)^{1-\beta}}. \]

As in Bernard et al. (2007), we redefine the unit so that the cost function (5) will be used throughout the paper.

Furthermore, the firm’s cost-minimizing choice of \( l \) and \( z \) must satisfy \( z = \lambda l \) if input price ratio \( w/\tau \) exceeds the marginal rate of technical substitution along the ray \( z = \lambda l \). With \( w \) normalized to 1, the condition can be written \( \tau \leq \beta/(1 - \beta) \lambda \). From here on, we assume that \( \tau \) is large enough to induce emissions reduction beyond the no-regulation level: \( \tau > \beta/(1 - \beta) \lambda \).
firm’s optimal markup:

\[ p(\phi) = \frac{\tau^\beta}{\rho \phi}. \]  

(7)
as well as output quantity, revenues, and variable part of emissions:

\[ q(\phi) = Q \left( \frac{P \rho \phi}{\tau^\beta} \right)^\sigma, \quad r(\phi) = R \left( \frac{P \rho \phi}{\tau^\beta} \right)^{\sigma-1}, \quad z_{pv}(\phi) = \frac{\rho^\beta \tau}{\tau} R \left( \frac{P \rho \phi}{\tau^\beta} \right)^{\sigma-1}. \]  

(8)

It then follows that the ratios of any two firms’ outputs, revenues, and emissions can be conveniently expressed as the functions of ratios of their productivity levels for all policy regimes.

\[ \frac{q(\phi_1)}{q(\phi_2)} = \left( \frac{\phi_1}{\phi_2} \right)^\sigma, \quad \frac{r(\phi_1)}{r(\phi_2)} = \left( \frac{\phi_1}{\phi_2} \right)^{\sigma-1}, \quad \frac{z_{pv}(\phi_1)}{z_{pv}(\phi_2)} = \left( \frac{\phi_1}{\phi_2} \right)^{\sigma-1}. \]  

(9)

Thus more productive firms are larger, not only in output and revenues (as in Melitz), but also in variable emissions. Moreover, the ratio of any two firms’ emissions rates (i.e., variable emissions per unit of output) is an inverse of the ratio of the two firms’ productivity levels.

\[ \frac{z_{pv}(\phi_1)/q(\phi_1)}{z_{pv}(\phi_2)/q(\phi_2)} = \frac{\phi_2}{\phi_1}. \]  

(10)

Because firms’ fixed emissions do not vary by productivity, these relationships imply that more productive firms emit more in absolute terms, yet emit less per unit of output.

Though it seems quite intuitive that more productive firms tend to be larger in all firm-level variables, it is not necessarily obvious why more productive firms need to have less emissions rates. But this follows directly from the Copeland-Taylor framework. Because firms use emissions as an input for production and because more productive firms can produce more given any input levels, more productive firms emit less for a given output level, including the profit-maximizing output level. Indeed, the empirical evidence suggests this result is consistent with observed firm behavior (Cui et al., 2012; Cole, Eliott, and Shimamoto, 2005; Mazzanti and Zoboli, 2009; and Shadbegian and Gray, 2006).

Lastly, the variable part of the profit equals \((p - \tau^\beta / \phi)q\). Therefore, we can rewrite firm’s profit as:

\[ \pi(\phi) = \frac{r(\phi)}{\sigma} - f \tau^\beta. \]  

(11)

Because firm’s revenue is increasing in \(\phi\), firm’s profit is also increasing in \(\phi\) per equation (11).

2.D. Entry-Exit Conditions
Prior to entry, each entrant first pays a fixed entry cost. This entry cost represents the unrecoverable cost of intangible and tangible resources devoted to entry such as research and development, learning about the industry, obtaining business licenses, and clearing environmental assessments. In the base model, we assume this entry activity discharges pollution at the same emissions intensity as production, so that the entry cost takes the following form:

\[ fe^{-\beta}w^{1-\beta}, \quad fe > 0, \tag{12} \]

where \( w \equiv 1 \) as before. Though the assumption is consistent with Bernard et al. (2007), by this, we are also assuming that firms need to buy permits for emissions generated not only through production but also through entry, and that the regulatory authority has the monitoring and enforcement capacity to ensure that. We shall discuss the implications of alternative assumptions on the entry cost in Section 8.

After paying the entry cost, the firm observes its productivity level \( \phi \), drawn from a common distribution \( G \) that has a positive support over \((0, \infty)\) with density \( g \). Each successful entrant produces a unique commodity in the commodity market and buy/sell pollution permits in the emissions market. The firms that make zero or negative profits exit the market immediately. Firms then face an exogenous probability \( \delta \) of adverse shocks each period that force them to exit the market. Let \( \phi^* \) be the cutoff productivity level such that \( \pi(\phi^*) = 0 \). Using firm’s profit (11), we see that \( \pi(\phi^*) = 0 \) implies:

\[ r(\phi^*) = \sigma f^{-\beta}. \tag{13} \]

Because \( \pi(\cdot) \) is increasing in \( \phi \), firms with \( \phi < \phi^* \) immediately exit and never produce.

The distribution of incumbent firms then is determined by the initial distribution \( G \) of productivity shocks, conditional on successful entry:

\[ \mu(\phi) = \begin{cases} \frac{g(\phi)}{1 - G(\phi^*)} & \text{if } \phi \geq \phi^* \\ 0 & \text{o.w.} \end{cases}, \tag{14} \]

Hence, the cutoff \( \phi^* \) uniquely defines the distribution of firm-level productivity, which also uniquely defines the distributions of all firm-level variables such as emissions, outputs, and revenues. Substituting (13) and (9) in (11) and taking the conditional average of firms’ profits, we see that the average profit \( \bar{\pi} \) satisfies:

\[ \bar{\pi} = \pi \left( \frac{\phi}{\bar{\phi}} \right) = \left[ \left( \frac{\phi}{\phi^*} \right)^{\sigma^{-1}} - 1 \right] f^{-\beta}, \tag{ZCP} \]
where $\tilde{\phi}$ is the weighted average productivity defined by:

$$
\tilde{\phi}(\phi^*) = \left[ \int \phi^{\sigma-1} \mu(\phi) d\phi \right]^{\frac{1}{\sigma-1}}.
$$

The equation (ZCP) implicitly defines the exit (and shutdown) condition, since it describes the relationship between the cutoff productivity $\phi^*$ and the average profit $\bar{\pi}$ implied by firms’ exit behavior.

To pin down the long-run equilibrium, we also need to derive the entry condition. To do so, we follow Melitz (2007) and Bernard et al. (2007), and focus on the stationary (and steady-state) equilibrium in which all aggregate variables as well as the mass and distribution of incumbent firms stay constant over time.\(^8\) The stationary equilibrium concept is useful for our analysis not only because of its tractability but also because it is a dynamic-model analogue of the long-run equilibrium concept employed in the conventional static models of environmental regulations.

Because a potential entrant is uncertain as to its productivity prior to entry, the entrant enters the market if and only if its \textit{ex ante} expected value of entry is higher than or equal to the fixed cost of entry. In the stationary equilibrium, a successful entrant with productivity $\phi$ earns $\pi(\phi)$ and faces the probability of death $\delta$ in all periods, so that its value of entry is equal to $\sum_{t=0}^{\infty} (1-\delta)^t \pi(\phi) = \pi(\phi) / \delta$. The \textit{ex ante} expected value of entry then is $E[\pi(\phi) / \delta] = p_{in} (\bar{\pi} / \delta)$, where $p_{in} = 1 - G(\phi^*)$. Free entry implies that entry should occur until all net expected profits are exhausted. Thus entry should occur until:

$$
(1 - G(\phi^*)) \frac{\bar{\pi}}{\delta} - \frac{fe}{\tau^2} = 0.
$$

The equation (FE) defines the entry condition.

Because (ZCP) and (FE) equations jointly constitute the entry-exit condition, any potential distortions due to the conditional allocation rules should, in principle, appear in these equations. Substituting (ZCP) into (FE), we obtain the equation that governs firms’ entry-exit behavior that must hold in equilibrium:

$$
(1 - G(\phi^*)) \left[ \left( \frac{\tilde{\phi}}{\phi^*} \right)^{\sigma-1} - 1 \right] = \delta \frac{fe}{f}.
$$

\(^8\)The stationary equilibrium concept has been employed in Melitz (2003) and a number of subsequent studies for several reasons. First, the empirical literature finds the size distribution of firms that persists over time. Second, the theoretical literature suggests that a history of firm-specific independent shocks in a dynamic process can generate such a stationary distribution of productivity (e.g., Luttmer, 2007). Third, while it is possible to incorporate an evolution of size distributions over time in the spirit of Ericson and Pakes (1995) or Hopenhayn (1992), such models tend to be substantially less tractable.
Note that the combined entry-exit condition is solely a function of exogenous parameters of the model, so is the cutoff productivity $\phi^*$. Hence, the existence and uniqueness of $\phi^*$ is ensured (see Melitz (2003) for the proof).

2.E. Aggregate Conditions

Once the cutoff productivity is determined, aggregate resource constraints must bind the mass of firms (or equivalently, mass of varieties) that can be supported in the stationary equilibrium. In the model, the economy is inherently endowed with the labor supply $L^s$, which must be allocated for use in either abatement, investment (by new entrants), or production. Because distortions in the entry-exit condition changes the distribution of firms, they may also affect the average behavior of the firms. It then follows that the entry-exit distortions may, through aggregate resource constraints, affect the equilibrium mass of firms $M$. Furthermore, as we shall see later, conditional allocation rules can directly change the aggregate resource constraints, because free distribution of permits under grandfathering may confer the pollution endowment $Z^s$.

In the case of auctioned ET, labor is used either in abatement, production, or investment by new entrants. Equivalently, in our model, emissions and labor are used in either production or investment by new entrants. (Recall that in the Copeland-Taylor framework, emissions as a by-product of production and investment are translated into an input for production and investment). Assume, as in Copeland and Taylor (1994) and other related literature, the government recycles back the revenues from auctioned permits, in a lump-sum manner, to the consumers. Then the sum of aggregate payments to labor and pollution permits used in production must equal the difference between the aggregate revenue and the aggregate profit: $L_p + \tau Z_p = R - \Pi$. On the other hand, the sum of aggregate payments to labor and pollution permits used in investment must equal the aggregate cost of entry: $L_e + \tau Z_e = N f_e$, where $N$ is a mass of new entrants. Because all aggregate variables remain constant in all periods, a mass of successful entrants $(1 - G(\phi^*))N$ must equal the mass of firms $\delta M$ that are hit by adverse shocks. Combining these with (FE), we have:

$$L_e + \tau Z_e = \frac{\delta M}{(1 - G(\phi^*))} = M \overline{\pi} = \Pi.$$

Hence, $L = L_p + L_e = R - \Pi - \tau Z_p + \Pi - \tau Z_e = R - \tau Z$. In other words, the aggregate income must equal the sum of aggregate payments to labor and pollution permits:

$$R = L + \tau Z.$$  \hfill (17)
Now consider the accounting equation for aggregate emissions:

\[ \tau Z = \tau Z_{pv} + \tau Z_{pf} + \tau Z_e, \]

where \( Z_{pv} \) and \( Z_{pf} \) stand for the variable part and the fixed part of aggregate emissions from production, respectively. From individual firms’ optimality conditions, \( \tau z_{pv}(\phi) = \rho \beta r(\phi) \) and \( \tau z_{pf} = \beta f \tau^\beta \). Integrating them over all firms, we have \( \tau Z_{pv} = \rho \beta R \) and \( \tau Z_{pf} = \beta M f \tau^\beta \). Moreover, the Cobb-Douglas specification of the entry cost implies \( \tau Z_e = \beta \Pi = \beta M \bar{\pi} \).

Hence, we can re-write this accounting equation as follows.

\[ \tau Z = \rho \beta R + \beta M f \tau^\beta + \beta M \bar{\pi}. \]  

(18)

Applying \( M = R/\bar{r} \) and \( \bar{r} = \sigma(\bar{\pi} + f \tau^\beta) \) and canceling terms, we obtain the demand for permits as a function of the economy-wide income:

\[ \tau Z = \beta R, \]  

(19)

which says that the share of the aggregate payments to pollution permits in the aggregate expenditure must equal the emissions intensity.

Using (17) and (19), we obtain the equilibrium price of permits under the auctioned ET:

\[ \tau = \frac{\beta L_s}{(1 - \beta) Z_s}. \]  

(20)

for a given cap on emissions \( Z_s \). Moreover, using \( M = R/\bar{r} \) and \( \bar{r} = \sigma(\bar{\pi} + f \tau^\beta) \) along with (17) and (19), the equilibrium mass of firms under auctioned ET is given by:

\[ M = \frac{L^s}{\sigma(1 - \beta)(\bar{\pi} + f \tau^\beta)}. \]  

(21)

Once the distribution and the mass of firms are identified, all aggregate variables can be readily determined as follows:

\[ P = M^{\frac{1}{\bar{r}}} p(\bar{\phi}), \quad Q = M^{\frac{1}{\bar{r}}} q(\bar{\phi}), \quad R = PQ = M r(\bar{\phi}), \quad \Pi = M \pi(\bar{\phi}), \quad Z = M z(\bar{\phi}). \]  

(22)

These relationships mean, as in Melitz (2003), that because the weighted average of the firm’s productivity levels \( \bar{\phi} \) is independent of the number of firms \( M \), an industry comprised of \( M \) with any distribution that yields the same average productivity \( \bar{\phi} \) behaves the same way as an industry with \( M \) representative firms having the same productivity \( \phi = \bar{\phi} \). Hence, the impacts of emissions trading on aggregate variables can be conveniently analyzed as if
they impact only the mass of firms and the average behavior of the firms, despite the fact that emissions trading may influence different firms differently.

3. Impact of Auctioned Emissions Trading

We shall start by analyzing the effects of an auctioned ET, relative to no regulation, so as to distinguish the effects of emissions trading from those of particular allocation schemes. For expository ease, we shall use subscript \( i = n \) to denote variables for no regulation and \( i = a \) for the auctioned ET.

Under no regulation, firms face no price of pollution, and therefore, produce at a maximum substitution possibility between labor and emissions: \( q = \phi Al \). Hence, the cost function becomes

\[
c_n(q) = \left[ \frac{q}{\phi} + f \right] w, \tag{23}
\]

where \( w \equiv 1 \) as before. Maximizing (6) along with (23) and following the same steps as before, we obtain the average profit:

\[
\bar{\pi}_n = \pi_n(\bar{\phi}_n) = \left[ \left( \frac{\bar{\phi}_n}{\phi_*} \right)^{\sigma^{-1}} - 1 \right] f. \tag{24}
\]

Combined with the same free-entry condition as before, this yields the entry-exit equation under no regulation:

\[
(1 - G(\phi_*^n)) \left[ \left( \frac{\bar{\phi}_n}{\phi_*^n} \right)^{\sigma^{-1}} - 1 \right] = \frac{\delta f_e}{f}. \tag{25}
\]

Note that this entry-exit condition is identical to that under the auctioned ET. Hence, we have \( \phi_*^n = \phi_*^a \).\(^9\) This implies that the auctioned ET does not alter the entry-exit condition.

\(^9\)To be more precise, \( f \)'s are different under no regulation and under emissions trading. In the absence of regulation, the fixed cost of production is given by:

\[
\min\{wl \mid Al \geq y_f, l \geq 0\} = f_n w,
\]

where \( f_n = y_f/A \), while under emissions trading:

\[
\min\{wl + \tau z \mid z^\beta l^{1-\beta} \geq y_f, l \geq 0, z \geq 0\} = f \tau^\beta w^{1-\beta},
\]

where \( f = y_f / \beta^\beta (1 - \beta)^{1-\beta} \). (Parameter \( y_f \) represents the size of the fixed cost). The fixed costs of entry, \( f_{ne} \) and \( f_e \), can be derived analogously. Then the ratios of these fixed costs would still satisfy \( f_e / f = f_{ne} / f_n \). Thus \( \phi_*^n = \phi_*^a \) holds.
The result that $\phi^*_n = \phi^*_a$ and that $\phi^*_a$ does not depend on $\tau$ may appear somewhat counter-intuitive, as it implies that a naive conjecture — that a positive price of emissions may induce exit of less productive (and more pollution-intensive) firms — fails here.

Though it may seem counter-intuitive, the result indeed closely parallels that of Spulber (1985) and of Baumol (1988), which states that both (auctioned) emissions trading and emissions tax induce efficient entry/exit in the long run. Indeed, the economic mechanism that underlies in Spulber is the same as in ours. The key to understanding the result is to see that firm’s profit can be completely written as the sole function of firm’s revenue as in (11). Because all firms face increased marginal cost of production due to the price of pollution and can readjust their markup prices in proportion to their productivity levels, the presence of the positive permit price affects all firms the same way, including those entrants who decided to exit the market. Hence, the auctioned emissions trading does not favor any particular firm, either productive or unproductive, and thus does not have any impact on industry-wide allocation of firm-level variables. As a result, exactly the same type of firms stay in the market, each with a higher price, a lower output quantity, and a lower labor input level due to increased marginal cost. Section 8 explains more fully that the factor intensity in entry is the real driver in determining whether or not an increased permit price would induce exit of less productive firms.

Though the auctioned ET does not alter the entry-exit condition, it does result in a smaller mass of firms and less entry in equilibrium, which again mirrors the result of Spulber and Baumol. With no regulation, labor is used either in production $L_p$ or investment $L_e$ by new entrants, but not in abatement. The aggregate payments to labor used in production must equal the difference between the aggregate revenue and the aggregate profit: $L_p = R - \Pi$. On the other hand, the aggregate payments to labor used in investment must equal the costs incurred by the new entrants: $L_e = N_f e$. Following the same logic as in the case of auctioning, we have $L_e = \Pi$. Hence, $L = L_p + L_e = R - \Pi + \Pi = R$. Using the fact that $M_n = R/\bar{r}_n$ and $\bar{r}_n = \sigma(\bar{\pi}_n + f)$, we have:

$$M_n = \frac{L^s}{\sigma(\bar{\pi}_n + f)}.$$  

(26)

To compare $M_a$ and $M_n$, first observe that $\bar{\pi}_a > \bar{\pi}_n$ because $\tau^\beta w^{1-\beta} > w$: i.e., the marginal cost of production is higher with than without regulation.\(^{10}\) It may seem counter-intuitive that the average firm profit is higher with emissions trading than without it. But

\(^{10}\)Inequality $\tau^\beta w^{1-\beta} > w$ also implies the relative factor price $\tau/w$ is greater than 1, but this is only a consequence of the definition of units used in our cost functions. All we require is $\tau/w > \beta/(1 - \beta)\lambda$, which can ensure the marginal cost under regulation, $\tau^\beta w^{1-\beta}/\phi \beta^\beta (1 - \beta)^{1-\beta}$, exceeds the marginal cost under no regulation, $w/\phi \lambda^\beta$. 

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this occurs because firms need to be more profitable to make up for the cost of pollution to stay active in the market. Now substitute (ZCP) and (24), respectively, into (21) and (26). We then see that:

\[ \frac{M_a}{M_n} < 1, \]

where the inequality follows because \( \tau > \frac{\beta}{1 - \beta} \lambda \) by assumption. Because \( N = \delta M/(1 - G(\phi^*)) \), this also implies new entry is smaller under the auctioned ET than no regulation.

**Proposition 1** An auctioned emissions trading does not alter the entry-exit condition, yet reduces the mass of firms and new entry relative to no regulation. That is, regardless of the size of \( Z^* \),

\[ \phi_a^* = \phi_n^*, \quad M_a < M_n, \quad N_a < N_n. \]

Furthermore, the average firm profit is higher under the auctioned ET than under no regulation (i.e., \( \bar{\pi}_a > \bar{\pi}_n \)) and is decreasing in the emissions cap \( Z^* \).

### 4. Impact of Permanent Allocation Rule

We now examine the impacts of grandfathering under the permanent allocation rule \( (i = PA) \), relative to auctioning \( (i = a) \). A point of departure for our analysis is the entry-exit condition embodied in (ZCP) and (FE). With permanent allocation, firms who receive permits upon entry retain the permits upon exit, whereas firms who did not receive permits need to buy permits every period from other firms who hold them. In other words, the initial distribution of permits is permanent regardless of firms’ entry/exit status. Such a rule is used in the U.S. Acid Rain Program.

Because the firms who receive permits upon entry (i.e., firms who enter in \( t = 0 \)) can sell permits upon exit, the exit condition for such firms is

\[ \frac{r(\phi^*)}{\sigma} - f \tau^\beta + \tau z^*(\phi^*) = \tau z^*(\phi^*). \]

On the other hand, the firms who do not receive permits upon entry (i.e., firms who enter in subsequent periods) must buy permits in the auction or emissions market. Hence, the exit condition for such firms is identical to the case of auctioning:

\[ \frac{r(\phi^*)}{\sigma} - f \tau^\beta = 0. \]
Either way, firm’s (economic) profit is $\pi(\phi) = r(\phi)/\sigma - f\tau^\beta$ so that $r(\phi^*) = \sigma f\tau^\beta$, which is identical to (13) in the case of auctioning. Following the same steps as under the auctioned ET, we obtain the entry-exit equation under permanent allocation:

$$
(1 - G(\phi^*_{PA})) \left[ \left( \frac{\phi^*_{PA}}{\phi^*_{PA}} \right)^{\sigma-1} - 1 \right] = \frac{\delta f_e}{f}.
$$

Because (27) is the same as (16), the cutoff productivity stays the same as under the auctioned ET.

How about the mass of firms? Under the permanent allocation rule, only firms that enter in period $t = 0$ are given some permits for free and retain them forever after (even after they exit), whereas no firms that enter in the subsequent periods are given permits so they must buy them in the auction or the emissions market. In the long run, therefore, no active firms hold permits for free, yet the firms who received permits but left the market continue to hold and sell the permits every period forever. The value of freely distributed permits to these inactive firms is preserved in the economy, but will be accounted for in the demand side, not in the supply side. Hence, the payments by new entrants to these inactive firms who hold permits work exactly as a lump-sum transfer of the auction revenues to the consumers in the case of auctioning. Therefore, in the (long-run) stationary equilibrium, all the aggregate accounting conditions are the same as under the auctioned ET. Importantly, because these arguments do not depend on how permits are distributed initially to which firms, the independence property holds under the permanent allocation rule.

**Proposition 2** Suppose that given the cap on aggregate emissions $Z^*$, the regulatory authority allocates permits freely with a permanent allocation rule. Then in the stationary equilibrium, the outcome of emissions trading is the same under grandfathering as under auctioning (i.e., $\phi^*_a = \phi^*_{PA}$, $M_a = M_{PA}$, $N_a = N_{PA}$, and $\tau_a = \tau_{PA}$) regardless of the initial distribution of permits.

5. Impact of Closure Provision

Under grandfathering with a closure provision ($i = CP$), incumbent firms are allocated some amount of permits freely, yet they lose the permits on a certain condition. The condition is usually firm’s exit, as with the case of the European Union Emission Trading Scheme (Ellerman and Buchner, 2007). The closure provision makes the initial distribution of permits non-permanent. The independence property under the permanent allocation rule is the
result of the fact that initial permits are distributed *unconditional* on all relevant economic decisions, including entry and exit. With the closure provision, however, the initial assignment of permits upon entry serves as an entry subsidy, whereas the loss of permits upon exit serves as an exit tax. We shall demonstrate that this *de facto* subsidy-tax scheme on entry and exit causes two types of effects, and as a result, at least one qualification for the independence property would fail. However, we shall also see that it is still possible for the regulatory authority to devise an allocation rule so that at least some of the allocative outcomes, such as the cutoff productivity and the permit price, would remain intact.

To demonstrate these points, let us consider a generic allocation scheme in which firms receive permits in proportion to a baseline “business-as-usual (BAU)” emissions level under no regulation. In practice, such a baseline could be a historical average or an industry average. All we require is that the baseline needs to be predetermined so that it is exogenous to all relevant decisions such as production and abatement. Furthermore, we assume that firms receive these permits only when they are in operation, and the closure provision requires firms to forego permits when they cease operation.\(^\text{11}\) More specifically in terms of our model, this allocation rule implies:

\[
z_{CP}^s(\phi) = \begin{cases} 
(\frac{\phi}{\phi^*})^{\chi-1} z_b & \text{if produce} \\
0 & \text{o.w.}
\end{cases}
\]  

where \(z_b\) is the baseline emissions level and \(\chi > 0\). Recall that firms’ variable emissions increase at the rate \(\sigma - 1\) in proportion to productivity \(\phi\). Hence, this formula means that more productive firms would be allocated disproportionately more permits if \(\chi > \sigma\) (i.e., allocation of permits is regressive), whereas less productive firms would be allocated more permits if \(\chi < \sigma\) (i.e., allocation of permits is progressive). We emphasize, however, that our objective here is *not* to evaluate the effects of any particular allocation rule. Rather, we consider this generic scheme to demonstrate the distortionary effects of *any conditional allocation rules that have such a closure provision*.

Because the firms must forego permits upon exit, the exit condition under this scheme is

\[
\pi(\phi^*) = \frac{r(\phi^*)}{\sigma} - f \tau^\beta + r(\phi^*) = 0,
\]

\(^{11}\)In this sense, this allocation scheme should be considered a production/closure provision rather than an entry/closure provision. While it may be of some interest to investigate the difference between the two, we leave that out to avoid undue complexity.
which implies \( r(\phi^*) = \sigma(f \tau^\beta - \tau z^*(\phi^*)) \). Using (9), we can re-write this as:

\[
\pi_{CP}(\phi) = \left( \frac{\phi}{\phi_{CP}} \right)^{\sigma-1} - 1 \right)f \tau^\beta - \tau \left[ \left( \frac{\phi}{\phi_{CP}} \right)^{\sigma-1} z^*(\phi_{CP}^*) - z^*(\phi) \right].
\]

Applying (28), the second term of the above equation can be re-written as:

\[
s(\phi; \chi, \sigma) \equiv s(\phi; \chi, \sigma) \equiv \tau \left[ \left( \frac{\phi}{\phi_{CP}} \right)^{\sigma-1} z^*(\phi_{CP}^*) - z^*(\phi) \right] = \tau z_b \left( \frac{\phi}{\phi_{CP}^*} \right)^{\chi-1} \left[ \left( \frac{\phi}{\phi_{CP}^*} \right)^{\sigma-\chi} - 1 \right].
\]

Because \( \phi \geq \phi_{CP}^* \) by assumption, we see that \( s(\phi; \chi, \sigma) \geq 0 \) if and only if \( \chi \leq \sigma \). It then follows that the ZCP condition is given as:

\[
\bar{\pi}_{CP} = \pi_{CP} \left( \tilde{\phi}_{CP} \right) = \left[ \left( \frac{\tilde{\phi}_{CP}}{\phi_{CP}^*} \right)^{\sigma-1} - 1 \right]f \tau^\beta - \tilde{s}(\chi, \sigma). \quad \text{(ZCP-CP)}
\]

Applying (33) and manipulating, we obtain:

\[
(1 - G(\phi_{i}^*)) \left\{ \left[ \left( \frac{\tilde{\phi}_{i}}{\phi_{i}^*} \right)^{\sigma-1} - 1 \right] - \tilde{s}(\chi, \sigma) \right\} = \frac{\delta f_e}{f} \quad \text{(30)}
\]

Because \( \tilde{s}(\chi, \sigma) \geq 0 \) if and only if \( \chi \leq \sigma \) and because the left-hand side without the term \( \tilde{s} \) is decreasing in \( \phi_{i}^* \) (see Melitz for its proof), we see that \( \phi_{CP}^*(\chi < \sigma) < \phi_{CP}^*(\chi = \sigma) = \phi_{PA}^* = \phi_a^* < \phi_{CP}^*(\chi > \sigma) \). \(^{12}\) That is, a progressive allocation would raise the average productivity of firms relative to auctioning while a regressive allocation would lower it.

\(^{12}\)It may appear that the entry-exit condition (30) depend on \( \tau \), and therefore, on the size of \( Z^* \). However, this is not true. The key here is that the size of \( z_b \) has to be restricted by the size of \( Z^* \):

\[
Z^* \equiv \int z^*(\phi)M\mu(\phi)d\phi = z_b M \int \left( \frac{\phi}{\phi_{n}} \right)^{\chi-1} \mu(\phi)d\phi.
\]

Applying this in \( \tilde{s} \) along with (33) and manipulating, we obtain:

\[
\tilde{s} = \frac{\Phi_1}{\Phi_2} \frac{\sigma \beta}{1 - \beta + \sigma \beta + \sigma \beta (\Phi_2 / \Phi_1)} f \tau^\beta,
\]

where

\[
\Phi_1 \equiv \int \left( \frac{\phi}{\phi_{n}} \right)^{\chi-1} \mu(\phi)d\phi, \quad \text{and} \quad \Phi_2 \equiv \int \left( \frac{\phi}{\phi_{n}} \right)^{\chi-1} \left[ \left( \frac{\phi}{\phi_{CP}^*} \right)^{\sigma-\chi} - 1 \right] \mu(\phi)d\phi.
\]

The term \( f \tau^\beta \) (and also \( z_b \)) in (30) cancels out. Hence, the entry-exit condition does not depend on either \( \tau \) or \( Z^* \).
The key to understanding this point is to recognize that firms receive implicit subsidies (taxes) if firms are distributed permits in such a way that enables them to be net sellers (buyers) of the permits in equilibrium. A progressive allocation would allow less (more) productive firms to be net sellers (buyers), whereas a regressive allocation would allow more (less) productive firms to be net buyers (sellers). For example, if permits are distributed based on a uniform emissions rate and the BAU output level, then given that firms’ emissions rates are decreasing in productivity, such an allocation rule would be regressive and induce exit of less productive firms.\textsuperscript{13} In contrast, if firms are allocated permits in proportion to their emissions, then the entry-exit condition is unaffected because such an allocation rule would result in the neutral distribution of permits and favor no particular firm at the intensive margin. This result mirrors that of Böhringer and Lange (2005) who find that allocating permits proportionally to past emissions allows firms to “face the same marginal benefits from emissions… in subsequent periods.”

We now examine another impact of the closure provision. To do so, we first derive the expression for the equilibrium mass of firms under the grandfathered ET with the closure provision that would hold for any value of $\chi$. One important distinction between the permanent and non-permanent allocation rules is that the value of freely distributed permits stay within the market (i.e., it remains with firms that operate in the market), because permits are given only to firms that stay active while the firms that exit must forego them. In terms of aggregate accounting conditions, this means that the sum of aggregate payments to labor and emissions used in production must equal the difference between the aggregate revenue (in this case, from sales of commodities and permits) and the aggregate profit: $L_p + \tau Z_p = R + \tau Z^* - \Pi$. That is, the incumbent firms receive the permits for free, which they can sell in the emissions market. On the other hand, the sum of aggregate payments to labor and pollution permits used in investment must equal the aggregate cost of entry: $L_e + \tau Z_e = N f_e$. Then following the same steps as under auctioning, we have $L_e + \tau Z_e = \Pi$. Hence, $L = L_p + L_e = R + \tau Z^* - \Pi - \tau Z_p + \Pi - \tau Z_e = R + \tau (Z^* - Z)$. Because the market clearing in the emissions market requires that total payments for permits equal the value of all permits, we have:

$$R = L.$$  

(31)

Comparing this with (17), we see that the aggregate revenue is lower under the closure

\textsuperscript{13}Such a rate-based allocation rule was used in the U.S. SO\textsubscript{2} Allowance Program, where each regulated unit received allowances roughly based on the fixed emissions rate (i.e., 2.5 lbs/mmBtu in Phase I and 1.2 lbs/mmBtu) and its historical fuel use (which has roughly one-to-one relationship to its electricity output), with some unit-specific bonus reserves. The allocation is permanent under the U.S. SO\textsubscript{2} Allowance Program. Instead, we are discussing the rate-based allocation in the context of entry/closure provision.
provision than under the auctioned ET given $L^s$.

Now consider the accounting equation for aggregate emissions (18). Unlike auctioning or permanent allocation, the cutoff profit (29) implies that the average revenue now has a term on the (average) value of permits: $ar{r} = \sigma(\bar{\pi} + f\tau^\beta - \tau \bar{z})$ instead of $\bar{r} = \sigma(\bar{\pi} + f\tau^\beta)$. Substituting this along with $M = R/\bar{r}$ into the accounting equation (18) and manipulating, we obtain the demand for permits as a function of the economy-wide income:

$$\tau Z = \frac{\beta R}{1 - \beta}.$$  \hspace{1cm} (32)

Moreover, (32) implies $\tau \bar{z}^s \equiv \tau Z^s/M = \tau Z/M = \beta L/(1 - \beta)M$. Applying this in $M = R/\bar{r}$, we obtain:

$$M_{CP} = \frac{L}{\sigma(\bar{\pi}_{CP} + f\tau^\beta - \tau \bar{z}_{CP}^s)} = \frac{(1 - \beta + \sigma\beta)L}{\sigma(1 - \beta)(\bar{\pi}_{CP} + f\tau^\beta)}.$$  \hspace{1cm} (33)

Now we see the two distortionary impacts of the closure provision on the mass of firms. First, there is a pure impact of the closure provision via its effect on aggregate resource constraints. That is, even when the regulator allocates permits in a neutral manner (i.e., $\chi = \sigma$, in which case $\phi_{CP}^* = \phi_a^*$ and $\bar{\pi}_{CP} = \bar{\pi}_a$), we still have:

$$M_a/M_g = \frac{1}{1 - \beta + \sigma\beta} < 1,$$

where the last inequality follows because $1 - \beta + \sigma\beta = 1 + (\sigma - 1)\beta > 1$ (recall $\sigma > 1$ by assumption). Second, there is a distortionary effect via its effect on the entry-exit condition. As we discussed above, the cutoff productivity and the average profit is increasing in $\chi$, so that $M_{CP}$ is decreasing in $\chi$. In other words, allocation rules that would induce exit of less productive firms (and induce entry of more productive firms) would support a smaller overall mass of firms.

Interestingly, though, such distortions on either the size distribution of firms or the mass of firms do not affect the equilibrium price of permits. Applying $R = L$ in (32) and solving for $\tau$, we obtain the price of permits with closure provision given the emissions cap $Z^s$:

$$\tau = \frac{\beta L^s}{(1 - \beta) Z^s}.$$  \hspace{1cm} (34)

This expression is identical to the price of permits under auctioning in (20). Hence, the price of permits is the same as under the auctioned scheme.

In sum, the closure provision may alter not only the entry-exit condition but also the aggregate resource constraints. Yet, the market forces completely absorb all these distortions, at least in the determination of the permit price. Grandfathering would endow firms with
transferable property rights, which raises the demand for permits relative to auctioning for a given economy-wide income $R$ (see (19) and (32)). With auctioning, on the other hand, the payments go to the government and eventually to the demand side, which raises the demand for permits relative to grandfathering (see (17) and (31)). In equilibrium, these two competing effects adjust perfectly to exactly offset each other, and hence, the price of permits is still unaffected. Because this result is independent of any particular allocation rules, the invariance of the permit price with respect to the initial distribution of permits still holds even with the closure provision, but allocative outcomes would still be different because the initial distribution matters for both the size distribution and mass of firms.\textsuperscript{14}

**Proposition 3** Suppose that given the cap on aggregate emissions $Z^s$, the regulatory authority allocates permits freely with a closure provision. Then neither the size distribution of firms nor the mass of firms is independent of the initial distribution of permits, yet the equilibrium price of permits still remains the same as under auctioning regardless of the initial distribution of permits.

6. Impact of Output-based Allocation Rule

With an output-based allocation rule ($i = OBA$), all new entrants are allocated some amount of permits freely as a rebate to their production. Firms forego the permits upon exit by definition because they do not produce after exit. Such a rule was proposed in the Waxman-Markey legislation, and several variations of it were investigated in the previous studies. For example, Fischer and Fox (2007) considered allocations based on firms’ output shares within each sector, whereas Fowlie et al. (2013) considered allocations based on an industry-specific emissions rate from a previous period. In this paper, we interpret the OBA as an allocation scheme based on firms’ output shares in the industry. Formally,

\[
z^{s*}_{OBA}(\phi) = \begin{cases} 
\frac{q_{OBA}(\phi)}{Q_{OBA}} Z^s & \text{if produce} \\
0 & \text{o.w.}
\end{cases}.
\] (35)

Note that in this formula, output shares are value-adjusted to account for the fact that products are differentiated across firms. Furthermore, the OBA rule serves as a \textit{de facto} rebate not only on firms’ \textit{production status} but also on \textit{production amounts}. Hence, the OBA

\textsuperscript{14}To be more precise, this permit-price invariance must be understood in terms of \textit{relative} input prices. That is, $\tau_a/w_a = \tau_g/w_g$, where wages are normalized to 1 in this economy.
rule may alter firms’ incentives directly not only at the extensive margin (i.e., entry/shutdown decisions) but also at the intensive margin (i.e., production/emissions decisions).

Before starting our analysis, we clarify one important assumption concerning how permit allocation is treated in our model. Given the emissions cap $Z^*$, the amount of permits each firm receives depends on its share in the aggregate output that would arise in the equilibrium under this allocation rule. Moreover, the emissions cap must equal the total amount of permits allocated for firms that enter and stay active in equilibrium. To ensure this, we assume that firms have perfect foresight about all aggregate economic variables, so that they can perfectly anticipate their own permit allocations $z^s$ prior to entry given the knowledge of $Z^*$ and the allocation rule. By this, we are implicitly assuming that firms only anticipate how many permits they would receive upon entering the market, and that firms do not expect either their entry/exit or their output/emissions to influence the distribution of permits. This behavioral assumption is employed in the study of the impact of permit allocation rules by Fowlie et al. (2013), and is also consistent with virtually all economic analyses of perfectly competitive markets concerning the equilibrium prices.

Under the OBA rule, the firm with productivity $\phi$ under the endogenous OBA maximizes, in place of (6),

$$p^r(q)q - c(q) + \tau \frac{q^\rho}{Q^\rho} Z^s,$$

where $p(q) = Q^\frac{1}{2} Pq^{-\frac{1}{2}}$ from (3). The necessary and sufficient condition for the firm’s optimization program yields the following

$$p(\phi) = \left(\frac{P \rho \phi \gamma}{\tau^\beta}\right)^{\frac{1}{\sigma}}, \quad q(\phi) = Q\left(\frac{P \rho \phi \gamma}{\tau^\beta}\right)^{\frac{1}{\sigma}}, \quad z_{pv}(\phi) = \frac{\rho \beta \gamma}{\tau} R \left(\frac{P \rho \phi \gamma}{\tau^\beta}\right)^{\frac{1}{\sigma}},$$

where $\gamma \equiv 1 + \tau (Z^*/R)$. Thus, again, the ratio of any two firms’ outputs and revenues are proportional to the ratio of the firms’ productivity. We can then re-write the firm’s profit as

$$\pi(\phi) = \left\{\left(\frac{\phi}{\phi^*}\right)^{\sigma-1} - 1\right\} f r^\beta - \tau \left\{z^s(\phi^*) \left(\frac{\phi}{\phi^*}\right)^{\sigma-1} - z^s(\phi)\right\},$$

where the expression inside the second braces cancels out because

$$z^s(\phi^*) \left(\frac{\phi}{\phi^*}\right)^{\sigma-1} - z^s(\phi) = \frac{q(\phi^*)^\rho}{Q^\rho} Z^s \left(\frac{\phi}{\phi^*}\right)^{\sigma-1} - \frac{q(\phi)^\rho}{Q^\rho} Z^s = 0.$$
Hence, the ZCP condition becomes identical to that under the auctioned ET:

\[ \bar{\pi}_{OBA} = \pi_{OBA} \left( \bar{\phi}_{OBA} \right) = \left[ \left( \frac{\bar{\phi}_{OBA}}{\bar{\phi}_{OBA}} \right)^{\sigma-1} - 1 \right] f \tau^\beta. \]  \tag{37} 

It follows that the cutoff productivity under the OBA rule is the same as that under the auctioned ET. This result indicates that the OBA does not alter the entry-exit condition, and as a result, the size distribution of firms remains the same as under the auctioned ET. The economic intuition behind this result is the same as the neutral allocation (\( \chi = \sigma \)) with the closure provision. Because firms are distributed permits in proportion to its output share (and conditioned on production rather than entry), and because each firm’s output is proportional to its productivity, the OBA rule results in the neutral distribution of permits and favors no particular firm. Put differently, the OBA rule serves like implicit subsidies on firms’ exogenous productivity levels.

Now consider the accounting equation for aggregate emissions:

\[ \tau Z = \tau Z_{pv} + \tau Z_{pf} + \tau Z_e. \]

From individual firms’ optimality conditions, \( \tau z_{pv}(\phi) = \rho \beta \gamma r(\phi) \) and \( \tau z_{pf} = \beta f \tau^\beta \). Integrating them over all firms, we have \( \tau Z_{pv} = \rho \beta \gamma R \) and \( \tau Z_{pf} = \beta M f \tau^\beta \). Moreover, the Cobb-Douglas specification of the entry cost implies \( \tau Z_e = \beta \Pi = \beta M \bar{\pi} \). Substitute these into the accounting equation above, and apply \( M = R/\bar{r} \) and \( \bar{r} = \sigma(\bar{\pi} + f \tau^\beta - \tau z^s) \). We then obtain:

\[ \tau Z = \rho \beta \gamma R + \frac{\beta R}{\sigma} + \beta \tau Z = \beta R + \rho \beta \tau Z + \beta \tau Z, \]

Solving this for \( \tau \) with \( R = L \), we obtain the price of permits with entry/closure provision given the emissions cap \( Z^s \):

\[ \tau = \frac{\beta L^s}{[1 - \beta(1 + \rho)] Z^s}. \]  \tag{38} 

Comparing (38) and (20), we see \( \tau_{OBA} > \tau_a \). Hence, the price of permits is higher under the OBA rule than under the auctioned ET.

Furthermore, following the same steps as before, we have:

\[ M_{OBA} = \frac{(1 - \beta(1 + \rho - \sigma)) L^s}{\sigma(1 - \beta(1 + \rho))(\bar{\pi}_{OBA} + f \tau^\beta)}. \]  \tag{39}
Comparing (39) with (21), we observe that:

\[
M_a/M_{OBA} = \frac{1}{1 - \beta (1 + \rho - \sigma)} \left[ \frac{1 - \beta (1 + \rho)}{1 - \beta} \right]^{1-\beta} < 1,
\]

where the last inequality follows because both the first and second terms are less than 1 (for the first term, note \(1 - \beta (1 + \rho - \sigma) = 1 + \beta \rho \sigma > 1\)).

In essence, firms face an incentive to increase output under the OBA rule because they can increase the receipts of permits by increasing their output shares. This perverse incentive in turn increases the demand for permits and thus the price of permits would be higher than under the auctioned ET for a given emissions cap \(Z^s\). The result is consistent with Fisher and Fox (2007).

In sum, though the OBA rule is indeed a conditional allocation rule, it would not affect the entry-exit condition, yet it would distort the aggregate resource constraints via its effect on firms’ output decisions. Consequently, it results in a violation of the independence property in terms of the price of permits. This result is in a sharp contrast to the case of the closure provision.

**Proposition 4** Suppose that given the cap on aggregate emissions \(Z^s\), the regulatory authority allocates permits freely in proportion to firms’ output shares. Then the size distribution of firms stays the same as under auctioning. Yet both the equilibrium price of permits and the mass of firms would be higher than under auctioning.

### 7. Welfare Implications

The main result of the previous sections — that design of emissions trading can affect size distribution, mass of firms, and permit price by altering the entry-exit and aggregate accounting conditions — has substantial implications for social welfare. In real-world settings, either the allocation scheme or the emissions cap or both are chosen primarily through political processes, and thus, are often outside the regulatory agency’s control. We shall see that given the emissions cap, social welfare can vary substantially across different allocation schemes, showing important interactions with size distribution, mass of firms, and permit price.

To demonstrate this point, we shall examine social welfare for a given emissions cap under three allocation schemes: auctioning, grandfathering with closure provision, and grandfathering with the OBA. As in Melitz, we use per capita utility of the aggregate consumer as the
measure of social welfare: \( W \equiv \frac{U}{L} = \frac{Q}{L} - h(Z) \). Observe that given the allowable economic resources \( L^* \) and \( Z^* \), social welfare depends entirely on the size of the aggregate output index \( Q \).

Recall first that \( Q = \frac{R}{P} \) and \( P = M^{\frac{1}{1-\sigma}} p(\tilde{\phi}) \). Applying the markup pricing rule (7) or (36), we see that the relative impacts of allocation schemes can be decomposed as follows: For any allocation schemes \( i \) and \( j \),

\[
\frac{Q_i}{Q_j} = \left( \frac{R_i}{R_j} \right) \times \left( \frac{M_i}{M_j} \right)^{\frac{1}{\sigma-1}} \times \left( \frac{\tilde{\phi}_i}{\tilde{\phi}_j} \right) \times \left( \frac{\mu_i}{\mu_j} \right)^{-1} \times \left( \frac{\tau_i}{\tau_j} \right)^{-\beta} . \tag{40}
\]

This equation indicates that the overall impact on aggregate output index is composed of five competing effects on: economy-wide income, mass of firms (or equivalently, product variety), weighted average productivity (or equivalently, size distribution), markup, and factor price (in this case, permit price only as \( w \) is normalized to 1).\(^{15}\) Higher aggregate income, mass of firms, and average productivity all tend to increase aggregate output, whereas higher markup factor and factor price tend to decrease it. Therefore, all of the identified intra-industry effects of allocation schemes discussed in previous sections will interact with one another in determining the aggregate output index and the social welfare. The question then is, which of the effects tends to dominate in each allocation scheme?

Let us first compare auctioning versus closure provision. Recall that the permit price stays the same between the two schemes and that the weighted average productivity \( \tilde{\phi} \) critically depends on the allocation rule \( (\chi) \) under the closure provision. To avoid undue complexity, let us consider the case of neutral allocation \( (\chi = \sigma) \), so we have \( \tilde{\phi}_a = \tilde{\phi}_{CP} \). Then the last two terms of (40) cancel out. As discussed in Section 5, the aggregate income is higher under auctioning than under closure provision by a factor \( (1 - \beta) \), which tends to favor auctioning, whereas the mass of firms is lower under auctioning than under closure provision, which tends to favor closure provision. It turns out the former dominates the latter. Substitute \( R_a = L/(1 - \beta) \) and \( R_{CP} = L \) and (21) and (33) into (40). We then have:

\[
\frac{Q_{CP}}{Q_a} = (1 - \beta) (1 - \beta + \sigma \beta)^{\frac{1}{\sigma-1}} \leq 1,
\]

where the last inequality follows because the ratio equals 1 when \( \beta = 0 \) and its derivative is negative.\(^{16}\) Hence, the aggregate output (and the welfare) is higher under auctioned ET

\(^{15}\)The markup factor \( \mu_i \) is \( 1/\rho \) for \( i = a, PA, \) and \( CP \), and \( 1/\rho \gamma = (1 - \beta (1 + \rho)) / \rho (1 - \rho \beta) \) for \( i = OBA \).

\(^{16}\)To see this, note that:

\[
\frac{\partial Q_{CP}}{\partial \beta} = \left[ 1 + (\sigma - 1) \beta \right]^{\frac{1}{\sigma-1}} \left\{ -1 + (1 - \beta) [1 + (\sigma - 1) \beta]^{-1} \right\}.
\]
than under grandfathered ET with closure provision for a given $Z^*$.

How about the OBA? As with the closure provision, the aggregate income is higher under auctioning than under OBA by a factor $(1 - \beta)$, the mass of firms is lower under auctioning than under the OBA, and the weighted average productivity is the same between the two schemes. In this case, however, the OBA also induces a higher price of permits than auctioning, which tends to favor auctioning, whereas the markup is smaller under the OBA than auctioning, which tends to favor the OBA. Thus, the aggregate output would be lower under the OBA than auctioned ET, unless the OBA induces a substantially larger mass of firms or a substantially smaller markup than auctioning. It turns out that it does. Substitute $R_a = L/(1 - \beta)$ and $R_{CP} = L$, mass-of-firms expressions (21) and (33), and permit-price expressions (20) and (38) into (40). Manipulating the terms, we obtain:

$$\frac{Q_a}{Q_{OBA}} = \frac{1}{1 - \rho \beta} \left[ \frac{1 - \beta (1 + \rho)}{1 - \beta} \right]^{\frac{1 - \beta}{\rho}} \left[ \frac{1 - \rho + \rho^2 \beta}{1 - \rho} \right]^{\frac{1}{1 - \sigma}} \leq 1,$$

where the last inequality follows because the ratio equals 1 when $\beta = 0$ and its derivative is negative.\(^\dagger\)

The discussion so far establishes that $Q_{CP} \leq Q_a \leq Q_{OBA}$: i.e., the OBA scheme induces the highest aggregate output (and social welfare) among the three allocation schemes given the emissions cap. In other words, emissions permits carry the highest welfare value under the OBA rule. The result is consistent with Fischer and Fox (2007) and Fowlie et al. (2013) as well as the rationale behind the Waxman-Markey legislation that the OBA is a viable means to compensate for the increased cost of pollution control.

A flip side of this result is that the emissions cap could be optimally adjusted to improve social welfare in second-best settings where choice over allocation schemes is politically constrained. A natural question then is, how should the regulator adjust the cap in such settings? To address this question, let us solve the second-best planner’s problem, in which the regulatory authority maximizes social welfare but is allowed to choose only $Z^*$, subject to a pre-determined allocation rule. Substituting relevant expressions for $R, M$, and $\tau$ as before where the first multiplicative term is positive and the second multiplicative term is:

$$-1 + (1 - \beta) \left[ 1 + (\sigma - 1) \beta \right]^{-1} \leq 0.$$

\(^\dagger\)The proof that the derivative is negative is rather involved and is available upon request.
and manipulating, we can rewrite the social welfare under each scheme:

\[
W_i = \begin{cases} 
\Gamma(\phi^*)Z^{\frac{\beta}{\rho}} - h(Z) & \text{if } i = a \\
\nu_{CP}\Gamma(\phi^*)Z^{\frac{\beta}{\rho}} - h(Z) & \text{if } i = CP \\
\nu_{OBA}\Gamma(\phi^*)Z^{\frac{\beta}{\rho}} - h(Z) & \text{if } i = OBA
\end{cases}
\]

where \(\nu_{CP} \equiv Q_{CP}/Q_a\) and \(\nu_{OBA} \equiv Q_{OBA}/Q_a\) are constants given the exogenous primitives of the model as shown above, and

\[
\Gamma(\phi^*) \equiv (1 - \beta)^{-1}\rho^* \left[ L/\{\sigma(1 - \beta)f\} \right]^{\frac{1}{\sigma - 1}} \left\{ (1 - \beta)/(\beta L) \right\}^{\frac{\beta}{\rho}}.
\]

Note that because \(\Gamma\) depends on \(\phi^*\), the optimal emissions cap should also depend on the size distribution in general.

As discussed in preceding sections, \(\phi^*\) does not depend on \(Z^s\) (regardless of \(\chi = \sigma\) or not). Hence, the first-order necessary condition for the optimum is: For scheme \(i\),

\[
(\beta/\rho) \nu_i \Gamma(\phi^*)Z_i^{\frac{\beta}{\rho} - 1} = h'(Z_i), \tag{41}
\]

where \(\nu_a = 1\). For ease of exposition, assume that the disutility from pollution \(h(\cdot)\) is given by \(h(Z) = Z^a\) \((a \geq 1)\). Because \(\beta\) and \(\rho \in (0, 1)\), \(\beta/\rho\) may or may not be less than 1. Hence, we need \(h\) to be sufficiently convex: i.e., \(a \geq \beta/\rho\). Then (41) implies:

\[
Z_{CP}^s/Z_a^s = (\nu_{CP})^{\frac{\beta}{\rho} - \frac{1}{\rho}} \quad \text{and} \quad Z_{OBA}^s/Z_a^s = (\nu_{OBA})^{\frac{\beta}{\rho} - \frac{1}{\rho}}.
\]

Because \(\nu_{CP}\) is less than 1 and \(\nu_{OBA}\) is greater than 1 as shown above, the second-best optimum satisfies \(Z_{CP}^s \leq Z_a^s \leq Z_{OBA}^s\) provided that \(a \geq \beta/\rho\). That is, because the OBA scheme carries the highest welfare value per unit of permits, the regulator should raise the cap for the OBA scheme relative to the auctioned scheme (and for the auctioned scheme relative to the closure provision). Or alternatively, we could impose curvature on the utility from \(Q\) and assume a constant marginal damage on \(h\). For example, if we instead assume

\[18\] To see this, note that the second-order condition is:

\[
(\beta/\rho)(\beta/\rho - 1)\nu_i \Gamma(\phi^*)Z_i^{\frac{\beta}{\rho} - 2} - a(a - 1)Z^{a - 2} \leq 0.
\]

Plugging in (41) and manipulating, we have

\[
\frac{\beta/\rho - 1}{a - 1} \leq 1,
\]

which requires \(a \geq \beta/\rho\).
\[ W = \log(Q/L) - hZ \text{ as a special case, then we would have } Z_{CP}^s = Z_a^s = Z_{OBA}^s. \]

**Proposition 5** Given emissions cap \( Z^s \), the social welfare and aggregate output index differ substantially across allocation schemes, due to changes in the entry-exit and aggregate accounting conditions: i.e., \( W_{OBA} \geq W_a \geq W_{CP(\chi=\sigma)} \) and \( Q_{OBA} \geq Q_a \geq Q_{CP(\chi=\sigma)} \). Furthermore, suppose \( h(Z) = Z^a \) and \( a \) is sufficiently large (i.e., \( a \geq \max\{1, \beta/\rho\} \)) to ensure the sufficient condition of the second-best planner’s problem in which choice over allocation schemes is politically constrained. Then the optimal emissions cap satisfies the following relationship: \( Z_{CP(\chi=\sigma)}^s \leq Z_a^s \leq Z_{OBA}^s \).

### 8. The Cost of Emissions in Entry

Some may argue that our main claim — the entry/closure provision can alter the size distribution and mass of firms, yet it still does not distort the price of permits — may depend substantially on the assumption of the same emissions intensities in production and entry. Indeed, it is plausible that firms may face different technologies between production and entry. Even if firms’ emissions intensities are the same, it may not be feasible, either economically or politically, for the regulatory authority to require entering firms to pay the full cost of emissions in entry prior to their operation. An important question then is, Is our independence result robust to different emissions intensities in production and entry. The answer turns out to be no—our result changes if firms face different emissions intensities in production and entry.

To demonstrate this point, let \( \beta_e \) be the emissions intensity in the fixed entry cost, and assume \( \beta_e \neq \beta \) in general. We shall focus on the case of \( \chi = \sigma \) under the closure provision discussed in Section 5, in comparison with the auctioned ET.

Consider first the entry-exit conditions. Because all the arguments excluding the fixed entry cost are still intact, the expressions for firm’s profit and the resulting zero-cutoff conditions remain the same as \((ZCP)\) and \((ZCP-CP)\), respectively, under auctioning and under the closure provision. In the free-entry condition, however, we have \( f_e \tau^{\beta_e} \) in place of \( f_e \tau^{\beta} \). Thus, substituting the zero cutoff condition into the free-entry condition, we obtain the equation that defines the entry-exit condition:

\[
(1 - G(\phi^*)) \left[ \left( \frac{\phi^*}{\phi^s} \right)^{\sigma-1} - 1 \right] \frac{f \tau^{\beta}}{\delta} = f_e \tau^{\beta_e}. \tag{42}
\]
Unlike (16) or (30), the cutoff productivity now depends on the price of permits $\tau$. The expected value of entry on the left-hand side and the cost of entry on the right-hand side depend on the factor intensities of production and entry, respectively. These factor intensities, together with the price of emissions (relative to wage), determines whether the cutoff productivity increases relative to the case where $\beta = \beta_e$. For example, suppose $\beta > \beta_e$, so that $\tau^\beta > \tau^{\beta_e}$ with $\tau/w > 1$. In this case, production would be relatively more costly than entry for a given price of permits. Hence, an increase in the permit price would increase the expected value of entry more than the cost of entry (given $\phi^*\tau$). This tends to raise the cutoff productivity level $\phi^*$. A key here is to recognize that firm’s productivity is associated only with the (marginal) cost of production. When entry is less pollution intensive than production, an increase in the permit price would make production relatively more costly than entry, thereby inducing exit of low productive firms. The reverse holds when entry is more pollution intensive than production, in which case the permit price increase would allow low-productivity firms to stay active upon entry.

Because (42) is the same under auctioning and the closure provision, the cutoff productivity would be the same if $\tau$ is the same. Now, let us show that the price of permits is indeed different between the auctioned ET and the closure provision. Let us first observe that virtually all the aggregate accounting conditions remain the same as before, so that $R = L + \tau Z$ under auctioning and $R = L$ under the closure provision. An exception is on the accounting equation for the aggregate emissions. The equation (18) now becomes:

$$\tau Z = \rho \beta R + \frac{\beta}{\bar{r}} f \tau^\beta + \beta_e \frac{R}{\bar{r}} \bar{\pi}.$$ 

In the case of auctioning, $\bar{r} = \sigma \bar{\pi} + f \tau^\beta = \sigma f \tau^\beta \bar{\phi}/\phi^*\sigma^{-1}$. Substituting this into (43) and manipulating, we have:

$$\tau Z = (\beta + \kappa(\phi^*)) R,$$

where $\kappa(\phi^*) \equiv (1 - \rho)(\beta_e - \beta) \left\{ 1 - (\bar{\phi}/\phi^*)^{1-\sigma} \right\} < 0$ if $\beta_e < \beta$. Using $R = L + \tau Z$ and solving this for $\tau$, we obtain the price of permits under auctioning given the emissions cap $Z^*$:

$$\tau_a = \frac{(\beta + \kappa(\phi^*)) L}{(1 - \beta - \kappa(\phi^*)) Z^*}.$$ 

Under the closure provision (with $\chi = \sigma$), $\bar{r} = \sigma \bar{\pi} + f \tau^\beta - \tau z^* = \sigma (f \tau^\beta - \tau z^* \bar{\phi}/\phi^*) \bar{\phi}/\phi^*\sigma^{-1}$. Substituting this into (43) and manipulating, we have:

$$\tau(1 - \beta_e) Z = (\beta + \eta(\phi^*)) R,$$

31
where $\eta(\phi^*) \equiv (1 - \rho)(\beta_e - \beta) \left\{ 1 - (\tilde{\phi}/\phi^*)^{1-\sigma} \left[ f\tau^\beta/(f\tau^\beta - \tau z^*(\phi^*)) \right] \right\} < 0$ if $\beta_e < \beta$. Using $R = L$ and solving for $\tau$, we obtain the price of permits under the closure provision given the emissions cap $Z^*$:

$$\tau_{CP} = \frac{(\beta + \eta(\phi^*_{CP})) L}{(1 - \beta_e) Z^*}. \tag{45}$$

Comparing (44) and (45), we see $\tau_{CP} \neq \tau_a$ and $\phi^*_{CP} \neq \phi^*_a$ in general.

**Proposition 6** If firms face different factor intensities in production and entry, then neither the size distribution of firms nor the price of permits would be the same under the auctioned and the grandfathered emissions trading with the closure provision.

### 9. Non-Pollution-Intensive Sector

Another important qualification for our main result is the full-employment assumption. We explicitly use this assumption in deriving the mass of firms and the price of permits. Presumably, though, an introduction of emissions trading would cause reallocation of employment from pollution-intensive industries to less pollution-intensive industries. Hence, the full employment assumption would be more appropriate in the model incorporating two or more industries with different pollution intensities. In this section, we shall examine if the price-invariance result still holds in such an economy.

It is sufficient to consider an economy with two sectors in order to convey our main points. Following Copeland and Taylor (1994) and Bernard et al. (2007), let the preferences of the representative consumer be given, in place of (1), by:

$$U = \sum_k \alpha^k \ln(Q^k) - L h(E),$$

where $\sum \alpha^k = 1$ and $Q^k$ is the composite good for industry $k$ defined by:

$$Q^k = \left[ \int_{\omega \in \Omega_k} q^k(\omega)^\rho d\omega \right]^{1/\rho}.$$

On the producer side, firms in each sector maximize profits (6) with the same production and entry costs (5) and (12) as before, except that firms in different sectors have different $\beta$'s. Assume sector 1 is more pollution-intensive than sector 2: $\beta_1 > \beta_2$. Labor and permits are freely mobile across sectors. We leave all other components of the model unchanged.

Under this setup, we shall compare the auctioned ET with the grandfathered ET with entry/closure provision with a neutral allocation ($\chi = \sigma$) for each sector. Let us first observe
that as long as factor intensities are the same in production and entry for each sector, cutoff productivities are the same between auction and grandfathering with closure provision for each sector: $\phi^*_a = \phi^*_{CP}$. Moreover, virtually all the aggregate accounting conditions remain the same as before, so that $R = L + \tau Z$ under auctioning and $R = L$ under grandfathering with entry/closure provision.

A difference occurs on the accounting equation for the aggregate emissions for each sector $k$:

$$\tau Z^k = \tau Z^p_k + \tau Z^f_k + \tau Z^e_k.$$  \hfill (46)

In the case of auctioning, (46) implies $\tau Z^k = \beta^k R^k$, where $R^k$ is the aggregate expenditures for industry $k$. Moreover, the Cobb-Douglas utility function implies that the consumer’s expenditure share for each composite good is $\alpha^k$. Hence, the aggregate demand for permits is given by:

$$\tau Z = \theta R,$$

where $\theta \equiv \alpha^1\beta^1 + \alpha^2\beta^2$. Substituting $R = L + \tau Z$ and the supply of labor and permits, we obtain the price of permits under auctioning:

$$\tau_a = \frac{\theta L^s}{(1 - \theta)Z^s}.$$  \hfill (47)

Analogously, the accounting equation of labor implies that the labor employed in each sector satisfies:

$$L^k_a = (1 - \beta^k)R^k = \frac{\alpha^k(1 - \beta^k)L^s}{(1 - \theta)}.$$  

Applying this in $M^k = R^k/\bar{r}^k$, we see:

$$M^*_a = \frac{\alpha^k(1 - \beta^k)L^s}{\sigma(1 - \theta)(\bar{\pi}_a + f\tau_a^k)}.$$  

On the other hand, under grandfathering with entry/closure provision, by applying analogous arguments in (46), we obtain:

$$\tau Z^k = \beta^k R^k + \tau \beta^k Z^{s,k},$$  \hfill (48)

where $Z^{s,k}$ is the amount of permits allocated for industry $k$. Let $\hat{\alpha}^k$ be the share of permits distributed to sector $k$, so that $Z^{s,k} = \hat{\alpha}^k Z^s$. Adding the above equation over the two sectors
and solving for $\tau$, we obtain the aggregate demand for permits under the closure provision:

$$\tau_{CP} = \frac{\theta L^s}{(1 - \hat{\theta})Z^s},$$

(49)

where $\hat{\theta} \equiv \hat{\alpha}^1 \beta^1 + \hat{\alpha}^2 \beta^2$. Comparing (47) with (49), the permit price is the same between auctioning and grandfathering with closure provision if and only if $\hat{\alpha}^k = \alpha^k$. Importantly, if the share of permits allocated to the pollution-intensive sector exceeds its aggregate income share (i.e., if $\hat{\alpha}^k > \alpha^k$), then it would raise the permit price relative to auctioning. This occurs because such an allocation would raise the overall demand for permits.

Even in the case of $\hat{\alpha}^k = \alpha^k$, there would still be active trading across sectors. To see this, plugging (49) in (48), we have:

$$Z^k_{CP} = \frac{\alpha^k \beta^k - (\alpha^k - \hat{\alpha}^k)\beta^1 \beta^2}{\theta}Z^s.$$

Therefore, if $\hat{\alpha}^k = \alpha^k$,

$$Z^k_{CP} - Z^{s,k}_{CP} = \frac{\alpha^k}{\theta} (\beta^k - \theta)Z^s,$$

where $\beta^1 > \theta$ and $\beta^2 < \theta$. Hence, the pollution-intensive sector would be the net buyer while the non-pollution-intensive sector would be the net seller of permits. In other words, for there to be no inter-industry trading of permits under grandfathering with closure provision, the permit price must be different from that under auctioned ET.

Importantly, the inter-industry allocations of labor and firms also depend on permit allocation. Manipulating the analogous accounting equation for labor, we see:

$$L^k_{CP} = \frac{[ (1 - \beta^k) \alpha^k (1 - \hat{\theta}) + (1 - \beta^k) \hat{\alpha}^k \theta ] L^s}{1 - \hat{\theta}},$$

which implies that employment in the pollution-intensive sector is increasing in the share of permits distributed to that sector.

As for the mass of firms, note that:

$$M^k_{CP} = \frac{\alpha^k L}{\sigma \left( \bar{\pi}^k_{CP} + \tau z^{s,k}_{CP} \right)} = \frac{\alpha^k L + \sigma \tau Z^{s,k}_{CP}}{\sigma (\bar{\pi}^k_{CP} + \tau z^{s,k}_{CP})}. $$
Applying $\bar{\pi}_i = \left(\frac{\hat{\phi}_i}{\phi_i^*}\right)^{\alpha - 1} f \tau_i^{\hat{\beta}_i}$ with $\phi_a^* = \phi_{CP}^*$ and (47) with (49), we have:

$$M^k_a/M^k_{CP} = \frac{\alpha^k}{\alpha^k + \sigma \hat{\alpha}^k \hat{\theta} - \alpha^k \hat{\theta}} \left(\frac{1 - \hat{\theta}}{1 - \theta}\right)^{1 - \beta^k}.$$ 

This equation implies, first, that $M^k_a < M^k_{CP}$ if $\hat{\alpha}^k = \alpha^k$ as expected from the one-sector model, and second, that whether $M^k_{CP}$ exceeds $M^k_a$ is, in general, indeterminate and depends on the size of $\hat{\alpha}^k$.

In sum, this section demonstrates that under the closure provision, initial distribution of permits across sectors has real impacts, not only on the permit price, but also on the inter-industry allocations of emissions and labor. On one hand, permit allocations conditioned on entry/production status increase firms’ incentive for entry and production, thereby affecting the demand for emissions in each sector. On the other hand, the expenditure share on each sector is determined by the consumer preferences (and is independent of the design of emissions trading). Consequently, unlike in the one-sector model, initial permit distribution can influence the equilibrium permit price, for it can shift labor and emissions away from one sector to another.

**Proposition 7** Consider an economy consisting of two sectors with different pollution intensities. Then the equilibrium price of permits would be the same under the auctioned emissions trading and the grandfathered emissions trading with entry/closure provision if and only if permits are allocated in such a way that the share of permits distributed in each sector equals the expenditure share for that sector. The sectoral emissions and labor employment, as well as the mass size of firms in each industry, all depend on the permit distribution across sectors.

10. Concluding Remarks

This paper examined the long-run impacts of conditional allocation rules under emissions trading in the Melitz-type economy that accounts for endogenous entry/exit of heterogeneous firms. The model allows us to make one important distinction in identifying the allocative impacts, i.e., a distinction between the effect on size distribution of firms versus that on the mass of firms. This distinction is important not only because we can clarify the nature of distortion in entry but also because it confers a distinction between the average firm behavior versus the aggregate behavior of the industry. We then considered a suit of allocation
schemes in a way to increment policy treatments: from auctioning to grandfathering with permanent allocation, to grandfathering with entry/closure provision, and finally to grandfathering with output-based allocation. The incremental policy treatment, combined with the aforementioned advantage of the model, allowed us to fully disentangle the sources and effects of distortions created through conditional allocation rules.

Our first set of results is that the auctioned ET does not alter the entry-exit condition, and therefore, the cutoff productivity (i.e., the lowest productivity of firms that enter the market in equilibrium) under the auctioned ET stays the same as under no regulation. However, as expected, a smaller mass of firms enter under the auctioned ET with a higher average profit relative to no regulation because firms faced with a positive price of pollution need to be more profitable in order to stay active in the industry.

Second, grandfathering, or free distribution of permits, per se is shown to have no effect on the entry-exit condition. With permanent allocation, firms who receive permits upon entry retain the permits upon exit, whereas firms who did not receive permits need to buy permits every period from other firms who hold them. Because the allocation of permits does not depend on firms’ entry-exit status, such a permanent allocation rule does not distort the entry-exit conditions or aggregate accounting conditions, regardless of how permits are allocated initially. As a result, the auctioned ET and the permanent allocation rule result in the same stationary equilibrium — despite the fact that the transferable property rights were freely distributed under the grandfathered ET.

Things change dramatically under conditional allocation rules, however. Under the entry/closure provision (as in the EUETS), new entrants are allocated some amount of permits freely while firms lose permits upon exit. Under such a provision, neither the cutoff productivity nor the mass of firms is independent of the initial distribution of permits. We show that the initial distribution of permits may alter the size distribution of firms if permits are distributed in a manner disproportionate to firms’ productivity levels (and therefore, their unconstrained emissions levels). If, for example, firms are allocated permits based on a uniform emissions rate (as in the U.S. Acid Rain Program), then the initial distribution of permits would be regressive (i.e., less productive firms would be distributed smaller amounts of permits relative to their unconstrained emissions), so that they would become net buyers of permits, whereas more productive firms would become net sellers. As a result, such a rate-based allocation rule would induce entry of more productive firms and raise the cutoff productivity.

Importantly, however, this distortion in the entry-exit conditions does not necessarily distort the price of permits — it still remains the same as under the auctioned ET. With entry/closure provision, the value of payments by net buyers of permits must equal that of sales.
by net sellers of permits in the stationary equilibrium. In contrast, with permanent allocation, the payments go to the government and eventually to all the market participants (i.e., consumers/firms), which raises the demand for permits relative to the case of entry/closure provision. However, with entry/closure provision, the free distribution of permits also encourages entry of any firms, which also raises the demand for permits. In equilibrium, these two effects exactly offset each other. Consequently, the price of permits is unaffected. With a two-sector model, however, initial distribution of permits across sectors is shown to have real impacts on the equilibrium price of permits, because it can influence the real demand for permits via inter-industry reallocations of employment, emissions, and firms.

The OBA rule further confounds these effects. Because the OBA rule allocates permits based on firms’ output, it serves as a rebate not only on production but also on entry because firms forgo permits upon exit as they cease their production. A priori then, one would expect the OBA rule to distort both entry/exit and production behavior. It turns out, however, while the OBA does distort production behavior and the mass and entry of firms, it does not distort the entry-exit behavior. Because firms receive a rebate on the amount of production, all firms face the incentive to increase their supply relative to the auctioned ET. This increases the demand for emissions, and as a result, raises the price of permits compared to the auctioned ET. However, because firms produce outputs according to their productivity levels, the allocation of permits in proportion to output shares constitutes an allocation of property rights based on their innate productivity levels. Hence, the OBA rule does not, in principle, distort the entry-exit condition.

These impacts of conditional allocations schemes on size distribution, mass of firms, and permit price have real implications for welfare under second-best settings, in which design of allocation rules is politically constrained. We demonstrate that in such a second-best setup, the (constrained) ‘optimal’ emissions cap must be adjusted for specific design features of emissions trading, taking into account of their impacts on size distribution, mass of firms, and permit price.

Furthermore, we investigate the implications of two important assumptions of the model: emissions cost in entry and full employment. When factor intensities differ between production and entry, the independence property with respect to the price of permits no longer holds even with the entry/closure provision. In a model with two sectors with different pollution intensities, the aggregate emissions, labor employment as well as mass of firms in each sector all are shown to depend on the initial distribution of permits across sectors. We thus conclude that whether the price-invariance property holds with conditional allocation rules or not depends not only on their specific design features but also on entry cost structures and the coverage of non-pollution-intensive sectors in emissions trading.
These results suggest a new and important pathway for future empirical studies in the growing field of empirical environmental economics. Taken at a different angle, our findings suggest, first, that environmental regulation that supports the same price of pollution may induce different size distributions, mass sizes, and new entries of firms within and across industries under different regulatory design features. Second, an increased cost of pollution may or may not induce exit of more pollution-intensive firms, depending on the technology and regulatory environment that defines the factor intensity of entry cost. And when it does, different designs of emissions trading should generally induce different prices of pollution even for the same emissions cap. The proposed framework offered herein is also widely applicable for assessing other regulatory instruments such as emissions taxes, abatement subsidies, and command-and-control policies. Hence, it would allow us to formulate rich testable hypotheses as to how an environmental regulation affects the industry size, entry/exit, and productivity distribution of regulated firms. Further exploration of such a pathway is left for future research.
References


