Center for Economic Institutions

Working Paper Series

No. 2017-8

"Fiscal Forward Guidance: A Case for Selective Transparency"

Ippei Fujiwara and Yuichiro Waki

March 2018

Center for Economic Institutions

Working Paper Series

Institute of Economic Research
Hitotsubashi University
2-1 Naka, Kunitachi, Tokyo, 186-8603 JAPAN
http://cei.ier.hit-u.ac.jp/English/index.html
Tel:+81-42-580-8405/Fax:+81-42-580-8333

Fiscal Forward Guidance:

A Case for Selective Transparency*

Ippei Fujiwara† — *Keio University / ANU*

Yuichiro Waki‡ — *University of Queensland*

July 20, 2017

Abstract

Should the fiscal authority use forward guidance to reduce future policy uncertainty perceived by private agents? Using dynamic general equilibrium models, we examine the welfare effects of announcing future fiscal policy shocks and show that *selective transparency* is desirable — announcing future policy shocks that are distortionary can be detrimental to ex ante social welfare, whereas announcing non-distortionary shocks generally improves welfare. Sizable welfare gains are found with constructive ambiguity regarding the timing of a tax increase in a realistic fiscal consolidation scenario. However, being secretive about distortionary shocks is time inconsistent, and welfare loss from communication may be unavoidable.

JEL Classification: D82; E62; H20; E30

Keywords: news shock; fiscal policy; private information; communication; forward guidance

*We thank Kosuke Aoki, Paul Beaudry, Andres Bellofatto, Efrem Castelnuovo, Masaru Inaba, Luigi Iovino, Olivier Jeanne, Keiichiro Kobayashi, Ryo Jinnai, Sagiri Kitao, Nobu Kiyotaki, Anton Korinek, Ellen McGrattan, Bruce Preston, and participants at the 2016 Southern Workshop In Macroeconomics (SWIM) at University of Auckland, the AJRC and HIAS Joint Conference on Recent Issues in Finance and Macroeconomics at ANU, and seminars at the RIETI, Bank of England, Bocconi University, University of Washington and Keio University for useful comments and discussions. This study is conducted as a part of the project "On Monetary and Fiscal Policy under Structural Changes and Societal Aging" undertaken at the Research Institute of Economy, Trade and Industry (RIETI). We are also grateful for financial support from JSPS KAKENHI Grant-in-Aid for Scientific Research (A) Grant Number 15H01939.

†Faculty of Economics, Keio University, Tokyo, Japan. Crawford School of Public Policy, Australian National University, Canberra, Australia. Email: ippei.fujiwara@keio.jp

‡School of Economics, University of Queensland, Brisbane, Australia. Email: y.waki@uq.edu.au

1 Introduction

Forward guidance has been extensively used in the conduct of monetary policy of many developed countries since the onset of the Great Recession and, in fact, it was used by some central banks even before the crisis as argued by Svensson (2014). Forward guidance is thought to increase social welfare either through an explicit commitment to a future course of policy action or through the direct communication of superior information possessed by a central bank.[1] The importance of commitment has been emphasized in the New Keynesian monetary policy literature in general (Woodford, 2003) and also in the zero lower bound situation (Eggertsson and Woodford, 2003). Recent theoretical studies such as Bassetto (2015) and Fujiwara and Waki (2015) investigate when the two types of forward guidance are useful or not.

This paper studies the role of the second type of forward guidance in the conduct of fiscal policy, which we call *fiscal forward guidance*, by asking whether and when the fiscal authority can improve welfare by providing its superior information about future policy actions. This question is relevant for two reasons. The first reason is that, in reality, fiscal policy changes are often pre-announced. The empirical literature of fiscal policy effects recognizes that economic variables respond differently to pre-announced and to announced policy changes (Mertens and Ravn, 2010, 2011, 2012; Leeper, Walker, and Yang, 2013). Differential effects of anticipated and unanticipated shocks open up the possibility that communication by the fiscal authority may be a useful policy tool. The second reason is that it is often argued that fiscal policy uncertainty is large: Baker, Bloom, and Davis (2015) construct newspaper-based uncertainty indices and indeed find that "[f]iscal matters, especially tax policy, stand out... as the largest source of policy uncertainty, especially in recent years." Government communication may be used to reduce such uncertainty by increasing the forecastability of

[1] Campbell, Evans, Fisher, and Justiniano (2012) call the former kind of forward guidance *Odyssean forward guidance* and the latter *Delphic forward guidance*.

its future policy actions.

We evaluate the welfare effects of fiscal forward guidance using a standard neoclassical growth model. To focus on the role of communication, fiscal policy actions themselves are assumed to be subject to exogenous shocks to spending and distortionary taxes. The government privately observes signals about future policy shocks and communicates the signals truthfully and credibly. Fiscal forward guidance makes economic agents more informed about future policy shocks, thereby effectively introducing news shocks. We consider uncertainty about the level of policy actions and about the timing of policy action changes.[2]

Our main finding is that the sign of the welfare effect depends on whether fiscal forward guidance is about distortionary taxes or about spending: ex ante social welfare decreases when the private sector is made more informed about future tax shocks, while it increases when made more informed about future spending shocks, as far as their correlation is not strong. We show this result analytically for a stylized three period model and confirm it in a general model through numerical exercises. Hence, ex ante optimal communication policy features *selective transparency* — it may require secrecy or less transparency if policy uncertainty is mainly about future distortionary taxes and transparency if spending is a primary source of policy uncertainty. An intuitive explanation is as follows. Imagine that shocks to spending and taxes are orthogonal. Spending news shocks improve ex ante welfare in the first-best allocation. Spending news shocks also move the equilibrium allocation with the first-best allocation, resulting in ex ante welfare increases. However, distortionary tax news shocks do not affect the first best when spending and taxes are orthogonal. Therefore, tax news shocks merely exacerbate inefficient fluctuations of the equilibrium allocation,

[2] In Appendix A.2 we consider uncertainty shocks following Bloom (2009). Born and Pfeifer (2014) and Fernandez-Villaverde, Guerron-Quintana, Kuester, and Rubio-Ramirez (2015) explore the impacts of fiscal uncertainty, expressed as the time-varying volatility shocks in a canonical DSGE model using the higher-order perturbation method. We use the model of Brock and Mirman (1972) instead and solve the model in a closed form to examine the effects of time-varying volatility *news* shocks.

which reduces ex ante welfare. We also demonstrate that the above welfare implications can be reversed when shocks to spending and taxes are sufficiently strongly correlated.

How large can the welfare effect of fiscal forward guidance be in a realistic setting? To answer this question, we augment a scenario of fiscal consolidation in Japan examined in Hansen and Imrohoroglu (2016) with uncertainty on the timing of a consumption tax increase. The welfare loss from being transparent about the future course of consumption taxes can be as large as 0.04% in consumption equivalent units, which is comparable to the welfare loss from business cycles in real business cycle models.

We also argue that selective transparency is time inconsistent. If the government can send verifiable information to the public, the government is tempted, ex post, to communicate its private information about future tax shocks if the future tax realizations happen to be less distortionary than have been expected by the public. Therefore, without the ability to commit, the gain from being secretive about tax news may not be fully achievable, and the government's ability to observe future tax shocks is actually harmful for ex ante welfare.

Throughout the paper we limit our attention to fiscal policy shocks because our interest is in the role of communication by the fiscal authority. Although we do not formally analyze non-policy shocks such as technology shocks, we conjecture that our main result generalizes to such shocks: news about non-distortionary shocks is likely to be welfare-enhancing while news about distortionary shocks is likely to reduce welfare.[3] We also, purposefully, focus on the representative-agent framework for two reasons. First, it allows us to identify the key mechanism at work behind the welfare effects of news shocks. Second, it makes it clear that our results do not rely on the

[3] Fujiwara and Waki (2015) examine the welfare effects of technology news shocks in various New Keynesian models. When the only friction is price stickiness, technology shocks are non-distortionary under the optimal monetary policy and technology news shocks improve welfare. However, with more frictions or under sub-optimal monetary policy, technology shocks can be distortionary and news about them can reduce welfare.

presence of specific heterogeneity, market incompleteness, or inefficiency that may arise from uninsurable idiosyncratic risk or from the overlapping generation structure or from inefficient coordination.

The remainder of this paper is organized as follows. In Section 2 we set up simple, three period models and show that selective transparency is desirable. Section 3 demonstrates the robustness of our results by numerical exercises using a general stochastic neoclassical growth model. In Section 4, we quantify welfare loss from transparency by extending the model of Hansen and Imrohoroglu (2016) to incorporate policy uncertainty. Section 5 discusses sustainability of selective transparency and possible consequences of introducing heterogeneity. Section 6 concludes.

1.1 Related literature

To the best of our knowledge, this is the first attempt to explore the welfare consequences of announcing future policy actions in a standard general equilibrium model with well-defined social welfare. There are a number of positive studies that emphasize the importance of anticipated fiscal shocks. Ramey (2011) reexamines the size of the fiscal multiplier considering the timing of the spending news, and Mertens and Ravn (2011, 2012) examine the effects of anticipated and pre-announced tax shocks in the U.S. Yang (2005) uses a real business cycle model with anticipated tax shocks to show that a standard SVAR exercise is misspecified under fiscal foresight and that it is unable to recover the correct impulse responses when applied to the model-generated data. Leeper, Walker, and Yang (2013) show that tax foresight creates difficulties in econometric inference on the effectiveness of fiscal policy and examine both narrative and DSGE approaches to resolve this problem. Bi, Leeper, and Leith (2013) examine whether fiscal consolidation leads to economic expansion under uncertainty about the timing of the consolidation, as well as its composition (tax rises/spending cuts). The authors investigate the role of anticipated consolidation, which is modeled as policy news shocks. Evans, Honkapohja, and Mitra (2009) analyze the responses to the anticipated fiscal policy under adaptive learning. These analyses are positive ones and do not ask whether making announcement is socially beneficial or not, while ours is normative.

There is a vast literature on the role of policymaker's communication when private agents are heterogeneously and privately informed, following the seminal work by Morris and Shin (2002). The literature has found that transparency, modeled as an increased precision of a publicly observed signal, is not always welfare-enhancing. The underlying mechanism of this finding is strategic complementarity of individuals' actions, which can result in excessive use of a public signal because it is useful to predict others' actions (Amato, Morris, and Shin, 2002). Quite obviously, the mechanism at work in our paper is different; there is no strategic interaction, and a public signal does not help predict others' actions in our representative-agent framework. In Section 5 we discuss potential implications of introducing heterogeneously informed private agents to our setting, and also relate our results with a conjecture made in Angeletos and Pavan (2007).

Fujiwara and Waki (2015) examine the role of Delphic forward guidance in monetary policy using New Keynesian models in which the central bank has private information about various future shocks.[4] As in the present paper, it is shown that secrecy about some shocks can be desirable from the ex ante point of view.[5] The main mechanism in their paper is the forward-looking New Keynesian Phillips curve that obtains under the sticky price friction. The present paper only concerns models without nominal rigidities but obtains the same results.

[4]Shocks can be either distortionary or efficient, and can affect not only policy variables but also economic fundamentals.

[5]Interestingly, in some model specifications, it is found that productivity news shocks reduce welfare. This is because productivity shocks can be distortionary when there are multiple frictions or when monetary policy is suboptimal.

2 A simple analytical framework

We begin our analysis with a simple three-period model in which a future fiscal policy action is stochastic. The representative household has access to linear saving technology and, therefore, fiscal forward guidance can in general affect the equilibrium outcome through the household's expectations and saving decision.

2.1 The model

Time is discrete and there are three periods, $t \in \{0, 1, 2\}$. There are two actors in the economy: the representative household and the government. Fiscal policy actions such as tax and spending take place in period 2. They are exogenous random variables and are the only source of uncertainty. In period 0, both parties are endowed with common information regarding the period-2 fiscal policy, which is represented by a σ-field $\mathcal{F}_0$. We will use a shorthand $\mathbb{E}_0[.]$ to denote the expectation operator conditional on the period-0 information, $\mathbb{E}[.|\mathcal{F}_0]$. At the beginning of time 1, the government receives private information about the time-2 policy actions and may communicate some information to the household. The representative household then chooses consumption and savings optimally, conditional on the available information, and an equilibrium realizes.

The information possessed by the household in period 1 after the government has conveyed some information is represented by a σ-field $\mathcal{F}_1^P$ with $\mathcal{F}_0 \subset \mathcal{F}_1^P$. Conditional expectation given $\mathcal{F}_1^P$, $\mathbb{E}[.|\mathcal{F}_1^P]$, is denoted by $\mathbb{E}_1^P[.]$. The superscript P is attached to emphasize that it is the private sector's expectation.

After the communication, the representative household solves

$$\max_{C_1, C_2, S_1} \ln C_1 + \mathbb{E}_1^P[\ln C_2]$$

subject to

$$\begin{aligned} C_1 + S_1 &= Y_1 \\ C_2 &= (1-\tau_2^K)S_1 + T_2 \end{aligned}$$

where Y_1 is exogenous income in period 1, T_2 is lump-sum transfer from the government in period 2, and τ_2^K is the savings tax in period 2.

All the fiscal policy actions take place in period 2, and the government budget constraint is given by

$$G_2 + T_2 = \tau_2^K S_1,$$

where G_2 denotes wasteful spending by the government in period 2.

The equilibrium condition is given by the Euler equation

$$\frac{1}{C_1} = \mathbb{E}_1^P[\frac{1}{C_2}(1-\tau_2^K)], \tag{1}$$

and the resource constraints in two periods:

$$C_2 = S_1 - G_2, \tag{2}$$

$$C_1 = Y_1 - S_1. \tag{3}$$

Let $s := S_1/Y_1$ be the saving rate. Then the equilibrium condition can be summarized by the following equation:

$$1 = \mathbb{E}_1^P[\frac{1-s}{s - G_2/Y_1}(1-\tau_2^K)]. \tag{4}$$

We consider three fiscal policy specifications. The first specification assumes that the saving tax τ_2^K is random and that the tax revenue is fully rebated back to the household: $T_2 = \tau_2^K S_1$ and $G_2 = 0$. In the second specification, the government spending G_2 is random and is financed fully by the lump-sum tax: $G_2 = -T_2$ and $\tau_2^K = 0$. In the last specification, the spending to output ratio, G_2/S_1, is random.

2.2 Distortionary tax only

First we consider the case in which the government rebates the tax revenue back to the household and $G_2 = 0$.[6] Equation (4) then reduces to

$$\frac{s}{1-s} = \mathbb{E}_1^P[1-\tau_2^K].$$

Let X denote the household's period-1 conditional expectation of the after-tax rate, i.e. $X = \mathbb{E}_1^P[1-\tau_2^K]$. Then we can write the household's equilibrium utility evaluated in period 1 as a function of X:

$$\ln C_1 + \mathbb{E}_1^P[\ln C_2] = \ln(1-s) + \ln s + 2\ln Y_1 = f(X),$$

where the function f is defined by $f(x) := \ln x - 2\ln(1+x) + 2\ln Y_1$.

Once the household receives its information about future taxes, $X = \mathbb{E}_1^P[1-\tau_2^K]$ takes a particular value, x, pinning down the above utility value at $f(x)$. We call it the *ex post* social welfare, as it corresponds to a particular realization of X. Figure 1 depicts the function f. It is single-peaked at $x = 1$, because $x = 1$ corresponds to the case where the expected tax rate is zero and where the first-best saving rate is chosen in equilibrium. Quite obviously, the household's equilibrium utility evaluated at time 1 is high (low) when the expected tax happens to be close to (away from, respectively) zero.

What we would like to understand, however, is whether it is good, from the *ex ante*

[6] As Ricardian equivalence holds, this kind of lump-sum transfer can also be interpreted as debt repayment when there is some outstanding government debt in period 1.

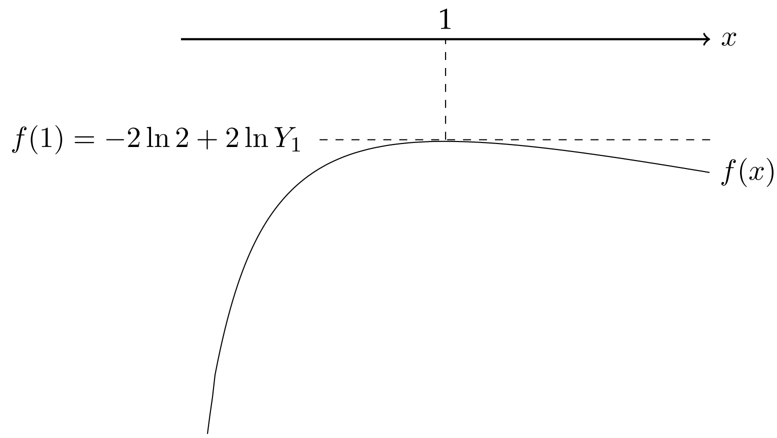


Figure 1: Ex post social welfare as a function of a realization x of $\mathbb{E}_1^P[1-\tau_2^K]$

point of view, to enable the household to forecast future policy actions better, and not whether it is good or bad that the household's tax expectations take a particular value. We use *ex ante* welfare as a criterion. Ex ante welfare is the household's equilibrium expected utility evaluated at time 0 and, therefore, equals

$$\mathbb{E}_0[f(X)] = \mathbb{E}_0\left[f\left(\mathbb{E}_1^P[1-\tau_2^K]\right)\right].$$

Interestingly, as shown in the following proposition, the more information the household has access to at time 1, the smaller is ex-ante welfare.

Proposition 1 *Suppose that $0 < 1-\tau_2^K < 1+\sqrt{2}$ almost everywhere. Let $\mathcal{F}_1^P$ and $\tilde{\mathcal{F}}_1^P$ be two σ-fields satisfying $\mathcal{F}_0 \subset \mathcal{F}_1^P \subset \tilde{\mathcal{F}}_1^P$ (i.e. $\mathcal{F}_1^P$ is coarser than $\tilde{\mathcal{F}}_1^P$). Then*

$$\mathbb{E}_0\left[f\left(\mathbb{E}[1-\tau_2^K|\mathcal{F}_1^P]\right)\right] \geq \mathbb{E}_0\left[f\left(\mathbb{E}[1-\tau_2^K|\tilde{\mathcal{F}}_1^P]\right)\right].$$

I.e. ex ante welfare is weakly bigger when the household's period-1 information set is given by $\mathcal{F}_1^P$ than it is when the household's period-1 information set is $\mathcal{F}_1^P$. The inequality is strict if and only if $\mathbb{E}[1-\tau_2^K|\mathcal{F}_1^P] \neq \mathbb{E}[1-\tau_2^K|\tilde{\mathcal{F}}_1^P]$ with strictly positive probability.

The assumption that $0 < 1 - \tau_2^K < 1 + \sqrt{2}$ is not at all restrictive as it allows any tax rate between $-100\sqrt{2}\%$ (i.e. more than 100% subsidy) and 100%.

The proof is based on two simple facts: (1) the function f is strictly concave on $[0, 1 + \sqrt{2}]$ and (2) $\mathbb{E}[1 - \tau_2^K | \tilde{\mathcal{F}}_1^P]$ is, loosely speaking, a mean-preserving spread of $\mathbb{E}[1 - \tau_2^K | \mathcal{F}_1^P]$. Let $X := \mathbb{E}[1 - \tau_2^K | \mathcal{F}_1^P]$ and $\tilde{X} := \mathbb{E}[1 - \tau_2^K | \tilde{\mathcal{F}}_1^P]$. Because $0 < 1 - \tau_2^K < 1 + \sqrt{2}$ almost everywhere, both X and $\tilde{X}$ always take values on $[0, 1 + \sqrt{2}]$. Because f is strictly concave on the same interval, Jensen's inequality implies

$$\mathbb{E}_0[f(\tilde{X})] = \mathbb{E}_0\big[\mathbb{E}[f(\tilde{X}) | \mathcal{F}_1^P]\big] \leq \mathbb{E}_0\big[f(\mathbb{E}[\tilde{X} | \mathcal{F}_1^P])\big] = \mathbb{E}_0\big[f(X)\big].$$

The intuition is as follows. Because the first-best allocation is constant and the representative household is risk-averse, it is socially undesirable when an equilibrium allocation becomes more risky. When the household is given more information about future taxes, its tax expectations $\mathbb{E}_1^P[1 - \tau_2^K]$ become more accurate and move with the newly provided information that was not forecastable originally. It is as if the expectations are hit with an additional, orthogonal shock. Formally, we have

$$\tilde{X} = X + \underbrace{\Big\{\mathbb{E}[1 - \tau_2^K | \tilde{\mathcal{F}}_1^P] - \mathbb{E}[1 - \tau_2^K | \mathcal{F}_1^P]\Big\}}_{\text{"updating" term}}. \tag{5}$$

The updating term in (5) is, by definition, orthogonal to X and has mean zero conditional on $\mathcal{F}_1^P$. Therefore, $\tilde{X}$ is a mean-preserving spread of X, and more information makes an equilibrium allocation more risky.

2.3 Spending shock only

When government spending is random and financed only by the lump-sum tax, it turns out that information revelation is weakly beneficial for ex ante welfare.

This is because the equilibrium allocation solves the following social planner's problem:

$$\max_{C_1,C_2,S_1} \mathbb{E}_0\left[\ln C_1 + \mathbb{E}_1^P[\ln C_2]\right] \quad (= \mathbb{E}_0[\ln C_1 + \ln C_2])$$

subject to the resource constraints (2) and (3) and the information constraint that the time-1 choice, (C_1, S_1), depends only on the information available to the household at time 1.[7] When the household's information set is improved, the last constraint is (weakly) relaxed and, therefore, ex ante welfare never decreases. If additional information changes the planner's optimal choice with non-zero probability, ex ante welfare strictly improves. The logic clearly extends to more general models in which, for example, the household's planning horizon is more than two periods, the household's utility is not log utility, etc., as far as an equilibrium outcome is efficient.

2.4 Spending-to-output ratio shock

Another common assumption made in DSGE models is that the government spending to output ratio follows an exogenous stochastic process. Now we denote $\psi_2 := G_2/S_1$ and assume it is random.

Let us assume first that spending is financed by lump-sum tax only: $G_2 = -T_2$ and $\tau_2^K = 0$. Equation (4) then reduces to

$$\frac{s}{1-s} = \mathbb{E}_1^P[\frac{1}{1-\psi_2}] = \mathbb{E}_1^P\left[1 - \left(1 - \frac{1}{1-\psi_2}\right)\right].$$

The equilibrium condition is identical to the above distortionary-tax-only case except that τ_2^K is replaced by $1-1/(1-\psi_2)$. Then Proposition 1 implies that more information about ψ_2, reduces welfare if $1/(1-\psi_2) \in (0, 1+\sqrt{2})$.

Why doesn't the result in Section 2.3 apply here, even though there is no distor-

[7] Formally, the last constraint is the "measurability" constraint that the choice (C_1, S_1) is *measurable* with respect to the σ-field that represents the household's information at time 1.

tionary tax? The reason is that the equilibrium is indeed inefficient due to the endogeneity of G_2. The social marginal return on saving equals $1-\psi_2$ because the fraction ψ_2 is used for wasteful spending, but the private return on saving equals 1 because the household does not internalize $G_2 = \psi_2 S_1$. The spending-to-output ratio shock is thus distortionary and more information about it is as harmful as is more information about distortionary tax.

An equilibrium becomes efficient if spending is fully financed by the linear saving tax τ_2^K. Note that the government budget constraint is given by $G_2 = \tau_2^K S_1$. Together with $\psi_2 = G_2/S_1$, it follows that $\psi_2 = \tau_2^K$. Then the private marginal return on saving becomes $1-\tau_2^K$, which equals the social marginal return on saving. The discussion in Section 2.3 then applies. However, in the present log utility model, equation (4) becomes

$$1 = \mathbb{E}_1^P\left[\frac{1-s}{s(1-\psi_2)}(1-\tau_2^K)\right] = \frac{1-s}{s},$$

and thus the equilibrium saving rate is invariant at the first-best level of $1/2$. Hence, ex ante welfare is constant regardless of the information possessed by the household at time 1. In Section 2.5.1 we argue that ex ante welfare indeed strictly improves with information for general CRRA utility functions.[8]

This case suggests that the welfare consequences of information may depend on the correlation between spending and distortionary tax. In Section 3.2 we examine the role of correlation.

2.5 Generalizations

In Appendix A we generalize the above results using the three-period model with a general CRRA utility function and the infinite horizon model of Brock and Mirman (1972). Conveying news about future spending shocks is welfare improving in both

[8] Note that the same implication obtains when the saving tax τ_2^K is random and all the revenue is used for spending.

models when there is no distortionary tax for the reason we have described above and, therefore, we focus on the implications of distortionary taxes below.

2.5.1 A general CRRA utility function

As in the log utility case, providing more information about future capital income taxes revenue from which is rebated back as a lump-sum transfer is detrimental to ex ante welfare, when the after-tax rate $1-\tau_2^K$ takes values within a certain range.[9] Ex post social welfare is concave within an interval around 1, as in the log-utility case.

However, ex ante welfare increases with information when the spending-to-output ratio, $\psi_2 = G_2/S_1$, always equals the saving tax rate, τ_2^K for a general CRRA utility case. Equation (4) becomes

$$\frac{s}{1-s} = \mathbb{E}_1^P[(1-\tau_2^K)^{1-\sigma}]^{\frac{1}{\sigma}},$$

where σ is the relative risk aversion parameter. It can be shown that the household's equilibrium utility is *convex* in a realization of $X = \mathbb{E}_1^P[(1-\tau_2^K)^{1-\sigma}]$ and, therefore, if the household is enabled to forecast $(1-\tau_2^K)^{1-\sigma}$ better (with positive probability), ex ante welfare is strictly improved. Figure 2 depicts ex post social welfare as a function of a realization x of X, with $\sigma = 0.2$ and $Y_1 = 1$. The log utility is a knife-edge case in which the information provision is irrelevant.

2.5.2 Brock-Mirman model

Although the fiscal forward guidance in the above three-period model is about the future tax rate, perhaps it is more interesting to examine the case in which fiscal forward guidance is about the timing of a policy change. Private agents in a highly-indebted

[9]The range varies with the relative risk aversion parameter. However, for each value of relative risk aversion, we can find such a range that contains 1 in its interior.

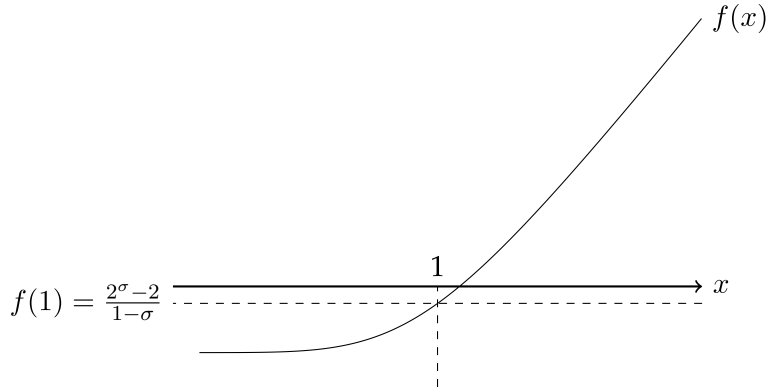


Figure 2: Social welfare as a function of a realization x of $\mathbb{E}^P[(1-\tau_2^K)^{1-\sigma}]$

country may expect that fiscal consolidation will take place at some point in future but may not be sure about the exact timing of its occurrence.

In Appendix A we show two results for the Brock-Mirman model. First, providing information reduces ex ante welfare not only when it is about the tax rate but also when it is about the timing of future tax changes. Second, revealing information about a future second-moment shock (an ”uncertainty” shock) to the saving tax volatility is also welfare-reducing. We generalize the first result in Section 3 for a general neoclassical growth model.

2.6 Discussion

So far we have seen that the welfare effect of forward guidance depends crucially on the nature of a policy shock — whether it is a distortionary tax or spending shock — and whether policy actions are orthogonal or correlated. In the next section we investigate this issue further in a more general stochastic general equilibrium model that incorporates various distortionary taxes.

It is important to note that the undesirability of revealing information about the saving tax shock does not require that an equilibrium has zero distortion when the household is "uninformed," i.e. $\mathbb{E}_1^P[1-\tau_2^K] = 1$ with probability one when no information is provided by the government. In such a case, any information moves its expectation away from 1, which is the peak of the function f, and, therefore, any news is bad news. This resembles a standard, well-known result in the capital taxation literature — non-zero capital income tax is non-distortionary when it is completely a surprise. However, the analogy is valid only in the special case where the private sector expects zero distortion on average. The negative welfare effect of news holds true even when $\mathbb{E}_1^P[1-\tau_2^K] \neq 1$ with some probability when the government provides no news, despite the fact that some news can move $\mathbb{E}_1^P[1-\tau_2^K]$ closer to one and improves the *ex post* welfare, i.e. some news may be good news.

Because it is possible that credible revelation of tax news increases ex post welfare for some realizations of news, being secretive about future policy shocks is time-inconsistent. When the tax distortion happens to be smaller than expected, such information is good news and ex post welfare increases if the household obtains the information. If the government is unable to commit to its communication policy, then it may be tempted to release some information to the private sector to raise ex post welfare even when it is undesirable ex ante.

3 A general neoclassical growth model

Now we set up a general standard neoclassical growth model that includes shocks to various distortionary taxes as well as to spending. The purpose is to examine the welfare effects of information dissemination in a case where the timing of policy change is uncertain and in a case where the spending and tax shocks are imperfectly correlated. We also use a version of this model to quantify the welfare effect using a tax reform considered in Hansen and Imrohoroglu (2016). Analytical results are also obtained but only for some special cases. We abstract from growth for simplicity. Sticky price models are examined in Fujiwara and Waki (2015).

As in the previous three-period model, all economic agents start their lives in period 0 with the same information; from period 1 on, while the private sector observes contemporaneous shocks perfectly, the government observes more information about future shocks than does the private sector.

We investigate the welfare effects of making the private sector more informed about future policy shocks. Specifically, the private sector is assumed to observe n-period ahead shocks each period, i.e. the household receives tax/spending news shocks, and we compare ex ante welfare by varying n from 0. To maintain symmetric information in period 0, we assume that no news is observed in period 0 and that in period 1 the private sector observes shocks that will hit the economy from period 1 to period $n+1$.

Hereafter we denote aggregate variables by uppercase letters and individual-level variables by lowercase letters: c_t, k_t, and l_t denote consumption, capital, and labor at the individual level in period t, and C_t, K_t, and L_t denote the same variables at the aggregate level.

The representative household maximizes

$$u(c_1) - v(l_1) + \mathbb{E}_1^P \sum_{t=2}^{\infty} \beta^{t-1} \{u(c_t) - v(l_t)\}$$

subject to

$$(1+\tau_t^C)c_t + k_{t+1} = (1-\tau_t^K)r_t k_t + (1-\tau_t^L)w_t l_t + (1-\delta)k_t + T_t,$$

taking the initial capital $k_1 = \overline{K}_1$ as given. Functions u and v are iso-elastic and given by, respectively, $u(c) = (c^{1-\sigma}-1)/(1-\sigma)$ and $v(l) = \chi l^{1+\eta}/(1+\eta)$, where χ and η are strictly positive.

The representative firm owns the Cobb-Douglas production technology $Y = K^{\alpha} L^{1-\alpha}$ and maximizes its profit taking the rental rate for capital, r, and wage, w, as given. Its optimality condition is given by $r_t = \alpha K_t^{\alpha-1} L_t^{1-\alpha}$ and $w_t = (1-\alpha) K_t^{\alpha} L_t^{-\alpha}$.

The government uses its tax revenue for spending, G, and lump-sum transfer to the household, T. Its budget constraint is

$$\tau_t^C C_t + \tau_t^K r_t K_t + \tau_t^L w_t L_t = G_t + T_t.$$

We omit the government debt because we assume either the Ricardian equivalence or balanced budget with $T_t = 0$.

Finally, the resource constraint is

$$C_t + K_{t+1} = K_t^{\alpha} L_t^{1-\alpha} + (1-\delta) K_t. \tag{6}$$

We again use ex ante welfare defined by $\mathbb{E}_0 \sum_{t=1}^{\infty} \beta^{t-1} \{u(c_t) - v(l_t)\}$.

3.1 Timing uncertainty

First we consider a case with timing uncertainty for a policy change. Our preferred interpretation is that the government may have some discretion over the exact timing of implementing policy changes.[10] Also, as illustrated by Mertens and Ravn (2012), implementation lags across tax liability changes are widely distributed, suggesting that the timing of implementation may be uncertain ex ante.[11]

To model the timing uncertainty, we assume that the private sector knows that

[10] In 2011, the then Japanese Prime Minister Noda announced the consumption tax hike to 8% in 2014 and then to 10% in 2015, and it was approved by the upper house plenary session. In 2014, after the first tax increase to 8%, the Prime Minister Abe postponed the second tax increase but made it explicit that it should be implemented in 2017. It was postponed further in 2016.

[11] Of course, one can find cases in which implementation timing have been clearly announced. For example, D'Acunto, Hoang, and Weber (2016) estimate the impact of unexpected VAT rate change, of which implementation timing was announced explicitly.

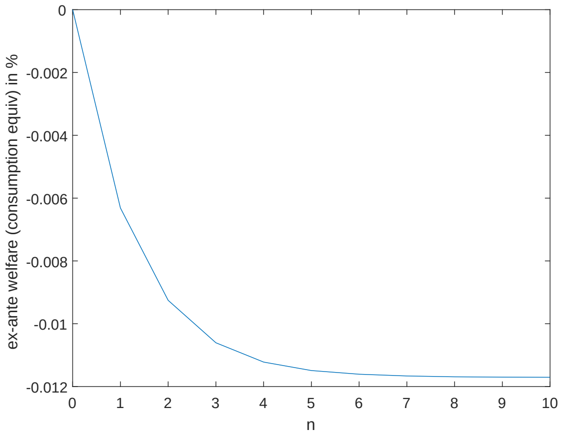
Table 1: Parameter values for the model with timing uncertainty

Preference parameters	
Discount factor, β	0.96
Relative risk aversion, σ	1
Labor disutility weight, χ	1
Inverse of the Frisch elasticity of labor supply, η	1
Technology parameters	
Capital share parameter, α	0.3
Depreciation rate, δ	0.1
Fiscal policy parameters in the steady state	
Spending share in output, G/Y	0.2
Consumption tax, τ^C	0.1
Capital income tax, τ^K	0.2
Labor income tax, τ^L	0.2

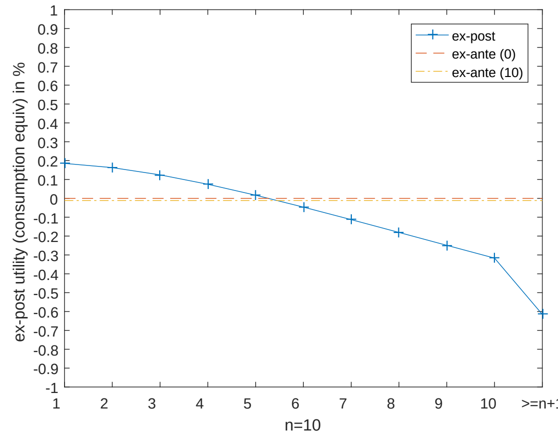
distortionary tax or government spending will change to a known level but does not know exactly when the change will occur. Although the model has four policy variables, $(\tau_t^C, \tau_t^K, \tau_t^L, G_t)$, we examine them individually by turning on only one of them at a time and fixing others at constant values. For qualitative illustration, the model is calibrated to annual frequency. Parameter values are reported in Table 1. The initial capital K_1 is set to 80% of the steady-state level.[12] We use the endogenous grid method to compute a recursive competitive equilibrium. Details of computation are given in Appendix B.

Our main finding is that the results in the three period model hold true in this general setting too. Namely, news about a future distortionary tax change reduces ex ante welfare but news about a future spending change increases it.

[12]Although the initial level of K_1 affects the welfare level, it has only a negligible effect on how welfare changes as more news is made available for the private sector.



(a) ex ante welfare vs. n



(b) ex post welfare

Figure 3: Ex ante and ex post welfare measured in consumption unit: the case of capital income tax cut

3.1.1 Capital income tax

Imagine that private agents, in period 0, know that capital income tax will be *reduced* at some point after period 1 but the timing of tax reduction is uncertain, because, for example, the government has left the exact timing of the tax cut at its discretion. To model this situation, we assume that τ_t^K follows a two-state Markov chain with initial condition $\tau_1^K = 30\%$, that τ_t^K switches from 30% to the steady state value of 20% with probability $1 - p$, and that 20% is the absorbing state. We set $p = 0.8$.

Figure 3a shows that ex ante welfare decreases with n. In particular, it decreases steeply for small n's, suggesting that the marginal welfare loss from additional information is bigger when the private agents do not possess much information about future shocks. For the purpose of comparison, Mertens and Ravn (2012) report that the median anticipation horizon for anticipated tax shocks is six quarters in the post-World War II US data, which is indeed where the marginal welfare loss is large in the figure. Secrecy emerges as an optimal communication policy regarding future tax changes. This optimality in secrecy also implies that the government should annouce future tax changes with a minimum implementation lag possible.

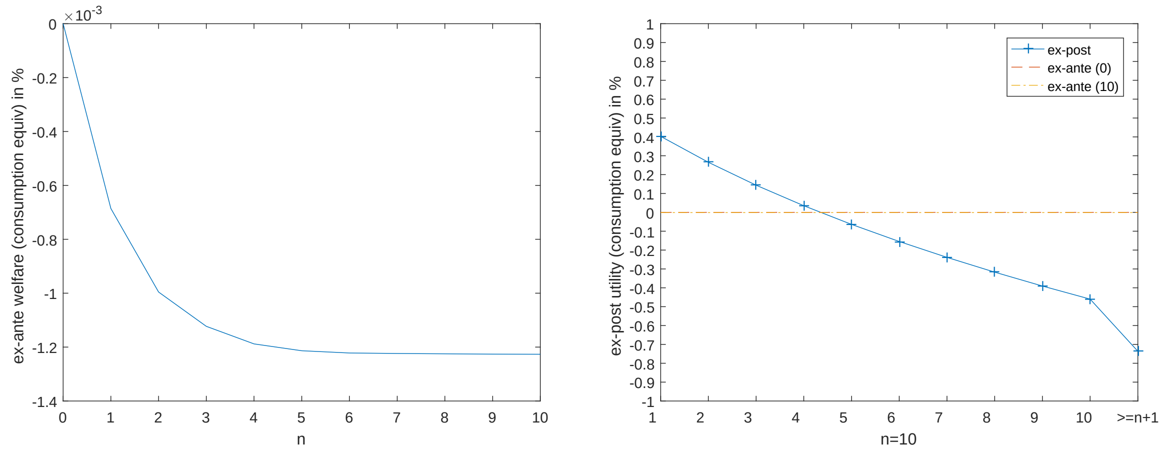
Now we use the $n = 10$ case to illustrate the time-inconsistency problem. At time 1, there are 11 possibilities: the household foresees either that the capital income tax will be reduced for sure at time $t \in \{2, 3, ..., 11\}$ or that the tax will be cut after $t = 12$. For each possibility we compute ex post welfare in period 1 and plot it in Figure 3b with the tax-cut timing on the horizontal axis. Ex post welfare in period 1 is decreasing in the tax-cut timing simply because social welfare increases when it is realized that a distortionary tax will be reduced quickly and decreases when it is realized that a distortionary tax will stay high for long periods.[13] If the tax cut happens to occur early (late), ex post welfare is higher (lower) than ex ante welfare. Therefore, if the government is unable to commit to secrecy, it is tempted to announce the timing of the tax cut when it is relatively early and a time-inconsistency problem arises.

3.1.2 Labor income tax

Now we switch the processes of capital and labor income taxes: the capital income tax rate is constant at 20% and the labor income tax is assumed to follow the aforementioned two-state Markov chain with the initial tax rate being 30%.

Figure 4 displays ex ante and ex post welfare. Results are qualitatively the same as before — ex ante welfare decreases with n and ex post welfare when $n = 10$ decreases with the anticipated timing of tax cut. Interestingly, although the effect on ex ante welfare is much smaller than in the capital income tax case, the effect on ex post welfare, as measured by the slope of ex post welfare graph, is larger. Unlike in the capital income tax case, learning that the tax cut will occur very soon has a strong positive effect on ex post welfare and weakens the negative effect of information on ex ante welfare.

[13] The slope of ex post welfare graph is reversed if we consider a tax hike instead of a tax cut.



(a) ex ante welfare vs. n

(b) ex post welfare

Figure 4: Ex ante and ex post welfare measured in consumption unit: the case of labor income tax cut

3.1.3 Consumption tax

For a consumption tax cut from 20% to 10%, the results are qualitatively similar to the capital income tax cut case as shown in Figure 5.

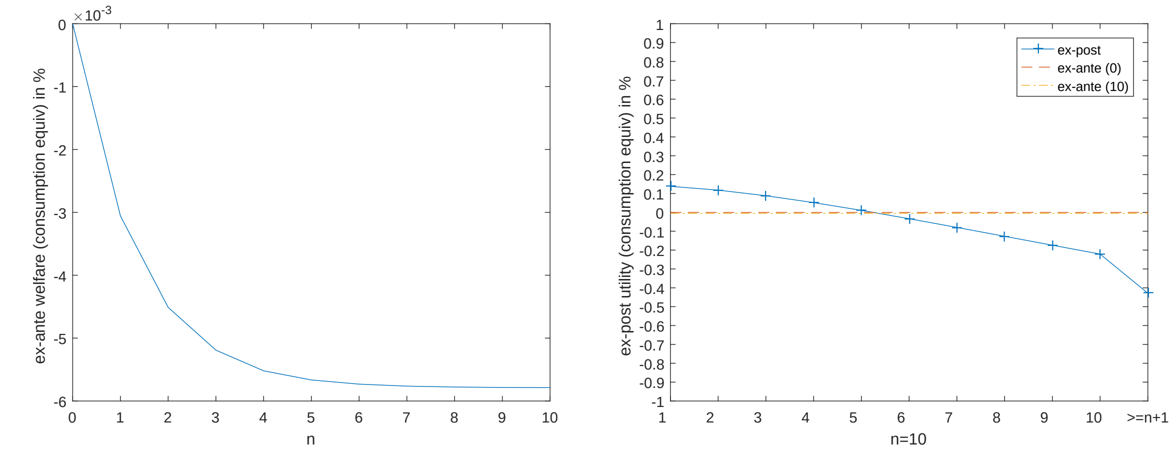
3.1.4 Spending

For the spending news case, we assume that government spending is initially 10% of the steady state output and then doubles. All distortionary taxes are constant.

Figure 6 displays ex ante and ex post welfare. Even though the equilibrium is not efficient, news about future spending increases ex ante welfare.

3.1.5 Robustness

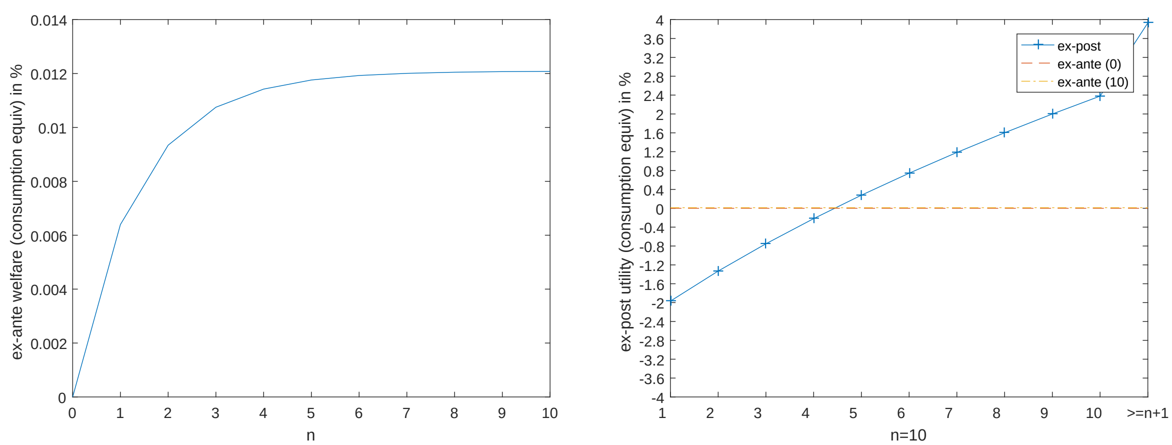
The welfare effect of announcement declines when taxes are made less distortionary. For example, when σ is raised from unity, it reduces the welfare effect by lowering the elasticity of intertemporal substitution. A lower elasticity of intertemporal substitution implies that the household's response to news about intertemporal distortion



(a) ex ante welfare vs. n

(b) ex post welfare

Figure 5: Ex ante and ex post welfare measured in consumption unit: the case of consumption tax cut



(a) ex ante welfare vs. n

(b) ex post welfare

Figure 6: Ex ante and ex post welfare measured in consumption unit: the case of government spending increase

is small, and thus the capital income tax news has a small effect on the equilibrium outcome. Another example is when the capital income tax is imposed on the after-depreciation rental rate, $r_t - \delta$, not on the rental rate itself, r_t. When depreciation is subtracted from taxable income, it reduces the intertemporal distortion from a given capital income tax rate and, again, the tax news shock has a smaller effect on the equilibrium.

3.2 Correlation between spending and taxes

Given that news has differential welfare effects depending on whether it is about spending or about distortionary taxes, it is of interest to examine cases where these shocks are, potentially imperfectly, correlated.

Here we examine the spectrum of correlation between -1 and 1. Perhaps it is easy to imagine when they have positive correlation. Positive correlation occurs, for example, when tax revenue adjusts to satisfy spending need. Negative correlation is also likely if we consider fiscal consolidation during which the government cuts spending while raising taxes.[14]

To examine imperfect correlation cases, it is convenient to assume that tax and spending shocks follow continuous stochastic processes.

The consumption tax is set to a constant value of 10%. We assume that the capital and labor income tax rates are the same $\tau_t^L = \tau_t^L = \tau_t$. Both G_t and τ_t are modeled as AR(1) processes:

$$\ln G_t = (1-\rho)\ln G_{ss} + \rho \ln G_{t-1} + \epsilon_t^g, \tag{7}$$

$$\ln \tau_t = (1-\rho)\ln \tau_{ss} + \rho \ln \tau_{t-1} + \epsilon_t^\tau. \tag{8}$$

[14] One may think that a positive correlation is the standard case. However, using the US narrative record, Romer and Romer (2010) find that spending-driven tax changes have been virtually nonexistent after 1975 and that deficit-driven tax changes occurred rather frequently between late 1970s and early 1990s.

We set parameters so that the two processes are roughly comparable. First, we use the same AR(1) coefficient, ρ, so that they are equally persistent. Second, the variances of innovations, ϵ_t^g and ϵ_t^τ, are set to the same number and the steady state relationship $\tau_{ss} = G_{ss}/Y_{ss}$ is imposed, so that spending, G_t, and a proxy for tax revenue, $\tau_t Y_{ss}$, follow the same stochastic process.

We consider two specifications. In the first specification, the n-period ahead tax shock is perfectly observed and is correlated with a shock to spending, but spending is also hit by a contemporaneous shock. In the second specification, it is the spending shock that is perfectly observed and a contemporaneous shock is to the tax rate. More formally, the two specifications are given as follows: for $v > 0$ and $\gamma \in [-1, 1]$,

$$\epsilon_t^\tau = \sqrt{v} u_{t-n}^\tau, \quad \epsilon_t^g = \sqrt{v}(\sqrt{1-\gamma^2} u_t^g + \gamma u_{t-n}^\tau) \tag{9}$$

$$\epsilon_t^g = \sqrt{v} u_{t-n}^g, \quad \epsilon_t^\tau = \sqrt{v}(\gamma u_{t-n}^g + \sqrt{1-\gamma^2} u_t^\tau) \tag{10}$$

where u_t^g and u_t^τ are IID white noise with unit variance and independent with each other. The subscript for u^τ and u^g denotes when these shocks are observed. Note that in both specifications $v = var(\epsilon_t^g) = var(\epsilon_t^\tau)$ and that γ is the correlation coefficient of ϵ_t^τ and ϵ_t^g.

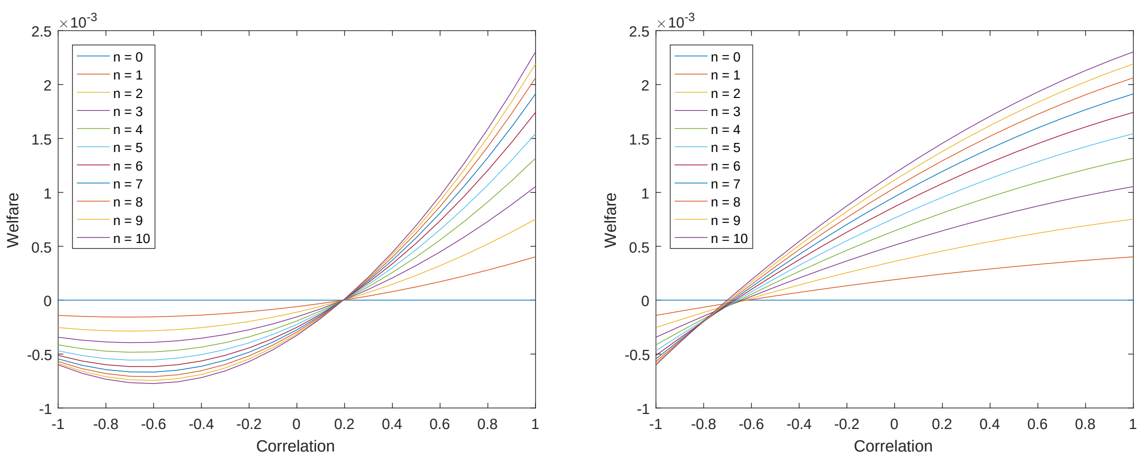
Parameter values are reported in Figure 2. The model is calibrated to quarterly frequency.

Because the state space is high dimensional for large n and is also continuous, we solve the model by a second order approximation using Dynare. By including the recursion $V_t = u(c_t) - v(l_t) + \beta E_t V_{t+1}$ in the system, we obtain the policy function for V as a quadratic function of the state variables and shocks. It is straightforward to compute its expected value.[15]

[15] Because $(u_{1-n}^i, u_{2-n}^i, ..., u_0^i)$ with $i \in \{\tau, g\}$ are assumed to be observed at the beginning of period 1, we take expectations over these shocks when computing ex ante welfare based on the time-0 information.

Table 2: Parameter values for the model with correlated shocks

Preference parameters	
Discount factor, β	0.99
Relative risk aversion, σ	1
Labor disutility weight, χ	1
Inverse of the Frisch elasticity of labor supply, η	1
Technology parameters	
Capital share parameter, α	0.3
Depreciation rate, δ	0.03
Fiscal policy parameters in the steady state	
Spending share in output, G/Y	0.2
Consumption tax, τ^C	0.1
Capital income tax, τ^K	0.2
Labor income tax, τ^L	0.2
Fiscal policy shock parameters	
AR(1) coefficient, ρ	0.9
Unconditional variance of $\ln G_t$ and $\ln \tau_t$, $v/(1-\rho^2)$	0.0025



(a) Welfare gain from τ news

(b) Welfare gain from G news

Figure 7: Welfare gain from news at different levels of correlation

Figure 7a shows, at each level of correlation, how ex ante welfare changes as we increase n in the first specification. Ex ante welfare when $n = 0$ is normalized at 0 for each value of the correlation coefficient and welfare for $n \geq 1$ are expressed in expressed in consumption-equivalence (%). When shocks are uncorrelated ($\gamma = 0$), receiving the tax news is still detrimental to ex ante welfare. Therefore, the mere presence of a spending shock does not change the welfare implication of distortionary tax news. The welfare effect remains negative unless the correlation coefficient is raised to a sufficiently positive value. However, very strong positive correlation is not required to generate a positive welfare effect of news. The welfare effect of news turns positive when the correlation exceeds approximately 0.2.

Figure 7b shows the results for the second specification. Again, even in the presence of a distortionary tax shock, the spending news improves ex ante welfare when two shocks are uncorrelated. For the welfare effect of news to be negative, strong negative correlation is needed. Note also that when the correlation coefficient is around -0.7 the welfare effect of news is non-monotonic in n. Welfare can decrease with n when n is small and then start increasing with n.

Comparison between Figure 7a and Figure 7b reveals that, for a given correlation structure and given n, ex ante welfare is higher when future spending shocks are made perfectly forecastable than it is when future tax shocks are made perfectly forecastable. The noisier the tax news, the higher the welfare. And the less noisier the spending news, the higher the welfare. Figure 8 shows the result when both shocks are perfectly observed in advance.[16]

In sum, in order to improve welfare through communication, the government should provide accurate information about future spending shocks but should refrain from doing so for future distortionary tax shocks. With such selective transparency, fiscal forward guidance can generate ex ante welfare gain unless the spending and tax

[16] In other words, $\epsilon_t^\tau = \sqrt{v} u_{t-n}^\tau$ and $\epsilon_t^g = \sqrt{v}(\sqrt{1-\gamma^2} u_{t-n}^g + \gamma u_{t-n}^\tau)$.

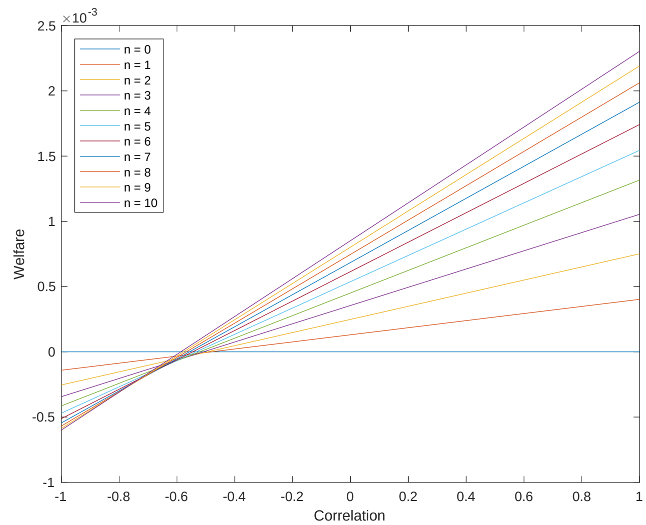


Figure 8: Welfare gain from news when both shocks are predictable

shocks have strong negative correlation. However, when these shocks are strongly and negatively correlated, the government should not reveal spending news either. This suggests that constructive ambiguity about future fiscal consolidation can be welfare improving, because spending and taxes are expected to have negative correlation under fiscal consolidation.

4 Quantifying the welfare effects

As the last exercise, we quantify the welfare effect of fiscal forward guidance based on a particular policy proposal for fiscal consolidation in Japan. Hansen and Imrohoroglu (2016) use a neoclassical growth model to quantify the fiscal adjustment needed for Japan to reduce the long-run debt to output ratio to 60%. One of their policy experiments involves a very sharp increase in consumption tax to approximately 60% when the debt-to-GDP ratio hits 250% and then a decline to 47% when the debt-to-GDP ratio reaches the assumed target of 60%.

We use the Markov switching model in Section 3.1 and choose most parameter values from Hansen and Imrohoroglu (2016). The parameter values are reported in Table

Table 3: Parameter values for the Hansen-Imrohoroglu experiment

Preference parameters	
Discount factor, β	0.9677
Relative risk aversion, σ	1
Labor disutility weight, χ	22.6331
Inverse of the Frisch elasticity of labor supply, η	2
Technology parameters	
Capital share parameter, α	0.3783
Depreciation rate, δ	0.0842
Fiscal policy parameters in the steady state	
Spending share in output, G/Y	0.2362
Consumption tax, τ^C	0.5
Capital income tax, τ^K	0.3557
Labor income tax, τ^L	0.3324

3.[17] Policy uncertainty is modeled as follows. All taxes other than the consumption tax, as well as government spending, are fixed at constant values. The consumption tax follows a two-state Markov chain. The initial tax rate is 8%. The tax rate changes to 50% with probability 0.3 each period, and 50% is the absorbing state. Because the fiscal reform takes place in year 2018 in Hansen and Imrohoroglu (2016), we choose the transition probability of 0.3 to make the probability of the fiscal reform *not* taking place for 10 years sufficiently low. It is $0.7^{10} \approx 0.028 < 0.03$ with our choice.[18]

As in the previous experiments, we allow private agents to observe the tax rates in the next n periods and measure ex ante welfare for different values of n. Figure 9 shows ex ante welfare as a function of n. Because the possible consumption tax change is large, the decline in ex ante welfare is also large. Ex ante welfare loss from observing the n-period-ahead tax rate monotonically increases to around 0.04% as we

[17] We do not incorporate utility from real bond holdings and, therefore, debt repayment is equivalent to lump-sum transfer. We also abstract from technology and population growth, but their values are almost zero in Hansen and Imrohoroglu (2016).

[18] We rule out the possibility that delayed consolidation requires higher tax rates. Bi, Leeper, and Leith (2013) take it into account and investigate the consequences from fiscal consolidations in a model where timing and composition of fiscal consolidation is uncertain.

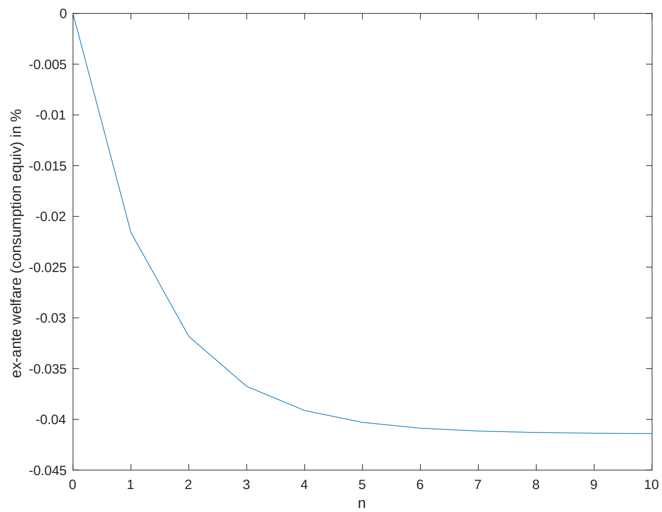


Figure 9: Ex ante welfare in consumption unit

increase n, and most of the loss materializes when n is raised from 0 to 3. There is a welfare gain from constructive ambiguity about the timing of fiscal consolidation, and this gain is comparable to the gain from business cycle smoothing in a real business cycle model.

5 Discussion

Now we discuss the sustainability of selective transparency and possible implications of introducing heterogeneity.

5.1 Is selective transparency sustainable?

Our results suggest that, from the ex ante point of view, it is socially desirable for the government to be selectively transparent about its future course of actions. However, selective transparency may not be sustainable for the following reasons.

First, as we have discussed already, being secretive about future tax shocks is time inconsistent. Therefore, sustaining secrecy may be difficult unless the government has a commitment ability or faces sufficiently strong punishment after revealing tax news.

Second, the government may indeed be subject to pressure to announce its private information. This is because an individual agent can profit from unilaterally obtaining more information including future tax shocks; holding fixed all variables that she takes as given, obtaining more information must weakly increase her objective function. Hirshleifer (1971), by considering a model with technological (i.e. not policy) shocks, shows that the private gain from information acquisition and dissemination may exist even if the information does not have any social value.[19] Hence, when information acquisition and dissemination are costly, these costs will be simply wasted. Things are worse in our setting: the social value of tax news is indeed negative, but the private sector nonetheless has the motivation and may waste resources trying to acquire that information.

A formal analysis is warranted to understand when selective transparency is not sustainable as an equilibrium outcome in a strategic setting, and how much information is revealed in equilibrium.

5.2 Departures from a representative agent framework

Throughout the paper we have focused on a representative-agent framework. Now we discuss the possibility that household heterogeneity may strengthen the case for the selective transparency.

There are numerous studies following Morris and Shin (2002) that examine the role of information provision by a policymaker when private agents receive idiosyncratic private signals — hence are heterogeneously informed — about the state of

[19] He points out that a privately informed agent can benefit from using the information to adjust her asset position, from taking a speculative asset position and then publicizing the information, and from resale of the information.

economy. When the dispersion of private agents' actions causes inefficiency, a public signal increases welfare by reducing the dispersion. More precise public information, however, can reduce social welfare when individuals have a stronger than socially desirable coordination motive. In a static example with a quadratic objective function, Angeletos and Pavan (2007) show that more precise public or private information increases welfare if the signal is about an "efficient" shock, which moves the social and private objectives in the same way, and that it decreases welfare if it is about an "inefficient" shock, which acts as a wedge between the social and private objectives. Based on the example, they conjecture that "if business cycles are driven primarily by shocks in markups or other distortions that induce a countercyclical efficiency gap, it is possible that providing markets with information that helps predict these shocks may reduce welfare." Although there is no clear mapping from our general dynamic economy to their example economy, perhaps it is natural to interpret a spending news shock as an efficient shock, and a distortionary tax news shock as an inefficient one. Then our main finding indeed provides supporting evidence for their conjecture, even though there is no heterogeneity across private agents in our model. Therefore, we conjecture that our main finding of differential welfare effects from the two types of news shocks could be strengthened when private agents are heterogeneously informed.

Heterogeneity may provide a channel through which selective transparency improves welfare. For example, Hirshleifer (1971) considers a complete market model in which households have heterogeneous state-contingent endowment patterns. Public information revelation that occurs before trading and that affects people's beliefs about future states makes households' wealth more risky in the following sense. Information that makes people believe that some states are less likely will lower the price of state-contingent claims for these states and thus lower the value of wealth for individuals who possess the corresponding claims disproportionately. From the ex ante point of view, risk-averse individual households may want to avoid such risk and to choose to prohibit public information revelation. Therefore, our conjecture is that when there is policy uncertainty regarding, or a shock to, resource redistribution across heterogeneous groups of households, revealing news about it can reduce ex ante welfare.

6 Conclusion

This paper studies the welfare consequences of fiscal forward guidance. Disclosing information about future fiscal policy changes can be detrimental to ex ante welfare when it is about future distortionary tax shocks while it can be welfare-enhancing if it is about future spending. Therefore, a benevolent fiscal authority may want to commit, ex ante, to being selectively transparent — being secretive about future tax shocks while being transparent about future spending shocks. This result is shown analytically in some models and demonstrated numerically in more general models. We also demonstrate that welfare gain from secrecy can be sizable for large, but not totally unrealistic, fiscal adjustments. In an accompanying paper we also examine the effect of fiscal forward guidance when taxes are chosen optimally to finance an exogenous series of spending (Fujiwara and Waki, 2016).

References

AMATO, J. D., S. MORRIS, AND H. S. SHIN (2002): "Communication and Monetary Policy," *Oxford Review of Economic Policy*, 18(4), 495–503.

ANGELETOS, G.-M., AND A. PAVAN (2007): "Efficient Use of Information and Social Value of Information," *Econometrica*, 75(4), 1103–1142.

BAKER, S. R., N. BLOOM, AND S. J. DAVIS (2015): "Measuring Economic Policy Uncertainty," NBER Working Papers 21633, National Bureau of Economic Research, Inc.

BARILLAS, F., AND J. FERNANDEZ-VILLAVERDE (2007): "A generalization of the endogenous grid method," *Journal of Economic Dynamics and Control*, 31(8), 2698–2712.

BASSETTO, M. (2015): "Forward Guidance: Communication, Commitment, or Both?," *manuscript, Federal Reserve Bank of Chicago and University College London*.

BI, H., E. M. LEEPER, AND C. LEITH (2013): "Uncertain Fiscal Consolidations," *Economic Journal*, 0, F31–F63.

BLOOM, N. (2009): "The Impact of Uncertainty Shocks," *Econometrica*, 77(3), 623–685.

BORN, B., AND J. PFEIFER (2014): "Policy risk and the business cycle," *Journal of Monetary Economics*, 68(C), 68–85.

BROCK, W. A., AND L. J. MIRMAN (1972): "Optimal economic growth and uncertainty: The discounted case," *Journal of Economic Theory*, 4(3), 479–513.

CAMPBELL, J. R., C. L. EVANS, J. D. FISHER, AND A. JUSTINIANO (2012): "Macroeconomic Effects of Federal Reserve Forward Guidance," *Brookings Papers on Economic Activity*, 44(1 (Spring), 1–80.

CARROLL, C. D. (2006): "The method of endogenous gridpoints for solving dynamic stochastic optimization problems," *Economics Letters*, 91(3), 312–320.

D'ACUNTO, F., D. HOANG, AND M. WEBER (2016): "Unconventional Fiscal Policy, Inflation Expectations, and Consumption Expenditure," *mineo*.

EGGERTSSON, G. B., AND M. WOODFORD (2003): "The Zero Bound on Interest Rates and Optimal Monetary Policy," *Brookings Papers on Economic Activity*, 34(2003-1), 139–235.

EVANS, G. W., S. HONKAPOHJA, AND K. MITRA (2009): "Anticipated fiscal policy and adaptive learning," *Journal of Monetary Economics*, 56(7), 930–953.

FERNANDEZ-VILLAVERDE, J., P. GUERRON-QUINTANA, K. KUESTER, AND J. RUBIO-RAMIREZ (2015): "Fiscal Volatility Shocks and Economic Activity," *American Economic Review*, 105(11), 3352–84.

FUJIWARA, I., AND Y. WAKI (2015): "Private news and monetary policy," Globalization and Monetary Policy Institute Working Paper 238, Federal Reserve Bank of Dallas.

——— (2016): "Optimal Taxation and News Shocks," .

HANSEN, G., AND S. IMROHOROGLU (2016): "Fiscal Reform and Government Debt in Japan: A Neoclassical Perspective," *Review of Economic Dynamics*, 21, 201–224.

HIRSHLEIFER, J. (1971): "The Private and Social Value of Information and the Reward to Inventive Activity," *American Economic Review*, 61(4), 561–574.

LEEPER, E. M., T. B. WALKER, AND S. S. YANG (2013): "Fiscal Foresight and Information Flows," *Econometrica*, 81(3), 1115–1145.

MERTENS, K., AND M. RAVN (2010): "Measuring the Impact of Fiscal Policy in the Face of Anticipation: A Structural VAR Approach," *Economic Journal*, 120(544), 393–413.

MERTENS, K., AND M. O. RAVN (2011): "Understanding the Aggregate Effects of Anticipated and Unanticipated Tax Policy Shocks," *Review of Economic Dynamics*, 14(1), 27–54.

MERTENS, K., AND M. O. RAVN (2012): "Empirical Evidence on the Aggregate Effects of Anticipated and Unanticipated US Tax Policy Shocks," *American Economic Journal: Economic Policy*, 4(2), 145–81.

MORRIS, S., AND H. S. SHIN (2002): "Social Value of Public Information," *American Economic Review*, 92(5), 1521–1534.

RAMEY, V. A. (2011): "Identifying Government Spending Shocks: It's all in the Timing," *The Quarterly Journal of Economics*, 126(1), 1–50.

ROMER, C. D., AND D. H. ROMER (2010): "The Macroeconomic Effects of Tax Changes: Estimates Based on a New Measure of Fiscal Shocks," *American Economic Review*, 100(3), 763–801.

SVENSSON, L. E. (2014): "Forward Guidance," NBER Working Papers 20796, National Bureau of Economic Research, Inc.

WOODFORD, M. (2003): *Interest and Prices: Foundations of a Theory of Monetary Policy*. Princeton University Press, Princeton.

YANG, S.-C. S. (2005): "Quantifying tax effects under policy foresight," *Journal of Monetary Economics*, 52(8), 1557 – 1568.

A Appendix: analytical results

A.1 A three-period model with a general CRRA utility function: distortionary tax only

The representative household chooses consumption and savings:

$$\max_{C_1,C_2,S_1} \frac{C_1^{1-\sigma}-1}{1-\sigma} + \mathbb{E}_1^P[\frac{C_2^{1-\sigma}-1}{1-\sigma}]$$

subject to the same set of constraint. Again, we first consider the case in which the government rebates the tax revenue back to the household. The equilibrium condition is given by the Euler equation

$$C_1^{-\sigma} \quad = \quad \mathbb{E}_1^P[C_2^{-\sigma}(1-\tau_2^K)]$$

and the resource constraints (2) and (3). Then, again, these three equations can be summarized by a single equation with the saving rate $s := S_1/Y_1$:

$$\frac{s}{1-s} = \mathbb{E}_1^P[1-\tau_2^K]^{\frac{1}{\sigma}}.$$

Let X denote $\mathbb{E}_1^P[1-\tau_2^K]$. Then ex ante welfare is written as a function of X:

$$\begin{aligned}
\mathbb{E}_0[\frac{C_1^{1-\sigma}-1}{1-\sigma}+\frac{C_2^{1-\sigma}-1}{1-\sigma}] &= Y_1^{1-\sigma}\mathbb{E}_0[\frac{(1-s)^{1-\sigma}}{1-\sigma}+\frac{s^{1-\sigma}}{1-\sigma}]-\frac{2}{1-\sigma} \\
&= Y_1^{1-\sigma}\mathbb{E}_0[g(X;\sigma)]-\frac{2}{1-\sigma},
\end{aligned}$$

where

$$g(x;\sigma) := \frac{1}{1-\sigma}(1+x^{\frac{1-\sigma}{\sigma}})\frac{1}{(1+x^{\frac{1}{\sigma}})^{1-\sigma}}.$$

Then, for each $\sigma > 0$, $g(.;\sigma)$ is strictly concave in a neighborhood of $x = 1$. If the support of $1-\tau_2^K$ is contained in the neighborhood, then for $\mathcal{F}_1^P \subset \tilde{\mathcal{F}}_1^P$ we obtain

$$Y_1^{1-\sigma}\mathbb{E}_0[g(\tilde{X};\sigma)] = Y_1^{1-\sigma}\mathbb{E}\left[\mathbb{E}[g(\tilde{X};\sigma)|\mathcal{F}_1^P]\right] \leq Y_1^{1-\sigma}\mathbb{E}[g(X;\sigma)],$$

where $X := \mathbb{E}[1-\tau_2^K|\mathcal{F}_1^P]$ and $\tilde{X} := \mathbb{E}[1-\tau_2^K|\tilde{\mathcal{F}}_1^P]$.

A.1.1 Spending-to-output ratio shock financed by the saving tax

When the spending-to-output ratio $\psi_2 = G_2/S_1$ is random and spending is financed only by the saving tax, we have $C_2 = (1-\psi_2)S_1 = (1-\tau_2^K)S_1$ and the equilibrium condition reduces to

$$\frac{s}{1-s} = \mathbb{E}^P[(1-\tau_2^K)^{1-\sigma}]^{\frac{1}{\sigma}}.$$

Denoting $\mathbb{E}^P[(1-\tau_2^K)^{1-\sigma}]$ by X, we obtain

$$\begin{aligned}
\mathbb{E}_0[\frac{C_1^{1-\sigma}-1}{1-\sigma} + \frac{C_2^{1-\sigma}-1}{1-\sigma}] &= Y_1^{1-\sigma}\mathbb{E}_0[\frac{(1-s)^{1-\sigma}}{1-\sigma} + \frac{s^{1-\sigma}(1-\tau_2^K)^{1-\sigma}}{1-\sigma}] - \frac{2}{1-\sigma} \\
&= Y_1^{1-\sigma}\mathbb{E}_0[\frac{(1+X^{\frac{1}{\sigma}})^{\sigma}}{1-\sigma}] - \frac{2}{1-\sigma}.
\end{aligned}$$

The term $(1+x^{\frac{1}{\sigma}})^{\sigma}/(1-\sigma)$ is convex in x and, therefore, ex ante welfare improves when more information is provided.

A.2 Analytical results: the Brock-Mirman model

In this section we argue that our analytical results for the three period model hold true in an infinite horizon stochastic growth model in Brock and Mirman (1972). Here we only consider shocks to capital income tax. Government spending is assumed to be zero.

The only restriction we impose on the stochastic process of capital income tax rates $\{\tau_t^K\}_{t=0}^{\infty}$ is that an equilibrium exists. Therefore, our specification encapsulates various scenarios that cannot be analyzed with three-period models. For example, we can consider a situation in which private agents know that the tax rate will be increased to a certain level at some future date but are unsure about the exact timing. Another example is that the autocovariance structure of the tax changes stochastically over time.

The representative household maximizes

$$\mathbb{E}_0^P[\sum_{t=0}^{\infty} \beta^t \ln c_t]$$

subject to the budget constraint:

$$c_t + k_{t+1} = (1-\tau_t^K) r_t k_t + w_t + T_t, \quad \forall t,$$

taking the prices $\{r_t, w_t\}_{t=0}^{\infty}$ and the fiscal policy $\{\tau_t^K, T_t\}_{t=0}^{\infty}$ as given. The household supplies one unit of labor inelastically, and capital fully depreciates after production. The representative firm owns a Cobb-Douglas production function, $Y = K^\alpha L^{1-\alpha}$, and its first-order condition is given by

$$r_t = \alpha K_t^{\alpha-1} \text{ and } w_t = (1-\alpha) K_t^\alpha. \tag{11}$$

The government budget constraint is

$$\tau_t^K r_t k_t = T_t. \tag{12}$$

The equilibrium condition is:

$$\begin{aligned} \frac{1}{C_t} &= \beta \mathbb{E}_t^P[\frac{1}{C_{t+1}}(1-\tau_{t+1}^K)\alpha K_{t+1}^{\alpha-1}], \quad \forall t, \\ C_t + K_{t+1} &= K_t^\alpha, \quad \forall t, \end{aligned}$$

and the transversality condition $\lim_{T\to\infty} \beta^T \mathbb{E}_t^P[K_{T+1}/C_T] = 0$.[20]

[20]When we have consumption taxes that converges almost surely, the equilibrium condition is identical except that the term $1-\tau_{t+1}^K$ in the Euler equation is replaced with $(1-\tau_{t+1}^K)(1+\tau_t^C)/(1+\tau_{t+1}^C)$. Therefore, all the following results hold in the presence of consumption tax if we replace $1-\tau_{t+1}^K$ with $(1-\tau_{t+1}^K)(1+\tau_t^C)/(1+\tau_{t+1}^C)$.

Let $Y_t = K_t^\alpha$ and $s_t := K_{t+1}/Y_t$. Then we have

$$\frac{s_t}{1-s_t} = \alpha\beta\mathbb{E}_t^P[\frac{1}{1-s_{t+1}}(1-\tau_{t+1}^K)] \Rightarrow \frac{1}{1-s_t} = 1 + \alpha\beta\mathbb{E}_t^P[\frac{1}{1-s_{t+1}}(1-\tau_{t+1}^K)].$$

Iterating forward, we obtain

$$\frac{1}{1-s_t} = 1 + \mathbb{E}_t^P\Big[\sum_{j=1}^{\infty}(\alpha\beta)^j\prod_{i=1}^{j}(1-\tau_{t+i}^K)\Big] + \lim_{J\to\infty}(\alpha\beta)^J\mathbb{E}_t^P[\frac{1}{1-s_{t+J}}\prod_{j=1}^{J}(1-\tau_{t+j}^K)].$$

Assume that the last term is zero.[21] Then we have

$$\frac{s_t}{1-s_t} = \mathbb{E}_t^P\Big[\sum_{j=1}^{\infty}(\alpha\beta)^j\prod_{i=1}^{j}(1-\tau_{t+i}^K)\Big].$$

This is clearly a generalization of the two-period model with log utility.

Let $X_t = \mathbb{E}_t^P[\sum_{j=1}^{\infty}(\alpha\beta)^j\prod_{i=1}^{j}(1-\tau_{t+i}^K)]$. Then ex ante utility of the representative household can be written in terms of $\{X_t\}_{t=0}^\infty$ and parameters only.

Lemma 1 *Ex ante welfare equals*

$$\frac{\alpha\beta}{1-\alpha\beta}\mathbb{E}\Big[\sum_{t=0}^{\infty}\beta^t f(X_t)\Big] + \frac{\alpha\ln K_0}{1-\alpha\beta},$$

where the function $f(x) := \ln x - (1/\alpha\beta)\ln(1+x)$ is strictly concave on $\big(0, \sqrt{\alpha\beta}/(1-\sqrt{\alpha\beta})\big)$ and is strictly convex on $(\sqrt{\alpha\beta}/(1-\sqrt{\alpha\beta}), \infty)$.

As in the three period model with log utility, when the after-tax rate $1-\tau_t^K$ resides in the interval on which f is strictly concave, more information is harmful for ex ante welfare.

Proposition 2 *Let $\mathcal{F}$ and $\mathcal{G}$ be filtrations such that $\mathcal{F}$ is coarser than $\mathcal{G}$ (i.e. $\mathcal{F}_t \subset \mathcal{G}_t$ for all*

[21] A sufficient condition for this is that $1-\tau_t^K$ is bounded above by $1/\alpha$. Then the transversality condition implies the convergence to zero.

t) and that $\{\tau_t^K\}$ is $\mathcal{F}$-adapted. Suppose for all t,

$$0 < \sum_{j=1}^{\infty} (\alpha\beta)^j \prod_{i=1}^{j} (1 - \tau_{t+i}^K) < \frac{\sqrt{\alpha\beta}}{1 - \sqrt{\alpha\beta}}$$

holds almost everywhere.[22] *Then,*

$$\mathbb{E}[f(X_t^{\mathcal{F}})] \geq \mathbb{E}[f(X_t^{\mathcal{G}})]$$

for all t, where $X_t^{\mathcal{F}} := \mathbb{E}[\sum_{j=1}^{\infty} (\alpha\beta)^j \prod_{i=1}^{j} (1 - \tau_{t+i}^K) | \mathcal{F}_t]$ and $X_t^{\mathcal{G}} := \mathbb{E}[\sum_{j=1}^{\infty} (\alpha\beta)^j \prod_{i=1}^{j} (1 - \tau_{t+i}^K) | \mathcal{G}_t]$. Inequality is strict when $X_t^{\mathcal{F}} \neq X_t^{\mathcal{G}}$ with positive probability.

If the representative household is provided with information that improves its expectations of future taxes with positive probability, then the stochastic process of $\{X_t\}_{t=0}^{\infty}$ changes with positive probability. In such situations, Proposition 2 together with Lemma 1 implies that ex ante utility of the household strictly decreases.

A.2.1 Uncertainty shock

The above model encapsulates specifications with uncertainty shocks. Consider the following stochastic process $\{z_t\}_{t=0}^{\infty}$ with time-varying risk:

$$z_t = z_{t-1} \times (1 + \sigma_t \epsilon_t),$$

where ϵ_t is an I.I.D. random variable that is distributed over $[-1, 1]$ symmetrically around 0. Here $\{\sigma_t\}_{t=0}^{\infty}$ is a potentially persistent process in the interval $[\underline{\sigma}, \overline{\sigma}] \subset \mathbb{R}_+$. Let $\underline{z}$ and $\overline{z}$ be such that $0 < \underline{z} < \overline{z} < 1/\sqrt{\alpha\beta}$. Suppose that $z_0 \in (\underline{z}, \overline{z})$. Let S be the

[22] A sufficient condition is that $1 - \tau_t^K$ is bounded above by $1/\sqrt{\alpha\beta}$, because

$$0 < \sum_{j=1}^{\infty} (\alpha\beta)^j \prod_{i=1}^{j} (1 - \tau_{t+i}^K) \leq \sum_{j=1}^{\infty} \frac{(\alpha\beta)^j}{(\sqrt{\alpha\beta})^j} = \sum_{j=1}^{\infty} (\sqrt{\alpha\beta})^j = \frac{\sqrt{\alpha\beta}}{1 - \sqrt{\alpha\beta}}$$

holds.

stopping time such that

$$S := \inf\{t \geq 0 : z_t < \underline{z} \text{ or } z_t > \overline{z}\}.$$

We assume

$$1 - \tau_t^K = \begin{cases} z_t & \text{if } S > t, \\ \underline{z} & \text{if } S \leq t \text{ and } x_S < \underline{z}, \\ \overline{z} & \text{if } S \leq t \text{ and } x_S > \overline{z}. \end{cases}$$

In other words, $1 - \tau_t^K$ follows the same process as z_t until the latter variable hits the boundary of $[\underline{z}, \overline{z}]$ and stays at the boundary after that. Therefore, when $\{\sigma_t, \epsilon_t\}$ is adapted to the filtration of the household, all the assumptions in Proposition 2 are satisfied. The term $\mathbb{E}_t^P[\sum_{j=1}^{\infty}(\alpha\beta)^j \prod_{i=1}^{j}(1 - \tau_{t+i}^K)]$ involves higher-order moments of $\{\sigma_{t+j}\epsilon_{t+j}\}_{j=1}^{\infty}$ and, therefore, it changes when the household is provided more information about $\{\sigma_{t+j}\epsilon_{t+j}\}_{j=1}^{\infty}$. Therefore, it follows from Proposition 2 that ex ante welfare decreases when the government provides information about future uncertainty shocks.

A.2.2 Proofs

Proof of Lemma 1. Because $K_{t+1} = s_t K_t^{\alpha}$ by definition, we have

$$\ln K_t = \alpha^t \ln K_0 + \sum_{j=0}^{t-1} \alpha^{t-1-j} \ln s_j.$$

Therefore,

$$\begin{aligned} \sum_{t=0}^{\infty} \beta^t \ln C_t &= \sum_{t=0}^{\infty} \beta^t \{\ln(1 - s_t) + \alpha \ln K_t\} \\ &= \sum_{t=0}^{\infty} \beta^t \{\ln(1 - s_t) + \alpha^{t+1} \ln K_0 + \sum_{j=0}^{t-1} \alpha^{t-j} \ln s_j\} \end{aligned}$$

Collecting the terms that involve s_t,

$$\begin{aligned} \beta^t \ln(1-s_t) + \sum_{l=t+1}^{\infty} \beta^l \alpha^{l-t} \ln s_t &= \frac{\beta^t}{1-\alpha\beta}\{(1-\alpha\beta)\ln(1-s_t) + \alpha\beta \ln s_t\} \\ &= \frac{\beta^t}{1-\alpha\beta}\Big\{\alpha\beta \ln X_t - \ln(1+X_t)\Big\}. \end{aligned}$$

Ex ante welfare of the household is thus

$$\mathbb{E}\Big[\sum_{t=0}^{\infty} \beta^t \frac{\alpha\beta}{1-\alpha\beta}\Big\{\ln X_t - \frac{1}{\alpha\beta}\ln(1+X_t)\Big\}\Big] + \frac{\alpha \ln K_0}{1-\alpha\beta}.$$

The function $f(x) := \ln x - (1/\alpha\beta)\ln(1+x)$ is strictly concave on $\big(0, \sqrt{\alpha\beta}/(1-\sqrt{\alpha\beta})\big)$ and is strictly convex on $(\sqrt{\alpha\beta}/(1-\sqrt{\alpha\beta}), \infty)$. ■

Proof of Proposition 2. Under the stated condition, $X_t^{\mathcal{F}}$ and $X_t^{\mathcal{G}}$ are in the interval $[0, \sqrt{\alpha\beta}/(1-\sqrt{\alpha\beta})]$. Because f is strictly concave on the same interval, Jensen's inequality implies that

$$\mathbb{E}[f(X_t^{\mathcal{G}})] = \mathbb{E}\left[\mathbb{E}[f(X_t^{\mathcal{G}})|\mathcal{F}_t]\right] \le \mathbb{E}\left[f\left(\mathbb{E}[X_t^{\mathcal{G}}|\mathcal{F}_t]\right)\right] = \mathbb{E}[f(X_t^{\mathcal{F}})].$$

($\mathbb{E}[X_t^{\mathcal{G}}|\mathcal{F}_t] = X_t^{\mathcal{F}}$ follows from the law of iterated expectations and $\mathcal{F}_t \subset \mathcal{G}_t$.) Because f is strictly concave, inequality is strict if and only if $X_t^{\mathcal{F}} \ne X_t^{\mathcal{G}}$ with positive probability.

■

B Appendix: computation

We use the endogenous grid method (see Carroll (2006)) for computation. We follow Barillas and Fernandez-Villaverde (2007), but the separability of utility in consumption and labor simplifies the algorithm.

We focus on an equilibrium in which aggregate capital K and aggregate labor L follow the laws of motion:

$$\begin{aligned} K' &= X^K(K, z) \\ L &= X^L(K, z). \end{aligned} \tag{13}$$

The representative household's problem is written recursively as follows:

$$V(k, K, z) = \max_{c,k',l} u(c) - v(l) + \beta \mathbb{E}[V(k', K', z')|z]$$

subject to

$$(1+\tau^C(z))c+k' = (1-\tau^K(z))\alpha K^{\alpha-1}L^{1-\alpha}k+(1-\tau^L(z))(1-\alpha)K^{\alpha}L^{-\alpha}l+(1-\delta)k+T(K,z)$$

and the aggregate law of motion (13). Because L depends only on K and z, we do not need to include L as a state variable for the household problem. The solution to this problem is given by $C(k, K, z)$, $K'(k, K, z)$, and $L(k, K, z)$.

The government budget constraint requires

$$\tau^K(z)\alpha K^{\alpha}L^{1-\alpha} + \tau^C(z)C + \tau^L(z)(1-\alpha)K^{\alpha}L^{1-\alpha} = G(K, z) + T(K, z).$$

The government spending is allowed to vary with (K, z). Our specification therefore nests one in which the government spending share in output fluctuates randomly:

$G(K, z) = \phi(z)K^{\alpha}(X^L(K, z))^{1-\alpha}$.

A recursive equilibrium consists of the policy function (C, K', L), the value function v, the aggregate law of motion (X^K, X^L), and the fiscal policy (G, T) such that (a) the pair of (C, K', L) and v solves the household's recursive problem given (X^K, X^L) and T, (b) for all (K, z), (X^K, X^L) satisfies $X^K(K, z) = K'(K, K, z)$ and $X^L(K, z) = L(K, K, z)$, and (c) markets clear.

The equilibrium condition is summarized by the following three equations:

$$\begin{aligned}
u'(C(K, K, z)) &= \beta\mathbb{E}[V_1(K', K', z')|z] \\
\frac{v'(L(K, K, z))}{u'(C(K, K, z))} &= \frac{(1-\tau^L(z))(1-\alpha)K^{\alpha}L(K, K, z)^{-\alpha}}{1+\tau^C(z)} \\
V_1(K, K, z) &= \frac{1}{1+\tau^C(z)}\{1-\delta+(1-\tau^K(z))\alpha K^{\alpha-1}L(K, K, z)^{1-\alpha}\}u'(C(K, K, z))
\end{aligned}$$

Let $W(K, z) := V(K, K, z)$, $D(K, z) := V_1(K, K, z)$, $\tilde{C}(K, z) := C(K, K, z)$, $\tilde{K}'(K, z) := K'(K, K, z)$, and $\tilde{L}(K, z) = L(K, K, z)$.

$$\begin{aligned}
u'(\tilde{C}(K, z)) &= \beta\mathbb{E}[D(K', z')|z](1+\tau^C(z)) \\
v'(\tilde{L}(K, z))\tilde{L}(K, z)^{\alpha} &= \frac{(1-\tau^L(z))(1-\alpha)K^{\alpha}}{1+\tau^C(z)}u'(\tilde{C}(K, z)) \\
D(K, z) &= \frac{1}{1+\tau^C(z)}\{1-\delta+(1-\tau^K(z))\alpha K^{\alpha-1}\tilde{L}(K, z)^{1-\alpha}\}u'(\tilde{C}(K, z)) \\
W(K, z) &= u(\tilde{C}(K, z)) - v(\tilde{L}(K, z)) + \beta\mathbb{E}[W(\tilde{K}'(K, z), z')|z]
\end{aligned}$$

Algorithm 1 (Endogenous grid method with endogenous labor) *First fix the grid $Kend$ for k'.*

- *Initial guess: D_0 and W_0.*
- *For $n \geq 0$, take (D_n, W_n) as given and compute, for each z and $K' \in Kend$,*

1. $c_{n+1}(k', z)$ *using*

$$c_{n+1}(K', z) = \left[\beta \mathbb{E}[D_n(K', z')|z](1 + \tau^C(z)\right]^{-1/\sigma},$$

2. $k_{n+1}(K', z)$ *and* $l_{n+1}(K', z)$ *using*

$$c_{n+1}(K', z) + K' = k_{n+1}(K', z)^{\alpha} l_{n+1}(K', z)^{1-\alpha},$$

   and

$$v'(l_{n+1}(K', z)) l_{n+1}(K', z)^{\alpha} = \frac{(1 - \tau^L(z))(1 - \alpha) u'(c_{n+1}(K', z))}{1 + \tau^C(z)} k_{n+1}(K', z)^{\alpha},$$

   [this part requires a nonlinear equation solver]

3. *The derivative of the value function:*

$$D_{n+1}(k_{n+1}(K', z), z) = \frac{u'(c_{n+1}(K', z))}{1 + \tau^C(z)} \{1 - \delta + (1 - \tau^K(z)) \alpha k_{n+1}(K', z)^{\alpha - 1} l_{n+1}(K', z)^{1-\alpha}\},$$

4. *The value function:*

$$W_{n+1}(k_{n+1}(K', z), z) = u(c_{n+1}(K', z)) - v(l_{n+1}(K', z)) + \beta \mathbb{E}[W_n(K', z')|z].$$

5. *For each* z*, interpolate* (D_{n+1}, W_{n+1}) *to obtain their values on* $Kend$*.*

6. *Terminate the iteration if* $||W_{n+1} - W_n||$ *becomes smaller than the pre-specified tolerance level. Otherwise increase* n *by 1 and repeat the previous computation.*

When $v(l) = \chi l^{1+\eta}/(1 + \eta)$, then in Step 2 we can express

$$\frac{k_{n+1}(K', z)}{l_{n+1}(K', z)} = \left[\frac{(1 - \tau^L(z))(1 - \alpha) u'(c_{n+1}(K', z))}{\chi(1 + \tau^C(z))}\right]^{\frac{-1}{\alpha + \eta}} k_{n+1}(K', z)^{\frac{\eta}{\alpha + \eta}},$$

and substitute this into the resource constraint to obtain

$$c_{n+1}(K',z)+K' = k_{n+1}(K',z)^{\frac{\alpha(1+\eta)}{\alpha+\eta}} \times \left[\frac{(1-\tau^L(z))(1-\alpha)u'(c_{n+1}(K',z))}{\chi(1+\tau^C(z))}\right]^{\frac{1-\alpha}{\alpha+\eta}} + (1-\delta)k_{n+1}(K',z),$$

and solve the second equation for $k_{n+1}(K',z)$ and then the first equation for $l_{n+1}(K',z)$.