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AN EMPIRICAL INVESTIGATION OF FUTURES CONTRACTS PRICING IN JAPAN*

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I. *Introduction*

If the number of hedgers selling futures contracts is greater than the number of buyers, speculators will buy futures assets in order to make a profit. That is, they obtain the risk premium of a futures transaction. Hicks and others have called this phenomenon 'normal backwardation.' The converse phenomenon is called 'contango.' Empirical tests of these phenomena have been attempted using the CAPM, but their results are not compatible.[1]

According to the CAPM, it is difficult to specify a market portfolio which is composed of all risky assets in the market, and it is not appropriate to include futures assets in the market portfolio, because their net supply is zero.[2] However when the APT (arbitrage pricing theory) is used, it does not depend on the market portfolio which is to be specified *a priori*. Ehrhardt, Jordan and Walkling (1987) have investigated the commodity futures markets in the U.S., and reached the conclusion that the prices in these markets were equal to the expected future spot prices of the commodities.

The purpose of this paper is to analyse the pricing of the Japanese futures markets, in particular, the market for government bonds, gold and soybeans, by means of the APT and to test whether there are premiums in these markets or not. In section 2, the test methodology is explained. Section 4 shows the results of the empirical investigation which uses the data described in section 3. Section 5 states the concluding remarks.

II. *Test Methodology*

According to Roll and Ross (1980), the APT is specified as follows. The i-th asset's rate of return during the t-th period is assumed to be

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1 Stein (1986) surveyed these results.

2 See Baxter et al. (1985).

(1) $R_{it}=E_{it}+B_{i1}D_{1t}+\ldots+B_{ik}D_{kt}+\varepsilon_{it}$,

where E_{it} is the expected value of R_{it}, $D_{ht}(h=1,\ldots, k)$ is the h-th common factor during the t-th period, the averages of which are zero, B_{ih} is the sensitivity (that is, the factor loadings) of the i-th asset to the h-th factor and ε_{it} is the disturbance term, whose average is zero. The relationship of the APT is

(2) $E_{it}=F_{ot}+B_{i1}F_{1t}+\ldots+B_{ik}F_{kt}$,

where $F_{ot},\ldots,F_{kt}$ are the weights, F_{ot} is the rate of return of the asset whose sensitivity B_{ih} is zero and F_{ht} $(h=1,\ldots,k)$ is the risk premium of the portfolio with risk only with respect to the h-th factor. From equation (1) and (2),

(3) $R_{it}=F_{ot}+B_{i1}F_{1t}+\ldots+B_{ik}F_{kt}+u_{it}$,

where

(4) $u_{it}=B_{i1}D_{1t}+\ldots+B_{ik}D_{kt}+\varepsilon_{it}$

is the disturbance term generated by D_{1t}, ..., D_{kt} and ε_{it}, and the average of u_{it} is zero.

Measuring equation (3), the rate of return of the futures asset R_t^f is used in substitution for R_{it}. If the estimated values of $F_{1t},F_{2t},\ldots$satisfy

(5) $\hat{F}_{ht}=0, \quad h=1,\ldots,k$,

then the risk premiums are zero and there is neither normal backwardation nor contango. The null hypothesis, equation (5), is tested as follows.[3] The test static is

$$\chi^2=T(\bar{\lambda}-\mu_0)'\Phi^{-1}(\bar{\lambda}-\mu_0),$$

whose distribution is chi-squared. With $\mu_0=0$, the null hypothesis can be tested, where T is the number of the observations and

$$\bar{\lambda}=\sum_t \hat{\lambda}_t/T,$$

$$\hat{\lambda}_1=(\hat{F}_{1t},\hat{F}_{2t},\ldots,\hat{F}_{kt})',$$

$$\Phi=\sum_t (\hat{\lambda}_t-\bar{\lambda})\,(\hat{\lambda}_t\ldots\bar{\lambda})'/T.$$

III. *Data*

The futures assets which are used in the estimation are long-term government bonds, gold and imported soybeans on the Tokyo Stock Exchange, the Tokyo Commodity Exchange for Industry and the Tokyo Grain Exchange. It is assumed that investors make similar contracts for these futures assets. The price of gold appears to be sensitive to the

[3] See Gultekin and Rogalski (1985).

interest rate. Soybeans imported from the U.S. are most actively traded on the futures markets in Japan.

The price data used are the settlement prices published every Friday. The Tokyo Stock Exchange began the trading of bond futures in October 1985. Bond futures have five delivery months, but only two of them are actively traded. Hence the weekly data of the same two delivery months are available after the second week of December 1985. The clearing days are the 20th of March, June, September and December, and the last trading days are the nine preceding days. Data for the same two delivery months can be continuously collected for about three months beginning from March, June, September and December. Gold has seven delivery months and soybeans six, and the last trading days are around the 25th, every two months. Hence five delivery months of data for these two futures are available during these three months. These twelve variables, that is, two for bonds, five for gold and five for soybeans, are used for the estimation.

The estimation periods consist of the following 89 weeks. They are

1) the 2nd week of December, 1985–the 3rd week of February, 1986 (11 weeks),
2) the 1st week of March, 1986–the 1st week of June, 1986 (14 weeks),
3) the 5th week of May, 1986–the 4th week of August, 1986 (13 weeks),
4) the 1st week of September, 1986–the 1st week of December, 1986 (14 weeks),
5) the 4th week of November, 1986–the 3rd week of February, 1987 (13 weeks),
6) the 2nd week of March, 1987–the 1st week of June, 1987 (13 weeks),
7) the 2nd week of June, 1987–the 3rd week of August, 1987 (11 weeks).

Parts of these sub-periods overlap in order to develop as many samples as possible. Different sub-periods have different combination of delivery months. The first and seventh sub-periods contain a fewer number of weeks, so data for December 9, 1985, February 24, 1986, June 10 and August 25, 1987 are included, providing for 93 samples.

Margin is required for futures trading.[4] This consists of the initial margin which is independent of the futures' prices and an additional margin which depends on price changes. Calculating the rate of return, it is difficult to take the very variable additional margin into account. Therefore, as a first approximation, it is assumed that the margin requirement is 100%[5] and that the margin is deposited at the concluding day of the contract (t) and is withdrawn at the delivery day (U). The rate of return of the futures contract R_t^f is calculated as follows.

$$R_t^f = (P_s^f - P_t^f)/P_t^f \times 365/(U-t) \times 100, \tag{6}$$

where P_s^f, P_t^f are the day S's and day t's prices of the futures asset whose last trading day is S. The number of investment days (U-t) is used to alter the rates of return to yearly rates. Sample statistics are shown in Table 1-1∼1-7.

IV. *Estimation Results*

As the first step of the estimation, factor analyses of the equation (1) were performed

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[4] As for margin, see Black (1976) and Telser (1981).

[5] Bodie and Rosansky (1980) made a similar assumption.

to gain the factor loadings B_{t1}, ..., B_{tk}. Because these factor loadings were assumed to be constant during each sub-period stated above,[6] the estimation was done at every sub-period. In order to determine that the number of factors is k, the chi-squared tests was used to investigate the null hypothesis.[7] However in the 2nd, 3rd, 4th and 7th sub-periods, the numbers of factors which insured that the degree of freedom was positive did not satisfy the five percent significance level[8] and to determine the number was not possible. Hence three other criteria were employed.[9]

(a) the number of factors that ensures that the eigenvalues of the correlation coefficient matrix are greater than one,

(b) the number of factors immediately before that which most effect the eigenvalue, substituting the squares of the multiple correlation coefficient for the communality,

(c) the number of factors that make the cumulative contribution greater than 0.8.

The calculation results of criterion (a) showed that the optimal numbers of the factors were three at the 1st, 2nd, 3rd and 4th sub-periods, two at the 5th and 7th sub-periods and one at the 6th sub-period. Criterion (b) gave the results of six at the 6th, five at the 2nd four at the 1st, two at the 4th and 7th and one at the 6th sub-period. Criterion (c) showed that they were two at the 1st, 2nd, 4th, 5th and 7th and one at the 6th sub-period. Because the chi-squared test of the equation (5) is collectively applied for all sub-periods, it is desirable to make the factors' numbers of all sub-periods equal. Considering this and the above results, the number of factors was set to two. Principal component methods were utilized to estimate the factor loadings. The factors were not rotated.

As the second step of the estimation, using the factor loadings gained at the first step as the explanatory variables, a cross sectional regression analysis of equation (3) was attempted by means of the Yule-Walker method to estimate F_{0t}, F_{1t} and F_{2t}.

The estimation results of the first step are shown in Tables 2-1~2-7 and results for the second step are shown as Table 3. The chi-squared test of equation (5) gave $\chi^2=0.6072$, which did not cause the rejection of the null hypothesis. That is, the estimates of F_{1t} and F_{2t} were equal to zero, meaning the risk premiums of the futures trading were zero. From these, there was neither normal backwardation nor contango, and the prices of the futures in the present were equal to the expected prices of the spot transaction in the future.

[6] The factor loadings are assumed to be different for different sub-periods, because the combinations of the delivery months are not equal.

[7] See Gultekin and Rogalski (1985).

[8] For the cases where the probability-values of the test are highest, the calculated chi-squared values and the critical values are as follows:

sub-period	number of factors	χ^2		$\chi^2_{0.05}$
1	5	25.657	$<$	26.3
2	6	16.950	$>$	16.92
3	6	27.324	$>$	16.92
4	7	9.103	$>$	7.81
5	4	32.446	$<$	36.4
6	7	4.224	$<$	7.81
7	7	9.312	$>$	7.81

[9] For example, see Okuno et al. (1971), Shiba (1979) and Ichikawa and Ohashi (1987).

V. *Concluding Remarks*

This paper has investigated futures contract pricing in Japan, and concludes that there is neither normal backwardation nor contango associated with this market. Using these results, it will be possible to obtain the expected values of spot prices and to test the hypotheses regarding expectation formation without survey data.

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TABLE 1-1. SAMPLE STATISTICS—RATES OF RETURN ON FUTURES CONTRACTS
1) 1st sub-period

Futures	Delivery Month	Mean	Standard Deviation
Bonds	Mar. '86	23.29	5.984
	Jun. '86	7.504	2.833
Gold	Feb. '86	− 44.20	53.26
	Apr. '86	− 44.99	15.44
	Jun. '86	− 34.40	9.190
	Aug. '86	− 22.90	6.842
	Oct. '86	− 5.993	6.386
Soybeans	Feb. '86	− 26.44	107.6
	Apr. '86	−120.8	24.53
	Jun. '86	− 78.36	9.601
	Aug. '86	− 52.34	4.762
	Oct. '86	− 1.276	5.752

Note: Rates of returns (yearly rate, %) are calculated using equation (6).

TABLE 1-2. SAMPLE STATISTICS—RATES OF RETURN ON FUTURES CONTRACTS
2) 2nd sub-period

Futures	Delivery Month	Mean	Standard Deviation
Bonds	Jun. '86	− 2.652	15.01
	Sep. '86	1.777	3.471
Gold	Jun. '86	− 20.71	14.70
	Aug. '86	− 6.028	10.16
	Oct. '86	16.41	10.72
	Dec. '86	6.852	7.042
	Feb. '87	− 0.0078	4.792
Soybeans	Jun. '86	−110.8	27.48
	Aug. '86	− 53.05	5.229
	Oct. '86	30.63	14.93
	Dec. '86	− 25.67	1.695
	Feb. '87	− 25.86	1.339

TABLE 1-3. SAMPLE STATISTICS—RATES OF RETURN ON FUTURES CONTRACTS
3) 3rd sub-period

Futures	Delivery Month	Mean	Standard Deviation
Bonds	Sep. '86	3.276	6.217
	Dec. '86	3.399	2.317
Gold	Aug. '86	24.93	40.63
	Oct. '86	50.01	16.72
	Dec. '86	24.21	8.818
	Feb. '87	9.744	5.518
	Apr. '87	13.26	4.667
Soybeans	Aug. '86	−26.37	127.6
	Oct. '86	120.5	59.19
	Dec. '86	3.579	24.48
	Feb. '87	− 8.612	15.40
	Apr. '87	8.609	15.30

TABLE 1-4. SAMPLE STATISTICS—RATES OF RETURN ON FUTURES CONTRACTS
4) 4th sub-period

Futures	Delivery Month	Mean	Standard Deviation
Bonds	Dec. '86	8.016	6.965
	Mar. '87	12.97	2.722
Gold	Dec. '86	−10.79	21.72
	Feb. '87	−18.65	6.842
	Apr. '87	− 4.591	5.621
	Jun. '87	− 5.212	4.032
	Aug. '87	− 3.426	3.288
Soybeans	Dec. '86	− 4.365	41.52
	Feb. '87	−12.81	12.11
	Apr. '87	19.80	6.194
	Jun. '87	6.862	4.348
	Aug. '87	3.569	3.401

TABLE 1-5. SAMPLE STATISTICS—RATES OF RETURN ON FUTURES CONTRACTS
5) 5th sub-period

Futures	Delivery Month	Mean	Standard Deviation
Bonds	Mar. '87	19.98	5.445
	Jun. '87	22.83	2.951
Gold	Feb. '87	− 8.599	20.98
	Apr. '87	9.591	9.811
	Jun. '87	3.377	5.021
	Aug. '87	3.565	3.665
	Oct. '87	5.111	3.145
Soybeans	Feb. '87	−86.67	89.74
	Apr. '87	29.57	10.20
	Jun. '87	8.694	4.825
	Aug. '87	5.932	3.858
	Oct. '87	−15.00	2.373

TABLE 1-6. SAMPLE STATISTICS—RATES OF RETURN ON FUTURES CONTRACTS
6) 6th sub-period

Futures	Delivery Month	Mean	Standard Deviation
Bonds	Jun. '87	6.902	16.26
	Sep. '87	−26.22	11.47
Gold	Jun. '87	1.272	15.32
	Aug. '87	4.224	6.673
	Oct. '87	6.740	4.435
	Dec. '87	−10.15	4.724
	Feb. '88	−17.40	4.075
Soybeans	Jun. '87	−35.14	63.36
	Aug. '87	− 9.423	19.05
	Oct. '87	−32.91	12.91
	Dec. '87	4.684	13.11
	Feb. '88	0.3581	8.024

TABLE 1-7. SAMPLE STATISTICS—RATES OF RETURN ON FUTURES CONTRACTS
7) 7th sub-period

Futures	Delivery Month	Mean	Standard Deviation
Bonds	Sep. '87	−40.81	12.61
	Dec. '87	−2.773	8.263
Gold	Aug. '87	−35.24	47.72
	Oct. '87	− 4.326	12.31
	Dec. '87	−25.49	8.566
	Feb. '88	−30.24	6.005
	Apr. '88	−24.66	4.350
Soybeans	Aug. '87	134.3	292.4
	Oct. '87	−40.18	15.53
	Dec. '87	8.061	18.05
	Feb. '88	0.2669	10.92
	Apr. '88	− 7.593	7.526

TABLE 2-1. ESTIMATES OF FACTOR LOADINGS
1) 1st sub-period

Futures	Delivery Month	B_1	B_2
Bonds	Mar. '86	−0.1424	−0.3675
	Jun. '86	−0.3432	0.6780
Gold	Feb. '86	0.9416	−0.0308
	Apr. '86	0.9448	0.2440
	Jun. '86	0.9770	0.1053
	Aug. '86	0.9821	−0.0778
	Oct. '86	0.9291	−0.3322
Soybeans	Feb. '86	0.6851	−0.4944
	Apr. '86	0.1517	0.9852
	Jun. '86	0.3977	0.9049
	Aug. '86	0.6901	0.6589
	Oct. '86	0.7263	−0.5015

Note: These tables show the results of factor analysis of equation (1). The number of factors is two.

TABLE 2-2. ESTIMATES OF FACTOR LOADINGS
2) 2nd sub-period

Futures	Delivery Month	B_1	B_2
Bonds	Jun. '86	0.4960	−0.5601
	Sep. '86	0.6211	−0.4192
Gold	Jun. '86	0.7319	0.5174
	Aug. '86	0.9441	0.2884
	Oct. '86	0.9816	0.1696
	Dec. '86	0.9715	0.2087
	Feb. '87	0.9612	0.2396
Soybeans	Jun. '86	−0.6151	0.7040
	Aug. '86	0.2549	0.8925
	Oct. '86	0.9175	0.1757
	Dec. '86	−0.4260	0.6913
	Feb. '87	−0.5656	0.6605

TABLE 2-3. ESTIMATES OF FACTOR LOADINGS
3) 3rd sub-period

Futures	Delivery Month	B_1	B_2
Bonds	Sep. '86	−0.6444	0.6640
	Dec. '86	−0.8010	0.5020
Gold	Aug. '86	0.3312	0.9120
	Oct. '86	0.8592	0.4833
	Dec. '86	0.6913	0.7162
	Feb. '87	0.5168	0.8479
	Apr. '87	0.5898	0.7938
Soybeans	Aug. '86	0.1554	−0.0064
	Oct. '86	0.9318	−0.2826
	Dec. '86	0.9097	−0.3668
	Feb. '87	0.9013	−0.3845
	Apr. '87	0.9193	−0.3744

TABLE 2-4. ESTIMATES OF FACTOR LOADINGS
4) 4th sub-period

Futures	Delivery Month	B_1	B_2
Bonds	Dec. '86	−0.4426	−0.6895
	Mar. '87	−0.0597	−0.5728
Gold	Dec. '86	0.8871	0.4054
	Feb. '87	0.8585	0.3762
	Apr. '87	0.9618	0.1843
	Jun. '87	0.9604	0.1829
	Aug. '87	0.9642	0.1502
Soybeans	Dec. '86	0.1943	0.8454
	Feb. '87	−0.6164	0.7552
	Apr. '87	−0.4827	0.7923
	Jun. '87	−0.6210	0.7445
	Aug. '87	−0.5431	0.6885

TABLE 2-5. ESTIMATES OF FACTOR LOADINGS
5) 5th sub-period

Futures	Delivery Month	B_1	B_2
Bonds	Mar. '87	0.9471	0.0227
	Jun. '87	0.9055	0.0495
Gold	Feb. '87	0.8437	−0.3705
	Apr. '87	0.9431	−0.3089
	Jun. '87	0.9325	−0.3120
	Aug. '87	0.9334	−0.2919
	Oct. '87	0.9418	−0.2746
Soybeans	Feb. '87	−0.9008	0.2955
	Apr. '87	0.8924	0.3719
	Jun. '87	0.8140	0.5426
	Aug. '87	0.7668	0.6230
	Oct. '87	0.4265	0.8784

TABLE 2-6. ESTIMATES OF FACTOR LOADINGS
6) 6th sub-period

Futures	Delivery Month	B_1	B_2
Bonds	Jun. '87	0.8440	−0.2564
	Sep. '87	0.9464	−0.1872
Gold	Jun. '87	0.9370	0.3261
	Aug. '87	0.9200	0.3856
	Oct. '87	0.8881	0.4460
	Dec. '87	0.9698	0.2290
	Feb. '88	0.9792	0.1585
Soybeans	Jun. '87	0.9118	−0.1946
	Aug. '87	0.9575	−0.1982
	Oct. '87	0.9773	−0.1949
	Dec. '87	0.9501	−0.2590
	Feb. '88	0.9597	−0.2463

TABLE 2-7. ESTIMATES OF FACTOR LOADINGS
7) 7th sub-period

Futures	Delivery Month	B_1	B_2
Bonds	Sep. '87	0.2469	−0.8574
	Dec. '87	0.9727	0.0136
Gold	Aug. '87	−0.7888	0.5541
	Oct. '87	−0.6369	0.7585
	Dec. '87	−0.9133	0.3804
	Feb. '88	−0.9485	0.2693
	Apr. '88	−0.9291	0.3296
Soybeans	Aug. '87	0.5402	0.8097
	Oct. '87	0.7360	0.5942
	Dec. '87	0.8782	0.4671
	Feb. '88	0.8954	0.4314
	Apr. '88	0.9043	0.3941

TABLE 3. ESTIMATION RESULTS FROM APT MODEL

Observation	F_1	F_2	Observation	F_1	F_2
1	-26.90	-44.43	48	-16.61	-12.49
2	-28.73	-44.35	49	-8.16	-25.65
3	-28.66	-43.25	50	-13.68	-27.64
4	-30.96	-44.45	51	-14.17	-21.30
5	-44.05	-47.10	52	-2.12	-20.16
6	-60.21	-49.05	53	1.83	-6.51
7	-60.66	-48.86	54	2.70	16.79
8	-51.10	-53.35	55	19.98	1.23
9	-35.20	-56.82	56	16.23	2.80
10	-24.53	-86.61	57	24.76	3.24
11	-40.91	-119.03	58	28.23	8.84
12	-34.75	-46.91	59	23.22	10.99
13	-47.02	-142.65	60	25.69	16.90
14	29.86	-28.82	61	32.31	15.26
15	30.80	-27.64	62	45.87	17.75
16	30.64	-27.64	63	50.88	11.72
17	35.96	-26.46	64	80.98	19.59
18	38.86	-26.03	65	93.62	20.46
19	38.30	-30.55	66	123.53	16.17
20	44.32	-26.76	67	194.44	36.77
21	37.57	-18.80	68	-232.21	-10.15
22	49.42	-17.73	69	-215.87	-0.46
23	64.51	-3.75	70	-227.90	-2.47
24	51.08	-15.64	71	-232.58	1.21
25	49.92	-21.41	72	-205.86	11.03
26	59.02	-47.21	73	-198.40	-2.79
27	74.86	-29.84	74	-177.59	-8.57
28	6.66	21.72	75	-134.09	18.16
29	12.74	27.71	76	-60.24	33.96
30	12.45	15.90	77	-41.52	56.44
31	17.05	28.28	78	-92.16	30.34
32	19.35	19.25	79	-20.55	43.52
33	23.87	2.90	80	-37.39	60.03
34	25.74	-3.02	81	-4.28	8.59
35	37.52	23.22	82	-5.59	5.93
36	36.45	23.51	83	-1.66	10.87
37	41.35	20.68	84	1.45	12.61
38	23.88	-22.43	85	7.39	10.53
39	-4.04	-84.85	86	13.58	14.39
40	77.52	20.99	87	14.18	18.66
41	-6.80	-0.71	88	14.95	5.41
42	-8.83	-5.27	89	43.86	41.27
43	-8.72	-0.39	90	52.79	56.19
44	-17.04	2.33	91	60.87	198.61
45	-19.76	-2.13	92	-3.75	12.01
46	-19.46	-0.56	93	76.22	234.57
47	-15.76	-7.38			

Note: This table shows the results of cross sectional regression of equation (3). The independent variables are the factor loadings obtained in Table 2. The estimation method is that developed by Yule and Walker. The first observation corresponds to December 9, 1985 and the last corresponds to August 25, 1987.