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WALD TESTS OF THE PURE EXPECTATIONS HYPOTHESIS OF THE TERM STRUCTURE OF INTEREST RATES*

HIROSHI KAMAE

I. Introduction

A number of empirical tests of the pure expectations hypothesis (PEH) have been attempted. Some researchers have regressed an equation representing the PEH, and time series analysis has often been applied. Sargent (1979), Campbell and Shiller (1987) and MacDonald and Speight (1988) conducted tests by means of bivariate vector autoregressive (BVAR) models. The term structure theory analyzes effects of the term to maturity on the yield, hence other factors such as the coupon must be constant. However, in Japan, coupon bearing government bonds have different rates, and it is difficult to collect samples of the same coupon and term to maturity. In this paper, the yields of bonds having the same specified coupon rate are determined by estimation.

The BVAR models can be applied to stationary data, hence the first differences of the interest rate data are often utilized as variables.¹ By using these variables, however, we lose information on the level of the interest rates, and if the long-term and short-term interest rates are cointegrated, there exists no finite order BVAR model in regard to their first differences.² Hence Campbell and Shiller (1987) and MacDonald and Speight (1988) used BVAR models that treated the short-term interest rate and the spread of the long-term and short-term rates as variables.³ In this paper, these variables are also employed. As will be shown later, the spreads and the first differences of the short-term rates are stationary.

The rational expectations hypothesis (REH) is assumed and a composite hypothesis of the PEH with the REH is tested in this paper. Hence if the composite hypothesis is rejected, at least one of the two hypotheses is not valid.

In section two, the PEH is specified by means of the BVAR model, composed of the

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¹ See, Sargent (1979), Baillie and McMahon (1985).

² See, MacDonald and Speight (1988), p. 290.

^a If two variables are assumed to be a jointly covariance-stationary process, it can be represented by a bivariate moving average process, and if the invertible condition is satisfied, it has an autoregressive representation. See, Baillie et al. (1983) p. 554, Judge et al. (1985) p. 658.

spreads and the first differences of short-term interest rates. In section 4-3, this hypothesis is examined using the Wald test and data which is explained in the third section. All the variables are shown to be stationary in section 4-1, and the numbers of order of the auto-regressive process are statistically determined in section 4-2.

II. Specification of the PEH

The PEH is expressed as follows in terms of the coupon bearing bonds,

(1)
$$R_t = [(1-\alpha)/(1-\alpha^m)] \sum_{k=0}^{m-1} \alpha^k E_t(r_{t+k})$$

where R_t denotes a long-term interest rate, that is, a yield to maturity of the coupon bearing bonds, r_t denotes a short-term interest rate, E denotes an operator expressing expectation, \overline{R} is the average of R_t and α is a 'discount factor'⁴ being equal to $1/(1+\overline{R})$. The interest rates are expressed per period. The REH is assumed, meaning that the expectations are formed using all available information. If $\overline{R} > 0$ then $\alpha < 1$, and when $m \to \infty$, then $\alpha^m \to 0$, hence the equation (1) becomes

(2)
$$R_t = (1-\alpha) \sum_{k=0}^{\infty} \alpha^k E_t(r_{t+k}).$$

The spread S_t is a deviation of R_t from r_t and⁵

(3)
$$S_t = \sum_{k=1}^{\infty} \alpha^k E_t(\triangle r_{t+k}).$$

An *n*-th order BVAR model using the $\triangle r_t$ and S_t is expressed as follows,

(4)
$$\begin{bmatrix} \triangle r_t \\ S_t \end{bmatrix} = \begin{bmatrix} a(L) & b(L) \\ c(L) & d(L) \end{bmatrix} \begin{bmatrix} \triangle r_{t-1} \\ S_{t-1} \end{bmatrix} + \begin{bmatrix} u_{1t} \\ u_{2t} \end{bmatrix},$$

where a(L), b(L), c(L) and d(L) denote the *n*-th order polynomial of a lag operator L.

$$a(L) = \sum_{k=1}^{n} a_{k}L^{k},$$

$$b(L) = \sum_{k=1}^{n} b_{k}L^{k},$$

$$c(L) = \sum_{k=1}^{n} c_{k}L^{k},$$

$$d(L) = \sum_{k=1}^{n} d_{k}L^{k}.$$

The disturbance term

⁴ See, Shiller et al. (1983) p. 178. However, as shown in Appendix, the discount factor must not be defined in terms of yields to matiruty, but in terms of spot rates.

⁵ See, the equation (3) in Campbell and Shiller (1987).

$$\begin{bmatrix} u_{1t} \\ u_{2t} \end{bmatrix} = v_t$$

is assumed to satisfy

(5)
$$\begin{cases} E(v_t) = 0, \\ E(v_t v_{t-i}) = \begin{cases} \Omega & (i=0) \\ 0 & (i \neq 0). \end{cases}$$

If equation (4) is transformed into the equation

$$(6) \begin{bmatrix} \Delta r_{t} \\ \Delta r_{t-1} \\ \vdots \\ \Delta r_{t-n+1} \\ S_{t} \\ \vdots \\ S_{t-n+1} \end{bmatrix} = \begin{bmatrix} a_{1} \dots & a_{n} & b_{1} \dots & b_{n} \\ 1 \dots & 0 & 0 \dots & 0 \\ \vdots \\ 0 \dots & 1 & 0 & 0 \dots & 0 \\ c_{1} \dots & c_{n} & d_{1} \dots & d_{n} \\ 0 \dots & 0 & 1 \dots & 0 \\ \vdots \\ 0 \dots & 0 & 0 \dots & 1 & 0 \end{bmatrix} \begin{bmatrix} \Delta r_{t-1} \\ \Delta r_{t-2} \\ \vdots \\ \Delta r_{t-n} \\ S_{t-1} \\ \vdots \\ S_{t-n} \end{bmatrix} + \begin{bmatrix} u_{1t} \\ 0 \\ \vdots \\ 0 \\ u_{2t} \\ 0 \\ \vdots \\ 0 \end{bmatrix},$$

then this can be written as follows.

(7)
$$X_t = AX_{t-1} + \varepsilon_t$$
.

Equation (3) is transformed into the equation

(8)
$$e'X_t = \sum_{k=1}^{\infty} \alpha^k E_t(f'X_{t+k}),$$

where e' and f' denote vectors of $1 \times 2n$, and the (n+1)th element of e' and the first element of f' are one and the others are zero.

From equation (7),

(9)
$$X_{t-1+k} = A^k X_{t-1} + A^{k-1} \varepsilon_t + A^{k-2} \varepsilon_{t+1} + \cdots + \varepsilon_{t-1+k}$$

is obtained. If the information set useful for forecasting at period (t-1) is composed of past values of S and Δr , then the disturbance terms in the future such as ε_t , ε_{t+1} are unknown, so the optimal k period forecast formed at period (t-1) is

(10) $E_{t-1}(X_{t-1+k}) = A^k X_{t-1},$

which is acquired by substituting zero for the disturbance terms.⁶ This optimal forecast utilizes all the available information and is the same as the expected value under REH.7 It follows from equations (8) and (10) that

⁶ See, Granger and Newbold (1986) p. 229 and Campbell and Shiller (1987) p. 1067.

⁷ See, Sargent (1972) p. 74 and Shiller (1973) p. 856. Modigliani and Shiller (1973, p. 29) and Pesando (1975, p. 851) have shown that the optimal forecast satisfies efficiency condition of the REH.

(11)
$$e'X_{t-1} = \sum_{k=1}^{\infty} \alpha^k E_{t-1}(f'X_{t-1+k})$$

= $\sum_{k=1}^{\infty} f' \alpha^k A^k X_{t-1},$

hence

(12)
$$e' = \sum_{k=1}^{\infty} f' \alpha^k A^k = f' \alpha A (I - \alpha A)^{-1}.$$

Transforming this equation, we get

(13)
$$e'(I-\alpha A)=f'\alpha A$$
.

Finally

(14)
$$e'I = (e'+f')\alpha A$$

is obtained.

III. Data

The discount factors and prices of coupon bearing bonds whose coupons are specified are estimated using monthly data for actual coupon bearing bonds, and the yield to maturity obtained from them are utilized for measurement. The estimation method developed by McCulloch (1975), which assumed continuous coupon payments, are modified by assuming discrete payments.⁸

Samples of coupon bearing bonds are all long-term government bonds which are listed on the Tokyo Stock Exchange, and the price data are collected from small lot bonds traded at the end of every month. If there are more than one issue of the same coupon and term to maturity, then the bonds with higher values are employed. The secondary spot trading market may have undergone structural changes after the futures market began to trade on October, 1985, hence the data is collected from the period, October, 1985 to March, The number of samples is 42. The coupon rates of the bonds used for estimation 1989. are set equal to 4, 6, and 8% and the terms to maturity are set equal to 4, 6, 8 and 10 years. Those bonds have been chosen for purpose of estimation, because the coupon rates of the issues which have been listed during the data collecting periods and whose terms to maturity were greater than four years are from 3.9 to 8.5%. Yield to maturity (yearly rate) of an issue whose coupon is z% and term to maturity is y years is written as Rzy. The estimation is performed quarterly. The short-term interest rate (r) is the bond trading yield with a three-month repurchase agreement, expressed by yearly rate per period. The spread Szy is Rzy minus r. \bar{R} which is used to calculate the discount factor α is substituted by the average of estimated Rzy's.

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⁸ See Appendix and Thies (1985).

IV. Test Methodology and Results

4-1. Test of unit root

By using Fuller's (1976) unit root tests, variables are examined to determine whether they are stationary or not. The null hypothesis is that a unit root exists. y_t denotes a variable at t-th period. If values of ρ , which are estimated from the following three equations,

- (15a) $y_t = \rho y_{t-1} + u_t$
- (15b) $y_t = \mu + \rho y_{t-1} + u_t$
- (15c) $y_t = \mu + \beta t + \rho y_{t-1} + u_t$,

are equal to one, then variable y has a unit root. These equations are estimated by the OLS, and $\tau = (\hat{\rho} - 1)/s$ is calculated, where $\hat{\rho}$ is ρ 's estimated value and s is its standard error. If τ is greater than the critical value published in Fuller (1976), the null hypothesis is not rejected. The results of these tests are given at Table 1. Equation (15a) shows that all

Variable	τα	tb	το	Variable	ta
r	-4.21	-12.17	-7.78	S810	-2.00*
R810	-1.41**	-6.20	-5.97	S808	-3.02
R808	-1.76**	- 5.74	-5.65	S806	-4.14
R806	-2.02*	- 5.68	-5.49	S804	-5.21
R804	-2.25*	-6.11	-4.98	S610	-1.96*
R610	-1.37**	6. 17	- 5.97	S608	-3.06
R608	—1.74**	-5.73	- 5.75	S606	-4.11
R606	-2.00*	-5.64	-5.64	S604	-5.12
R604	-2.25*	-6.13	-5.22	S410	
R410	1.32**	-6.12	5. 92	S408	-2.93
R408	-1.72**	- 5. 79	- 5.91	S406	-4.04
R406	-1.99*	-5.61	5. 79	S404	-5.04
R404	-2.22*	-6.12	-5.44	Note: Ta. Th	and τ_c are ca
Δr	-6.52	-6.12	6. 69	using ρ 's which	are estimated
△R 810	-7.78	-7.70	-7.58	(15b) and (15c) i	respectively an
△R 808	-7.97	-7.30	8. 14	tion s's. If τ is	s greater than
△R 806	-7.59	-7.55	-7.87	lished in Fuller	r (1976), the
△R804	-7.07	-7.01	-7.65	critical values a	at 5% and 1
△R610	-7.73	- 7.64	- 7. 54	-1.95, -2.62	for τ_a , -2 .
△R608	-8.03	- 7. 98		-3.30, -4.13 1	of te respectiv
△R606	-7.60	-7.61	-7.84		
△R604	7.14	-7.05	-7.62		
△R410	-7.66	-7.62	7. 50		
△R408	-8.08	-8.08	-8.17		
△R406	- 7.67	-7.68	-7.85		
△R404	-7.21	-7.14	-7.60		

TABLE 1. RESULTS OF UNIT ROOT TESTS

Variable	ta	τδ	τc
S810	-2.00*	-5.81	-4.93
S808	-3.02	6. 86	-6.45
S806	-4.14	-6.46	-6.03
S804	-5.21	-6.47	-6.29
S610	-1.96*	-5.75	-4.74
S608	-3.06	-6.81	-6.33
S606	-4.11	-6.30	-5.93
S604	-5.12	-6.26	-6.01
S410		-5.63	-4.52
S408	-2.93	6. 69	-6.24
S406	-4.04	-6.17	-5.78
\$404	-5.04	-6.02	5, 72

alculated as $\tau = (\rho - 1)/s$ l from equations (15a), nd their standard deviathe critical value pubunit root exists. The % significance levels are .93, -3.58 for τ_b and vely.

R's have the unit roots at 5 or 1% significance level and that the r, $\triangle r$, all of $\triangle R$'s and all of *S*'s except one *S* whose term to maturity is ten years. Equations (15b) and (15c) show that all the variables do not have the unit roots at the 5% level. Hence it seems possible that $\triangle r$ and almost all of the *S*'s are stationary.

4-2. Identification of the models

Akaike's Information Criterion (AIC) and likelihood ratio tests are employed in order to determine the numbers of orders of the autoregressive process of the BVAR models expressed by the equation (4). In this paper, monthly data are used and estimation is made quarterly, hence the numbers of order (m) of the BVAR models which must be compared are 3, 6, 9, 12 and 15.

The tests using the AIC are attempted as follows. AIC(m) represents the AIC which is obtained from *m*-th order BVAR model. Hence,

$$\operatorname{AIC}(m) = T \cdot \ln |\sum_{m}| + 2mr^2$$

where T denotes the numbers of the samples, \sum_m denotes the estimate of the variance-covariance matrix composed of the *m*-th order BVAR model's disturbance vectors, and r denotes the number of the variables of the model. In our cases r=2, hence

(16) AIC(m)= $T \cdot \ln |\sum_m| + 8m$.

As shown in Table 2a, the third order resulted in an AIC with the lowest value in all cases.

The likelihood ratio tests are applied as follows.⁹ If m > k, the null hypothesis is that the autoregressive process is of the *m*-th order, and the alternative one is that the autoregressive process is of the *k*-th order. The likelihood ratio,

S∖ m	3	6	9	12	15
S810	-1130.2	-1112.9	-1097.4	-1084.7	*
S808	-1139.7	-1124.7	-1118.0	-1103.6	*
S806	-1140.0	-1126.0	-1118.7	-1106.2	*
S804	-1147.2	-1135.8	-1127.5	-1115.7	*
S610	-1127.5	-1110.2	-1094.2	-1081.9	-1072.6
S608	-1138.5	-1123.2		-1111.6	*
S606	-1137.9	-1123.6	-1115.7	-1103.5	*
S604	-1143.4	-1131.4	-1122.2	-1110.8	*
S410	-1124.3	1107. 0	-1090.6	-1078.5	1068. 7
S408	-1137.0	-1121.4	-1114.0	1099. 0	*
S406	-1135.4		-1112.2	-1100.5	*
S404	-1138.6	-1125.9	-1116.2	-1106.0	*

TABLE 2a. MODEL IDENTIFICATION RESULTS BY THE AIC

Note: This table shows the values of the AIC obtained from equation (16) when the number of order is m. * indicates that the variance-covariance matrix composed of the BVAR model's disturbance vectors is singular, hence calculation of the AIC is not possible.

⁹ See, Judge et al. (1985) p. 686-.

S	m	LR(m-3)	LR(m)
S810	6	109.3	6.7
S808	6	115.9	9.0
S806	6	117.9	10.0
S804	6	109.1	12.6
S610	6	109. 2	4.7
S608	6	115.7	7.7
S606	6	122.9	9.7
S604	6	108.3	12.0
S410	6	109.3	6.7
S408	6	115.7	8.4
S406	6	118.0	9.4
S404	6	107.7	11.3

TABLE 2b. MODEL IDENTIFICATION RESULTS BY THE LIKELIHOOD RATIO TESTS

Note: LR(m-3) and LR(m) are likelihood ratios obtained from equation (18). They allow us to make likelihood ratio tests in order to determine the numbers of orders of autoregressive processes. If LR(m-3) and LR(m) are greater than the critical values of the chi-squared distribution whose degree of freedom is 12, the null hypotheses which state that the numbers of orders of autoregressive processes are m-3 and m respectively are rejected. The critical values are 21.0 for a 5% significance level and 26.2 for a 1% level.

$$LR = T(\ln |\sum_{k}| - \ln |\sum_{m}|),$$

has chi-squared distribution whose degree of freedom is 4(m-k). If this ratio is greater than the critical value, the null hypothesis is rejected. From equation (16),

(17) LR = AIC(k) - AIC(m) + 8(m-k)

is obtained.

The model is estimated quarterly, hence in equation (17), k is set equal to m-3, and the LR obtained is written as LR(m). That is,

(18)
$$LR(m) = AIC(m-3) - AIC(m) + 24.$$

Estimation results are shown on Table 2b. The fewest numbers of the order of the autoregressive process which does not cause rejection of the null hypothesis are six in all cases.

The number of order n is equal to m/3. Hence the Wald tests will be made for n=1 and 2.

4-3. Tests of the PEH

The PEH derived equation

(14) $e'I = (e' + f')\alpha A$,

which is tested by means of the Wald tests. θ and r are defined as

 $\theta' \equiv (a_1 \dots a_n \ b_1 \dots \ b_n \ c_1 \dots \ c_n \ d_1 \dots \ d_n)$

$$r(\theta)' \equiv e'I - (e' + f')\alpha A.$$

Equation (14) becomes

 $r(\theta)'=0.$

By estimating the BVAR model,

(4)
$$\begin{bmatrix} \bigtriangleup r_t \\ S_t \end{bmatrix} = \begin{bmatrix} a(L) & b(L) \\ c(L) & d(L) \end{bmatrix} \begin{bmatrix} \bigtriangleup r_{t-1} \\ S_{t-1} \end{bmatrix} + \begin{bmatrix} u_{1t} \\ u_{2t} \end{bmatrix},$$

 θ 's estimated value $\hat{\theta}$ is obtained. If the Wald test statistic,

(19) $WD = r(\hat{\theta})'[D' \{\Omega \otimes (Y'Y)^{-1}\} D]^{-1}r(\hat{\theta}),$

is greater than the critical value of the chi-squared distribution, the null hypothesis is rejected, where \otimes denotes the Kronecker product, T denotes the number of the sample, Y denotes a matrix of $(T \times 2n)$,

$$D = \partial r(\theta) / \partial \theta,$$

$$\Omega = \begin{bmatrix} E(u_{1t}^2) & E(u_{1t}u_{2t}) \\ E(u_{1t}u_{2t}) & E(u_{2t}^2) \end{bmatrix}$$

S	n	WDy	WD _o
S810	1	6.13	2. 56**
	2	14.10	8.55*
S808	1	10.78	7.32
	2	22.74	10.15
S806	1	13,08	11.35
	2	17.53	10.67
S804	1	8.56	10, 22
	2	18.11	11.43
S610	1	6.06	2.47**
	2	13.10	8.26*
S608	1	10. 57	7.17
	2	22.24	10.12
S606	1	12.81	11.27
	2	16. 59	10.40
S604	1	8.28	9.90
	2	16. 78	10.69
S410	1	5.99	2.38**
	2	11.98	7.92*
S408	1	10.34	7.03
	2	21.64	10. 10
S406	1	12, 54	11, 22
	2	15.60	10.17
S404	1	8.09	9.66
	2	15.62	10.26

TABLE 3. TESTS RESULTS OF THE PEH

Note: *n* is number of order of the autoregressive process. WD_y and WD_0 show Wald test statistics calculated from equations (4) and (19). WD_y is obtained by the Yule and Walker method and WD_0 by the OLS. When these statistics are greater than the critical values of chi-squared distribution whose degree of freedom is 2*n*, the PEH is rejected. The critical values are 5.99 for a 5% significance level and 4.61 for a 1% level when n=1, and 9.49 for 5% and 7.78 for 1% when n=2. * and ** attached to the WD's indicate that the PEH is not rejected at a 5% and a 5% and 1% significance level respectively.

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and

$$Y = \begin{bmatrix} \Delta r_{t-1} & \Delta r_{t-2} & \dots & \Delta r_{t-n} & S_{t-1} & S_{t-2} & \dots & S_{t-n} \\ \Delta r_{t-1+1/3} & \dots & S_{t-1+1/3} & \dots & \\ \vdots & & & & \\ \Delta r_{t-1+(T-1)/3} & \dots & S_{t-1+(T-1)/3} & \dots & \end{bmatrix}$$

Table 3 shows the test results. The PEH is rejected at the 5% significance level when equation (4) is estimated by the Yule and Walker method. For cases where the terms to maturity are less than 10 years, the PEH is also rejected by means of the OLS. When the term to maturity is 10 years, the hypothesis is not rejected by the OLS, but from Table 1 its spread is not stationary, hence this result is not to be accepted at its face value.

Finally, in this paper, the REH is assumed and the composite hypothesis is tested. However, there is no evidence that the REH is valid, hence we cannot reject the PEH completely.

V. Concluding Remarks

In this paper, the PEH is tested by means of time series analysis, specifically the Wald tests, using the yields of long-term bonds with similar coupons determined by estimation. The results show that the PEH is rejected for almost all cases. This suggests there are risk premiums. An analysis of these risk premiums will be attempted in a future paper.

HITOTSUBASHI UNIVERSITY

APPENDIX: ESTIMATION OF YIELD TO MATURITY OF SPECIFIED COUPON BEARING BONDS

Interest is paid biannually for coupon bearing government bonds in Japan. C denotes the coupon of a bond per year, M denotes the term to maturity at the present and N denotes the integer part of 2M. Interest will be paid after (M-N/2), (M-N/2+1/2), ..., (M-1/2) and M years. If accrued interest is taken into account, the first interest is (2M-N)(C/2). The price of this bond is equal to

(A1)
$$P = (2M - N)(C/2) \cdot \delta(2M - N) + (C/2) \cdot \delta(2M - N + 1) + \dots + (C/2) \cdot \delta(2M - 1) + (C/2 + 100) \cdot \delta(2M)$$
$$= (2M - N)(C/2) \cdot \delta(2M - N) + (C/2) \sum_{r=1}^{N} \delta(2M - N + r) + 100 \cdot \delta(2M),$$

where δ (s), which is a discount factor for s periods, is indicated as $1/(1 + R_{st}^*)^s$ and R_{st}^* is a spot rate for s periods at t-th period. t is deleted in cross-sectional regressions. It is assumed that the δ (s) is approximated to a third-order spline function of s,

(A2)
$$\delta(s) = d_0 + d_1 \cdot s + d_2 \cdot s^2 + d_3 \cdot s^3 + d_4 \cdot z_1^3 + d_5 \cdot z_2^3 + d_6 \cdot z_3^3,$$

where

$$z_i = \begin{cases} s - k_i & \text{(if } s - k_i > 0) \\ 0 & \text{(if } s - k_i \le 0), \end{cases}$$

 k_i 's are knot points, and it is assumed *a priori* $k_1=2$, $k_2=4$ and $k_3=8$ (periods). From equations (A1) and (A2), next equation is obtained.

$$\begin{array}{ll} (A3) \quad P = d_0[(C/2)(2M-N) + C \cdot N/2 + 100] + d_1 \{(C/2)(2M-N)^2 + (C \cdot N/2)[(2M-N) \\ &+ (N+1)/2)] + 200M\} + d_2 \{(C/2)(2M-N)^3 + (C \cdot N/2)[(2M-N)^2 \\ &+ (N+1)(2M-N) + (N+1)(2N+1)/6] + 400M^2\} + d_3 \{(C/2)(2M-N)^4 \\ &+ (C \cdot N/2)[(2M-N)^3 + 3(N+1)(2M-N)^2/2 + (N+1)(2N+1)(2M-N)/2 \\ &+ N(N+1)^2/4] + 800M^3\} + d_4 \{(C/2)(2M-N)(2Z_1-N)^3 + (C \cdot N/2)[(2Z_1-N)^3 \\ &+ 3(N+1)(2Z_1-N)^2/2 + (N+1)(2N+1)(2M-N)/2 + N(N+1)^2/4] + 800M^3\} \\ &+ d_5 \{(C/2)(2M-N)(2Z_2-N)^3 + (C \cdot N/2)[(2Z_2-N)^3 + 3(N+1)(2Z_2-N)^2/2 \\ &+ (N+1)(2N+1)(2M-N)/2 + N(N+1)^2/4] + 800M^3\} \\ &+ d_6 \{(C/2)(2M-N)(2Z_3-N)^3 + (C \cdot N/2)[(2Z_3-N)^3 + 3(N+1)(2Z_3-N)^2/2 \\ &+ (N+1)(2N+1)(2M-N)/2 + N(N+1)^2/4] + 800M^3\} , \end{array}$$

where

$$Z_{i} = \begin{cases} M - k_{i}/2 & \text{(if } M - k_{i}/2 > 0) \\ 0 & \text{(if } M - k_{i}/2 \le 0), \end{cases}$$

and unit of $k_i/2$ is a year. By regressing monthly data cross-sectionally, the monthly estimates of coefficients such as d_0, \ldots, d_6 are determined.

Estimates of monthly prices P of the coupon bearing bonds of specified coupons and terms to maturity are gained by substituting the estimates of coefficients and the C and M. With these, by solving equation,

(A4)
$$P = \sum_{s=2M-N}^{2M} \frac{(C/2)}{(1+R)^s} + \frac{100}{(1+R)^{2M}},$$

yield to maturity R of the coupon bearing bonds is obtained.

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