Increasing Risk and Robinsonian Investment Function (1)

by Toichiro Asada

I. Introduction

In her *Essays in the Theory of Economic Growth* [9], Mrs. Robinson writes: "It seems reasonably plausible to say that, given the general characteristics of an economy, to sustain a higher rate of accumulation requires a higher level of profits, both because it offers more favourable odds in the gamble and because it makes finance more readily available. For purposes of our model, threfore, the 'animal spirits' of the firms can be expressed in terms of a function relatively the desired rate of growth of the stock of productive capital to the expected level of profits." (pp. 37~8.) "The central mechanism of our model is the desire of firms to accumulate, and we have assumed that it is influenced by the rate of profit. The rate of investment that they are planning for the future is, therefore, higher the greater the rate of profit on investment (estimated on the bases of current prices). Valueing the existing stock of capital on the bases of the same rate of profit, we can then express their plans in teɹms of a rate of accumulation." (p. 47.)

These statements may simply be formulated as follows.

(I－1)　$g = g(r^e)$　;　$g'(r^e) > 0$

; where g is the rate of capital accumulation planned by the entrepreneur and r^e is the expected rate of profit on capital stock.

This type of investment function, which is called 'animal spirit function', plays a central role in several Post-Keynesian models of economic growth. (See, for example, Asimakopulos [3] and Aoki & Marglin [2].) However, 'ad-hocness' of this approach has often been criticized by some writers. (See, for example, Aoki [1] chap. 1.)

The purpose of this note is to present a 'choice-theoretic' foundation to this type of investment function.

It is worth to note that Uzawa [11] already derived this type of investment function introducing the assumption of the increasing cost of the investment (the Penrose effect). Our approach is formally similar to Uzawa's but its economic implication is considerably different.

The key concept in our approach is the assumption of the 'increasing risk' rather than the 'increasing cost' of the investment, and the attitude of the entrepreneur towards risk is intimately related to his investment policy. In this respect, the stance of this note relative to the Robinsonian investment function may somewhat resemble to that of Tobin's classical paper [10] relative to the Keynesian demand function for money.

II. Preliminary of the model

Let us imagine the representative entrepreneur who has the following Neumann-Morgenstern type of utility function.

(II-1) $U_t(\pi_t, K_t) = \alpha V(\pi_t, K) + \beta$

; where α is the arbitrary positive real number and β is the arbitrary real number which is not necessarily positive. π_t is the real profit and K_t is the real capital stock measured by a suitable numeraire (for example, by 'wage unit' in the sense of Keynes [6] chap. 4). The subscript t denotes 'time'.

We further assume that $U_t(\pi_t, K_t) > U_t(0, 0)$ if $[\pi_t, K_t] > [0, 0]$, and that $U_t(\pi_t, K_t) - U_t(0, 0) \equiv \alpha\{V(\pi_t, K_t) - V(0, 0)\} \equiv \alpha u(\pi_t, K_t)$ is linear homogeneous ; i. e.

$$\text{(II—2)} \quad [\pi_t, K_t] > [0, 0] \longrightarrow u(\pi_t, K_t) > 0, \quad \lambda u(\pi_t, K_t) = u(\lambda\pi_t, \lambda K_t) \text{ for all } \lambda \geqq 0.$$

Linear homogeneity of $u(\pi_t, K_t)$ inplies that $U_t(\pi_t, K_t)$ is homothetic ; i. e.

$$\text{(II—3)} \quad U_t(\pi_t^1, K_t^1) \geqq U_t(\pi_t^2, K_t^2) \longrightarrow U_t(\lambda\pi_t^1, \lambda K_t^1) \geqq U_t(\lambda\pi_t^2, \lambda K_t^2) \text{ for all } \lambda \geqq 0.$$

From the above assumptions, $U_t(\pi_t, K_t)$ can be rewritten as follows if $K_j \neq 0$.

$$\text{(II—4)} \quad U_t(\pi_t, K_t) = \alpha v(r_t) K_t + U_t(0, 0) \equiv W(r_t, K_t)$$

; where $r_t \equiv \pi_t / K_t$ is the 'rate of profit' at the period t.

We assume that $v(r_t)$ is at least three times continuously differentiable and that $v'(r_t) \equiv \partial u / \partial \pi_t > 0$.

Next, suppose that the entrepreneur has a sort of subjective estimation on the profitability of the investment. In the present context, we assume that he supposes that r_t is the normally distributed random variable which has the following properties.

$$\text{(II—5)} \quad r_t \sim N(r^e, \sigma_r^2(g_t)) \ \forall t \geqq 0.$$

$$\text{(II—6)} \quad \sigma_r^{2\prime}(g_t) > 0, \ \sigma_r^{2\prime\prime}(g_t) > 0, \ \sigma_r^{2\prime\prime\prime}(g_t) = 0$$

; where r^e is the positive constant.

(II—6) implies that 'marginal risk' $\sigma_r^{2\prime}(g_t)$ as well as 'risk' $\sigma_r^2(g_t)$ increases as g_t increases. Roughly speaking, this assumption says that the entrepreneur supposes subjectively that the more rapid the speed of the alteration of the environment, the less certain is the possibility of the return of the investment. (2)

Now, the investment policy of the entrepreneur is given as the path of the rate of accumulation g_t which maximizes the discounted present value of his 'expected utility'. The maximization problem of the entrepreneur is formulated as follows.

$$\text{(II—7)} \quad \underset{-\delta \leqq g_t \leqq \bar{g}}{\text{Max}} \int_0^{\infty} E_t U_t(\pi_t, K_t) \cdot \exp(-\rho t)\, dt$$

$$\text{subject to } \dot{K}_t = g_t K_t, \ K_o \equiv \text{given.} > 0$$

; which is equivalent to

$$\text{(II—7)}' \quad \underset{-\delta \leqq g_t \leqq \bar{g}}{\text{Max}} \int_0^{\infty} E_t\, v(r_t) K_t \cdot \exp(-\rho t)\, dt$$

$$\text{subject to } \dot{K}_t = g_t K_t, \ K_o \equiv \text{given.} > 0$$

; where E_t is the expactation operator, and $\bar{g} > 0$ is the maximum rate of capital accumulation which is given exogenously from the outside of the system, and δ is the (constant) rate of capital depreciation, and ρ is the subjective discount rate of the entrepreneur.

For simplicity we confine the objects of the analysis to the situations where Pontryagin's maximum principle is applicable. Hence, we set the additional constraint that g_t must be continuous with respect to time.

Furthermore, on account of the technical demand, we assume that ρ is the constant which satisfies the following inequality.

$$(II-8) \quad \rho > \bar{g}.$$

Expanding $v(r_t)$ in a Taylor series around $v(r^e)$ and neglecting the terms of higher order than third, we have ;

$$(II-9) \quad v(r_t) = v(r^e) + v'(r^e)(r_t - r^e) + (1/2)v''(r^e)(r_t - r^e)^2 + (1/6)v'''(r^e)(r_t - r^e)^3.$$

From (II—5), (II—6) and (II—9), we have ;

$$(II-10) \quad E_t v(r_t) = v(r^e) + (1/2)v''(r^e)\sigma_r^2(g_t) \equiv f(g_t ; r^e).$$

(Note that since r_t is normally distributed, $E_t(r_t - r^e)^3 = 0$.)

If the distribution of r_t is concentrated around r^e, this approximation can be rationalized. Through this note, we adopt this approximation.

Before proceed to the next section, let us refer briefly to the measures of the risk aversion.

The 'certainty equivalence' (r_a) of the uncertain rate of profit r_t is defined as follows.

$$(II-11) \quad v(r_a) = E_t v(r_t).$$

Expanding the left hand side of (II—11) in a Taylor series around $v(r^e)$ and approximating by the first term, we obtain ;

$$(II-12) \quad v(r_a) = v(r^e) + v'(r^e)(r_a - r^e).$$

From (II—4), (II—10), (II—11) and (II—12), we have the following approximate expression if $K_t \neq 0$.

$$(II-13)\quad A(r^e) \equiv -\frac{\partial^2}{\partial r^2} W(r_t, K_t) \Big/ \frac{\partial}{\partial r} W(r_t, K_t) \Bigg|_{r_t = r^e}$$

$$\equiv -v''(r^e) / v'(r^e) = (r^e - r_a) / \{\sigma_r^2(g_t)/2\}.$$

From (II—4), (II—10) and (II—11), we also have the following equality if $K \neq 0$.

$$(II-14)\quad B(r^e) \equiv -\frac{\partial^2}{\partial r^2} W(r_t, K_t) \Big/ \Big\{ W(r_t, K_t) - U\,0,\ 0)) \Big\} \Bigg|_{r_t = r^e}$$

$$\equiv -v''(r^e) / v(r^e) = \{1 - v(r_a) / v(r^e)\} / \{\sigma_r^2(g_t)/2\}.$$

$A(r^e)$ is nothing but the Arrow-Pratt measure of the absolute risk aversion of the entrepreneur. (See Pratt [8].) We can consider $B(r^e)$ to be the alternative measure of the risk aversion. (Note that both of $A(r^e)$ and $B(r^e)$ are invariant with respect to the linear transformation of $U_t(\pi_t, K_t)$.) For simplicity we say $A(r^e)$ the 'A-measure' and $B(r^e)$ the 'B-measure'.

III. Working of the model

we shall now examine the working of the model. First let us prove the following lemma.

<**Lemma** 1.>

If $-\delta \leqq g \leqq \bar{g}\ \forall t \geqq 0$ and g_t is continuous with respect to t,

$$\int_0^\infty f(g_t; r^e) K_t \cdot \exp(-\rho t)\, dt < \infty.$$

(Proof.) For convenience let us write $\max_{-\delta \leqq g_t \leqq \bar{g}} f(g\ ;r) \equiv f_{max}$.

Then, from (II—7)′, (II—8) and (II—10) we have ;

$$\int_0^\infty f(g_t\ ;r^e)\,K_t\cdot \exp(-\rho t)\,dt$$

$$= K_o \int_0^\infty f(g_t\ ;r^e)\cdot \exp\left\{\int_0^t (g_\tau-\rho)\,d\tau\right\}dt$$

$$\leqq \begin{cases} K_o\ f_{max} \displaystyle\int_0^\infty \exp\left\{\int_0^t (\bar{g}-\rho)\,d\tau\right\}dt \\ K_o\ f_{max} \displaystyle\int_0^\infty \exp\left\{\int_0^t -(\delta+\rho)\,d\tau\right\}dt \end{cases}$$

$$= K_o\ f_{max}\ /\ (\rho-\bar{g}) > \infty\ (\text{if } f_{max} \geqq 0).$$

$$= K_o\ f_{max}\ /\ (\rho+\delta) < \infty\ (\text{if } f_{max} < 0).$$

(q. e. d.)

This lemma assures that we need not be troubled with the problem of 'infinity'. Next we shall analyze the model under the following classification.

(1) The case of the non risk avertor ($v''(r) \geqq 0$).

(2) The case of the risk avertor ($v''(r) < 0$).

Whether we adopt the A-mersure or the B-measure, this standard of the classification is invariant.

(1) The case of the non risk avertor

In this case, we have $f'(g_t) \geqq 0$ from (II—10). Hence, if we denote $\bar{g}_t$ the arbitrary continuous feasible path of capital accumulation other than the path $g_t=\bar{g}\ \forall_t \geqq 0$, we obtain the following inequality from the continuity of $\bar{g}_t$ with respect to t.

$$(\text{III}-1)\quad K_0\int_0^\infty f(\tilde{g}_t;r^e)\cdot\exp\left\{\int_0^t(\tilde{g}_\tau-\rho)\,d\tau\right\}dt$$

$$\leqq K_0 f(\bar{g};r^e)\int_0^\infty \exp\left\{\int_0^t(\tilde{g}_\tau-\rho)\,d\tau\right\}dt$$

$$< K_0 f(\bar{g};r^e)\int_0^\infty \exp\left\{\int_0^t(\bar{g}-\rho)\,d\tau\right\}dt$$

$$= K_0 f(\bar{g},r^e)/(\rho-\bar{g}).$$

This result can be summarized as the following

PROPOSITION 1.

If the entrepreneur is the non risk avertor ($v''(r^e)\geqq 0$), *his optimal policy is to fix the rate of capital accumlation to the maximum level* $\bar{g}$ *for all times.*

(2) The case of the risk avertor

In this case at first glance the problem is complicated, but Pontryagin's maximum principle will serve. Let define the Hamiltonian function as follows.

$$(\text{III}-2)\quad H_t(g_t,K_t,\lambda_t,t)\equiv\{f(g_t;r^e)+\lambda_t g_t\}K_t\cdot\exp(-\rho t)$$

; where $\lambda_t \exp(-\rho t)$ is the co-state variable.

Then, a set of the necessary conditions for a maximum of (II—7) is given as follows. (See, for example, Intriligator [4] chap. 14.)

$$(\text{III}-3)\quad \text{(i)}\quad \max_{-\delta\leqq g\leqq\bar{g}} H_t(g_t,K_t,\lambda_t,t)\quad \forall t\geqq 0.$$

$$\text{(ii)}\quad (\lambda_t\exp(-\rho t)) = -\partial H_t/\partial K_t\quad \forall t\geqq 0.$$

(iii) $\lim_{t\to\infty} (\lambda_t \exp(-\rho t)) = 0.$

We can rewrite the condition (III—3) (ii) as follows.

$$\text{(III—3) (ii)}' \quad \dot{\lambda} = -f(g_t; r^e) - \lambda_t g_t + \lambda_t \rho \quad \forall t \geqq 0.$$

Since $\partial^2 H_t / \partial g_t^2 = f''(g_t) K_t \cdot \exp(-\rho t) < 0$ from (II—6), (II—10) and (III—2); H_t is the strict concave function of g_t. Hence, $\partial H_t / \partial g_t = 0$
is the sufficient condition for (III—3) (i) if we ignore the constraint on g_t. We are interested in the case of the interior solution and from now on the interior solution will be assumed. The condition $\partial H_t / \partial g_t = 0$ can be rewritten as follows.

$$\text{(III—3) (i)}' \quad \lambda_t = -f'(g_t).$$

Differentiating (III—3) (i) with respect to time and substituting to (III—3) (ii)', we have the following differential equation with respect to g_t.

$$\text{(III—4)} \quad \dot{g}_t = \{f(g_t; r^e) + f'(g_t)(\rho - g_t)\} / f''(g_t) \equiv F(g_t).$$

From (II—6) and (II—10), $f'''(g_t) = (1/2) v''(r^e) \sigma_r^{2'''}(g) = 0$. Therefore, if $g_t \leqq \bar{g}$ we have ;

$$\text{(III—5)} \quad F'(g_t) = \rho - g_t > 0.$$

(III—5) implies that the stationary solution of (III—4) is, if exists, unique and unstable. (See Fig. 1.) The sufficient condition for the existence of the economically meaningful stantionary solution of (III—4) (g^*) is given as follows.

(III—6)　$F(-\delta) < 0$,　$F(\bar{g}) > 0$.

Since $f''(g_t) < 0$, the condition (III—6) is equivalent to the following condition. (See (II—10).)

(III—6)′　$v(r^e) + (1/2)\, v''(r^e)\, \{\sigma_r^2(-\delta) + \sigma_r^{2\prime}(-\delta)(\rho + \delta)\} > 0,$

$v(r^e) + (1/2)\, v''(r^e)\, \{\sigma_r^2(\bar{g}) + \sigma_r^{2\prime}(\bar{g})(\rho - \bar{g})\} < 0.$

We assume that this condition is fulfilled.

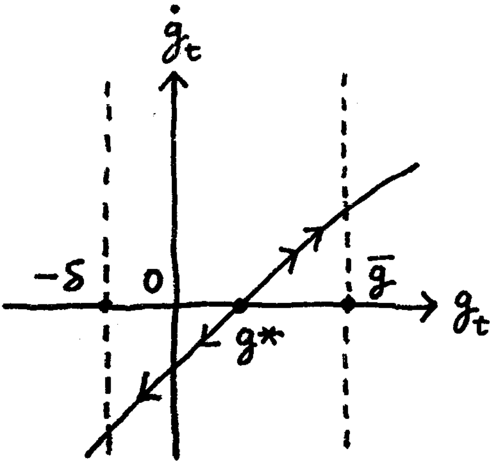

Fig. 1.

In this case, the only feasible path of g_t which satisfies both of (III—3) (i)′ and (III—3) (ii)′ is;

(III—7)　$g_t = g^* \quad \forall t \geqq 0$

; where g^* must satisfy the following equality.

(III—8) $v(r^e) + (1/2)\, v''(r^e)\, \{\sigma_r^2(g^*) + \sigma_r^{2\prime}(g^*)(\rho - g^*)\} = 0.$

Since $\lim_{t\to\infty} \lambda^* \exp(-\rho t) \equiv \lim_{t\to\infty} \{-f'(g^*) \cdot \exp(-\rho t)\} = 0,$ (III—3) (iii) is also fulfilled if (III—7) and (III—8) are satisfied.

Now we can prove that the conditions (III—7) ~ (III—8) are also the sufficient conditions for a maximum of (II—7)′. Namely,

PROPOSITION 2.

Suppose that the entreprneur is uhe risk avertor ($v''(r^e) < 0$) *and that* (III—6)′ *is satisfied. Then, the necessary and sufficient conditions for a maximum of* (II—7)′ *are given by* (III—7) *and* (III—8).

(Proof.) Let write $[f^*, K_t^*, g^*]$ the path of $[f_t, K_t, g_t]$ which satisfies (III—7) and (III—8), and write $[\tilde{f}, \tilde{K}_t, \tilde{g}_t]$ the arbitrary continuous feasible path of $[f_t, K_t, g_t]$. Then, we obtain the following equality.

(III—9)
$$M \equiv \int_0^\infty \{f_t^* K_t^* - \tilde{f}_t \tilde{K}_t\} \exp(-\rho t)\, dt$$
$$= \int_0^\infty \{f_t^* K_t^* - \tilde{f}_t K_t + \lambda^* (g^* K_t^* - \dot{K}_t^*)$$
$$- \lambda^* (\tilde{g}_t \tilde{K}_t - \dot{\tilde{K}}_t)\} \cdot \exp(-\rho t)\, dt$$
$$= \int_0^\infty \{f_t^* K_t^* - \tilde{f}_t \tilde{K}_t + \lambda^* g^* K^* - \lambda^* \tilde{g}_t \tilde{K}_t\} \exp(-et)\, dt$$
$$+ \lambda^* \cdot \exp(-et) \cdot (\tilde{K}_t - K_t^*) \Big|_0^\infty - \int_0^\infty (\lambda^* \dot{\exp}(-et))(\tilde{K}_t - K_t^*)\, dt$$

; where $\lambda^* \equiv -f'(g^*)$. From (II—7)′, (II—8) and (III—3) (i)′, we have the following relationship.

(III—10)
$$\lambda^* \cdot \exp(-\rho t) \cdot (\tilde{K}_t - K^*) \Big|_0^\infty$$
$$= \lim_{t\to\infty} \left\{\lambda^* \cdot \exp(-\rho t) \cdot (\tilde{K}_t - K^*)\right\} - \lambda^* (K_o - K_o)$$

$$= -f'(g^*) K_0 \left(\lim_{t\to\infty} \exp\left\{ \int_0^t (\tilde{g}_t - \rho)\, d\tau \right\} - \lim_{t\to\infty} \exp\left\{ (g^* - \rho)\, t \right\} \right) = 0 .$$

From (III—3) (ii) we also have ;

(III—11) $\quad (\lambda^* \dot{\exp} (-\rho t)) = -(f_t^* + \lambda^* g^*) \cdot \exp(-\rho t).$

Substituting (III—10) and (III—11) into (III—9), we obtain the following expression.

(III—12) $$M \equiv \int_0^\infty \left\{ (f_t^* + \lambda^* g^*) - (\tilde{f}_t + \lambda^* g_t) \right\} \tilde{K}_t \exp(-\rho t)\, dt.$$

Since $G(g_t) \equiv f(g_t) + \lambda^* g$ is the strict concave function of g (i. e. $G''(g) = f''(g) < 0$'), the following inequality is established.

(III—13) $\quad G(g^*) - G(\tilde{g}_t) \geqq G'(g^*) \cdot (g^* - \tilde{g}_t)$

; where equality holds if and only if $g^* = \tilde{g}_t$ · From (III—3) (i)' we know that $G'(g^*) = f'(g^*) + \lambda^* = 0$. Hence, from (III—12) and (III—13) we can establish the following result.

(III—14) $$M \equiv \int_0^\infty (f^* K^* - \tilde{f}_t \tilde{K}_t) \cdot \exp(-\rho t)\, dt \geqq 0.$$

Furthermore, if $g_\tau \neq g^*$ for some $\tau \geqq 0$, the strict inequality in (III—14) is established from the continuity of $\tilde{g}_t$ with respect to t. This completes the proof. (q. e. d.)

IV. Comparative dyuamic analysis in the case of the risk avertor

Next, we shall investigate how the changes of the parameters ρ or r^e affect on the optimal policy of the entrepreneur in the case of the risk avertor. Totally differentiating (III—8) with respect to ρ and g*, and rearranging terms, we have the following result.

(Iv—1) $dg^*/d\rho = -\sigma_r^{2\prime}(g^*)/\{\sigma_r^{2\prime\prime}(g^*)(\rho - g^*)\} < 0$.

Similarly totally differentiating (III—8) with respect to r^e and g*, we have ;

(Iv—2) $$dg^*/dr = -\frac{v'(r^e) + (1/2)v'''(r^e)\{\sigma_r^{2\prime}(g^*)(\rho - g^*) + \sigma_r^2(g^*)\}}{(1/2)v''(r^e)\{\sigma_r^{2\prime\prime}(g^*)(\rho - g^*)\}}$$

From (Iv—2) it is assured that if $v'''(r^e) \geqq 0$, $dg^*/dr > 0$. The economic implication of this condition is given by the following lemma. (3)

<**Lemma 2.** >

If the entrpreneur's absolute risk aversion is non-increasing in the sense of Arrow &Pratt (i. e. $A'(r^e) \leqq 0$), $v'''(r^e) > 0$.

(Proof.) Differentiating (II—13), we have ;

$A'(r^e) = -\left[v'''(r^e)v'(r^e) - \{v''(r^e)\}^2\right]\{v'(r^e)\}^2$.

Hence, if $A'(r^e) \leqq 0$, $v'''(r^e) \geqq (v''(r^e)\}^2 / v'(r^e) > 0$.

(q. e. d.)

The above results can be summarized as follows.

PROPOSITION 3.

Suppose that the entrepreneur is the risk avertor and that (III—6)′ *is satisfied, Then. the higher the subjective rate of discnunt* (ρ), *the lower is the optimal rate of capital accumulation* (g*).

Furthermore, assume that the entreprneur's absolute risk aversion *is non-increasing in the sense of Arrow & Pratt. Then, the higher the expected rate of profit* (r^e), *the higher is* g^*.

This proposition is our version of the Robinsonian investment function.

Lastly, we shall confirm the another interesting property of the model.

Using (II—14), we can rewrite (III—8) as follows.

(Iv—3) $1 - (1/2)\, B\,(r^e)\, \{ \sigma_r^2\,(g^*) + \sigma_r^{2\prime}\,(g^*)\,(\rho - g^*) \} = 0.$

Totally differentiating this equation, we have ;

(Iv—4) $dg^*/dB\,(r^e)\Big|_{\substack{\rho=\text{const.}\\ r^e=\text{const.}}}$

$$= -\frac{\sigma_r^2\,(g^*) + \sigma_r^{2\prime}\,(g^*)\,(\rho - g^*)}{B\,(r)\,\sigma_r^{2\prime\prime}\,(g^*)\,(\rho - g^*)} < 0.$$

Hence, the following proposition is obtained.

PROPOSITION 4.

Suppose that the entrepreneur is the risk avertor and that (III—6)′ *is satisfied. Then, ceteris paribus, the higher his degree of the risk aversion measured by the B-meusure, the lower is the optimal rate of accumulation* (g^*).

V. Summary and conjectures

In this note we have shown that under some conditions the Robinsonian investment function can be derived choice-theoretically. Prposition 3 in this note, together with propositions 1 and 4, links the investment policy of the entrepreneur under uncertainty closely to his attitude towards risk. And the assumption of the 'increasing risk' of the investment plays a central role in the process of the derivation of these propositions.

In this model, the 'animal spirit' of the entrepreneur has consistently been interpreted by his degree of the risk aversion.

The model which has been developed in this note will, therefore, serve as a microfoundation of the Posk-Keynesian models of the steady growth equilibrium.

A full examination of the multi-sectoral Posk-Keynesian model of the growth equilibrium involving this type of investment function will be tried in the another paper of the author. Therefore, here we shall merely express some conjectures concerning the re-interpretation of the model in the context of the economy-wide perspective. Proposition 1 says that if the entrepreneur is not risk avertor, his 'animal spirit' is so high that he decides to invest up to the bottle neck irrespective of his expectation about the profitability of the investment. Hence, if all of the entrepreneurs in the economiy are the 'non risk avertors' and the bottle neck of the investment is given by the full employment ceiling of the labour, the rate of capital accumulation in this economy will be pushed up to the 'natural rate of growth' and so the full employment of the labour will be attained. Therefore, proposition 1 will serve as a microfoundation of the full employment growth equilibrium model which was developed by Pasinetti [7]. On the other hand, proposition 3 implies that if the entrepreneur is the risk avertor and furthermore some plausible properties are fulfilled, he will not decide to accumulate the capital up to the bottle neck unless enough profitability of the investment is expected. Hence, if all of the entrepreneurs in the economy are the 'risk avertors' and the ecoaomy is in a position of the Robinsonian 'internal equilibrium' where their dismal expectation is self-realized and is self reproducible, the unemployment of the labour will persist (even if the financial constraint doesn't exist). (See Robinson [9] chap. II · and Aoki & Marglin [2].

Now, in conclusion let us point out two major limitatioas of the model which has been developed in this note.

First, the change of the expectation has not been analyzed in this note. If the expectation of the entrepreneur is deceived for a long time, his expectation will be modified and so his investment policy will change. The analysis of the dynamics of these 'disequilibrium' situations has not been challanged in this note, and it is the theme to be treated carefully in an independent article.

Second, there is more subtle and difficult problem concerning the economic treatment of the 'uncertainty'. In the famous chapter 12 of the "General Theory" [6], Keynes writes: "Enterprise only prtends to itself to be mainly actuated by the statements in its own prospectus, however candid and sincere. Only a little more than an expedition to the South Pole, is it based on an exact calculation of benefits to come. Thus if the animal spirits are dimmed and the sponaneous optimism falters, leaving us to depend on nothing but a mathematical expectation, enterprise will fade and die; ——though fears of loss may have a basis no more reasonable than hopes of profit had before", (pp. 161~2.)

This statement seems to be less applicable to the behaviours of the modern corporations than those in the age of Keynes. For example, in our age even the 'expedition to the South Pole' is probably the result of the exact and deliberate calculation of the benefit compared to the risk.

Nevertheless, Keynes' indication that the problem of the 'true' uncertainty cannot be disposed of by a simple mathematical treatment is important as before. But the full consideration of this unresolved problem is beyond the scope of this short note.

Finally, let us allow to quote from Keynes [6] once more.

"We should not conclude from this that everything depends on waves of irrational psychology. On the contrary, the state of lonn-term expectation is often steady, and, even when it is not, the other factors exert their compensatory effects. ……it is our innate urge to activity which makes the wheels go round, our rational selves choosing between the alternatives as best we are able, calculating where we can, but often falling back for our motive og whim or sentiment or chance." (pp. 162~3.)

Footnotes

(1) Thanks are due to the valuable comments by Prof. K. Ara and Dr. S. Takekuma. Needless to say, however, the auther alone is responsible for the remaining errors and for the views expressed here.

(2) We owe this basic idea to Kalecki [5]. He argues: "A firm considering expansion must face the fact that, given the amount of the entrepreneurial capital, *the risk increases with the amount invested*. The greater the investment in relation to the entrepreneurial capital, the greater is the reduction of the entrepreneur's income in the event of an unsuccessful business venture." ([5] p. 106 ; *Italics* added.) Hence, the relation (II—6) may be named the 'Kalecki effect' by analogy of the Penrose effect.

(3) In a sense, the assumption of the 'non-increasing' (absolute) risk aversion implies that the entrepreneur doesn't become more cowardly if the expected rate of profit increases. of course, this is the considerably plausible assumption.

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