ON THE DEFINITION OF MONEY

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I. The Function of Money

As the function of Money, we can consider the following three functions of money that is (1) the function as a unit of account, (2) the function as a medium of exchange, and (3) the function as a store of value. The former is the abstract function of money, and the latter two are the phenomenal functions of money.

The classical school proposing Gold Standard recognized that money had three functions and the important roles in the real world. Rather because they considered that the stability of value of money was important in real world as a medium of exchange and a store of value, they were eagering in the stability of value of money by Gold Standard. Nevertheless, classical school is called to have the money-veil view and the classical dichotomy.

In classical economics, relative prices and the rate of interest are determined in real economy and the demand for money and supply of money have less effect on them. While the supply of money have some effect on the absolute price only, with no effect on real factors through the rate of interest. Monetary analysis of classical school shows itself in the Quantity Theory of Money and lays stress on the medium of exchange as the function of money.

Now, we can show a simple model as following

\[ I(r) = S(y, r) \]  
\[ \bar{M} = P \times L(y, r) \]

Equation (1) shows the equilibrium of production market, and Equation (2) shows the equilibrium of money market. There are only 2 equations for 3 variables \( y, r, P \), therefore we have to add one more equation to get a complete system. M. Friedman proposed that classical model add \( y = y_0 \ldots (3) \) and Keynesian model \( P = P_0 \ldots (4) \).

Firstly we consider classical model. Substituting (3) into (1), we have the equilibrium rate of interest \( r_0 \). In this case, \( M \) is indifferent from the decision of \( r_0 \). Substituting \( r_0 \) and (3) into (2), we have the equilibrium price level \( P_0 \). Thus \( M \) have some effect on General Price level only, with no effect on real economy. Therefore it is called that they have the money-veil view and the classical dichotomy. Then equation (2), replacing income velocity \( V = y/L(y, r) \), is simply the classical quantity equation \( MV = Py \ldots \)  

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understand that the classical quantity equation emphasizes the function as a medium of exchange of money. \( M = k P Y \), which uses Marshallian \( k \) in stead of \( V \) in \( \delta \), shows a little stress on the store of value. In order to understand fully the function as a store of value, therefore, we had to wait for the liquidity preference theory by J.M. Keynes.

Simple Keynesian model consists of \( \delta, \eta \), and \( \varphi \) equations. Substituting \( \delta \) into \( \eta \), gives an equation in two variables, \( y, r \). This is the IS curve of Hicks’s famous IS-LM analysis. Substituting \( \varphi \) into \( \delta \) gives an equation in two variables, \( y, r \). This is Hicks’s LM curve. The simultaneous solution of the two curves determines \( r_0 \) and \( Y_0 \). This means getting free from classical dichotomy.

Keynes introduced the motive analysis of holding money, that is the transactions-motive, the precautionary-motive and the speculative-motive. The demand for money to satisfy the transactions-motive can be classified as the income-motive and the business-motive. The former is to bridge the interval between the receipts of income and its disbursement, and the latter is to held to bridge the interval the receipts of the sale-proceeds and its cost. The demand for money to satisfy the precautionary-motive are considered the following 2 types. (a) To provide for contingencies requiring sudden expenditure and for unforeseen opportunities of advantageous purchase. (b) To hold an asset of which the value is fixed in terms of money to meet a subsequent liability fixed in terms of money. Keynes considers specially (a) as the precautionary-motive, with (b) under the heading of the speculative-motive, but modern monetary economists consider specially (a) under the heading of the precautionary-motive, with (b) as the transactions-motive. The Demand for money to satisfy the speculative motive is the central definition of understanding the liquidity preference theory, and Keynes laid stress on this last motive.

Let the amount of money held to satisfy the transactions- and precautionary-motives be \( M_1 \), and the amount held to satisfy the speculative-motive be \( M_2 \). Corresponding to these two compartments of money, we then have two liquidity functions \( L_1 \) and \( L_2 \). \( L_1 \) mainly depends on the level of income, while \( L_2 \) mainly depends on the rate of interest.

\[
M = M_1 + M_2 = L_1 (Y) + L_2 (r)
\]

The function of money connecting with this speculative motive is those of money as a store of value.

**II. Modern Monetary Theory and The Functions of Money**

The first important development in Keynesian theory has been the introduction of risk into the individual decision-making process, which is known as the portfolio selection theory. And the second development concerns the attack on Keynesian theory by M. Friedman, and the introduction of real assets into the portfolio.

(a) Theory of Portfolio Selection and The Function of Money.

The theory of portfolio selection has been developed by Markowitz, Tobin and Hicks. It is normally assumed that a positive return enters positively into an investor’s utility function, while risk enters negatively. Therefore there is a trade-off between return and risk.

In this theory expected return and risk are measured by the mean and standard deviation of the subjective probability of the return of any investment. So the theory of portfolio selection is called as either the mean-variance approach or the two parameter ap-
An investor's object is to get the result, in terms of yield and risk, over his whole portfolio. There are three types of investors; risk averters, risk lovers, and risk neutral. This is illustrated in Fig. 1.

In general, normal investors will be risk averters so that the object of the portfolio analysis will be the risk aversor's behavior on the selection of portfolio.

Assuming that all wealth, \( W \), is held either in the form of money, \( M \), or bonds, \( B \), i.e.

\[
W = M + B
\]

it follows that

\[
\alpha + \beta = 1 \quad [\because \alpha = M/W, \beta = B/W]
\]

\( M \) is called a safety asset (\( \mu_0, \sigma_0 \)) and \( B \) is a risky asset (\( \mu_1, \sigma_1 \)). A safety asset is not necessary a rigid money (\( \mu_0 = 0, \sigma_0 = 0 \)) but a reasonable monetary asset (\( \mu_0 > 0 \), \( \sigma_0 = 0 \)).

\( \mu_R \): the expected return of investments

\( \sigma_R \): the standard deviation of its expected return; risk of investments

Line \( ZB \) in Fig. 2. shows the investors' opportunity locus, i.e. the combination of expected risk and returns on the portfolio which investor will selects.
In order to get the optimum portfolio between a safety asset and a risky asset, we need to know his attitudes to return and risk, which is shown by the indifference curves, and \( P \) in Fig. 2 is the optimum portfolio of investor with risk aversion.

Next, let's consider the portfolio-selection among risky assets. Let both risky assets are \( B(\mu_1, \sigma_1) \) and \( C(\mu_2, \sigma_2) \). Assume that the proportion of wealth allocated to \( B \) is \( \alpha \) and the proportion to \( C \) is \( (1-\alpha) \). Then the expected return of the portfolio \( (\mu_R) \) and the standard deviation \( (\sigma_R) \) is following.

\[
\begin{align*}
\mu_R &= E[R] = \alpha \mu_1 + (1-\alpha) \mu_2 \\
\sigma_R^2 &= E[(R-E[R])^2] \\
&= \alpha^2 \sigma_1^2 + (1-\alpha)^2 \sigma_2^2 + 2\alpha(1-\alpha)R_{12}\sigma_1\sigma_2
\end{align*}
\]

**Fig. 3. Investment Opportunity Locus**

\[
\begin{align*}
(1) & \quad R_{12} = + \text{perfect positive correlation} \\
(2) & \quad R_{12} = -1 \text{perfect negative correlation} \\
(3) & \quad R_{12} = 0 \text{ perfectly independent} \\
(4) & \quad -1 < R_{12} < +1 \text{ imperfect correlation} \\
(5) & \quad 0 < R_{12} < +1 \text{ positive correlation}
\end{align*}
\]

Equation (1) and Equation (2) will give the relationship between \( \mu_R \) and \( \sigma_R \), i.e.

\[
\begin{align*}
\sigma_R^2(\mu_1-\mu_2)^2 - \mu_R^2(\sigma_1^2 + \sigma_2^2 - 2R_{12}\sigma_1\sigma_2) \\
+ 2\mu_R(\mu_2\sigma_1^2 + \mu_1\sigma_2^2 - (\mu_1 + \mu_2)R_{12}\sigma_1\sigma_2) \\
- (\mu_2^2\sigma_1^2 + \mu_1^2\sigma_2^2 - 2\mu_1\mu_2R_{12}\sigma_1\sigma_2) &= 0
\end{align*}
\]

This shows the quadratic equation concerning \( \mu_R \) and \( \sigma_R \) and are called the investment opportunity locus. The point \( P \) in Fig. 4 gives the optimum portfolio among risky assets.

**Fig. 4. Optimum Portfolio: Risky Assets**
Finally let's consider the optimum portfolio among risky assets \((B, C)\) and a safety asset \((Z)\). This is illustrated in Fig. 5.

**FIG. 5. OPTIMUM PORTFOLIO: GENERAL CASE**

The point \(M\) in Fig. 5 will give the optimum portfolio among \(BCZ\) assets.

In the monetary analysis of portfolio selection, the function of money as a store of value is stressed. In other words, the portfolio analysis seems to be the investor's investment behavior, but not the entrepreneur’s investment behavior. I think that Hicks understood this rightly in saying that \(M_1\) was not voluntary demand for money, but \(M_2\) was voluntary demand for money.

(b) Introduction of Real Asset into Portfolio

Friedman extend the theory of demand for money in introducing of real asset. His wealth \((W)\) is viewed as the present value of the discounted flow of permanent income \((Y)\) i.e. \(W = \frac{Y}{r}\) and this shows budget constraint. Major assets are money, bonds, equities and physical goods.

Money is a safety asset, and bonds, equities, real assets are risky assets.

- \(r_m\) is the expected nominal rate of return on money.
- \(r_b\) is the expected nominal rate of return on bonds.
- \(r_e\) is the expected nominal rate of return on equities.

\[
\frac{1}{r_b} \frac{dr_b}{dt} \quad \text{is the capital gain or loss resulting from a rise, or fall, in the price of the bonds.}
\]

Similarly \(\frac{1}{r_e} \frac{dr_e}{dt}\) is the capital gain or loss for any change in the nominal price of the equities. Nominal yield of physical goods is given by the rate of inflation \(\left(\frac{1}{P} \frac{dP}{dt}\right)\) and the constant real yield. Let the ratio of non-human to human wealth be \(h\), and tastes and preferences be \(u\).

We can write an implicit demand for money function as following.

\[
M_d = f(P; r_b, -\frac{1}{r_b} \frac{dr_b}{dt}; r_e, \frac{1}{P} \frac{dP}{dt}; -\frac{1}{r_e} \frac{dr_e}{dt}; \frac{1}{P} \frac{dP}{dt}; h; \frac{Y}{r}; \mu)
\]

we can simplify the demand for money function.
This model was presented in the spirit of the old quantity theory and was called the new quantity theory. But this model has more in common with Keynes' liquidity preference theory than the old quantity theory, in stressing on the store of value of money.

In Keynes' theory, the selection of money and bond only is taken, while in Friedmans' theory money is viewed as a direct substitute over a wide range of assets, typically bonds, equities and real goods. So, Friedmans' theory is said to be more general than Keynes' theory.

According to the own-rate of interest by Keynes (chap. 17), money is viewed as a direct substitute of real assets. The own-rate of interest of any commodity is yield \((q)\)—carring cost \((c)\) + liquidity-premium \((f)\).

Let's consider three types of assets, i.e., house, wheat and money. The yield on house is \(q\), and its carrying cost and liquidity-premium is negligible, and the carrying cost of wheat is \(-c_2\) and its yield and liquidity-premium is negligible, and \(l_3\) is the liquidity-premium of money and its yield and carrying-cost is negligible. Therefore, \(q,\) is the house-rate of interest, \(-c_2\) the wheat-rate of interest, and \(l_3\) the money-rate of interest.

To determine the relationships between the expected returns on different types of assets, we take money as our standard of measurement. Let the expected percentage appreciation of houses be \(a_1\) and of wheat be \(a_2\). It will be useful to call \(a_1+q_1, a_2-c_2\) and \(l_3\), the house-rate of money-interest, the wheat-rate of money-interest, and the money-rate of money-interest respectively. In equilibrium, \(a_1+q_1, a_2-c_2\) and \(l_3\) will be equal.

Thus, Keynes considered carrying cost as well as introduced the real goods as the object of portfolio selection. Therefore, Keynes' Theory seems to be rather general than Friedmans' Theory.

III. On the Misunderstanding of the Economics of Keynes

Modern monetary theory have been developed in stressing the store of value of money. Two misunderstanding of the Economics of Keynes have risen.

a) On the zero elasticity of substitution of money.
b) On the less emphasis of the exchange-function of money. Let's consider (a) first.

Keynes showed the following three fundamental properties of money. (chap. 17)

(i) Money has a zero, or small, elasticity of production.

(ii) Money has an elasticity of substitution equal or nearly equal to zero.

(iii) Money has the very high elasticity of the demand for money as a store of value.

The problem is on (ii) property of money. Keynes explained that as the exchange value of money rises there is no tendency to substitute some other factor for money i.e. it is money only to perform the function of money. While, in liquidity preference theory, there are high substitutability of money and bonds, i.e. this means the portfolio selection balance.

Own-rate of interest theory shows that there is no dilemma in introducing of real goods into portfolio. This is investors' or wealth holders' investment behavior, but not entrepreneurs' one which uses time to produce goods by capital and labour.
Next, let's consider of (b), which communicate with (a). The importance of the exchange function of money in Keynesian Theory was reassured by R. Clower and A. Leijonhufvud. They maintained that "money buys goods, while goods buy money, but goods does not buy goods" is the central proposition of monetary theory, and tried to emphasize the exchange function of money. And they contended that we had to separate the notional demand and effective demand, and under uncertainty it was important to consider so called income constrained process. This is a fundamental core of "dual decision hypothesis". The function of money in this theory is the exchange-function, under uncertainty, which shows reappraisal of the exchange function in modern monetary theory. According to Clower and Leijonhufvud, the uniqueness of the economics of Keynes is seen in the discovery of information failure.

P. Davidson laid stress on the exchange function of money under uncertainty, too. He considered the importance of "finance motive" which was introduced by Keynes and ignored since then. The finance motive is a dynamic definition, regardless of a static model of Keynesian Economics. The ordinary view that the demand for money as a medium of exchange is directly related to income is somewhat misleading simplification. The introduction of the finance motive concept involves relating the demand for transactions balances to planned, expected spending propensities during the period. That is, the demand for money is a function of the aggregate planned demand for goods. Thus, we can not ignore the importance of the function of a medium of exchange of money, especially under uncertainty of the real world.

IV. The Definition of Money

Either the bias stress on the exchange function or the store of value function leads to the wrong concept of money. We needs to get together three functions to make the right concept of money. Hicks tried to get the right concept of money by using both the three functions of money and the three motives for holding money. Hicks's two triads is this, as following. The precautionary motive have two meaning as showing in Chap. I. Hicks's precautionary motive concept is subject to (9) (page 8). In this paper, we will call the precautionary motive concept subject to (9), the precautionary motive, according to Keynes, and the precautionary motive concept subject to (5), the asset motive.

To analyse the demand for money, therefore, we break it down into four components.

1) The transactions demand \( M_P(T) \)
2) The precautionary demand \( M_P(P) \)
3) The asset demand \( M_A \)
4) The speculative demand \( M_S \)

\[
M_T = L_1(y, P) \\
\frac{\partial M_T}{\partial y} > 0, \quad \frac{\partial M_T}{\partial P} > 0, \\
M_P = L_1(y, P, \mu_y) \\
\frac{\partial M_P}{\partial y} > 0, \quad \frac{\partial M_P}{\partial P} > 0, \quad \frac{\partial M_P}{\partial \mu_y} > 0
\]
$M_D^P + M_D^P = M_D^\text{active} = L_1(y, P, \mu_y)$

$M_D^A = L_2(r, \mu_r, W)$

$\frac{\partial M_A^D}{\partial r} < 0, \quad \frac{\partial M_A^D}{\partial \mu_r} > 0, \quad \frac{\partial M_A^D}{\partial W} > 0,$

$M_D^S = L_2(r, \hat{r}, \mu_r)$

$\frac{\partial M_S^D}{\partial r} < 0, \quad \frac{\partial M_S^D}{\partial \hat{r}} > 0, \quad \frac{\partial M_S^D}{\partial \mu_r} > 0,$

$M_D^A + M_D^S = M_D^\text{idle} = L_2(r, \hat{r}, \mu_r, W)$

$M_D = M_D^\text{active} + M_D^\text{idle}$

$= L_1(y, P, \mu_y) + L_2(r, \hat{r}, \mu_r, W)$

**notation**

- $y$: real income
- $P$: general price level
- $r$: market rate of interest
- $W$: Wealth
- $k$: marshallian $k$
- $\hat{r}$: normal rate of interest

Let $P, \mu_y, \mu_r, \hat{r}, W$ be given. 

$M_D = L_1(y) + L_2(r)$

$= k \cdot y + L_2(r)$

According to the 4 motives of holding money, we need to amend the functions of money, also. Hicks’s function of money as a medium of exchange seems to include not only the function of pure exchange, but also the one of deferred payment, which connects with lending and borrowing or credit. Let the function as deferred payment separate from the function of money as a medium of exchange, so that the Hicks’s two triads figure gives two squares figure. (Fig. 7). The right part of the Fig. 7 is connected with the industrial circulation and the money balance in this circulation will be accommodated with the active money balance.

**FIG. 6. THE TWO TRIADS BY HICKS**

- function as a unit of account
  - speculative motive
  - transactions motive
  - function as a store of value
  - precautionary motive
  - function as a medium of exchange

- $\mu_y$: an index of uncertainty regarding future receipts and payments
- $\mu_r$: an index of uncertainty regarding the future value of $r$
While the left part of the figure is connected with the financial circulation and the money balance in this circulation will be accomodated with the idle money balance.

The money concept is shown systematically like this. According to this two squares figure, Keynes's money concept is illustrated as Fig. 8.

Firstly we started from the Hicks's two triad figure and reached the two squares figure. Two figures have the similarities, but are different from each other fundamentally. Hicks's two triad figure consists from the viewpoint of the function of money, while my two square figure (also, Keynes' triangular figure) consists from the viewpoint of the motives for holding money.

Hicks's explained that "Keynes's classification is essentially 'close-up' it refers to the behavior of a single individual, or decision-maker, operating within a monetary system that is already defined. The other triad has a stance that is entirely different. It is most illuminating when we are standing right back, so that even the monetary system itself is allowed to vary."

Hicks seems to think about that the viewpoint of the function of money is better. Considering that he is a promulgator of the general equilibrium approach, his assension can be
understood. But, Economics is a social science, so that I think that the viewpoint of the motive for holding money is better. The money concept of typical economic subjects, i.e. firms, household, investors and financial institutions etc. will have to be investigated empirically.

REFERENCES