ENDOGENOUS TIMING IN A VERTICALLY DIFFERENTIATED DUOPOLY WITH QUANTITY COMPETITION*

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Abstract

This paper examines the endogenous choice of timing in a vertically differentiated duopoly with fixed cost of quality improvement when firms compete in quantities in the market. Using an extended game with observable delay, it is shown that when firms can choose the timing of quality choice, simultaneous play equilibria arise. By contrast, when firms can choose their relative positions in the quality space before deciding the timing, the game yields sequential play equilibria in which the low quality firm moves first and the high quality firm moves second.

Keywords: endogenous timing; quality choice; quantity competition; vertical product differentiation.


I. Introduction

The models of vertically differentiated oligopoly have been extensively investigated in the industrial organization literature (e.g., Musa and Rosen, 1979; Gabszewicz and Thisse, 1979; Shaked and Sutton, 1982, 1983; Ronnen, 1991; Motta, 1993; Aoki and Prusa, 1997; Lehmann-Grube, 1997; Valletti, 2000; Aoki, 2003). Under duopoly, a stylized framework is a two stage game in which firms choose their product qualities in the first stage and compete in either prices or quantities in the second stage. Two established results in that framework are: (i) Firms choose distinct qualities and (ii) The firm that produces a higher quality product earns higher profits than the firm that produces a low quality product (Motta, 1993; Aoki and Prusa, 1997; Lehmann-Grube, 1997; Aoki, 2003). These results hold, independent of the mode of competition at the final stage. Since the high quality firm can earn higher profits, the first mover chooses to produce a higher quality product when the first stage of quality choice is played sequentially (Aoki and Prusa, 1997; Aoki, 2003).

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Lambertini (1999) demonstrated that when firms endogenously choose the timing of quality choice, only simultaneous play equilibria can arise. He showed that the above result holds regardless of whether firms choose the timing at the very first stage or whether they choose their relative position in the quality space before deciding the timing. This result can be explained by lack of a second mover advantage in a vertically differentiated duopoly with price competition, which was indicated by Aoki (1998). Without a second mover advantage, neither firm prefers to move second. This result questions the relevance of analyzing the sequential move game under vertical differentiation, as in Aoki and Prusa (1997) and Aoki (2003). Lambertini (1999), however, only examined the case in which firms compete in prices.

This paper re-examines the endogenous choice of timing in a vertically differentiated duopoly by focusing on the case in which firms compete in quantities at the final stage of the game. As in Lambertini (1999), I embed endogenous quality choice with quantity competition into an extended game with observable delay in the spirit of Hamilton and Slutsky (1990). I restrict my attention to the fixed cost model of quality improvement. Two alternative games are considered. In the first game, in stage 1, firms simultaneously choose the timing of quality choice. In stage 2, firms choose their product qualities in the order determined in stage 1 and in stage 3, firms compete in quantities. In the second game, before deciding the timing of moves, firms choose their relative positions in the quality space simultaneously. The rest of the game proceeds in the same order as in the first game.

The main results are as follows. First, as in Lambertini (1999), only simultaneous play equilibria arise in the first game. Similar to the case of price competition, the result can be explained by lack of a second mover advantage. Since the high quality producer earns higher profits and the first mover chooses to produce a higher quality product, both firms prefer to move first. Firms also prefer a sequential move to a sequential move with the rival leading. Second, when firms choose their relative positions in the quality space before determining the timing of moves, only sequential play equilibria are obtained. In particular, in any pure-strategy subgame perfect Nash equilibria, the low quality firm chooses to move first and the high quality firm moves second. This result sharply contrasts with Lambertini’s (1999) result in the case of price competition. That is, only simultaneous play equilibria emerge even if firms choose their relative positions in the quality space at the very first stage of the game. In the case of quantity competition, the reaction function of the low quality firm in the quality space is downward-sloping, while that of the high quality firm is upward-sloping. In other words, qualities are strategic substitutes for the low quality firm and strategic complements for the high quality firm. Thus, my result is consistent with Hamilton and Slutsky’s (1990) theorem (Theorem V (B)). That is, the player whose reaction function is downward-sloping prefers to move first and the player whose reaction function is upward-sloping prefers to move second. Once firms’ relative positions in the quality space are fixed, a first mover advantage exists for the low quality firm and a second mover advantage exists for the high quality firm. Consequently, a natural order of leadership arises, as shown by Hamilton and Slutsky (1990). This result is also consistent with the results in Gal-Or (1985) and Dowrick (1986).

Lambertini (1996) is closely related to this paper. He also analyzes the endogenous timing choice in a vertically differentiated duopoly. However, he focuses on the case in which firms compete in prices. In this paper, I extend the analysis to the case in which firms compete in quantities.
in a vertically differentiated duopoly by combining several factors including (i) price or quantity competition at the final stage, (ii) the market being fully covered or not, (iii) the order of play in each stage being characterized by simultaneous moves, price (or quantity) leadership, quality leadership, repeated leadership, or alternate leadership. However, this paper differs from his on several points. First, he only considers the case in which relative positions of firms in the quality space are pre-determined. Second, I examine the case of fixed cost of quality improvement, while he focuses on the case of variable cost of quality improvement. Third, in terms of the results, he shows that under partial market coverage a unique subgame perfect equilibrium is characterized by simultaneous moves in both quality and price (or quantity) stages. In this paper, by contrast, sequential play equilibria emerge. This may be due to the difference in the cost of quality improvement. Since he uses a specific functional form for the cost function, it is not clear how robust his result is.

The remainder of the paper is organized as follows. Section 2 sets up the basic model of vertical differentiation. Section 3 presents the games in which the timing of moves is exogenously given. Section 4 analyzes the equilibrium outcomes in two alternative games of endogenous timing. Section 5 concludes the paper.

II. The Basic Model

The model is an extended version of the standard model of vertical differentiation. There is a continuum of consumers indexed by \( \theta \), which is uniformly distributed on \([0, \bar{\theta}]\) with density one. Each consumer is assumed to either buy one unit of the vertically differentiated good or nothing. Consumer \( \theta \)’s (indirect) utility is given by \( u = \theta q - p \) if he buys one unit of a product of quality \( q \in [0, \infty) \) at price \( p \in [0, \infty) \). His utility is zero if he buys nothing. In this model, the market is not fully covered in the sense that consumers with lower values of \( \theta \) do not buy the product in equilibrium.

There are two firms in the market, denoted as 1 and 2. Each firm offers a single product. The marginal and average production costs are assumed to be invariant with respect to both quality and quantity. For simplicity, I let these costs be zero. The cost of quality improvement is given by \( F(q_i) = k(q_i)^n \), where \( q_i \) is the level of firm \( i \)’s product quality, \( k > 0 \) and \( n \geq 2 \) is an integer.

I consider two alternative extended games with observable delay proposed by Hamilton and Slutsky (1990). The game structure of the first extended game (hereafter called ‘Game 1’) is as follows: In stage 1, firms simultaneously choose the sequence of moves at the stage of quality choice; in stage 2, firms choose their product qualities in the order determined in stage 1; and in stage 3, firms compete in quantities. In the second extended game (hereafter called ‘Game 2’), before stage 1 (called stage 0), firms simultaneously choose their relative positions in the quality space. The rest of the game proceeds in the same way as Game 1 except that the strategy space in stage 2 is restricted by the firms’ choice in stage 0.

Throughout the paper, I use a subgame perfect Nash equilibrium (SPNE) as an equilibrium concept. I restrict my attention to pure-strategy equilibria.

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III. Fixed Timing Games

Before examining the endogenous timing game, it is useful to analyze outcomes of the games in which the timing of quality choice is exogenously given.

At the final stage, firms simultaneously choose quantities. Each firm’s equilibrium revenue at the final stage is given by

\[ R_i^c(q_i, q_j) = \begin{cases} 
\frac{\partial^2 q_i (2q_i - q_j)^2}{(4q_i - q_j)^2}, & \text{if } q_i > q_j, \\
\frac{\partial^2 q_j (q_j - q_i)^2}{(4q_j - q_i)^2}, & \text{if } q_i < q_j, 
\end{cases} \]

for \( i, j = 1, 2 \). \( R_i^c(q_i, q_j) \) has the following properties (Aoki, 2003, Lemma 3): When \( q_i > q_j \), \( \partial R_i^c/\partial q_i > 0 \), \( \partial R_i^c/\partial q_j < 0 \), \( \partial^2 R_i^c/\partial q_i^2 < 0 \), and \( \partial^2 R_i^c/\partial q_i \partial q_j > 0 \); when \( q_i < q_j \), \( \partial R_i^c/\partial q_i < 0 \), \( \partial R_i^c/\partial q_j > 0 \), and \( \partial^2 R_i^c/\partial q_i \partial q_j < 0 \). Firm \( i \)'s profits are then given by \( \Pi_i(q_i, q_j) = R_i^c(q_i, q_j) - F(q_i) \), \( i = 1, 2 \).

There are three possible games associated with the move in stage 2: (i) a simultaneous move; (ii) a sequential move with the high quality firm leading; and (iii) a sequential move with the low quality firm leading.

In the simultaneous move game, taking the rival’s product quality as given, firm \( i \)'s quality best-response correspondence \( q_i = B_i(q_j) \) in stage 2, which is characterized by the first-order condition (FOC), is given by \( B_i(q_j) = q_i^H(q_j) \) if \( q_i \leq \hat{q}_i \) and \( B_i(q_j) = q_i^L(q_j) \) if \( q_i \geq \hat{q}_i \), where \( q_i^H(q_j) < q_i < q_i^L(q_j) \) and \( \hat{q}_i \) satisfies \( \Pi_i(q_i^H(\hat{q}_i), \hat{q}_i) = \Pi_i(q_i^L(\hat{q}_i), \hat{q}_i) \). The properties of \( B_i(q_j) \) are as follows (Aoki, 2003, Lemma 4): (i) \( B_i(q_j) \neq q_j \) \( \forall q_j \); (ii) \( B_i(q_j) \) is discontinuous at \( q_j = \hat{q}_i \); (iii) \( dB_i(q_j)/dq_j > 0 \) for \( q_i \leq \hat{q}_i \); and (iv) \( dB_i(q_j)/dq_j < 0 \) for \( q_i \geq \hat{q}_i \). The third and fourth properties imply that qualities are strategic complements for the higher quality producer and strategic substitutes for the lower quality producer.

Let \( q_i^H \) and \( q_i^L \) be product qualities of the high and low quality products, respectively, in Nash equilibria (NEs) of the simultaneous move game. In the simultaneous move game, there are two pure-strategy NEs in stage 2, namely, \( (q_1^H, q_2^H), (q_1^L, q_2^L) \) (Aoki, 2003, Proposition 5). In the sequential move game with the high quality firm leading, there is a unique pure-strategy NE in stage 2, in which the high quality firm chooses \( q_i^H \) and the low quality firm chooses \( q_i^L \) with \( q_i^L > q_i^H \) and \( q_i^H > q_i^N \) (Aoki, 2003, Proposition 6). In the sequential move game with the low quality firm leading, on the other hand, there is a unique pure-strategy NE in stage 2, as shown in the following lemma:

**Lemma 1** In the sequential move game with the low quality firm leading, there is a unique pure-strategy NE in stage 2, in which the high quality firm chooses \( q_i^H \) and the low quality firm chooses \( q_i^L \) with \( q_i^L < q_i^H \) and \( q_i^H < q_i^N \).

**Proof.** Evaluate the low quality firm’s FOC along the high quality firm’s quality best-response correspondence at the NE point under simultaneous choice to obtain

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3 Although the profit function is not locally concave for some qualities, the quality best-responses are, as Aoki (2003) showed, characterized by FOCs rather than corner solutions.
Table 1. Numerical results

<table>
<thead>
<tr>
<th></th>
<th>Simultaneous choice</th>
<th>Firm H is the leader</th>
<th>Firm L is the leader</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q_H$</td>
<td>$q_H^N = 0.125971/\phi/k$</td>
<td>$q_H^H = 0.136348/\phi/k$</td>
<td>$q_H^L = 0.125964/\phi/k$</td>
</tr>
<tr>
<td>$q_L$</td>
<td>$q_L^N = 0.045112/\phi/k$</td>
<td>$q_L^H = 0.043198/\phi/k$</td>
<td>$q_L^L = 0.044941/\phi/k$</td>
</tr>
<tr>
<td>$\Pi_H$</td>
<td>$\Pi_H^N = 0.009735/\phi/k$</td>
<td>$\Pi_H^H = 0.009884/\phi/k$</td>
<td>$\Pi_H^L = 0.009758/\phi/k$</td>
</tr>
<tr>
<td>$\Pi_L$</td>
<td>$\Pi_L^N = 0.001366/\phi/k$</td>
<td>$\Pi_L^H = 0.001318/\phi/k$</td>
<td>$\Pi_L^L = 0.001366/\phi/k$</td>
</tr>
</tbody>
</table>

\[
\frac{d\Pi_L}{dq_L} \bigg|_{q_H = q_H^H(q_L), (q_L, q_L^N)} = \frac{\partial R_H^L}{\partial q_H} \frac{dq_H}{dq_L} \bigg|_{q_H = q_H^H(q_L)} + \left( \frac{\partial R_L^L}{\partial q_L} - F'(q_L) \right)_{(q_L, q_L^N)}<0,
\]

(2)

because $\partial R_H^L/\partial q_H < 0$ and $dq_H/dq_L \big|_{q_H = q_H^H(q_L)} > 0$. Thus, $q_L^H < q_L^N$ holds. Since $dq_H/dq_L \big|_{q_H = q_H^H(q_L)} > 0$ holds, $q_L^H < q_L^N$ implies that $q_H^H < q_H^N$. □

As shown in the above lemma, when the low quality firm is the leader, both firms choose lower qualities than those in the NE of the simultaneous move game.

Table 1 summarizes the outcomes associated with the three cases when the fixed cost function is given by $F(q_i) = k(q_i)^2$. In the table, I denote $q_i = q_H, q_j = q_L, \Pi_i = \Pi_H$, and $\Pi_j = \Pi_L$ for $q_I > q_j$. While I present the numerical results in Table 1 as a reference, the results in the next section are not restricted to the case of $F(q_i) = k(q_i)^2$. These results hold for $F(q_i) = k(q_i)^n$, with any integer $n \geq 2$.

IV. Endogenous Timing Games with Quantity Competition

In this section, I investigate the two alternative extended games formulated in Section 2. I first examine Game 1.

In stage 1, firms simultaneously choose the timing of quality choice. The set of possible times at which firms can choose to move is $T = \{F, S\}$, i.e., first and second. If both firms choose to move at the same time, the second stage is played as a simultaneous move game. If one firm chooses $F$ and the other firm chooses $S$, on the other hand, the second stage is played as a sequential move game.

As shown in the previous section, when the second stage is played as a simultaneous move game, there are two pure-strategy NEs, which are identical except for the identity of firms. When the second stage is played as a sequential move game, on the other hand, the first mover not only chooses a quality higher than that of the rival, but also chooses a quality higher than the higher quality in NE under simultaneous choice. That is, the first mover chooses $q^H_L$ (Aoki, 2003, Proposition 6). Consequently, in the sequential move game, the second mover chooses $q^H_L$. These observations imply that when either $(F, F)$ or $(S, S)$ are chosen in stage 1, each firm’s equilibrium payoff is either $\Pi_H^L$ or $\Pi_L^L$, depending on the equilibrium in stage 2. When either $(F, S)$ or $(S, F)$ are chosen in stage 1, the first mover’s payoff is $\Pi_H^L$ and the second mover’s payoff is $\Pi_L^L$. Thus, SPNEs in Game 1 are as follows:
Proposition 1. In Game 1, there are two pure-strategy SPNEs. In either SPNE, both firms choose F in stage 1, resulting in simultaneous play in stage 2.

Proof. Since the analysis based on the backward induction has already been performed up to stage 2, I have only to characterize NEs in stage 1. In stage 1, given firm j’s choosing F, firm i’s payoff is given by $\Pi^N_i$ or $\Pi^F_i$ if it chooses F and $\Pi^H_i$ if it chooses S. As Aoki (2003, Proposition 5) showed, $\Pi^N_i > \Pi^H_i$ holds. Moreover, since $q^H_i > q^N_i$ and

$$\frac{dH}{dq_H} \bigg|_{q_i = q_i^-} = \frac{dR_i}{dq_H} \bigg|_{q_i = q_i^-} + \left( \frac{dR_i}{dq_L} - F'(q_L) \right) \bigg|_{q_i = q_i^-} \frac{dq_L}{dq_H} < 0,$$

then it yields that $\Pi^N_i > \Pi^H_i$, which implies that F is the best response for firm i. Given firm j’s choosing S, on the other hand, firm i’s payoff is given by $\Pi^H_i$ if it chooses F and $\Pi^N_i$ or $\Pi^L_i$ if it chooses S. Since $\Pi^H_i > \Pi^N_i$ holds, F is the best response for firm i, which implies that F is the dominant strategy for firm i. By symmetry, the same argument holds for firm j and hence F is the dominant strategy for firm j.

The result in Proposition 1 can be explained by lack of a second mover advantage. Since the high quality producer can earn higher profits, each firm has an incentive to move first in order to become a high quality producer. Furthermore, given the rival’s choice of F, a firm prefers a simultaneous move (i.e., choosing F) to followership (i.e., choosing S). This is because the second mover becomes the low quality producer whose reaction function is downward-sloping. As Gal-Or (1985) and Dowrick (1986) showed, there is no second mover advantage for the player with a downward-sloping reaction function.4

Lambertini (1999, Proposition 1) also showed that simultaneous play equilibria arise in the case of price competition at the final stage. Thus, I can state that when firms endogenously choose the timing of quality choice, only simultaneous play equilibria are obtained, regardless of the mode of competition at the final stage.

I now turn to Game 2. In Game 2, in stage 0, firms simultaneously choose a subspace of the quality space they play in stage 2. The set of possible subspaces in the quality space is $\Sigma = \{1H, 2H\}$, where $1H$ is a subspace defined by $q_1 \geq q_2$ and $2H$ is a subspace of $q_1 < q_2$. When one firm chooses $1H$ and the other firm chooses $2H$, firms play in different subspaces in stage 2, resulting in no pure-strategy NEs in stage 2. Thus, in order for an outcome to be a pure-strategy SPNE, firms have to choose the same subspace in stage 0.

Consider the subgame following $(1H, 1H)$. Since firm 1 is the high quality producer in this subgame, when $(F, F)$ or $(S, S)$ are chosen in stage 1, firm 1 earns $\Pi^H_i$ and firm 2 earns $\Pi^L_i$. When $(F, S)$ is chosen in stage 1, firm 1 and 2’s payoffs are given by $\Pi^F_H$ and $\Pi^F_L$, respectively. When $(S, F)$ is chosen in stage 1, firm 1 and 2’s payoffs are given by $\Pi^L_H$ and $\Pi^L_L$, respectively. Payoffs in the subgame following $(2H, 2H)$ can be obtained in a similar way.

The outcome in Game 2 is presented in the following proposition:

4 Aoki (1998) showed that a second mover advantage does not exist in the case of price competition, despite the fact that the quality best-response correspondence is upward-sloping for both high and low quality producers.
**Proposition 2** In Game 2, there are two pure-strategy SPNEs, in which the firm that chooses to be the low quality producer in stage 0 moves first and the firm that chooses to be the high quality producer in stage 0 moves second at the stage of quality choice.

**Proof.** Since \((1H, 2H)\) and \((2H, 1H)\) in stage 0 result in no pure-strategy NEs in stage 2, I have only to examine \((1H, 1H)\) and \((2H, 2H)\). Consider first the subgame following \((1H, 1H)\). The normal form representation of this subgame is presented below.

<table>
<thead>
<tr>
<th></th>
<th>(F)</th>
<th>(S)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Firm 1</td>
<td>(1H)</td>
<td>(1H), (\Pi_H^L), (\Pi_H^H)</td>
</tr>
<tr>
<td>Firm 2</td>
<td>(F)</td>
<td>(1H), (\Pi_H^L), (\Pi_H^H)</td>
</tr>
</tbody>
</table>

Since \(q_L^L < q_H^N\) and
\[
\left. \frac{d\Pi_H}{dq_L} \right|_{q_L=q_H(q_L)} = \left. \frac{\partial R_H}{\partial q_L} \right|_{q_L=q_H(q_L)} + \left( \left. \frac{\partial R_H}{\partial q_H} \right|_{q_L=q_H(q_L)} - F'(q_H) \right) \left. \frac{dq_H}{dq_L} \right|_{q_L=q_H(q_L)} < 0,
\]
then it yields that \(\Pi_H^L < \Pi_H^H\), which implies that \(S\) is firm 1’s best response for firm 2’s \(F\). It holds that \(\Pi_H^H > \Pi_H^L\) and hence \(F\) is firm 1’s best response for firm 2’s \(S\). For firm 2, as shown in the proof of Proposition 1, \(\Pi_L^L > \Pi_L^H\) holds, which implies that \(F\) is firm 2’s best response for firm 1’s \(F\). It holds that \(\Pi_L^H > \Pi_L^L\) and hence \(F\) is firm 2’s best response for firm 1’s \(S\). Thus, \((S, F)\) is a unique NE in this subgame. The subgame following \((2H, 2H)\) can be analyzed analogously. \((F, S)\) is a unique NE in the subgame following \((2H, 2H)\). \(\square\)

Unlike in Game 1, firms choose to move sequentially in the stage of quality choice. This is because by restricting the strategy space to either \(q_1 > q_2\) or \(q_1 < q_2\), only upward-sloping or downward-sloping part of the reaction function remains and the discontinuity disappears. Each firm’s reaction function satisfies Hamilton and Slutsky’s (1990) conditions. As Hamilton and Slutsky (1990) showed, when strategic variables are strategic substitutes for one player and strategic complements for another player, the outcome is unique whereby the former prefers to move first and the latter prefers to move second. In my model, as shown in the previous section, qualities are strategic substitutes for the low quality firm and strategic complements for the high quality firm. Thus, the low quality firm prefers to move first and the high quality firm prefers to move second.\(^5\)

The result in Proposition 2 contrasts with what Lambertini (1999) showed in the case of price competition. Lambertini (1999) demonstrated that simultaneous play emerges in equilibrium even if firms choose their relative positions before determining the timing of moves. This\(^5\)

\(^5\) As Gal-Or (1985) and Dowrick (1986) showed, a second mover advantage exists for those who have upward-sloping reaction functions.
is mainly because in the case of price competition, the high quality firm prefers a simultaneous move to followership. When the low quality firm is the leader, it chooses a higher quality than its quality in the NE under simultaneous choice, which is harmful to the high quality firm due to intensified price competition.

V. Concluding Remarks

In this paper, I examined the endogenous choice of timing in a vertically differentiated duopoly with fixed cost of quality improvement. I focused on the case of quantity competition at the final stage. The framework of the extended game with observable delay proposed by Hamilton and Slutsky (1990) was employed.

The outcome of the endogenous timing game depends on whether or not firms can choose their relative positions in the quality space before deciding the timing of quality choice. When firms cannot choose their relative positions at the very first stage of the game, only simultaneous move equilibria are obtained. This is because each firm has an incentive to move first in order to earn higher profits by producing a higher quality product. Moreover, since a second mover advantage does not exist, each firm prefers a simultaneous move to followership.

When firms can choose their relative positions in the quality space before deciding the timing of quality choice, by contrast, only sequential play equilibria emerge. Once a firm obtains the position of the higher quality producer, it has an incentive to move second due to the second mover advantage. Since qualities are strategic complements for the higher quality firm and strategic substitutes for the lower quality firm, the order of moves emerging in subgame perfect equilibrium is consistent with the findings in the existing literature (Gal-Or, 1985; Dowrick, 1986; Hamilton and Slutsky, 1990).

The result in this paper implies that when the relative position of firms in the quality space is pre-determined under Cournot duopoly, firms will choose their product qualities sequentially. Otherwise, they will choose their product qualities simultaneously.

In this paper, I only investigated the endogenous timing of quality choice. Following Lambertini (1996), it may be interesting to extend the analysis to the endogenous timing at the stage of quantity choice and also at both stages.

REFERENCES


