Abstract

This paper explores the nature of vertical intra-industry trade: the exchange of an intermediate good and a final good that requires the intermediate good in the same industry. A factor endowment model is extended to a setting with a technological difference in the production of the intermediate good between countries. Unlike the result of the existing work, the share of intra-industry trade does not reach a peak when countries have identical factor endowment ratios. This paper shows that a difference in factor intensities between intra-industry goods plays a crucial role in deriving this result.

Keywords: vertical intra-industry trade, factor proportions, and technological differences

JEL Classification: F11

I. Introduction

Most of the theoretical work on intra-industry trade focused on horizontal intra-industry trade, the exchange of differentiated products that have the same factor intensity in their production. In recent years, another type of intra-industry trade has increased its importance empirically. Vertical intra-industry trade is the exchange of intermediate goods and final goods that use the intermediate goods for their production. The production processes of manufacturing, which used to be integrated within one country, have been fragmented across countries due to the improvement of transport and communication technology and the movement of trade liberalization in developing as well as developed nations. Such fragmentation causes
vertical intra-industry trade since, in some manufacturing sectors, final goods, parts, and components are classified in the same industry. For example, in automobile industries, “Sixty percent of U.S. auto exports to Canada are engines and parts, while seventy five percent of U.S. auto imports from Canada are finished cars and trucks” (Hummels et al. (1998)).

In this paper, I explore the nature of vertical intra-industry trade. For this purpose, I develop a factor endowment model with a technological difference. Let us consider a typical factor endowment model with two factors, two goods, and two countries. If countries are identical except for factor endowment ratios, comparative advantage is determined according to a factor endowment difference. In free trade equilibrium, a capital abundant country exports a capital-intensive good and imports a labor-intensive good. Suppose that a production process for one good consists of two stages. In the first stage, an intermediate good is produced with production factors. In the second stage, the intermediate good is transformed into a final good by using production factors. If another good is produced only with production factors and does not require multiple production stages, then the basic model is extended to a setting with two final goods and one intermediate good. I introduce a technical difference into this extended setting. In addition to a factor endowment difference, there is asymmetry in production technology of the intermediate good between countries. In this extended setting, there are two causes for trade. A technical difference determines the direction of trade flows in the intermediate good. A factor endowment difference determines the pattern of trade in two final goods. I show that vertical intra-industry trade arises under this mechanism, and examine its nature in trade equilibrium.

This paper is related to the recent theoretical work on trade such as Dixit and Norman (1981), Helpman and Krugman (1985), and Davis (1995). Dixit and Norman (1981) and Helpman and Krugman (1985) emphasized the role of imperfect competition and increasing returns as driving forces for intra-industry trade. On the other hand, Davis (1995) developed a model with perfect competition and constant returns and showed that intra-industry trade arises due to comparative advantage. Their work investigated only horizontal intra-industry trade and reached the same proposition: the share of intra-industry trade in total trade is maximized when countries have identical factor endowment ratios. I modify a framework developed by Davis (1995) to focus on vertical intra-industry trade. I show that the share of vertical intra-industry trade has a single peak in a graph showing its dependence on factor endowment ratios. However, unlike the case of horizontal intra-industry trade, the share does not reach the peak at a point in which countries have identical factor endowment ratios. I examine a cause for this divergence and show that a difference in the factor intensities of intra-industry goods plays a crucial role in deriving this result.

The rest of this paper is organized as follows. In Section 2, I develop a factor endowment model with a technological difference. Using an integrated equilibrium approach, I illustrate a factor price equalization set. In Section 3, I examine the pattern of trade and show that vertical intra-industry trade arises in trade equilibrium. In Section 4, I investigate the value of trade by...
illustrating the level curves of trade values. I also examine a relation between the share of intra-industry trade and factor endowment ratios. In Section 5, I close this paper with concluding remarks.

II. The Model

I develop a model with two countries, two factors and three goods. There are two final goods $X$ and $Y$ and one intermediate good $Z$. The final good $X$ uses two production factors, capital $K$ and labor $L$, and one intermediate good $Z$ in its production. The production of the final good $Y$ uses only capital and labor. The intermediate good $Z$ also requires capital and labor in its production. I assume that the final good $X$ and the intermediate good $Z$ are classified in the same industry. Therefore, a direct exchange of $X$ and $Z$ is vertical intra-industry trade.

Two countries, Home and Foreign, share identical technologies in the production of $X$ and $Y$. I assume there exists a Hicks-neutral technical difference in the production of $Z$ between countries.\(^4\) In Home, $X$, $Y$, and $Z$ are produced with

$$X = F_X(K_X, L_X, Z), \quad (1)$$
$$Y = F_Y(K_Y, L_Y), \quad (2)$$
$$Z = AF_Z(K_Z, L_Z). \quad (3)$$

where $F_i(i = X, Y, Z)$ is a constant returns to scale production function. In Foreign, production functions are as follows:

$$X^* = F_X(K^*_X, L^*_X, Z^*), \quad (4)$$
$$Y^* = F_Y(K^*_Y, L^*_Y), \quad (5)$$
$$Z^* = F_Z(K^*_Z, L^*_Z). \quad (6)$$

I assume that $A > 1$ holds. This implies that Home has a technical advantage in the production of the intermediate good $Z$.

1. Integrated Equilibrium

Let us consider the equilibrium of an integrated world economy. In an integrated world, production factors as well as goods can move freely between countries. The allocation of integrated equilibrium can be reproduced through trade in goods if factor endowment distribution belongs to a factor price equalization set. The goal of this section is to characterize the factor price equalization set of endowment distributions.\(^5\)

I assume that every market is perfectly competitive. In an integrated world, factor prices

\(^4\) Davis (1995) introduced a Hicks-neutral technical difference to the Heckscher-Ohlin framework and showed that “increasing returns are not necessary for intra-industry trade.” He assumed that there exists a technical difference in the production of a final good so that he could analyze the horizontal intra-industry trade in final goods.

\(^5\) This approach is also used by Dixit and Norman (1980), Helpman and Krugman (1985), and Davis (1995).
are equalized between countries and thus the intermediate good Z is produced only at Home. Let the final good Y be the numeraire. The competitive conditions for the goods markets are

\[ 1 = c_Y(w, r), \]  
\[ p_X = c_X(w, r, p_Z), \]  
\[ p_Z = c_Z(w, r), \]

where \( p_i(i = X, Y, Z) \) is a price of a good \( i \) and \( c_i(i = X, Y, Z) \) is a unit cost function of a good \( i \). The unit cost of \( X \), \( c_X \), depends on the intermediate good price \( p_Z \), wages \( w \), and rental rates \( r \).

The production factors are fully employed in the integrated factor markets. Let \( XX \), \( YY \) and \( ZZ \) denote the outputs of \( X \), \( Y \) and \( Z \) in the integrated equilibrium. The resource constraints are

\[ a_{LX}(w, r, p_Z)X + a_{LY}(w, r)Y + a_{LZ}(w, r)Z = L, \]  
\[ a_{KX}(w, r, p_Z)X + a_{KY}(w, r)Y + a_{KZ}(w, r)Z = K, \]

where \( a_{Lj} = \partial c_j / \partial w \) and \( a_{Kj} = \partial c_j / \partial r \). The vector \((L, K)\) denotes fixed amounts of labor and capital in an integrated world economy. I assume that the factor intensity ranking is as follows.\(^6\)

\[ \frac{a_{KZ}}{a_{LZ}} > \frac{a_{KX}}{a_{LX}} > \frac{a_{KY}}{a_{LY}} \]  

The intermediate good \( Z \) is the most capital intensive and the final good \( Y \) is the most labor intensive.

Preferences are assumed to be homothetic and identical. Let \( \alpha(p_X) \) be a share of spending on the final good \( X \). The equilibrium conditions for the goods markets are

\[ \alpha_X = \frac{p_X X}{p_X X + Y}, \]  
\[ \alpha_Z(w, r, p_Z)X = Z, \]

where \( \alpha_Z = \partial c_Z / \partial p_Z \). Let \( V = (K, L) \) and \( V^* = (K^*, L^*) \) be factor endowments of Home and Foreign respectively. Now we are ready to construct the factor price equalization set of endowment distributions \((V, V^*)\). Let \( E(i) (i = X, Y, Z) \) denote an employment vector of a sector \( i \) in the integrated equilibrium.

\[ E(X) = (a_{LX}X, a_{KX}X) \]  
\[ E(Y) = (a_{LY}Y, a_{KY}Y) \]  
\[ E(Z) = (a_{LZ}Z, a_{KZ}Z) \]

The factor price equalization set \( FTP \) is characterized as follows:

\(^6\) In the following section, I shall discuss the role of this assumption in deriving results.
In Fig. 1, the FPE set is drawn as a parallelogram \(O'OQ'O'\). Recall that the intermediate good \(Z\) is produced only at Home due to its technical advantage. This implies that the employment vector \(E(Z)\) is drawn as a ray from Home’s origin \(O\), and as a result, Home’s origin for the employment vectors \(E(X)\) and \(E(Y)\) is \(O'\). If the distribution of factor endowments belongs to the FPE set, the allocation of the integrated equilibrium is reproduced through international trade in goods. In the next section, I shall examine the exact relationship between trade patterns and factor endowments.

III. The Pattern of Trade

Suppose that the distribution of factor endowments is \(E_1\) in Fig. 2. The intersection of the diagonal \(OO'\) and the isoincome line \(BB'\) determines the relative GDP level of Home: \(\frac{GDP}{GDP^*} = \frac{OC}{O'C}\). This relative GDP level also equals \(\frac{OC}{O'C}\) because \(CC'\) is parallel to \(OO'\). Since preferences are homothetic and identical, the relative consumption level of Home is defined as 

\[
FPE = \left\{(V, V^*): \exists (\lambda_i, \lambda_i^*) \geq 0, \lambda_i + \lambda_i^* = 1 \text{ for } i = X, Y, \lambda_Z = 0, \sum_{i = X, Y, Z} \lambda_i E(i) = V, \sum_{i = X, Y, Z} \lambda_i^* E(i) = V^* \right\}
\]

(18)

The configuration of the FPE set in Fig. 1 is similar to that obtained in the case of oligopoly (Helpman and Krugman (1985, Ch.5)). Suppose that the \(Z\) sector is oligopolistic with a given number of firms in each country and countries have the same technology for the production of \(Z\). Then each country’s fraction of the world output of \(Z\) equals the share of each country in the total number of firms. Thus, the FPE set in Fig. 1 is equivalent to that obtained in the case in which the share of the home country in the total number of firms equals one, and thus, \(Z\) is produced exclusively by home firms.

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is proportional to its relative GDP level. The home consumption of $X(Y)$ is therefore described by $OCX(OCY)$. The pattern of production is also derived easily. Home specializes in the production of $X$ and $Z$, the outputs of which are given by $OCE_1$ and $OCO$ respectively. Now we can derive the pattern of trade: Home exports $Z$ and $X$, and imports $Y$ when the endowment allocation is $E_1$. Home exports the capital intensive goods $X$ and $Z$. This is consistent with the prediction of the Heckscher-Ohlin model. Observe that there is no vertical intra-industry trade.\footnote{It is also easy to obtain the net factor content of trade. The factor content of consumption at Home is described as $OC$. Therefore, $E_1C$ is the net factor content of trade. The capital abundant home country is a net exporter of capital services and a net importer of labor services. Note that a country never becomes a net importer or exporter of all factors' services. The reason is that each country's share of world spending equals its share of world factor income (i.e. trade is balanced).}

Let us move the endowment allocation to the southeast along the isoincome line $BB'$. The pattern of consumption does not change because the relative GDP level of each country remains constant along the isoincome line $BB'$. The pattern of trade is affected only through a change in the pattern of production. Home is still the exporter of the capital intensive goods $X$ and $Z$ until the endowment allocation reaches $E_2$. When the endowment allocation reaches $E_2$, Home produces as much $X$ as it consumes. That is, the production of $X$ is lowered to the level of self-sufficiency. The capital abundant home country exports the capital intensive good $Z$ for the imports of the labor intensive good $Y$.

If the distribution of factor endowments is in the interior of $E_2E_3$, Home exports $Z$ for the imports of the both $X$ and $Y$. Now we observe vertical intra-industry trade. Foreign imports the intermediate good $Z$ and combines it with production factors to make the final good $X$. Some outputs of the final good $X$ produced at Foreign are exported to Home. Note that the capital abundant home country now imports the capital intensive final good $X$. This seems to be paradoxical from the viewpoint of the Heckscher-Ohlin model, but it is really not. Home
earns income from the exports of the most capital intensive good $Z$ and spends its income on importing $X$ and $Y$, which are relatively more labor intensive than $Z$. If the endowment allocation reaches $E_3$, the home consumption of $Y$ equals its own production so that there is no trade in $Y$. The capital abundant home country now exports $Z$ and imports $X$. Trade consists purely of vertical intra-industry trade.

Finally, if the endowment allocation is in the interior of $E_3E_4$ or coincides with $E_4$, Home exports $Y$ and $Z$, and imports $X$. This trade pattern also seems to be surprising because, at any endowment point between $E_3$ and $E_4$, the capital abundant home country exports the most labor intensive good $Y$. However, this can be interpreted as follows. Suppose we subtract the employment vector of the intermediate good from the factor endowments of Home. Then, Home is regarded as a relatively more labor abundant country in terms of net factor endowments. In terms of net factor endowments, this pattern of trade is consistent with the Heckscher-Ohlin theorem.

IV. The Value of Trade

In this section, I examine the effects of factor endowment ratios on the value of trade. Trade is composed of intra-industry trade and inter-industry trade. The value of each trade component as well as the total value of trade is analyzed by illustrating level curves in the factor price equalization set. I also investigate the effects of factor endowment ratios on the share of intra-industry trade. Unlike the proposition derived in the existing work, the share does not reach a peak when countries have identical factor endowment ratios. I examine why there is a difference in results between this paper and the existing work.

1. The Value of Intra-Industry Trade

The previous section shows that trade patterns depend on the distribution of factor endowments. According to these trade patterns, we can divide the FPE set into three regions by the two dividing lines, $AO^*$ and $A'O^*$ (see Fig. 3). The dividing line $AO^*$ is the locus of self-sufficiency of $X$ and the other line $A'O^*$ is the locus of the self-sufficiency of $Y$. It is convenient to label these subsets as follows: $AQO^*: S_1$, $O'AO^*A': S_2$, and $A'O'O': S_3$. The value of intra-industry trade $VT_{intra}$ is defined here as a value of direct exchange of $X$ and $Z$. If the distribution of factor endowments is in $S_1$, there is no intra-industry trade. Therefore, $VT_{intra}$ equals zero in $S_1$.

In $S_2$, Home exports $Z$ for the imports of $X$ and $Y$. The value of intra-industry trade is

$$VT_{intra} = p_X(sX - X)$$

where $s = GDP/(GDP + GDP^*)$ is the relative size of Home as measured by $GDP$ and

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9 Melvin (1989) also showed the similar kind of “surprising” trade pattern in his article on trade in producer services. In this article, he showed that a capital abundant country imports a capital intensive good for the export of capital services. He argued that the Heckscher-Ohlin theorem holds in this case because capital services are more capital intensive than any other goods.

10 These two dividing lines are linear because $E_3E_4$ falls (rises) proportionally with the relative size of Home (Foreign).
The Value of Intra-industry Trade

It is useful for the later analyses to construct the level curves of the value of trade. In Fig. 3, the segment $AO^*$ is a locus of self-sufficiency of $X$. If the distribution of factor endowments is on this locus, the value of intra-industry trade always equals zero. This implies that the level curves of the value of intra-industry trade in $S_2$ are parallel to the self-sufficiency line $AO^*$ (see Fig. 3). Observe that the value of vertical intra-industry trade increases as we move the endowment point toward $E_3$ along the isoincome line $BB'$. If the factor endowment point is in $S_3$, Home exports $Y$ and $Z$ for the import of $X$. The value of intra-industry trade in $S_3$ is therefore represented as follows:

$$VT_{intra} = p_Z \left( Z - \frac{X}{\overline{X}} \right)$$

The ratio of $X$ to $\overline{X}$ keeps constant if the endowment point moves parallel to $O'Q'$. Therefore, the level curves of the value of intra-industry trade are parallel to $O'Q'$ (see Fig. 3). The value of intra-industry trade becomes larger, as we move the factor endowment point along the isoincome line $BB'$ toward $E_4$.\footnote{This is because $VT_{intra}$ decreases with $X/\overline{X}$ that declines as Home becomes relatively more labor abundant.}

In short, as we move the distribution of factor endowments along the isoincome line from $E_1$ to $E_4$, $VT_{intra}$ equals zero in $S_1$, and increases in $S_2$ and $S_3$. If the relative size of each country is constant and the endowment point is in $S_2$ and $S_3$, the value of intra-industry trade decreases with the capital labor ratio of Home.

2. The Value of Inter-Industry Trade

Let us turn to inter-industry trade. The value of inter-industry trade is represented here as the value of the export of $Y$. Above the self-sufficiency line of $Y$, $A'O^*$, Foreign exports $Y$, ...
but below $AO^*$, Home is the exporter of $Y$. Therefore, the value of inter-industry trade is derived as follows.

$$S_1 \text{ and } S_2: \quad VT_{\text{inter}} = Y^* - s^* Y$$

$$S_3: \quad VT_{\text{inter}} = Y - s Y$$

On the self-sufficiency line of $Y$, $A'O^*$, $VT_{\text{inter}}$ is constant (equals zero). Thus, the level curves of the value of inter-industry trade are parallel to $A'O^*$ (see Fig. 4). Observe that the value of inter-industry trade increases as we move the factor endowment point further away from the self-sufficiency line of $Y$, $A'O^*$.

3. The Total Value of Trade

The total value of trade equals the export value of either country. In $S_1$, Home exports $X$ and $Z$ for the import of $Y$, so the total value of trade is derived as follows.

$$VT = p_X(X - sX) + p_Z \left( Z - \frac{X}{X} Z \right)$$

Notice that $VT$ increases with $X$ as long as the relative size of Home, $s$, is constant. In other words, the total value of trade decreases as Foreign produces more of $X$. In $S_1$ there is no intra-industry trade and thus the total value of trade equals the value of inter-industry trade. This implies that the level curves of $VT$ are identical to those of inter-industry trade (see Fig. 5).

In $S_2$, Home exports $Z$ for the imports of $X$ and $Y$. The total value of trade is as follows.

12 We can rewrite $VT$ as follows, $VT = \frac{X}{X} (p_X X - p_Z Z) - p_X X + p_Z Z$. This implies that $VT$ increases with $X$. 

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**FIG. 4. THE VALUE OF INTER-INDUSTRY TRADE**

![Diagram showing the value of inter-industry trade]
In contrast to the case of $S_1$, $VT$ decreases with $X$ in $S_2$. The intermediate good $Z$ is produced only at Home and thus the export of $Z$ increases with the production of $X$ at Foreign. In $S_2$, the level curves are parallel to $O'Q'$ because $X/\bar{X}$ is constant on $O'Q'$. In $S_3$, Home imports $X$ and exports $Y$ and $Z$. The total value of trade has the following representation.

$$VT = p_Z \left( \frac{Z}{\bar{X}} \frac{X}{\bar{X}} Z \right)$$

$$= p_X (s\bar{X} - X).$$

The result is summarized as follows. Suppose the size of each country does not change. Then, the total value of trade reaches the minimum level on the self-sufficiency line of $X$, $AO^*$, and rises as the endowment point moves further away from the self-sufficiency line $AO^*$.

4. The Share of Intra-Industry Trade

Now let us examine the share of intra-industry trade. The share is represented as the ratio $\frac{VT}{VT}$. Phillips (1991, Ch.4) showed that the similar kind of result can be obtained in a model with increasing returns-to-scale in homogeneous goods.
of the value of intra-industry trade to the total value of trade.

\[
SI = \frac{VT_{\text{intra}}}{VT}
\]

Since the sum of the value of intra-industry and inter-industry trade equals the total value of trade, \( VT = VT_{\text{intra}} + VT_{\text{inter}} \), we can rewrite \( SI \) as follows.

\[
SI = 1 - \frac{VT_{\text{inter}}}{VT}
\]

\( SI \) is maximized when the share of inter-industry trade is minimized. Fig. 4 shows that the value of inter-industry trade reaches the minimum level, zero, on the self-sufficiency line of \( Y, A'O^* \). Thus, on the self-sufficiency line \( A'O^* \), \( SI \) reaches the value of one and thus it is maximized. This result suggests that the share of vertical intra-industry trade has a single peak, but it does not reach the peak on the diagonal of the box diagram \( OO^* \) (see Fig. 3). Unlike the proposition obtained in the exiting work, the share of intra-industry trade does not reach 100 percent when countries have identical factor endowment ratios. In the rest of this section, I shall examine why we obtain this result.

In Fig. 3, the share of intra-industry trade is maximized on \( A'O^* \). For instance, if the endowment point is given by \( E_3 \), the home output of \( Y \) equals the self-sufficiency level, and thus, trade is purely vertical. Home exports the intermediate good \( Z \) for the import of the final good \( X \). Since the endowment point \( E_3 \) is not on the diagonal \( OO^* \), the share of intra-industry trade is not maximized when countries have identical endowment ratios. A crucial reason for this result is a difference in the factor intensity between \( X \) and \( Z \). If \( X \) and \( Z \) have identical factor intensities, \( OO' \) has the same slope as \( O'Q \). Then, the self-sufficiency line \( A'O^* \) coincides with the diagonal \( OO^* \). This implies that the share of intra-industry trade is maximized on the diagonal \( OO^* \).

Davis (1995) focused on horizontal intra-industry trade, the exchange of different final goods having the same factor intensity. He showed that the share of intra-industry trade is maximized when countries have identical factor endowment ratios. We can reach the same proposition as derived in Davis (1995) if the intermediate good \( Z \) has the same factor intensity as the final good \( X \). Nonetheless, if there is a difference in factor intensity between them, the proposition fails to hold.\(^1\)

Therefore, whether the proposition holds or not crucially depends on a factor intensity difference between goods that are classified in the same industry. Factor intensity differences between intermediate goods and final goods that use the intermediate inputs could be much larger as compared to those between final goods that are closely substituted with each other. It is more likely that the proposition fails to hold if the vertical aspect of intra-industry trade is taken into account.

\(^1\) The assumption on the factor intensity ranking (12) does not matter for this result. For example, suppose that good \( Y \) is relatively more capital intensive than other two goods \( X \) and \( Z \). Then, the share of intra-industry trade is not maximized on the diagonal as long as \( X \) and \( Z \) have different factor intensities.
V. Concluding Remarks

In this paper, I develop a simple model that is used to investigate the nature of vertical intra-industry trade. This paper departs from the existing work in that it has focused on vertical intra-industry trade although most existing work investigated only horizontal intra-industry trade. I modify a framework developed by Davis (1995) that assumes constant returns and perfect competition. As a result, a mechanism that causes intra-industry trade is a principle of comparative advantage. Unlike the result derived by Davis (1995), the share of intra-industry trade does not reach a peak at a point in which countries have identical factor endowment ratios. The reason for this result is in an assumption that an intermediate good and a final good that are in the same industry do not have the same factor intensity. This suggests that whether goods that are classified in the same industry have identical factor intensities or not plays an important role in explaining the behavior of the share of intra-industry trade.

This paper does not make any empirical investigation of vertical intra-industry trade. Using input-output tables, Hummels et al. (1998) empirically investigated the volume of vertical trade in intermediate goods and final goods. However, they did not examined the nature of vertical intra-industry trade. Input-output tables might be useful to do some empirical work on vertical intra-industry trade. For example, it seems to be an easy exercise to calculate the indices of intra-industry trade. Furthermore, it is an interesting attempt to compare them with the indices of horizontal intra-industry trade and explore a systematic difference between vertical and horizontal intra-industry trade.

References


