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STRATEGIC ASPECTS OF INTERNATIONAL LENDING AND BORROWING: A TWO-COUNTRY DYNAMIC GAME MODEL*

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Abstract

This paper is a consideration of strategic aspects of national saving policies in a game theory setting. In pure exchange economy involving two countries, each government chooses a future time path of the national consumption-wealth ratio in order to maximize its citizens' utility. When private time preference rates of two countries are different, the government of the country with the lower time preference rate has an incentive to slow down the national asset accumulation. The government of the higher time preference rate country has an incentive to slow down the national asset reduction. In Cournot-Nash equilibria of this dynamic game, international capital flows are depressed as compared with competitive equilibria, which are Pareto-optimal. It is shown that the governments achieve the Cournot-Nash equilibrium path of consumption-wealth ratio using a set of taxes and subsidies. A large country intervenes into the market more than a small one. In this sense, a large country exploits a small one. Dynamics of world interest rates are also analyzed.

Key words: Dynamic Game, International Lending, Intertemporal Terms of Trade
JEL Classification: D91, F34

I. *Introduction*

As global integration of national financial markets advances, international aspects of saving and investment policies become more important. To solve the current account deficit problem, the United States is advised to reform its saving restraining tax system. The new Japanese consumption tax is sometimes criticized for its encouraging effect on savings and current accounts. In integrated and interdependent financial markets, there is no assurance that each country's self-seeking savings policies attain globally Pareto-optimal allocation.

International aspects of saving and investment policies, despite their importance, have not been analyzed within a convincing theoretical framework.[1] As the above two examples show,

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[1] Hamada (1965) studies this problem within a framework of a two country growth model with perfect capital mobility. Only a steady state is analyzed. He shows that the Cournot-Nash equilibrium of the savings policy game

the ordinary arguments seems to be based partly on the dubious presumption that current account imbalances are undesirable and all countries had better coordinate their policies to reduce current account imbalances. This presumption seems to contradict the common sense of microeconomics and policy game theory. In the international trade theory, it is well-known that each country's self-seeking trade policies result in shrinking international trade. Because international capital flows are exchanges of present goods for future goods, it seems that capital flows are undesirably reduced in non-cooperative equilibria. If the world economy tends to fall into such non-cooperative equilibria, then the cooperative equilibrium will be achieved by amplifying current account imbalances.

In this paper this point is illustrated within the framework of a two-country dynamic game model with infinite time horizon consumers. Each government decides on a time path of a national consumption-wealth ratio as a strategic variable in order to maximize its citizens' welfare. The present study is an application of strategic analysis of international interdependence.[2] In contrast to usual studies of policy games on current account balances,[3] this model has a microeconomic foundation of the policy target.

It will be shown that uncoordinated actions by national governments are most likely to result in shrinking international capital flows. Countries with low savings and current account deficits have an incentive to tax private consumption or subsidize private saving in order to restrain international borrowing. By restraining borrowing, they can bring down world interest rates and improve their intertemporal terms of trade. Countries with high savings and current account surpluses have an incentive to introduce capital income tax or subsidize private consumption in order to restrain saving and consequently cut down capital outflows and bring up world interest rates.

Because our main concern is with the transition process of international capital flows, the dynamics of the model are analyzed not by taking a linear approximation around the steady state but by solving nonlinear dynamic equations globally. For this purpose, a drastically simple model is used. First, production is entirely exogenous; the output of consumption goods is constant. There is one consumption good and one production factor. Although the production factor is immovable, the consumption goods and ownership claim to the production factor is traded internationally. Second, international capital flows take place because of the difference between two countries' time preference rates, each of which is assumed to be constant. In this case, the country with the higher time preference rate uses up assets while the country with the lower rate accumulates assets continuously in competitive equilibria.[4]

in a steady state is Pareto-optimal. A similar model is studied by Ohyama (1989). As in Hamada (1965), only a steady state is analyzed. In spite of several microeconomic refinements, Ohyama (1989) gets fundamentally the same results as Hamada (1965). It is nevertheless premature to conclude that there is no need for international policy coordination of national savings policies. We do not know about process of transition to the steady state. In the transition process, national governments may have an incentive to exploit their oligopolistic power on world interest rates. The present paper focuses on this problem.

[2] See Cooper (1985), Hamada (1986), Buiter and Marston (1986), and Turnovsky et al. (1988).

[3] For example, in Oudiz and Sachs (1984) the current account goal for the United States is taken to be zero and for Germany and Japan to be 2 percent of GNP. They do not present any basis for this assumption.

[4] This phenomenon has usually been analyzed in a context of a closed economy with heterogeneous consumers. The divergence was originally pointed out by Ramsey (1928) and rigorously analyzed by Becker (1980). Becker (1980) did not explicitly solve the dynamics of asset holding and interest rates. If time preference rated depend on the instantaneous utility level (see Koopmans et al., 1965, and Uzawa, 1968), then heterogeneous consumers coexist in the long-run. For details, see Fukao and Hamada (1989). Constant time preference rates are assumed here because of the difficulty of analyzing dynamic games with variable time preference rates.

Thanks to these simplifications, the dynamic system is solved globally and explicit time paths for each countries' savings, world interest rates, and other variables are derived. Such paths in Cournot-Nash equilibria will be compared with those in cooperative equilibria.

The remaining sections of the paper are organized as follows. The next section presents the model and studies dynamics of competitive equilibria with no government intervention. In section three, the dynamic game between two countries is studied with the assumption that each government decided on a time path of a national consumption-wealth ratio as a strategic variable in order to maximize its citizens' welfare. It will be shown that competitive equilibria are Pareto-optimal and that Cournot-Nash equilibrium are suboptimal. In Cournot-Nash equilibria, the amount of international lending is smaller than the socially optimal level.

In market economies, governments can not directly control the national consumption-wealth ratio. In section four, it is shown that each government can accomplish the Cournot-Nash equilibrium time path of consumption-wealth ratio by using a natural set of taxes and subsidies. In the final section, the implications of this analysis and possible extensions of it are summarized.

II. *The Model*

Though the main concern of this paper is with dynamic games of national saving policies, it is convenient to begin with an analysis of competitive equilibria with no government intervention.

Consider a world with two countries, one consumption good, and one production factor. We assume that production is entirely "exogenous"; there is no possibility of affecting the output of consumption goods and the existing amount of the production factor. One unit of the consumption goods is produced from one unit of the production factor each period. There is one unit of the production factor in the world. $\overline{K}$ unit is located in the home country. $1-\overline{K}$ is located in the foreign country. The production factor is immovable between the countries.

Ownership to this production factor is determined in a competitive stock market. One equity share corresponds to one unit of the production factor and earns each period one unit of the consumption goods as a dividend. The equity shares and the consumption goods are traded internationally without cost. Let q_t denote the price of the equity shares measured by the consumption goods at time t. We assume that consumers anticipate the future path of q_t exactly.

Let r_t denotes the real rate of return of the equity shares measured by the consumption goods. The rate of return consists of dividends and capital gains;

$$r_t = \frac{1}{q_t} + \frac{\dot{q}_t}{q_t}.$$

If there is a loan market, arbitrage will equalizes the instantaneous real interest rate with r_t. Therefore we regard r_t as the instantaneous interest rate.

The population of each country stays constant. Let A_t denote the number of equity shares owned by citizens in the home country at time t. A_t also denotes the income of the home country, because one equity share earns one unit of consumption goods as a dividend. The citizens in the home country are endowed with $\bar{A}_0$ shares of the equity at time 0.

The representative individual in the home country has an infinite time horizon and solves an optimization problem:

$$\max_{\{C_t\}} \int_0^\infty e^{-\beta t}\, ln\, C_t\, dt,$$

subject to

$$\dot{A}_t = \frac{1}{q_t}(A_t - C_t), \tag{1}$$

$$A_0 = \bar{A}_0, \tag{2}$$

$$C_t \geq 0,$$

and

$$A_t \geq 0,$$

where β denotes the time preference rate of the representative individual in the home country. We assume β is positive. C_t denotes home country's real consumption. A_t denotes both the home country's wealth and the dividend income.

Because our main concern is in the transition process, the dynamics of the model are analyzed not by taking a linear approximation around the steady state but by solving nonlinear dynamic equations globally. To get a manageable dynamics, we need to assume a log linear instantaneous utility function.[5]

The necessary and sufficient conditions[6] for the optimal behavior are (1), (2), non-negativity conditions of C_t, A_t,

$$\dot{C}_t = \left(\frac{1}{q_t} + \frac{\dot{q}_t}{q_t} - \beta\right) C_t, \tag{3}$$

$$\lim_{t\to\infty} \frac{q_t}{C_t} e^{-\beta t} \geq 0, \tag{4}$$

and

$$\lim_{t\to\infty} \frac{q_t A_t}{C_t} e^{-\beta t} = 0, \tag{5}$$

where (4) and (5) are transversality conditions. The Euler equation (3) implies that optimal

[5] For example, if we assume a Paretian instantaneous utility function:

$$U_t = \frac{C_t^\gamma}{\gamma} \quad \text{for} \quad 0 < \gamma < 1,$$

then, we get the following Euler equation instead of equation (3).

$$\dot{C}_t = \frac{1}{1-\gamma}\left(\frac{1}{q_t} + \frac{\dot{q}_t}{q_t} - \beta\right) C_t,$$

In this case, C_t depends not only on A_t but also on the whole future time path of the interest rate. And we do not obtain a simple relationship between wealth and consumption like equation (6).

[6] See Arrow-Kurz (1970) page 49.

consumption fluctuates according to the difference between the real interest rate and the time preference rate. To understand the equation (3) intuitively, we transform it into

$$\hat{C}_t + \beta = r_t, \tag{3'}$$

where the circumflex accent denotes the growth rate of the variable. Differentiating instantaneous utility regarding time, we can ascertain that the left-hand side of the above equation denotes the marginal rate of substitution between the consumption at time t and the consumption at time $t+dt$. On the right-hand side, r_t denotes the relative price of consumption goods at time t and time $t+dt$. For competitive individuals, the optimal behavior is to equalize their marginal rate of substitution with the market price r_t. As we shall see later, for national governments the optimal condition is different from (3′) because of their oligopolistic power.

From the budget constraint (1′), the Euler Equation (3), and the transversality condition (5), we obtain a consumption function, which relates domestic consumption and the wealth:

$$C_t = \beta q_t A_t, \tag{6}$$

This consumption function also satisfies the other conditions of optimality. Individuals keep their consumption-wealth ratio equal to their time preference rate. Substituting the optimal consumption (6) into the budget constraint (1) yields a dynamic equation of the number of equity shares that are owned by the home country:

$$\hat{A}_t = \frac{1}{q_t} - \beta, \tag{7}$$

Foreign households solve a similar problem:

$$\max_{\{C_t^*\}} \int_0^\infty e^{-\beta^* t} \, ln \, C_t^* dt,$$

subject to

$$\dot{A}_t^* = \frac{1}{q_t}(A_t^* - C_t^*), \tag{1*}$$

$$A_0^* = 1 - \bar{A}_0, \tag{2*}$$

$$C_t^* \geq 0,$$

and

$$A_t^* \geq 0,$$

where C_t^* denotes real consumption of the foreign country. A_t^* denotes both the foreign country's wealth and the dividend income. β^* is the time preference rate of the foreign representative citizen. β^* can be different from β.

In the same way as the home country case, a fundamental relationship between foreign consumption and wealth can be written as

$$C_t^* = \beta^* q_t A_t^*. \tag{6*}$$

The dynamic equation of the foreign country's equity shares is derived from (1*) and (6*):

$$\hat{A}_t^* = \frac{1}{q_t} - \beta^*. \tag{7*}$$

We consider here the competitive equilibria of our model. There are two markets, a consumption goods market and a stock market. The equilibrium condition of the consumption goods market is

$$C_t + C_t^* = 1. \tag{8}$$

The equilibrium condition of the stock market is

$$A_t + A_t^* = 1. \tag{9}$$

Because of Walras's law, one of the two equations is redundant.

Now, we solve the differential equations. Owing to the simplification of the production and the assumption of log linear utility functions, global and explicit solution can be derived.

First, we derive the solution of each country's wealth. Subtracting (7*) from (7) and integrating the result from 0 to t yield

$$\frac{A_t}{A_t^*} = \frac{A_0}{A_0^*} e^{(\beta^* - \beta)t}.$$

From initial conditions (2), (2*), the equilibrium condition of the stock market (9), and the above equation, the explicit solution of the home country's equity shares can be derived:

$$A_t = \frac{\overline{A}_0 e^{-\beta t}}{\overline{A}_0 e^{-\beta t} + (1 - \overline{A}_0) e^{-\beta^* t}}. \tag{10}$$

The foreign country has $1 - A_t$ shares.

The dynamics of the equity share holding depend on the time preference rates of the two countries. If the time preference rates are identical, the two countries' asset and consumption levels will stay constant.[7] If the time preference rates are different, divergence will occur. The country with the lower time preference rate accumulates equity shares continuously and finally obtains all the equity shares in the world. As the budget constraints (1) and (1*) indicate, the increase in one country's share holding means that that country saves a positive amount. In the foregoing process, the country with the lower time preference rate saves positive amount. And the country with the higher time preference rate always dissaves.

Let β_t denote the net foreign asset of the home country. The net foreign asset is equal to the difference between the national wealth of the home country and the amount of the production factor that is located in the home country:

$$\begin{aligned} B_t &= A_t - \overline{K} \\ &= \frac{\overline{A}_0 e^{-\beta t}}{\overline{A}_0 e^{-\beta t} + (1 - \overline{A}_0) e^{-\beta^* t}} - \overline{K}. \end{aligned} \tag{11}$$

If the time preference rate of the home country is smaller than that of the foreign country, the

[7] If tax rates or fiscal expenditures are changed over time, then the private consumption level fluctuates even in this case. See Frenkel and Razin (1987).

net foreign asset of the home country increases continuously, which means that the home country continues to have a current account surplus.

The equilibrium equity share price can be derived from the two countries' consumption functions (6), (6*), and the equilibrium condition of the consumption goods market (8):

$$q_t = \frac{1}{A_t\beta + (1-A_t)\beta^*} = \frac{\bar{A}_0 e^{-\beta t} + (1-\bar{A}_0)e^{-\beta^* t}}{\bar{A}_0 \beta e^{-\beta t} + (1-\bar{A}_0)\beta^* e^{-\beta^* t}}. \tag{12}$$

The equilibrium equity share price is equal to the weighed harmonic average of the reciprocal of the two countries' time preference rates. The weights are the two countries' wealth shares in the world.

The equilibrium interest rate can be derived from (12) and the definition of the interest rate:

$$r_t = \frac{\bar{A}_0 \beta^2 e^{-\beta t} + (1-\bar{A}_0)\beta^{*2} e^{-\beta^* t}}{\bar{A}_0 \beta e^{-\beta t} + (1-\bar{A}_0)\beta^* e^{-\beta^* t}}. \tag{13}$$

The equilibrium interest rate is equal to the weighted average of the time preference rates of the two countries. The weights are the two countries' consumption shares in the world.

The solutions of the two countries' consumption expenditure can be derived from (10), (12), and the consumption functions (6) and (6*):

$$C_t = \frac{A_t\beta}{A_t\beta + (1-A_t)\beta^*} = \frac{\bar{A}_0 \beta e^{-\beta t}}{\bar{A}_0 \beta e^{-\beta t} + (1-\bar{A}_0)\beta^* e^{-\beta^* t}}, \tag{14}$$

$$C_t^* = \frac{(1-\bar{A}_0)\beta^* e^{-\beta^* t}}{\bar{A}_0 \beta e^{-\beta t} + (1-\bar{A}_0)\beta^* e^{-\beta^* t}}. \tag{14*}$$

The country with the lower preference rate consumes a relatively small amount at first. As it accumulates the wealth, it comes to consume more.

III. *The Policy Game*

In this section, the dynamic game between the two countries is analyzed. Let x_t and x_t^* denote consumption-wealth ratios of the home country and the foreign country respectively:

$$x_t \equiv \frac{C_t}{q_t A_t},$$

$$x_t^* \equiv \frac{C_t^*}{q_t A_t^*}.$$

We assume that at an initial date each government chooses an all-future time path of the national consumption-wealth ratio as a strategic variable in order to maximize the utility of its representative citizen.[8] In section four, it will be shown that each government can control its national consumption-wealth ratio using a set of taxes and subsidies.

If the governments choose a time path of some other ratio as a strategic variable (for example, a saving-wealth ratio or some tax rate), then the character of Cournot-Nash equilibria will differ from the following result. This arbitrariness is well known as Bertrand's Critique. Notice that some seemingly plausible variables are not appropriate for use as a strategic variable in our model. For example, we can not take a time path of consumption as a strategic variable. The world output of consumption goods is constant and equal to one. Therefore the two countries can not decide their consumption level independently. By the same reason, we can not use a path of saving-income ratio as a strategic variable as in Hamada (1965). We shall discuss this problem further in section four.

Under the equilibrium condition of the consumption market (10), the equity share price q_t on the two countries' consumption-wealth ratios x_t, x_t^* in the following way:

$$q_t = \frac{1}{A_t x_t + (1-A_t) x_t^*}. \tag{15}$$

The more the two countries increase their consumption-wealth ratios, the cheaper the equity shares become. The home country's consumption level in competitive equilibrium is derived from the definition of x_t and the above equation:

$$C_t = \frac{A_t x_t}{A_t x_t + (1-A_t) x_t^*}. \tag{16}$$

The budget constraint of the home country under the policy game can be derived from the equation (15), the definition of x_t, and the budget constraint of the home country (1):

$$\dot{A}_t = A_t (1-A_t)(x_t^* - x_t). \tag{17}$$

(17) is a transition equation of the system.

The one unit increase of the consumption-wealth ratio x_t costs a proportional decrease of $\dot{A}_t$ as the equation (17) denotes. By the one unit increase of x_t, how much the home country can expand the consumption level? As the equation (16) indicates, it depends on the two countries' consumption-wealth ratios, x_t and x_t^*. If x_t^* is zero, then the home country will not be able to expand the consumption level by increasing x_t. In this case, the increase of x_t causes only a proportional decrease of the equity share price. The greater the foreign country's consumption-wealth ratio x_t^*, and the smaller the x_t, the less expensive the home country's consumption becomes. As we shall see later, this duopolistic character of the market makes

[8] It is assumed that at the initial date each government must take a binding commitment regarding the policies it will take at all future dates. Another possible approach is to model governments as choosing decision rule strategies instead of path strategies. That is, governments are assumed to observe the values of relevant state variables and respond instantaneously by choosing their current actions. For detail, see Basar and Olsder (1982), Reinganum and Stokey (1985), and Turnovsky et al. (1988). In dynamic game models with decision rule strategies, the players' time horizon is usually assumed to be finite. And non-cooperative equilibria are found by using backward induction. It seems inappropriate to assume finite time horizon in the saving policy problem. For decision rule strategies with infinite time horizon, see Oudiz and Sachs (1985).

each government behave in a strategic way.

The government of the home country chooses a time path of the consumption-wealth ratio x_t in order to maximize its citizens' welfare. We assume that the home country government regards the time path of the foreign consumption-wealth ratio as given and that the foreign country government takes the symmetrical view with respect to the home country government's actions. That is, we analyze non-cooperative open loop solutions of our policy game.

The optimization problem of the home country government is

$$\max_{\{x_t\}} \int_0^\infty e^{-\beta t} \, ln \, \frac{A_t x_t}{A_t x_t + (1-A_t) x_t^*} dt,$$

subject to

$$\dot{A}_t = A_t (1-A_t)(x_t^* - x_t), \tag{17}$$

$$A_0 = \bar{A}_0, \tag{2}$$

$$x_t \geq 0,$$

and

$$A_t \geq 0,$$

The current-value Hamiltonian can be written as

$$\Psi(A_t, x_t, \theta_t, t) \equiv ln \, \frac{A_t x_t}{A_t x_t (1-A_t) x_t^*} + \theta_t \{A_t (1-A_t)(x_t^* - x_t)\},$$

where the costate variable θ_t denotes the current value of the marginal contribution of the state variable A_t to the utility. Because

$$\Psi^0(A_t, \theta_t, t) = \max_{x_t} \Psi(A_t, x_t, \theta_t, t)$$

is a concave function of A_t for given θ_t and t, the necessary and sufficient conditions for the optimal behavior are the equation (17), nonnegativity conditions of x_t and A_t,

$$\frac{\partial \Psi}{\partial x_t} = \frac{(1-A_t) x_t^*}{x_t \{A_t x_t + (1-A_t) x_t^*\}} - \theta_t A_t (1-A_t) = 0, \tag{18}$$

$$\begin{aligned} \dot{\theta}_t &= \beta \theta_t - \frac{\partial \Psi}{\partial A_t} \\ &= \beta \theta_t - \frac{x_t^*}{A_t \{A_t x_t + (1-A_t) x_t^*\}} + \theta_t (2A_t - 1)(x_t^* - x_t), \end{aligned} \tag{19}$$

$$\lim_{t \to \infty} \theta_t e^{-\beta t} \geq 0, \tag{20}$$

and

$$\lim_{t \to \infty} \theta_t A_t e^{-\beta t} = 0. \tag{21}$$

Eliminating θ_t from (18) and (19) and arranging the result with (15) and (17), we obtain a necessary condition of the optimal x_t under a given time path of x_t^*:

$$\hat{x}_t - \hat{x}_t^* = (x_t + x_t^*) + (x_t \hat{+} x_t^*) - \beta + x_t - x_t^*. \tag{22}$$

In order to understand the meaning of the above equation, we rearrange it into an equation of $C_t$and C_t^*. Substituting the definitions of x_t and x_t^*, (15) and (17) into (22) yields

$$\hat{C}_t + \beta = r_t + \hat{C}_t^*. \tag{23}$$

This necessary condition of optimality, (23) corresponds to the equation (3′), which is a necessary condition of the optimal behavior of the home country citizens in the competitive market. On the right-hand side of (23), there is a new term, which denotes the growth rate of the foreign consumption.

An intuitive argument goes as follows. Like (3′), the left hand side of the equation (23) denotes the marginal rate of substitution for the home country between consumption at time t and consumption at time $t+d_t$. The right hand side of the equation (23) denotes the marginal cost of consumption at time t measured by consumption at time $t+d_t$. From the equation (16), we know that if the foreign consumption increases over time, then the marginal cost of the future consumption will become cheaper for the home country relative to the present consumption. Because of this effect, the marginal cost differs from the market price r_t. Notice that this effect makes the home country's consumption move more closely to the foreign country's consumption than in the competitive equilibria.

The conditions of the optimal behavior of the home country government will now be summarized. The necessary and sufficient conditions are the nonnegativity conditions of x_t and A_t, the budget constraint (17), the equation (22), and transversality conditions. Substituting (18′) into (20) and (21) yields the following transversality conditions:

$$\lim_{t\to\infty} \frac{x_t^*}{A_t\,(x_t + x_t^*)\,x_t} e^{-\beta t} \geq 0, \tag{20′}$$

$$\lim_{t\to\infty} \frac{x_t^*}{(x_t + x_t^*)\,x_t} e^{-\beta t} = 0. \tag{21′}$$

The government of the foreign country chooses a time path of the consumption-wealth ratio x_t^* in order to maximize the utility of its representative citizen. The government of the foreign country regards the time path of the home country's consumption-wealth ratio x_t as given.

$$\max_{\{x_t^*\}} \int_0^\infty e^{-\beta^* t}\, ln \frac{(1-A_t)\,x_t^*}{A_t x_t + (1-A_t)\,x_t^*} dt,$$

subject to

$$\dot{A}_t = A_t\,(1-A_t)(x_t^* - x_t), \tag{17}$$

$$A_0 = \bar{A}_0, \tag{2}$$

$$x_t^* \geq 0,$$

and

$$1-A_t \geq 0,$$

Like the home country case, the necessary and sufficient conditions of the optimal behavior of the foreign country are the budget constraint (17), the nonnegativity conditions of x_t^*, $1-A_t$,

$$\hat{x}_t^* - \hat{x}_t = (x_t + x_t^*) + (x_t \hat{+} x_t^*) - \beta^* + x_t^* - x_t, \tag{22*}$$

$$\lim_{t\to\infty} \frac{x_t}{(1-A_t)(x_t+x_t^*)\,x_t^*} e^{-\beta^* t} \geq 0, \tag{20*}$$

and

$$\lim_{t\to\infty} \frac{x_t}{(x_t+x_t^*)\,x_t^*} e^{-\beta^* t} = 0. \tag{21*}$$

As in the home country case, the equation (22*) can be arranged into

$$\hat{C}_t^* + \beta^* = r_t + \hat{C}_t, \tag{23*}$$

Time paths of x_t, x_t^*, and A_t in Cournot-Nash equilibria are determined by the budget constraint (17); the equation (22) and (22*); the transversality conditions (20′), (21′), (20*), and (21*); the non-negativity conditions of x_t, x_t^*, A_t and $1-A_t$; and the initial condition (2). Time paths of C_t, C_t^* and q_t are determined by the equilibrium time paths of x_t, x_t^*, and A_t; equation (15); and the definitions of x_t, and x_t^*.

Without solving the dynamic system, the interest rate and the equity share price in Cournot-Nash equilibria can be derived from (23) and (23*):

$$r_t = \frac{1}{2}(\beta + \beta^*), \tag{24}$$

$$q_t = \frac{2}{\beta + \beta^*}. \tag{25}$$

The interest rate and the equity share price are constant in equilibria. The constant interest rate will be studied further in section four.

Because it is difficult to solve the nonlinear differential equation (17), (22), and (22*) directly, we rearrange the equilibrium conditions. Substituting the definitions of x_t, x_t^* into the preceding conditions of Cournot-Nash equilibria and using (15), the equilibrium conditions of x_t, x_t^* and A_t can be transformed into the following equations of C_t, C_t^* and A_t:

$$\dot{A}_t = \frac{\beta + \beta^*}{2}(A_t - C_t), \tag{26}$$

$$\hat{C}_t - \hat{C}_t^* = \frac{1}{2}(\beta^* - \beta), \tag{27}$$

$$C_t + C_t^* = 1, \tag{8}$$

$$\lim_{t\to\infty} \frac{C_t^*}{(1-A_t)C_t} e^{-\beta t} \geq 0, \tag{20″}$$

$$\lim_{t\to\infty} \frac{A_t C_t^*}{(1-A_t)C_t} e^{-\beta t}=0, \tag{21''}$$

$$\lim_{t\to\infty} \frac{C_t}{A_t C_t^*} e^{-\beta^* t}\geq 0, \tag{20*'}$$

$$\lim_{t\to\infty} \frac{(1-A_t)C_t}{A_t C_t^*} e^{-\beta^* t}=0, \tag{21*'}$$

$$C_t\geq 0,$$

$$C_t^*\geq 0,$$

$$0\leq A_t\leq 1, \tag{28}$$

and

$$A_0=\bar{A}_0. \tag{2}$$

The above conditions are equivalent to the preceding conditions of x_t, x_t^* and A_t. From (27) and (8), we obtain solutions of the two countries' consumption in the dynamic game:

$$C_t=\frac{C_0 e^{-\frac{1}{2}\beta t}}{C_0 e^{-\frac{1}{2}\beta t}+(1-C_0)e^{-\frac{1}{2}\beta^* t}}, \tag{29}$$

$$C_t^*=\frac{(1-C_0)e^{-\frac{1}{2}\beta^* t}}{C_0 e^{-\frac{1}{2}\beta t}+(1-C_0)e^{-\frac{1}{2}\beta^* t}}. \tag{29*}$$

In Cournot-Nash equilibria the country with the lower time preference rate increases its consumption over time and the country with the higher time preference rate decreases its consumption in a way similar to that in equilibria. But the divergence speed of the two countries' consumption are different in the two cases. From (14) and (14*) we know that the gap between the growth rates of the two countries' consumption levels is $\beta-\beta^*$ in competitive equilibria. In the policy game, the gap is $(\beta-\beta^*)/2$. This slowdown of the divergence comes from the second terms on the right-hand side of (23) and (23*). As we have discussed the meaning of the equation (23), each country has an incentive to move its consumption more closely to the other country's consumption in the duopolistic situation.

The initial consumption level of the home country C_0 is determined in the following way. From (2), (26), (27), and (29), we get

$$\bar{A}_0=\frac{\beta+\beta^*}{2}\int_0^\infty \frac{C_0 e^{-\frac{1}{2}\beta s}}{C_0 e^{\frac{1}{2}\beta^* s}+(1-C_0)e^{\frac{1}{2}\beta s}}\,\mathrm{ds}. \tag{30}$$

C_0 is determined by this equation. C_0 is uniquely determined and satisfies $0\leq C_0\leq 1$ under a given $\bar{A}_0$, which satisfies $0\leq\bar{A}_0\leq 1$. C_0 increases as $\bar{A}_0$ increases.

In the same way as (30), we get a relationship between the home country's wealth and its consumption at time t:

$$A_t = \frac{\beta+\beta^*}{2}\int_0^{\infty}\frac{C_t e^{-\frac{1}{2}\beta s}}{C_t e^{\frac{1}{2}\beta^* s}+(1-C_t)e^{\frac{1}{2}\beta s}}\,\mathrm{ds}. \tag{31}$$

Substituting (29) into (31) yields the solution of A_t:

$$A_t = \frac{\beta+\beta^*}{2}\int_0^{\infty}\frac{C_0 e^{-\frac{1}{2}\beta(s+t)}}{C_0 e^{-\frac{1}{2}\beta(s+t)}+(1-C_0)e^{-\frac{1}{2}\beta^*(s+t)}}\,e^{-\frac{1}{2}(\beta+\beta^*)s}\,\mathrm{ds}. \tag{32}$$

If the time preference rates are identical, the two countries' asset and consumption levels will stay constant. And the two countries' consumption shares, C_t and C_t^* are equal to their initial endowment shares, A_0 and A_0^*. That is, if $\beta=\beta^*$, then the resource allocation in the Cournot-Nash equilibrium will be identical with that in the competitive market. As we shall see later, the competitive equilibrium is Pareto-optimal. Therefore, there is no need for policy coordination in this case.

If the time preference rate of the home country β is smaller than that of the foreign country β^*, the home country's equity share A_t will increase as time elapses. As seen later, the speed of increase is slower than that in competitive equilibria.

It is not difficult to confirm that the time paths of C_t and $C_t^*$and A_t, defined by (29), (29*), (30), and (32), satisfy all the conditions of Cournot-Nash equilibria, which we have summarized. Therefore, these time paths are Cournot-Nash equilibrium paths.

The time path of x_t is derived from (29), (31), and the definition of the consumption-wealth ratiox_t:

$$\begin{aligned} x_t = \frac{C_t}{q_t A_t} &= \frac{1}{\displaystyle\int_0^{\infty}\frac{e^{-\frac{1}{2}\beta s}}{C_t e^{\frac{1}{2}\beta^* s}+(1-C_t)e^{\frac{1}{2}\beta s}}\,\mathrm{ds}} \\ &= \frac{1}{\displaystyle\int_0^{\infty}\frac{\left\{C_0 e^{-\frac{1}{2}\beta t}+(1-C_0)e^{-\frac{1}{2}\beta^* t}\right\}e^{-\frac{1}{2}\beta s}}{C_0 e^{-\frac{1}{2}\beta t}e^{\frac{1}{2}\beta^* s}+(1-C_0)e^{-\frac{1}{2}\beta^* t}e^{\frac{1}{2}\beta s}}\,\mathrm{ds}}. \end{aligned} \tag{33}$$

The consumption-wealth ratio of the foreign country can be derived in the same way.

The solution (33) implies the following strategy of the home country government. If the time preference rate of the home country is higher than that of the foreign country, the home country will decreases its consumption-wealth ratio as time elapses. This ratio finally approaches to β as t$\to\infty$. In competitive equilibria, the consumption-wealth ratio of the home country is always β, as the consumption function (6) indicates. In Cournot-Nash equilibria the higher time preference country comes to behave like a price-taker as its economic share decreases.

In contrast, if the time preference rate of the home country is lower than that of the foreign country, then the home country will increase its consumption-wealth ratio as time

FIG. 1.

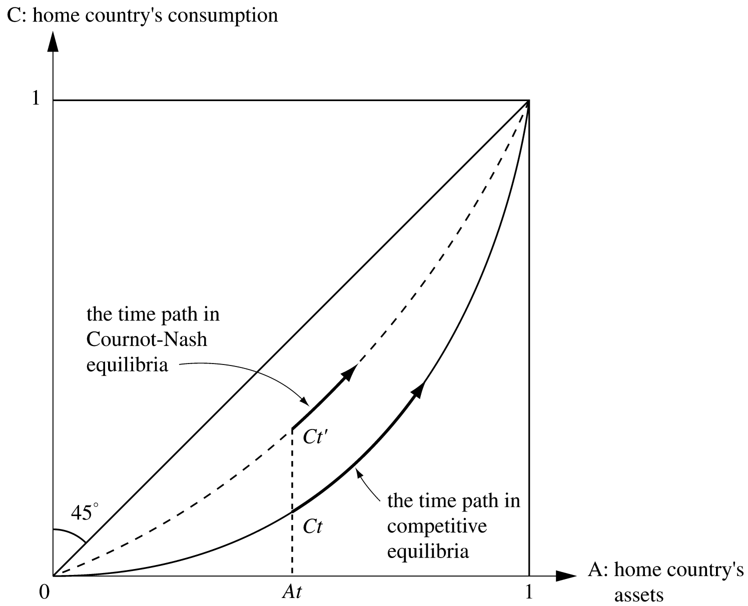

Figure 1. The dynamics of the home country's consumption and assets. (The time preference rate of the home country is assumed to be lower than that of the foreign country)

elapses. This ratio finally approaches to $(\beta+\beta^*)/2$. The lower time preference country does not behave like a price-taker even after it obtains almost all the wealth of the world. It continues to exploit the other country. This result depends on our assumption that production is entirely exogenous. If production is endogenous, then government intervention in private savings will causes a harmful side effect: the intervention will cause the marginal productivity of capital to deviate from the time preference rate. As the lower time preference country increases its wealth and consumption, the benefit from the exploitation of the other country diminishes. But the harmful side effect remains. Therefore the government will reduce its intervention after obtains almost all the wealth of the world.

The relationship between the consumption and the equity share holding in Cournot-Nash equilibria is compared with that in competitive equilibria (Figure 1). The time preference rate of the home country is assumed to be lower than that of the foreign country. The dotted curve is the locus of the home country's consumption and equity shares in Cournot-Nash equilibria. This relationship is defined by the equation (31). The solid curve is the locus of the home country's consumption and equity shares in competitive equilibria. This relationship is defined by the equation (14). In both cases, $1-C_t$ and $1-A_t$ denote the foreign country's consumption and equity shares respectively. The two curves are upward sloping and upward concave.[9] On

[9] In the case of competitive equilibria, this is obvious from the equation (14). In the case of Cournot-Nash equilibria, the upward sloping is obvious from the equation (31). We can confirm the concavity by differentiating the equation (31). Differentiating the equation (31) twice yields

the 45° line, the home country's consumption C_t is equal to its income A_t. The two curves are located below the 45° line. It indicates that the home country, which is assumed to have the lower time preference rate, continuously accumulates equity shares in both equilibria.

The curve of competitive equilibrium is located below the curve of Cournot-Nash equilibria (Figure 1), which can be confirmed in the following way.

Assume one consumption level C_t^0 at time t. Let A_t^N denotes the amount of equity shares that brings the home country to consume C_t^0 in Cournot-Nash equilibria. Let A_t^M denotes the amount of equity shares that brings the home country to consume C_t^0 in competitive equilibria. From (14) and (31), we get

$$A_t^M - A_t^N = A_t^M - \frac{(\beta+\beta^*)}{2}\int_0^\infty \frac{C_t^0 e^{-\frac{1}{2}\beta s}}{C_t^0 e^{\frac{1}{2}\beta^* s} + (1-C_t^0)e^{\frac{1}{2}\beta s}}\,\mathrm{ds}$$

$$= \frac{(\beta+\beta^*)}{2} A_t^M(1-A_t^M)\int_0^\infty \frac{\beta^* e^{-\frac{1}{2}\beta^* s} - \beta e^{-\frac{1}{2}\beta s}}{A_t^M \beta e^{\frac{1}{2}\beta^* s} + (1-A_t^M)\beta^* e^{\frac{1}{2}\beta s}}\,\mathrm{ds},$$

where

$$\int_0^\infty \left(\beta^* e^{-\frac{1}{2}\beta^* s} - \beta e^{-\frac{1}{2}\beta s}\right)\mathrm{ds} = 0,$$

and

$$\frac{1}{A_t^M \beta e^{\frac{1}{2}\beta^* s} + (1-A_t^M)\beta^* e^{\frac{1}{2}\beta s}},$$

is a decreasing function of s. If $\beta < \beta^*$, then

$$\beta^* e^{-\frac{1}{2}\beta^* s} - \beta e^{-\frac{1}{2}\beta s},$$

will also be a decreasing function of s. Therefore, $A_t^M - A_t^N$ is positive under conditions, $\beta < \beta^*$, $A_t^M \neq 1$, and $A_t^M \neq 0$. It is shown that the curve of competitive equilibria is located below the curve of Cournot-Nash equilibria.

At the home country's given equity shares A_t, the home country, which is assumed to have the lower time preference rate, consumes C_t in the competitive equilibrium and consumes C_t' in the Cournot-Nash equilibrium (Figure 1). By consuming more, the home country can bring down the equity share price measured by the consumption goods. The lower equity price means that the lender country can acquire the equity share in exchange for a lesser amount of the consumption goods. On the other hand, the country with the higher time preference rate has an incentive to consume less than what it consumes in the competitive equilibrium.

Because of the promotion of consumption, the equity share holding of the lower time

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$$\frac{d^2 A_t}{dC_t^2} = \frac{\beta+\beta^*}{2}\int_0^\infty \frac{2\left(e^{-\frac{1}{2}\beta^* s} - e^{-\frac{1}{2}\beta s}\right)e^{-(\beta+\beta^*)s}}{\left\{C_t e^{-\frac{1}{2}\beta^* s} + (1-C_t)e^{-\frac{1}{2}\beta s}\right\}^3}\,\mathrm{ds},$$

If $\beta > \beta^*$, then $d^2A_t/dC_t^2 < 0$.
It is shown that the curve of Cournot-Nash equilibria in Figure 1 is upward concave.

preference country grows more slowly in Cournot-Nash equilibria than in competitive equilibria. The two arrows of different length in Figure 1 indicate this.

As the equation (11) indicates, the net foreign asset is equal to the difference between the national wealth of the home country and the amount of the production factor that is located in the home country. Because the location of the production factor is constant in our model, the net foreign asset of the home country moves in parallel with the national wealth of the home country. Therefore the net foreign asset of the lower time preference country grows more slowly in Cournot-Nash equilibria than in competitive equilibria; this means that the current account imbalance, which is equal to the international capital flow, is smaller in Cournot-Nash equilibria than in the competitive equilibria. The uncoordinated actions by the national governments result in shrinking international lending and borrowing.

In the international trade theory, it is well known that each country's self-seeking trade policies result in shrinking international trade. Keeping in mind that international capital flows are international exchanges of present goods for future goods, we see that the present analysis is a natural extension of standard trade theory.

Because there is no market distortion, the allocation in competitive equilibria is Pareto-optimal, which can be confirmed by comparing the consumption paths of competitive equilibria with solution paths of a social optimization problem:

$$\max_{\{C_t,\, C_t^*\}} \omega \int_0^\infty e^{-\beta t}\, ln\; C_t\, dt + (1-\omega) \int_0^\infty e^{-\beta^* t}\, ln\; C_t^* dt,$$

subject to

$$C_t + C_t^* = 1,$$

$$C_t \geq 0,$$

and

$$C_t^* \geq 0,$$

where ω denotes the weight given to the home country consumers by the central planner. If $\omega = \beta^* \overline{A}_0 / (\beta^* \overline{A}_0 + \beta(1-\overline{A}_0))\}$, then the solutions of the social optimization problem will become identical with the path of the foregoing competitive equilibrium. As compared with the socially optimal path, the consumption of expenditures of the two countries diverge more slowly in Cournot-Nash equilibria.[10]

IV. *Tax Policies in Cournot-Nash Equilibria*

In a market economy, government cannot directly determine the national saving ratio. But the saving ratio can be controlled with appropriate tax policies. As shown in this section, each government can accomplish the foregoing Cournot-Nash equilibrium time path of consumption-wealth ratio by using a set of taxes and subsidies. Strictly, there is tax policies, by which each country can accomplish the consumption-wealth ration path (33) of Cournot-

[10] It is not difficult to confirm that both countries can improve their welfare simultaneously by deviating their consumption paths appropriately from the Cournot-Nash paths.

Nash equilibrium under a given time path of the other country's consumption-wealth ratio.

Assume that the home country government levies a capital income tax on its residents.[11] Let τ_t denotes the tax rate at time t. τ_t can be negative. The negative tax rate τ_t denotes that the government is subsidizing asset holding. The government is assumed to impose the tax purely for reallocation purposes and reimburse the revenue to the public in a lump-sum fashion. Under this tax policy, the budget constraint on the home country's citizens becomes

$$\dot{A}_t = \frac{1}{q_t}(A_t - C_t + T_t) - \tau_t A_t, \tag{34}$$

where T_t denotes the reimbursement. The budget constraint of the government can be written as

$$T_t = \tau_t q_t A_t \tag{35}$$

Now the Euler equation of the home country citizens becomes

$$\dot{C}_t = \left(\frac{1}{q_t} + \frac{\dot{q}_t}{q_t} - \tau_t - \beta\right) c_t, \tag{36}$$

instead of (3).

The consumption-wealth ratio of the competitive citizens in the home country can be derived from a transversality condition, (34), (35), and (36):

$$X_t \equiv \frac{C_t}{q_t A_t} = \frac{1}{\int_t^{+\infty} e^{-\int_t^s (\beta + \tau_v) dv} ds} \tag{37}$$

Notice that the consumption-wealth ratio of the competitive citizens does not depend on the foreign strategy $\{x_t^*\}$ nor on any market conditions. It is not difficult to ascertain that if the capital income tax rate moves as

$$\tau_t = \frac{1}{2}(\beta^* - \beta) \frac{C_0 e^{-\frac{1}{2}\beta t}}{C_0 e^{-\frac{1}{2}\beta t} + (1 - C_0) e^{-\frac{1}{2}\beta^* t}}, \tag{38}$$

then the consumption-wealth ratio path of the competitive citizens (37) will become identical with the optimal strategy of the home country in Cournot-Nash equilibrium (33).

The equation (38) indicates that if the home country's time preference rate is lower than that of the foreign country, then the home country will accomplish the optimal consumption-saving path by taxing the domestic citizens' asset holding. Under the capital income tax, the home country citizens consume more as compared with the discretionary case.

In the same way, it can be shown that the foreign government can accomplish the optimal strategy (33*) by controlling its capital income tax as

[11] As is well known, capital income taxes and consumption taxes are sometimes equivalent. In this model, governments can achieve the optimal consumption-wealth ratio path not only by using capital income taxes but also by using consumption taxes.

FIG. 2.

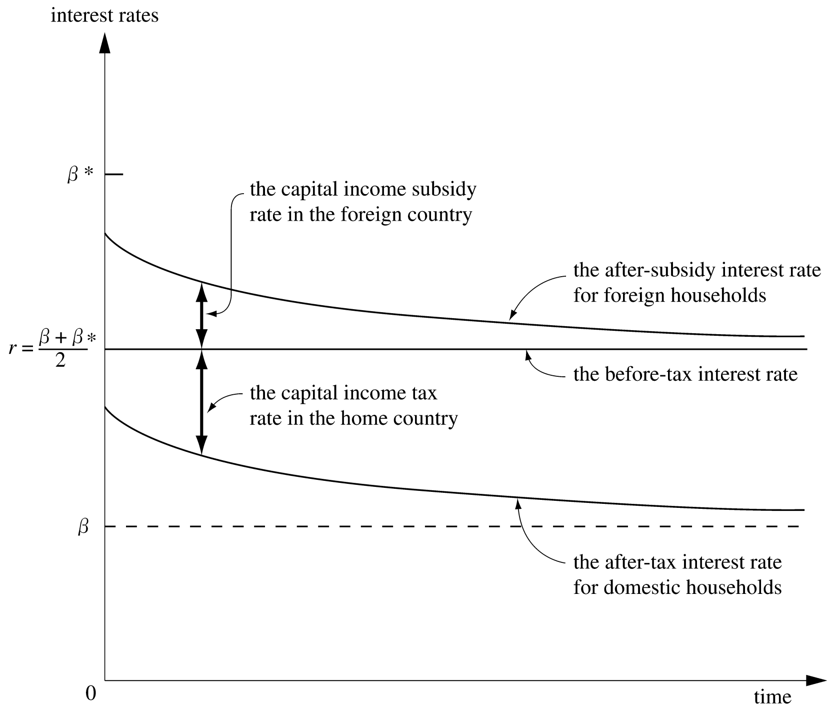

Figure 2. The dynamics of interest rates, tax rates, and subsidy rates in the Cournot-Nash equilibrium: the capital income tax case. (The time preference rate of the home country β is assumed to be lower than that of the foreign country β^*.)

$$\tau_t^* = -\frac{1}{2}(\beta^* - \beta)\frac{(1-C_0)e^{-\frac{1}{2}\beta^* t}}{C_0 e^{-\frac{1}{2}\beta t} + (1-C_0)e^{-\frac{1}{2}\beta^* t}}, \qquad (38^*)$$

where τ_t^* denotes the tax rate in the foreign country at time t.

The tax policies of the two countries in Cournot-Nash equilibrium are compared in Figure 2. The time preference rate of the home country is assumed to be lower than that of the foreign country. Notice that the difference between the two countries' after-tax interest rates is constant over time. The difference is always $(\beta^* - \beta)/2$. In Cournot-Nash equilibria of this model, the relative size of the two countries will not affect the magnitude of the market distortion if the market distortion is measured by the interest differential between the two countries. The higher time preference country comes to behave like a price-taker as its economic share decreases (Figure 2). In contrast, the lower time preference country intensifies its intervention into the market as its economic share increases. In this sense, a large country exploits a small one.

V. *Concluding Remarks*

By introducing a theory of non-cooperative dynamic games into the international lending and borrowing problem, it is seen that each country's self-seeking savings policies produce suboptimal allocation in the transition process. In Cournot-Nash equilibria of this policy game, governments of high time preference countries depress private consumption by, for example, subsidizing asset holding. By depressing consumption, they can bring down the world interest rate and improve their intertemporal terms of trade. Governments of low time preference countries boost private consumption by, for example, taxing asset holdings. As a result, international capital flows shrink to an undesirable level Cournot-Nash equilibria. If the world economy tends to fall into such non-cooperative equilibria, the cooperative equilibria, the cooperative equilibrium will be achieved by amplifying current account imbalances.

The world interest rate also moves differently as compared with that in competitive equilibria. In competitive equilibria, the world interest rate is equal to the weighted average of the two countries' time preference rates. And the weights are equal to each country's consumption shares in the world. As the lower time preference country expands its consumption share over time, the world interest rate falls. In Cournot-Nash equilibria, the world interest rate stays constant, because the government of the lower time preference country intensifies its capital income tax rate and continues to exploit the higher time preference country.

Some qualification as well as generalization of the above results may be noted. First, the story becomes more involved, if the opportunity for physical investment is introduced. Assume that the production function has constant returns to scale and is identical in both countries. There is only one production factor, capital stock. The capital stock and consumption goods are identical. In this case, the governments have no incentive to introduce savings policy, because they can not affect the world interest rate, which is always equal to the constant marginal productivity of capital. The non-cooperative equilibrium becomes identical with the competitive equilibrium. Probably the real economy lies between the two extreme cases, the constant output case and the constant marginal productivity case.

Secondly, it is possible that other formulations of governments' strategy spaces bring about different non-cooperative equilibria. For example, it will be fruitful to compare the case of territorial principle taxation,[12] in which the domestic government taxes investment income from domestic assets regardless of who invested in them, with the case of nationality principle taxation, in which the government taxes domestic residents' investment income regardless of where the income is derived. Another possible approach is to model governments as choosing decision rule strategies instead of path strategies.

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[12] See Hamada (1966).

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