COOPERATIVE AND NON-COOPERATIVE R&D IN AN OLGOPOLY WITH SPILLOVERS: STRATEGIC SUBSTITUTES VERSUS STRATEGIC COMPLEMENTS

KOTARO SUZUMURA AND NORIYUKI YANAGAWA

Abstract

The positive as well as normative effects of cooperative R&D are examined vis-a-vis non-cooperative R&D, socially first-best R&D, and socially second-best R&D within a two-stage model of oligopolistic competition with product differentiation. It is shown, among others, that both non-cooperative and cooperative R&D levels are socially insufficient at the margin in terms of the first-best as well as second-best criterion if firms compete in prices, irrespective of whether the number of firms and the degree of R&D spillovers are large or small.

I. Introduction

Two features of research and development (R&D) activities of a firm engaging in an oligopolistic competition deserve particular emphases. In the first place, the outcome of a firm's R&D activities may spread out cost-reducing benefits to other firms without any compensation, thereby negatively affecting a firm's incentive to make R&D commitment. In the second place, there are many concrete examples of R&D joint ventures or explicit coordinations of R&D activities among oligopolistic competitors that are fiercely competing in the product market. The first feature has been the focus of researches in theoretical industrial organization, as typified by Dasgupta and Stiglitz (1980) and Spence (1984), as well as in the recent endogenous growth literature, as typified by Romer (1990) and Grossman and Helpman (1991). The second feature is more difficult to capture despite an upsurge of theoretical researches focussing precisely on the role played by R&D joint ventures. Notable examples of theoretical researches in this arena are d'Aspremont and Jacquemin (1988), Katz (1986) and Suzumura (1992). In particular, it was shown by Suzumura (1992) in terms of a two-stage model of oligopolistic competition that:

(a) In the presence of sufficiently large R&D spillovers, neither non-cooperative nor cooperative equilibria achieve even the second-best R&D level;

Some of the salient works along similar lines include Beath, Poyagou-Theotoky and Ulph (1990), Kamien, Muller and Zang (1992), Katz and Ordover (1990) and Ulph (1990). See also Grossman and Shapiro (1986), Jacquemin (1988), Jorde and Teece (1990), Levin and Reiss (1988), and Okuno-Fujiwara and Suzumura (1993) for some related important works on R&D.
(b) In the absence of spillover effect, while cooperative R&D level remains socially insufficient, non-cooperative level may overshoot the first- and second-best levels of R&D.

Although these analytical results are certainly not without interest, it deserves emphasis that they are crucially dependent on the assumption that firms compete in the product market in terms of quantities. In other words, it is assumed that the strategic variables in the product market oligopoly are strategic substitutes. Recollect that, after the seminal works of Bulow, Geanakoplos and Klemperer (1985) and Fudenberg and Tirole (1984), it is well known that the play of the first stage game is much affected whether the second stage strategies are strategic substitutes or strategic complements. In view of this fact, it may well be worthwhile to examine how the results of Suzumura (1992) will be affected if the second stage strategies are prices, viz., strategic complements. This is precisely what we set out to verify in this paper. It may not be out of place to emphasize that this problem cannot be mechanically settled by the simple application of the foregoing analyses. Nevertheless, we can show that the non-cooperative equilibrium R&D level as well as the cooperative equilibrium R&D level is socially insufficient in sharp contrast with the result established by Suzumura (1992) with rather important policy implications. It is our hope that our result adds some further insight on the contrast between strategic substitutes and complements.

The structure of this paper is as follows. Our basic two-stage model of R&D competition is introduced in Section 2. Section 3 is devoted to the analysis of displacement of the second stage Bertrand-Nash equilibrium in response to an exogenous change in the first stage R&D commitment. Section 4 presents our welfare analysis on the subgame perfect equilibrium and our main result is established therein. In Section 5 we conclude with some clarifying observations.

II. The Two-Stage Model of Oligopolistic Competition

Consider an industry involving \( n \) firms \( (2 \leq n < +\infty) \) producing related but differentiated products. Let \( q_i = f_i(p) \), where \( p := (p_1, \ldots, p_n) \), denote the demand function for the \( i \)-th product, where \( q_i \) and \( p_i \) are, respectively, the output and price of the \( i \)-th product. The cost of producing \( q_i \) of the \( i \)-th product and the expenditure on cost-reducing R&D incurred by firm \( i \) are \( c(x_i; x_{-i})q_i \) and \( x_i \), respectively, where \( x_{-i} = (x_1, \ldots, x_{i-1}, x_{i+1}, \ldots, x_n) \). For the sake of expositional simplicity, we are assuming that the amount of R&D is measured by its expenditure.

Throughout this paper, we assume the following:

\( A(1) \): The demand function \( f_i(p) \) is defined on \( R^n_+ \) and twice continuously differentiable on \( P := \{ p \in R^n_+ | f'_i(p) > 0 \ (i=1, 2, \ldots, n) \} \) with

\( 2 \) The concept of strategic substitutes and strategic complements is due originally to Bulow, Geanakoplos and Klemperer (1985). When firms compete in quantities (resp. prices), it is quite natural to assume that the strategic variables are strategic substitutes (resp. strategic complements) as it corresponds to the assumption of downward (resp. upward) sloping reaction curves in the quantity (resp. price) space.
A(2): The average variable cost function $c(x_i; x_{-i})$ is defined on $\mathbb{R}^n_-$ and twice continuously differentiable on $X := \{x \in \mathbb{R}^n_+ | c(x_j; x_{-j}) > 0 \}$ with

(a) $\frac{\partial}{\partial x_i} c(x_i; x_{-i}) < 0$ and $\frac{\partial}{\partial x_j} c(x_i; x_{-i}) = 0$ (i~j);
(b) $\frac{\partial}{\partial x_j} c(x_i; x_{-i}) < (\partial/\partial x_j) c(x_i; x_{-i}) (i \neq j)$ for any symmetric $x \in X$.

The meaning of these assumptions is easy to interpret. The products of the industry in question are substitutes, and the own price effect dominates the aggregate cross price effects. A firm's R&D is cost-reducing and can benefit other firms without payment, but the cost-reducing benefit of the own R&D expenditure outweighs the benefits accruing freely from other firms when all firms are spending the same amount on R&D activities.

We are now ready to describe the structure of our two-stage model of oligopolistic competition. We assume that firms compete in two stages. In the first stage, firms make an irrevocable commitment to a level of R&D expenditure, whereas firms compete in the second stage by choosing prices.

The payoff function of firm $i$ in the second stage game is

\[
\pi_i(p; x) := [p_i - c(x_i; x_{-i})] f^i(p) - x_i.
\]

In what follows, $p^N(x) = (p_1^N(x), \ldots, p_i^N(x), \ldots, p_n^N(x))$ stands for the Bertrand-Nash equilibrium of the second stage game corresponding to the specified R&D profile $x$, which is defined by

\[
p_i^N(x) := \arg \max_{p_i>0} \pi_i((p_i, p^N_{-i}(x)); x) \quad (i = 1, 2, \ldots, n),
\]

where $p^N_{-i}(x) = (p_1^N(x), \ldots, p_{i-1}^N(x), p_{i+1}^N(x), \ldots, p_n^N(x))$. Assuming the interior optimum and second-order conditions, $p^N(x)$ can be characterised by $(\partial/\partial p_i) \pi_i(p^N(x); x) = 0$, viz.,

\[
f^i(p^N(x)) + \{p_i^N(x) - c(x_i; x_{-i})\} f^i_j(p^N(x)) = 0 \quad (i = 1, 2, \ldots, n).
\]

Throughout this paper, it is assumed that $p^N(x)$ is symmetric for any given symmetric R&D profile $x$.

To facilitate our subsequent analysis, let us define

\[
\alpha_i(p; x) := (\partial^2/\partial p_i^2) \pi_i(p; x) \quad (i = 1, 2, \ldots, n)
\]

and

\[
\beta_{ij}(p; x) := (\partial^2/\partial p_i \partial p_j) \pi_i(p; x) \quad (i \neq j; i, j = 1, 2, \ldots, n).
\]
Since the second stage game is played by firms which use prices as strategic variables, it is natural to require the following:

A(3): The second stage strategic variables are strategic complements, viz., $\beta_{ij}(p; x) > 0$ ($i \neq j; i, j = 1, 2, \ldots, n$) holds for all $(p; x) > 0$.

By carrying out the required second-order differentiation, we may verify that

(6) $\alpha_{i}(p; x) = 2f_{i}^{\prime}(p) + \{p_{i} - c(x_{i}; x_{-i})\} f_{ii}^{\prime}(p)$

and

(7) $\beta_{ij}(p; x) = f_{ij}^{\prime}(p) + \{p_{i} - c(x_{i}; x_{-i})\} f_{jj}^{\prime}(p)$

hold, where $f_{ii}^{\prime}(p) = (\partial^{2}/\partial p_{i}^{2}) f_{i}(p)$ and $f_{ij}^{\prime}(p) = (\partial^{2}/\partial p_{i} \partial p_{j}) f_{i}(p)$.

Turning to the first stage game, the first stage payoff function of firm $i$ is defined by

(8) $H_{i}(x) = \sum_{j=1}^{n} \left( x_{i} - c(x_{i}; x_{-i}) \right) f_{i}(p_{N}(x)) - x_{i}$ ($i = 1, 2, \ldots, n$).

In terms of the first stage payoff function (8), we may introduce two crucial equilibrium concepts. The first concept is the non-cooperative equilibrium a la Nash, denoted by $x^{N} = (x_{1}^{N}, \ldots, x_{i}^{N}, \ldots, x_{n}^{N})$ and defined under the assumption of interior optimum and second-order conditions by $(\partial/\partial x_{i}) H_{i}(x^{N}) = 0$ ($i = 1, 2, \ldots, n$). It is clear that the equilibrium pair $(x^{N}, p^{N}(x^{N}))$ is precisely the subgame perfect equilibrium of the entire game. The second concept is the cooperative equilibrium, denoted by $x^{C} = (x_{1}^{C}, \ldots, x_{i}^{C}, \ldots, x_{n}^{C})$, where firms coordinate their R&D levels to maximize their joint profits, and is defined by $(\partial/\partial x_{i}) \sum_{j=1}^{n} H_{j}(x^{C}) = 0$ ($i = 1, 2, \ldots, n$) under the assumption of interior optimum and second-order conditions with the proviso that $x_{1} = x_{2} = \ldots = x_{n}$. It is assumed in what follows that $x^{N}$ as well as $x^{C}$ is symmetric, so that $p^{N}(x^{N})$ as well as $p^{C}(x^{C})$ is symmetric too.

It is easy to verify that $x^{N}$ and $x^{C}$ are characterized by

(9) $-f_{i}^{\prime}(p^{N}(x^{N}))(\partial/\partial x_{i}) c(x_{i}^{N}; x_{-i}^{N}) - 1 + (n - 1)(\partial/\partial x_{i}) p_{i}^{N}(x^{N}) f(p^{N}(x^{N}))(p_{i}^{N}(x^{N}) - c(x_{i}^{N}; x_{-i}^{N})) = 0$ ($i \neq j; i, j = 1, 2, \ldots, n$),

and

(10) $-f_{i}^{\prime}(p^{C}(x^{C}))(\partial/\partial x_{i}) c(x_{i}^{C}; x_{-i}^{C}) - 1 - (n - 1)f_{i}^{\prime}(p^{N}(x^{C}))(\partial/\partial x_{i}) c(x_{i}^{C}; x_{-i}^{C}) + (n - 1)(\partial/\partial x_{i}) p_{i}^{N}(x^{C}) + (n - 1)(\partial/\partial x_{i}) p_{i}^{C}(x^{C}) = 0$ ($i \neq j; i, j = 1, 2, \ldots, n$),

respectively.

III. Displacement of the Second Stage Nash Equilibrium

As an auxiliary step in carrying out our welfare analysis, let us examine how the Bertrand-Nash equilibrium $p^{N}(x)$ reacts to an exogenous change in $x_{i}$. For any symmetric R&D profile $x$, let us define $\omega^{N}(x) := (\partial/\partial x_{i}) p_{i}^{N}(x)$ and $\theta^{N}(x) := (\partial/\partial x_{i}) p_{i}^{N}(x)$ ($i \neq j; i, j = 1,$
2, . . . , n). The method of analysis we have developed in Suzumura (1992) can be modified suitably to yield the following expression for $\omega^B(x)$ and $\theta^B(x)$:

\begin{equation}
\omega^B(x) = \frac{f'_i(p^N(x))}{\Delta^B(x)} \left[ (\partial/\partial x_i) c(x_i; x_{-i}) \{ \alpha^B(x) + (n-2)\beta^B(x) \} 
- (n-1)(\partial/\partial x_i) c(x_i; x_{-i}) \beta^B(x) \right] < 0
\end{equation}

and

\begin{equation}
\theta^B(x) = \frac{f'_i(p^N(x))}{\Delta^B(x)} \left[ \alpha^B(x)(\partial/\partial x_i) c(x_i; x_{-i}) - \beta^B(x)(\partial/\partial x_i) c(x_i; x_{-i}) \right] < 0
\end{equation}

hold, where $\alpha^B(x) := \alpha_i(p^N(x); x)$, $\beta^B(x) := \beta_i(p^N(x); x)$ and $\Delta^B(x) := \{ \alpha^B(x) + (n-1)\beta^B(x) \} > 0$. Thus, under the basic assumption A(3) of strategic complementarity, an increase in a firm's cost-reducing R&D unambiguously decreases the Bertrand-Nash equilibrium price of all firms irrespective of whether the R&D spillovers are large or small. This is in sharp contrast with the equilibrium response when A(3) is replaced by the assumption of strategic substitutability.

Figure 1 and Figure 2 are meant to crystallize this sharp contrast in the context of Bertrand-Nash duopoly equilibrium. Recollect that the reaction curves are upward (resp. downward) sloping if strategic complementarity (resp. strategic substitutability) prevails. In Figure 1 and Figure 2, $R_i(x)$ denotes the reaction curve of firm $i$ in the price space ($i=1, 2$) when the R&D profile $x$ prevails. The second stage Bertrand-Nash equilibrium is represented by the point $E^0$ where the two reaction curves have a stable cross. Since the R&D expenditures are cost-reducing, an increase in $x_1$ shifts up $R_i(x)$, but it also shifts up $R_2(x)$ through spillovers of the cost-reducing benefits. In the case of Figure 1, the Bertrand-Nash equilibrium changes from $E^0$ to $E^1$ accordingly. It is clear that the equilibrium

**Figure 1. DUOPOLY EQUILIBRIUM UNDER STRATEGIC COMPLEMENTARITY**

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5 It is easy to verify that the Bertrand-Nash equilibrium $p^N(x)$ is stable with respect to the myopic adjustment mechanism $p_i = \sigma(\partial/\partial p_i) \Pi_i(p; x)$ ($i=1, 2, \ldots, n$), where $\sigma>0$ denotes the adjustment coefficient. See Dixit (1986) among many others.
price is lower at $E^1$ than at $E^0$ for both products irrespective of whether R&D spillovers are large or small. In the case of Figure 2, however, the Bertrand-Nash equilibrium changes from $E^0$ to either $E^1$ or $E^2$ depending on whether R&D spillovers are small or large. Therefore, the equilibrium price of firm 2 increases (resp. decreases) if R&D spillovers are large (resp. small). It is this contrast in the comparative statical implications which sharply separate strategic complementarity from strategic substitutability.

IV. Welfare Analysis of Cooperative and Non-Cooperative R&D

With the purpose of gauging the welfare performance of mixed cooperative-non-cooperative equilibrium $\{x^c, p^N(x^c)\}$ vis-a-vis overall non-cooperative equilibrium $\{x^N, p^N(x^N)\}$, let the market surplus function $W(x, q)$ be defined by

$$W(x, q) = V(q) - \sum_{j=1}^{n} [c(x_j, x_j)q_j + x_j],$$

where $V(q)$ is the gross benefit function of the representative consumer and $q=(q_1, \ldots, q_n)$ denotes a specified output vector. Let $q^F(x)=(q_1^F(x), \ldots, q_n^F(x))$ and $q^S(x)=(q_1^S(x), \ldots, q_n^S(x))$ be defined, respectively, by $q_i^F(x) = f(c(x); x_i)$ and $q_i^S(x) = f^F(p^N(x))$ ($i=1, 2, \ldots, n$). Then the socially first-best welfare function $W^F(x)$ and the socially second-best welfare function $W^S(x)$ are defined by $W^F(x) = W(x, q^F(x))$ and $W^S(x) = W(x, q^S(x))$, respectively. To simplify our notation, let $p_i^F(x)$ and $p_i^S(x)$ be defined by $c(x_i; x_{-i})$ and $p_i^N(x)$, respectively ($i=1, 2, \ldots, n$).

By differentiating $W^r(x)$ ($r=F, S$) with respect to $x_i$ and invoking $(\partial/\partial q_i)V(q^r(x)) = p_i^r(x)$ ($j=1, 2, \ldots, n$), we obtain

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6 Clearly, $q_i^F(x)$ [resp. $q_i^S(x)$] denotes the socially first-best output (resp. the Bertrand-Nash equilibrium output) of firm $i$. 

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(14) \( \frac{\partial}{\partial x_t} W^*(x) = \gamma^*(x) + \delta^*(x) + \sigma^*(x) \),
where

\begin{align*}
(15) \quad \gamma^*(x) & := -q^*_t(x) \frac{\partial}{\partial x_t} c(x_t; x_{-t}) - 1, \\
(16) \quad \delta^*(x) & := \sum_{j=1}^{n} \left( p^*_j(x) - c(x_j; x_{-j}) \right) \left( \frac{\partial}{\partial x_t} q^*_j(x) \right) \\
\text{and} \\
(17) \quad \sigma^*(x) & := -\sum_{j=1}^{n} q^*_j(x) \left( \frac{\partial}{\partial x_t} c(x_j; x_{-j}) \right).
\end{align*}

As in Suzumura (1992), the term \( \gamma^*(x) \), which is non-zero only because firms commit themselves to the cost-reducing R&D expenditure, may be called the commitment effect. The term \( \delta^*(x) \), which is nothing other than the sum of marginal distortions induced by an exogenous change in \( x_t \), may be called the distortion effect. Finally, the term \( \sigma^*(x) \), which is precisely the sum of R&D spillovers induced by an exogenous change in \( x_t \), may be called the spillover effect. Our task is to evaluate these economically meaningful terms for \( x = x^F \) and \( x = x^S \), with the purpose of determining whether \( \frac{\partial}{\partial x_t} W^*(x) > 0 \) (resp. \( < 0 \)) means that a marginal increase (resp. decrease) in \( x_t \) increases the value of the welfare function \( W^*(x) \) marginally, where \( x \in \{ x^F, x^S \} \) and \( T \in \{ F, S \} \).

Let us begin with the welfare analysis in terms of the first-best welfare function \( W^F(x) \). It is clear in this case that \( \delta^F(x) = 0 \) and \( \sigma^F(x) \geq 0 \). We can also rewrite \( \gamma^F(x) \) as follows:

\begin{align*}
(18) \quad \gamma^F(x) & = -q^F_t(x) \frac{\partial}{\partial x_t} c(x_t; x_{-t}) - 1 \\
& = \gamma^S(x) - \left( q^F_t(x) - q^S_t(x) \right) \left( \frac{\partial}{\partial x_t} c(x_t; x_{-t}) \right).
\end{align*}

Invoking (9), we can verify that

\begin{align*}
(19) \quad \gamma^S(x^N) & = -(n-1)\theta^S(x^N) \sum_{j=1}^{n} f^j(\zeta(x^N)) \{ p^N_j(x^N) - c(x^N_j; x^N_{-j}) \} > 0,
\end{align*}

where use is made of A(1) and (12). Furthermore, by virtue of the mean value theorem and A(1), there exists a number \( \zeta(x) \), where \( 0 < \zeta(x) < 1 \), such that

\begin{align*}
(20) \quad q^F_t(x) - q^S_t(x) & = - \left( p^N_t(x^N) - c(x_t; x_{-t}) \right) \sum_{j=1}^{n} f^j(\zeta(x)) \{ p^N_j(x^N) + \left( 1 - \zeta(x) \right) c(x) \} > 0.
\end{align*}

Invoking A(2), we may now assert on the basis of (18), (19) and (20) that \( \gamma^F(x^N) > 0 \). This, in its turn, insures that \( \frac{\partial}{\partial x_t} W^F(x^N) > 0 \).

Let us now turn to the welfare performance of the cooperative equilibrium \( x^C \) in terms of \( W^F(x) \). Invoking (10), we may obtain

\begin{align*}
(21) \quad \gamma^F(x^C) & = -(n-1)q^S_t(x^C) \frac{\partial}{\partial x_t} c(x^C_t; x^C_{-t}) \\
& - \left( n - 1 \right) \{ p^N_t(x^C) - c(x^C_t; x^C_{-t}) \} f^j(\psi^F(x^C)) \psi^F(x^C),
\end{align*}

where \( \psi^F(x^C) := \omega^F(x^C) + (n-1)\theta^F(x^C) < 0 \) by virtue of (11) and (12). It then follows that
where use is made of (20) for \( x = x_c \).

We have thus established the first main result of this paper.

**Theorem 1**

Suppose that A(1), A(2), A(3) and A(4) hold. Then the non-cooperative equilibrium R&D as well as the cooperative equilibrium R&D is socially insufficient at the margin in terms of the first-best welfare function in the sense that \( (\partial/\partial x_i) W^F(x^N) > 0 \) as well as \( (\partial/\partial x_i) W^F(x^C) > 0 \) hold.

Although the welfare verdicts in terms of the first-best criterion are not without analytical interest, a basic doubt remains as to the empirical relevance of these verdicts. Since the criterion \( W^F(x) \) presupposes that the socially first-best marginal cost principle is enforceable, it is relevant to ask if the foregoing verdicts may be kept intact if this unrealistic presupposition is deleted. This is precisely the purpose of our further analysis in terms of the second-best welfare criterion \( W^S(x) \).

Note that we have already shown that \( \gamma^S(x) > 0 \) holds, whereas \( \sigma^S(x^N) \geq 0 \) holds in view of the basic assumption A(2). In sharp contrast with the first-best case, however, the distortion effect \( \delta^S(x) \) does not vanish in the second-best case, which can be evaluated as follows:

\[
(23) \quad \delta^S(x) = \{ p_i^N(x) - c(x_i; x_{-i}) \} \left[ f_j^j(p^N(x)) + (n-1)f_j^j(p^N(x)) \right] \Psi^S(x) > 0,
\]

where use is made of A(1) and \( \Psi^S(x) < 0 \). We have thus verified that the crucial inequality \( (\partial/\partial x_i) W^S(x^N) > 0 \) holds.

Turning to the second-best performance of cooperative R&D \( x^C \), note that the first term in the RHS of (21) cancels with \( \delta^S(x^N) \), whereas the second term thereof is unambiguously positive. Invoking \( \delta^S(x^C) > 0 \) which follows from (23) for \( x = x_C \), we may assert that \( (\partial/\partial x_i) W^S(x^C) > 0 \) holds.

We have thus established the second main result of this paper.

**Theorem 2**

Suppose that A(1), A(2), A(3) and A(4) hold. Then the non-cooperative equilibrium R&D \( x^N \) as well as the cooperative equilibrium R&D \( x^C \) is socially insufficient at the margin in terms of the second-best welfare function in the sense that \( (\partial/\partial x_i) W^S(x^N) > 0 \) as well as \( (\partial/\partial x_i) W^S(x^C) > 0 \) hold.

Thus, if firms compete in the second stage game using prices as their strategic variables, the non-cooperative equilibrium R&D as well as the cooperative equilibrium R&D is socially insufficient at the margin even when we replace the first-best criterion by the second-best criterion.

For the sake of easy comparison, this verdict is summarized in Table 1. It clearly stands out that, unlike the case of quantity competition in the second stage game where Suzumura (1992) and Okuno-Fujiwara and Suzumura (1993) have demonstrated that the marginal welfare verdicts hinge squarely on the extent of R&D spillovers, the marginal welfare verdicts in the second stage price competition case are quite unambiguous and
Table 1. Price Competition with or without R&D Spillovers

\[
\begin{array}{c|c|c}
\frac{\partial}{\partial x_i} W^P(x) & x^N & x^G \\
\frac{\partial}{\partial x_i} W^S(x) & + & + \\
\end{array}
\]

Table 2. Quantity Competition with Large R&D Spillovers

\[
\begin{array}{c|c|c}
\frac{\partial}{\partial x_i} W^P(x) & x^N & x^G \\
\frac{\partial}{\partial x_i} W^S(x) & + & + \\
\end{array}
\]

Table 3. Quantity Competition with No R&D Spillovers

\[
\begin{array}{c|c|c}
\frac{\partial}{\partial x_i} W^P(x) & x^N & x^G \\
\frac{\partial}{\partial x_i} W^S(x) & -a & + \\
\end{array}
\]

\[a\] This requires that the number of firms is at least 3.
\[b\] This requires that the number of firms is sufficiently large.

Independent of the extent of R&D spillovers and the number of firms. To highlight this contrast, the previous results on the case of second stage quantity competition are succinctly reproduced in Table 2 and Table 3.

V. Concluding Remarks

In concluding this paper, two clarifying comments seem to be in order.

In the first place, the formal framework of our analysis is restrictive in that it is strictly geared with the case of cooperative R&D activities among competitors in the product market in full neglect of the case of R&D cooperations among vertically related firms, e.g., material suppliers, equipment suppliers and assemblers. It goes without saying that the R&D collaborating among vertically related firms is interesting enough to warrant an extensive study of its own. We should also note that our formulation of R&D spillovers in terms of the average variable cost function which remains the same whether or not firms collaborate may be seriously inadequate. For fuller analysis, we should somehow endogenous the spillover function by making the cost-reducing technology dependent on the extent to which firms pool their complementary R&D resources. Furthermore, care should be taken with the important aspect of R&D cooperation viz. the stability of R&D cartels. These problems which are left unanalysed in this and preceding papers must be the natural targets of our subsequent research.

In the second place, care should be taken with the complexity of the alternative institutional arrangements actually in use for implementing R&D collaborations among firms which compete in the product market. It may well be worthwhile to specify the contractual and organizational details of collaborative R&D and examine their comparative performance. An initial step along this line was recently taken by Kamien, Muller and Zang (1992), but much more work certainly remains to be done in the future.
A. Derivation of the First-Order Condition for $x^C$

By differentiating $\sum_{j=1}^{n} \pi_j(p^N(x); x)$ with respect to $x_i$, and taking the fact that all firms coordinate their R&D expenditures symmetrically into consideration, we may obtain

\[(1^*) \sum_{j=1}^{n} \left( \sum_{k=1}^{n} (\partial/\partial x_k) \pi_j(p^N(x); x) (\partial/\partial x_i) p^N_k(x) + (\partial/\partial x_i) \pi_j(p^N(x); x) \right) + \sum_{k \neq j} \left( \sum_{k=1}^{n} (\partial/\partial x_k) \pi_j(p^N(x); x) (\partial/\partial x_h) p^N_k(x) + (\partial/\partial x_h) \pi_j(p^N(x); x) \right) \right] = 0.\]

Invoking $(\partial/\partial x_i) \pi_i(p^N(x); x) = 0$ $(i = 1, 2, \ldots, n)$ and

\[ (2^*) \frac{(\partial/\partial x_h) \pi_j(p^N(x); x)}{q_j(x)} = \begin{cases} -q_j^N(x)(\partial/\partial x_j)c(x_j; x_{-j}) - 1 & \text{if } h = j \\ -q_j^N(x)(\partial/\partial x_h)c(x_j; x_{-j}) & \text{if } h \neq j, \end{cases} \]

we may reduce $(1^*)$ into (10) in the main text.

B. Derivation of the Formulas for $\omega^B(x)$ and $\theta^B(x)$

By differentiating the first-order condition (3) characterizing $p^N(x)$ with respect to $x_i$ and $x_h$ $(i \neq h; i = 1, 2, \ldots, n)$, we may obtain the following simultaneous equations for $\omega^B(x)$ and $\theta^B(x)$:

\[(3^*) \alpha^B(x) \omega^B(x) + (n-1) \delta^B(x) \theta^B(x) = f_i^j(p^N(x))(\partial/\partial x_i)c(x_i; x_{-i}) \]

and

\[(4^*) \beta^B(x) \omega^B(x) + (\alpha^B(x) + (n-2) \delta^B(x)) \theta^B(x) = f_i^j(p^N(x))(\partial/\partial x_i)c(x_i; x_{-i}). \]

We can solve $(3^*)$ and $(4^*)$ for $\omega^B(x)$ and $\theta^B(x)$ to obtain (11) and (12) in the main text. The signs of $\omega^B(x)$, $\theta^B(x)$ and $\delta^B(x)$ are determined by $A(1)$, $A(2)$, $A(3)$ and $A(4)$.

REFERENCES


