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REAL AND MONEY INCOME MULTIPLIER

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I. *A Dilemma in the Keynesian Dynamics*

The most conspicuous feature of the prevailing trade cycle theories since Keynes' *General Theory*, is that trade cycles or economic fluctuations have been largely explained in terms of such an aggregate concept as *real* national income. Although Keynes himself did analyze the fluctuations by an indicator such as employment or national income in wage-terms, the post-Keynesian has customarily adopted the real terms concept. This tendency seems to be unavoidable to a degree, as far as the consumption function, a keystone of the Keynesian economics, is concerned.

Why must it be expressed in real terms? If we assume a linear form of consumption function, the consumption function *in money terms* which connects money consumption expenditure C and money national income Y, will be

$$C = a + bY \quad \text{.............................}(1)$$

where a and b=const. From this equation, we can derive

$$C/Y = b + a/Y \quad \text{........................}(2)$$

in which is involved a false implication that a particular average propensity to consume will correspond to a given *money* income. However, provided that no money illusion exists among consumers, it will be more plausible to deal with the average propensity to consume as a particular function of *real* income $\left(\frac{Y}{P}\right)$, i.e.,

$$C/Y = \beta + \alpha\Big/\left(\frac{Y}{P}\right) \quad \text{.....................}(2')$$

If we transform (2′) into the following form,

$$\left(\frac{C}{P}\right) = \alpha + \beta\left(\frac{Y}{P}\right) \quad \text{.....................}(1')$$

the *real consumption function* will be obtained, which represents the ordinary Keynesian way of thinking.

Once the consumption function, the nucleus of the Keynesian economics, however, has been expressed in real terms, it will naturally represent all other function too in real terms. Thus the Keynesian economics could not but follow the way towards the *real output dynamics*.[1] Nevertheless, the multiplier process itself, even though derived from the real consumption function, is essentially a *monetary phenomenon*, because the multiplier sequence is nothing but a self-expanding process of monetary expenditures and even when a change of the real output should not move correspondingly, the price level would increase as if it were a variant in a more general sence of the multiplier process. Nevertheless, it is a serious contradiction that the consumption function is usually composed in real terms. Notwithstanding the fact that the 'multiplier rolling' is a monetary phenomenon, the consumption function, the computing origin of the above, is obliged to wear a *real* garment, in order to maintain the *stability* of the function. The trade cycle is now analyzed largely in terms of real output, in spite of the concurrent severe fluctuations of prices. The exposition of price fluctuations is treated as if it were a causally isolated phenomenon from the multiplier process, although both are actually interwoven with each other.

If output follows behind the effective demand faithfully like a shadow, and if the reversed Say's law that 'the demand creates its own supply' is valid, the Keynesian real output dynamics may quite naturally be constructed on the basis of a real consumption function. The actuality, however, is extremely different. Formerly the Wicksellian analyzed the price fluctuation process as being caused by the investment-saving gap. Now the Keynesian Revolution elucidated that IS gap will promote a change of output. But *in fact*, the gap generates both fluctuations of prices and production.

Keynes, in Chapter 21 of his *General Theory*, deals with the problem how a change in monetary effective demand will be absorbed between price and output alterations. His analysis in terms of price and output elasticities, $e_0 \cdot e_p$, nevertheless, is not in harmoney with his theory of the multiplier. One of our major problems lies in this harmonization.

One dilemma in the Keynesian economics originates from the above-mentioned inconsistency between monetary multiplier and real consumption function, but his economics also faces another dilemma in the exposition of reality. As a matter of fact, in the United States and other countries, during the inter-war period, there occured an extraordinary severe oscillation

[1] Cf. J.R. Hicks, *A Contribution to the Theory of the Trade Cycle*, 1950. As Hicks pointed out, the investment function, in addition to the consumption function, must be expressed in real terms, because the *acceleration principle* will be more conveniently developed in real terms than in wage terms. "For what that principle is concerned with is the effect of change in *output* on investment; it is not evident that a rise in output will have any different effect in this direction when it is due to increasing employment from what it will have when it is due to many of the various causes which can be grouped together as increasing productivity" (p. 8).

of production, and so far as that period is concerned, the real output approach is fairly realistic. But in prewar Japan or in Great Britain in the 19th century, the Juglar cycle of industrial production or real national income, lacks distinctness. Prices are rather more cyclical than production, and very frequently the Juglar cycles of production disappear in its secular upward trend, still more so with reference to the Kondratieff long wave. We cannot find the Kondratieff waves in terms of the index of industrial production or real GNP. Nevertheless, Higgins has even attempted to draw the long waves in terms of GNP in one of his essays.[2] This is quite erroneous and unrealistic. But his is an interesting example of Keynesian adherence to real output fluctuations.

It is one of the shortcomings of the present Keynesian dynamics that it fails to synthesize price fluctuations with those of output. Of course, we do not attempt to solve the whole problem here. We are merely satisfied to grasp a simple formulation of the inter-relation between multiplier and price changes. This is the task in the next section.

II. *Relation between Income Multiplier and Price Fluctuations*

It has been generally assumed that the theory of multiplier is only valid within the economic sphere of a constant price level, or when the elasticity of production with reference to effective demand is always unity. In other words, it postulates the hypothetical case where the whole increment of effective demand is necessarily absorbed by production increases with the result of no price changes. Generally, it is called the theory of under-employment multiplier. In fact, however, prices usually fluctuate even in the stage of under-employment, and in the statistical determination of multiplier, such series as income, consumption, investment, and so on are often deflated by the index of the general price level.

The theoretical assumption of a constant price level is already abandoned when income multiplier is statistically computed, because if the assumption of a constant price level is an indispensable condition, its statistical computation is nonsense.

Although Keynes adopted the wage term, many post-Keynesians use the real term. Consequently we shall henceforward use the multiplier in real terms. When national income is expressed by Y, investment by I, savings by S, general price index by P, price elasticity with reference to

[2] B. Higgins, "Concepts and Criterion of Secular Stagnation," in *Income, Employment and Public Policy*, Essays in Honour of Alvin H. Hansen, 1948, p. 83.

money national income by e_p, and output elasticity with reference to Y by e_0 $(=1-e_p)$, we are able to construct the following relation between real income multiplier $k_r\left[=d\left(\frac{Y}{P}\right)\Big/d\left(\frac{I}{P}\right)\right]$ and money income multiplier $k_m[=dY/dI]$,

$$k_r=\frac{d\left(\frac{Y}{P}\right)}{d\left(\frac{I}{P}\right)}=\frac{\frac{dY}{P}-Y\frac{dP}{P^2}}{\frac{dI}{P}-I\frac{dP}{P^2}}=\frac{dY-Y\frac{dP}{P}}{dI-I\frac{dP}{P}}=\frac{1-e_p}{\frac{dI}{dY}-\frac{I}{Y}e_p}=\frac{e_0}{\frac{1}{k_m}-\frac{S}{Y}e_p}$$

$$\therefore\ \frac{k_r}{k_m}=e_0+k_re_p\frac{S}{Y}\ \dots\dots\dots\dots\dots\dots\dots\dots(3)$$

We call this *the fundamental equation of real and money income multiplier.* The meaning of this is as follows: The ratio of real to money income multiplier $\left(\frac{k_r}{k_m}\right)$ largely depends on the value of e_0, i.e., upon the intensity of inflation, for the value of k_r is usually 2–3, S/Y the savings ratio 10–15 %, and e_p in normal times does not rise above 0.5, thus the product of the above three coefficients $k_re_p\frac{S}{Y}$ is 0.1 or thereabout. $\frac{k_r}{k_m}$, therefore, is dominantly determined by the value of e_0.

As a matter of course, $1=e_p+e_0$, because it can be derived by the total differential of the identity $Y=O\cdot P$, (where O is the physical output). Therefore the nearer e_0 is to zero, the closer e_p will be to unity, and the economic conditions will be inflationary. On the contrary, the nearer e_0 is to unity, the closer e_p will be to zero, and k_r will get close to k_m.

Consequently, the severer an inflation becomes, the closer e_0 will be to zero, with the consequence of the difference k_m-k_r growing greater. Formerly, Richard and W.M. Stone computed the marginal propensity to consume and the multiplier with the reference to the six countries, i. e., Germany, Great Britain, Holland, Poland, Sweden and America, but they used not k_r but k_m.[3] Although an international comparison of k_r is meaningful, that of k_m, is nonsense, because even if the computed value of k_r should be equivalent between two countries, k_m in the more inflationary country would be relatively greater than that in the less inflationary country. Colin Clark once compared k_m from time series with k_r from budget studies,[4] utilizing Stone's results. This also lacks economic meaning.

The above is a formal relation between k_r, k_m and e_0, and, in a sense, the bridge between Keynes' multiplier theory and his $e_0\cdot e_p$ theory. If the multiplier theory be constructed on the constant price level hypothesis,

[3] Richard and W.M. Stone, "The Marginal Propensity to Consume and the Multiplier, A Statistical Investigation," *The Review of Economic Studies*, October 1938.

[4] Colin Clark, *The Conditions of Economic Progress*, 1940, Second edition, Chapter 15.

Keynes' two theoretical apparatus could by no means be harmonized. However, if the multiplier is calculated by the *deflated* income-consumption data, they can be satisfactorily combined, though k_r in this case is not in a position causally to determine real national income relatively to an initially performed investment in money terms.

As the real consumption function is relatively stable, k_r is also stable. As J. Duesenberry pointed out,[5] this may be a *fundamental relation*. On the contrary, the monetary consumption function or money income multiplier is a *derived relation*, because it is difficult to forecast how much prices will change in the forthcoming period, whereas it is comparatively conjecturable how much real consumption will change when a given real income changes. Therefore the money income multiplier as a derived relation depends upon the degree of price fluctuation when the real income multiplier, as a fundamental relation, is once determined.

In the fundamental equation (3), e_0 may be assumed to change in a volatile manner in each phase of business cycle.

If we assume $e_0=1$, and substitute it into (3), we obtain,

$$\frac{k_r}{k_m} = 1 \quad ; \quad k_r = k_m$$

When the price level is perfectly stable, the money income multiplier will concide with the real income multiplier.

But when to the contrary $k_r=k_m$, is e_0 always 1? Surely such will be a special case, but not always, since the relation is not symmetrical or reversible. If we put $k_r=k_m$ into (3), remembering $e_0=1-e_p$, we have

$$1 = \frac{S}{Y}k_r\,; \quad (\text{since } k_r = k_m) \quad 1 = \frac{S}{Y}k_m$$

$$\therefore \quad k_r = k_m = \frac{1}{S/Y}$$

Consequently when there is no intercrept in the two-dimensional diagram composed of S and Y, the condition $K_r=K_m$ will always be satisfied however much the price level changes.

The above, however, is too imaginary case. In fact, not only prices are variable, but also there cannot be supposed any two dimensional diagram without intercept with reference to S and Y. If we compute the values of K_r and K_m from the U.S. Department of Commerce national income data and the BLS consumer's price index 1929–40, we obtain the result $K_r=2.8$ and $K_m=3.5$. This coefficient K_r is not a genuine multiplier, since S and Y are not reduced to a *per capita* basis, but it will probably be adequate as a coefficient indicating the connection with the value $e_0 \cdot e_p$. If we further assume that $S/Y=0.1$ and put these values into (3), e_0 thus

[5] J. Duesenberry, *Income, Saving and the Theory of Consumer Behavior*, 1949, pp. 73–75.

derived indirectly becomes 0.72. On the other hand, e_0 in the United States surpasses 0.8 for 6 years and is below 0.8 for 5 years. For these periods, there are several years when S/Y is negative and S/Y actually fluctuated very violently. Therefore, if we put annually changing S/Y into (3), there will be established a clear consistent relation among K_r, K_m, e_0 and S/Y.

The multiplier under price fluctuations can be formally explained by the above formulation. But in the customary theory of multiplier, the assumption is generally made that before full employment prices do not entirely change ($e_0=1$) and only after full employment, the increase of effective demand will be wholly absorbed by price-rise ($e_p=1$). Goodwin's procedure is one example.[6]

We are interested, then, in deriving the so-called full employment multiplier, i.e., the money income multiplier when $e_p=1$. Substituting $e_p=1$ into (3), we obtain

$$\frac{K_r}{K_m}=K_r\frac{S}{Y} \qquad \therefore \quad K_m=\frac{Y}{S}=\frac{Y/P}{S/P}$$

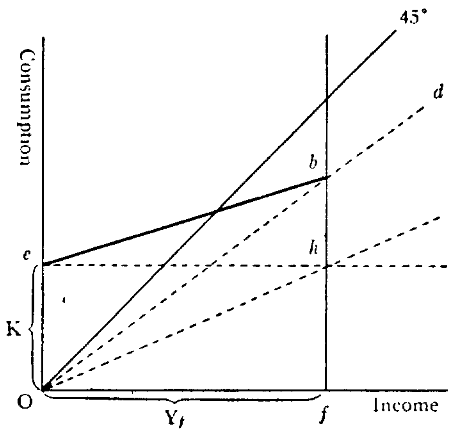
In the form of the propensity to consume,

$$\frac{dC}{dY}=\frac{C}{Y}=\frac{C/P}{Y/P}=\frac{C_f}{Y_f}$$

Where C_f and Y_f is full employment consumption and income respectively, and will be equivalent to C/P and Y/P at and after full employment, if, until full employment, a constant price level is maintained.

If we assume, in Chart 1, that Y_f is the full employment income and real consumption schedule before full employment is eb, the monetary consumption function after full employment will be bd. This reveals that the ratio between real consumption and real income at the point $f(=C_f/Y_f)$ will remain unchanged, however money income may be nominally swelled.

Chart 1



As generally observed, the marginal propensity to consume after full employment bd is greater than that before full employment eb, and a slope of bd represents not only the marginal but also the average propensity to consume.

Goodwin's full employment

[6] R.M. Goodwin, "The Theory of Multiplier," in Harris ed. *The New Economics*, 1947, pp. 482–499.

multiplier[7] was formulated as, $\frac{1}{1-(a+K/Y_f)}$,
where a is the pre-full-employment marginal propensity to consume and K is the intercrept of the real consumption function eb. Our result perfectly coincides with his formula, because, as the graph distinctly shows,

$$\begin{array}{ccccc} bd & = & a & + & K/Y_f \\ \parallel & & \parallel & & \parallel \\ \frac{bhf}{Y_f} & = & \frac{bh}{Y_f} & + & \frac{hf}{Y_f} \end{array}$$

The above graph indicates the following interesting consequence. The closer the point where the consumption schedule intersects with the 45° line, approaches towards the full employment level f, the wider the distance bf becomes, with the inevitable result that the income-consumption ratio at full employment (ob) as well as the marginal propensity to consume (bd) becomes greater. As the slope bd becomes steeper, K_m after full employment grows greater. And just when the break-even point of income= consumption coincides with the full employment level, bd becomes the 45° line itself. This is the case of the marginal propensity to consume=1 and the money income multiplier=infinity, where the economy is at the verge of instability. Provided that private investment and government expenditures are added to this level of consumption, the inflationary price-wage spiral will necessarily begin. We can easily understand from the above exposition why in the post-war devastated countries where real income has greatly declined, the hyper-inflation was inevitable.

We must now recognize the hitherto postulated assumption that relative prices are invariable over time, i.e., the parallel movement of the price level of investment goods and the general price level. But such an assumption, of course, is unrealistic. In this respect, J.R. Hicks has made the following suggestion. The value of real investment in terms of consumption goods, he says, is the multiplicand, to which the multiplier has to be applied. Thus, without any expansion in real investment in terms of investment goods, there will be an expansion in consumption, due to the multiplier effect, *merely as a result of the price rise*.[8] This is the Hicks method of solution which he proposed as an inspection of ceiling. But the unparallel movement between investment goods prices and consumption goods prices occurs not only in the neighborhood of full employment, but also at the phase of recovery from depression. Consequently, his method contradicts his fundamental procedures of real output dynamics, especially with reference to this exposition of acceleration principle *in physical terms*. However, it is easy to see why he had no other course than choose that

[7] Idem, p. 497.
[8] J.R. Hicks, *op. cit.*, p. 130.

way. The essence of real income multiplier probably obliged him to do so.

As a matter of course, the multiplier is essentially monetary. Therefore the multiplier will lose its proper character if we pursue the truly real relation when relative prices change, but we shall proceed to follow the consequences of the real income multiplier, remembering this limitation.

If we assume the general price level as P_y, the investment goods price level as P_i, then the real income multiplier K_R which takes into consideration these two deflators becomes,

$$K_R=\frac{d\left(\frac{Y}{P_y}\right)}{d\left(\frac{I}{P_i}\right)}=\frac{\left(dY-Y\frac{dP_y}{P_y}\right)P_i}{\left(dI-I\frac{dP_i}{P_i}\right)P_y}=\frac{1-e_p}{\left(1-\frac{dP_i}{P_i}\Big/\frac{dI}{I}\right)}\cdot\frac{P_i}{P_y}\cdot\frac{dY}{dI}$$

If we assume the elasticities of investment goods prices and investment goods output with respect to monetary investment are I_p and I_0 respectively,

$$K_R = \frac{P_i}{P_y}\cdot\frac{e_0}{I_0}K_m \quad \text{..........................(4)}$$

P_i and P_y, however, are in terms of index numbers and are 1 respectively at the base point of time. Therefore we can neglect P_i/P_y. Then

$$\frac{K_R}{K_m}=\frac{e_0}{I_0}=\frac{\frac{dY}{Y}-\frac{dP_y}{P_y}}{\frac{dI}{I}-\frac{dP_i}{P_i}}\cdot\frac{\frac{dI}{I}}{\frac{dY}{Y}} \quad \text{..................(5)}$$

It is evident that the ratio K_R/K_m will be determined, if the values of $\frac{dY}{Y}$ and $\frac{dI}{I}$ are given, solely by the relative variations of the two price levels. In this case the relative price will exert a dominant influence upon the gap of K_R and K_m.

Further if we assume $I_0=1$, i.e., that in the investment goods industry, prices are constant, the real income multiplier in this case $K_{\bar{R}}$ will be

$$K_{\bar{R}} = K_m e_0 \quad \text{................................(6)}$$

This means that although the initial investment is instantaneously embodied in real capital equipment, $K_{\bar{R}}$ relative to K_m declines owing to the price-rise in other industries. This fact will happen even if the price-rise of investment goods is steeper than that of consumer's goods, since, while the investment will be embodied at every current price level in physical capital goods, the consumers will be in a position to purchase consumer's goods only after the lapse of a further price-rise. Keynes criticized the forced saving theory by his own new theory of multiplier.[9] but his criticism is inadequate because the multiplier process will ordinarily accompany a price-

[9] J.M. Keynes, *General Theory*, pp. 79–81.

rise and inevitably cause a time lag, thus enforcing upon the masses forced saving to some degree. The ratio $K_{\overline{R}}/K_m$, therefore might be an indicator of the degree of forced saving.

The equation (6) $K_{\overline{R}}=K_m e_0$ is valid when a price-rise does not occur in the investment goods industry, as above explained, but it may be used for another purpose, i.e., when we are mainly concerned with the problem of the production effect of monetary investment or monetary government expenditures. Then, by multiplying the money income multiplier by e_0, we can estimate the proportional contribution of monetary investment to the rise of production as compared with the successive increase of money income.

The statistically computed multiplier we frequently come across, however, is not the money income multiplier K_m but the real income multiplier in the form of K_r. The next problem is how to derive $K_{\overline{R}}$ when K_r is known.

By transforming (3)

$$K_m = \frac{K_r}{1-e_p\left(1-K_r\dfrac{S}{Y}\right)} \quad \text{.....................}(3')$$

and remembering the equation (6), we derive

$$K_{\overline{R}} = \frac{K_r e_0}{1-e_p\left(1-K_r\dfrac{S}{Y}\right)} \quad \text{.....................}(7)$$

From this equation,

$$\text{if} \quad e_0 = 1 \longrightarrow K_{\overline{R}} = K_r = K_m$$
$$\text{if} \quad e_p = 1 \longrightarrow K_{\overline{R}} = 0$$

and, if for example $e_p=0.3$, $\dfrac{S}{Y}=0.1$ and $K_r=2.8 \rightarrow K_{\overline{R}}=2.5$. Consequently the ordinary real income multiplier K_r is greater than *the production multiplier of monetary investment* $K_{\overline{R}}$ which is causally more significant. It is erroneous to think that monetary investment or government expenditures will yield its K_r times real income. By the equation (7), we should consider, instead, that its $K_{\overline{R}}$ times of real income would be generated. In this sense, the Keynesian theory of multiplier, has so far unconsciously given serious illusion to economic students. It is not essential for the *ordinary* multiplier whether the multiplier process is an output expanding process or price-rise process, since the nucleus of the multiplier is substantially monetary.

III. *Keynes' Mistake on the Theory of Price and Output Elasticities*

Our simple attempt at harmonization of the multiplier theory, on the one hand, and the $e_o \cdot e_p$ theory, on the other hand, was developed in section II. We intend, here, to digress from the main description, and to give a close inspection as to Keynes' own theory of price and output elasticities with respect to effective demand.[10] Present-day Keynesians are quite oblivious of Lord Keynes' brilliant Chapter 20, "The Employment Function" and Chapter 21 "The Theory of Prices," as if they were one of the "red herrings" of the General Theory, but it may be necessary to make the trade cycle theory a vivid picture of actuality to utilize those chapters. In this section, however, we confine ourselves within the criticism of Keynes $e_o \cdot e_p$ analysis.

In the preceding section we defined $e_o \cdot e_p$ in money terms, i.e., $e_o = \frac{dO}{O} \cdot \frac{Y}{dY}$ and $e_p = \frac{dP}{P} \cdot \frac{Y}{dY}$, by the total differential of the identity $Y = O \cdot P$. Keynes, however, defines them in wage terms, i.e., $e_o = \frac{dO}{O} \frac{Y_w}{dY_w}$, and $e_p' = \frac{dP_w}{P_w} \cdot \frac{Y_w}{dY_w}$, by the total differential of the identity $Y_w = O \cdot P_w$, where $Y_w = \frac{Y}{w}$ and $P_w = \frac{P}{w}$ and w is the money wage level. Henceforth, we shall use D and D_w in place of Y and Y_w, following Keynes' notations. Consequently, Keynes' first formula is

$$1 = e_o + e_p' \qquad \text{(8)}$$

$e_o = \frac{dO}{O} \frac{D_w}{dD_w}$ and $e_p' = \frac{dP_w}{P_w} \frac{D_w}{dD_w}$. This means the percentage change of the effective demand in wage terms will be absorbed by the percentage change of prices in wage terms and that of output $\left(\text{i.e., } \frac{dD_w}{D_w} = \frac{dP_w}{P_w} + \frac{dO}{O}\right)$. But the rise of P_w is equal to the decline of real wages $\frac{w}{P}$, and the decline of P_w to the rise of $\frac{w}{P}$. Now if we define $W_p = \frac{w}{P}$,

$$e_p' = -\frac{dW_p}{W_p} \Big/ \frac{dD_w}{D_w} \qquad \text{(9)}$$

because $\frac{dP_w}{P_w} \Big/ \frac{dD_w}{D_w} = \left(\frac{dP}{P} - \frac{dW}{W}\right) \Big/ \frac{dD_w}{D_w} = -\left(\frac{dW}{W} - \frac{dP}{P}\right) \Big/ \frac{dD_w}{D_w} =$

[10] I previously tackled this problem, in my book "*Employment and Wages*," (written in Japanese) 1949, but there is another small essays in 1946, which first paid attention to the error in Keynes' $e_0 \cdot e_p$ theory.

$-\frac{dW_p}{W_p}\Big/\frac{dD_w}{D_w}$. e_p' is now transformed into an indicator of real wage variation from that of price flexibility. Therefore, we can see nothing of the theory of prices in the equation (9). It may rather be a formula of decreasing returns in response to the rise of output or employment. It is quite natural that Keynes himself rewrites the notations in (8), before going from Chapter 20 "The Employment Function" to Chapter 21 "The Theory of Prices." But in order to express them in money terms, we may replace the equation (8) by the following equation.

$$1=\frac{dO}{O}\Big/\frac{dD}{D}+\frac{dP}{P}\Big/\frac{dD}{D} \quad \text{......................}(10)$$

Strangely enough, Keynes obtains different formula in money terms,

$$1=e_p+e_o(1-e_w) \quad \text{..........................}(11)$$

where $e_p=\frac{dP}{P}\Big/\frac{dD}{D}$ and $e_w=\frac{dW}{W}\Big/\frac{dD}{D}$. The equations (10) and (11) clearly contradict. Which is correct? As will be noticed, e_p in this equation is not in money terms. From (11), we derive

$$e_o=\frac{1-e_p}{1-e_w}=\frac{\frac{dD}{D}-\frac{dP}{P}}{\frac{dD}{D}-\frac{dW}{W}}=\frac{dO}{O}\Big/\frac{dD_w}{D_w}$$

e_o is not $\frac{dO}{O}\Big/\frac{dD}{D}$ but $\frac{dO}{O}\Big/\frac{dD_w}{D_w}$ in the notations of Keynes, i.e., in wage terms. Consequently, the equation (11) is expressed in mixed terms. Formally Keynes himself is right, because e_o is the same not only in (8) but also in (11). A few problems, however, remain.

Firstly, in Chapter 21, section 6, he says "The condition $e_w=1$ means that the wage-unit in terms of money rises in the same proportion as the effective demand, since $e_w=\frac{DdW}{WdD}$; and the condition $e_o=0$ means that output no longer shows any response to a further increase in effective demand, since $e_o=\frac{D_dO}{O_dD}$."[11] He clearly recognizes $e_o=\frac{DdO}{OdO}$, i.e., in money terms, if so, the equation (11) is mathematically erroneous. If we take $e_o=\frac{D_wdO}{OdD_w}$, the above verbal explanation becomes incorrect. At any rate he causes an unnecessary confusion.

The *second* problem is substantial, as against the formal dilemma in the first. The problem is why only e_o in (11) must be expressed in wage terms. In fact, e_o in wage terms cannot be compared with e_p or e_w in

[11] J.M. Keynes, *op. cit.*, p. 304.

peried	1	2
D	100	110
P	100	109
O	100	101
W	100	109
D_w	100	101
P_w	100	100

money terms on an equal footing. For instance, if there occurs a price rise of 9%, a production increase of 1%, and 10% rise in effective demand, and further the parallel movement of wages and prices, then e_o in wage terms becomes 1, while e_o in money terms is 0.1. This is an astonishing difference. When e_o in wage terms $(=1)$ is compared with e_p in money terms $(=0.9)$, we cannot judge whether the economy is inflationary or not.

As already explained, e_o in wage terms$=\frac{1-e_p}{1-e_w}$ or $\frac{\frac{dO}{O}\Big/\frac{dD}{D}}{1-e_w}$. From this we derive a few consequences.

1. If $\frac{dP}{P}=\frac{dW}{W}$, $e_p=e_w$. $\therefore$ e_o in wage terms$=1$, except when $e_p=e_w=1$. Therefore, except in the indeterminate case when the percentage change of production is zero, the parallel movement of prices and wages will always make e_o in wage terms$=1$, however severe the inflation may be.

2. When production is constant, $\frac{dO}{O}\Big/\frac{dD}{D}$ is always zero. Therefore e_o in wage terms is always zero, except when $e_w=1$. In the case when $\frac{dO}{O}=0$, $\frac{dD}{D}=\frac{dP}{P}$, and $\frac{dP}{P}\neq\frac{dW}{W}$ (inflationary case), we have $e_p=1$ and $e_o=0$. But at the critical point when e_w is unity, e_o becomes indeterminate.

3. Mathematically the above formal argument may be possible. But, as an actual problem, the constancy of production and a 0.2% rise of production may be identified. On the other hand, it does not make any difference $\frac{dP}{P}=\frac{dW}{W}$ or $\frac{dP}{P}-\frac{dW}{W}=0.2\%$. e_o in wage terms is thus a very delicate and unstable coefficient which may be unity or zero at random by an operation of raising or dropping fractions. Whereas the economy itself might be in a state of hyper-inflation, whether e_o is zero or unity. It is unrealistic that $\frac{dO}{O}$ becomes completely zero, but it may be more probable that the percentage increase of P and W is close to equivalence in hyper-inflation. Consequently, we shall not go too far in saying that e_o in wage terms is in the neighborhood of unity, since the difference be-

tween $\frac{dP}{P}$ and $\frac{dW}{W}$ may usually be negligible in true inflation, notwithstanding the trivial positive or negative changes of production.

IV. *Some Statistical Analysis*

We shall now stop our theoretical consideration of the relation of real and money income multiplier, but attempt to supplement our article with some statistical, though preliminary, analysis.

Before analysing the Japanese economy, we shall dwell on the American economy. Since the recent Consumption Function Controversy, we have many consumption functions computed in relation to the American economy, but they are usually personal consumption functions with personal disposable income as their major variable. Consequently they are probably as inadequate as those from which we derive a national wide multiplier, because they do not take into consideration "business savings," as a leakage besides personal savings. For this reason, we shall prefer national income to disposable income as a determinant of consumption expenditure thus composing two sorts of consumption functions, one in real terms, the other in money terms. Utilizing the Department of Commerce national income estimates and the BLS consumers' price index, the real consumption function would be (suffix r means real term)

1929–1940 and 1946–47 $C_r=20.5+0.641\ Y_r;\ k_r=2.786$
1941–1943 $C_r=64.7+0.1254\ Y_r;\ k_r=1.143$

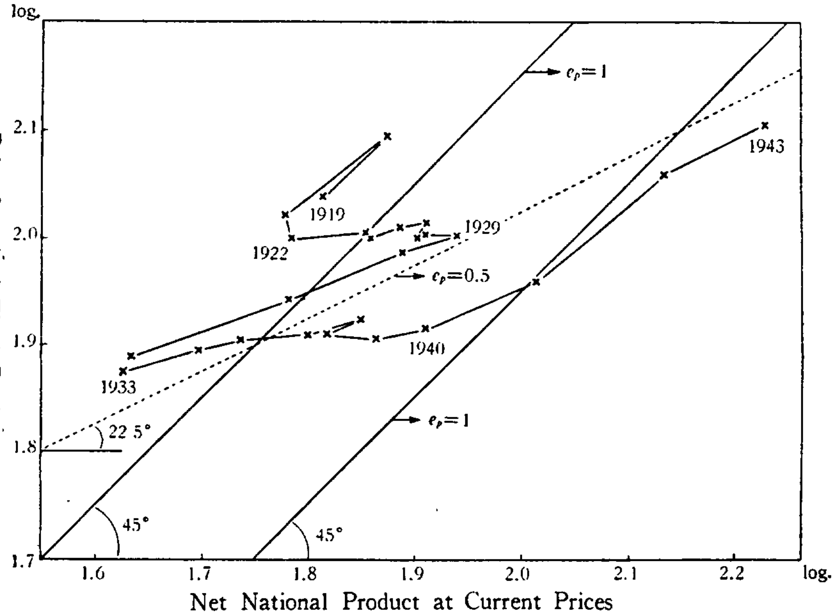
On the other hand, in money consumption function, composed of national income and consumption expenditures in current prices, the marginal propensity to consume is 0.714 in the former period (k_m=3.5) and 0.333 in the period 1941–44 (k_m=1.5)

What are, then, computed values of $e_o \cdot e_p$? We have tried to draw a Chart 2, with NNP on the horizontal axis, and with a price level on the vertical axis, each measured in logarithms, in order to grasp the behavior of e_p graphically.[12] As is easily seen, the slope of the curve shows the elasticity e_p, because it represents $\frac{d\log P}{d\log D}$. The results are as follows. At the end of the First Great War, e_p becomes a relatively high value, showing a very flexible price responsiveness to the monetary effective demand (1920=0.795; 1921=0.735). On the contrary, during the period 1922–1929, the price level was extremely stable, notwithstanding the rise of American

[12] The data is taken from S. Kuznets: *National Products since 1869*, 1946, pp. 55–56. The index of the price-level for final products is derived from a comparison of NNP in current prices with NNP in 1929 prices.

production, as if and $e_p \doteqdot 0$ (1923=0.09; 1929=−0.114). Owing to the general increase of capacity output, and the invariable level of employment during this period, the bottlenecks, such as full empleyment and full capacity, did not come into view, which made possible the extraordinary high stability of the price level. Almost zero value of e_p shows not a stage of depression but a phase of high productivity enhanced by successive investments.

Chart 2. Effective Demand and Price Level in America



Net National Product → from S. Kuznets: *National Product since 1869*

Price-level of final product → $\frac{\text{NNP in current prices}}{\text{NNP in 1929 prices}}$

Next, in the recession from 1929–1933, we obtain $e_p=0.41$, and by a comparison of 1929 and 1932, $e_p=0.37$ is obtained. This exhibits the *asymmetry* of e_p between the prosperity and recession of business cycles. Such asymmetry reappears in the recovery phase of 1933–1940, causing a smoother slope of $e_p(e_p=0.095)$ than in the recession. The reader will find *the Z-shaped behavior of price-effective demand curve* during one business cycle. This asymmetry is different from what Lord Keynes once pointed out. The asymmetry between Inflation and Deflation, pointed out by Keynes,[13] was

[13] J.M. Keynes, *op. cit.*, p. 291.

the fact that whilst a deflation of effective demand below the level required for full employment will diminish employment as well as prices, an inflation of it above this level will merely affect prices. But the fact that such inflation did not occur in 1920's, has made the Z-shape behavior of prices different from what Keynes once expected. After the Second World War, e_p becomes greater, 1941=0.418; 1942=0.879; 1943=0.438. Such regularities of e_p during business cycles will be a first step not only in the synthesis of real and money income multiplier, but also in grasping in an organic manner the whole behavior of prices and output.

We can draw a similar type of graph (Chart 3) also with reference to Japan. It is interesting that we again find the *Z-shape pattern* (*but tilting to the left*). The characteristics of price behavior in Japan lies in the very fact of this tilting. What causes this tilting? First cause is the secular upward trend of the price level since the Meiji era, second is the extraordinary downward flexibility of prices in the period 1920–1931, in spite of the still continuing rapid growth of industrial production during the same period. The fluctuation of e_p is as follows. During the first World War, 1915–1920, e_p=0.868. In the price-fall period, 1925–31. e_p=2.214. (this is an abnormal case when prices and output exhibit an opposite movement). In the recovery phase 1931–33, e_p becomes the very high value 1.238. But in the period 1933–36, e_p=0.344, and in 1937–39 period, e_p=0.433. Only in 1937, e_p=1.309. In the war period, 1939–42, e_p amounted to 1.438. From

Chart 3. Effective Demand and Price Level in Japan

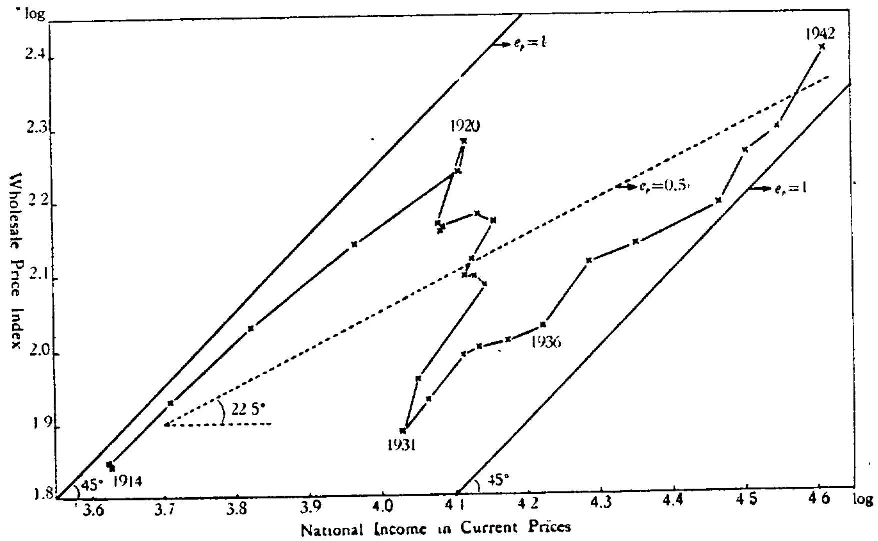
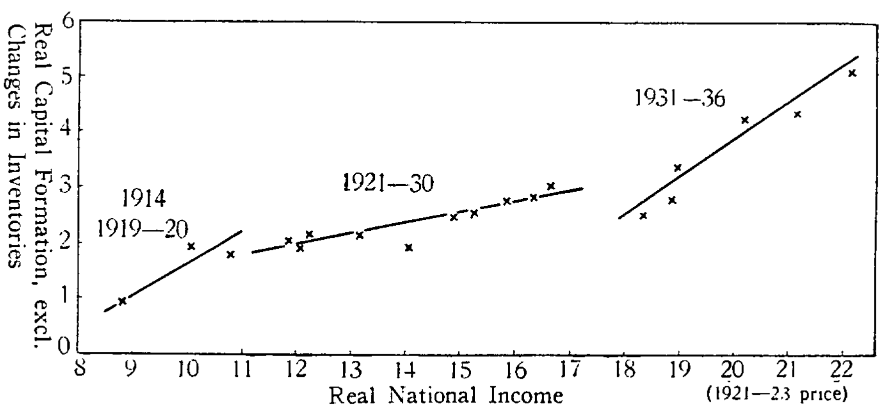


Chart 3, we can see a steeper slope of e_p in Japan, as compared with the American economy. The breadth of the Z-shape pattern is also greater, which shows the greater flexibility of the Japanese price level. These present a striking contrast with the American case (Chart 2). These peculiarities are also the consequence of the almost cycleless rapid tempo of economic growth in Japan (especially with reference to the growth of the manufacturing industry).

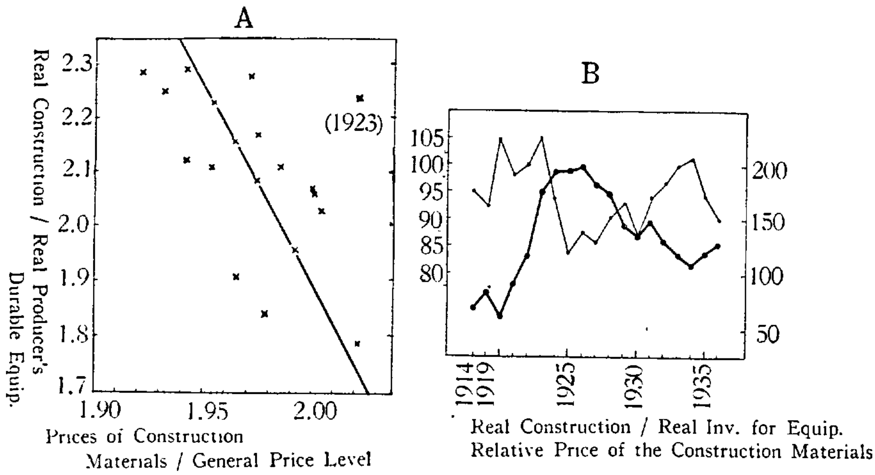
Now what is the value of multiplier? To our regret, there has been no reliable estimate of capital formation or consumption expenditures in Japan. Especially the estimate of capital formation by the commodity-flow method has never been undertaken. Recently I have ventured to work with an estimate and obtained some results, (which was published in the "Keizai Kenkyu" (the Economic Review) Vol. 4, No. 1, Jan. 1953, in Japanese, but an English "Digest of Statistical Research," 'Capital Formation of Japan' is attached to it), but my work is confined to an estimate of private and public producers' durable equipment and construction, and excludes inventory-changes. Moreover, we could not estimate capital consumption. Therefore, as a first preliminary approach, we have compared the net national income series with the gross capital formation excl. inventory-changes both in real terms.[14] (We have omitted the graph in money terms, since the stability of the investment-income schedule in money terms is smaller than in real terms). From this we shall not be able to grasp the true value of the multiplier, but its likely behavior during every phase of business cycle will be observable.

Chart 4. National Income and Capital Formation in Japan



[14] The real national income and capital formation in Chart 4 is obtained by using a common deflator, the wholesale price index. Therefore the graph shows some difference from the graph (Chart 1) in "Keizai Kenkyu" (The Economic Review) Jan. 1935, because in the latter the price index of construction materials is used in deriving the real value of construction.

Chart 5



Our study throws light upon the following interesting relations.

Firstly, when, as in Chart 4, the real capital formation (excluding inventory changes) is compared with the real national income, three sets of investment curve can be fitted, 1914–20; 1921–30; 1931–36. Each curve is a linear straight line and the variance of scattered points is extremely small. In the two periods, 1914–20. and 1931–36, which can be characterized as prosperous price-rise phases, the slopes of investment curves are fairly steep, while during the price-fall period of over 10 years (1921–30), the slope becomes smooth. In other words, in Japan the investment curve (and perhaps the savings function too) alters its slope sensitively in response to the trade cycle. This result appears to correspond contrariwise to the cyclical behavior of labor's relative share in Japan, that is, the latter decreases during a price rise phase and increases during a price-fall phase.

Secondly, a comparison between the real construction/real producer's durable equipment ratio and the relative price index of construction materials is made in Chart 5. In Chart 5, A, there is depicted the negative correlation between them, thus explicitly indicating the strong elasticity of substitution or the large relative price effect upon the relative proportion of the two forms of investments. As a matter of course, public works by the government and reconstruction after the Kanto earthquake disaster in 1923, played a prominent role in promoting the relative weight of construction, but the relative price fall of construction materials exerted a more extensive and permeating influence. In Chart 5, A, one scattered point is 1923. In this year, the Kanto earthquake disaster took place. Chart 5, B, too, exhibits opposite movements of the two forms of investments and their relative prices.

Thirdly, during about 10 years (1920–31), the price level fell by 55%, while on the other hand industrial production rose by 90%. To this puzzling movements the answer may be that during this period the Japanese textile industry achieved a high rate of expansion, which was accelerated by the steep curve of its export. However, during my work upon capital formation, I found another answer, that is, the construction activity during the price-fall period surpassed the decreasing production of equipment, thus leading to the mysterious opposite movement of prices and production. Although I do not know how powerful this effect may be, I believe the discovery of this fact is the most fruitful by-product of our study. Producer's durable equipment, which amounted to ¥ 1,184 milion in 1920 in 1921–24 prices, fell to ¥ 742 million in 1925 in the same const. prices, while real construction increased from ¥ 732 million to ¥ 1,452 million during the same period. The high growth rate of industrial production was not slowed down by this construction activity.

Why the titling Z-shape pattern is embodied in the price level in Japan, is now evident. Although it may be advisable to postpone the analysis of the money and real income multiplier, until an accurate estimate of net capital formation is completed, we may, preliminarily, conclude that even during the price-fall period, expansion of production was still possible not only due to the counteracting rise of construction, and to the extraordinary extension of exports of textile goods, but also due to the rising investment multiplier in the price-fall period.

There is an important limitation with reference to $e_o \cdot e_p$ apparatus, since it assumes that output and price-level are the only functions of effective demand, whereas output is also a function of foreign trade, etc. In Japan the mechanism of prices plays a prominent role in the steady promotion of industry, but a further exposition must be postponed to another article.