EQUILIBRIUM IN INTERNATIONAL TRADE: A DIAGRAMMATIC ANALYSIS OF THE CASE OF INCREASING COST

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The diagrammatic analysis of the equilibrium condition of and the gain from international trade has been developed with reference to the case of increasing cost.[1] The case of increasing cost is most convenient to deal with, because a double maximum condition required for the maximisation of entrepreneurial surplus and satisfactions are explained by the equi-marginal principle.

It is the purpose of this paper to attempt a simple and exact diagrammatic representation of equilibrium and gain in international trade in the case of increasing cost. A rigorous mathematical model is provided in Appendix. It is not the purpose of this paper to add new findings to the analysis hitherto developed, but to present a fundamental chart, particularly a compound offer curve, for the analysis of transfer problem, optimum tariff, technological improvement, economic growth, and so forth.

Although the analysis is extended to a three country trade in the last part of this paper, we shall confine ourselves to the models in which two countries (say England and German as assumed by J. S. Mill) trade with respect to two commodities (say, E-goods and G-goods) which are produced with increasing costs. Each country is, however, assumed to be a single unit, as if it were an individual, or to be consisted of individuals who are exactly alike. Each country is further assumed to have the utility function or community preference scale of its own, although we do not have any intention

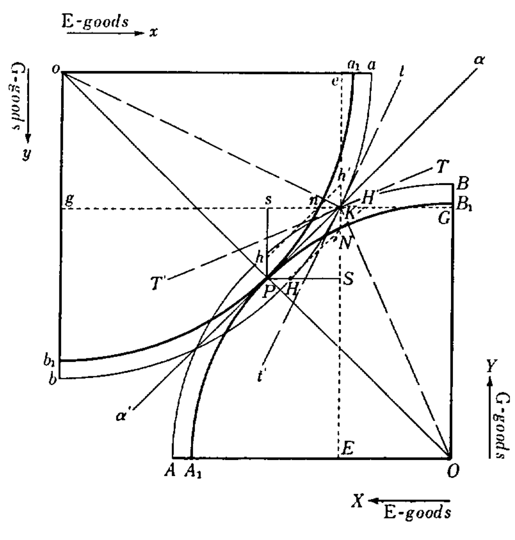
[1] Among the contributions to this subject, the following are important:
Wassily W. Leontief, "The Use of Indifference Curves in the Analysis of Foreign Trade," *Quarterly Journal of Economics*, 1933, reprinted in *Readings in the Theory of International Trade*, ed. H. S. Ellis and Lloyd A. Metzler, 1949.
Abba P. Lerner, "The Diagrammatical Representation of Cost Conditions in International Trade," *Economica*, Aug. 1932; "The Diagrammatical Representation of Demand Conditions in International Trade," *Economica*, Aug. 1934; both are reprinted in *Essays in Economic Analysis*, Macmillan, 1953.
R. E. Baldwin, "Equilibrium in International Trade: A Diagrammatic Analysis," *Quarterly Journal of Economics*, Nov. 1948.
Gottfried Haberler, "Some Problems in the Pure Theory of International Trade," *Economic Journal*, June 1950.
J. E. Meade, *A Geometry of International Trade*, George Allen & Unwin, 1952.
Charles F. Haywood and Theodore K. Ruprecht, "A Note on the Use of the Box Diagram in International Trade Theory," *American Economic Review*, June 1954.

to inquire into the way of constructing the community preference scale. All the models in this paper represent a static general equilibrium system, in which production function, utility function, and pre-trade quantity of production and consumption and prices are given at the outset. It is assumed that both countries behave to maximise the sum of their gains from trade and to keep the balance of trade in equilibrium. It is also assumed, as usually done, that transportation costs and trade barriers do not exist.

I. *Model I: Pure Specialisation Exchange*

Let us suppose that two countries exchange between each other the increment of production, in which each country has a comparative advantage, so as to maximise their entrepreneurial surplus. The entrepreneurial surplus means the difference between the revenue from exports and cost needed to produce the exports, the revenue and cost being measured in terms of numéraire (E-goods). We have then a pure specialisation exchange model. It serves to explain the gain from the international division of labor carried along the line of comparative advantage, since it does not take into consideration changes in the consumption of both countries.

Fig. 1



In Fig. 1, the German production fan, *oab*, is superimposed upon the English production fan, *OAB*, so as to make each pre-trade equilibrium point coincide at *K*. Curves *AB*, A_1B_1, etc. are transformation (or opportunity cost) curves for England, and *ab*, a_1b_1, etc. are those for Germany. Each transformation curve shows constant returns to scale. Any combination of the two commodities on the curve is produced with a constant amount of resources (or "bales" in Marshallian terminology). The transformation curve is concave to the origin because of the law of diminishing returns to varying proportions of production factors, or, because of the increasing cost for each commodity.[2] For the simplicity and exactness of charting, the transformation curves are supposed to be a concentric circle, the center of which is the origin, O for England and o for Germany.

The pre-trade equilibrium is seen at K for both countries. England produced OE of E-goods and OG of G-goods at a price ratio shown by the slope of TT' line, which is tangent to the transformation curve AB. Germany produced oe of E-goods and og of G-goods at a price ratio shown by the slope of tt' line, which is tangent to the transformation curve ab.

Connecting the origins of both countries, we have a line Oo, which may be called the *producer's contract curve.* The transformation curve of the two countries is always tangent to each other on Oo line, and, consequently, the equilibrium of trade falls on this line.

We may consider two cases for Model I.

Model IA. Let us suppose that the two countries minimise the resources required to obtain through trade the same combination of the two commodities as they had before the opening of trade.[3] Then, the trade gives rise to the economy of resources for both countries.

In Fig. 1, the above assumption takes the form that both countries are ready to obtain the combination of commodities at K. Let us draw a line $\alpha\alpha'$, the international price line, passing through K perpendicularly at P to Oo line. P is the new production point for both countries. Since the line $\alpha\alpha'$ is tangent at P to the transformation curves of the two countries, A_1B_1 for England and b_1a_1 for Germany, the new production point P satisfies such a required equilibrium condition that the marginal rate of production substitution between the two commodities is equal in both countries to the international price ratio.

Because of the above assumption, the difference of quantity of the two commodities at P and at K is counterbalanced in the following way. England exports PS of E-goods in exchange for SK of G-goods, while Germany exports Ps $(=SK)$ of G-goods in exchange for sK $(=PS)$ of E-goods. The ratio of PS of E-goods to SK of G-goods is equal to the international price ratio, which is represented by the slope of the line $\alpha\alpha'$. The trade is therefore balanced.

The gain from the trade may be seen as the economy of resources for

[2] See, Paul A. Samuelson, *Foundations of Economic Analysis*, Cambridge, Harvard University Press, 1948, pp. 233-235.

[3] Harrod provides us with a formula which determines the equilibrium price ratio from ordinary cost-and-quantity supply schedules under simplified assumptions. One of his assumptions is that the total production of the participants as a whole remains the same. See R. F. Harrod, *International Economics*, 1939, pp. 22-35 and Appendix (pp. 201-203). Our Model IA is accord with his assumptions.

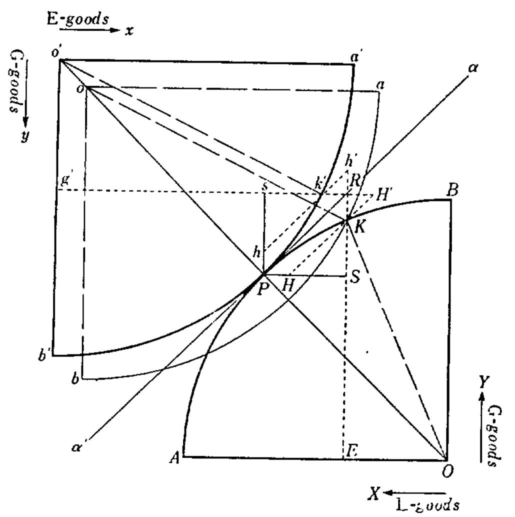
both countries, which is shown by the lowering of the transformation curve, from AB to A_1B_1 for England and from ba to b_1a_1 for Germany.

The gain from the trade may, however, be measured more exactly as the entrepreneurial surplus. Let the intersection of the transformation curve A_1B_1 with vertical line KE be N. Passing through N, let us draw line HNH' in parallel to the international price line $\alpha\alpha'$. As is clear from the transformation curve, PS of E-goods is produced with the same resources as required to produce SN of G-goods. In other words, the cost of PS of E-goods is SN of G-goods. SN of G-goods is, however, equal to HS of E-goods in terms of the international price ratio. Therefore, the entrepreneurial surplus for England is $PS-HS=PH$ or KH' in terms of E-goods (numéraire).

If we calculate the entrepreneurial surplus for Germany in a similar way as above, it will be nK in terms of E-goods. The sum of the entrepreneurial surplus of the two countries is $nK+KH'=nH'$ in terms of E-goods. If we measure the entrepreneurial surplus in terms of G-goods, it will be NK for England and Kh' for Germany, their sum being Nh'. It is clear that the entrepreneurial surplus in this case is the same as the resources saved.

Model IB, Let us suppose that the two countries maximise the sum of their revenues in terms of numéraire or of the other commodities by using the same amount of resources as before the opening of trade.[4] The trade has the effect to increase the quantity of commodities to be obtained by each country.

Fig. 2



[4] Model IB is originating from J. R. Hicks, "The Foundation of Welfare Economics," *Economic Journal*, Dec. 1939, p. 702, Fig. 2.

In Fig. 2, the English production fan, OAB, and German production fan, oab, as well as the initial equilibrium point for both countries, K, are the same as in Fig. 1. Let us move the German production fan from oab to $o'a'b'$ until it touches the English production fan at P. As a result of this movement, the German origin o and its initial equilibrium point K move respectively to o' and k'. Let us connect the English origin, O, with the new German origin, o'. Then we have a new producer's contract curve Oo'.

Let us extend a vertical line through K, the English initial equilibrium point, and a horizontal line through k', the German initial equilibrium point. Let their intersection be R. Let us further draw the international price line, $\alpha\alpha'$ passing through R perpendicularly at P to the producer's contract curve, Oo'. Then, the equilibrium conditions of trade are explained in the same way as in Model IA.

At P, the new production point of the two countries, the marginal rate of production substitution between the two commodities is equal with respect to both countries to the international price ratio. PS of English E-goods is exchanged for SR of German G-goods, the ratio of them being equal to the international price ratio. The trade is therefore balanced.

The gain from the trade may be seen as the increase of importable commodities in both countries. For England, the combination of commodities at R as compared with that at K shows the same amount of exportable commodities (OE of E-goods) and a larger amount of importable commodities (ER of G-goods which is larger than EK by the amount of KR). For Germany, the combination of commodities at R as compared with that at k' shows the same amount of exportable commodities ($o'g'$ of G-goods) and a larger amount of importable commodities ($g'R$ of E-goods which is larger than $g'k'$ by the amount of $k'R$).

The gain from trade may, however, be measured more exactly as the entrepreneurial surplus. Passing through K, let us draw a line HKH' in parallel to the international price line $\alpha\alpha'$. Then, as in Model IA, it is clear that the cost of PS of E-goods for England is SK of G-goods or HS of E-goods. Therefore, the entrepreneurial surplus for England is $PS-HS=PH$ or RH' in terms of E-goods (numéraire). By reasoning in similar way, we find that the entrepreneurial surplus for Germany is $k'R$ in terms of E-goods. The sum of the entrepreneurial surplus of these two countries is $k'R+RH'=k'H'$ in terms of E-goods. If we measure the entrepreneurial surplus in terms of G-goods, it will be KR for England and Rh' for Germany, their sum being Kh'. It is clear that the sum of the entrepreneurial surplus of both countries in this case is the same as the sum of increments of importable commodities for both countries, which in turn is equal to vector oo', or Kk'.

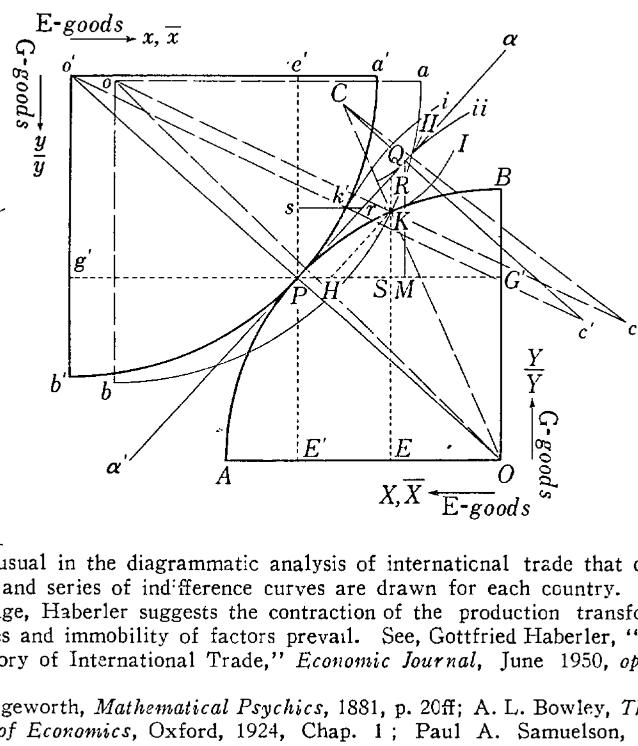
The resources are economised in Model IA, while the importable commodities are increased in Model IB. But, each of them shows that the sum of entrepreneurial surplus of both countries is maximised for different levels of employment of each country. Since the sum of entrepreneurial surplus is larger in Model IB than in Model IA, the former is more desirable than the latter from the standpoint of trade. The level of employment of an economy as a whole depends, however, upon effective demand or the relation between saving and investment. It is possible that one of or both countries may attain a trade equilibrium by leaving some resources unused[5] as shown in Model IA. It is important to know that an equilibrium of trade is attained for each level of employment of the participating countries.

II. *Model II: Compound Exchange*

Let us suppose that two countries maximise their entrepreneurial surplus, spending their revenue which includes the maximised entrepreneurial surplus, and also maximise their satisfactions. We have then a compound exchange model.

It has been set up representing a *pure exchange*[6] or pure consumer's

Fig. 3



[5] It has been usual in the diagrammatic analysis of international trade that only one transformation curve and series of indifference curves are drawn for each country. To the best of author's knowledge, Haberler suggests the contraction of the production transformation curve if the rigid prices and immobility of factors prevail. See, Gottfried Haberler, "Some Problems in the Pure Theory of International Trade," *Economic Journal*, June 1950, *op. cit.*, p. 232, Fig. 2.

[6] See F. Y. Edgeworth, *Mathematical Psychics*, 1881, p. 20ff; A. L. Bowley, *The Mathematical Groundwork of Economics*, Oxford, 1924, Chap. 1 ; Paul A. Samuelson, *Foundations of Economic Analysis*, *op. cit.*, pp. 237-238.

exchange which is based upon consumption (or utility) indifference curves and initial stocks and which aims at the maximisation of satisfactions (or total utilities) of participants of exchange. We have set up in the previous section the pure specialisation exchange model which aims at the maximisation of entrepreneurial surplus of participants of exchange. The compound exchange model in this section aims at the double maximisation of entrepreneurial surplus and satisfactions of participants of exchange.

In Fig. 3, the German production fan, *oab*, and its utility surface are superimposed upon the English production fan, *OAB*, and its utility surface so as to make each pre-trade equilibrium point coincide at *K*. The rule of charting is the following:

(1) It is supposed as before that the production transformation curves for each country are concentric circles with the center at *O* for England and *o* for Germany. By connecting the center of both countries, the producer's contract curve *Oo*, may be drawn.

(2) It is supposed that consumption indifference curves for each country are concentric circles with the center (bliss-point) at *C* for England and *c* for Germany. By connecting the center of both countries, the consumer's contract curve *Cc*, may be drawn.

(3) The pre-trade equilibrium for each country requires such conditions that the marginal rate of consumption substitution equals the marginal rate of production substitution and both of them equal the pre-trade price ratio of the two commodities. This is seen for England at *K*, where English transformation curve, *AB*, is tangent to one of its consumption indifference curves, *I*, and the slope of the tangent is the pre-trade price ratio for England. Similar situation is seen at *K* for Germany, but it is not shown in Fig. 3 for avoiding the complexity of chart. German indifference curves, *i*, *ii*, etc., correspond to its production fan, *o′a′b′*, which is moved from *oab*. Therefore, the initial equilibrium condition for Germany can clearly be seen at *k′*.

(4) The above condition required for the pre-trade or closed-system equilibrium is always satisfied with any point on line *OC* for England and on line *oc* for Germany. We may call *OC* or *oc* line the *economic growth path*. According to the amount of resources put in, a country will grow or shrink along the economic growth path unless there happens any foreign trade or change in utility function (or tastes) and in production function (or technological improvements).

Before the opening of trade, as it is seen in Fig. 3, the consumer's contract curve *Cc* is not parallel to the producer's contract curve *Oo*. Let us move German production fan and utility surface keeping the former in touch with English production fan until the consumer's contract curve, *Cc′*, is parallel to the producer's contract curve *Oo′*. As a result of the movement, German origin *o*, its initial equilibrium point *K*, and its bliss-point *c*, will respectively move to o', k', and c'.

The German production fan, $o'a'b'$, touches the English production fan, OAB, at P which is on the producer's contract curve Oo'. Passing through P, let us draw a line $\alpha\alpha'$ perpendicularly at Q to the consumer's contract curve Cc'. At Q one of. English indifference curves, II, touches one of German indifference curves, ii, and both are tangent to the international price line $\alpha\alpha'$. P is the post-trade production point and Q is the post-trade consumption point for both countries. P and Q satisfy such double equi-marginal condition of international equilibrium that the marginal rate of production substitution for each country and the marginal rate of consumption substitution for each country are common and the same as the international price ratio. The double equi-marginal condition satisfies the double maximisation of entrepreneurial surplus and satisfactions.

The difference between production and consumption in one country will be exchanged for that in other. In other words, PM of English E-goods is exchanged for MQ of German G-goods, the ratio of them being equal to the international price ratio. The trade is therefore balanced.

The gain from the trade may be seen in two aspects. Firstly, the trade gives rise to the entrepreneurial surplus for both countries. Passing through K, let us draw a line HK in parallel to the line $\alpha\alpha'$. Then, we find the entrepreneurial surplus for England as PH in terms of E-goods (numéraire). Let us draw a horizontal line through k', and let the intersection of that line with $\alpha\alpha'$ line be r. Then, we find the entrepreneurial surplus for Germany as $k'r$ in terms of E-goods.

Secondly, the trade gives more satisfactions to both countries. This is shown by the fact that the indifference curve has moved from the initial one to a higher order one, i.e., from I to II for England and from i to ii for Germany. The increase in satisfactions is also shown by the fact that the radius of consumption indifference curves which are represented by concentric circle becomes smaller, i.e., from CK to CQ for England and from $cK=c'k'$ to $c'Q$ for Germany.

It is important to recognise that the increase in satisfactions cannot be attained without the maximisation of the entrepreneurial surplus.

We have explained in the above with reference to Fig. 3 a case in which both countries maintain full employment and attain the equilibrium of international trade. It is, however, possible that, because of the shortage of effective demand on the part of a country as a whole, one of or both countries attain the equilibrium of international trade by leaving some resources unused.

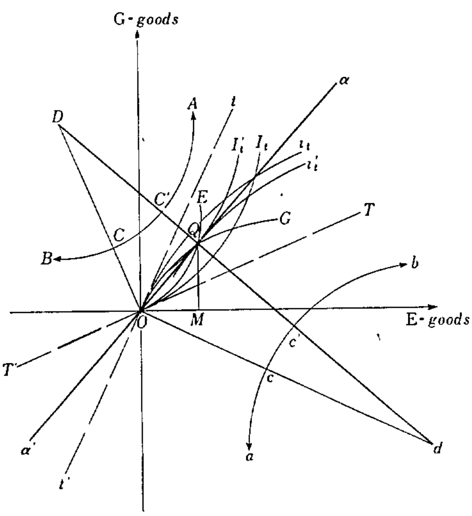
III. *The Compound Offer Curve*

If a reciprocal demand and supply curve or offer curve is drawn as the locus of points of tangency of consumption indifference curves to price lines which pass through an origin, it is Hicksian price-consumption curve.[7] The offer curve of that nature is presented by Leontief.[8] Since it does not involve any change in production due to the opening of trade, it may be called a *simple offer curve.* It is questionable whether the offer curve drawn by Marshall[9] or Edgeworth[10] is that kind of cffer curve or a more complex one which comprises not only changes in consumption but also changes in production due to the opening of trade. It seems to the author that the latter alternative is true.

Let us call the offer curve involving changes both in consumption and production a *compound offer curve.* The compound offer curve is, first, drawn by Baldwin[11] as the locus of vectors between production and consumption points for each price ratio, when those vectors are redrawn from an origin.

Meade[12] was the second who drew the compound offer curve. He draws the *trade indifference curve* as the locus of movement of the origin of production fan when the production fan is slided along a consumption indiffer-

Fig. 4



[7] J. R. Hicks, *Value and Capital*, Oxford, 1939, p. 30.
[8] Wassily W. Leontief, "The Use of Indifference Curves in the Analysis of Foreign Trade," *Readings, op. cit.*, p. 231, Fig. 2.
[9] Alfred Marshall, *The Pure Theory of Foreign Trade*, 1879, reprinted in No. 1 in Series of Reprints of Scarce Tracts in Economics and Political Science, London School of Economics, 1930; and *Money, Credit and Commerce*, London, 1923, Appendix J.
[10] F. Y. Edgeworth, "The Pure Theory of International Trade," *Economic Journal*, 1894, p. 425, Fig. 1.
[11] R. E. Baldwin, "Equilibrium in International Trade: A Diagrammatic Analysis," *op. cit.*, p. 751, Fig. 2.
[12] J. E. Meade, *A Geometry of International Trade, op. cit.*, Figs. I-IV and Chap. II.

ence curve by keeping them in touch. Then, the compound offer curve is drawn as the locus of tangency of the trade indifference curves to price lines which pass through an origin.

The trade indifference curves and compound offer curve may be easily drawn under our simplified assumption that consumption indiffernce curves and production transformation curves are both concentric circles.

In Fig. 4, let us draw the pre-trade equilibrium price line for England, TT'. Let the economic growth path for England be OD, which is perpendicular at O to TT' line. The English economic growth path, OD, consists of two parts: the pre-trade radius of consumption indifference curves, OC, which is KC in Fig. 3, and the pre-trade radius of production transformation curve, which is OK in Fig. 3. The production transformation curve is shown as AB. Let us draw a concentric circle with D as its center. Then, we have trade indifference curves for England, It, It', etc.

The pre-trade price line TT', is tangent at O to the trade indifference curve It. The line TT' is parallel to a tangent line at C to the production transformation curve, AB. The line TT' is also tangent at O to one of consumption indifference curves, the center of which is C. Therefore, the double equi-marginal condition required for the pre-trade equilibrium is satisfied.

Similarly, let us re-draw the economic growth path for Germany, oc in Fig. 3, as Od in Fig. 4. Then, we may draw the trade indifference curves for Germany, it, it', etc., with d as its center.

It is now easy to draw the compound offer curve for each country as the locus of points of tangency of the trade indifference curves to price lines which pass through the origin O, i.e., the curve OE for England and curve OG for Germany. The international equilibrium point may be found as the intersection of the two compound offer curves, Q.

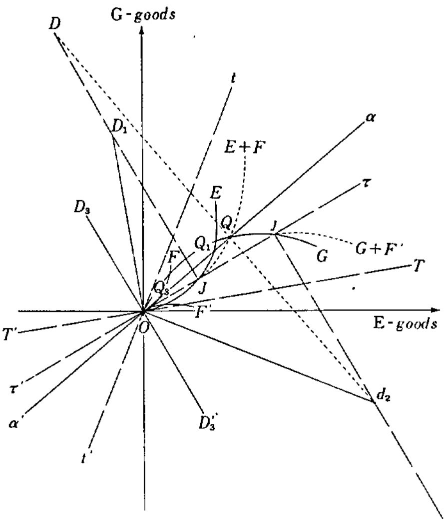
The international equilibrium point Q may, however, be found in another way. Let us connect the center of trade indifference curves of the two countries. Then we have a compound contract curve Dd. Let us draw the international price line $\alpha\alpha'$ passing through O perpendicularly at Q to the line Dd. Let us mark M as the intersection of vertical line through Q with horizontal line through O. Thus, it is clear that OM of English E-goods is exchanged for MQ of German G-goods at the price ratio, the slope of the line $\alpha\alpha'$.

At the international equilibrium point Q, the trade indifference curve of each country, It' for England and it' for Germany, is tangent to each other and to the international price line $\alpha\alpha'$. Also at Q, one of English consumption indifference curves, the center (bliss-point) of which is C', touches one of the German consumption indifference curves, the center of which is c', and both are tangent to the international price line $\alpha\alpha'$. A tangent at C' to an English production transformation curve AB, is parallel to a tangent at c' to a German production transformation curve ab, and both are parallel to the international price line, $\alpha\alpha'$. Therefore, the double equi-marginal condition required for the international equilibrium is satisfied.

A movement from C to C' along an English production transformation curve or from c to c' along a German production transformation curve is the same as the movement of production point from K to P for England or from k' to P for Germany in Fig. 3. It is important to recognise in Fig. 4 that the utility surface for each country moves according to the changes in production as it is shown by the movement of the bliss-point for each country.

The gain from the trade is shown by the fact that the trade indifference curve changes from the pre-trade one to a higher order one, i.e., from It to It' for England and from it to it' for Germany. This fact is also measured by the shortening of the radius of the trade indifference curves from DO to DQ for England and from dO to dQ for Germany.

Fig. 5



A three (or many) country trade[13] can be explained in similar way as in Fig. 4. In Fig. 5, the pre-trade price line, the economic growth path, and the compound offer curve for England are respectively TT', OD_1, and OE; and those for Germany are tt', od_2, and OG respectively. Let us suppose a third country, say France. The pre-trade price line for France is $\tau\tau'$, the economic growth path is either OD_3 or OD_3', both being equal, while the compound offer curve is either OF or OF'. If the international price ratio becomes more favorable to E-goods than the slope of the line

[13] *Cf.* G. S. Becher, "A Note on Multi-Country Trade," *American Economic Review*, Sept. 1952, pp. 567-568.

$\tau\tau'$, then France will trade in the same line as England does and, consequently, OD_3 and OF are effective. If the international price line becomes more favorable to G-goods than the slope of the line $\tau\tau'$, then France will trade in the same line as Germany does and, consequently, OD_3' and OF' are effective.

Let us draw a line DJ passing through D_1 perpendicularly at J to the line $\tau\tau'$, the line DJ being necessarily parallel to the line OD_3. Further let DD_1 be equal to D_3O. Until the international price ratio changes from the slope of the line TT' (English pre-trade price line) to the slope of the line $\tau\tau'$, France cannot compete with England. Therefore, the quantity of trade of E- and G-goods shown by vector OJ is the non-competing quantity of England over France. Once the international price ratio becomes more favorable to E-goods than the slope of the line $\tau\tau'$, France can export E-goods and import G-goods in competition with England in exporting E-goods and importing G-goods. Then, the total of offer curves of England and France will be shown by a dotted curve $E+F$, which is drawn from the total trade indifference curves of England and France which take D as center.

Similarly, let us draw a line dj so as to pass through d_2 and to be perpendicular at j to the line $\tau\tau'$, line dj being necessarily parallel to the line OD_3', and let dd_2 be equal to $D_3'O$. Until the international price ratio changes from the slope of the line tt' (German pre-trade price line) to the slope of the line $\tau\tau'$, France cannot compete with Germany in exporting G-goods and importing E-goods. Therefore, the quantity of E- and G-goods shown by vector Oj is the non-competing quantity of trade of Germany over France. Once the international price ratio becomes more favorable to G-goods than the slope of the line $\tau\tau'$, France can export G-goods and import E-goods in competition with Germany. Then, the total of offer curves of Germany and France will be shown by a dotted curve $G+F'$, which is drawn from the total trade indifference curves of Germany and France which take d as center.

Now the international equilibrium between the three countries may be easily determined. Firstly, the trade equilibrium point is found as the intersection Q of English-and-French offer curve $E+F$, with German offer curve OG.[14] Secondly, let us connect the center of English-and-French trade indifference curves, D, and the center of German trade indifference curves, d_2. We have then a compound contract curve between the three countries, Dd_2. Let us draw the international price line $\alpha\alpha'$, so as to pass through the origin O, and to be perpendicular at Q to the line Dd_2. Q is the international equilibrium point.

At the international eqilibrium point Q, one of English-and-French trade

[14] It may be of no need to mention that OF' and $G+F'$ offer curves are not effective, since the dotted offer curve, $G+F'$, does not intersect with OE offer curve.

indifference curves, the center of which is D, touches one of German trade indifference curves, the center of which is d_2, and both are tangent to the international price line aa'. The equilibrium quantity of trade of England-and-France for Germany is shown by vector OQ. One of English trade indifference curves, the center of which is D_1, is tangent at Q_1 to the international price line aa'. Therefore, the quantity of trade for England is shown by vector OQ_1. One of French trade indifference curves, the center of which is D_3, is tangent at Q_3 to the international price line aa'. Therefore, the quantity of trade for France is shown by vector OQ_3. The total of vectors OQ_1 and OQ_3 equals the vector OQ.

Under such simplified assumtions that both production transformation curves and consumption indifference curves are concentric circles, the maximum principle of the entrepreneurial surplus and that of satisfactions for the two countries, which are required for the equilibrium of international trade, are geometrically analysed, first each principle separately and then simultaneously.

The simplified geometrical analysis makes charting easy and exact. It is particularly useful to draw the compound offer curve and to extend the analysis to a three or many country trade. It will be efficiently applied to the further analysis of transfer problem, optimum tariff, technological improvement, economic growth, and so forth.

If it is assumed that both production transformation curves and consumption indifference curves are not concentric circles, the analysis in this paper ought to be modified accordingly.[15] The equilibrium conditions required, however, remain unaltered as in this paper.

APPENDIX

Model I: Pure Specialisation Exchange

Let us suppose that two countries exchange between each other the increment of production, in which each country has comparative advantage, so as to maximise their entrepreneurial surplus. The entrepreneurial surplus is the difference between the revenue from export and its cost required. Then, we have a pure specialisation exchange model.

Let A, B stand for the initial quantities of production of E-goods and G-goods for the first country (England) and a, b for the second conntry (Germany), X for the increment of production of E-goods in England and Y for the quantity of G-goods which can be produced with the same amount of resources as required to produce X. Similarly, let y stand for the increment of production of G-goods in

[15] Such an equilibrium condition that the international price line is perpendicular to contract lines ought to be shown that an international price line is perpendicular to the tangent of contract curves.

Germany and x for the quantity of E-goods which can be produced with the same amount of resources as required to produce y. Let us further suppose that X is exchanged for y at a price ratio, p, which is the quantitative exchange ratio of E-goods (numéraire) per unit of G-goods.[16]

The production functions for each country are written

(1.1) $G(A+X,\ B-Y)=N$ (constant),

where $\frac{d\,(B-Y)}{d\,(A+X)}<0$, and $\frac{d^2\,(B-Y)}{d\,(A+X)^2}<0$, or, since A and B are constant, $\frac{dY}{dX}>0$, and $\frac{d^2Y}{dX^2}>0$.

(1.2) $g(a-x,\ b+y)=n$ (constant),

where $\frac{d(b+y)}{d(a-x)}<0$, and $\frac{d^2(b+y)}{d(a-x)^2}<0$, or $\frac{dy}{dx}>0$ and $\frac{d^2y}{dx^2}>0$.

N and n stand for a given amount of composite unit of factors of production for each country.

Let the entrepreneurial surplus for each country be

(1.3) $V=X-p\,Y$,

(1.4) $v=p\,y-x$.

Subject to (1.1), maximise V. Then, we have

$$W=V+\Lambda[N-G(A+X,\ B-Y)],$$

(1.5) $\frac{\partial W}{\partial X}=0, \quad 1-\Lambda\frac{\partial G}{\partial(A+X)}=0, \quad \therefore\ \Lambda=1\Big/\frac{\partial G}{\partial(A+X)}$.

(1.6) $\frac{\partial W}{\partial Y}=0, \quad -p+\Lambda\frac{\partial G}{\partial(B-Y)}=0, \quad \therefore\ \Lambda=p\Big/\frac{\partial G}{\partial(B-Y)}$.

(1.7) $p=\frac{\partial G}{\partial(B-Y)}\Big/\frac{\partial G}{\partial(A+X)}$.

Similarly, subject to (1.2), maximise v. Then, we have

$$w=v+\lambda[n-g(a-x,\ b+y)],$$

(1.8) $\frac{\partial w}{\partial x}=0, \quad -1+\lambda\frac{\partial g}{\partial(a-x)}=0, \quad \therefore\ \lambda=1\Big/\frac{\partial g}{\partial(a-x)}$.

(1.9) $\frac{\partial w}{\partial y}=0, \quad p-\lambda\frac{\partial g}{\partial(b+y)}=0, \quad \therefore\ \lambda=p\Big/\frac{\partial g}{\partial(b+y)}$.

(1.10) $p=\frac{\partial g}{\partial(b+y)}\Big/\frac{\partial g}{\partial(a-x)}$.

From (1.1) and (1.7), it follows

(1.11) $X=X(p), \qquad Y=Y(p)$.

Similarly, from (1.2) and (1.10), it follows

(1.12) $x=x(p), \qquad y=y(p)$.

We have an exchange equation

(1.13) $X-p\,y=0$, or $X(p)-p\,y(p)=0$,

which determines the equilibrium value of p. By substituting the equilibrium value of p in (1.11) and (1.12), we have the equilibrium values of X, Y, x and y. Then,

[16] In this appendix, capital letters refer to the first country (England) and small letters to the second country (Germany) except p.

we obtain the equilibrium value of revenue, $A+p(B-Y+y)$ for England and $(a-x+X)+p\,b$ for Germany.

Model II: Compound Exchange

Let us suppose that two countries maximise their entrepreneurial surplus, spending their revenue which includes the maximised entrepreneurial surplus, and also maximise their total utilities. Then, we have a compound exchange model.

As in Model I, subject to production functions

(2.1) $G(A+X,\ B-Y)=N$,

(2.2) $g(a-x,\ b+y)=n$,

maximise V and v. Then, we have

(2.3) $p=\dfrac{\partial G}{\partial(B-Y)}\Big/\dfrac{\partial G}{\partial(A+X)}$.

(2.4) $p=\dfrac{\partial g}{\partial(b+y)}\Big/\dfrac{\partial g}{\partial(a-x)}$.

(2.5) $X=X(p),\qquad Y=Y(p)$.

(2.6) $x=x(p),\qquad y=y(p)$.

Let us suppose that consumers in England choose to consume $A+X-\bar{X}$ and $B-Y+\bar{Y}$ and consumers in Germany $a-x+\bar{x}$ and $b+y-\bar{y}$ so as to maximise their satisfactions. $\bar{X}$ is English excess supply of E-goods and $\bar{Y}$ its excess demand for G-goods, and $\bar{x}$ is German excess damand for E-goods and $\bar{y}$ its excess supply of G-goods.

Budget equations for each country with reference to the foreign trade may be written

(2.7) $\bar{X}=p\,\bar{Y}$.

(2.8) $\bar{x}=p\,\bar{y}$.

Let the utility functions for each country be

(2.9) $U=F(A+X-\bar{X},\ B-Y+\bar{Y})$,

where $\dfrac{d(B-Y+\bar{Y})}{d(A+X-\bar{X})}<0$, or, since A and B are constant and since X and Y are given for consumers, $-\dfrac{d\bar{Y}}{d\bar{X}}<0$, and $\dfrac{d^2(B-Y+\bar{Y})}{d(A+X-\bar{X})^2}=\dfrac{d^2\bar{Y}}{d\bar{X}^2}>0$.

(2.10) $u=f(a-x+\bar{x},\ b+y-\bar{y})$,

where $\dfrac{d(b+y-\bar{y})}{d(a-x+\bar{x})}=-\dfrac{d\bar{y}}{d\bar{x}}<0$, and $\dfrac{d^2(b+y-\bar{y})}{d(a-x+\bar{x})^2}=\dfrac{d^2\bar{y}}{d\bar{x}^2}>0$.

Subject to (2.7), maximise U. Differentiating $F+\Lambda(\bar{X}-p\,\bar{Y})$ with reference to $\bar{X}$ and $\bar{Y}$, we have

$$-\frac{\partial F}{\partial(A+X-\bar{X})}+\Lambda=0,\qquad \frac{\partial F}{\partial(B-Y+\bar{Y})}-\Lambda p=0.$$

Therefore,

(2.11) $p=\dfrac{\partial F}{\partial(B-Y+\bar{Y})}\Big/\dfrac{\partial F}{\partial(A+X-\bar{X})}$.

Similarly, subject to (2.8), maximise u. Then, we have

(2.12) $p=\dfrac{\partial f}{\partial(b+y-\bar{y})}\Big/\dfrac{\partial f}{\partial(a-x+\bar{x})}$.

From (2.7) and (2.11), it follows

(2.13) $\bar{X}=\bar{X}(p).\qquad \bar{Y}=\bar{Y}(p)$.

Similarly, from (2.8) and (2.12), it follows

(2.14) $\bar{x}=\bar{x}(p)$. $\bar{y}=\bar{y}(p)$.

Since in the equilibrium of international trade English export (or excess supply) of E-goods is equal to German import (or excess demand) for the same commodity, we have

(2.15) $\bar{X}(p)=\bar{x}(p)$.

By substituting (2.15) in (2.7) and (2.8), we obtain

(2.16) $\bar{Y}(p)=\bar{y}(p)$.

Thus, the equilibrium value of p is determined either by (2.15) or by (2.16).

Let us denote equilibrium values by suffix $_0$. Then, for England, the equilibrium value of production is

$$(A+X_0)+p_0(B-Y_0),$$

and that of consumption is

$$(A+X_0-\bar{X}_0)+p_0(B-Y_0+\bar{Y}_0).$$

The two values are equal, since

$$\bar{X}_0=p_0\bar{Y}_0,$$

by equation (2.7).

Similarly, for Germany, the equilibrium value of production is

$$(a-x_0)+p_0(b+y_0),$$

and that of consumption is

$$(a-x_0+\bar{x}_0)+p_0(b+y_0-\bar{y}_0).$$

The two values are equal, since

$$\bar{x}_0=p_0\bar{y}_0,$$

by equation (2.8).