

ON BIASES OF THE PRICE-DEFLATOR

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I

With the increasing role that price-deflators play in econometric analysis, the delicate numerical effects caused by using deflator-series must be closely scrutinized. In some cases another model-setting of analysis might be preferred at seeing some unexpected results of one model due to the biases of deflator.

The biases of price-deflators¹ can be classified into three types: the model bias, the formula bias and the data bias. By the model bias is meant the sort of biasing effect caused by a model in which the deflator and the deflatand² are related in some function. In ordinary uses the deflator-series are taken as denominators for the deflatand series under the assumption that the factor of price-level is included in the deflatand money value as a product-factor. This assumption, however, is not due where, for instance, the price-level acts as one of the regressors explaining the money value of the regressand. If we apply the denominator form of the deflator to this regression analysis, a sort of model bias is found in this misapplication.

Secondly, the formula type of bias has a connection to the formula used for the construction of the price-deflator. The Laspeyres formula is well-known to have a limitation in its power of expressing the change of the price-level, to the effect that its use as deflator includes this second type of bias. We can, however, neglect these first two types of bias when possible. But the third type of bias can we never neglect in any case. That is the data-bias which stems inevitably from the discrepancies on the nature of data between the deflatand and the deflator. As a matter of course we try to select appropriate deflator-series so as to fit well to the nature of the deflatand-series. Nevertheless there are apt to remain some discrepancies between the two series—e.g. the total expenditure of a family as the deflatand and the Consumers' Price Indices as deflator, in the point of their "coverage". This sort of discrepancies will be ever deepened so long as the Laspeyres formula will hold without any revision of the weight-system.

Among these three the third type, the data bias type, will be treated in this paper, the other two being put aside for the time being.

¹ The writer intends to include under the term "deflator" many sorts.....such as population-deflator, family-size deflators, quality-deflators, etc., so that the commonly used one to change money-term into real term is duly specified as the "price-deflator."

² The term "deflatand" is used in this paper after the usage of "regressand" and "predictand" in the regression analysis (Wald) and forecasting (Hotelling) respectively.

II

The biasing effect of the third type, the data bias, can be decomposed into a few elements, each of which bears its economic meaning as shown below. Denoting the deflatand (some sort of money value in t-period) by $V_t = \hat{\Sigma} \hat{p}_t \hat{q}_t$ and the deflator (some sort of price-index) by P_{0t} , we can set the deflating operation as

$$V'_t = V_t / P_{0t}$$

where the real value V'_t gets influenced by the data-biases from the existence of discrepancies which may arise between the Σ 's, p 's and q 's in the deflator and the $\hat{\Sigma}$'s, \hat{p} 's and \hat{q} 's in the deflatand. The signs (\wedge) on the symbol of the deflatand are called the "deflatand-signs".

By inserting necessary intermediate terms, we can derive the following identity which expresses an analysis of the deflator-bias in the data type:

$$V'_t = V_t / P_{0t} = \alpha \cdot \beta \cdot \gamma \cdot Q_{0t} \cdot V_0 \dots\dots\dots (A)$$

The process of derivation together with the definitions used are as follows: the price-deflator P_{0t} is assumed for the time being to be the Laspeyres type, $P_{0t} = \Sigma p_t q_0 / \Sigma p_0 q_0$. This whole argument still holds, if the type of the index be different.

$$\begin{aligned} V'_t &= \frac{V_t}{P_{0t}} = \frac{\hat{\Sigma} \hat{p}_t \hat{q}_t}{\frac{\Sigma p_t q_0}{\Sigma p_0 q_0}} \\ &= \frac{\hat{\Sigma} \hat{p}_t \hat{q}_t}{\hat{\Sigma} \hat{p}_t \hat{q}_t} \cdot \frac{\hat{\Sigma} \hat{p}_t q_t}{\hat{\Sigma} \hat{p}_t q_t} \cdot \frac{\hat{\Sigma} \hat{p}_t q_t}{\Sigma p_t q_t} \cdot \frac{\Sigma p_t q_t}{\Sigma p_t q_0} \cdot \frac{\Sigma p_0 q_0}{\hat{\Sigma} \hat{p}_0 q_0} \cdot \frac{\hat{\Sigma} \hat{p}_0 q_0}{\hat{\Sigma} \hat{p}_0 q_0} \cdot \frac{\hat{\Sigma} \hat{p}_0 \hat{q}_0}{\hat{\Sigma} \hat{p}_0 \hat{q}_0} \cdot \hat{\Sigma} \hat{p}_0 \hat{q}_0 \end{aligned}$$

(1) (2) (3) (4) (5) (6) (7) (8)

Now we may, firstly, take up the (I)-term and the (7)-term in the right hand side of this analysing equation. Comparing the denominator with the numerator of these two ratios, we find only one difference in the sign put upon the q 's. The deflatand may be some total value of production and, if so, the \hat{q} 's therein must consequently denote the quantity of production; while the q 's in the deflator may commonly be expected to denote the quantity of transactions, not that of production, since the usual price-index (e.g. the whole sale price indices) is designed according to the transaction standard. Under such a circumstance the (7)-term turns to be a sort of composite ratio which shows how many percent out of the total quantity of production were put to market in the base period, while the (I)-term is the inverse ratio of the same nature for the current period. Therefore, denoting those composite ratios for both periods respectively by β_0 and β_t , we get from (7) and (I)

$$\beta_0 = \hat{\Sigma} \hat{p}_0 q_0 / \hat{\Sigma} \hat{p}_0 \hat{q}_0, \text{ and } \beta_t = \hat{\Sigma} \hat{p}_t q_t / \hat{\Sigma} \hat{p}_t \hat{q}_t.$$

Each of these two ratios, which means in their general form some sort of quantity structure, in each period may be unity and disappear from the identity (A), if we are to deflate total expenditures of average family by the consumers'

price indexes, where both q's included in the deflatand and deflator are the same in nature, namely, the quantity purchased by the family. In general, however, both β_0 and β_t would remain other than unity. so that it is worth while giving them the name of "quantity structure ratios" for the respective period. And the β in the identity (A) is nothing but an inverse index of the over-time change of the quantity structure ratio, i.e.

$$\beta = \beta_0 / \beta_t = (I) \times (7).$$

This inverse index, β , we tentatively call the "coefficient of quantity structure change," or " β coefficient".

Next, we can derive in the similar way from (2)-and (6)-terms what we should like to call the "price diversity ratios," γ_0 and γ_t , and, by combining the two ratios, get the "coefficient of price diversity change" or simply the " γ coefficient":

$$\gamma = \gamma_0 / \gamma_t = (2) \times (6).$$

Quite similarly we can get the "coefficient of coverage change" or " α coefficient" by combining the two coverages, α_0 and α_t , for the base and current periods, which come directly from (3)-and (5)-terms :

$$\alpha = \alpha_0 / \alpha_t = (3) \times (5).$$

There still remain two unexplained terms, (4) and (8). The latter $\hat{\Sigma} \hat{p}_0 \hat{q}_0$ indicates the money value for the deflatand in the base period, that is V_0 , according to the notation of $V_t = \hat{\Sigma} \hat{p}_t \hat{q}_t$. The former, the (4) item, is nothing but a sort of quantity index in the form of Paasche: $Q_{0t} = \Sigma p_t q_t / \Sigma p_t q_0$. This quantity index, however, is what we can get directly from the price deflator itself by using the same p 's, q 's and Σ 's included in the deflator. Therefore, if the deflator is of the Laspeyres type, the derived quantity index is of the the Paasche type and, if the deflator is of the Paasche type, the derived index is the Laspeyres, and so on.

Thus each of the analysed terms can be explained and combined as the product-component into the identity (A). What the identity tells us is that the deflated amount, V'_t , can be explained as the multiplication-product of those five elements: the coefficient coverage change " α ", the coefficient of quantity structure change " γ ", the coefficient of price diversity change " β ", the derived quantity index " Q_{0t} " and the base-period money value of the deflatand, " V_0 ". Of course it is not true that all of the five elements always appear in deflating operations. We may easily see that the first three elements, α , β and γ , can take the value of unity if we choose the deflator so as to keep away any sort of discrepancies against the deflatand in the sense of data biases. But the latter two, Q_{0t} and V_0 , do not disappear in any case. The identity (A) can be shrunk into the identity (B), when the first three components take the value of unity at the same time:

$$V'_t = Q_{0t} \cdot V_0 \dots\dots\dots (B)$$

The identity (B) shows us the fundamental interpretation of the "real term" or the "deflated amount" which is essentially still a money amount (namely, the money amount, V_0 ; multiplied by a quantity-change, Q_{0t}), but its over-time change is parallel to the quantity-change expressed by Q_{0t} (because the base-period amount V_0 is constant over-time). It is for this reason that the combina-

tion of Q_{0t} and V_0 should be duly called the essential part of the deflator-effect, while the other three coefficients, α , β and γ , are better called the biasing parts, because the fundamental part may be biased by their non-unity values, if any.

III

Let us now consider how these parts work in a dynamic economy.

The first bias-coefficient α was defined as a ratio of the base-period coverage α_0 to the current-period coverage α_t : the coverage was in its turn conceived after the common usage, that is, the ratio of the total money amount of index-items to the total money amount of the whole items from among which the index items are selected.

It will be convenient to start from a situation in which we are going to deflate a series of national total amounts of production by a series of wholesale price indices. In designing a price index number, items are usually so selected as to achieve as high a coverage as possible, but as time goes on, new goods may appear, so that the whole extent of production grows larger and larger, and the current coverage α_t becomes smaller than the base-period coverage α_0 . Thus the following relations come into effect:

$$\alpha_0 > \alpha_t, \quad \alpha = \frac{\alpha_0}{\alpha_t} > 1.$$

The possible biasing effect caused by the degree in which α is found larger than unity, may be all the more marked when we deal with the long-term growth of economy.

The result, however, may be somewhat different when we start from another model of deflating operation. If the deflatand-series be some sort of money amount with smaller scope than included in the deflator, e.g. a series of money amount of construction, the α coefficient could be smaller than unity. For, in such a model the extent of the denominator amount in α_0 and α_t , i.e. $\hat{\Sigma} p_0 q_0$ and $\hat{\Sigma} p_t q_t$, is small and definite so that the possible over-time changes come merely from "p" and "q".

The second coefficient β , which was defined as the ratio of the quantity structure ratio β_0 for the base-period to that for the current period β_t , is of the nature that it can vanish away in ordinary deflating operation, if we carefully select deflator series appropriate to the situation. But in such a case as deflating the national total amount of agricultural production by the price-indices of agricultural products, this coefficient plays an important role. Here the \hat{q} 's in the deflatand denote the quantity produced where as the \hat{q} 's in the deflator denote their quantity put to market, so that β_0 and β_t are qualified to denote the rate of merchandization for the respective period. And it is easy to deduce that β_t grows larger than β_0 as the money economy develops in rural society and consequently that the β coefficient tends to become smaller than unity.

$$\beta_0 < \beta_t, \quad \beta = \frac{\beta_0}{\beta_t} < 1.$$

By the way, the β for the industrial products can be interpreted as the synthetic rate of shipment, rather than of marchandization, and may, as such, fluctuate around unity as the business cycles proceed. Thus it is highly probable that $\beta > 1$ in a boom, while $\beta < 1$ in a depression.

The third bias coefficient is the one of price diversity change, γ . Formally speaking, this is what we can derive by substituting the p 's for the q 's in the context of β coefficient. But as the p 's are the central factor in the price-deflator problem, we can select a deflator-series just appropriate to the deflator-series so that the price diversity may vanish. It seems, therefore, that there arises no problem. But we know many cases where we are obliged to use a wrong deflator conciously, e.g. when consistent deflators are desirable for each part of the gross national expenditures, some part of which requires to use the consumers' price index and some the wholesale price index, or when we want to deflate a money amount for some remote area and have no deflator-series appropriate to that area, and so on.

Once this γ coefficient matters, it shows no definite inclination as seen in α and β . We have no other way than to consider its special behavior according to each case.

IV

Thus we are in the position to say that the deflated money amount V'_t suffers biases in value to the extent that those bias-coefficients reviewed above are other than unity. For instance, when $\alpha = 1.3$, other coefficients keeping unity, then the size of V'_t is found to contain a 30% upward-bias. In order to make such an interpretation effective, it is now highly necessary to examine what the fundamental part of the deflated amount ($Q_{0t} \cdot V_0$) does imply. By a slight change of the identity (B) we get

$$V_{0t} = V_t / V_0 = P_{0t} \cdot Q_{0t} \dots\dots\dots (C).$$

This is the well-known identity showing the index of total amounts equals to the product of price index and quantity index. Of course the type of price index must hold a close relation to that of quantity index in this identity in the following way: if P_{0t} is of the Laspeyres type, Q_{0t} necessarily turns out to be of the Paasche, and if P_{0t} is the Paasche, type Q_{0t} is in its turn Laspeyres.³ We have to pay special attention to the latter case, where Q_{0t} takes the type of Laspeyres and, in combination with V_0 , can transform the V'_t amount in (B) into the so-called "amount revaluated by the constant prices":

$$V'_t = Q_{0t} \cdot V_0 = (\sum p_0 q_t / \sum p_0 q_0) \cdot \sum p_0 q_0 = \sum p_0 q_t.$$

Some writers with this relation recommend that the price index as deflator is to

³ Moreover, if P_{0t} = Fisher's type, then Q_{0t} = Fisher's too and if P_{0t} is of the Edgeworth type, then Q_{0t} takes the following form:

$$V'_t = \sum p_t q_t / \frac{\sum p_t (q_0 + q_t)}{\sum p_0 (q_0 + q_t)} = \frac{(1 + Q_{(L)}) \cdot Q_{(P)}}{1 + Q_{(P)}} \cdot V_0,$$

which keeps the necessary dimension for the quantity index.

take the type of Paasche's so that the derived quantity index can take the Laspeyres type. Since the deflatand-signs on the p 's and q 's of V_0 are missed here, this recommendation must be called premature. But we would like to say that this recommendation is powerful enough, not in its numerical effect, but by its existence, to show the nature of the real value or the amount in the real term.

The problem here, however, lies in the discrepancies in the deflatand-signs between Q_{0t} and V_0 . If, in the identity (A), the three bias coefficients are all unity, no problem will arise. On the contrary, if any of those coefficients be other than unity, the discrepancy is sure to arise between Q_{0t} and V_0 . Essentially speaking, the quantity index Q_{0t} must be constructed in accordance with the contents of V_0 as follows: $\hat{Q}_{0t} = \hat{\Sigma} \hat{p}_0 \hat{q}_t / \hat{\Sigma} \hat{p}_0 \hat{q}_0$, for example. \hat{Q}_{0t} is not always Q_{0t} . Therefore in order to make the interpretation effective that the amount in real term is nothing but the base-period amount of the deflatand multiplied by the over-time quantity-change, the original identity (A) should be written as

$$V'_t = \alpha \cdot \beta \cdot \gamma \cdot \delta \cdot \hat{Q}_{0t} \cdot V_0, \quad \delta = Q_{0t} / \hat{Q}_{0t}.$$

A new coefficient δ has appeared here and the biasing part comes to consist of four elements, the product of which can be rewritten as

$$\alpha \beta \gamma \delta = \hat{P}_{0t} / P_{0t}, \quad \text{where } \hat{P}_{0t} = \hat{\Sigma} \hat{p}_t \hat{q}_0 / \hat{\Sigma} \hat{p}_0 \hat{q}_0 \text{ for instance.}$$

Consequently $V'_t = (\hat{P}_{0t} / P_{0t}) \cdot \hat{Q}_{0t} \cdot V_0$.

This tells us a self-evident story that the whole bias problem dealt with above was caused by the simple fact that we used a wrong deflator P_{0t} , instead of the right one, \hat{P}_{0t} . And to make the matter worse, the fourth coefficient appeared on the scene with a disturbing effect to the above-developed analysis: that is, δ does not change its value dependently on α , β and γ .

So the theory runs. In truth, the situation comes from the mere manipulation of an identity. The whole argument developed, however, is detailed enough, we believe, to imply something to the practise of econometric analysis. For one thing, our identity (A) can work efficiently enough, if $\hat{Q}_{0t} \doteq Q_{0t}$, or $\delta \doteq 1$. If the assumption does not hold, we will be able to improve the exactness of the deflating operations through some possible short-cut estimations of the coverage change, the rates of merchandization, etc., without designing specific \hat{P}_{0t} 's for ever-changing individual cases.⁴

⁴ The writer obtained numerical facts from the C.P.I. data in Japan to the effect that the coverage-change coefficient α for 1955, the base-year being 1951, showed 1.12 for the general index and reached as high as 1.4 for housing, clothing, etc.

(wrong) (correct)

p.8 (near center) $\frac{\widehat{\Sigma} p_t q_t}{\widehat{\Sigma} p_t q_t} \rightarrow \frac{\widehat{\Sigma} \widehat{p}_t q_t}{\widehat{\Sigma} p_t q_t}$
(2) (2)

p.11 (IV 3. line) then \rightarrow than