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Abstract

In this paper, we examine what appropriate formulations for labor exploitation are, in order to explain the emergence of class and exploitation status in capitalist economies. Given the well-known controversy on plausible formulations for labor exploitation in joint production economies, we propose an axiom, *Axiom for Labor Exploitation* (**LE**), which every formulation of labor exploitation should satisfy to be considered 'Marxian.' Using this axiom, the necessary and sufficient condition for plausible formulations of Marxian exploitation is characterized to verify Class-Exploitation Correspondence Principle (CECP) [Roemer (1982)]. According to this, if some labor exploitation formulations, such as the well-known formulations of Morishima (1974) and Roemer (1982; Chapter 5) are applied, CECP no longer holds in general convex cone economies. Based upon this argument, we propose two new definitions of labor exploitation, each of which verifies CECP as well as Fundamental Marxian Theorem (FMT).

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1 Introduction

Marxian exploitation of labor is the difference between labor hours an individual provides and labor hours necessary to produce a consumption bundle the individual can purchase via his income. This concept is used as an index of 'unjust' distribution. That is, the existence of labor exploitation should reflect the existence of 'unjust' distribution in some sense.

During the 1970's and 1980's, there were remarkable developments in the debate about this concept in mathematical Marxian economics. Fundamental Marxian Theorem (FMT) was originally proved by Okishio (1963) and later named as such by Morishima (1973). FMT shows a correspondence between the existence of positive profit and the existence of labor exploitation. It gives us a useful characterization for *non-trivial* equilibria, where a trivial equilibrium is one such that its social production point is zero.[1] [2]

There has also been a substantial development in the works on exploitation of labor; "General Theory of Exploitation and Class" promoted by Roemer (1982, 1982a). It argues that in a capitalist economy with inequal distribution of productive assets, where the productive assets (capital) are *scarce* relative to labor, *if the labor supplied by agents is inelastic with respect to their wealth* (that is, the monetary value of their own capital), then the Class-Exploitation Correspondence Principle (CECP) can hold. This argument, which seems to support the Marxian perspective on the capitalist economy, implies that under the inelastic labor supply condition, if the capitalist economy is under an equilibrium with positive profits, then class and exploitation status *logically* emerge in that economy, accurately reflecting inequality in

[1] Note that FMT was originally considered to prove the classical Marxian argument that the exploitation of labor is the source of positive profits in the capitalist economy. However, it does not follow from FMT that the exploitation of labor is the unique source of positive profits. The reason is that any commodity can be shown to be exploited in a system with positive profits whenever the exploitation of labor exists. This observation was pointed out by Brody (1970), Bowles and Gintis (1981), Samuelson (1982), and was named "Generalized Commodity Exploitation Theorem (GCET)" by Roemer (1982).

[2] After the seminal work by Morishima (1973), there were many generalizations and discussions of FMT. While the original FMT is discussed in simple Leontief economies with homogeneous labor, the generalization of FMT to Leontief economies with heterogeneous labor was made by Fujimori (1982), Krause (1982), etc. The problem of generalizing FMT to von Neumann economies was discussed by Steedman (1977) and one solution was proposed by Morishima (1974). Furthermore, Roemer (1980) generalized the theorem to convex cone economies. These arguments may reflect the robustness of FMT.

the distribution of wealth. That is, the wealthier agents are *exploiters*, and they can rationally choose from all classes in society to belong to the *capitalist class*. In contrast, the least wealthy agents are *exploited*, and they cannot but choose to belong to the *working class*: there is no other available option for the least wealthy agents. Thus, the exploiting agents of the capitalist class have the richest life options, whereas the exploited agents of the working class have the poorest life options: the existence of labor-exploiters and labor-exploited reflects *unequal opportunity of life options*.[3]

This analysis presumed the simple Leontief-type production economy, in which the formulation of Marxian labor exploitation was given by the Okishio (1963) type. However, once we presume a more *general convex cone economy*, then the Okishio type formulation of labor exploitation is known to be ill-defined. Thus, in more general convex cone economies such as the von Neumann economy, the issue of what is a plausible formulation for Marxian exploitation of labor is controversial. There are two formulations for Marxian exploitation of labor, which are well-known in the literature of mathematical Marxian economics; one is Morishima (1974), and the other is Roemer (1982; Chapter 5). In addition to these, there are probably other potential formulations that are plausible definitions for Marxian exploitation of labor.

Given this background controversy, in this paper, we first propose a plausible axiom, *Axiom for Labor Exploitation* (**LE**), that every formulation of labor exploitation should satisfy to be considered 'Marxian.' By using this axiom, we characterize what kinds of formulations for 'Marxian' labor exploitation can verify CECP as a theorem in general convex cone economies. Based upon this characterization, we show that if Roemer's (1982; Chapter 5) or Morishima's (1974) formulation of Marxian exploitation of labor is applied, CECP no longer holds true in general convex cone economies.[4]

[3] This argument was criticized by some Marxian theorists, such as Bowles and Gintis (1990) and Devine and Dymski (1991, 1992), since it assumed a standard neoclassical labor market, which was regarded as not a *real*, but an *ideal* model of capitalist economies by these critics. However, as Yoshihara (1998) showed, CECP essentially holds true even if the neoclassical labor market is replaced by a non-neoclassical labor market with efficiency wage contracts, which was interpreted as a more realistic aspect of capitalist economies by those same critics.

[4] Note that Roemer (1982) argued that the epistemological role of CECP in our understanding of the capitalist economy is as an axiom, although the formal version of it emerges as a theorem. So, if we wish to verify CECP, we must seek an appropriate model which will preserve this principle as a theorem. By this reason, Roemer (1982) insisted that the Roemer (1982) definition of labor exploitation is superior to the Morishima (1974)

Moreover, we propose two new definitions of Marxian exploitation of labor, each of which satisfies **LE** and is given as the difference between one unit of labor supplied by an agent per day and the minimal amount of labor socially necessary to achieve the agent's *income* per day. In contrast to the two traditional definitions, CECP can be shown to hold true in general convex cone economies under the two *new* definitions. We could also resolve, under the two new definitions, most of the difficulties that Marxian economic theory has faced. That is, the difficulty of FMT in the general convex cone economy, that Petri (1980) and Roemer (1980) discussed, is resolved.

In the following paper, section 2 defines a basic economic model with convex cone production technology, and also introduces the equilibrium notion in this paper and alternative formulations, including our new definitions, for Marxian exploitation of labor. Section 3 discusses the robustness of CECP under the various definitions of labor exploitation in general convex cone economies. Section 4 discusses the performance of the new definitions in terms of FMT. Finally, section 5 provides some concluding remarks.

2 The Basic Model

2.1 Production

Let P be the production set. P has elements of the form $\alpha = (-\alpha_0, -\underline{\alpha}, \overline{\alpha})$ where $\alpha_0 \in \mathbb{R}_+$, $\underline{\alpha} \in \mathbb{R}^m_+$, and $\overline{\alpha} \in \mathbb{R}^m_+$. Thus, elements of P are vectors in $\mathbb{R}^{2m+1}$. The first component, $-\alpha_0$, is the direct labor input of the process α; and the next m components, $-\underline{\alpha}$, are the inputs of goods used in the process; and the last m components, $\overline{\alpha}$, are the outputs of the m goods from the process. We denote the net output vector arising from α as $\widehat{\alpha} \equiv \overline{\alpha} - \underline{\alpha}$. We assume that P is a closed convex cone containing the origin in $\mathbb{R}^{2m+1}$. Moreover, it is assumed that:

A 1. $\forall \alpha \in P$ s.t. $\alpha_0 \geq 0$ and $\underline{\alpha} \geqq 0$, $[\overline{\alpha} \geq 0 \Rightarrow \alpha_0 > 0]$;[5] and
A 2. $\forall$ commodity m vector $c \in \mathbb{R}^m_+$, $\exists \alpha \in P$ s.t. $\widehat{\alpha} \geqq c$.

one. Based upon this argument, which he made himself, however, the Roemer (1982) type of labor exploitation will also be shown to be invalid, since CECP fails to hold even in the model with the Roemer (1982) exploitation.

[5] For all vectors $x = (x_1, \ldots, x_p)$ and $y = (y_1, \ldots, y_p) \in \mathbb{R}^p$, $x \geqq y$ if and only if $x_i \geqq y_i$ $(i = 1, \ldots, p)$; $x \geq y$ if and only if $x \geqq y$ and $x \neq y$; $x > y$ if and only if $x_i > y_i$ $(i = 1, \ldots, p)$.

Given such P, we will sometimes use the following notations:

$$\begin{aligned} P(\alpha_0 = 1) &\equiv \{(-\alpha_0, -\underline{\alpha}, \overline{\alpha}) \in P \mid \alpha_0 = 1\}, \\ \widehat{P}(\alpha_0 = 1) &\equiv \{\widehat{\alpha} \in \mathbb{R}^m \mid \exists \alpha = (-1, -\underline{\alpha}, \overline{\alpha}) \in P \text{ s.t. } \overline{\alpha} - \underline{\alpha} \geq \widehat{\alpha}\}. \end{aligned}$$

As a notation, we use, for any set $S \subseteq \mathbb{R}^m$, $\partial S \equiv \{x \in S \mid \nexists x' \in S \text{ s.t. } x' > x\}$.

Given a market economy, any price system is denoted by $p \in \mathbb{R}_+^m$, which is a price vector of m commodities. Moreover, a *subsistence vector* of commodities $b \in \mathbb{R}_+^m$ is also necessary in order to supply one unit of labor per day. We assume that the nominal wage rate is normalized to unity when it purchases the subsistence consumption vector only, so that $pb = 1$ holds.

2.2 A Model of Accumulation and Marxian Equilibrium Notion

For the sake of simplicity, we follow the same setting as that in Roemer (1982; Chapter 5). That is, our schematic model of a capitalist economy is that all agents are accumulators who seek to expand the value of their endowments as rapidly as possible. Let us denote the set of agents by N with generic element ν. All agents have access to the same technology P, but they differ in their bundles of endowments. An agent $\nu \in N$ can engage in three types of economic activity: he can sell his labor power γ_0^ν, he can hire the labor powers of others to operate $\beta^\nu = \left(-\beta_0^\nu, -\underline{\beta}^\nu, \overline{\beta}^\nu\right) \in P$, or he can work for himself to operate $\alpha^\nu = (-\alpha_0^\nu, -\underline{\alpha}^\nu, \overline{\alpha}^\nu) \in P$. His constraint is that he must be able to afford to lay out the operating costs in advance for the activities he chooses to operate, either with his own labor or hired labor, funded by the value of his endowment. He can choose the activity level of each of α^ν, β^ν, and γ_0^ν under the constraints of his capital and labor endowments. Thus, given (p, w), where w is a nominal wage rate, his program is:

$$\max_{\left(\alpha^\nu;\, \beta^\nu;\, \gamma_0^\nu\right) \in P \times P \times \mathbb{R}_+} [p(\overline{\alpha}^\nu - \underline{\alpha}^\nu)] + \left[p\left(\overline{\beta}^\nu - \underline{\beta}^\nu\right) - w\beta_0^\nu\right] + [w\gamma_0^\nu]$$

such that

$$\begin{aligned} p\underline{\alpha}^\nu + p\underline{\beta}^\nu &\leqq p\omega^\nu \equiv W^\nu, \\ \alpha_0^\nu + \gamma_0^\nu &\leqq 1. \end{aligned}$$

Given (p,w), let $\mathcal{A}^{\nu}(p,w)$ be the set of actions $(\alpha^{\nu};\ \beta^{\nu};\ \gamma_0^{\nu}) \in P \times P \times [0,1]$ which solve ν's program at prices (p,w).

Based on Roemer (1982; Chapter 5), the equilibrium notion of this model is given as follows:

Definition 1: A *reproducible solution* (RS) for the economy specified above is a pair $\left((p,w), (\alpha^{\nu};\ \beta^{\nu};\ \gamma_0^{\nu})_{\nu\in N}\right)$, where $p \in \mathbb{R}^m_+$, $w \geqq pb = 1$, and $(\alpha^{\nu};\ \beta^{\nu};\ \gamma_0^{\nu}) \in P \times P \times [0,1]$, such that:

(a) $\forall \nu \in N$, $(\alpha^{\nu};\ \beta^{\nu};\ \gamma_0^{\nu}) \in \mathcal{A}^{\nu}(p,w)$ (revenue maximization);

(b) $\underline{\alpha} + \underline{\beta} \leqq \omega$ (social feasibility),
where $\underline{\alpha} \equiv \sum_{\nu\in N} \underline{\alpha}^{\nu}$, $\underline{\beta} \equiv \sum_{\nu\in N} \underline{\beta}^{\nu}$, and $\omega \equiv \sum_{\nu\in N} \omega^{\nu}$;

(c) $\beta_0 \leqq \gamma_0$ (labor market equilibrium)
where $\beta_0 \equiv \sum_{\nu\in N} \beta_0^{\nu}$ and $\gamma_0 \equiv \sum_{\nu\in N} \gamma_0^{\nu}$; and

(d) $\widehat{\alpha} + \widehat{\beta} \geqq \alpha_0 b + \beta_0 b$ (reproducibility),
where $\widehat{\alpha} \equiv \sum_{\nu\in N}(\overline{\alpha}^{\nu} - \underline{\alpha}^{\nu})$, $\widehat{\beta} \equiv \sum_{\nu\in N}(\overline{\beta}^{\nu} - \underline{\beta}^{\nu})$, and $\alpha_0 \equiv \sum_{\nu\in N} \alpha_0^{\nu}$.

The three parts except (a) need some comments. Part (d) says that net outputs should at least replace employed workers' total consumption. This is equivalent to requiring that the vector of social endowments does not decrease in terms of components, because (d) is equivalent to $\omega - (\underline{\alpha} + \alpha_0 b) + \overline{\alpha} \geqq \omega$, where the right hand side is the social stocks at the beginning of this period, the left hand side is the stocks at the beginning of the next period. Part (b) says that intermediate inputs must be available from current stocks. Here, we assume that wage goods are dispensed at the end of each production period, therefore stocks need not be sufficient to accommodate them as well. Finally, (c) is the condition of labor market equilibrium. This condition allows strict inequality between labor demand β_0 and labor supply γ_0. If it holds in strict inequality, then the nominal wage rate is driven down to the subsistence wage $w = pb = 1$. If it holds in equality, then it might hold that $w \geqq pb = 1$.

Let $P(\omega) \equiv \{\alpha = (-\alpha_0, -\underline{\alpha}, \overline{\alpha}) \in P \mid \underline{\alpha} \leqq \omega\}$ and $\alpha_0(\omega) \equiv \max\{\alpha_0 \mid \exists \alpha = (-\alpha_0, -\underline{\alpha}, \overline{\alpha}) \in P(\omega)\}$. Then:

Proposition 1: *Let $b \in \mathbb{R}^m_{++}$ and $\alpha_0(\omega) \leqq |N|$. Under A1, A2, a* reproducible solution (RS) *of Definition 1 exists for the economy specified above.*

Proof. It follows from Theorem 2.5 of Roemer (1980; 1981) that for any non-negative values $(W^{\nu})_{\nu\in N}$, a *quasi-reproducible solution* (QRS) $\left((p,1), (\alpha^{\nu};\ \beta^{\nu};\ \gamma_0^{\nu})_{\nu\in N}\right)$

[Roemer (1980; 1981)] exists. (Note that QRS is defined by Definition 1(a), 1(c), and 1(d).) Thus, it suffices to show that for each $(\omega^{\nu})_{\nu\in N}$, there exists a QRS, $\left((p,1),(\alpha^{\nu};\ \beta^{\nu};\ \gamma_0^{\nu})_{\nu\in N}\right)$, which constitutes an RS under the initial endowments $(\omega^{\nu})_{\nu\in N}$. Take any $(\omega^{\nu})_{\nu\in N}$ such that for any $\nu\in N$, $\omega^{\nu}\in\mathbb{R}_+^m$ and $\alpha_0(\omega)\leqq|N|$. Let $S\equiv\left\{p\in\mathbb{R}_+^m\mid pb=1\right\}$. Given $(\omega^{\nu})_{\nu\in N}$, let $\mathfrak{W}:S\to\mathbb{R}_+^n$ such that for any $p\in S$, $\mathfrak{W}(p)=(p\omega^{\nu})_{\nu\in N}$. Let $\varphi:\mathfrak{W}(S)\twoheadrightarrow S$ be a correspondence such that for any $\mathbf{W}=(W^{\nu})_{\nu\in N}\in\mathbb{R}_+^n$, $p\in\varphi(\mathbf{W})$ implies that there exists $(\alpha^{\nu};\ \beta^{\nu};\ \gamma_0^{\nu})_{\nu\in N}\in(P\times P\times[0,1])^n$ such that $\left((p,1),(\alpha^{\nu};\ \beta^{\nu};\ \gamma_0^{\nu})_{\nu\in N}\right)$ is a QRS under $\mathbf{W}$. Then, define $\Psi\equiv\varphi\circ\mathfrak{W}$. Thus, Ψ is a correspondence from S into itself.

We show that Ψ is upper hemi-continuous with non-empty, convex-compact valued. First, it is obvious that Ψ is non-empty compact-valued. Since $\mathfrak{W}$ is a continuous function, it suffices to show φ is upper hemi-continuous. Let $\mathbf{W}^{\mu}\to\mathbf{W}$ as $\mu\to\infty$, $p^{\mu}\in\varphi(\mathbf{W}^{\mu})$ for each μ, and $p^{\mu}\to p$. Suppose $p\notin\varphi(\mathbf{W})$. Then, by definition of QRS, it implies that for any (α,β) with $\alpha\equiv\sum_{\nu\in N}\alpha^{\nu}$ and $\beta\equiv\sum_{\nu\in N}\beta^{\nu}$ such that $(\alpha^{\nu};\ \beta^{\nu};\ \gamma_0^{\nu})\in\mathcal{A}^{\nu}(p,1)$ for any $\nu\in N$, $\widehat{\alpha}+\widehat{\beta}\ngeqq(\alpha_0+\beta_0)\,b$ holds. Then, for large enough μ, p^{μ} has only $(\alpha^{\mu},\beta^{\mu})\in\mathcal{A}(p^{\mu},1)\equiv\sum_{\nu\in N}\mathcal{A}^{\nu}(p^{\mu},1)$ with $\widehat{\alpha}^{\mu}+\widehat{\beta}^{\mu}\ngeqq(\alpha_0^{\mu}+\beta_0^{\mu})\,b$. This is a contradiction, since $p^{\mu}\in\varphi(\mathbf{W}^{\mu})$. Thus, $p\in\varphi(\mathbf{W})$. Finally, we can show that $\varphi(\mathbf{W})$ is convex-valued for any $\mathbf{W}\in\mathfrak{W}(S)$. Let $p,p'\in\varphi(\mathbf{W})$ such that $\left((p,1),(\alpha^{\nu};\ \beta^{\nu};\ \gamma_0^{\nu})_{\nu\in N}\right)$ and $\left((p',1),(\alpha'^{\nu};\ \beta'^{\nu};\ \gamma_0'^{\nu})_{\nu\in N}\right)$ are QRSs under $\mathbf{W}$. Note that p supports $\widehat{\alpha}+\widehat{\beta}$ as an efficient net production, whereas p' supports $\widehat{\alpha}'+\widehat{\beta}'$. Let $\widehat{\alpha}_p^N+\widehat{\beta}_p^N\equiv\frac{\widehat{\alpha}+\widehat{\beta}}{\alpha_0+\beta_0}$ and $\widehat{\alpha}_{p'}'^N+\widehat{\beta}_{p'}'^N\equiv\frac{\widehat{\alpha}'+\widehat{\beta}'}{\alpha_0'+\beta_0'}$. Take any $p''\equiv tp+(1-t)\,p'$, where $t\in(0,1)$. Since P is convex-cone, there exists $\widehat{\alpha}_{p''}''^N+\widehat{\beta}_{p''}''^N\in\partial\widehat{P}(\alpha_0=1)$ such that $\widehat{\alpha}_{p''}''^N+\widehat{\beta}_{p''}''^N\geqq s\left(\widehat{\alpha}_p^N+\widehat{\beta}_p^N\right)+(1-s)\left(\widehat{\alpha}_{p'}'^N+\widehat{\beta}_{p'}'^N\right)$ for some $s\in[0,1]$, and p'' supports $\widehat{\alpha}_{p''}''^N+\widehat{\beta}_{p''}''^N$. Then, there exists $u>0$ such that $p''u\left(\underline{\alpha}_{p''}''^N+\underline{\beta}_{p''}''^N\right)=W\equiv\sum_{\nu\in N}W^{\nu}$. Let $\alpha''+\beta''\equiv u\left(\alpha_{p''}''^N+\beta_{p''}''^N\right)$. Thus, there exists $(\alpha''^{\nu};\ \beta''^{\nu};\ \gamma_0''^{\nu})_{\nu\in N}\in\times_{\nu\in N}\mathcal{A}^{\nu}(p'',1)$ under the capital constraint W^{ν} for any $\nu\in N$, such that $\sum_{\nu\in N}\alpha''^{\nu}+\sum_{\nu\in N}\beta''^{\nu}=\alpha''+\beta''$. Note that $\widehat{\alpha}_{p''}''^N+\widehat{\beta}_{p''}''^N\geqq b$ follows from $\widehat{\alpha}_p^N+\widehat{\beta}_p^N\geqq b$ and $\widehat{\alpha}_{p'}'^N+\widehat{\beta}_{p'}'^N\geqq b$, so that $\widehat{\alpha}''+\widehat{\beta}''\geqq(\alpha_0''+\beta_0'')\,b$ holds. Moreover, since $\underline{\alpha}_{p''}''^N+\underline{\beta}_{p''}''^N=s\left(\underline{\alpha}_p^N+\underline{\beta}_p^N\right)+(1-s)\left(\underline{\alpha}_{p'}'^N+\underline{\beta}_{p'}'^N\right)$ by definition, $u\leqq s(\alpha_0+\beta_0)+(1-s)(\alpha_0'+\beta_0')$ follows from $p\left(\underline{\alpha}+\underline{\beta}\right)=W=p'\left(\underline{\alpha}'+\underline{\beta}'\right)$ and $p''=tp+(1-t)\,p'$. Since

$\alpha_0'' + \beta_0'' = u$, $\alpha_0'' + \beta_0'' \leqq |N|$ holds, so that $\beta_0'' \leqq \gamma_0'' = |N| - \alpha_0''$. Thus, $\left((p'', 1), (\alpha''^{\nu};\ \beta''^{\nu};\ \gamma_0''^{\nu})_{\nu \in N}\right)$ is a QRS under $\mathbf{W}$. This implies $p'' \in \varphi(\mathbf{W})$.

Hence, by the Kakutani's fixed point theorem, there exists $p^* \in S$ such that $p^* \in \Psi(p^*)$. By the construction of Ψ, p^* has its $(\alpha^{\nu};\ \beta^{\nu};\ \gamma_0^{\nu})_{\nu \in N}$ such that $\left((p^*, 1), (\alpha^{\nu};\ \beta^{\nu};\ \gamma_0^{\nu})_{\nu \in N}\right)$ constitutes an RS under $(\omega^{\nu})_{\nu \in N}$. ■

Roemer (1980, 1981) shows that, for any $\mathbf{W} \in \mathbb{R}_+^n$, there *exists* an endowment $(\omega^{\nu})_{\nu \in N}$ such that an RS exists with respect to $(\omega^{\nu})_{\nu \in N}$, in which the monetary values of $(\omega^{\nu})_{\nu \in N}$ coincide with $\mathbf{W}$. In contrast, Proposition 1 shows that for any $(\omega^{\nu})_{\nu \in N} \in \mathbb{R}_+^{nm}$ with $\sum_{\nu \in N} \omega^{\nu} = \omega$ and $\alpha_0(\omega) \leqq |N|$, an RS exists. Thus, the existence of an RS is shown independently of endowments, under the assumption of *capital-limited* economies, $\alpha_0(\omega) \leqq |N|$.

Given an RS, $\left((p, w), (\alpha^{\nu};\ \beta^{\nu};\ \gamma_0^{\nu})_{\nu \in N}\right)$, let $\alpha^{p,w} \equiv \sum_{\nu \in N} \alpha^{\nu} + \sum_{\nu \in N} \beta^{\nu}$, which is the aggregate production activity *actually* accessed in this RS. Thus, the pair $((p, w), \alpha^{p,w})$ is the summary information of this RS. In the following, we sometimes use $((p, w), \alpha^{p,w})$ or only (p, w) for the representation of the RS, $\left((p, w), (\alpha^{\nu};\ \beta^{\nu};\ \gamma_0^{\nu})_{\nu \in N}\right)$.

2.3 Various Formulations for Marxian Exploitation of Labor

In this subsection, we discuss a general condition that every formulation for labor exploitation has to satisfy to be considered Marxian. Then, by this condition, the class of plausible formulations for Marxian exploitation of labor is identified. We show that both the Morishima (1974) and the Roemer (1982) definitions meet these conditions. We also introduce three alternative definitions for Marxian exploitation of labor, which also meet the condition.

In the following, we assume an RS with full employment (that is, Definition 1(c) holds in equality) for the sake of simplicity. Under any such RS, $((p, w), \alpha^{p,w})$, every agent $\nu \in N$ gets a revenue $\Pi^{\nu}(p, w) \equiv \pi^{\max}(p, w)\, p\omega^{\nu} + w$, as Roemer (1982) shows, where

$$\pi^{\max}(p, w) \equiv \max\left\{\frac{p\overline{\alpha} - (p\underline{\alpha} + w\alpha_0)}{p\underline{\alpha}} \mid \alpha = (-\alpha_0, -\underline{\alpha}, \overline{\alpha}) \in P\right\}.$$

Given any economy $\langle N; (P, b); (\omega^{\nu})_{\nu \in N}\rangle$, and any RS, $((p, w), \alpha^{p,w})$, let $N^{ter} \subseteq N$, $N^{ted} \subseteq N$, and $N^{ter} \cap N^{ted} = \varnothing$. Also, let $B(p, \Pi^{\nu}(p, w)) \equiv \left\{f^{\nu} \in \mathbb{R}_+^m \mid pf^{\nu} = \Pi^{\nu}(p, w)\right\}$, $B_+(p, \Pi^{\nu}(p, w)) \equiv \left\{f^{\nu} \in \mathbb{R}_+^m \mid pf^{\nu} \geqq \Pi^{\nu}(p, w)\right\}$, and $B_{-}(p, \Pi^{\nu}(p,w)) \equiv \left\{ f^{\nu} \in \mathbb{R}^{m}_{+} \mid pf^{\nu} \leqq \Pi^{\nu}(p,w) \right\}$. Let $c \in \mathbb{R}^{m}_{+}$ be a vector of produced commodities. Let

$$\phi(c) \equiv \{\alpha \in P \mid \widehat{\alpha} \geqq c\},$$

which is the set of the production points which produce, as net output vectors, at least c. Let $\zeta \equiv \partial\widehat{P}(\alpha_0 = 1) \cap \mathbb{R}^{m}_{+}$. Then:

Axiom for Labor Exploitation (LE): *Two subsets N^{ter} and N^{ted} constitute* the set of exploiters *and* the set of exploited agents *if and only if there exist $\overline{c} \in \zeta$ and $\underline{c} \in \widehat{P}(\alpha_0 = 1) \cap \mathbb{R}^{m}_{+}$ such that $p\overline{c} \geqq p\underline{c}$, and for any $\nu \in N$,*

$$\begin{aligned}
\nu &\in N^{ter} \Leftrightarrow \exists c^{\nu} \in B_{-}(p, \Pi^{\nu}(p,w)) \text{ s.t. } c^{\nu} \geq \overline{c} \text{ and } \exists \alpha \in \phi(c^{\nu}) \text{ with } \alpha_0 > 1; \\
\nu &\in N^{ted} \Leftrightarrow \exists c^{\nu} \in B_{+}(p, \Pi^{\nu}(p,w)) \text{ s.t. } c^{\nu} \leq \underline{c} \text{ and } \exists \alpha \in \phi(c^{\nu}) \text{ with } \alpha_0 < 1.
\end{aligned}$$

The axiom **LE** requires choosing two commodity vectors $\overline{c}, \underline{c} \in \mathbb{R}^{m}_{+}$, each of which can be produced as a net output by supplying one unit of labor. These $\overline{c}$ and $\underline{c}$ are considered as reference consumption bundles to identify the income range of *non-exploited non-exploiting* agents: any agent $\nu \in N$ with income $p\underline{c} \leqq \Pi^{\nu}(p,w) \leqq p\overline{c}$, who supplies one unit of labor, is regarded as neither exploited nor exploiting, since the amount of socially necessary labor that he can receive from consumption through his income is exactly one unit. Thus, if an agent $\nu \in N$ supplies one unit of labor and receives $\Pi^{\nu}(p,w) < p\underline{c}$, then he has a consumption bundle $c^{\nu} \in B_{+}(p, \Pi^{\nu}(p,w))$ with $c^{\nu} \leq \underline{c}$ such that c^{ν} is produced as a net output with less than one unit of labor. Then, **LE** requires that such an agent should be defined as 'exploited.'[6] The parallel argument can be also applied to the case of 'exploiter.' We think all potential formulations for Marxian notion of labor exploitation should have this property.

We can see that both the Morishima (1974) and the Roemer (1982) definitions of labor exploitation, which we will provide below, satisfy this axiom. First:

[6] This argument is independent of whether he really supplies one unit of labor or not, though we now assume full employment. Even if the economy is in equilibrium with unemployment, and an agent does not work at all, we can still apply the same argument to identify whether he is exploited or not. In fact, we can see that if he were to supply one unit of labor, he would receive his income $\Pi^{\nu}(p,w)$ from which the amount of 'socially necessary labor' for his income would be identified.

Definition 2: The Morishima (1974) labor value of commodity vector c, $l.v.(c)$, is given by

$$l.v.(c) \equiv \min \{\alpha_0 \mid \alpha = (-\alpha_0, -\underline{\alpha}, \overline{\alpha}) \in \phi(c)\}.$$

It is easy to see that $\phi(c)$ is non-empty by A2. Also,

$$\{\alpha_0 \mid \alpha = (-\alpha_0; -\underline{\alpha}; \overline{\alpha}) \in \phi(c)\}$$

is bounded from below by 0, by the assumption $\mathbf{0} \in P$ and A1. Thus, $l.v.(c)$ is well-defined since P is compact. Moreover, by A1, $l.v.(c)$ is positive whenever $c \neq \mathbf{0}$, so that $e(c)$ is well-defined.

Then:

Definition 3: A producer $\nu \in N$ is *exploited in the Morishima (1974) sense* if and only if:

$$\max_{f^\nu \in B(p, \Pi^\nu(p,w))} l.v.(f^\nu) < 1,$$

and he is an *exploiter in the Morishima (1974) sense* if and only if:

$$\min_{f^\nu \in B(p, \Pi^\nu(p,w))} l.v.(f^\nu) > 1.$$

Given an RS, (p, w), let $\overline{c} \in \zeta$ be such that $p\overline{c} \geqq pc$ for all $c \in \zeta$. Also, let $\underline{c} \in \zeta$ be such that $p\underline{c} \leqq pc$ for all $c \in \zeta$. We can check that ν is an exploiter in the Morishima (1974) sense if and only if $\Pi^\nu(p,w) > p\overline{c}$. Also, ν is exploited in the Morishima (1974) sense if and only if $\Pi^\nu(p,w) < p\underline{c}$. This argument implies that Definition 3 satisfies **LE**.

In contrast to the Morishima (1974) labor value, the definition of labor value in Roemer (1982) depends, in part, on the particular equilibrium the economy is in. Given a price system (p, w), let

$$\overline{P}(p,w) \equiv \left\{ \alpha = (-\alpha_0, -\underline{\alpha}, \overline{\alpha}) \in P \mid \frac{p\overline{\alpha} - (p\underline{\alpha} + w\alpha_0)}{p\underline{\alpha}} = \pi^{\max}(p,w) \right\}.$$

Then, let

$$\phi(c; p, w) \equiv \left\{ \alpha \in \overline{P}(p,w) \mid \widehat{\alpha} \geqq c \right\},$$

which is the set of those profit-rate-maximizing actions which produce, as net output vectors, at least c. Then:

Definition 4: The Roemer (1982) labor value of commodity vector c, $l.v.\,(c; p, w)$, is given by

$$l.v.\,(c; p, w) \equiv \min \{\alpha_0 \mid \alpha = (-\alpha_0, -\underline{\alpha}, \overline{\alpha}) \in \phi\,(c; p, w)\}\,.$$

Then:

Definition 5: Let (p, w) be a price of RS. A producer $\nu \in N$ is *exploited in the Roemer (1982) sense* if and only if:

$$\max_{f^{\nu} \in B(p, \Pi^{\nu}(p,w))} l.v.\,(f^{\nu}; p, w) < 1,$$

and he is an *exploiter in the Roemer (1982) sense* if and only if:

$$\min_{f^{\nu} \in B(p, \Pi^{\nu}(p,w))} l.v.\,(f^{\nu}; p, w) > 1.$$

It is easy to verify that $l.v.\,(c; p)$ is well-defined, and has a positive value whenever $c \neq \mathbf{0}$. Also, $l.v.\,(c; p) \geqq l.v.\,(c)$ holds.

To see that Definition 5 satisfies **LE**, let us define for any (p, w),

$$\theta_{(p,w)} \equiv \left\{c \in \mathbb{R}^m_+ \mid \exists \alpha \in \phi\,(c; (p, w)) : \alpha_0 = 1 \;\&\; \alpha_0 \text{ is minimized over } \phi\,(c; (p, w))\right\}.$$

Then, given an RS, (p, w), let $\overline{c} \in \theta_{(p,w)}$ be such that $p\overline{c} \geqq pc$ for all $c \in \theta_{(p,w)}$. Also, let $\underline{c} \in \theta_{(p,w)}$ be such that $p\underline{c} \leqq pc$ for all $c \in \theta_{(p,w)}$. We can check that ν is an exploiter in the Roemer (1982) sense if and only if $\Pi^{\nu}(p, w) > p\overline{c}$. Also, ν is exploited in the Roemer (1982) sense if and only if $\Pi^{\nu}(p, w) < p\underline{c}$. This argument implies that Definition 5 satisfies **LE**.

In addition to the above two definitions of labor exploitation, we also propose two new definitions. Following Roemer (1982), we still adopt the definition of labor value of commodities as in Definition 4. However, we refine the definition of labor exploitation from Roemer's (1982). The first new definition is given as follows:

Definition 6: Let $((p, w), \alpha_{p,w})$ be an RS. A producer $\nu \in N$ is *exploited* if and only if:

$$\min_{f^{\nu} \in B(p, \Pi^{\nu}(p,w))} l.v.\,(f^{\nu}; p, w) < 1,$$

and he is an *exploiter* if and only if:

$$\min_{f^{\nu} \in B(p, \Pi^{\nu}(p,w))} l.v.\,(f^{\nu}; p, w) > 1.$$

We can see that Definition 6 satisfies **LE** by choosing $\overline{c} \in \theta_{(p,w)}$ as $p\overline{c} \geqq pc$ for all $c \in \theta_{(p,w)}$, and $\underline{c} = \overline{c}$.

Note that $\min_{f^\nu \in B(p,\Pi^\nu(p,w))} l.v.\left(f^\nu; p, w\right)$ in Definition 6 can be regarded as the *indirect labor value of ν's income.* This implies that the labor value in Definition 6 is concerned *not* with an agent's *consumption vector*, *but* rather with an agent's *income* earned. Thus, this new definition implies the following: Suppose an economy is under a reproducible solution $((p, w), \alpha^{p,w})$. Then, if the *minimal expenditure* of labor socially necessary to reach an agent ν's income $\Pi^\nu(p,w)$ under the RS, $((p, w), \alpha^{p,w})$, is less (*resp.* more) than unity, then ν is exploited (*resp.* exploiter). [7]

The second new definition is given as follows. Given any RS, $((p, w), \alpha^{p,w})$, let $\widehat{\alpha}^N_{p,w} \equiv \frac{\widehat{\alpha}^{p,w}}{\alpha_0^{p,w}}$. Moreover, for any $\nu \in N$, let $t^\nu > 0$ be such that $pt^\nu \widehat{\alpha}^N_{p,w} = \Pi^\nu(p,w)$. Then:

Definition 7: Let $((p, w), \alpha_{p,w})$ be an RS. A producer $\nu \in N$ is *exploited* if and only if:

$$l.v.\left(t^\nu \widehat{\alpha}^N_{p,w}; p, w\right) < 1,$$

and he is an *exploiter* if and only if:

$$l.v.\left(t^\nu \widehat{\alpha}^N_{p,w}; p, w\right) > 1.$$

We can see that Definition 7 satisfies **LE** by choosing $\overline{c} = \widehat{\alpha}^N_{p,w}$ and $\underline{c} = \widehat{\alpha}^N_{p,w}$.

Definition 7 is also concerned *not* with an agent's *consumption vector*, *but* rather with an agent's *income* earned. The difference of Definition 7 from Definition 6 is that the minimal expenditure of labor socially necessary to reach an agent ν's income $\Pi^\nu(p,w)$ is given by examining the ray passing through the *actually accessed* social production point $\alpha^{p,w}$ solely, rather than the minimizer over $\overline{P}(p,w)$.[8] Under this definition, the following relationship holds:

total labor employed = labor value of national income (= net product).

[7] This interpretation of $\min_{f^\nu \in B(p,\Pi^\nu(p,w))} l.v.\left(f^\nu; p, w\right)$ is analogous to the notion of the minimal expenditure of wealth required to reach a given utility level in the expenditure minimization problem of the standard micro theory of consumer behavior.

[8] Note also that Definition 7 is an extension of Lipietz's (1982) formulation of labor exploitation defined in Leontief models to general convex cone models, although the background idea of his formulation is much different from ours.

This *macroeconomic identity* has been required as a basic property of labor value in Marxian economic theory.[9]

We may also consider a more subjective notion of labor exploitation. Suppose that there is a representative agent of this economy, and introduce this agent's welfare function $U : \mathbb{R}^m_+ \to \mathbb{R}$. This U is continuous and strictly monotonic on $\mathbb{R}^m_+$, and it should have the following property: for any RS, $((p,w), \alpha^{p,w})$, $\widehat{\alpha}^N_{p,w}$ is the maximizer of $U(c)$ over $B\left(p, p\widehat{\alpha}^N_{p,w}\right)$. Given this welfare function U, let $c^{\max}_U \in \mathbb{R}^m_+$ be the maximizer of $U(c)$ over ζ. Then:

Definition 8: Let $((p,w), \alpha^{p,w})$ be an RS. A producer $\nu \in N$ is *exploited* if and only if:

$$\Pi^\nu(p,w) < pc^{\max}_U,$$

and he is an *exploiter* if and only if:

$$\Pi^\nu(p,w) > pc^{\max}_U.$$

We can see that Definition 8 satisfies **LE** by choosing $\overline{c} = c^{\max}_U$ and $\underline{c} = c^{\max}_U$. This definition is extended from Matsuo (2004), although Matsuo provides only the definition of exploited agents in order to discuss FMT.

3 CECP in Accumulation Economies

In the following discussion, we will examine the viability of the above five definitions of labor exploitation respectively by checking whether CECP [Roemer (1982; Chapter 5)] holds true under each of these definitions, and show that only Definitions 6 and 7 verify CECP in general convex cone economies.

Following Roemer (1982; Chapter 5), let us define possible *classes*. At every RS in the model of section 2.2, different producers relate differently to the means of production. An individually optimal solution for an agent ν at the RS consists of three vectors $(\alpha^\nu; \beta^\nu; \gamma^\nu_0)$. According to whether these vectors are either zero or nonzero at the RS, all producers are classified into

[9] The macroeconomic identity is also satisfied by the labor value formulation of Flaschel (1983), although his method to derive labor values is extremely different from that of Definition 7: in Flaschel (1983), *additive labor values* are derived from the *square* matrices of input and output coefficients, which are defined by the maximally profitable production processes at a RS. In contrast, the labor value formulation in Definition 7 is given by Definition 4. Based on Flaschel's (1983) labor value formulation, we can consider another formulation of labor exploitation which satisfies **LE** with $\overline{c} = \widehat{\alpha}^N_{p,w} = \underline{c}$.

the following four types: that is, $(+,+,0)$, $(+,0,0)$, $(+,0,+)$, and $(0,0,+)$, where "+" means a nonzero vector in the appropriate place. Here, the notation $(+,+,0)$ implies, for instance, that an agent works for his own 'shop' and hires others' labor powers; while $(+,0,+)$ implies that an agent works for his own 'shop' and also sells his own labor power to others, etc.

Let us define four disjoint classes as follows:

$$
\begin{aligned}
C^H &= \left\{\nu \in N \mid \mathcal{A}^\nu(p,w) \text{ has a solution of the form } (+,+,0) \setminus (+,0,0)\right\}, \\
C^{PB} &= \left\{\nu \in N \mid \mathcal{A}^\nu(p,w) \text{ has a solution of the form } (+,0,0)\right\}, \\
C^S &= \left\{\nu \in N \mid \mathcal{A}^\nu(p,w) \text{ has a solution of the form } (+,0,+) \setminus (+,0,0)\right\}, \\
C^P &= \left\{\nu \in N \mid \mathcal{A}^\nu(p,w) \text{ has a solution of the form } (0,0,+)\right\}.
\end{aligned}
$$

We can see that the set of producers N can be partitioned into these four classes at any RS.[10]

Then:

Proposition 2 [Roemer (1982; Chapter 5)]**:** *Let* $(p,1)$ *be a price of* RS *with* $\pi^{\max}(p,1) > 0$*. Then,*

$$
\begin{aligned}
\nu &\in C^H \Leftrightarrow W^\nu > \max_{\alpha \in \overline{P}(p,1)} \left[\frac{p\underline{\alpha}}{\alpha_0}\right], \\
\nu &\in C^{PB} \Leftrightarrow \min_{\alpha \in \overline{P}(p,1)} \left[\frac{p\underline{\alpha}}{\alpha_0}\right] \leqq W^\nu \leqq \max_{\alpha \in \overline{P}(p,1)} \left[\frac{p\underline{\alpha}}{\alpha_0}\right], \\
\nu &\in C^S \Leftrightarrow 0 < W^\nu < \min_{\alpha \in \overline{P}(p,1)} \left[\frac{p\underline{\alpha}}{\alpha_0}\right], \\
\nu &\in C^P \Leftrightarrow W^\nu = 0.
\end{aligned}
$$

Now, CECP, which is a principle we would like to verify, is introduced as follows:

Class-Exploitation Correspondence Principle (CECP)[Roemer (1982)]**:** *For any economy defined as in section 2, and any reproducible solution with a positive profit rate, it holds that:*

[10] The partition of N into C^H, C^{PB}, C^S, and C^P is independent of whether the corresponding RS is with full employment or not. In fact, even if the economy is in equilibrium with unemployment, and an agent does not supply one unit of labor at all, we can still develop a hypothetical argument that indicates what class he would rationally choose to belong to if he were to supply one unit of labor.

(A) every member of C^H *is an* exploiter.
(B) every member of $C^S \cup C^P$ *is* exploited.

First, we discuss that under any definition of labor exploitation which satisfies **LE**, **CECP** holds true if the production possibility set is given by the *Leontief technology.* Let A be an $m \times m$ non-negative, indecomposable square matrix with input-output coefficients $a_{ij} \geqq 0$ for any $i, j = 1, \ldots, m$, and L be a positive $1 \times m$ vector with labor input coefficients $L_j > 0$ for any $j = 1, \ldots, m$. Then, let $P_{(A,L)} \equiv \left\{(-Lx, -Ax, x) \mid x \in \mathbb{R}^m_+\right\}$. Then:

Theorem 1: *Under A1, A2, let* $(p, 1)$ *be an* RS *with* $\pi^{\max}(p, 1) > 0$ *for an economy* $\langle N; \left(P_{(A,L)}, b\right); (\omega^\nu)_{\nu \in N}\rangle$*. Then, under any definition of labor exploitation satisfying **LE**, **CECP** holds true if and only if* $\underline{c} \in \zeta$*.*

The complete proof of this theorem will be given after Theorem 2 is discussed.

Note that **CECP** holds under any of the five definitions of labor exploitation in economies with Leontief technology, since any of them satisfies **LE** with $\underline{c} \in \zeta$ in those economies. Note also that in economies with Leontief technology, Definitions 3 and 5 are equivalent.

Insert Figure 1 around here.

Figure 1 illustrates that **CECP** holds under Definitions 3 and 5 in a two-goods economy with Leontief technology.

Second, we characterize, in general convex cone economies, what types of definitions of labor exploitation satisfying **LE** can preserve **CECP** as a theorem. Let $\overline{\Gamma}(p, w) \equiv \left\{\alpha \in \overline{P}(p, w) \mid \alpha_0 = 1\right\}$ and $\widehat{\overline{\Gamma}}(p, w) \equiv \left\{\widehat{\alpha} \in \mathbb{R}^m_+ \mid \alpha \in \overline{\Gamma}(p, w)\right\}$. For any set $S \subseteq \mathbb{R}^m_+$, let $co\{S\}$ denote the convex hull of S, and $comp\{S\}$ denote the *comprehensive hull* of S. Given any economy $\langle N; (P, b); (\omega^\nu)_{\nu \in N}\rangle$, and any RS, $((p, w), \alpha^{p,w})$, note that $\pi^{\max}(p, w) = \frac{p\widehat{\alpha}^{p,w} - w\alpha_0^{p,w}}{p\underline{\alpha}^{p,w}}$ follows from the definition of RS. Thus, there exists $\alpha^{p,w*} \in \overline{\Gamma}(p, w)$ such that for some $t > 0$, $t\alpha^{p,w*} = \alpha^{p,w}$. Moreover, there exists $c^{p,w} \in \zeta$ such that $pc^{p,w} \geqq pc$ for any $c \in \zeta$. Since $\widehat{\alpha}^{p,w*} \in \zeta$ by Definition 1(d), we have $pc^{p,w} \geqq p\widehat{\alpha}^{p,w*}$. Then:

Lemma 1: *Under A1, A2, there exists an economy* $\langle N; (P, b); (\omega^\nu)_{\nu \in N}\rangle$ *which has an* RS, $((p, 1), \alpha^{p,1})$*, such that* $pc^{p,1} > p\widehat{\alpha}$ *for any* $\alpha \in \overline{\Gamma}(p, 1)$.

Proof. Let us consider the following von Neumann system:

$$B = \begin{bmatrix} 5 & 3 & 9.8 & 0 \\ 5.25 & 4.5 & 0 & 5.25 \end{bmatrix}, A = \begin{bmatrix} 3.5 & 2 & 8 & 0 \\ 4.5 & 3 & 0 & 3.5 \end{bmatrix}, L = \begin{pmatrix} 0.75 & 1 & 0.6 & 1 \end{pmatrix}.$$

Define a production possibility set $P_{(B,A,L)}$ by

$$P_{(B,A,L)} \equiv \left\{ (-Lx, -Ax, Bx) \in \mathbb{R}_- \times \mathbb{R}_-^2 \times \mathbb{R}_+^2 \mid x \in \mathbb{R}_+^4 \right\}.$$

This $P_{(B,A,L)}$ is a closed convex cone in $\mathbb{R}_- \times \mathbb{R}_-^m \times \mathbb{R}_+^m$ with $\mathbf{0} \in P_{(B,A,L)}$. Moreover, $P_{(B,A,L)}$ is shown to satisfy A1 and A2.

Let $\mathbf{e}_j \in \mathbb{R}_+^m$ be a unit column vector with 1 in the j-th component and 0 in any other component. Then, $\alpha^1 \equiv (-L\mathbf{e}_1, -A\mathbf{e}_1, B\mathbf{e}_1)$, $\alpha^2 \equiv (-L\mathbf{e}_2, -A\mathbf{e}_2, B\mathbf{e}_2)$, $\alpha^3 \equiv (-L\mathbf{e}_3, -A\mathbf{e}_3, B\mathbf{e}_3)$, and $\alpha^4 \equiv (-L\mathbf{e}_4, -A\mathbf{e}_4, B\mathbf{e}_4)$. Moreover,

$$\begin{aligned}
\widehat{\alpha}^1 &\equiv (B-A)\,\mathbf{e}_1 = \begin{pmatrix} 1.5 \\ 0.75 \end{pmatrix}, \widehat{\alpha}^2 \equiv (B-A)\,\mathbf{e}_2 = \begin{pmatrix} 1 \\ 1.5 \end{pmatrix}, \\
\widehat{\alpha}^3 &\equiv (B-A)\,\mathbf{e}_3 = \begin{pmatrix} 1.8 \\ 0 \end{pmatrix}, \widehat{\alpha}^4 \equiv (B-A)\,\mathbf{e}_4 = \begin{pmatrix} 0 \\ 1.75 \end{pmatrix}.
\end{aligned}$$

Also, we have $\widehat{P}\,(\alpha_0 = 1) = co\,\{(2,1), (1,1.5), (3,0), (0,1.75), \mathbf{0}\}$.

Let $b = (1,1)$, and the social endowment of capital be given by $\omega = (2\,|N|, 3\,|N|)$. Then, for any economy $\langle N; \left(P_{(B,A,L)}, b\right); (\omega^\nu)_{\nu \in N}\rangle$ with $\sum_{\nu \in N} \omega^\nu = \omega$, a pair $((p,1), |N|\,\alpha^2)$ with $p = (0.5, 0.5)$ constitutes an RS. Note that

$$\begin{aligned}
\frac{[p\,(B-A) - L]\,\mathbf{e}_1}{pA\mathbf{e}_1} &= \frac{3}{32}; \frac{[p\,(B-A) - L]\,\mathbf{e}_2}{pA\mathbf{e}_2} = \frac{1}{10}; \\
\frac{[p\,(B-A) - L]\,\mathbf{e}_3}{pA\mathbf{e}_3} &= \frac{3}{40}; \frac{[p\,(B-A) - L]\,\mathbf{e}_4}{pA\mathbf{e}_4} = \frac{-1}{14}.
\end{aligned}$$

This implies that $\overline{\Gamma}\,(p,1) = \{\alpha^2\}$ and $\theta_{(p,1)} = \partial comp\left\{\widehat{\alpha}^2\right\}$. Thus,

$$\min_{\alpha \in \overline{P}(p,1)} \left[\frac{p\underline{\alpha}}{\alpha_0}\right] = \min_{\alpha \in \overline{\Gamma}(p,1)} p\underline{\alpha} = \max_{\alpha \in \overline{\Gamma}(p,1)} p\underline{\alpha} = \max_{\alpha \in \overline{P}(p,1)} \left[\frac{p\underline{\alpha}}{\alpha_0}\right] = p\underline{\alpha}^2.$$

Let $H\left(p, \widehat{\alpha}^2\right) \equiv \left\{c \in \mathbb{R}_+^2 \mid pc = p\widehat{\alpha}^2\right\}$, $H_+\left(p, \widehat{\alpha}^2\right) \equiv \left\{c \in \mathbb{R}_+^2 \mid pc > p\widehat{\alpha}^2\right\}$, and $H_-\left(p, \widehat{\alpha}^2\right) \equiv \left\{c \in \mathbb{R}_+^2 \mid pc < p\widehat{\alpha}^2\right\}$. Moreover, $\zeta_+ \equiv \zeta \cap H_+\left(p, \widehat{\alpha}^2\right)$, $\zeta_- \equiv \zeta \cap H_-\left(p, \widehat{\alpha}^2\right)$. Note that $\zeta_+ = [co\,\{(1,1.5), (2,1)\} \cup co\,\{(2,1), (3,0)\}] \setminus \{(1,1.5)\}$, $\zeta_- = co\,\{(0,1.75), (1,1.5)\} \setminus \{(1,1.5)\}$, and $\widehat{\alpha}^2 = (1,1.5)$. Since $c^{p,1} \in \zeta$ implies $c^{p,1} = (2,1)$, we have $pc^{p,1} > p\widehat{\alpha}^2$. Thus, we obtain a desired result. ■

Insert Figure 2 around here.

Then, the following theorem gives us a necessary and sufficient condition for formulations of labor exploitation satisfying **LE** in order to preserve **CECP** as a theorem:

Theorem 2: *Under A1, A2, let* $\langle N; (P, b); (\omega^\nu)_{\nu \in N} \rangle$ *be an economy with an* RS, $((p, 1), \alpha^{p,1})$, *with* $\pi^{\max}(p, 1) > 0$. *Then, for any definition of labor exploitation satisfying* ***LE***, ***CECP*** *holds true under this definition if and only if its corresponding* $\overline{c} \in \zeta$ *and* $\underline{c} \in \widehat{P}(\alpha_0 = 1) \cap \mathbb{R}^m_+$ *imply* $\overline{c}, \underline{c} \in \widehat{\overline{\Gamma}}(p, 1)$.

Proof. Let $\langle N; (P, b); (\omega^\nu)_{\nu \in N} \rangle$ be an economy with an RS, $((p, 1), \alpha^{p,1})$, such that $pc^{p,1} > p\widehat{\alpha}$ for any $\alpha \in \overline{\Gamma}(p, 1)$. Let

$$\alpha^{\max(p,1)} \equiv \arg \max_{\alpha \in \overline{\Gamma}(p,1)} p\underline{\alpha}; \text{ and } \alpha^{\min(p,1)} \equiv \arg \min_{\alpha \in \overline{\Gamma}(p,1)} p\underline{\alpha}.$$

Then, by Proposition 2, we have

$$\begin{aligned}
C^H &= \left\{\nu \in N \mid \Pi^\nu(p, 1) > \pi^{\max}(p, 1)\, p\underline{\alpha}^{\max(p,1)} + 1\right\}; \\
C^{PB} &= \left\{\nu \in N \mid \pi^{\max}(p, 1)\, p\underline{\alpha}^{\min(p,1)} + 1 \leqq \Pi^\nu(p, 1) \leqq \pi^{\max}(p, 1)\, p\underline{\alpha}^{\max(p,1)} + 1\right\}; \\
C^S &= \left\{\nu \in N \mid 1 < \Pi^\nu(p, 1) < \pi^{\max}(p, 1)\, p\underline{\alpha}^{\min(p,1)} + 1\right\}; \\
C^P &= \{\nu \in N \mid \Pi^\nu(p, 1) = 1\}.
\end{aligned}$$

Insert Figure 3 around here.

Let $H(p, \widehat{\alpha}) \equiv \left\{c \in \mathbb{R}^m_+ \mid pc = p\widehat{\alpha}\right\}$, $H_+(p, \widehat{\alpha}) \equiv \left\{c \in \mathbb{R}^m_+ \mid pc > p\widehat{\alpha}\right\}$, and $H_-(p, \widehat{\alpha}) \equiv \left\{c \in \mathbb{R}^m_+ \mid pc < p\widehat{\alpha}\right\}$. Moreover, $\zeta_+ \equiv \zeta \cap H_+\left(p, \widehat{\alpha}^{\max(p,1)}\right)$, $\zeta_- \equiv \zeta \cap H_-\left(p, \widehat{\alpha}^{\min(p,1)}\right)$. Then, $\zeta = \zeta_+ \cup \widehat{\overline{\Gamma}}(p, 1) \cup \zeta_-$.

1. Proof of the necessity.

Case 1): Consider any definition of labor exploitation satisfying **LE**, and for this definition, its corresponding $\overline{c} \in \zeta$ and $\underline{c} \in \widehat{P}(\alpha_0 = 1) \cap \mathbb{R}^m_+$ have the property that $\overline{c} \in \zeta_+$. Thus, $p\overline{c} > p\widehat{\alpha}^{\max(p,1)}$. Since $p\widehat{\alpha}^{\max(p,1)} = \pi^{\max}(p, 1)\, p\underline{\alpha}^{\max(p,1)} + 1$, we can construct an economy $\langle N; (P, b); (\widetilde{\omega}^\nu)_{\nu \in N} \rangle$ with $\sum_{\nu \in N} \widetilde{\omega}^\nu = \omega$, such that for some $\nu \in N$, $p\overline{c} > \pi^{\max}(p, 1)\, p\widetilde{\omega}^\nu + 1 > p\widehat{\alpha}^{\max(p,1)}$ holds. This agent ν belongs to C^H, as per Proposition 2. However, $p\overline{c} > \pi^{\max}(p, 1)\, p\widetilde{\omega}^\nu + 1 = \Pi^\nu(p, 1)$ implies that ν is not an exploiter.

Case 2): Consider any definition of labor exploitation satisfying **LE**, and for this definition, its corresponding $\overline{c} \in \zeta$ and $\underline{c} \in \widehat{P}(\alpha_0 = 1) \cap \mathbb{R}^m_+$ have the property that $\overline{c} \in \zeta_-$. Then, since $p\overline{c} \geqq p\underline{c}$ by **LE**, $p\underline{c} < p\widehat{\alpha}^{\min(p,1)}$ holds. Then, we can construct an economy $\langle N; (P, b); (\widetilde{\omega}^\nu)_{\nu \in N} \rangle$ with $\sum_{\nu \in N} \widetilde{\omega}^\nu = \omega$, such that for some $\nu \in N$, $p\underline{c} < \pi^{\max}(p,1) p\widetilde{\omega}^\nu + 1 < p\widehat{\alpha}^{\min(p,1)}$ holds. Note that this agent ν belongs to C^S, as per Proposition 2. However, $p\underline{c} < \pi^{\max}(p,1) p\widetilde{\omega}^\nu + 1 = \Pi^\nu(p,1)$ implies that ν is not exploited.

Case 3): Finally, consider any definition of labor exploitation satisfying **LE**, and for this definition, its corresponding $\overline{c} \in \zeta$ and $\underline{c} \in \widehat{P}(\alpha_0 = 1) \cap \mathbb{R}^m_+$ have the property that $\overline{c} \in \widehat{\widehat{\Gamma}}(p,1)$ and $p\overline{c} > p\underline{c}$. If $p\underline{c} < p\widehat{\alpha}^{\min(p,1)}$, then the argument of **Case 2)** can be applied.

In summary, the arguments of the above three cases imply that if a definition of labor exploitation satisfying **LE** preserves **CECP** as a theorem, then its corresponding $\overline{c} \in \zeta$ and $\underline{c} \in \widehat{P}(\alpha_0 = 1) \cap \mathbb{R}^m_+$ imply $\overline{c}, \underline{c} \in \widehat{\widehat{\Gamma}}(p,1)$.

2. Proof of the sufficiency.

Since the definition of labor exploitation satisfies **LE**, there are $\overline{c} \in \zeta$ and $\underline{c} \in \widehat{P}(\alpha_0 = 1) \cap \mathbb{R}^m_+$ such that $p\overline{c} \geqq p\underline{c}$ under the RS, $((p,1), \alpha^{p,1})$. Note that if $\overline{c}, \underline{c} \in \widehat{\widehat{\Gamma}}(p,1)$ under this definition of labor exploitation, then

$$\pi^{\max}(p,1) p\underline{\alpha}^{\min(p,1)} + 1 \leqq p\underline{c} \leqq p\overline{c} \leqq \pi^{\max}(p,1) p\underline{\alpha}^{\max(p,1)} + 1.$$

By **LE**, any agent $\nu \in N$ with W^ν under this RS such that $\Pi^\nu(p,1) < p\underline{c}$ is exploited, whereas any agent $\nu \in N$ with W^ν under this RS such that $\Pi^\nu(p,1) > p\overline{c}$ is an exploiter. Thus, any $\nu \in C^H$ becomes an exploiter, whereas any $\nu \in C^S \cup C^P$ is exploited in this economy. Thus, **CECP** holds under this definition of labor exploitation. ■

Proof of Theorem 1: In economies with Leontief technology, $\widehat{\widehat{\Gamma}}(p,1) = \zeta$ holds. If a definition of labor exploitation satisfies **LE** with $\underline{c} \in \zeta$, then there exists $\overline{c}, \underline{c} \in \widehat{\widehat{\Gamma}}(p,1)$ such that $p\overline{c} \geqq p\underline{c}$ under the RS, $((p,1), \alpha^{p,1})$. Thus, by Theorem 2, **CECP** holds under this definition. ■

Theorem 1 implies that any formulation of labor exploitation satisfying **LE** should have $\overline{c}, \underline{c} \in \zeta$ in order to verify **CECP** in models with Leontief technology, in which $\zeta = \widehat{\widehat{\Gamma}}(p,1)$ holds. Since $\overline{c}, \underline{c} \in \zeta$ is independent of the infomation about market equilibria, this characterization justifies a *price-independent* formulation of labor exploitation in capitalist economies with

Leontief technology. In contrast, according to Thoerem 2, price-independent formulations can no longer be valid in models with general convex cone technology, in which $\widehat{\overline{\Gamma}}(p,1)$ is just a subset of ζ. In such a case, since $\widehat{\overline{\Gamma}}(p,1)$ is the set of net outputs produced at profit-maximizing production points, any plausible formulation of labor exploitation should be *price-dependent* in order to verify **CECP**, which we will show below.

By the above Theorem 2, we can show that both the Morishima (1974) and the Roemer (1982) formulations for Marxian labor exploitation cannot preserve **CECP** as a theorem:

Corollary 1: *Under A1, A2, **CECP** cannot hold under Definition 3.*

Proof. Let $\langle N; (P,b); (\omega^{\nu})_{\nu \in N} \rangle$ be the economy constructed in Lemma 1. Then, this economy has an RS, $((p,1), \alpha^{p,1})$, such that $pc^{p,1} > p\widehat{\alpha}$ for any $\alpha \in \overline{\Gamma}(p,1)$. In this economy, if the Morishima (1974) formulation of labor exploitation (Definition 3) is applied, then $\underline{c} = \widehat{\alpha}^4$ and

$$\overline{c} \in \left\{ c \in \mathbb{R}^2_{++} \mid \exists t \in [0,1] : c = t\,(2,1) + (1-t)\,(3,0) \right\}.$$

Insert Figure 4 around here.

Note $\widehat{\overline{\Gamma}}(p,1) = \left\{ \widehat{\alpha}^2 \right\}$. Then, by Theorem 2, **CECP** violates under Definition 3. ■

Corollary 2: *Under A1, A2, **CECP** cannot hold under Definition 5.*

Proof. Let $\langle N; (P,b); (\omega^{\nu})_{\nu \in N} \rangle$ be the economy constructed in Lemma 1 as in the proof of Corollary 1. In this economy, if the Roemer (1982) formulation of labor exploitation (Definition 6) is applied, then $\underline{c} = (1,0)$ and $\overline{c} = \widehat{\alpha}^2$.

Insert Figure 5 around here.

Then, since $\widehat{\overline{\Gamma}}(p,1) = \left\{ \widehat{\alpha}^2 \right\}$, by Theorem 2, **CECP** violates under Definition 5. ■

We can also show that even Definition 8 cannot preserve **CECP** as a theorem.

Corollary 3: *Under A1, A2, **CECP** cannot hold under Definition 8.*

Proof. Let $\langle N; (P, b); (\omega^{\nu})_{\nu \in N} \rangle$ be the economy constructed in Lemma 1 as in the proof of Corollary 1. In this economy, if Definition 8 is applied as a formulation of labor exploitation, and the welfare function U of the representative agent has the following properties: $\widehat{\alpha}_{p,1}^{N} = \widehat{\alpha}^{2}$ and $c_{U}^{\max} = (2, 1)$. Thus, $\underline{c} = (2, 1)$ and $\overline{c} = (2, 1)$.

Insert Figure 6 around here.

Then, since $\widehat{\overline{\Gamma}}(p, 1) = \left\{ \widehat{\alpha}^{2} \right\}$, by Theorem 2, **CECP** violates under Definition 8. ■

Next, we show that in general convex cone economies, **CECP** holds true under Definitions 6 and 7:

Corollary 4: *Under A1, A2, **CECP** holds true under Definition 6.*

Proof. Given an RS, $(p, 1)$, let $\widetilde{c} \in \theta_{(p,1)}$ be such that $p\widetilde{c} \geqq pc$ for all $c \in \theta_{(p,1)}$. Note that in Definition 6, $\overline{c} = \widetilde{c}$ and $\underline{c} = \widetilde{c}$. Since $\widetilde{c} \in \widehat{\overline{\Gamma}}(p, 1)$ by definition, the desired result follows from Theorem 2. ■

Insert Figure 7 around here.

Corollary 5: *Under A1, A2, **CECP** holds true under Definition 7.*

Proof. Note that in Definition 7, $\overline{c} = \widehat{\alpha}_{p,1}^{N}$ and $\underline{c} = \widehat{\alpha}_{p,1}^{N}$. Since $\widehat{\alpha}_{p,1}^{N} \in \widehat{\overline{\Gamma}}(p, 1)$ by definition, the desired result follows from Theorem 2. ■

Insert Figure 8 around here.

There may potentially be another formulation of labor exploitation which satisfies **LE** and the condition $\overline{c}, \underline{c} \in \widehat{\overline{\Gamma}}(p, 1)$. For instance, we can consider a formulation of labor exploitation based on the labor value formulation of Flaschel (1983), as discussed in footnote 7. In this case, since the labor value of $\widehat{\alpha}_{p,1}^{N}$ is *unity* under Flaschel's (1983) formulation, the corresponding labor exploitation can be formulated to satisfy **LE** with $\overline{c} = \widehat{\alpha}_{p,1}^{N} = \underline{c}$. Then, **CECP** holds under this formulation. Except for this formulation, however, at least to the best of my knowledge in the current literature of mathematical Marxian economics, there are no other explicit formulations of labor exploitation than Definitions 6 and 7, which satisfy **LE** with $\overline{c}, \underline{c} \in \widehat{\overline{\Gamma}}(p, 1)$. In this sense, each of Definitions 6 and 7 could represent one of the most plausible formulations for Marxian exploitation of labor.

4 FMT in general convex cone economies

In this section, we discuss that the new formulations of labor exploitation given by Definitions 6 and 7 resolve the well-known difficulty in FMT under joint production economies. Let us consider an economy $\langle N; (P, b); (\omega^{\nu})_{\nu \in N} \rangle$ in which there is a partition N_1 and N_2 of the society N. That is, $N_1 \cup N_2 = N$ and $N_1 \cap N_2 = \varnothing$. Let us assume that for any $\nu \in N_1$, $\omega^{\nu} \in \mathbb{R}^m_{++}$, and for any $\nu \in N_2$, $\omega^{\nu} = \mathbf{0}$. Furthermore, every agent $\nu \in N_1$ is assumed to engage solely in operating $\beta^{\nu} \in P$ so as to maximize his profit revenue, whereas every agent $\nu \in N_2$ is assumed to engage solely in selling $\gamma^{\nu} \in [0, 1]$ so as to maximize his wage revenue.

In such a framework, Morishima (1974) showed that if the economy is under the von Neumann balanced growth equilibrium, then the *warranted profit rate*[11] is positive if and only if the Morishima (1974) labor exploitation is positive (that is, $l.v.(b) < 1$). However, Petri (1980) and Roemer (1980) showed that if the economy is under the RS, then FMT cannot hold: there is a case that the *maximal profit rate* is positive under no exploitation in the sense of Morishima (1974) (that is, $l.v.(b) = 1$). Furthermore, Roemer (1980; 1981) showed that the following assumption is the necessary and sufficient condition for FMT to hold true under the RS and the Morishima (1974) labor exploitation:

A 3. (Independence of Production) $\forall (-\alpha_0, -\underline{\alpha}, \overline{\alpha}) \in P$, $\forall \widehat{\alpha} \geqq 0$, and $\forall 0 \leqq c \leq \widehat{\alpha}$, $\exists (-\alpha'_0, -\underline{\alpha}', \overline{\alpha}') \in P$ s.t. $\overline{\alpha}' - \underline{\alpha}' \geqq c$ and $\alpha'_0 < \alpha_0$.

This assumption is rather strong, since every production set having inferior production processes is eliminated by it. Moreover, just excluding such production sets is not the real resolution since the failure of FMT occurs in production sets with inferior production processes.

However, if the Morishima (1974) labor exploitation is replaced by our Definitions 6 and 7, then, without A3, FMT can hold true even under RS. The following theorems illustrate this:

Theorem 3: *Under A1, A2, let* $((p, 1), \alpha^{p,1})$ *be a* reproducible solution (RS). *Then, the* RS *yields positive total profits if and only if every worker in* N_2 *is exploited in the sense of Definition 6.*

[11] That is, the minimal value of uniform profit rates. See Morishima (1974) for a more detailed discussion.

Proof. ($\Rightarrow$): Let $((p,1),\alpha^{p,1})$ be an RS with a positive total profit. Thus,

$$\begin{aligned} p\cdot\left(\sum_{\nu\in N_1}(\overline{\beta}^{\nu}-\underline{\beta}^{\nu})\right)-\sum_{\nu\in N_1}\beta_0^{\nu} &= p\cdot\left(\sum_{\nu\in N_1}(\overline{\beta}^{\nu}-\underline{\beta}^{\nu})-\sum_{\nu\in N_1}\beta_0^{\nu}b\right) \\ &= p\cdot\left(\widehat{\alpha}^{p,1}-\alpha_0^{p,1}b\right)>0. \end{aligned}$$

Since $p\in\mathbb{R}_+^m$ and $\widehat{\alpha}^{p,1}\geqq\alpha_0^{p,1}b$ by Definition 1(d), the last strict inequality implies $\widehat{\alpha}^{p,1}\geq\alpha_0^{p,1}b$. Let $f\in\mathbb{R}_+^m$ be such that $pf=pb$ and $\alpha_0^{p,1}f=t\widehat{\alpha}^{p,1}$ for some $0<t<1$. Then, by the convex cone property of the production set, $l.v.\left(\alpha_0^{p,1}f;p,1\right)\leqq l.v.\left(\alpha_0^{p,1}b;p,1\right)$, and $l.v.\left(\alpha_0^{p,1}f;p,1\right)<l.v.\left(\alpha_0^{p,1}b;p,1\right)$ holds whenever $f\neq b$. Thus, $l.v.\left(\alpha_0^{p,1}f;p,1\right)<\alpha_0^{p,1}$. By linearity, $l.v.\left(f;p,1\right)<1$, which implies $\min_{\widehat{f}\in B(p,1)}l.v.\left(\widehat{f};p,1\right)<1$, so that every worker is exploited in the sense of Definition 6.

($\Leftarrow$): Since there is no RS with a negative total profit, it suffices to discuss only the case of zero profit. Let $((p,1),\alpha^{p,1})$ be an RS with a zero total profit. Thus, $p\cdot\left(\widehat{\alpha}^{p,1}-\alpha_0^{p,1}b\right)=0$. By Definition 1(d), $\widehat{\alpha}^{p,1}\geqq\alpha_0^{p,1}b$. Let $f\in\mathbb{R}_+^m$ be such that $pf=pb$ and $\alpha_0^{p,1}f=t\widehat{\alpha}^{p,1}$ for some $0<t\leqq1$. Then, $p\cdot\left(\widehat{\alpha}^{p,1}-\alpha_0^{p,1}f\right)=0$ and $\alpha_0^{p,1}f=t\widehat{\alpha}^{p,1}$ imply that $t=1$. Thus, $\widehat{\alpha}^{p,1}=\alpha_0^{p,1}b$ holds whenever $p>0$. Note for this RS, $((p,1),\alpha^{p,1})$, any profit-maximizing production points $\alpha'\in\overline{P}(p,1)\cap\partial P(\alpha_0=1)$ has the property that $p\widehat{\alpha}'=1$ by $\pi^{\max}(p,1)=0$. Thus, for any $\alpha'\in\overline{P}(p,1)\cap\partial P(\alpha_0=1)$, $p\widehat{\alpha}'=\frac{p\widehat{\alpha}^{p,1}}{\alpha_0^{p,1}}=pb$. This implies for any $f\in\mathbb{R}_+^m$ such that $pf=pb$, $l.v.\left(f;p,1\right)\geqq1$ holds. Hence, $\min_{\widehat{f}\in B(p,1)}l.v.\left(\widehat{f};p,1\right)=1$, so that no worker is exploited in the sense of Definition 6.

If $p\geq0$, it may be the case that $\widehat{\alpha}^{p,1}\geq\alpha_0^{p,1}b$ and $\widehat{\alpha}^{p,1}\neq\alpha_0^{p,1}b$. However, as $p\cdot\left(\widehat{\alpha}^{p,1}-\alpha_0^{p,1}b\right)=0$ and $\{\beta^{\nu}\}_{\nu\in N_1}$ constitutes a profit-maximizing production plan at p, $b\in\partial\widehat{P}(\alpha_0=1)$ holds. By the same argument as above, for any $f\in\mathbb{R}_+^m$ such that $pf=pb$, $l.v.\left(f;p,1\right)\geqq1$ holds. Thus, $\min_{\widehat{f}\in B(p,1)}l.v.\left(\widehat{f};p,1\right)=1$, so that no worker is exploited in the sense of Definition 6. ■

Theorem 4: *Under A1, A2, let* $((p,1),\alpha^{p,1})$ *be a* reproducible solution (RS). *Then, the* RS *yields positive total profits if and only if every worker in* N_2 *is exploited in the sense of Definition 7.*

The proof of Theorem 4 is analogous to that of Theorem 3, and we therefore omit it.

Note that FMT cannot hold under RS if the labor exploitation is given by the Roemer (1982) type (Definition 5 in this paper). This difficulty cannot be resolved even if A3 is imposed. The proof is easily obtained by considering the economy that we constructed in the proof of Lemma 1. (See Figure 2.) In that economy, we can see that $l.v.\,(b;p,1)=1$, that implies every worker is not exploited in the sense of Roemer (1982), though the maximal profit rate is positive under the RS of that economy. Since that economy satisfies A3, we obtain the proof of the above statement.

Note also that FMT cannot hold true in general convex cone economies with heterogeneous consumption demands of workers if the definition of labor exploitation is either the Morishima (1974) type or the Roemer (1982) type. We can see that this difficulty is also resolved under Definitions 6 and 7, a detailed discussion for which is presented by Yoshihara (2006).

5 Concluding Remarks

As shown in the theorems in this paper, we characterized the condition for the plausible formulation of labor exploitation to verify CECP, and also proposed two new definitions of labor exploitation, each of which performed well in terms of both FMT and CECP. However, the new definitions have exclusively distinct characteristics in comparison with the previous definitions such as Morishima (1974) and Roemer (1982), which may give us new insights on the Marxian theory of labor exploitation and the theory of labor value.

First, Roemer (1982) claimed that prices should emerge logically prior to labor values so as to preserve CECP as a theorem in general convex cone economies. Though he did not succeed in proving this claim with his own price-dependent labor value formulation (Definition 4 in this paper), Theorem 2 in this paper proves that his claim itself is true. In fact, in order to verify CECP as a theorem, any formulation of labor exploitation satisfying **LE** should be price-dependent, as we discussed in section 3. This implies that the classic transformation problem in Marxian economic theory is no longer worth investigating, since any price-independent labor value formulation causes the failure of CECP. In other words, according to Theorems 1 and 2, the scope of the classical Marxian perspective on labor exploitation, that the exploitative relationship between capital and labor was considered to be logically independent of which prices constitute an equilibrium in the capitalist economy,[12] should be limited to models with Leontief technology.

Second, in the orthodox Marxian argument, labor exploitation was explained by using the concept of the labor value of labor power. The labor value of labor-power could be defined in the Morishima (1974) framework as the minimal amount of direct labor necessary to produce the subsistence consumption vector as a net output. This could be accepted by orthodox Marxism as the formulation of the *socially necessary labor time to reproduce labor power*. In such an argument, the subsistence consumption vector plays a crucial role in the formulation of the labor value of labor power. In Definition 6 of this paper, however, the labor value of labor power might be defined as the minimal amount of direct labor socially necessary to achieve workers' income by which they can respectively purchase at least the subsistence consumption vector. Also, in Definition 7, the labor value of labor power might be defined as the minimal amount of direct labor socially necessary to achieve workers' income, which is evaluated via the actually used social production path. In both of these formulations, the subsistence consumption vector is used, *at most indirectly*, to define the labor value of labor power. Thus, the labor value of labor power also no longer emerges logically prior to the price of labor power (wage income). Hence, the concepts of labor value in these new definitions are completely irrelevant to theories of *exchange values of commodities and labor power*.

In spite of such a significant difference of these new definitions from the orthodox Marxian notion of labor exploitation, they would be justified, according to the scenario Roemer (1982) offered, since both FMT and CECP hold true for these new definitions. Note that we still need a further conceptual argument about which of Definitions 6 and 7 is more appropriate formulation for Marxian notion of labor exploitation. We leave this for future occasions.

Note that there have been recently some papers published, such as Skillman (1995) and Veneziani (2007), which address the issue of whether the class and exploitation structure is (logically) *persistent* or not in the long run. They argue that if savings are explicitly introduced in an intertemporal setting, then positive profits and exploitation tend to disappear over time.

[12] In the classical Marxian perspective on the capitalist economy, the phenomenon of market movements was regarded as one reflection of the so-called *class struggle* between capital and labor, and the rate of labor exploitation was considered to measure the strength of the class struggle.

Though we did not address this issue in this paper, because we worked only on a temporary equilibrium model, it is worth commenting on it. As Skillman (1995) and Veneziani (2007) pointed out, the introduction of savings without population growth easily diminishes the scarcity of capital relative to labor in accumulation economies, which makes capital accumulation drive profits and the rate of exploitation to zero over time. To take the issue of persistent exploitation seriously, we should introduce, in addition to savings, the factor of population growth explicitly in an intertemporal model, as Skillman (1995) and Veneziani (2007) also pointed out. This kind of perspective is also shared with the classical Marxian argument for the *progressive production of a relative surplus population*, and it is beyond the scope of this paper. The objective of this paper is not to discuss the persistence of exploitation, but to discuss, under the presumption of market equilibrium with positive profits, what appropriate formulations of labor exploitation are, in order to explain the emerging mechanism of class and exploitation in capitalist economies.

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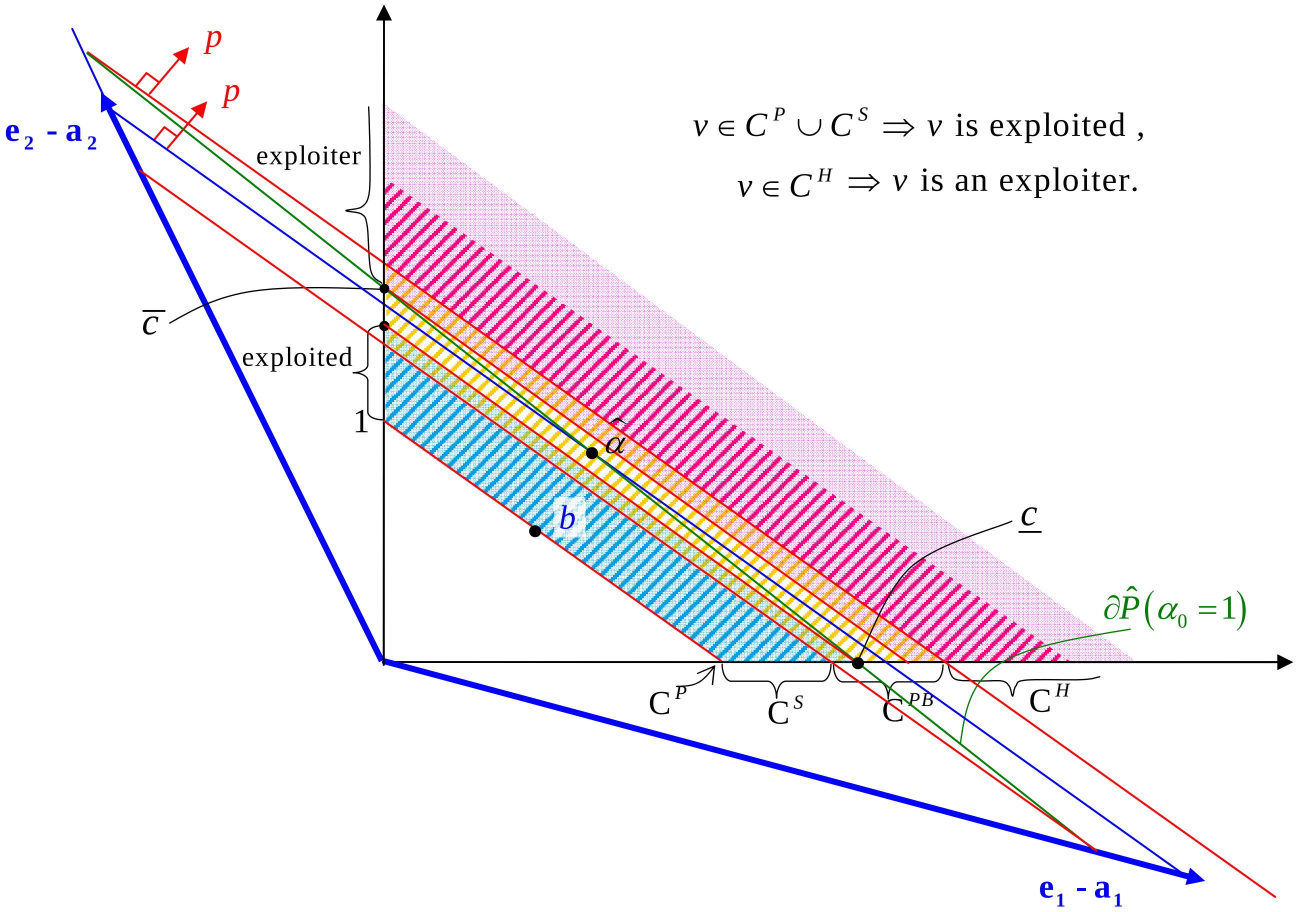


Figure1: Class-Exploitation Correspondence Principle holds under Definitions 3 and 5 in economies with Leontief technology

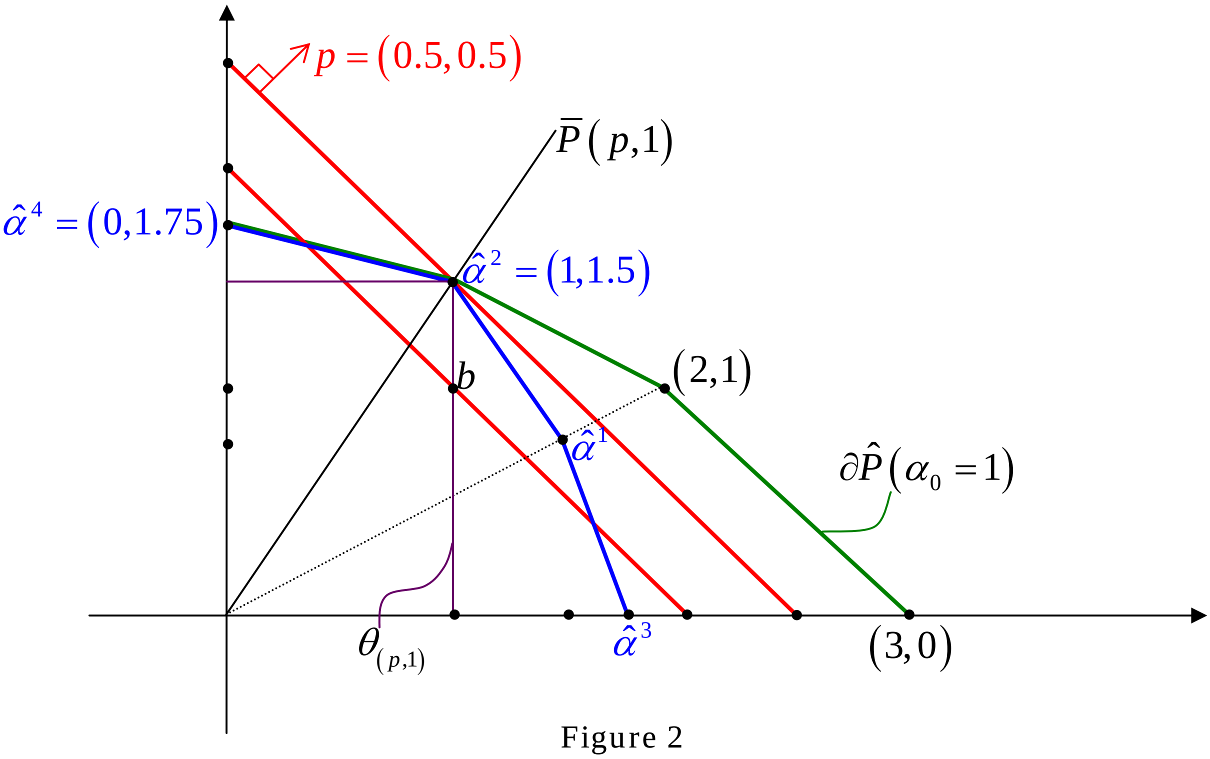
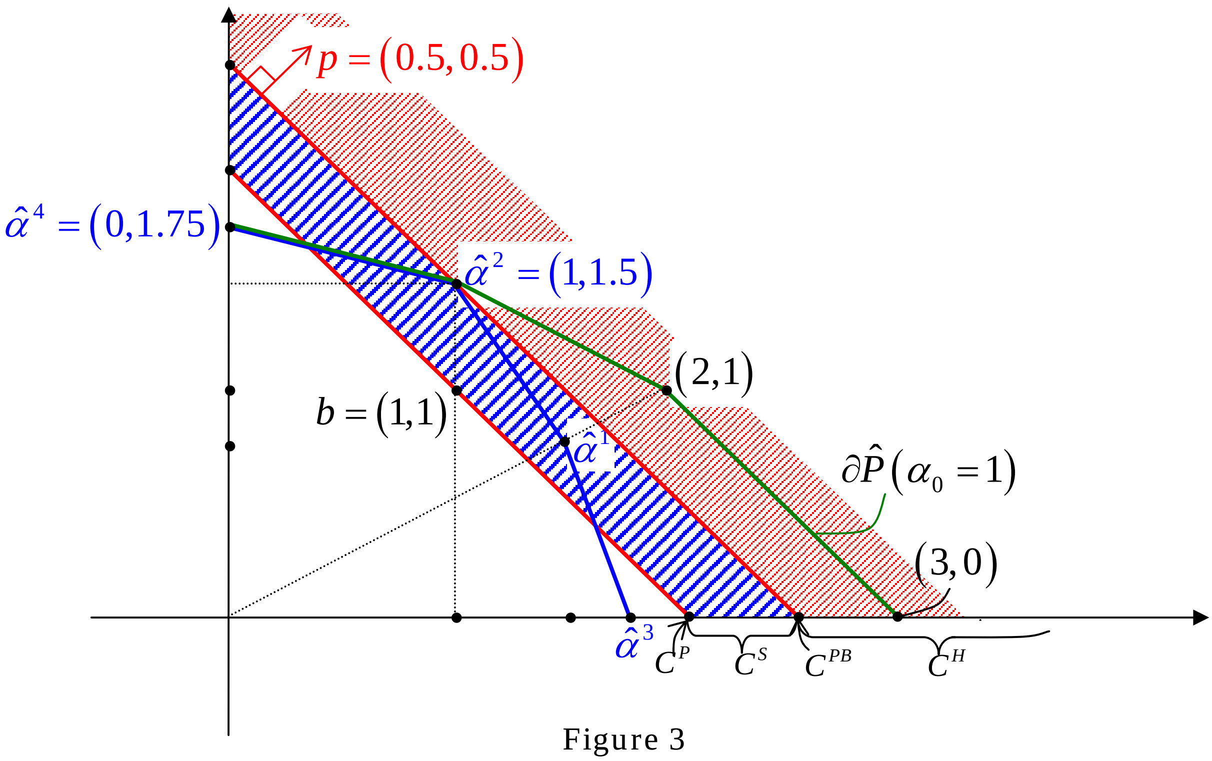

$p=(0.5,0.5)$
$\bar{P}(p,1)$
$\hat{\alpha}^4=(0,1.75)$
$\hat{\alpha}^2=(1,1.5)$
$(2,1)$
b
$\hat{\alpha}^1$
$\partial\hat{P}(\alpha_0=1)$
$\theta_{(p,1)}$
$\hat{\alpha}^3$
$(3,0)$

Figure 2



$p=(0.5,0.5)$
$\hat{\alpha}^4=(0,1.75)$
$\hat{\alpha}^2=(1,1.5)$
$(2,1)$
$b=(1,1)$
$\hat{\alpha}^1$
$\partial\hat{P}(\alpha_0=1)$
$(3,0)$
$\hat{\alpha}^3$
C^P
C^S
C^{PB}
C^H

Figure 3

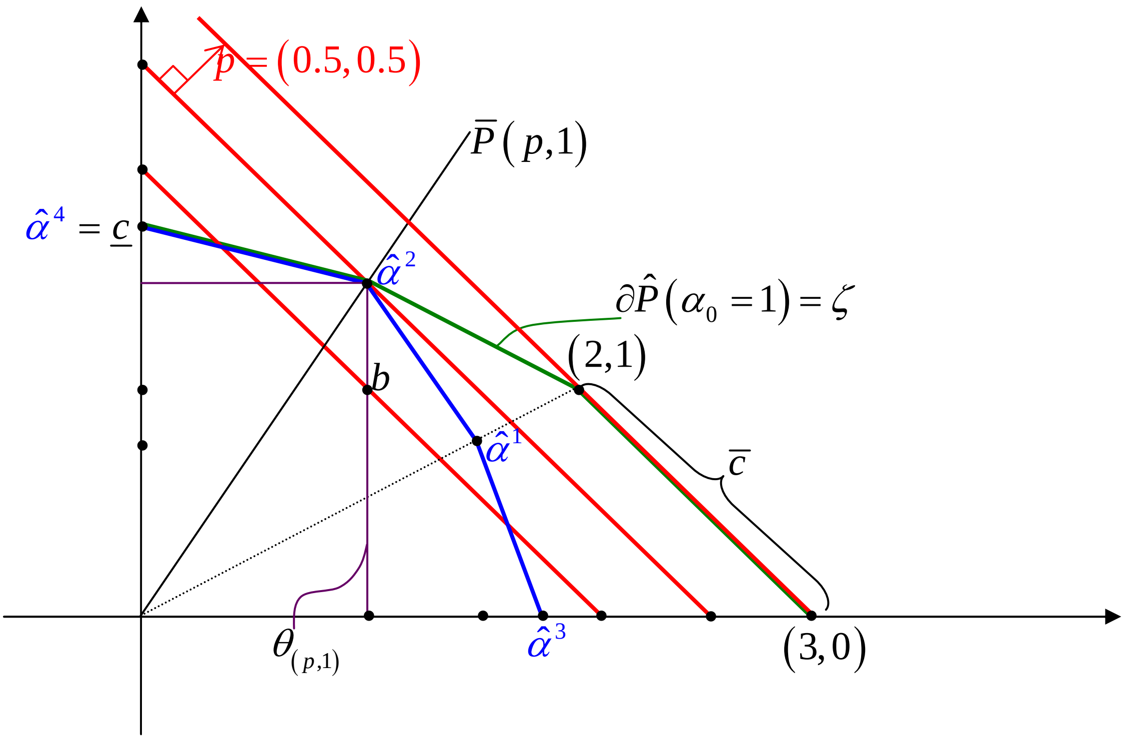


Figure 4: The Morishima (1974) definition for Marxian Labor Exploitation meets **LE**, but violates **CECP**.

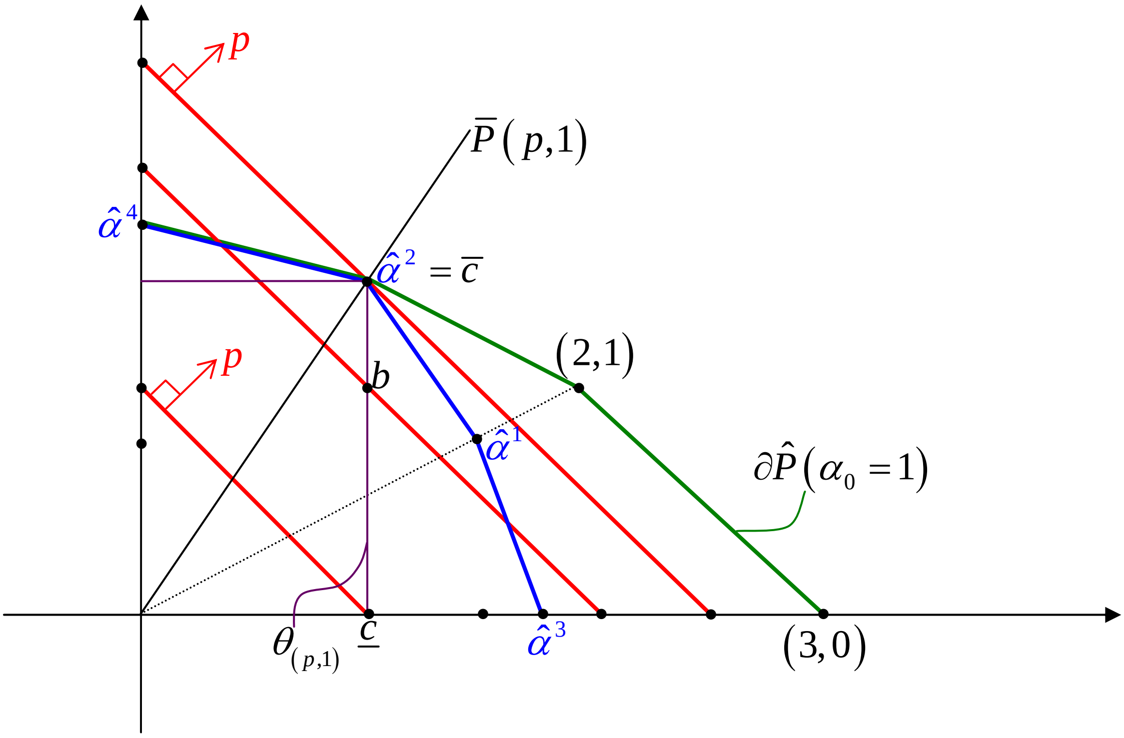


Figure 5: The Roemer (1982) definition for Marxian Labor Exploitation meets **LE**, but violates **CECP**.

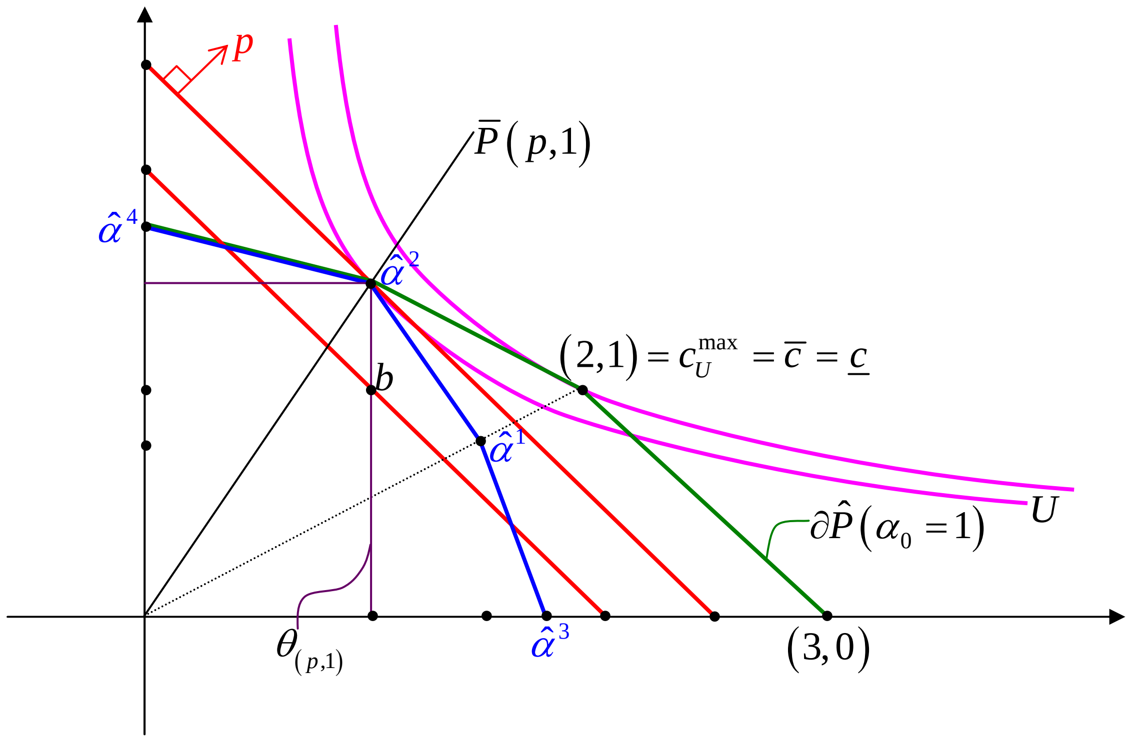


Figure 6: Definition 8 for Marxian Labor Exploitation meets **LE**, but violates **CECP**.

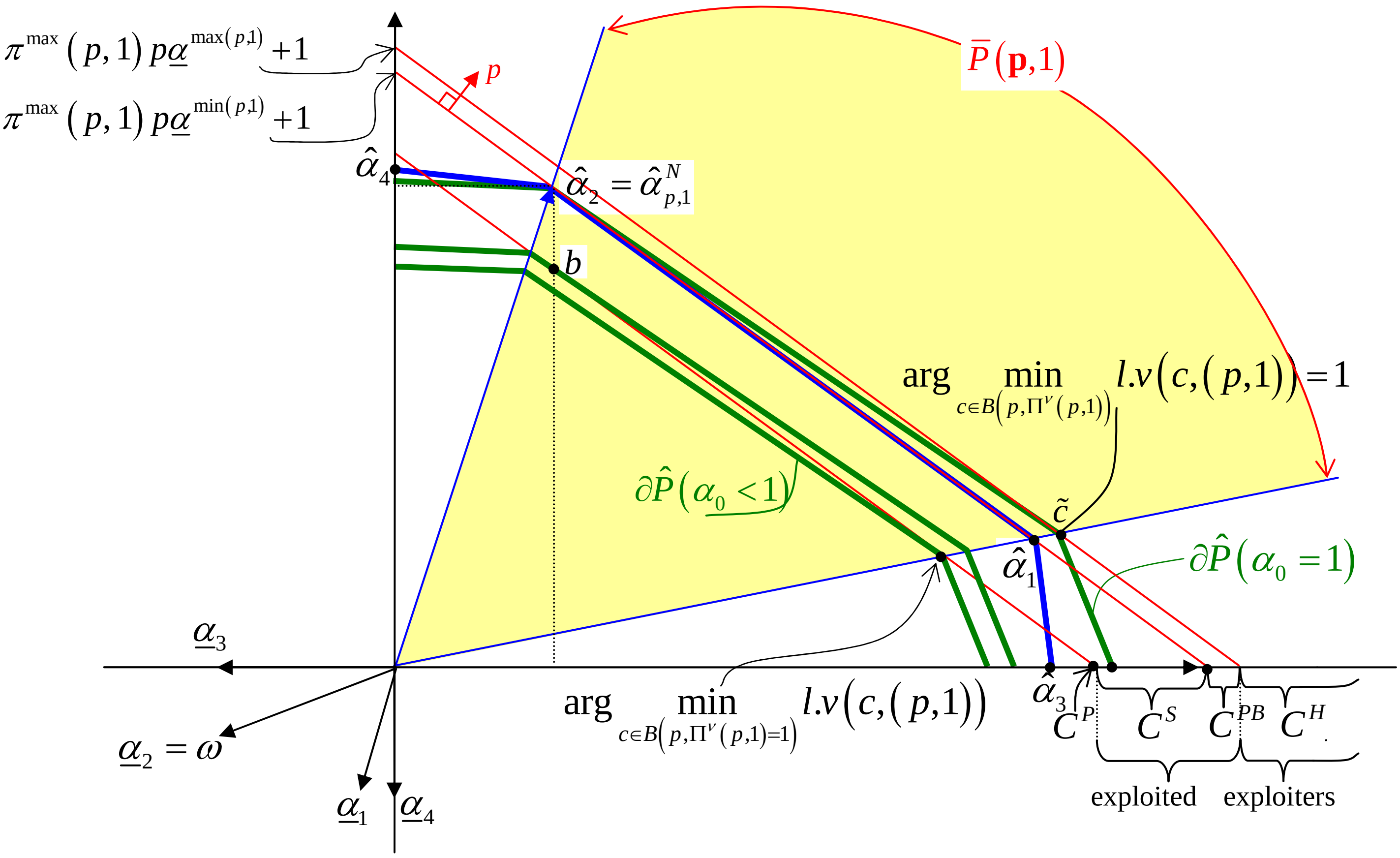


Figure 7: Class-Exploitation Correspondence Principle in a general convex cone economy when the formulation of exploitation is given by Definition 6

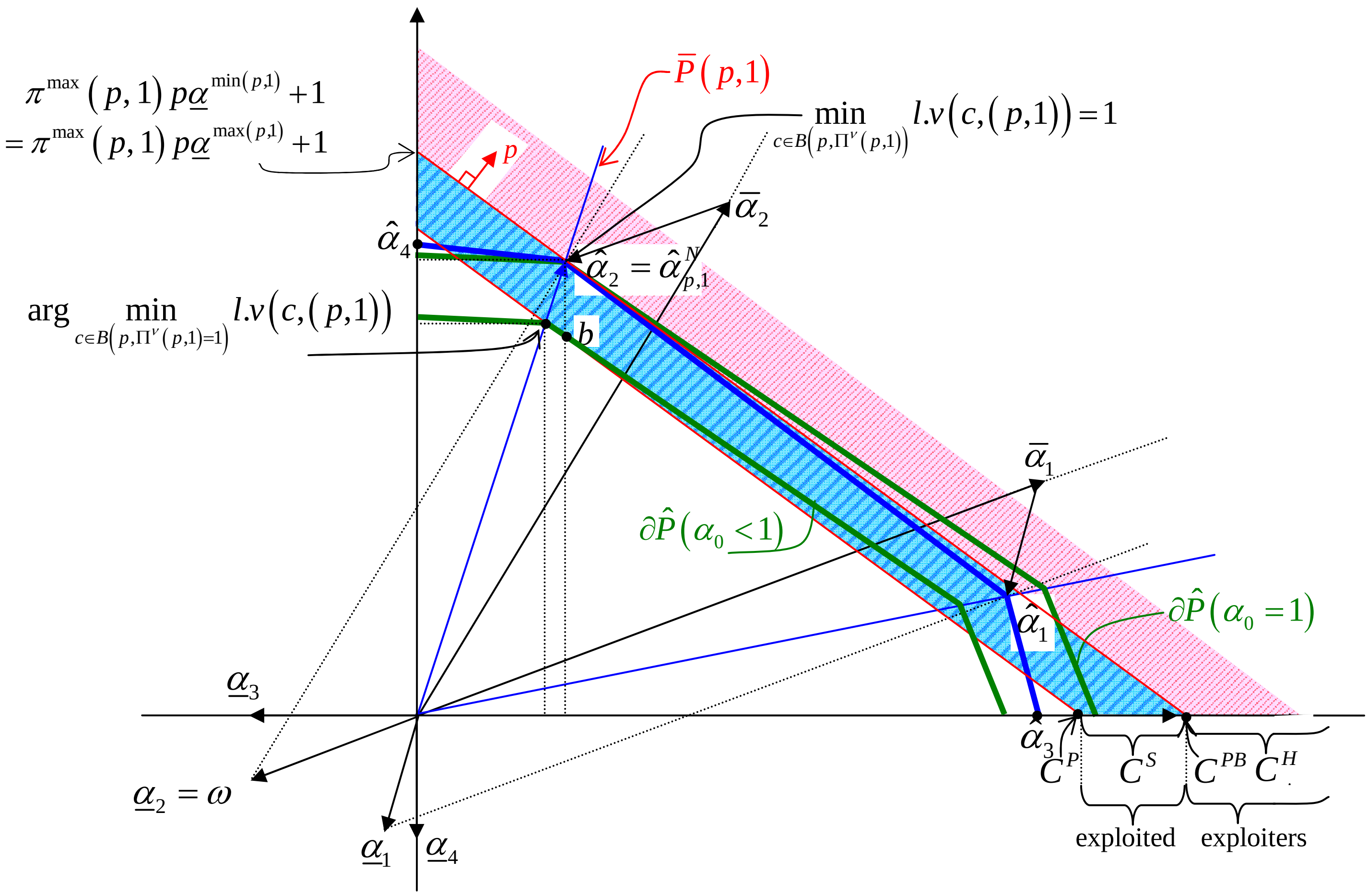


Figure 8: Class-Exploitation Correspondence Principle in a general convex cone economy when the formulation of exploitation is given by Definition 7.