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Reexamination of the Marxian Exploitation Theory

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Abstract

In this paper, we reexamine the mathematical analysis of Marxian exploitation theory. First, we reexamine the validity of the two types of Marxian labor exploitation, Morishima's (1974) type and Roemer's type (1982), in the argument of Fundamental Marxian Theorem (FMT). We show that the FMT does not hold true if we adopt the Roemer exploitation, and equilibrium notions are the reproducible solution [Roemer (1980)]. Also, we show that the FMT does not hold true for the Morishima exploitation if there exist heterogeneous demand functions among workers. Second, we reexamine the Class-Exploitation Correspondence Principle (CECP) [Roemer (1982)]. We show that the CECP does not hold true in the general convex cone economy even if we adopt the Roemer exploitation. Finally, we propose a new definition of Marxian labor exploitation, and show that all of the above difficulties can be resolved under this new definition.

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1 Introduction

During the 1970's and 1980's, there were remarkable developments in the debate about the exploitation of labor in Marxian economic theory. The "Fundamental Marxian Theorem (FMT)" was originally proved by Okishio (1963) and later named as such by Morishima (1973). The FMT showed a correspondence between the existence of positive profit and the existence of exploitation. It gives us a useful characterization for *non-trivial* equilibria, where a trivial equilibrium is one such that its social production point is zero.[1] After the seminal work by Morishima (1973), there were many generalizations and discussions of the FMT. While the original FMT is discussed in the simple Leontief economy with homogeneous labor, the generalization of the FMT to the Leontief economy with heterogeneous labor was made by Fujimori (1982), Krause (1982), etc. The problem of generalizing the FMT to the von Neumann economy was discussed by Steedman (1977) and one solution was proposed by Morishima (1974). Furthermore, Roemer (1980) generalized the theorem to a convex cone economy. These arguments may reflect the robustness of the FMT.

There has also been a remarkable development in works on Marxian economic theory. This is the "General Theory of Exploitation and Class" promoted by Roemer (1982, 1986). It argues that in a capitalist economy *if the labor supplied by agents is inelastic with respect to their wealth* (that is, the value of their own capital), then the Class-Exploitation Correspondence Principle (CECP), the Class-Wealth Correspondence Principle, and the Wealth-Exploitation Correspondence Principle can be proven. These theorems imply that under identical preferences of agents, class and exploitation status in the capitalist economy accurately reflect inequality in the distribution of wealth if the labor supply by any agent is inelastic with respect to his wealth at equilibrium prices. This argument was criticized by some Marxian theorists,[2] since it assumed a standard neoclassical labor market, which was

[1] Note that the FMT was originally considered to prove the classical Marxian argument that the exploitation of labor is the source of positive profits in the capitalist economy. However, it does not follow from the FMT that the exploitation of labor is the unique source of positive profits. The reason is that any commodity can be shown to be exploited in a system with positive profits whenever the exploitation of labor exists. This observation was pointed out by Brody (1970), Bowles and Gintis (1981), Samuelson (1982), and was named the "Generalized Commodity Exploitation Theorem (GCET)" by Roemer (1982).

[2] For instance, Bowles and Gintis (1990) and Devine and Dymski (1991, 1992).

regarded as not a *real*, but an *ideal* model of the capitalist economy by these critics. However, as Yoshihara (1998) showed, those theorems essentially hold true even if the neoclassical labor market is replaced by a non-neoclassical labor market with efficiency wage contracts, which was interpreted as a more realistic aspect of the capitalist economy by those same critics.

In this paper, we reexamine the above theorems in Marxian exploitation theory. We introduce two types of Marxian exploitation of labor: one is Morishima's (1974) type and the other is Roemer's type (1982). Then, first, we show that the FMT does not hold for the Roemer type of labor exploitation if equilibrium notions are the *reproducible solution* [Roemer (1980)]. We also show that the FMT does not hold for the Morishima type of labor exploitation if there exist heterogeneous demand functions among workers. Second, we show that the CECP no longer holds true in a convex cone economy, whichever of the Roemer and the Morishima types of exploitation we assume. Finally, we propose a new definition of Marxian exploitation of labor, and show that all of these difficulties can be resolved under this new definition.

In the following paper, section 2 defines a basic economic model with convex cone production technology, and also introduces the two types of Marxian labor exploitation. Section 3 discusses the failure of the FMT by using the two types of Marxian labor exploitation respectively. Section 4 discusses the failure of the CECP by using the two types of exploitation respectively. Finally, section 5 introduces the new definition of exploitation, and examines its performance in terms of the FMT and the CECP.

2 The Basic Model

Let P be the production set, which is assumed to be a convex cone. P has elements of the form $\alpha = (-\alpha_0, -\underline{\alpha}, \overline{\alpha})$ where $\alpha_0 \in \mathbb{R}_+$, $\underline{\alpha} \in \mathbb{R}^m_+$, and $\overline{\alpha} \in \mathbb{R}^m_+$. Thus, elements of P are vectors in $\mathbb{R}^{2m+1}$. The first component, $-\alpha_0$, is the direct labor input of the process α; and the next m components, $-\underline{\alpha}$, are the inputs of goods used in the process; and the last m components, $\overline{\alpha}$, are the outputs of the m goods from the process. We denote the net output vector arising from α as $\widehat{\alpha} \equiv \overline{\alpha} - \underline{\alpha}$. We assume that P is a closed convex cone containing the origin in $\mathbb{R}^{2m+1}$. Moreover, it is assumed that:

A 1. $\forall \alpha \in P$ s.t. $\alpha_0 \geq 0$ and $\underline{\alpha} \geqq 0$, $[\overline{\alpha} \geq 0 \Rightarrow \alpha_0 > 0]$; and

A 2. $\forall$ commodity m vector $c \in \mathbb{R}_{+}^{m}$, $\exists \alpha \in P$ s.t. $\widehat{\alpha} \geqq c$.

Given such P, we will sometimes use the notations like $P(\alpha_0 = 1)$ and $\widehat{P}(\alpha_0 = 1)$, where

$$P(\alpha_0 = 1) \equiv \{(-\alpha_0, -\underline{\alpha}, \overline{\alpha}) \in P \mid \alpha_0 = 1\}$$

and

$$\widehat{P}(\alpha_0 = 1) \equiv \{\widehat{\alpha} \in \mathbb{R}^m \mid \exists \alpha = (-1, -\underline{\alpha}, \overline{\alpha}) \in P \text{ s.t. } \overline{\alpha} - \underline{\alpha} \geq \widehat{\alpha}\}.$$

Given a market economy, any price system is denoted by $p \in \mathbb{R}_{+}^{m}$, which is a price vector of m commodities. Moreover, a *subsistent vector* of commodities $b \in \mathbb{R}_{+}^{m}$ is also necessary in order to supply one unit of labor per day. We assume that the nominal wage rate is normalized to unity when it purchases the subsistent consumption vector only. By assumption, $pb = 1$ holds.

2.1 Two Definitions for Marxian Exploitation of Labor

To define Marxian exploitation of labor in terms of surplus value, we must first define the labor value of a vector of commodities. In the economic model with joint production such as the von Neumann model and the convex cone model, there have been two candidates for the definition of Marxian exploitation of labor: one is of Morishima (1974) and Roemer (1981), and the other is Roemer (1982). The difference between them comes from the different definitions of labor value between them.

Morishima's definition of labor value of a vector of commodities is given independently of the particular equilibrium the economy is in. Let $c \in \mathbb{R}_{+}^{m}$ be a vector of produced commodities. Let

$$\phi(c) \equiv \{\alpha \in P \mid \widehat{\alpha} \geqq c\},$$

which is the set of the production points which produce, as net output vectors, at least c. Then:

Definition 1: The Morishima (1974) labor value of commodity vector c, $l.v.(c)$, is given by

$$l.v.(c) \equiv \min\{\alpha_0 \mid \alpha = (-\alpha_0, -\underline{\alpha}, \overline{\alpha}) \in \phi(c)\}.$$

Given the consumption vector, $c \in \mathbb{R}^m_+$, of the worker which is purchased by his wage revenue per day, the Morishima rate of labor exploitation is defined as follows:

Definition 2: The Morishima (1974) rate of labor exploitation at the consumption bundle $c \in \mathbb{R}^m_+$ is

$$e(c) \equiv \frac{1 - l.v.(c)}{l.v.(c)}.$$

It is easy to see that $\phi(c)$ is non-empty by A2. Also,

$$\{\alpha_0 \mid \alpha = (-\alpha_0; -\underline{\alpha}; \overline{\alpha}) \in \phi(c)\}$$

is bounded from below by 0, by the assumption $\mathbf{0} \in P$ and A1. Thus, $l.v.(c)$ is well-defined since P is compact. Moreover, by A1, $l.v.(c)$ is positive whenever $c \neq \mathbf{0}$, so that $e(c)$ is well-defined.

In contrast to the Morishima (1974) labor value, the definition of labor value in Roemer (1982) depends, in part, on the particular equilibrium the economy is in. Given a price vector $p \in \mathbb{R}^m_+$ and $\alpha \in P$, let $\pi(p; \alpha) \equiv \frac{p\widehat{\alpha} - \alpha_0}{p\underline{\alpha} + \alpha_0}$. Given the consumption bundle $c \in \mathbb{R}^m_+$ and a price vector $p \in \mathbb{R}^m_+$, let $\overline{P}(p) \subseteq P$ be the set of profit-maximizing production points when the price system is $(p, 1)$.[3] Then, let

$$\phi(c; p) \equiv \left\{\alpha \in \overline{P}(p) \mid \widehat{\alpha} \geqq c\right\},$$

which is the set of those profit-rate-maximizing actions which produce, as net output vectors, at least c. Then:

Definition 3: The Roemer (1982) labor value of commodity vector c, $l.v.(c; p)$, is given by

$$l.v.(c; p) \equiv \min \{\alpha_0 \mid \alpha = (-\alpha_0, -\underline{\alpha}, \overline{\alpha}) \in \phi(c; p)\}.$$

[3] That is, $\overline{P}(p) \equiv \arg\max \{\pi(p; \alpha) \mid \alpha \in P\}$.

Definition 4: The Roemer (1982) rate of labor exploitation at the consumption bundle $c \in \mathbb{R}^m_+$ is

$$e(c;p) \equiv \frac{1 - l.v.(c;p)}{l.v.(c;p)}.$$

It is easy to verify that $l.v.(c;p)$ is well-defined, and has a positive value whenever $c \neq \mathbf{0}$, so that $e(c;p)$ is well-defined. Also, $l.v.(c;p) \geq l.v.(c)$ which indicates that $e(c;p) > 0$ implies $e(c) > 0$.

In the following discussion, we will introduce two types of Marxian economic models; one is for discussing the Fundamental Marxian Theorem (FMT) (Roemer (1981; Chapter 2)), and the other is for discussing the Class Exploitation Correspondence Principle (Roemer (1982; Chapter 5)). We will, then, examine the viability of Morishima's (1974) and Roemer's (1982) definitions of labor exploitation respectively in both of the Marxian economic models.

3 Failure of the Fundamental Marxian Theorem

3.1 The Model for the Fundamental Marxian Theorem

In the model for the FMT, it is assumed that there are two classes, the working class and the capitalist class. The working class is characterized by agents, each of who has no initial endowment of input goods, while the capitalist class is made up of agents having some non-negative and non-zero amount of input goods. Moreover, the consumption of any agent in the working class is given exogenously by the subsistent consumption bundle b, while any capitalist is implicitly assumed to save all his revenue for investing in the next production period.

There are $|N|$ capitalists; the ν-th one is endowed with a vector of produced commodity endowments ω^ν. Workers have no endowments of produced commodities; they have only one unit of labor endowment, which is homogenous among the workers. We assume that every worker has the same level of labor skill.

Under the assumption of *stationary expectations* on prices [Roemer (1980, 1981; Chapter 2)], capitalist ν's program is given, facing a market price vector

p and the wage rate 1, by:

$$\begin{aligned} \text{choose } \alpha^{\nu} &\in P \text{ to maximize } p\overline{\alpha}^{\nu} - (p\underline{\alpha}^{\nu} + \alpha_0^{\nu}) \\ \text{s.t. } p\underline{\alpha}^{\nu} + \alpha_0^{\nu} &\leq p\omega^{\nu}. \end{aligned}$$

The set of the production processes which are the optimal solutions of the above problem is denoted by $\mathcal{A}^{\nu}(p, 1)$. Given this, we are ready to discuss the definition of equilibrium in this economic system:

Definition 5 [Roemer (1980, 1981; Chapter 2)]**:** A *reproducible solution* (RS) for the economy specified above is a pair $\left(p, \{\alpha^{\nu}\}_{\nu \in N}\right)$, where $p \in \mathbb{R}_+^m$ and $\alpha^{\nu} \in P$, such that:

(a) $\forall \nu \in N$, $\alpha^{\nu} \in \mathcal{A}^{\nu}(p, 1)$ (profit maximization);

(b) $\widehat{\alpha} \geqq \alpha_0 b$ (reproducibility),
where $\widehat{\alpha} \equiv \sum_{\nu \in N}(\overline{\alpha}^{\nu} - \underline{\alpha}^{\nu})$ and $\alpha_0 \equiv \sum_{\nu \in N} \alpha_0^{\nu}$;

(c) $pb = 1$ (subsistent wage); and

(d) $\underline{\alpha} + \alpha_0 b \leqq \omega$ (social feasibility),
where $\underline{\alpha} \equiv \sum_{\nu \in N} \underline{\alpha}^{\nu}$ and $\omega \equiv \sum_{\nu \in N} \omega^{\nu}$.

The three parts except (a) need some comments. Part (b) says that net outputs should at least replace employed workers' total consumption. This is equivalent to requiring that the vector of social endowments does not decrease in terms of components, because (b) is equivalent to $\omega - (\underline{\alpha} + \alpha_0 b) + \overline{\alpha} \geqq \omega$, where the right hand side is the social stocks at the beginning of this period, the left hand side is the stocks at the beginning of the next period. Part (d) says that intermediate inputs and workers' consumption must be available from current stocks. Here, we assume that wage goods are dispensed at the beginning of each production period, therefore stocks must be sufficient to accommodate them as well. Finally, Part (c) says that the equilibrium wage rate is equal to the subsistent level. It implies that workers are plentiful relative to the social endowments of capital stocks, so that a percentage of workers should be unemployed (industrial reserve army), by which the equilibrium wage rate is driven down to the subsistent level.

The existence of the reproducible solution is guaranteed as the following proposition shows:

Proposition 1 [Roemer (1980; 1981)]**:** *Let $b \in \mathbb{R}_{++}^m$. Under A1, A2, and stationary expectation of prices, a* reproducible solution (RS) *of Definition 5 exists for the economy specified above.*

3.2 Failure of the FMT, using Roemer's Definition

Morishima (1974) showed that, in the balanced growth equilibrium of the von Neumann economy, the warranted rate of profit is positive, if and only if the Morishima (1974) rate of labor exploitation is positive. This theorem (FMT) is robust, even if the von Neumann system of production technology contains some inferior production processes. In contrast, Roemer (1981) showed that the equivalent relationship between the positive rate of Morishima's (1974) labor exploitation and the positive profit rate no longer holds true, if the production technology contains an inferior process. This difference comes from the different equilibrium notions between them: Roemer (1981) discussed the profit rate which prevails under the reproducible solution defined in Definition 5 of this paper. Furthermore, Roemer (1981) gave us a condition, under which Morishima's (1974) labor exploitation is equivalent to profit-making even in the reproducible solution of a general convex production economy. This condition requires non-existence of an inferior production process, and is called *Independence of Production*.

In this subsection, we also discuss the validity of FMT when the equilibrium notion of the convex cone production economy is the reproducible solution. Here, we examine the case of Roemer's (1982) exploitation of labor. We set up the same situation as Roemer (1981; Chapter 2) to discuss this problem. Up to this point, it is not obvious at all, even under the assumption of the independence of production, whether the positive profit rate entails the positive rate of the Roemer (1982) exploitation or not, since $e(c) > 0$ does not necessarily imply $e(c;p) > 0$. Note that Roemer (1982; Chapter 5, p. 158) already wrote that the FMT continued to hold using the Roemer (1982) exploitation. We will show that this conjecture of him is wrong.

At the first place, let us introduce the assumption of the independence of production:

A 3. (Independence of Production)
$(-\alpha_0, -\underline{\alpha}, \overline{\alpha}) \in P$, $\widehat{\alpha} \geqq 0$, and $0 \leqq c \leq \widehat{\alpha}$, then $\exists\,(-\alpha_0', -\underline{\alpha}', \overline{\alpha}') \in P$ such that $\overline{\alpha}' - \underline{\alpha}' \geqq c$ and $\alpha_0' < \alpha_0$.

Under this assumption, Roemer (1981) showed that the FMT holds true when the labor exploitation is Morishima's (1974) type:

Proposition 2 [Roemer (1980; 1981)]**:** *Let $b \in \mathbb{R}^m_{++}$. Under A1, A2, A3, and stationary expectation of prices, the following statements are equivalent:*

(i) $e(b) > 0$;
(ii) *there exists a reproducible solution yielding positive total profits*;
(iii) *all reproducible solutions yield positive total profits.*

Now, let us examine the robustness of FMT, when the definition of labor exploitation is replaced by the Roemer (1982) type. The following theorem, however, implies that the answer is negative:

Theorem 1: *Let $b \in \mathbb{R}^m_{++}$. Under A1, A2, A3, and stationary expectation of prices, there exists an economy with convex cone production technology such that every reproducible solution $\left(p, \{\alpha^\nu\}_{\nu \in N}\right)$ yields positive total profits and zero rate of the* Roemer (1982) exploitation $e(b;p) = 0$.

Proof. Let $m = 2$, $\#N = 1$, and ν is the unique capitalist in this society. Also let us define an economy as follows: in the first place, when the worker provides one unit of labor, then his subsistent consumption bundle is given by $b = (1,1)$. The social endowment of capital in the economy is given by $\omega = (2,1)$, which is owned by the unique capitalist ν. Second, let us define three production points:

$$\begin{aligned} \alpha^1 &= \left(-\alpha^1_0, -\underline{\alpha}^1, \overline{\alpha}^1\right) = (-1, (-2,-1), (2,3)); \\ \alpha^2 &= \left(-\alpha^2_0, -\underline{\alpha}^2, \overline{\alpha}^2\right) = (-1, (-1,0), (3,1)); \text{ and} \\ \alpha^3 &= \left(-\alpha^3_0, -\underline{\alpha}^3, \overline{\alpha}^3\right) = (-1, (-1,-1), (4,1)). \end{aligned}$$

Now, let P be a *closed, convex cone* subset of $\mathbb{R}^{2m+1}$ such that
1) $\mathbf{0} \in P$; and
2) $co\{\alpha^1, \alpha^2, \alpha^3\} = P(\alpha_0 = 1)$, where coX is the convex hull of a set X.

Given the above defined economy, we will show that there is a unique reproducible solution (RS). Note that:

$$\forall p \in \triangle^m \setminus \{(1,0)\},\ p\omega < p(\underline{\alpha}' + b)\ \ (\forall \underline{\alpha}' \in co\left\{\underline{\alpha}^1, \underline{\alpha}^2, \underline{\alpha}^3\right\} \setminus \left\{\underline{\alpha}^2\right\}),\ (*)$$

where $\triangle^m$ is the simplex set. Also, for $p = (1,0)$, $p\omega = p(\underline{\alpha}' + b)$ for any $\underline{\alpha}' \in co\{\underline{\alpha}^2, \underline{\alpha}^3\}$. Note that if (p,α) is a RS, then $\underline{\alpha} + \alpha_0 b \leq \omega$ by Definition 5(d). Thus, if (p,α) is a RS with $\alpha \neq \alpha^2$, then $\alpha_0 < 1$. Moreover, by Definition 5(b), if (p,α) is a RS, then $\widehat{\alpha} \geqq \alpha_0 b$. Let $\alpha^{12} \equiv \frac{1}{2}\alpha^1 + \frac{1}{2}\alpha^2$, and $\partial P(\alpha_0 = 1)$ be the set of upper-boundary of $P(\alpha_0 = 1)$. Then, $co\{\alpha^{12}, \alpha^2\} = \left\{\alpha' \in \partial P(\alpha_0 = 1) \mid \widehat{\alpha}' \geqq b\right\}$. Thus, by linearity of P, if (p, α) is a RS, then there exist $t \in (0,1]$ and $\alpha' \in co\{\alpha^{12}, \alpha^2\}$ such that $\alpha = t\alpha'$.

Note that if $p = \left(\frac{1}{3}, \frac{2}{3}\right)$, then $p\widehat{\alpha}^1 - \alpha_0^1 = p\widehat{\alpha}^2 - \alpha_0^2 = p\widehat{\alpha}' - \alpha_0' > p\widehat{\alpha}'' - \alpha_0''$ for any $\alpha' \in co\{\alpha^1, \alpha^2\}$ and any $\alpha'' \in co\{\alpha^2, \alpha^3\}$. However, because of the property (*), the existence of capital constraint in the profit maximization problem implies that α^2 is the unique profit maximizer at the price $p = \left(\frac{1}{3}, \frac{2}{3}\right)$.

Note also that if $p = \left(\frac{1}{2}, \frac{1}{2}\right)$, then $p\widehat{\alpha}^2 - \alpha_0^2 = p\widehat{\alpha}'' - \alpha_0'' > p\widehat{\alpha}' - \alpha_0'$ for any $\alpha' \in co\{\alpha^1, \alpha^2\}$ and any $\alpha'' \in co\{\alpha^2, \alpha^3\}$. Thus, by the same reasons as the above paragraph, α^2 is the unique profit maximizer at the price $p = \left(\frac{1}{2}, \frac{1}{2}\right)$ also.

Note that if p has the property that $\frac{1}{2} < p_1 < \frac{2}{3}$, $\frac{1}{2} > p_2 > \frac{1}{3}$, then α^2 is the unique profit maximizer at that price. Moreover, for any other price p, there is no $\alpha' \in co\{\alpha^{12}, \alpha^2\}$ such that for some appropriate $t \in (0,1]$, $t\alpha'$ constitutes a profit maximizer at that price.

In summary, if there exists a RS, say (p, α), then $\alpha = \alpha^2$ should hold. In fact, for any $p^* \in \triangle^m(\alpha^2) \equiv \left\{p \in \triangle^m \mid \frac{1}{2} \leqq p_1 \leqq \frac{2}{3}, \frac{1}{2} \geqq p_2 \geqq \frac{1}{3}\right\}$, (p^*, α^2) constitutes a RS. Moreover, in this case, $\pi(p^*; \alpha^2) > 0$.

Insert Figure 1 around here.

Finally, we can check that in (p^*, α^2), where $p^* \in \triangle^m(\alpha^2)$, $e(b; p^*) = 0$ holds, whereas its corresponding profit is positive. This is because $\overline{P}(p^*) = \{t\alpha^2 \in P \mid t \in \mathbb{R}_+\}$ and $\phi(\alpha_0^2 b; p^*) = \{\alpha^2\}$, which implies $l.v.(\alpha_0^2 b; p^*) = \alpha_0^2$, so that $l.v.(b; p^*) = 1$. ■

The above proof constructs an economy such that for any reproducible solution (p, α), α is the unique profit maximizer at this price p. Moreover, the corresponding net output $\widehat{\alpha}$ of this production point does not strongly dominate the subsistent consumption vector b: $\widehat{\alpha} \not> b$, as described in Figure 1. In such a case, the minimum amount of direct labor to produce at least b over the profit-maximizing production set is equal to the amount of direct labor at the reproducible solution.

Note that Roemer (1982; Chapter 5; footnote 6) pointed out that the assumption of independence of production must be made concerning the restricted production set $\overline{P}(p)$ so as to keep the FMT holding using the Roemer (1982) exploitation. The constructed economy in the proof of Theorem 1 does not meet this revised version of independence of production. But, it seems that such a restriction for possible types of production sets is uncompelling.

A3 in this paper implies the elimination of inferior production processes from possible production sets, which is an acceptable restriction, whereas the revised version of independence of production in Roemer (1982) does not necessarily have such an implication. Alternatively, the Roemer (1982) "independence of production" eliminates the types of production sets whose corresponding net output possibility sets, $\widehat{P}\left(\alpha_0 = 1\right)$, are representable by strictly concave and continuously differentiable functions. This is because in such types of production sets, any reproducible solution has only the unique profit-maximizing path, as described in Figure 1, and so the Roemer (1982) "independence of production" is violated. That seems to be too restricted. Moreover, we cannot identify whether or not the targeted economy satisfies the Roemer (1982) "independence of production" in advance to discover which of price vectors constitutes a reproducible solution in this economy.

The failure of the FMT in the above theorem comes from the explicit introduction of capital constraints into the equilibrium notion rather than the violation of the Roemer (1982) "independence of production." In fact, if we define an equilibrium notion without the explicit capital constraint as in Definition 1(d), then the FMT continues to hold even under the Roemer (1982) exploitation of labor and without the Roemer (1982) "independence of production." To discuss this, let:

Definition 5*: A *reproducible solution** (RS*) for the economy specified above is a pair $\left(p, \{\alpha^\nu\}_{\nu \in N}\right)$, where $p \in \mathbb{R}^m_+$ and $\alpha^\nu \in P$, such that the conditions (b) and (c) of Definition 5 hold, and:

(a*) $\forall \nu \in N$, $\exists \alpha^\nu \in \mathcal{A}^{*\nu}\left(p, 1\right)$ (profit maximization),
where $\mathcal{A}^{*\nu}\left(p^*, 1\right) \equiv \arg\max \left\{p^*\left(\widehat{\alpha}' - \alpha'_0 b\right) \mid \alpha' \in P \text{ and } \alpha'_0 \leq W^\nu\right\}$ and W^ν is ν's financial endowment;

(d*) $\alpha_0 \leqq \overline{L}$ (social feasibility of labor demand),
where $\alpha_0 \equiv \sum_{\nu \in N} \alpha^\nu_0$, and $\overline{L} \equiv \sum_{\nu \in N} W^\nu$ is the aggregate financial endowments for purchasing labor.

This equilibrium notion is based upon Roemer (1981; Chapter 2; Appendix 2).

Theorem 2: *Let $b \in \mathbb{R}^m_{++}$. Under A1, A2, A3, and stationary expectation of prices, there exists a reproducible solution* $\left(p, \{\alpha^\nu\}_{\nu \in N}\right)$ such that it yields positive total profits if and only if $e\left(b; p\right) > 0$.*

Proof. By Theorem 2.17 [Roemer (1981; Chapter 2; Appendix 2)], there exists a RS*, $\left(p, \{\alpha^{\nu}\}_{\nu \in N}\right)$, so that $\widehat{\alpha} \geqq \alpha_0 b$. Note that in this RS*, $\alpha_0 = \overline{L}$ by profit maximization, and p is an efficiency price which supports $\widehat{\alpha} \in \widehat{P}\left(\alpha_0 = \overline{L}\right)$ as an efficient production. In other words, $p\widehat{\alpha} \geqq p\widehat{\alpha}'$ holds for any $\widehat{\alpha}' \in \widehat{P}\left(\alpha_0 = \overline{L}\right)$. Note that by $b \in \mathbb{R}^m_{++}$ and $\widehat{\alpha} \geqq \alpha_0 b$, $\widehat{\alpha} \in \mathbb{R}^m_{++}$. Then, by A3, $p \in \mathbb{R}^m_{++}$. Thus, since the total profits of this RS* is equal to $p\left(\widehat{\alpha} - \alpha_0 b\right)$, it holds that

$$p\left(\widehat{\alpha} - \alpha_0 b\right) \lesseqqgtr 0 \Leftrightarrow \widehat{\alpha} - \alpha_0 b \lesseqqgtr 0.$$

If $\widehat{\alpha} - \alpha_0 b = 0$, then it is easy to see that $l.v.\left(\alpha_0 b\right) = \alpha_0$ by A3. Thus, $l.v.\left(\alpha_0 b; p\right) \geqq \alpha_0$ since $l.v.\left(\alpha_0 b; p\right) \geqq l.v.\left(\alpha_0 b\right)$, so that $e\left(b; p\right) \leqq 0$. If $\widehat{\alpha} - \alpha_0 b \geq 0$, then there exists $\alpha^* \in P$ such that $\alpha_0^* = \alpha_0$ and $\widehat{\alpha}^* - \alpha_0^* b > 0$ by A3. Let $p^* \in \mathbb{R}^m_{++}$ be a price vector which supports $\widehat{\alpha}^* \in \widehat{P}\left(\alpha_0 = \overline{L}\right)$ as an efficient production. Then, we can constitute a new RS* $\left(p^*, \{\alpha^{*\nu}\}_{\nu \in N}\right)$ such that $\sum_{\nu \in N} \alpha^{*\nu} = \alpha^*$. Since the only constraint for each capitalist is his budget for purchasing his employers, it is easy to find a division $\{\alpha^{*\nu}\}_{\nu \in N}$ of α^* such that $\alpha^{*\nu} \in \mathcal{A}^{*\nu}\left(p^*, 1\right)$ and $\sum_{\nu \in N} \alpha_0^{*\nu} = \overline{L}$. In this new RS*, since $\widehat{\alpha}^* - \alpha_0^* b > 0$, it holds $p^*\left(\widehat{\alpha}^* - \alpha_0^* b\right) > 0$. Moreover, by A3, there exists $\alpha^{**} \in P$ such that $\widehat{\alpha}^{**} - \alpha_0^* b \geqq 0$ and $\alpha_0^{**} < \alpha_0^*$. In particular, since $\widehat{\alpha}^* - \alpha_0^* b > 0$, we can choose $\alpha^{**} = t\alpha^*$ with $t \in (0,1)$ by the cone property of P. In this case, $\alpha^{**} \in \overline{P}\left(p^*\right)$ so that $l.v.\left(\alpha_0^* b; p^*\right) < \alpha_0^*$. This implies $e\left(b; p^*\right) > 0$.

Conversely, if $e\left(b; p^*\right) > 0$, then $e\left(b\right) > 0$. Thus, by Proposition 2, all RS*s yield positive total profits. ■

Although the above theorem seems to show the validity of the Marxian theory of labor exploitation and profits using the Roemer (1982) definition, it cannot be regarded as such: the theorem permits the situation that although the workers continue to supply a constant amount of labor and receive a constant amount of real wage goods, they are exploited or not according to the aggregate production points chosen in equilibrium. For instance, given $\alpha_0 > 0$ and $\alpha_0 b > 0$, if $\widehat{\alpha} - \alpha_0 b > 0$ in a reproducible solution, say, $(p, (\widehat{\alpha}, \alpha_0))$, then they should be exploited at $(p, (\widehat{\alpha}, \alpha_0))$, while if $\widehat{\alpha}' - \alpha_0 b \geq 0$ in another reproducible solution, say, $\left(p', \left(\widehat{\alpha}', \alpha_0\right)\right)$, then they might not be exploited at $\left(p', \left(\widehat{\alpha}', \alpha_0\right)\right)$; in fact, they are not exploited at $\left(p', \left(\widehat{\alpha}', \alpha_0\right)\right)$ if $\widehat{P}\left(\alpha_0 = 1\right)$ has such an upper-boundary as representable by a strictly concave and continuously differentiable function, because $\overline{P}\left(p'\right)$ consists only of the type of

$\left(t\widehat{\alpha}', t\alpha_0\right)$ where $t \in \mathbb{R}_{++}$. This is unacceptable, since the status of workers in terms of exploitation has to be identified only based upon their *objective* conditions of labor like α_0 and $\alpha_0 b$. But, the above example indicates that the choice of net output components may influence the exploitation status of workers even under the same labor condition, which is inappropriate.

3.3 The case of Workers' Heterogenous Consumption Demands

In this subsection, we go back to Morishima (1974) type of labor exploitation, and examine the validity of FMT in reproducible solutions with heterogeneous consumption demands among workers. We can see that the FMT holds true in the Leontief economy even if workers have heterogeneous consumption demands. That is, in that economy, the positive rate of profit prevails in the reproducible solution, if and only if the average rate of exploitation of all workers is positive [Roemer (1981)]. Moreover, the latter condition holds, if and only if each and every worker's rate of exploitation is positive. Thus, in the economic model with Leontief production technology and heterogeneous consumption demands, the scenario of Marxian exploitation of labor is completely consistent. Such a result, however, can no longer hold once we discuss it in a general convex cone economy with heterogeneous consumption demands.

Let I be the finite set of types of the workers with heterogeneous consumption demands with a generic element υ. Let $F(\upsilon) \in [0,1]$ be the fraction of the υ-type workers. By definition, $\sum_{\upsilon \in I} F(\upsilon) = 1$. Let $\upsilon^* \equiv \min_{\upsilon \in I} \{F(\upsilon)\}$, and let us assume that the aggregate labor endowments of the υ^*-type workers is normalized to one. Thus, for any type $\upsilon \in I$, the aggregate labor endowments of this type of workers is given by $\frac{F(\upsilon)}{F(\upsilon^*)}$. Then, let

$$P_0(\upsilon) \equiv \left\{ \alpha_0^\upsilon \in \mathbb{R}_+ \mid \exists \alpha^\upsilon = (-\alpha_0^\upsilon, -\underline{\alpha}^\upsilon, \overline{\alpha}^\upsilon) \in P \text{ s.t. } \alpha_0^\upsilon \leq \frac{F(\upsilon)}{F(\upsilon^*)} \right\}$$

be the set of feasible production points by the υ-type workers.

Given a price system $p \in \mathbb{R}_+^m$, let us denote the consumption demand of the υ-type worker per one unit of income by $d^\upsilon(p) \in \mathbb{R}_+^m$. The demand function $d^\upsilon(\cdot)$ is assumed to be derived from a continuous, strictly monotonic, quasi-concave, and homothetic utility function of the υ-type worker, and $pd^\upsilon(p) = 1$ for any $p \in \mathbb{R}_+^m$ normalized to $\sum_{j=1}^m p_j = 1$.

Given a price system $p \in \mathbb{R}^m_+$ and a production plan $\alpha = (-\alpha_0, -\underline{\alpha}, \overline{\alpha}) \in P$, let the aggregate labor demand α_0 consist of $(\alpha_0^\upsilon)_{\upsilon \in I}$, where α_0^υ is the employed labor amount of the υ-type workers, and $\sum_{\upsilon \in I} \alpha_0^\upsilon = \alpha_0$. Then, the average consumption demand of the employed workers is defined by:

$$d\left(p; (\alpha_0^\upsilon)_{\upsilon \in I}\right) \equiv \frac{\sum_{\upsilon \in I} \alpha_0^\upsilon d^\upsilon (p)}{\alpha_0}.$$

Note that $p \cdot d\left(p; (\alpha_0^\upsilon)_{\upsilon \in I}\right) = 1$ by definition.

An economy is specified by a list $\mathcal{E} = \left(P; N; (\omega^\nu)_{\nu \in N}; I; (F(\upsilon))_{\upsilon \in I}; (d^\upsilon(\cdot))_{\upsilon \in I}\right)$. Now, we are ready to define reproducible solutions with heterogeneous consumption demands among workers:

Definition 6: A *reproducible solution* (RS) for the economy with heterogeneous consumption demands among workers $\mathcal{E}$ is a pair $\left(p, \{\alpha^\nu\}_{\nu \in N}\right)$, where $p \in \mathbb{R}^m_+$ and $\alpha^\nu \in P$, such that:

(a) $\forall \nu \in N$, $\exists \alpha^\nu \in \mathcal{A}^\nu (p, 1)$ (profit maximization);

(b) $\widehat{\alpha} \geqq \alpha_0 d\left(p; (\alpha_0^\upsilon)_{\upsilon \in I}\right)$ (reproducibility),
where $\widehat{\alpha} \equiv \sum_{\nu \in N} (\overline{\alpha}^\nu - \underline{\alpha}^\nu)$ and $\alpha_0 \equiv \sum_{\nu \in N} \alpha_0^\nu = \sum_{\upsilon \in I} \alpha_0^\upsilon$;

(c) $\forall \upsilon \in I$, $pd^\upsilon(p) = 1$ and $\alpha_0^\upsilon \in P_0(\upsilon)$; and

(d) $\underline{\alpha} + \alpha_0 d\left(p; (\alpha_0^\upsilon)_{\upsilon \in I}\right) \leqq \omega$ (social feasibility),
where $\underline{\alpha} \equiv \sum_{\nu \in N} \underline{\alpha}^\nu$ and $\omega \equiv \sum_{\nu \in N} \omega^\nu$.

This definition is essentially the same as Definition 5, except the point that the aggregate consumption demands of employed workers are endogenously given by each employed worker's demand function.

Proposition 3: *Under A1, A2, and stationary expectation of prices, a* reproducible solution (RS) *of Definition 6 exists for the economy $\mathcal{E}$.*

Theorem 3: *Under A1, A2, A3, and stationary expectation of prices, let $\left(p, \{\alpha^\nu\}_{\nu \in N}\right)$ be the* reproducible solution (RS) *with the average consumption demand of the employed workers, $d\left(p; (\alpha_0^\upsilon)_{\upsilon \in I}\right)$, for the economy $\mathcal{E}$. Then, the* RS *yields positive total profits if and only if $e\left(d\left(p; (\alpha_0^\upsilon)_{\upsilon \in I}\right)\right) > 0$.*

Proof. ($\Rightarrow$): Let $\left(p, \{\alpha^\nu\}_{\nu \in N}\right)$ be a RS with a positive total profit. Thus,

$$\begin{aligned} p \cdot \left(\sum_{\nu \in N} (\overline{\alpha}^\nu - \underline{\alpha}^\nu)\right) - \sum_{\nu \in N} \alpha_0^\nu &= p \cdot \left(\sum_{\nu \in N} (\overline{\alpha}^\nu - \underline{\alpha}^\nu) - \sum_{\upsilon \in I} \alpha_0^\upsilon d^\upsilon (p)\right) \\ &= p \cdot \left(\widehat{\alpha} - \alpha_0 d\left(p; (\alpha_0^\upsilon)_{\upsilon \in I}\right)\right) > 0. \end{aligned}$$

Since $p \in \mathbb{R}^m_+$ and $\widehat{\alpha} \geqq \alpha_0 d\left(p; (\alpha_0^\upsilon)_{\upsilon \in I}\right)$ by Definition 6(b), the last strict inequality implies $\widehat{\alpha} \geq \alpha_0 d\left(p; (\alpha_0^\upsilon)_{\upsilon \in I}\right)$ and $\widehat{\alpha} \neq \alpha_0 d\left(p; (\alpha_0^\upsilon)_{\upsilon \in I}\right)$. Thus, by A3, $l.v. \left(\alpha_0 d\left(p; (\alpha_0^\upsilon)_{\upsilon \in I}\right)\right) < \alpha_0$. By linearity, $l.v. \left(d\left(p; (\alpha_0^\upsilon)_{\upsilon \in I}\right)\right) < 1$, which implies $e\left(d\left(p; (\alpha_0^\upsilon)_{\upsilon \in I}\right)\right) > 0$.

($\Leftarrow$): Since there is no RS with a negative total profit, it suffices to discuss only the case of zero profit. Let $\left(p, \{\alpha^\nu\}_{\nu \in N}\right)$ be a RS with a zero total profit. Thus, $p \cdot \left(\widehat{\alpha} - \alpha_0 d\left(p; (\alpha_0^\upsilon)_{\upsilon \in I}\right)\right) = 0$. By Definition 6(b), $\widehat{\alpha} \geqq \alpha_0 d\left(p; (\alpha_0^\upsilon)_{\upsilon \in I}\right)$. If for some commodity j, $\widehat{\alpha}_j - \alpha_0 d_j\left(p; (\alpha_0^\upsilon)_{\upsilon \in I}\right) > 0$, then it follows that $p_j = 0$. However, since every worker has a strictly monotonic preference on $\mathbb{R}^m_+$, $p_j = 0$ implies $d_j\left(p; (\alpha_0^\upsilon)_{\upsilon \in I}\right) = +\infty$, a contradiction. Thus, $\widehat{\alpha} = \alpha_0 d\left(p; (\alpha_0^\upsilon)_{\upsilon \in I}\right)$.

Suppose $l.v. \left(\alpha_0 d\left(p; (\alpha_0^\upsilon)_{\upsilon \in I}\right)\right) < \alpha_0$. Then, $l.v. (\widehat{\alpha}) < \alpha_0$, which implies that $\widehat{\alpha} \notin \partial \widehat{P}(\alpha_0)$, where $\partial \widehat{P}(\alpha_0)$ is the set of upper-boundary of $\widehat{P}(\alpha_0) \equiv \{\widehat{\alpha}\text{"} \in \mathbb{R}^m \mid \exists \alpha' = (-\alpha_0, -\underline{\alpha}', \overline{\alpha}') \in P \text{ s.t. } \overline{\alpha}' - \underline{\alpha}' \geq \widehat{\alpha}\text{"}\}$. This also implies $\alpha \notin \partial P$, so that there exists at least one capitalist $\nu \in N$ such that $\alpha^\nu \notin \mathcal{A}^\nu(p, 1)$, a desired contradiction. Thus, $l.v. \left(\alpha_0 d\left(p; (\alpha_0^\upsilon)_{\upsilon \in I}\right)\right) = \alpha_0$, so that $e\left(d\left(p; (\alpha_0^\upsilon)_{\upsilon \in I}\right)\right) = 0$. ■

Let $\partial \widehat{P}(\alpha_0 = 1)$ is the set of upper-boundary of $\widehat{P}(\alpha_0 = 1)$, and let $\widehat{P}^\circ(\alpha_0 = 1) \equiv \widehat{P}(\alpha_0 = 1) \setminus \partial \widehat{P}(\alpha_0 = 1)$.

Theorem 4: *Under A1, A2, A3, and stationary expectation of prices, let* $\left(p, \{\alpha^\nu\}_{\nu \in N}\right)$ *be the* reproducible solution (RS) *with the average consumption demand of the employed workers,* $d\left(p; (\alpha_0^\upsilon)_{\upsilon \in I}\right)$, *for the economy* $\mathcal{E}$. *Then, the following two statements are equivalent:*
(I) *the* RS *yields positive total profits if and only if* $e(d^\upsilon(p)) > 0$ *for any* $\upsilon \in I$;
(II) $d^\upsilon(p) \in \widehat{P}^\circ(\alpha_0 = 1)$ *for any* $\upsilon \in I$.

Proof. On [(II)$\Rightarrow$(I)]. Let $d^\upsilon(p) \in \widehat{P}^\circ(\alpha_0 = 1)$ hold for any $\upsilon \in I$. This implies for any $\upsilon \in I$, there exists $\widehat{\alpha} \in \partial \widehat{P}(\alpha_0 = 1)$ such that $\widehat{\alpha} > d^\upsilon(p)$. Then, since P is a convex cone with A 2, there exists $\alpha^* \in P$ with $\alpha_0^* < 1$, such that $\widehat{\alpha}^* \in \partial \widehat{P}(\alpha_0^*)$ and $\widehat{\alpha}^* \geq d^\upsilon(p)$. This also implies $e(d^\upsilon(p)) > 0$ for any $\upsilon \in I$. Conversely, let us suppose there exists $\upsilon' \in I$ such that $d^{\upsilon'}(p) \notin \widehat{P}^\circ(\alpha_0 = 1)$. Then, for any $\alpha' \in P$ with $\alpha_0' = 1$, either $\widehat{\alpha}' = d^{\upsilon'}(p)$ or $\widehat{\alpha}' \not\geq d^{\upsilon'}(p)$ holds for this $\upsilon' \in I$. This implies $e\left(d^{\upsilon'}(p)\right) \leq 0$ for this $\upsilon' \in I$. Thus, the condition (II) holds if and only if $e(d^\upsilon(p)) > 0$ holds for any $\upsilon \in I$.

Thus, if the condition (II) holds, then $e\left(d\left(p;(\alpha_0^{\upsilon})_{\upsilon\in I}\right)\right)>0$ which implies that the RS yields positive total profits by Theorem 3. If the RS yields positive total profits, then given $d^{\upsilon}(p)\in\widehat{P}^{\circ}(\alpha_0=1)$ for any $\upsilon\in I$, $e(d^{\upsilon}(p))>0$ trivially holds for any $\upsilon\in I$.

On [(I)$\Rightarrow$(II)]. Suppose (II) does not hold. Let the RS yield positive total profits. Since the condition (II) is equivalent to $e(d^{\upsilon}(p))>0$ holds for any $\upsilon\in I$, there exists $\upsilon'\in I$ such that $e\left(d^{\upsilon'}(p)\right)\leq 0$ in spite of the RS with positive total profits. Thus, (I) is violated. ■

By the above theorem, we derive the following situation:

Corollary: *Under A1, A2, A3, and stationary expectation of prices, there exists an economy with convex cone technology and heterogeneous consumption demands among workers, such that there exists a* reproducible solution (RS), $\left(p,\{\alpha^{\nu}\}_{\nu\in N}\right)$, *with the average consumption demand of the employed workers,* $d\left(p;(\alpha_0^{\upsilon})_{\upsilon\in I}\right)$, *which yields positive total profits and a negative rate of the* Morishima (1974) exploitation *of some* υ^**-type workers,* $e\left(d^{\upsilon^*}(p)\right)<0$.

Proof. Let $m=2$, $\#N=1$, and ν is the unique capitalist in this society. Also let us define an economy as follows: at the first place, there are two types of workers, the type υ^* and the type μ: the υ^*-type (*resp.* μ-type) workers are characterized by their common consumption demand functions $d^{\upsilon^*}(\cdot)$ (*resp.* $d^{\mu}(\cdot)$). Assume that each of these two demand functions is respectively derived from a continuous, monotonic, quasi-concave, and homothetic utility function, and that

$$d^{\upsilon^*}(p)=(0.5,1.25)\ \text{ and } d^{\mu}(p)=(2.5,0.25)\ \text{ if } p=\left(\frac{1}{3},\frac{2}{3}\right).$$

Moreover, $F(\upsilon^*)=0.5=F(\mu)$. The social endowment of capital in the economy is given by $\omega=(2.5,0.75)$, which is owned by the unique capitalist ν. Second, let us define four production points:

$$\begin{aligned}
\alpha^1 &= \left(-\alpha_0^1,-\underline{\alpha}^1,\overline{\alpha}^1\right)=(-1,(-1,0),(2.5,1))\,;\\
\alpha^2 &= \left(-\alpha_0^2,-\underline{\alpha}^2,\overline{\alpha}^2\right)=(-1,(0,-1),(2.5,1.5))\,;\\
\alpha^3 &= \left(-\alpha_0^3,-\underline{\alpha}^3,\overline{\alpha}^3\right)=(-1,(-1.5,0),(1.5,1.01))\,;\text{ and}\\
\alpha^4 &= \left(-\alpha_0^4,-\underline{\alpha}^4,\overline{\alpha}^4\right)=(-1,(0,-1.5),(2.6,1.5))\,.
\end{aligned}$$

Now, we are ready to define a production possibility set of this economy. Let P be a *closed, convex cone* subset of $\mathbb{R}^{2m+1}$ such that
1) $\mathbf{0} \in P$;
2) $co\{\alpha^1, \alpha^2, \alpha^3, \alpha^4\} = P(\alpha_0 = 1)$;
3) the net output possibility set at one unit of labor input of P, $\widehat{P}(\alpha_0 = 1)$, is defined by:

$$\widehat{P}(\alpha_0 = 1) = co\left\{\widehat{\alpha}^1, \widehat{\alpha}^2, \widehat{\alpha}^3, \widehat{\alpha}^4, \mathbf{0}\right\}.$$

Given the above defined economy, we will show the existence of one reproducible solution (RS). Let us consider $(p^*, \alpha^*) = \left(\left(\frac{1}{3}, \frac{2}{3}\right), \alpha^1\right)$. We will show it is a RS. Since $d^{\upsilon^*}(p^*) = (0.5, 1.25)$ and $d^{\mu}(p^*) = (2.5, 0.25)$, we can define the aggregate consumption demand of all the employed workers by $d\left(p^*; \left(\alpha_0^{\upsilon^*}, \alpha_0^{\mu}\right)\right) = F(\upsilon^*) d^{\upsilon^*}(p^*) + F(\mu) d^{\mu}(p^*) = (1.5, 0.75)$, assuming the ratio of employed workers between the two types are equal. Thus, at (p^*, α^*), we have $\widehat{\alpha}^* = (1.5, 1) \geq (1.5, 0.75) = \alpha_0^* \cdot d\left(p^*; \left(\alpha_0^{\upsilon^*}, \alpha_0^{\mu}\right)\right)$, which implies that Definition 6(b) holds. Also, $\underline{\alpha}^* + \alpha_0^* \cdot d\left(p^*; \left(\alpha_0^{\upsilon^*}, \alpha_0^{\mu}\right)\right) = (1, 0) + (1.5, 0.75) = (2.5, 0.75) = \omega$, which implies that Definition 6(d) holds. Moreover, by construction, we can check that p^* is an efficiency price for α^*. Thus, since $p^*\underline{\alpha}^* + \alpha_0^* = p^*\omega^{\nu} < p^*\underline{\alpha} + \alpha_0$ for any $\alpha \in co\{\alpha^1, \alpha^2, \alpha^3, \alpha^4\} \setminus \{\alpha^1\}$, it follows that $\alpha^* \in \mathcal{A}^{\nu}(p^*, 1)$, Definition 6(a) holds. Finally, by construction, Definition 6(c) holds.

Insert Figure 2 around here.

Note that this RS yields a positive profit: $p^*\overline{\alpha}^* - (p^*\underline{\alpha}^* + \alpha_0^*) = \frac{1}{6} > 0$. Also, $e\left(d\left(p^*; \left(\alpha_0^{\upsilon^*}, \alpha_0^{\mu}\right)\right)\right) > 0$, since $l.v.\left(d\left(p^*; \left(\alpha_0^{\upsilon^*}, \alpha_0^{\mu}\right)\right)\right) < 1$. The last inequality follows from $l.v.(\widehat{\alpha}^*) = 1$, $\widehat{\alpha}^* \geq d\left(p^*; \left(\alpha_0^{\upsilon^*}, \alpha_0^{\mu}\right)\right)$, and A3. At the same time, in this RS, the υ^*-type employed workers are not exploited in the Morishima (1974) sense, since $e\left(d^{\upsilon^*}(p^*)\right) < 0$. The last inequality follows from $d^{\upsilon^*}(p^*) > \widehat{\alpha}^3$, $l.v.\left(\widehat{\alpha}^3\right) = 1$, and A3. In other words, $d^{\upsilon^*}(p^*) \notin \widehat{P}(\alpha_0 = 1) = co\{\mathbf{0}, (0, 1.01), (1.5, 1), (2.5, 0.5), (2.6, 0)\}$ implies $l.v.\left(d^{\upsilon^*}(p^*)\right) > 1$. ■

The above theorem implies that once we consider a more general model than the Leontief economy, the Marxian theory of labor exploitation can no longer be a consistent argument for explaining the source of positive profits even in the case of the Morishima (1974) definition of labor exploitation. This is because all workers are homogeneous in their labor endowments and their labor skills: they engage in the same working hours with the same labor skill and the same wage rate. They are mutually identical except for their consumption demands when they are faced with the same budget constraint. In this situation, we would expect that every worker is in the same position with respect to labor exploitation, since the exploitation status seems to reflect only workers' *objective* labor conditions. However, the above theorem shows that some types of workers are not exploited due to their *subjective* consumption demands, even if they are under the same *objective* labor condition as others who are exploited.

Although the above theorem assumes the notion of reproducible solutions proposed by Roemer (1981), the same type of conclusion could hold true even if the positive profit is not associated with the reproducible solution, but rather associated with the balanced growth equilibrium as in Morishima (1974). Thus, the above negative conclusion is not based on the choice of equilibrium notions.

4 Failure of the Class Exploitation Correspondence Principle in Convex Cone Economies

4.1 The Model for the Class Exploitation Correspondence Principle

In the model for the CECP, a fixed class structure like the model for the FMT is not assumed in advance of economic analysis. For the sake of simplicity, let us follow the same setting as that in Roemer (1982; Chapter 5). That is, our schematic model of a capitalist economy is that all agents are accumulators who seek to expand the value of their endowments as rapidly as possible. Let us denote the set of agents by N with generic element ν. All agents have access to the same technology P, but they differ in their bundles of endowments. An agent $\nu \in N$ can engage in three types of economic activity: he can sell his labor power γ_0^ν, he can hire the labor powers of others to operate $\beta^\nu = \left(-\beta_0^\nu, -\underline{\beta}^\nu, \overline{\beta}^\nu\right) \in P$, or he can work for himself to operate $\alpha^\nu = (-\alpha_0^\nu, -\underline{\alpha}^\nu, \overline{\alpha}^\nu) \in P$. His constraint is that he must be able to afford to lay out the operating costs in advance for the activities he chooses to operate, either with his own labor or hired labor, funded by the value of his endowment. He can choose the activity level of each of α^ν, β^ν, and γ_0^ν under the constraints of his capital and labor endowments. Thus, given (p, w), where w is a nominal wage rate, his program is:

$$\max_{\left(\alpha^\nu;\, \beta^\nu;\, \gamma_0^\nu\right) \in P \times P \times \mathbb{R}_+} \left[p\left(\overline{\alpha}^\nu - \underline{\alpha}^\nu\right)\right] + \left[p\left(\overline{\beta}^\nu - \underline{\beta}^\nu\right) - w\beta_0^\nu\right] + \left[w\gamma_0^\nu\right]$$

such that

$$\begin{aligned} p\underline{\alpha}^\nu + p\underline{\beta}^\nu &\leqq p\omega^\nu \equiv W^\nu, \\ \alpha_0^\nu + \gamma_0^\nu &\leqq 1. \end{aligned}$$

Given (p, w), let $\mathcal{A}^\nu(p, w)$ be the set of actions $(\alpha^\nu;\ \beta^\nu;\ \gamma_0^\nu) \in P \times P \times \mathbb{R}_+$ which solve ν's program at prices (p, w).

The equilibrium notion of this model is given as follows:

Definition 7 [Roemer (1982; Chapter 5)]**:** A *reproducible solution* (RS) for the economy specified above is a pair $\left((p, w), \{(\alpha^\nu;\ \beta^\nu;\ \gamma_0^\nu)\}_{\nu \in N}\right)$, where $p \in \mathbb{R}_+^m$, $w \geqq pb = 1$, and $(\alpha^\nu;\ \beta^\nu;\ \gamma_0^\nu) \in P \times P \times \mathbb{R}_+$, such that:

(a) $\forall \nu \in N$, $(\alpha^\nu;\ \beta^\nu;\ \gamma_0^\nu) \in \mathcal{A}^\nu(p, w)$ (profit maximization);

(b) $\underline{\alpha} + \underline{\beta} \leqq \omega$ (social feasibility),
where $\underline{\alpha} \equiv \sum_{\nu \in N} \underline{\alpha}^\nu$, $\underline{\beta} \equiv \sum_{\nu \in N} \underline{\beta}^\nu$, and $\omega \equiv \sum_{\nu \in N} \omega^\nu$;

(c) $\beta_0 \leqq \gamma_0$ (labor market equilibrium)
where $\beta_0 \equiv \sum_{\nu \in N} \beta_0^\nu$ and $\gamma_0 \equiv \sum_{\nu \in N} \gamma_0^\nu$; and

(d) $\widehat{\alpha} + \widehat{\beta} \geqq \alpha_0 b + \beta_0 b$ (reproducibility),
where $\widehat{\alpha} \equiv \sum_{\nu \in N}(\overline{\alpha}^\nu - \underline{\alpha}^\nu)$, $\widehat{\beta} \equiv \sum_{\nu \in N}(\overline{\beta}^\nu - \underline{\beta}^\nu)$, and $\alpha_0 \equiv \sum_{\nu \in N} \alpha_0^\nu$.

The essential concept of the reproducible solution here is almost the same as that for the model of the FMT. Each condition of profit maximization, social feasibility, and reproducibility in Definition 7 has the same implication as each of those conditions in Definition 5, except for the following two points: First, rational agents have the three components of actions in Definition 7, and secondly wages should be paid at the end of the current period. The only different factor from Definition 5 is (c), the condition of labor market equilibrium. This condition allows strict inequality between labor demand β_0 and labor supply γ_0. If it holds in strict inequality, then the nominal wage rate is driven down to the subsistent wage $w = pb = 1$. If it holds in equality, then it might hold that $w \geqq pb = 1$.

The existence of the reproducible solution in this definition is also guaranteed in a similar way to Proposition 1. Let $P(\omega) \equiv \{\alpha = (-\alpha_0, -\underline{\alpha}, \overline{\alpha}) \in P \mid \underline{\alpha} \leq \omega\}$ and $\alpha_0(\omega) \equiv \max\{\alpha_0 \mid \exists \alpha = (-\alpha_0, -\underline{\alpha}, \overline{\alpha}) \in P(\omega)\}$. Then:

Proposition 4: *Let $b \in \mathbb{R}^m_{++}$ and $\alpha_0(\omega) \leqq |N|$. Under A1, A2, and stationary expectation of prices, a* reproducible solution (RS) *of Definition 7 exists for the economy specified above.*

This proposition can be shown in a similar way to Proposition 1.

4.2 Failure of the CECP both in the Morishima and the Roemer Definitions of Exploitation

Following Roemer (1982; Chapter 5), let us define possible classes in the model of section 4.1. At every RS in the model of section 4.1, different producers relate differently to the means of production. An individually optimal solution for an agent ν at the RS consists of three vectors $(\alpha^\nu; \beta^\nu; \gamma_0^\nu)$. According to whether these vectors are either zero or nonzero at the RS, all producers are classified into the following four types: that is, $(+,+,0)$, $(+,0,0)$, $(+,0,+)$, and $(0,0,+)$, where "+" means a nonzero vector in the appropriate place. Here, the notation $(+,+,0)$ implies, for instance, that an agent works for his own 'shop' and hires others' labor powers; while $(+,0,+)$ implies that an agent works for his own 'shop' and also sells his own labor power to others, etc.

Let us define four disjoint classes as follows:

$$\begin{aligned}
C^H &= \{\nu \in N \mid \mathcal{A}^\nu(p,w) \text{ has a solution of the form } (+,+,0) \setminus (+,0,0)\}, \\
C^{PB} &= \{\nu \in N \mid \mathcal{A}^\nu(p,w) \text{ has a solution of the form } (+,0,0)\}, \\
C^S &= \{\nu \in N \mid \mathcal{A}^\nu(p,w) \text{ has a solution of the form } (+,0,+) \setminus (+,0,0)\}, \\
C^P &= \{\nu \in N \mid \mathcal{A}^\nu(p,w) \text{ has a solution of the form } (0,0,+)\}.
\end{aligned}$$

We can see that the set of producers N can be divided into these four classes at any RS.

Given a price system (p,w), let

$$\pi^{\max}(p,w) \equiv \max\left\{\frac{p\overline{\alpha} - (p\underline{\alpha} + w\alpha_0)}{p\underline{\alpha}} \mid \alpha = (-\alpha_0, -\underline{\alpha}, \overline{\alpha}) \in P\right\}$$

and

$$\overline{P}(p,w) \equiv \left\{ \alpha = (-\alpha_0, -\underline{\alpha}, \overline{\alpha}) \in P \mid \frac{p\overline{\alpha} - (p\underline{\alpha} + w\alpha_0)}{p\underline{\alpha}} = \pi^{\max}(p,w) \right\}.$$

In the following discussion, let us restrict our attention to a RS with a positive profit. Then:

Proposition 5 [Roemer (1982; Chapter 5)]**:** *Let* (p, w) *be a* RS *with* $\pi^{\max}(p,w) > 0$. *Then,*

$$\begin{aligned}
\nu &\in C^H \Leftrightarrow W^\nu > \max_{\alpha \in \overline{P}(p,w)} \left[\frac{p\underline{\alpha}}{\alpha_0} \right], \\
\nu &\in C^{PB} \Leftrightarrow \min_{\alpha \in \overline{P}(p,w)} \left[\frac{p\underline{\alpha}}{\alpha_0} \right] \leqq W^\nu \leqq \max_{\alpha \in \overline{P}(p,w)} \left[\frac{p\underline{\alpha}}{\alpha_0} \right], \\
\nu &\in C^S \Leftrightarrow 0 < W^\nu < \min_{\alpha \in \overline{P}(p,w)} \left[\frac{p\underline{\alpha}}{\alpha_0} \right], \\
\nu &\in C^P \Leftrightarrow W^\nu = 0.
\end{aligned}$$

Definition 8: Let (p, w) be a reproducible solution with net output $\widehat{\alpha} + \widehat{\beta}$. A *feasible assignment* is any collection of nonnegative vectors $(f^\nu)_{\nu \in N}$ such that:
(i) $\sum_{\nu \in N} f^\nu = \widehat{\alpha} + \widehat{\beta}$;
(ii) $pf^\nu = \pi^{\max}(p,w) W^\nu + w$.

Let us denote the class of feasible assignments by $\mathcal{F}$, and the set of bundles assigned to ν in $\mathcal{F}$ by $\mathcal{F}^\nu$. Then:

Definition 2*: A producer $\nu \in N$ is *exploited in the Morishima (1974) sense* if and only if:

$$\max_{f^\nu \in \mathcal{F}^\nu} l.v.(f^\nu) < 1,$$

and he is an *exploiter in the Morishima (1974) sense* if and only if:

$$\min_{f^\nu \in \mathcal{F}^\nu} l.v.(f^\nu) > 1.$$

Definition 4*: Let (p, w) be a reproducible solution with full employment of labor. A producer $\nu \in N$ is *exploited in the Roemer (1982) sense* if and only if only if:

$$\max_{f^{\nu}\in\mathcal{F}^{\nu}} l.v.\left(f^{\nu};p,w\right) < 1,$$ [4]

and he is an *exploiter in the Roemer (1982) sense* if and only if:

$$\min_{f^{\nu}\in\mathcal{F}^{\nu}} l.v.\left(f^{\nu};p,w\right) > 1.$$

Now, CECP, which is a principle we would like to verify, is introduced as follows:

Class Exploitation Correspondence Principle (CECP): *For any economy defined as in section 4.1, and any reproducible solution with a positive profit rate, it holds that:*
(A) every member of C^H *is an* exploiter.
(B) every member of $C^S \cup C^P$ *is* exploited.

We are ready to show that the CECP cannot hold if the Morishima (1974) definition of labor exploitation is used. Let us define:

$$\widehat{\theta} \equiv \left\{c \in \mathbb{R}^m_+ \mid \exists \alpha \in \phi\left(c\right) : \alpha_0 = 1 \text{ \& } \alpha_0 \text{ is minimized over } \phi\left(c\right)\right\}.$$

Then, given a RS (p,w), let $\widehat{\widehat{c}} \in \widehat{\theta}$ be such that $p\widehat{\widehat{c}} \geqq pc$ for all $c \in \widehat{\theta}$. Also, let $\underset{\sim}{\widehat{c}} \in \widehat{\theta}$ be such that $p\underset{\sim}{\widehat{c}} \leqq pc$ for all $c \in \widehat{\theta}$. We can check that ν is an exploiter in the Morishima (1974) sense if and only if $\pi^{\max}\left(p,w\right)W^{\nu} + w > p\widehat{\widehat{c}}$. Also, ν is exploited in the Morishima (1974) sense if and only if $\pi^{\max}\left(p,w\right)W^{\nu} + w < p\underset{\sim}{\widehat{c}}$. To verify that C^H consists of exploiters in the Morishima (1974) sense, we have to show

$$\max_{\alpha\in\overline{P}(p,w)} \left[\frac{p\underline{\alpha}}{\alpha_0}\right] \geqq \frac{p\widehat{\widehat{c}} - w}{\pi^{\max}\left(p,w\right)}. \quad (1)$$

Since $\overline{P}\left(p,w\right)$ is a cone, we can normalize the left hand side of the inequality (1) by taking $\alpha \in \overline{P}\left(p,w\right)$ for which $\alpha_0 = 1$. Thus, (1) can be reduced to

$$\max_{\alpha\in\overline{\Gamma}(p,w)} p\underline{\alpha} \geqq \frac{p\widehat{\widehat{c}} - w}{\pi^{\max}\left(p,w\right)}, \quad (2)$$

[4]Here the labor value of the consumption bundel f^{ν} is defined as:

$$l.v.\left(f^{\nu};p,w\right) \equiv \min\left\{\alpha_0 \mid \alpha = \left(-\alpha_0, -\underline{\alpha}, \overline{\alpha}\right) \in \phi\left(f^{\nu};p,w\right)\right\},$$

where $\phi\left(f^{\nu};p,w\right) \equiv \left\{\alpha \in \overline{P}\left(p,w\right) \mid \widehat{\alpha} \geqq f^{\nu}\right\}$.

where $\overline{\Gamma}(p,w) \equiv \left\{\alpha \in \overline{P}(p,w) \mid \alpha_0 = 1\right\}$. Note that

$$\max_{\alpha\in\overline{\Gamma}(p,w)} p\underline{\alpha} \geqq \max_{\alpha\in\theta^*} p\underline{\alpha} \quad (3)$$

where $\theta^* \equiv \left\{\alpha \in \overline{P}(p,w) \mid \exists c \in \mathbb{R}^m_+ : l.v.(c;p,w) = 1,\ \widehat{\alpha} \geqq c,\ \&\ \alpha_0 = 1\right\}$. This is because $\overline{\Gamma}(p,w) \supseteq \theta^*$. We proceed to show:

$$\max_{\alpha\in\theta^*} p\underline{\alpha} \geqq \frac{p\widehat{\widehat{c}} - w}{\pi^{\max}(p,w)}, \quad (4)$$

which, by (3) and (2), will prove (1). Notice that $\pi^{\max}(p,w)\,p\underline{\alpha} \equiv p\widehat{\alpha} - w$ for any $\alpha \in \overline{\Gamma}(p,w)$. Hence, (4) is equivalent to:

$$\max_{\alpha\in\theta^*} p\widehat{\alpha} - w \geqq p\widehat{\widehat{c}} - w. \quad (5)$$

However, as the following theorem shows, the CECP cannot hold whenever the Morishima (1974) exploitation of labor is applied:

Theorem 5: *Under A1, A2, A3, and stationary expectation of prices, there exists an economy with convex cone technology such that there exists a reproducible solution (p,w) with $\pi^{\max}(p,w) > 0$, in which there exists $\nu \in C^H$ who is not an exploiter in the Morishima (1974) sense.*

Proof. let us define four production points:

$$\begin{aligned}
\alpha^1 &= \left(-\alpha_0^1, -\underline{\alpha}^1, \overline{\alpha}^1\right) = (-1, (-1, 0.5), (1, 2.25)), \ \widehat{\alpha}^1 = (0, 1.75);\\
\alpha^2 &= \left(-\alpha_0^2, -\underline{\alpha}^2, \overline{\alpha}^2\right) = (-1, (-1, -1), (2, 2.5)), \ \widehat{\alpha}^2 = (1, 1.5);\\
\alpha^3 &= \left(-\alpha_0^3, -\underline{\alpha}^3, \overline{\alpha}^3\right) = (-1, (-3, -4), (5, 5)), \ \widehat{\alpha}^3 = (2, 1); \text{ and}\\
\alpha^4 &= \left(-\alpha_0^4, -\underline{\alpha}^4, \overline{\alpha}^4\right) = (-1, (-5, -3), (8, 3)), \ \widehat{\alpha}^4 = (3, 0).
\end{aligned}$$

Now, we are ready to define a production possibility set of this economy. Let P be a *closed, convex cone* subset of $\mathbb{R}^{2m+1}$ such that
1) $\mathbf{0} \in P$;
2) $co\left\{\alpha^1, \alpha^2, \alpha^3, \alpha^4\right\} \subseteq P$;
3) the net output possibility set at one unit of labor input of P, $\widehat{P}(\alpha_0 = 1)$, is defined by:

$$\widehat{P}(\alpha_0 = 1) = co\left\{\widehat{\alpha}^1, \widehat{\alpha}^2, \widehat{\alpha}^3, \widehat{\alpha}^4, \mathbf{0}\right\}.$$

Let $N = \{\lambda, \mu, \nu'\}$, and $\omega^{\lambda} = \omega^{\nu'} = \mathbf{0}$, and $\omega^{\mu} = (2, 2)$. Also, let $b = (1, 1)$ be the subsistent consumption bundle for any agent supplying one unit of labor. Let us consider the situation that $(p, w) = ((0.5, 0.5), 1)$ and

$$\begin{aligned} \left(\alpha^{\lambda}; \beta^{\lambda}; \gamma_0^{\lambda}\right) &= ((\mathbf{0}, \mathbf{0}, 0); (\mathbf{0}, \mathbf{0}, 0); 1), \\ \left(\alpha^{\mu}; \beta^{\mu}; \gamma_0^{\mu}\right) &= (((2, 2.5), (1, 1), 1); ((2, 2.5), (1, 1), 1); 0), \text{ and} \\ \left(\alpha^{\nu'}; \beta^{\nu'}; \gamma_0^{\nu'}\right) &= ((\mathbf{0}, \mathbf{0}, 0); (\mathbf{0}, \mathbf{0}, 0); 0). \end{aligned}$$

This indicates that under the price system $(p, w) = ((0.5, 0.5), 1)$, the agent λ is an employed worker who sells his one unit of labor to the agent μ; the agent μ operates his owned capital by his own working and buying one unit of labor from the agent λ; and the agent ν' is an unemployed worker. Thus, $C^H = \{\mu\}$ and $C^P = \{\lambda, \nu'\}$. We will show that this combination of the price system and the list of every agent's action constitutes a RS with $\pi^{\max}(p, w) > 0$. Moreover, we will show that the agent μ cannot be an exploiter in the Morishima (1974) sense.

First, we will show that the above list of economic actions $\{(\alpha^{\nu}; \beta^{\nu}; \gamma_0^{\nu})\}_{\nu \in N}$ is the list of optimal solutions for all agents at $(p, w) = ((0.5, 0.5), 1)$. Note that $\pi((p, w); \alpha^1) < 0$, $\pi((p, w); \alpha^2) = \frac{1}{4}$, $\pi((p, w); \alpha^3) = \frac{1}{7}$, and $\pi((p, w); \alpha^4) = \frac{1}{8}$, when $(p, w) = ((0.5, 0.5), 1)$. Thus, $\pi^{\max}(p, w) = \pi((p, w); \alpha^2)$, and so operating with only $t\alpha^2$ up to the budget constraint is the optimal solution for the agent μ with nonnegative endowment of capital ω^{μ}. Thus, $\{(\alpha^{\nu}; \beta^{\nu}; \gamma_0^{\nu})\}_{\nu \in N}$ constitutes the list of optimal solutions at $(p, w) = ((0.5, 0.5), 1)$. Second, the conditions (b) and (c) of Definition 7 are satisfied by the list $\{(\alpha^{\nu}; \beta^{\nu}; \gamma_0^{\nu})\}_{\nu \in N}$ at that price system. Finally, $2b = \left(\alpha_0^{\mu} + \gamma_0^{\lambda}\right) \cdot b \leq \widehat{\alpha}^{\mu} + \widehat{\beta}^{\mu} = 2\widehat{\alpha}^2$, which implies the condition (d) holds. Thus, $\left((p, w), \{(\alpha^{\nu}; \beta^{\nu}; \gamma_0^{\nu})\}_{\nu \in N}\right)$ is a RS with $\pi^{\max}(p, w) > 0$.

By the above definition, it holds that $\overline{P}(p, w) = \{t\alpha^2 \in P \mid t \in \mathbb{R}_+\}$, $\theta^* = \overline{\Gamma}(p, w) = \{\alpha^2\}$, and $\widehat{\theta} = co\left\{\widehat{\alpha}^1, \widehat{\alpha}^2\right\} \cup co\left\{\widehat{\alpha}^2, \widehat{\alpha}^3\right\} \cup co\left\{\widehat{\alpha}^3, \widehat{\alpha}^4\right\}$. Then, $\max_{\alpha \in \overline{\Gamma}(p,w)} p\underline{\alpha} = p\alpha^2$, so that $\max_{\alpha \in \theta^*} p\widehat{\alpha} - w = p\widehat{\alpha}^2 - 1 = \frac{1}{4}$, while $\widehat{\widehat{c}} = \widehat{\alpha}^3$, which implies that $\frac{1}{4} = p\widehat{\alpha}^2 - 1 = \max_{\alpha \in \theta^*} p\widehat{\alpha} - w < p\widehat{\widehat{c}} - w = p\widehat{\alpha}^3 - w = \frac{1}{2}$. Thus, the inequality (1) cannot be satisfied. In fact, $\pi^{\max}(p, w) p\omega^{\mu} + w = 1.5 = p\widehat{\widehat{c}}$, which implies that μ cannot be an exploiter in the Morishima (1974) sense. ■

The above theorem was pointed out by Roemer (1982; Chapter 5). Based on this result, he criticized the Morishima (1974) definition of labor exploitation, since the CECP is false under that definition. In contrast, as Roemer (1982; Chapter 5; Theorem 5.3) showed, all agents in C^H become exploiters whenever the Roemer (1982) exploitation is used. However, we can also show the existence of a RS, in which there could be an agent in $C^S \cup C^P$ who is not exploited, even if the Roemer (1982) exploitation is used.

Given (p, w), let

$$\theta \equiv \left\{c \in \mathbb{R}_+^m \mid \exists \alpha \in \phi\left(c; (p, w)\right) : \alpha_0 = 1 \text{ \& } \alpha_0 \text{ is minimized over } \phi\left(c; (p, w)\right)\right\}.$$

Thus, θ is the collection of nonnegative consumption bundles whose labor values in the Roemer (1982) sense are unity. Then, given a RS (p, w), let $\widetilde{c} \in \theta$ be such that $p\widetilde{c} \geqq pc$ for all $c \in \theta$. Also, let $\underset{\sim}{c} \in \theta$ be such that $p \underset{\sim}{c} \leqq pc$ for all $c \in \theta$. We can check that ν is an exploiter in the Roemer (1982) sense if and only if $\pi^{\max}(p, w) W^\nu + w > p\widetilde{c}$. Also, ν is exploited in the Roemer (1982) sense if and only if $\pi^{\max}(p, w) W^\nu + w < p \underset{\sim}{c}$. Then:

Theorem 6: *Under A1, A2, A3, and stationary expectation of prices, there exists an economy with convex cone technology such that there exists a reproducible solution (p, w) with $\pi^{\max}(p, w) > 0$, in which there exists $\nu \in C^S \cup C^P$ who is not exploited in the Roemer (1982) sense.*

Proof. Let us consider the same economic environment as in the proof of Theorem 5. Then, $\overline{P}(p, w) = \{t\alpha^2 \in P \mid t \in \mathbb{R}_+\}$, and

$$\theta = \left\{(t_1, 1.5) \in \mathbb{R}_+^2 \mid t_1 \in [0, 1]\right\} \cup \left\{(1, t_2) \in \mathbb{R}_+^2 \mid t_2 \in [0, 1.5]\right\}.$$

Insert Figure 3 around here.

Thus, $\underset{\sim}{c} \in \theta$ for $(p, w) = ((0.5, 0.5), 1)$ is given by $\underset{\sim}{c} = (1, 0)$. Thus, $p \underset{\sim}{c} = \frac{1}{2} < 1 = w$ for $\lambda \in C^P$, which implies that λ is not exploited in the Roemer (1982) sense at $(p, w) = ((0.5, 0.5), 1)$. ■

Although Roemer (1982) proposed to adopt the Roemer (1982) definition of labor exploitation instead of the Morishima (1974) one to preserve the CECP, the above two theorems show that both of the definitions cannot preserve the CECP in general convex cone models. In particular, an agent in the capitalist class may not be an exploiter if the Morishima (1974) definition of exploitation is adopted; whereas an agent in the working class may not be exploited if the Roemer (1982) definition is adopted.

5 A New Definition of Labor Exploitation

Roemer (1982) argued that the epistemological role of the CECP in our understanding of the capitalist economy is as an axiom, although the formal version of it emerges as a theorem. So, if we wish to verify the CECP, we must seek an appropriate model which will preserve this principle as a theorem. By this reason, Roemer (1982) insisted that the Roemer (1982) definition of labor exploitation is superior to the Morishima (1974) one. Based upon this argument made by himself, however, the Roemer (1982) type of labor exploitation is unable to be justified, since the CECP fails to hold even in the model with the Roemer (1982) exploitation as shown in Theorem 6. Thus, we seek further for a new definition of Marxian labor exploitation.

In this section, following Roemer (1982), we still adopt the definition of labor value of commodities as in Definition 3. However, we refine the definition of labor exploitation from Roemer's (1982). The definition of labor exploitation we propose is the difference between one unit of labor supplied by an agent per day and the minimal amount of direct labor socially necessary to provide the agent with his *income* per day.

Note that in Roemer (1982), labor exploitation was the difference between the supplied labor and the minimal amount of labor which was socially necessary to produce a commodity vector as a net output, where the commodity vector can be regarded as the subsistent consumption vector or as an agent's demand vector subject to his budget constraint. In this case, exploitation status of agents would be influenced by the characters of the subsistent consumption vector or of agents' demand vectors, as Theorems 1, 4, and 6 indicated. This seems unconvincing from the Marxian point of view, because the exploitation status of agents should reflect their objective conditions of labor only. Any agents who earn the same income by supplying the same amount of homogenous labor should be placed at the same status in terms of exploitation, regardless of their consumption vectors. This problem could be overcome by the new definition we propose.

The new definition is formally given by the following:

Definition 9: *Let (p, w) be a reproducible solution. Then, an agent $\nu \in N$ is* exploited *if and only if:*

$$\min_{f^\nu \in \mathcal{F}^\nu} l.v.\left(f^\nu; p, w\right) < 1.$$

In contrast, an agent $\nu \in N$ *is an* exploiter *if and only if:*

$$\min_{f^{\nu} \in \mathcal{F}^{\nu}} l.v. \left(f^{\nu}; p, w\right) > 1.$$

Insert Figure 4 around here.

Note that if an agent $\nu \in N$ is a worker who purchases a subsistent consumption vector $b \in \mathbb{R}_{+}^{m}$ by his wage income under a RS $\left((p, w), \{\alpha^{\nu}\}_{\nu \in N}\right)$, then for any $f^{\nu} \in \mathbb{R}_{+}^{m}$ such that $pf^{\nu} = pb = w$,

$$l.v. \left(f^{\nu}; p, w\right) = \min \left\{\alpha_0 \mid \alpha = \left(-\alpha_0, -\underline{\alpha}, \overline{\alpha}\right) \in \phi\left(f^{\nu}; p\right)\right\}.$$

Thus, $\min_{f^{\nu} \in \mathcal{F}^{\nu}} l.v. \left(f^{\nu}; p, w\right)$ in Definition 9 implies the minimizer of $l.v. \left(f^{\nu}; p, w\right)$ over the budget set

$$B(p, w) \equiv \left\{f^{\nu} \in \mathbb{R}_{+}^{m} \mid pf^{\nu} = pb = w\right\}.$$

This implies that the labor value in Definition 9 is concerned *not* with an agent's *consumption vector*, *but* rather with an agent's *income* earned. Thus, the new definition implies the following: Suppose an economy is under a reproducible solution (p, w). Then, if the minimal amount of labor socially necessary to provide each agent ν with income $\pi^{\max}(p, w) W^{\nu} + w$ is less (*resp.* more) than unity, then ν is exploited (*resp.* exploiter).

Noting that this fact, we can show the following theorems:

Theorem 7: *Under A1, A2, and stationary expectation of prices, let* $\left(p, \{\alpha^{\nu}\}_{\nu \in N}\right)$ *be the* reproducible solution (RS) *in the sense of Definition 5. Then, the* RS *yields positive total profits if and only if every worker is exploited in the sense of Definition 9.*

Proof. ($\Rightarrow$): Let $\left(p, \{\alpha^{\nu}\}_{\nu \in N}\right)$ be a RS with a positive total profit. Thus,

$$\begin{aligned} p \cdot \left(\sum_{\nu \in N} (\overline{\alpha}^{\nu} - \underline{\alpha}^{\nu})\right) - \sum_{\nu \in N} \alpha_0^{\nu} &= p \cdot \left(\sum_{\nu \in N} (\overline{\alpha}^{\nu} - \underline{\alpha}^{\nu}) - \sum_{\nu \in N} \alpha_0^{\nu} b\right) \\ &= p \cdot (\widehat{\alpha} - \alpha_0 b) > 0. \end{aligned}$$

Since $p \in \mathbb{R}_{+}^{m}$ and $\widehat{\alpha} \geqq \alpha_0 b$ by Definition 5(b), the last strict inequality implies $\widehat{\alpha} \geq \alpha_0 b$ and $\widehat{\alpha} \neq \alpha_0 b$. Let $f \in \mathbb{R}_{+}^{m}$ be such that $pf = pb$ and $\alpha_0 f = t\widehat{\alpha}$

for some $0 < t < 1$. Then, by the convex cone property of the production set, $l.v.(\alpha_0 f; p, 1) \leqq l.v.(\alpha_0 b; p, 1)$, and $l.v.(\alpha_0 f; p, 1) < l.v.(\alpha_0 b; p, 1)$ holds whenever $f \neq b$. Thus, $l.v.(\alpha_0 f; p, 1) < \alpha_0$. By linearity, $l.v.(f; p, 1) < 1$, which implies $\min_{\widehat{f} \in \mathcal{F}} l.v.\left(\widehat{f}; p, 1\right) < 1$, so that every worker is exploited in the sense of Definition 9.

($\Leftarrow$): Since there is no RS with a negative total profit, it suffices to discuss only the case of zero profit. Let $\left(p, \{\alpha^\nu\}_{\nu \in N}\right)$ be a RS with a zero total profit. Thus, $p \cdot (\widehat{\alpha} - \alpha_0 b) = 0$. By Definition 5(b), $\widehat{\alpha} \geqq \alpha_0 b$. Let $f \in \mathbb{R}^m_+$ be such that $pf = pb$ and $\alpha_0 f = t\widehat{\alpha}$ for some $0 < t \leq 1$. Then, $p \cdot (\widehat{\alpha} - \alpha_0 f) = 0$ and $\alpha_0 f = t\widehat{\alpha}$ imply that $t = 1$. Thus, $\widehat{\alpha} = \alpha_0 b$ holds whenever $p > 0$. In this case, $\min_{\widehat{f} \in \mathcal{F}} l.v.\left(\widehat{f}; p, 1\right) = 1$, so that no worker is exploited in the sense of Definition 9. If $p \geq 0$, it may be the case that $\widehat{\alpha} \geq \alpha_0 b$ and $\widehat{\alpha} \neq \alpha_0 b$. However, as $p \cdot (\widehat{\alpha} - \alpha_0 b) = 0$ and $\{\alpha^\nu\}_{\nu \in N}$ constitutes a profit-maximizing production plan at p, $\alpha_0 b \in \partial \widehat{P}(\alpha_0 = 1)$ holds. Thus, $\min_{\widehat{f} \in \mathcal{F}} l.v.\left(\widehat{f}; p, 1\right) = 1$, so that no worker is exploited in the sense of Definition 9. ■

Note that when showing Theorem 7, A3 is no longer indispensable.

Theorem 8: *Under A1, A2, and stationary expectation of prices, let* $\left(p, \{\alpha^\nu\}_{\nu \in N}\right)$ *be the* reproducible solution (RS) *in the sense of Definition 6, with the average consumption demand of the employed workers,* $d\left(p; (\alpha_0^\upsilon)_{\upsilon \in I}\right)$. *Then, the* RS *yields positive total profits if and only if every type of worker is exploited in the sense of Definition 9.*

Proof. By Theorem 3, if it is shown that every individual has a common exploitation rate in the sense of Definition 9, we can show the desired result. By definition, $\forall \upsilon \in I$, $pd^\upsilon(p) = p \cdot d\left(p; (\alpha_0^\upsilon)_{\upsilon \in I}\right) = 1$. Given the RS $\left(p, \{\alpha^\nu\}_{\nu \in N}\right)$, let $\widehat{\alpha} \equiv \sum_{\nu \in N} (\overline{\alpha}^\nu - \underline{\alpha}^\nu)$ and $\alpha_0 \equiv \sum_{\nu \in N} \alpha_0^\nu$. Let $\widehat{\alpha}^0 \equiv \frac{\widehat{\alpha}}{\alpha_0}$. Then, as shown in the proof of Theorem 7, there exists some $t \in (0, 1]$ such that $\forall \upsilon \in I$, $\min_{f^\upsilon \in \mathcal{F}^\upsilon} l.v.(f^\upsilon; p, 1) = l.v.\left(t\widehat{\alpha}^0; p, 1\right)$. This implies that all individuals have a common exploitation rate in the sense of Definition 9. ■

We can check that ν is an exploiter in the sense of Definition 9 if and only if $\pi^{\max}(p, w) W^\nu + w > p\widetilde{c}$. In addition, ν is exploited in the sense of Definition 9 if and only if $\pi^{\max}(p, w) W^\nu + w < p\widetilde{c}$. Then:

Theorem 9: *Under A1, A2, A3, and stationary expectation of prices, let*

(p, w) *be the* reproducible solution (RS) *in the sense of Definition 7, with* $\pi^{\max}(p, w) > 0$. *Then, it holds that:*

(A) every member of C^H *is an* exploiter *in the sense of Definition 9.*

(B) every member of $C^S \cup C^P$ *is* exploited *in the sense of Definition 9.*

Proof. The part (A) is already shown by Roemer (1982). Thus, we only show the part (B). To verify $C^S \cup C^P$ consisted of exploited agents in the sense of Definition 9, we have to show

$$\frac{p\widetilde{c} - w}{\pi^{\max}(p, w)} \geqq \min_{\alpha \in \overline{P}(p,w)} \left[\frac{p\underline{\alpha}}{\alpha_0}\right]. \quad (1^*)$$

Since $\overline{P}(p, w)$ is a cone, we can normalize the right hand side of the inequality (1^*) by taking $\alpha \in \overline{P}(p, w)$ for which $\alpha_0 = 1$. Thus, (1^*) can be reduced to

$$\frac{p\widetilde{c} - w}{\pi^{\max}(p, w)} \geqq \min_{\alpha \in \overline{\Gamma}(p,w)} p\underline{\alpha}, \quad (2^*)$$

as in the case for inequality (2). Note that

$$\min_{\alpha \in \overline{\Gamma}(p,w)} p\underline{\alpha} \leqq \min_{\alpha \in \theta^*} p\underline{\alpha} \quad (3^*),$$

as in the case for the inequality (3). Then, it is sufficient to show:

$$\frac{p\widetilde{c} - w}{\pi^{\max}(p, w)} \geqq \min_{\alpha \in \theta^*} p\underline{\alpha}. \quad (4^*)$$

Taking $\pi^{\max}(p, w)\, p\underline{\alpha} \equiv p\widehat{\alpha} - w$ for any $\alpha \in \overline{\Gamma}(p, w)$, the inequality (4^*) is equivalent to:

$$p\widetilde{c} - w \geqq \min_{\alpha \in \theta^*} p\widehat{\alpha} - w. \quad (5^*)$$

Note that $\widehat{\alpha} \in \theta$ for any $\alpha \in \theta^*$. Thus, for any $\widehat{\alpha} \in \theta$ such that $\alpha \in \theta^*$, $p\widetilde{c} \geq p\widehat{\alpha}$, which implies (5^*) holds true. This implies the part (B) is shown. ■

As shown in the above theorems in this section, the new definition of labor exploitation performs well in terms of both the FMT and the CECP. However, the new definition has exclusively distinct characteristics in comparison with the previous definitions, which may give us new insights on the Marxian theory of labor exploitation and the theory of labor value.

First, Roemer (1982) claimed that prices should emerge logically prior to labor values so as to preserve the CECP as a theorem in a general convex cone economy. According to Theorems 7, 8, and 9, we would also follow the above claim of Roemer (1982) in order to verify not only the CECP, but also the FMT in more general economic models.

Secondly, in the orthodox Marxian argument, labor exploitation was explained by using the concept of the labor value of labor power. The labor value of labor power could be defined in the Morishima (1974) framework as the minimal amount of direct labor necessary to produce the subsistent consumption vector as a net output. This could be accepted by the orthodox Marxist as the formulation of the *socially necessary labor time to reproduce labor power*. Here, the subsistent consumption vector plays a crucial role in the formulation of the labor value of labor power. In the new definition of this section, however, the labor value of labor power might be defined as the minimal amount of direct labor socially necessary to provide workers with the income by which they can respectively purchase at least the subsistent consumption vector. In this formulation, the subsistent consumption vector is used, *at most indirectly*, to define the labor value of labor power. Thus, even the labor value of labor power no longer emerges logically prior to the price of labor power (wage income). Hence, the concept of labor value in this new definition is completely irrelevant to theories of *exchange values of commodities and labor power*.

In spite of such a significant difference of this new definition from the orthodox Marxian notion of labor exploitation, it would be justified, according to the scenario Roemer (1982) offered, since both of the FMT and the CECP hold true for this new definition.

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Figure1 Proof of Theorem1

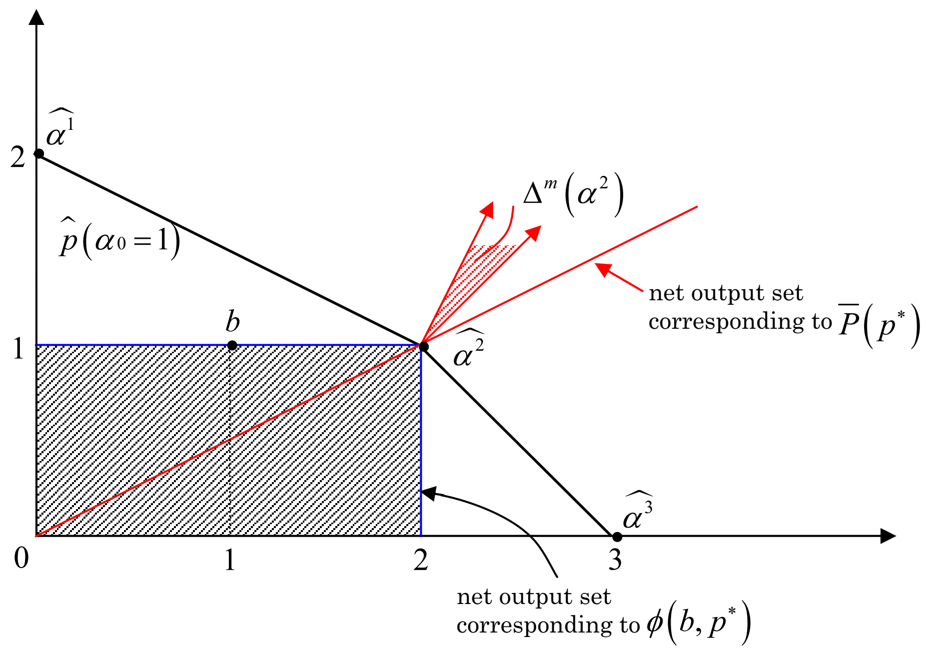
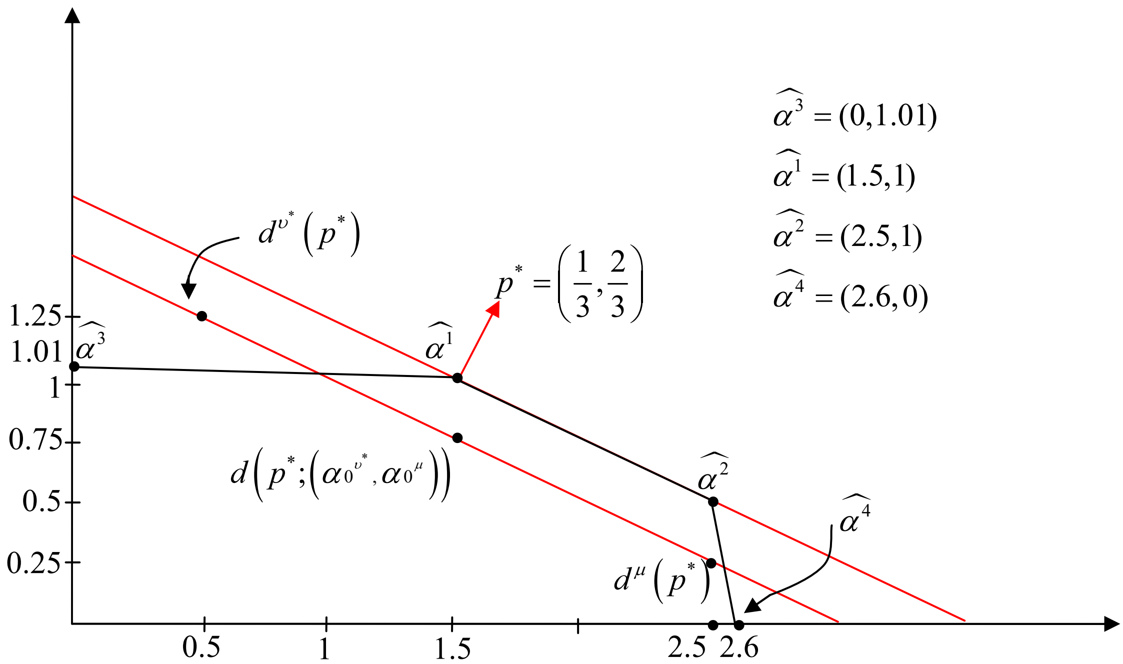


Figure2



$\widehat{\alpha^3} = (0, 1.01)$
$\widehat{\alpha^1} = (1.5, 1)$
$\widehat{\alpha^2} = (2.5, 1)$
$\widehat{\alpha^4} = (2.6, 0)$
$d^{\upsilon^*}(p^*)$
$p^* = \left(\frac{1}{3}, \frac{2}{3}\right)$
$d\left(p^*; \left(\alpha_0{}^{\upsilon^*}, \alpha_0{}^{\mu}\right)\right)$
$d^{\mu}(p^*)$
$\widehat{\alpha^3}$
$\widehat{\alpha^1}$
$\widehat{\alpha^2}$
$\widehat{\alpha^4}$
1.25
1.01
1
0.75
0.5
0.25
0.5
1
1.5
2.5
2.6

Figure3 Proof of Theorems 5 and 6

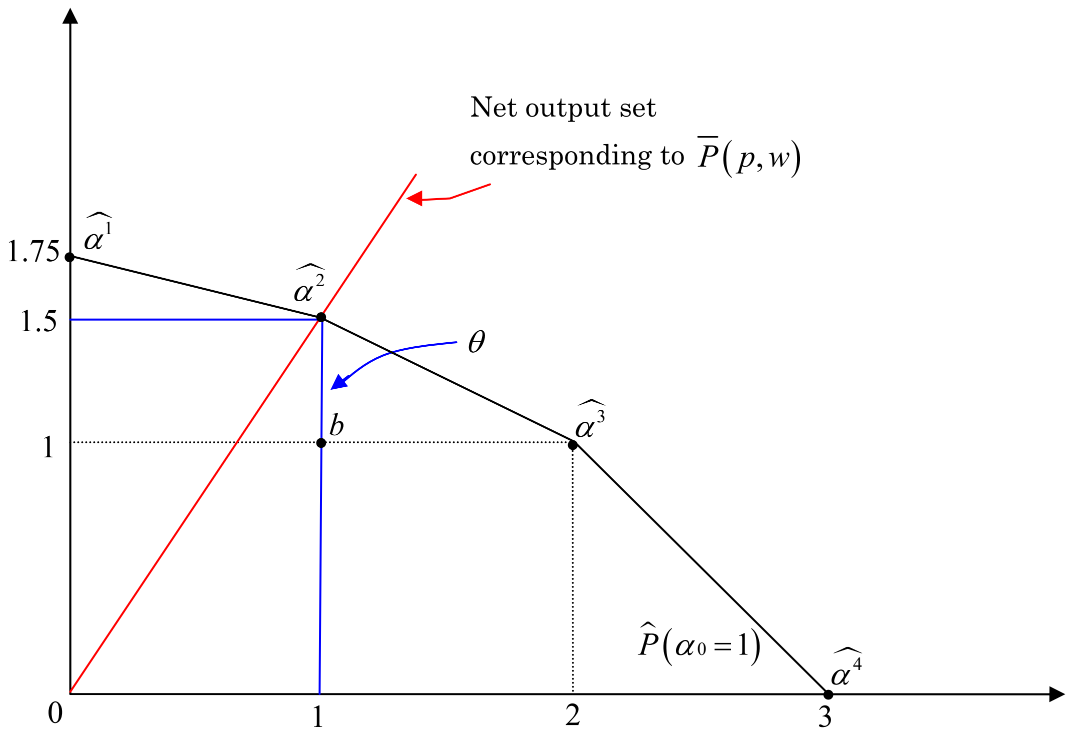


Figure4 Definition9