Hitotsubashi Journal of Economics 47 (2006), pp.265-283.

THE VARIETY EXPANDING GROWTH MODEL WITH CHANGE IN SUBSTITUTION (COMPLEMENTARY) AMONG GOODS*

NOBUHIRO HOBARA

Graduate school of Economics, Hitotsubashi University
Kunitachi, Tokyo186-8601, Japan
ged0110@srv.cc.hit-u.ac.jp

Received August 2005; Accepted September 2006

Abstract

Grossman and Helpman (1991) show that the spillover effect sustains the expansion of product variety. Considering the current economy, however, the conclusion seems to be over optimistic. While they assume that the elasticity of substitution among goods is constant, we assume it is a function of the variety of goods. We show that, when products become more substitutive as the variety increases, economic growth stops despite the spillover effect. Gali (1995) does not consider the case that products become more complementary as the variety increases. While, we show that, when they become complementary, the economy suddenly begins to grow.

Keywords: endogenous growth, variety expanding growth, elasticity of substitution among goods, complementary, Big Push
JEL Classification: D10, L16, O11, O32, O41

I. *Introduction*

Dixit and Stiglitz (1977) build monopolistic competition models with imperfect substitute goods, using the CES function. Using the results of Dixit and Stiglitz (1977), Grossman and Helpman (1991) show that the endogenous growth of the economy continues as long as the variety of goods keeps increasing as a spillover effect working on the invention of new goods.

Their argument, however, depends on the two characteristics of CES functions. First, the invention of new goods equally influences all the goods in the economy. Secondly, the elasticity of substitution among goods is constant regardless of the variety of goods in the economy.

Young (1993) modifies the former characteristic and differentiates the influence of the introduction of new goods from the point of view of adaptability to new goods. As a result, he shows that in some cases economic growth does not continue but the growth rate depends on the parameter showing the adaptability.

* Special thanks to Takatoshi Ito, Kyoji Fukao, Masakatsu Nakamura, Kazumi Asako, Tadahiko Tokita, and Kan Takeuchi for their comments and suggestions. I am also grateful to an anonymous referee for helpful comments.

In this paper, we modify the latter feature and reconsider the results of Grossman and Helpman (1991). In our model, the elasticity of substitution among goods is not constant, but we assume that it depends on the number of goods. Specifically, the elasticity of substitution ε is

$$\varepsilon = 1/(1-f(n)) \tag{1}$$

where n is the number of goods and f is a continuous, differentiable and monotone function. If $f'(n)>0$, then ε increases in the number of goods and the elasticity of substitution becomes stronger. Otherwise, $f'(n)<0$, ε decreases in the number of goods. In developed countries, introduction of a new attractive good sometimes alters the relationship among previously existing goods and makes it more competitive. This situation corresponds to the substitutive case ($f'(n)>0$). On the other hand, in developing countries, the invention of a good that causes a Big Push towards economic growth may make the existing goods more complementary, which corresponds to the latter case ($f'(n)<0$).

Gali (1995) introduces only the substitutive case ($f'(n)>0$) into the model of Grossman and Helpman (1991) and calculates the mark up rate in steady state.

In this paper, we incorporate both cases and examine the growth path. As a result, we reach different conclusions and draw different phase diagrams from those of Grossman and Helpman. It is shown that, when the elasticity becomes substitutive, the economic growth does not necessarily continue even if the standard conditions for the endogenous growth are satisfied. Conversely, when the existing goods become more complementary, economic growth suddenly occurs, even if the elasticity is strong and the conditions for growth are not satisfied at the beginning.[1]

We describe our model and the points of difference from the existing models in Section II. Next, we present the model and the dynamics in Section III, IV and V. In Section VI and VII, we consider that the mechanism of dynamics in our model, policy implications and the balanced growth path in our model. Finally we discuss the results in Section VIII.

II. *The Model and Points of Difference from the Existing Models*

As mentioned in Section I, Gali (1995) considers only the elasticity of substitution among intermediate goods and does not directly consider the elasticity of substitution among final goods.

Table 1, which is quoted from Toshihiro Horiuchi (2000), shows the relationship between the changes in the scale of market and the changes in the share of the three recent top firms in Japanese industries. Many industries top firm's share declines over time or whose market size declines. This fact meets the setting in Gali (1995).

However, among industries whose market scale has declined, there are many industries that produce final goods. This does not meet the setting in Gali (1995).

[1] Notice that Gali (1995) does not consider the change of the substitution in utility function or the complementary case ($f'(n)<0$) regarding both utility function and product function. We consider these cases and show their importance for the analysis of economic growth.

TABLE 1. RELATIONSHIP BETWEEN MARKET SIZE AND THE SHARE OF THE TOP THREE FIRMS

	Market size decline	Market size expansion
Share expansion	A game machine for families	A movie
	Advertisements	A mini component
	A color television	A CD Player
	A washing machine	An inkjet printer
	A room air-conditioner	Carrying/a car telephone
	H-rolled rolling steel	Pasta
	An aluminum article to roll	A cold medicine
	Polyester length fiber	A NC lathe
	A compact camera	A digital camera
	A city incinerator	A watch
	Cement	A car navigation system
	Detergent for kitchens	Women's underwear
	A golf ball etc.	etc.
Share decline	Publishing	A provider
	Optical fiber	Music software
	An electric refrigerator	A PC server
	A facsimile	A PC
	Low density polyethylene	A DVD player
	Print/Information paper	A video camera
	Flat glass	MD machinery
	A plastic ejection molding opportunity	Lithium ion battery
		LCD
	A super hard tool	64 mega DRAM
	Bearing	Gasoline
	A car	Frozen food
	A light car	Lunch trust
	An apartment	A ship
	Women's clothes	Home delivery
	Overseas travel	A credit card
	A hotel	Domestic aviation
	etc.	etc.

Source: "Industry Organization Theory" by Toshihiro Horiuchi (2000)

This fact means that consumer's utility deceases even though new products expand.[2] In the first place, a firm develops new goods not only to offer the new goods to consumers but also to compete with other companies belonging to the same industry. As a result, consumers' evaluation of the final goods often decreases as a result of excessive new products brought about by the competition.

In order to describe such a situation, it is necessarily that, also against the utility function, we make the same assumption as Gali (1995) in which the elasticity of substitution increases as the variety of goods expands.

Next, Gali (1995) considers only the substitution case ($f'(n) > 0$) but does not consider

[2] The marginal utility of each good increases as the expansion of goods described in Grossman and Helpman (1991), in which the elasticity of substitution is assumed to be constant. But the situation shown in Table 1 does not always correspond to the increase in marginal utility because the market size usually corresponds to consumer's utility.

the complementary case ($f'(n) < 0$). Grossman and Helpman (1991) consider that the growth continues if there is a spillover effect and it was considered unnecessary to consider other growth promotion factors of from the spillover effect, such as complementary.

However, Sollow (2000) shows that the growth does not continue when the elasticity of substitution is larger than a certain level even if there is a spillover effect as in Grossman and Helpman (1991) (See Section VI). In other words, in Grossman and Helpman (1991), there is a "dark zone" where growth does not occur. So, in the existing models, once the elasticity of substitution becomes larger than a certain level, the expansion of variety goods does not occur.

On the other hand, there is a different growth promotion factor to the spillover effect, complementary.[3] So in our model, the condition for variety expansion can become satisfied, even if the elasticity of substitution is larger than a certain level with a small number of product varieties. We present a phase diagram that expresses the Big Push, which changes the economy from a situation that is not conducive to growth to a situation that is conducive to growth, by the accumulation of goods with the change in elasticity. (See Section V).

New home electric appliances such as the black and white TV, the washing machine and the refrigerator (Three Sacred Treasures) appeared during the first period of high growth in the Japanese economy (the vacuum cleaner is another important example). As noted by Becker (1965), these new home electric appliances made housework easier. Table 2 shows average housewife's working hours and GDP with the spread refrigerators, vacuum cleaners and washing machines in Japan. Housework hours decreased from 1960 to 1965, while the spread of these electric appliances rose towards 50%.

The effect of reducing housework may have been greater because these electric appliances appeared simultaneously. Each electric appliance is concerned with a major, difficult aspect of housework: cooking, washing and cleaning. The simultaneous appearance of these machines provided the opportunity to effectively plan housework. For example, vacuum cleaning can be performed whilst the washing machine is operating. In fact, the plural simultaneous use induced simultaneous increases in the spread of these electric appliances.

Furthermore, televisions (one of the Three Sacred Treasures) became widespread and television watching hours also rose during the same period. Freed from some of the hard work by the introducing of washing machines, refrigerators and washing machines, housewives had more time to watch television, and at the same time, were subjected to television advertising of electrical appliances. These home electric appliances and television combined to produce substantial home time and are complementary in this way. Complementarity contributed to the first period of Japanese high growth. The existing models cannot describe such a plural appearance or use of new final goods causes complementary among goods that is conducive to economic growth.

In addition, there are examples that such a surge in complimentary among final goods give birth to an entirely new product. In the second period of high growth in Japan, the car and the air conditioner appeared as New Sacred Treasures. Later, cars were produced with air-

[3] Young (1993) limits the range of complementary influence from the appearance of new goods and distinguishes the complementary influence operating among goods from the substitutive influence operating among goods. However, he does not consider the change in elasticity of substitution and he only treats the spillover effect as complementary and the rent declining effect as substitution, the same as Grossman and Helpman (1991). In his model, it is impossible to consider the influence of the difference in the variety of goods appearing in the economy or the overall influence on the economy.

TABLE 2. SPREAD OF NEW ELECTRIC APPLIANCES AND JAPANESE WOMEN'S LIVING HOURS

	The spread of electric appliances					Living hours	
	Electric refrigerator	Electric washer	Vacuum cleaner	Black and white TV	Color TV	House working hours	TV watching hours
1957	2.8	20.2	0	0	0		
1958	3.2	24.6	0	0	0		
1959	5.7	33	0	23.6	0		
1960	10.1	40.6	7.7	44.7	0	6.66	2.54
1961	17.2	50.2	15.4	62.5	0		
1962	28	58.1	24.5	79.4	0		
1963	39.1	66.4	33.1	88.7	0		
1964	38.2	61.4	26.8	87.8	0		
1965	51.4	68.5	32.2	90.0	0	4.14	3.02
1966	61.6	75.5	41.2	94.4	0		
1967	69.7	79.8	47.2	96.2	0		
1968	77.6	84.8	53.8	96.4	0		
1969	84.6	88.3	62.6	94.7	0		
1970	89.1	91.4	68.3	90.2	26.3	4.37	3.28
1971	91.2	93.6	74.3	82.3	42.3		
1972	91.6	96.1	79.8	75.1	61.1		
1973	94.7	97.5	85.2	65.4	75.8	4.32	3.37
1974	96.5	97.5	89.6	55.7	85.9		
1975	96.7	97.6	91.2	48.7	90.3	4.33	3.44

Source: "White Paper on the National Lifestyle Fiscal Year 1990" by The Economic Planning Agency (the spread of new electric appliances) and "Survey on National Living Hours" by NHK (1980)" (Japanese women's living hours)

conditioners, which may have led to the increased use of cars. More recently, portable telephones have adopted the functions of PCs and cameras. According to NTT data, sales of the portable telephone reached particularly high levels in 2001 and 2003, when telephones with such added functions were launched.

III. *The Model*

Suppose that the representative household has the following CES utility function

$$D=\left[\int_0^n x(j)^{f(n)}dj\right]^{1/f(n)} \qquad 0<f(n)<1 \tag{2}$$

where D is the consumer's utility , j is the brand index of goods, $x(j)$ is the final goods output or the intermediate goods output of j, and n is the number of variety of goods. Note that we replace parameter α in Grossman and Helpman (1991) with the function $f(n)$.

From the first order condition for utility maximization, the demand function for good j is

$$x(j)=E/p_D\, p(j)^{-1/(1-f(n))}\left[\int_0^n p(j')^{1-1/(1-f(n))}dj'\right]^{-1/f(n)},\ j\in[0, n] \tag{3}$$

where P_D is the price index and is defined as follows

$$\left(\frac{1}{N}\int_0^n p(j)^{f(n)/(f(n)-1)}dj\right)^{(f(n)-1)/f(n)}=P_D. \tag{4}$$

Suppose $f'(n)>0$. The demand for each good decreases as n increases, since the household can obtain the same kind of utility from each good and the expenditure of the household become dispersed among each good. On the other hand, the demand for each good increases in the case of $f'(n)<0$, because then the relationship in utility or production among goods becomes strong and the household tries to consume a lot of each good.

A monopolistic firm produces each good j. As it has constant-return-to-scale technology and uses one production factor, labor, the profit function of the firm is

$$\pi(j)=P(j)x(j)-w(t)x(j) \tag{5}$$

where $x(j)$ is input or output and $w(t)$ is a common wage rate at time t.

New goods are developed by the R&D activity of firms. A firm that succeeds in obtaining the rent of a new good can enter the economy. The R&D function is

$$dn=\left(l_R/a\right)dt, \quad \text{if there is no spillover effect.} \tag{6a}$$

$$dn=n\left(l_R/a\right)dt, \quad \text{if there is a spillover effect.} \tag{6b}$$

where l_R is the labor input for R&D and $1/a$ is the productivity of R&D. When there is a spillover effect, the knowledge about the R&D of one firm can be common to all other firms in the economy.

The optimal price of good j is

$$p(j)=w/f(n) \quad \text{or} \quad p(j)=w/f(n) \tag{7}$$

Note that firms charge a lower (higher) mark up rate, as the elasticity becomes larger (smaller).

The change in the mark up rate affects the value of the R&D as next.

$$v=\frac{1-f(n)}{\rho n} \tag{8}$$

The firm obtains additive value as the number of goods increases in the case of $f'(n)<0$, while the value of R&D does not increase as the number of goods increases in the case of $f'(n)>0$. Compared to the model of Grossman and Helpman, the value of the R&D diminishes faster (slower) when the elasticity becomes larger (smaller).

The change in optimal prices also influences the cost of R&D, as follows.

$$v=\frac{L}{an}f(n) \quad \text{if there is a spillover effect.} \tag{9a}$$

$$v=\frac{L}{a}f(n) \quad \text{if there is no spillover effect.} \tag{9b}$$

Due to $f(n)$, the cost decreases faster (slower) when the relationship among goods becomes complementary (substitutive), compared to Grossman and Helpman (1991). If the relationship among goods becomes substitutive, the emergence of new goods causes crowding effects among goods in the economy. While, if the relationship becomes complementary, there are two external positive effects on economic growth, the complementary effect and the spillover effect.

IV. *Dynamics* (1) (*Substitutive Case*)

Grossman and Helpman (1991) show that endogenous growth appears when there is a spillover in the economy, while economic growth stops when there is no spillover in the economy.

In our model, the dynamic equations of v and n are

$$\frac{\dot{v}}{v}=\rho-\frac{1-f(n)}{n} \quad \text{(Concerning the value of R\&D in the spillover case)} \qquad (10)$$

$$\frac{\dot{n}}{n}=\frac{L}{a}-\frac{f(n)}{vn} \quad \text{(Concerning the cost of R\&D in the spillover case)} \qquad (11a)$$

$$\frac{\dot{n}}{n}=\frac{L}{a}-\frac{f(n)}{v} \quad \text{(Concerning the cost of R\&D in the no spillover case)} \qquad (11b)$$

First, we draw a phase diagram, assuming the following specific form of $f(n)$

$$f(n)=\frac{zn}{xn+y} \qquad (12)$$

Note that this form has the character of $f'(n)>0$ and we assume that there is a spillover in the economy. As shown in Figure 1, product expansion stops even though there is spillover in the economy. The value of R&D is larger than the cost of R&D and the trajectory of economic growth could appear with relatively smaller n. However, n converges to a certain level, as the elasticity of substitution becomes larger. Note that the condition of endogenous growth in traditional theory is satisfied. In an advanced economy, the crowding of new products by the severe competition for R&D in some cases does not increase but decrease the benefit of R&D in each firm. Such a crowding by competition for R&D does not seem to contribute to economic growth.

As shown in Toshihiro Horiuchi (2000), lately in Japan, the market size of the game industry concerning both software and hardware has been decreasing over successive years. According to “A Research for Information and Media Society” (2004), the market size of the game industry in 2002 was equivalent to that in 1990. (See Table 3)

On the other hand, R&D in the game industry remained active after 2000 and the number of applications for patents increased. Table 3 also shows the number of titles of the newly released games chiefly after 1990. Although the number of new titles was not so remarkable in the 1980s,[4] a large number of new titles appeared in the 1990s, exceeding 1000.[5] The

[4] The number of applications for game patents comprises only 18.3% of the total number of applications for patents in 1980s.

[5] The number of applications for game patents comprised 64.9% of the total application for patents after1995.

FIG. 1. DYNAMICS WHEN THERE IS SPILLOVER AND $f'(n)>0$ IS SATISFIED

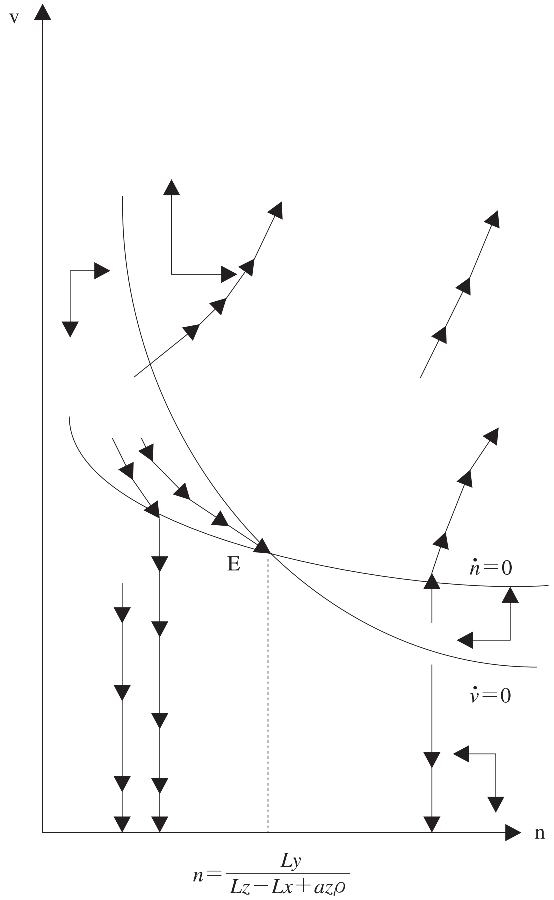


TABLE 3. MARKET SIZE OF THE GAME INDUSTRY AND THE NUMBER OF TITLES OF NEWLY RELEASED GAMES

Year	Market Size	Number of New Titles	Year	Market Size	Number of New Titles
1983		9	1996	7193	1080
1985		67	1997	7582	1066
1990		485	1998	6586	1120
1991	5660	582	1999	6040	1291
1992	6470	643	2000	6232	1329
1993	6720	638	2001	6134	1261
1994	6670	862	2002	5014	1051
1995	6930	1003			

Source: "A Research for Information and Media Society" (2004) by Dentsu Communication Institute Inc.

tendency continued until 2000 and the numbers of new titles was considerable also in 2001 and 2002 although the numbers decreased compared with the each of the previous years.

This means that only the race in R&D for new games between firms has continued activity although the market in the game industry has peaked. The competition between firms to obtain a share induced a heated race in R&D, unconnected to consumer's utility.[6]

Such a crowd of newly release games reduces the evaluation of consumers for each game, and creates difficult conditions for selling games.

In the case of $f'(n) > 0$, the ratio of the value of R&D to the cost decreases in increase of n (see Appendix A). The case of $f'(n) > 0$ may not be suitable for economic growth.

V. *Dynamics* (2) (*Complementary Case*)

Next, we assume the opposite case, in which there is a spillover in the economy and goods become complementary as n increases. We specify $f(n)$ as follows,

$$f(n) = \frac{e}{cn+d} \tag{13}$$

Figure 2 shows a phase diagram, which has not been presented in traditional growth theories. In previous models, the trajectory of economic growth appears with a smaller n whether economic growth continues or stops in process.

While, in our model, the trajectory of economic growth does not appear with a relatively smaller n of good, but the growth pass suddenly appears at the right hand of the crossed point. In short, it is not until a certain number of goods are settled in the economy that the product expansion starts. This figure is symmetrical concerning crossed points against the spillover case in Grossman and Helpman (1991).

In Section II, we considered the case that final goods newly appearing in the economy has directly complementary. Furthermore, we can find other cases that goods appearing in the economy have additive complementary.

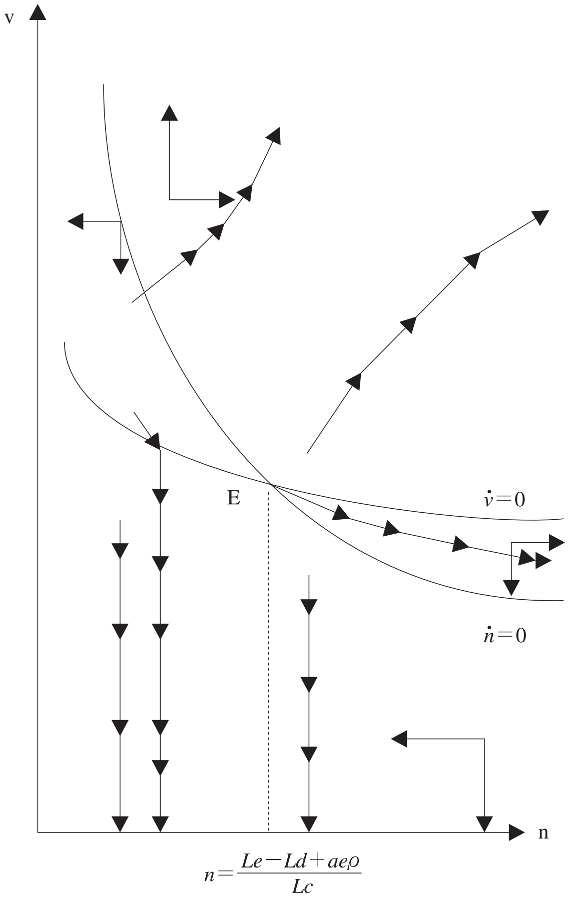
The appearance of attractive goods or industry often strengthens the relationship among goods or industries existing in the economy. In economic development, the introduction of roads and railways strengthens the relationship among industries in the economy. For example, the introduction of the automobile industry strengthens the relationship between the iron and steel industry and the glass industry. Table 4 shows the spread of car ownership (one of the Sacred Treasures), and the input coefficient of glass, tire, plastic, iron and steel industries to the car industry. While the spread of car ownership rose towards 50%, various industries such as iron and steel, glass, tire and plastic were able to sell their own products as a result of the appearance of the final good, cars. This means that the iron and steel industry, the glass industry and the tire industry were combined to produce cars and were in a complementary relationship due to the appearance of the final goods, car.[7]

Here we find that these input coefficients tend to decrease during this period. The merit of

[6] In fact, the main firms such as Namco, Sega, Konami, SONY and Taito comprise 37% of total game patent applications.

[7] We can also find such a situation in the appearance of other final goods. We recognize such a fact as the second example of the complementary generating by expansion of the variety of goods.

FIG. 2. DYNAMICS WHEN THERE IS SPILLOVER AND $f'(n)<0$ IS SATISFIED



scale is one of the causes of this situation, in using materials from these industries and producing cars, because the style of cars does not largely change during this time. Note that iron and steel were also used in the railroad industry and the input coefficient of iron and steel towards the railroad industry also decreases during this time. The introduction of the car industry enabled the iron and steel industry to increase supply and realize low price because of the merit of scale. And so both the car industry and the railroad industry enjoyed low material cost and become complementary in sharing common scale merit in the iron and steel industry.

Similarly the input coefficient of oil into synthetic fiber tends to decrease. The scale merit operates both in the car industry and in the synthetic fiber industry because the plastic used in the car industry is made from oil. Shinohara (1973) claimed that oil and oil-related industries are important in post-war Japanese economy because they have potential merit of scale and the appearance of various final industries, (car, synthetic fiber, housing, and so forth) help in

TABLE 4. INPUT COEFFICIENT OF VARIOUS INDUSTRIES INTO CAR, RAILWAY AND SYNTHETIC FIBER INDUSTRY

(4-1) Spread of cars and the input coefficient of various industries, tire/tube, plastic, steel and iron and glass into the car industry

	1970	1975	1980	1985	1990
Tire/tube	0.033368	0.023516	0.012905	0.007081	0.045949
Plastic		0.022353	0.020947	0.023322	0.019438
Steel and iron	0.068289	0.070767	0.053506	0.024721	0.020541
Glass	0.005715	0.005819	0.005296	0.004054	0.004152
The spread rate of cars	22.1	41.2	57.2	67.4	77.3

(4-2) Coefficient of steel and iron into the railway industry

	1970	1975	1980	1985	1990
Steel and iron/ railway industry	0.067116	0.056178	0.038001	0.036799	0.034348

(4-3) Coefficient of oil into the synthetic fiber industry

	1970	1975	1980	1985	1990
Oil/ synthetic fiber industry	0.066398	0.067911	0.020676	0.028419	0.021769

Source: "Linked input-output tables 1970-75-80, 1975-80-85 and 1985-90-95" by the Management and Coordination Agency.

Note that these data are recalculated regarding related articles using the standard of 1980 for comparison.

drawing out such potential.[8]

Such a strengthened relationship among goods may contribute to to economic growth. In the case of $f'(n)<0$, the ratio of the value of R&D to the cost increases as n increases (see Appendix A). The case of $f'(n)<0$ may be suitable for economic growth.

VI. *The Movement between Grossman and Helpman's Light Zone and Dark Zone*

In our model, the change in the elasticity of substitution changes depending on the variety of goods. We bring new insight into the relationship between the elasticity and economic growth. As shown in Solow (2000), the cost is larger than the value and economic growth does not occur when the elasticity of substitution between any pair of goods is larger than

$$\frac{L}{a\rho+L} \tag{14}$$

In Grossman and Helpman (1991), where the elasticity of substitution is constant, once the elasticity is larger than (14) at relatively smaller n, the value of R&D never becomes larger than the cost of R&D even after a certain number of goods is settled in the economy. And so in Grossman and Helpman (1991), such a situation is not considered in their analysis of economic growth.

But in our model, although the condition of economic growth is not satisfied at a

[8] We may recognize such a fact as the third example of the complementary generating by expansion of the variety of goods, to say, final goods enjoying the merit of scale in using common materials.

TABLE 5. BACKWARD / FORWARDS LINKAGE EFFECTS OF INDUSTRIES IN JAPAN, ITALY AND THE U.S.A.

	Backward / forwards linkage effects			Backward / forwards linkage effects		
Industries manufacturing intermediate goods	Iron and Steel	66	78	Coal products	63	67
	Non-ferrous Metals	61	81	Chemicals	60	69
	Paper and Products	57	78	Textiles	69	57
	Petroleum products	65	68	Rubber Products	51	48
	Print and Publishing	49	46			
Industries manufacturing final goods	Grain Mill Products	89	42	Transportation Equipment	60	12
	Leather and Products	66	37	Machinery	51	28
	Wood and Wood Products	61	38	Non-metallic Mineral product	47	30
	Apparel	69	12	Other manufacturing industry	43	20
	Shipbuilding	58	14			
	Processed Foods	60	15			
Industries producing intermediate primary goods	Metal Mining	21	93	Agriculture and Forestry	31	27
	Petroleum and Natural Gas	15	97	Electricity Power	27	59
	Coal Mining	23	87	Non-metallic Minerals	17	52
Industries producing final primary goods	Fishing	24	36	Services	19	34
	Transport	31	26	Trade	16	17

Source: Chenery, A.H. and T. Watanabe (1958), "International comparison of the structure of production," *Econometrica* 26, pp. 487-521.

relatively smaller n, the complementary may become stronger as expansion of the variety of goods and the condition of economic growth in some cases becomes satisfied. So, even if the elasticity is larger than (14), such a situation can be analyzed for economic growth.

In development economics, "Big Push" means the sudden or discontinuous jump from a situation that is not conducive to economic growth to a situation that is conducive to growth. Our model expresses the threshold towards the Big Push by the elasticity of substitution and the number of goods. Hirshman (1958) emphasizes the importance of the relationship among industries or goods in economic development. Our model reflects his idea and shows the threshold towards economic growth by the elasticity of substitution.

Table 5 is taken from Chenery and Watanabe (1958). It surveys the average interdependent relationship among each industry in Japan, Italy and the U.S.A.

Here forward linkage effects are interdependent relationships that act in purchasing the goods from other industries. On the other hand, backward linkage effects are interdependent relationships that act in selling their own goods to other industries. Such a demand would occur in an economy if each industry newly appears in the economy.

Notice that the backward linkage effects of final goods also are large. This fact is not supported by Gali (1995), who does not consider the change in the elasticity of final goods and, in the first place, Gali (1995) does not consider complementary cases.

Shinohara (1976) claimed that these effects become larger when they act in plural industries sequentially (to say, these effect acting in Industry A, acting next in Industry B and acting next in Industry C) than when they act separately in each industry. The appearance of such a demand creation effect may replace a situation that is not conducive to economic growth with a situation that is conducive to economic growth.

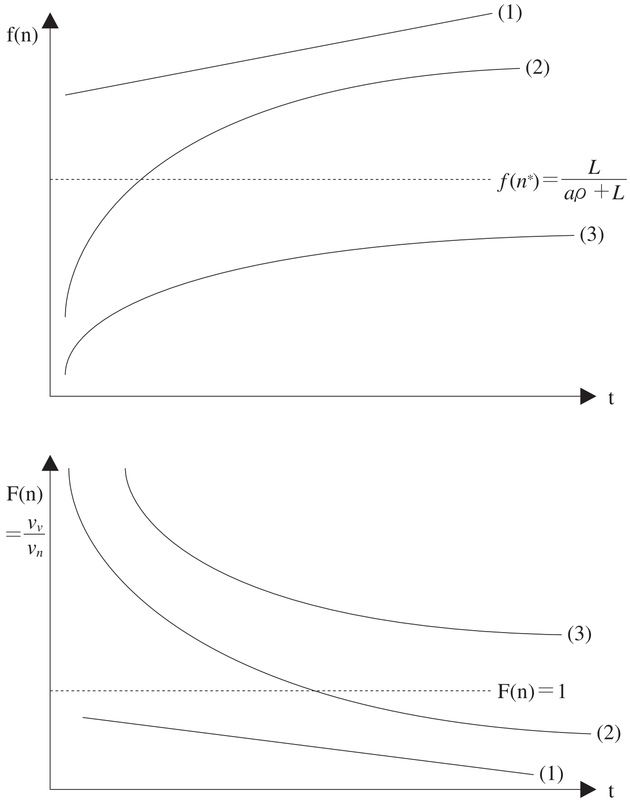
Once the elasticity of substitution becomes larger than (14), the economy falls into the situation that economic growth does not occur. Such a situation is also important for our analysis. Jones (1993) emphasizes that the effect of R&D in an advanced economy will be reduced so that he modifies an R&D-based endogenous growth model to underestimate the effect of R&D. And he shows that economic growth in some cases stops in process even though the traditional condition of endogenous growth is satisfied. But, in his model, the effect of R &D includes both the positive effect that is caused by the productivity of R&D and the negative effect that is caused by crowding of new products in the market. Here the productivity of R&D does not weaken in an advanced economy but rather becomes stronger as shown in the game industry as described in Section IV. Our model can divide these two effects and the cause of the slowdown of the effect of R&D in the advanced economy can be explained by the increase in the elasticity of substitution.

VII. *The Productivity of R&D, the Scale Effect and the Spillover Effect, General Functional Form*

In our model, we can examine not only the impact of the elasticity of substitution on the economic growth but also that of the productivity of R&D or the scale effect (population effect) on the economic growth. So, we can show that both the productivity of R&D and the scale effect (population effect) have positive impacts on economic growth (see Appendix A). While Jones (1993) cannot differentiate the productivity of R&D activity, the spillover effect of R&D and the crowded effect with R&D, we can divide the productivity of R&D from other effects and we can explain the decline in economic growth even if the productivity of R&D is high. Next, as with Sollow (1956) and Jones (1993), we can show that the scale effect has a positive impact on the economic growth in our model. Also in our model, we can examine the spillover effect itself, which is an important concept in endogenous growth theory (see Appendix A).

Next, we examined the impact of the change in the elasticity of substitution on the growth mainly in the case of a specific form. But we can examine such an impact also in the case of a general functional form. As shown in Appendix B, in our model, the ratio of the value of R &D to the cost of R&D always increases (decreases) in the case of $f'(n)<0$ ($f'(n)>0$).

FIG. 3. RELATIONSHIP BETWEEN THE VALUE (v_v) AND THE COST (v_n) IN THE CASE OF $f'(n)>0$



Concerning this result and the necessary condition[9] for endogenous growth in Section IV, we can show the conditions for the growth continuity both in the case of $f'(n)>0$ and $f'(n)<0$.[10]

---

[9] In order that the value of R&D is larger than its cost, the next condition about the elasticity of substitution has to be satisfied.

$$f(n)>\frac{L}{a\rho+L} \tag{15}$$

As shown in Section IV, this condition is a necessary condition for endogenous growth.

[10] As in Case (3) in Figure 3, in case of $f'(n)>0$, the growth is sustained only when the next conditions are satisfied.

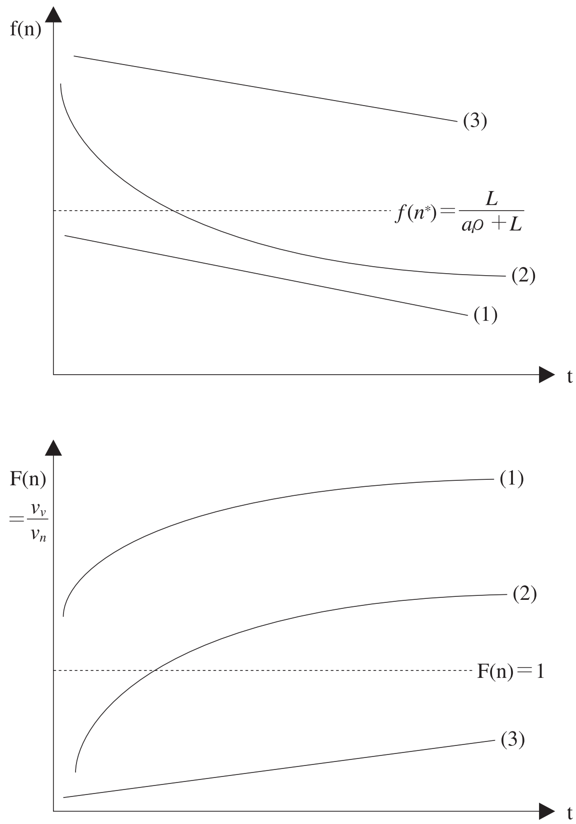
$$f(0)<\frac{L}{a\rho+L} \tag{16a}$$

$$\lim_{n\to\infty} f(n)<\frac{L}{a\rho+L} \tag{16b}$$

While, as in Case (3) in Figure 4, in case of $f'(n)<0$, the growth is not sustained only when the next conditions are satisfied.

$$f(0)>\frac{L}{a\rho+L} \tag{17a}$$

FIG. 4. RELATIONSHIP BETWEEN THE VALUE (v_v) AND THE COST (v_n) IN THE CASE OF $f'(n) < 0$



Finally, we can examine the growth rate g in steady state[11] and, under some assumptions, we show that there is a converging path in our model.[12] In short, under some assumptions, we can obtain a growth rate similar to that of Grossman and Helpman (1991).

$$g = (1-\varepsilon)\frac{L}{a} - \varepsilon\rho \tag{18}$$

VIII. *Conclusion*

We assume that the elasticity of substitution depends on the variety of goods, based on the

$$\lim_{n\to\infty} f(n) > \frac{L}{a\rho + L} \tag{17b}$$

[11] Note that we mean by the steady state in endogenous growth theory that the growth rate converges to a certain level.

[12] By showing a converging path, we can maintain that our model does not express an unrealistic economy.

model of Grossman and Helpman (1991). The following result was obtained: When the elasticity is increasing in the variety of goods, economic growth does not necessarily continue forever, while Grossman and Helpman claim so in their original model. On the other hand, when elasticity decreases in variety, the growth in some cases suddenly occurs under some conditions.

In such a setting, we bring new insight into the analysis for a developing economy as well as an advanced economy. Regarding the Big Push in economic development, we can give a threshold for economic growth using the relationship among goods. And by the elasticity of substitution we can explain the cause of the slowdown of the effect of R&D.

Finally under certain conditions, we show that our model has a converging path and our model does not express an unrealistic economy.

Appendix A

Here we assume that there is a spillover in the economy and we show how the productivity of R&D or the scale effect (population) has an influence on the economic growth in our model.

If $f'(n)<0$ (complementary case), the starting point for economic growth is

$$n=\frac{Le-Ld+ae\rho}{Lc} \tag{A1}$$

The starting point moves more backwards as population L becomes larger or the productivity of R&D $1/a$ becomes stronger. In short, as the population becomes larger or the productivity becomes stronger, the growth path appears at a smaller n.

While, if $f'(n)>0$ (substitutive case), the end point for economic growth is

$$n=\frac{Ly}{Lz-Lx+az\rho} \tag{A2}$$

The end point moves further forward as population L becomes larger or the productivity of R&D $1/a$ becomes stronger. In short, as the population becomes larger or the productivity becomes stronger, the economic growth continues until a larger n.

It is shown by the above observation that a larger population or a stronger productivity of R&D has a positive impact on economic growth.

Next, we examine the spillover effect in our model. We have to compare the spillover case with the no spillover case in order to examine the spillover effect itself. Here we concentrate on the case of $f'(n)>0$ and assume the specific functional form as shown next:

$$f(n)=\frac{n}{n+1} \tag{A3}$$

We obtain the starting point for economic growth using the next equation.

$$n=\sqrt{\frac{L}{a\rho}} \text{ in the no spillover case} \tag{A4}$$

$$n=\frac{L}{a\rho} \text{ in the spillover case} \tag{A5}$$

Because the next inequality is satisfied, the starting point is further forward when there is a spillover.

$$\frac{L}{a\rho} > \sqrt{\frac{L}{a\rho}} \tag{A6}$$

In short, the spillover has a positive impact on economic growth.

Appendix B

If we divide Equation (8) by Equation (9a), we obtain the next result.

$$\frac{v_v}{v_n} = \frac{L}{\rho a}\frac{1-f(n)}{f(n)} \equiv F(n) \tag{A7}$$

Here, if $F(n)$ increases as n increases, the ratio of the value of R&D to the cost of R&D becomes relatively higher as n increases. Otherwise, such a ratio becomes lower.

If we differentiate $F(n)$ by n, we obtain the next result.

$$F'(n) = \frac{-f'(n)f(n)-f'(n)(1-f(n))}{f^2(n)}\frac{L}{\rho a} \tag{A8}$$

$F(n)$ always decreases (increases) as the number of the variety of goods increases, if the elasticity of substitution among goods becomes stronger (smaller), because $f'(n)>0$ and $0<f(n)<1$ is satisfied.

Appendix C

Here we show that there is a converging path in our model. By showing a converging path, we can maintain that our model does not express an unrealistic economy.

Dynamic Equation (11a) is

$$\frac{\dot{n}}{n} = \frac{L}{a} - \frac{f(n)}{vn} \tag{11a}$$

Here we use $g \equiv \frac{\dot{n}}{n}$ to denote the growth rate and $V \equiv \frac{1}{vn}$ to represent the inverse of the aggregate equity value.

If we differentiate the logarithm of V, we obtain the next equation.

$$\frac{\dot{V}}{V} = f(\dot{n}) - g - \frac{\dot{v}}{v} \tag{A9}$$

The change rate of V depends on the differentiation of elasticity of substitution by time, growth rate g and the rate of change in each value of R&D. As the change rate of value of R&D or growth rate g increases, the rate of V decreases. While, as the differentiation of elasticity increases, the rate of V increases.

Dynamic Equation (10) is

$$\frac{\dot{v}}{v}=\rho-\frac{1-f(n)}{n} \tag{10}$$

Furthermore, if we differentiate the logarithm of $f(n)$, the next equation is satisfied.

$$\frac{f(\dot{n})}{f(n)}=\frac{df(n)}{dn}\frac{dn}{dt}\frac{1}{f(n)}=\frac{f'(n)}{f(n)}g \tag{A10}$$

In steady state, the next equation is satisfied, because the inverse of aggregate value does not change.

$$\dot{V}=0 \tag{A11}$$

From the above equation (A3), (10), (A4) and (A5), we obtain the following growth rate:

$$g=\frac{(1-f(n))\frac{L}{a}-f(n)\rho}{1-nf'(n)} \tag{A12}$$

Note that this growth rate depends on the level of substitution and the elasticity of substitution.

Finally in order to get a growth rate that converges, we assume the following conditions.

$$\lim_{n\to\infty} f(n)=\varepsilon \tag{A13a}$$

$$\lim_{n\to\infty} f'(n)=0 \tag{A13b}$$

And under such additive conditions, we obtain the next growth rate:

$$g=(1-\varepsilon)\frac{L}{a}-\varepsilon\rho \tag{18}$$

REFERENCE

Becker, G.S. (1965), "A theory of the allocation of time," *The Economic Journal* 75, pp. 493-517.

Chenery, A.H. and T. Watanabe (1958), "International comparison of the structure of production," *Econometrica* 26, pp. 487-521.

Dentsu-Soken [Dentsu Communication Institute Inc.], "A Research for Information and Media Society" (2004).

Dixit, A.K. and Stiglitz, J.E. (1977), "Monopolistic competition and optimal product diversity," *American Economic Review* 67, pp. 297-308.

Gali, J. (1995), "Product diversity, endogenous markups and development traps," *Journal of Monetary Economics* 31, pp. 39-63.

Grossman, G.M. and E. Helpman (1991), *Innovation and Growth in the Global Economy,* Cambridge, MA, MIT Press.

Hirschman, A.O. (1957), *The Strategy of Economic Development,* New Haven Yale University

Press.
Horiuchi, T. (2000), *Sangyo Soshiki Ron* [*Industry Organization Theory*], Minerva Press.
Jones, C.I. (1995), "R&D-based model of economic growth," *Journal of Political Economy* 103, pp. 759-784.
Keizaikikaku-chyo [The Economic Planning Agency], "Kokumin Seikatsu Hakusyo (1990) [White Paper on the National Lifestyle Fiscal Year 1990]".
NHK, Yoron Chosa Ka [public opinion poll place], "Kokumin Seikatsu Jikan Chosa [The Survey on National Living Hours](1980)".
Nurkse, R. (1953), Problems of Capital Formation in Underdevelopment Countries, New York, Oxford University Press.
Romer, P.M. (1990), "Endogenous technological change," *Journal of Political Economy* 98, pp. 71-102.
Shinohara, M. (1973), *Sangyo Kozo Ron* [*Industry Structure Theory*], Chikuma Press.
Solow, R.M. (1956), "A contribution to the theory of economic growth," *Quarterly Journal of Economics* 70, pp. 65-71.
Soumu-syo, Tokei-Kyoku [Ministry of Internal Affairs and Communications, Statistics Bureau], "Shakai Seikatsu Kihon Chosa [Survey on Time Use and Leisure Activities] (1976)".
Soumu-chyo Tokei-Kyoku [Ministry of Internal Affairs and Communications, Statistics Bureau], "Linked input-output tables 1970-75-80, 1975-80-85 and 1985-90-95".
Young, A. (1993), "Substitution and complementarily in endogenous innovation," *Quarterly Journal of Economics* 108, pp. 775-807.