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SHAPLEY BARGAINING AND MERGER INCENTIVES IN NETWORK INDUSTRIES WITH ESSENTIAL FACILITIES*

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Abstract

I construct a model of the network industry in which an upstream operator provides essential facilities for downstream operators. Assuming Shapley bargaining over access charges, I find that the main condition for mergers, either vertical or horizontal, to be beneficial to the merging parties is such that the network industry exhibits decreasing returns to network size.

Keywords: Shapley value, Merger, Network Industry, Essential Facilities,
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I. *Introduction*

A typical feature in network industries is that upstream firms provide essential facilities or inputs for downstream firms. Downstream firms, with access to essential factors, supply final services for end-users. We observe this structure in many network industries such as the Internet, cable, telecommunications, gas, and electricity industries. Essential bottleneck facilities in the relevant industries are backbones for ISPs (Internet Service Providers), channel providers for local cable system operators, LECs (Local Exchange Carriers) for long-distance telecom operators, distribution pipes for local gas suppliers, and transmission grids for local electricity firms.

In the paper, I consider the network industry in which one upstream operator supplies essential facilities for K downstream operators. Assuming Shapley bargaining over access charges, I examine merger incentives. The main condition for mergers, either vertical or horizontal, to be beneficial to the merging parties is that the aggregate profit function of a coalition is concave in the size of the network that the coalition covers, i.e., that the network industry exhibits decreasing returns to network size. It seems counter-intuitive that decreasing returns to network size imply merger benefits. However, notice that Shapley bargaining

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approach is a cooperative game theory; firms always achieve efficiencies whether they separate or integrate. Hence, I consider merger incentives in terms of bargaining advantage, not in terms of efficiency gains. I explain the results with two considerations of fair-share and bargaining-power incorporated in the Shapley value.

Since Shapley (1953) introduced the concept, there have been many theoretical works to prove its superiority as a bargaining solution concept, e.g., Harsanyi (1977), and Gul (1989). However, because of the complexity of computing it for large games, the Shapley value has received few applications in industrial economics. A notable exception is Hart and Moore (1990). They addressed the issue of the boundary of the firm in a general set-up, and considered ex ante relation-specific investment decisions of firms as well as ex post Shapley bargaining. The current paper, focusing only on Shapley bargaining, provides more thorough characterizations of merger incentives in the specific context of the network industry. Chipty and Snyder (1999) also addressed an issue similar to the current paper, a bargaining motivation for horizontal mergers, in the cable television industry. But, they adopted a bilateral Nash bargaining solution. Jeon and Wildman (2002) adopted Shapley bargaining solution to address size effects in the cable television industry. This paper extends their work into a more general set-up of network industries. Stole and Zwiebel (1996) showed that non-cooperative solution for intra-firm wage bargaining is consistent with the Shapley value when the firm and workers can renegotiate anytime before production. The firm is regarded as the provider of essential facilities for workers. They noticed that unionization is desirable from the workers' point of view when the production technology is concave. They just illustrated a numerical example with two employees. The results in this paper may be regarded as generalizing it in a broader context.

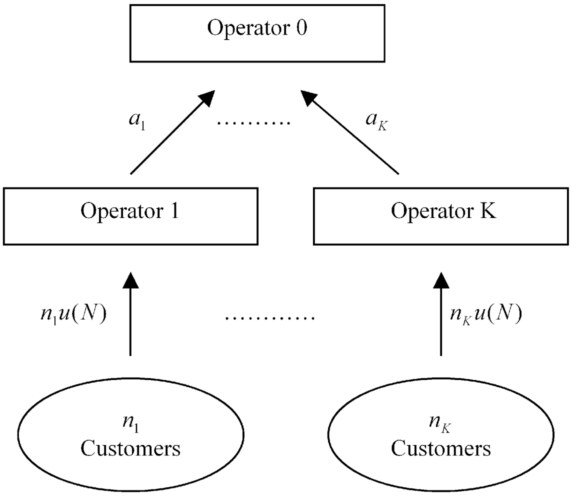
The paper is organized as follows. Section II constructs a model of the network industry with one upstream firm and K downstream firms, and explains the nature of Shapley bargaining over access charges for the essential factor that the upstream firm provides for downstream firms. Section III shows the main results, and provides intuitive explanation. Section IV illustrates some applications of the model: industries in which Metcalfe's Law characterizes network externalities, deregulated local-monopoly industries such as cable television as in Jeon and Wildman (2002), and intra-firm bargaining between the firm and employees as in Stole and Zwiebel (1996). Section V concludes by summarizing the results and discussing their implications and limitations.

II. *Model*

A network industry is composed of $K+1$ operators: operator 0 provides the essential facilities or inputs for downstream operator $i(=1, 2, \ldots, K)$, which supplies final services for end-users. Figure 1 shows the structure of the network industry.[1] Operator i covers a network with size n_i, and obtains gross profits before the payment of access charges, $n_i u(N)$, where N

[1] The upstream operator may be vertically integrated, and serves its own network with size n_0. But, my results do not depend on whether the essential facility provider is vertically integrated or not. To save notation, I will assume that the initial state is vertical separation: i.e., $n_0=0$.

FIG. 1. NETWORK INDUSTRY STRUCTURE



$\equiv \sum_{i=1}^{K} n_i$. The size of network may represent the number of end-users (user groups or service areas). If $u(N)$ represents net benefit per end-user, we expect $u'(N) > 0$ due to network externalities. Each downstream operator i pays access charge a_i to operator 0.

I assume that the costs of facilities investment are sunk. Then, operator 0's profit is $\sum_{i=1}^{K} a_i$, and operator i's net profit $n_i u(N) - a_i$ $(i = 1, 2, \dots, K)$. I do not model explicitly the process of bargaining that determines access charges. Instead I assume that each obtains its share according to the Shapley value. Given Shapley values Sh_i $(i = 0, 1, \dots, K)$, access charges a_i $(i = 1, 2, \dots, K)$ are determined by:

$$Sh_0 = \sum_{i=1}^{K} a_i \text{ and } Sh_i = n_i u(N) - a_i (i = 1, 2, \dots, K). \tag{1}$$

Subsequently, I will focus on determining each operator's Shapley value.

Denote the grand coalition as $I \equiv \{0, 1, 2, \dots, K\}$ and the value function as $v(S)$ for any partial coalition $S \subseteq I$. Any coalition's value is the aggregate of its members' profits. The network structure, specified as above, simplifies it to a great extent. Due to the nature of essentiality of operator 0's facilities, we can reduce the value function as follows:

$$\begin{aligned} v(S) &= 0 \qquad \forall S \subseteq I \text{ such as } 0 \notin S. \\ v(S) &= (n_{i_1} + \dots + n_{i_k}) u(n_{i_1} + \dots + n_{i_k}) \\ &= \pi(n_{i_1} + \dots + n_{i_k}) \, \forall S \subseteq I \text{ such as } S = \{0, i_1, \dots, i_k\} \end{aligned} \tag{2}$$

where $\pi(n) \equiv n u(n)$.

Notice that $\pi(n)$ represents the aggregate profit of coalition S with the network size of n. Obviously, $\pi(0) = 0$, and $\pi'(n) > 0$.

Given the characteristic value function $v(S)$, the Shapley values are defined as follows:

$$Sh_i = \sum_{\substack{i \in S \\ S \subseteq I}} \frac{(s-1)!(K+1-s)!}{(K+1)!} [v(S) - v(S \backslash \{i\})] \, (i=0, 1, \ldots, K), \tag{3}$$

where s and $K+1$ are the sizes of coalitions S and I, respectively. As is well known, the Shapley values, as defined by (3), can be given the following heuristic interpretation. Suppose that $K+1$ players line up in a random order. It is assumed that all orders of lining up have the same probability: viz., $1/(K+1)!$. Suppose that if a player, i, finds the members of coalition $S \backslash \{i\}$ (and no others) in front of him, he receives the amount $v(S) - v(S \backslash \{i\})$, i.e., the marginal amount which he contributes to the coalition, as payoff. Then, Shapley value Sh_i is the expected payoff to player i under this randomization scheme.[2]

In general, the Shapley values are too complicated to work with when K is large. However, in the set-up of network industries with essential facilities, we can exploit the properties of value function $v(S)$ in (2) to obtain:

$$Sh_0 = \sum_{k=1}^{K} \sum_{\substack{i_1 < \ldots < i_k \\ \{i_1, \ldots, i_k\} \subseteq I \backslash \{0\}}} \frac{k!(K-k)!}{(K+1)!} \pi(n_{i_1} + \ldots + n_{i_k}) \tag{4}$$ [3]

$$Sh_1 = \sum_{k=0}^{K-1} \sum_{\substack{i_1 < \ldots < i_k \\ \{i_1, \ldots, i_k\} \subseteq I \backslash \{0, 1\}}} \frac{(k+1)!(K-k-1)!}{(K+1)!} \{\pi(n_1 + n_{i_1} + \ldots + n_{i_k}) - \pi(n_{i_1} + \ldots + n_{i_k})\} \tag{5}$$ [4]

We can determine $Sh_2, \ldots, Sh_K$ by adapting (5) appropriately. For example,

$$Sh_2 = \sum_{k=0}^{K-1} \sum_{\substack{i_1 < \ldots < i_k \\ \{i_1, \ldots, i_k\} \subseteq I \backslash \{0, 2\}}} \frac{(k+1)!(K-k-1)!}{(K+1)!} \{\pi(n_2 + n_{i_1} + \ldots + n_{i_k}) - \pi(n_{i_1} + \ldots + n_{i_k})\} \tag{6}$$

III. *Main Results*

We are interested in merger incentives, i.e., whether two operators i and j can gain by merging into one. Denote the merged operator by $i+j$, and its share by Sh_{i+j}. Then, we can say that the merger is beneficial for the merging parties if $Sh_i + Sh_j < Sh_{i+j}$. We will consider the representative case of a vertical merger between 0 and 1. In this case, we can derive Sh_{0+1} by modifying (4) slightly:

$$Sh_{0+1} = \sum_{k=0}^{K-1} \sum_{\substack{i_1 < \ldots < i_k \\ \{i_1, \ldots, i_k\} \subseteq I \backslash \{0, 1\}}} \frac{k!(K-k-1)!}{K!} \pi(n_1 + n_{i_1} + \ldots + n_{i_k}) \tag{7}$$

[2] This explanation is adapted from Owen (1982, p. 197).

[3] To interpret this in terms of the general formula (3), consider $i=0$ and $S=\{0, i_1, \ldots, i_k\}$. The summation is over all $(i_1, \ldots, i_k)$ such that $i_1 < \ldots < i_k$ and $\{i_1, \ldots, i_k\} \subseteq I \backslash \{0\}$.

[4] The interpretation here is analogous to that in footnote 3. I.e., consider $i=1$ and $S=\{0, 1, i_1, \ldots, i_k\}$. The summation is over all $(i_1, \ldots, i_k)$ such that $i_1 < \ldots < i_k$ and $\{i_1, \ldots, i_k\} \subseteq I \backslash \{0, 1\}$.

Also, we will consider whether two downstream operators can benefit from a horizontal merger. For the representative case of a horizontal merger between 1 and 2, we can derive Sh_{1+2} by modifying (5):

$$Sh_{1+2}=\sum_{k=0}^{K-2}\sum_{\substack{i_1<\ldots<i_k\\ \{i_1,\ldots,i_k\}\subseteq I\setminus\{0,1,2\}}}\frac{(k+1)!(K-k-2)!}{K!}\{\pi(n_1+n_2+n_{i_1}+..+n_{i_k})-\pi(n_{i_1}+..+n_{i_k})\} \tag{8}$$

1. Incentives of Vertical Mergers

Suppose that there do not exist dominant downstream operators whose network sizes are considerably larger than others. More specifically, consider:

Assumption 1. If $s>s'$, then $\sum_{i\in S}n_i\geq\sum_{i\in S'}n_i$ $\forall S, S'\subseteq\{1, 2, ... K\}$.

In the above, s and s' denote the number of members in coalitions S and S', respectively. Hence, Assumption 1 means that any coalition with s members has a network size no less than does any other coalition with $s-1$ members. An extreme version of this assumption is that all operators have the same size of networks. With Assumption 1, we can establish the following result.[5]

Theorem 1. With Assumption 1, we have: $Sh_0+Sh_1\lesseqgtr Sh_{0+1}$ if $\pi''(n)\lesseqgtr 0$.

Assumption 1 is not always necessary for Theorem 1; it is just a sufficient condition. In fact, it is not necessary when $K=2$ or 3.[6]

Theorem 2. For $K=2$ or 3, we have: $Sh_0+Sh_1\lesseqgtr Sh_{0+1}$ if $\pi''(n)\lesseqgtr 0$.

2. Incentives of Horizontal Mergers

For any K, I can establish the following result on the sufficient conditions for a horizontal merger to be beneficial or not.

Theorem 3. $Sh_1+Sh_2\lesseqgtr Sh_{1+2}$ if $\pi''(n)\lesseqgtr 0$ and $\pi'''(n)\substack{\leq\\ \geq}0$.

In case of $K=2$ or 3, we do not need the third-order derivative condition.

Theorem 4. For $K=2$ or 3, we have: $Sh_1+Sh_2\lesseqgtr Sh_{1+2}$ if $\pi''(n)\lesseqgtr 0$.

To obtain an intuitive explanation for the results, decompose Shapley value for i of a game with the set of players, $I\equiv\{0, 1, 2, ..., K\}$, and the value function, $v(S)$, $S\subseteq I$ into:[7]

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5 All proofs are provided in the Appendix.

6 Even in case of $K=4$, I can construct an example which shows that Assumption 1 is not necessary for the result.

7 See Mas-Collel, Whinston and Green (1995, p. 681).

$$Sh_i(I, v) = \frac{1}{K+1} v(I) + \frac{1}{K+1} \left[\sum_{k \neq i} Sh_i(I \backslash \{k\}, v) - \sum_{k \neq i} Sh_k(I \backslash \{i\}, v) \right]. \tag{9}$$

The first term in (9) is the equal division among all players, which I call *fair-share consideration*. The second term in (9), which I call *bargaining-power consideration*, represents player i's bargaining power relative to other players; $Sh_i(I \backslash \{k\}, v)$ is i's share in the absence of k, while $Sh_k(I \backslash \{i\}, v)$ is k's share in the absence of i. Mergers reduce the total value of merging parties because all players share equally the grand coalition outcome in a fair-share consideration of the Shapley value. On the other hand, mergers will enhance the merging parties' influence, and hence increase their shares in a bargaining-power consideration of the Shapley value.

We may interpret the convexity (concavity) of industry profit function in network size as increasing (decreasing) *returns to network size*. Then, our results read that mergers are costly (beneficial) when returns to network size are increasing (decreasing). Why do increasing returns to network size lead to unprofitable mergers? Recall that in Shapley bargaining, fair-share consideration favors separation, while bargaining-power consideration favors merger. A noticeable feature in decomposition (9) is that the former is based on the grand coalition outcome, and the latter is determined according to outcomes of various partial coalitions. Hence, the former consideration based on the grand coalition outcome can be more important than the latter consideration based on partial coalitions' outcomes when there exist increasing returns to network size.

I emphasize that I adopt a cooperative approach of the Shapley value in addressing merger incentives. Efficiency is always achieved whether firms are integrated or separated. Hence, in comparison with separated firms, integrated firms cannot take advantage of additional efficiencies due to economies of scale or scope. The above results imply that in case of increasing returns to network size, mergers are rather costly to the merging parties in terms of Shapley bargaining.

IV. *Applications*

In this section, I provide some applications of the model that show its relevance.

1. Metcalfe's Law

Metcalfe's Law, named after Robert N. Metcalfe, co-inventor of Ethernet, states that the value of a network, defined as its utility to a population, grows with the square of the number of its users.[8] It seems to be mostly relevant to communication networks, where the value of a network depends on the number of possible connections between its members or nodes. Given the network size of n, each member can connect to $(n-1)$ other members, and the total number of two-way connections is ${}_nC_2 \times 2 = n(n-1)$. Suppose that network externalities obey a weak version of Metcalfe's Law such as $u(n) = n-1$; each consumer's benefits of subscribing to a network depend on the number of his own connections to other members. Then, with the definition of $\pi(n)$, we have:

[8] Refer to Newton (2001, p.436)

$$\pi(n) = nu(n) = n^2 - n, \quad \pi''(n) > 0, \quad \pi'''(n) = 0.$$

That is, Metcalfe's Law implies that the network industry shows increasing returns to network size.

Moreover, if network externalities obey a strong version of Metcalfe's Law such as $u(n) = (n-1) + \beta n(n-1)(0 < \beta < 1)$; each subscriber benefits from other members' communications as well as his own. Then, we have:

$$\pi(n) = nu(n) = n^2 - n + \beta(n^3 - n^2), \quad \pi''(n) > 0, \quad \pi'''(n) > 0.$$

The results in the paper suggest that operators do not benefit from mergers, either vertical or horizontal, in terms of Shapley bargaining outcome.

2. Cable Television Industry

Another application of the model, which actually motivated this work, is bargaining between programming network and multi-system operators in the cable industry.[9] In this context, upstream operator 0 is a programming network, and downstream operator i is a system operator with n_i local franchises. Each franchise unit has the following identical demand function:

$$Q = Q(P, B), \quad Q_1 < 0, \quad Q_2 > 0,$$

where Q denotes the number of subscribers in the franchise unit, P is subscription fee, and B represents network's expenditure on programming. The increase in network's budget in programming boosts up subscription demand, while the increase in subscription fee dampens it. System operators receive subscription fee P, and incur cost c, per subscriber. Hence, downstream operator i obtains net revenues from subscribers in their respective franchises, $n_i(P-c)Q(P, B)$. On the other hand, they pay programming fees a_i to network 0. Therefore, downstream operator i's net profits are $n_i(P-c)Q(P, B) - a_i (i = 1, 2, \ldots, K)$. The upstream programming network, network 0, has two sources: revenues from system operators, and revenues from advertisers. I assume that advertising revenues are proportional to the total number of subscribers who have access to the network with r per each subscriber. Firm 0 incurs programming cost B. Therefore, firms 0's net profits are: $\sum_{i=1}^{K} \{a_i + n_i rQ(P, B)\} - B$. Figure 2 recapitulates the industry structure and income flows. Given P and B, a_i's are determined according to the Shapley value.

Notice that network 0 is essential for any coalition in producing positive value. The value for a partial coalition such as $S = \{0, i_1, \ldots, i_k\}$ is:

$$v(S) = \max_{P, B} \; nR(P, B) - B, \tag{10}$$

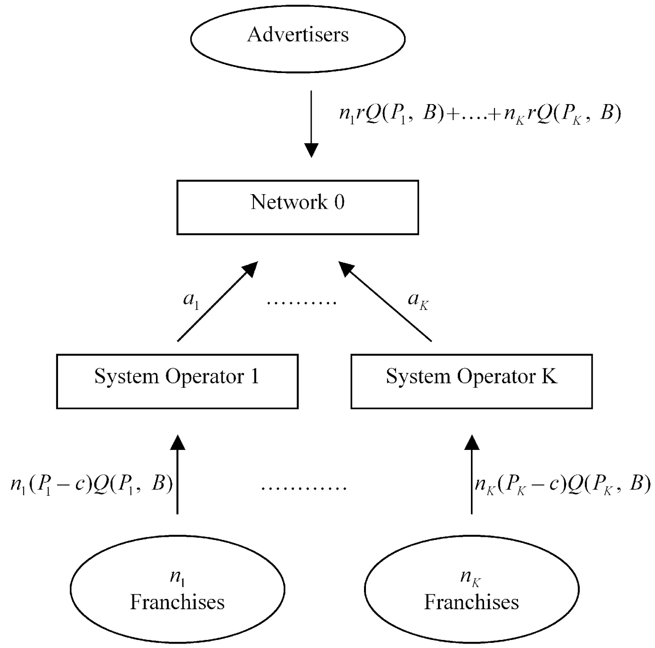
where $n = n_{i_1} + \ldots + n_{i_k}$, $R(P, B) \equiv (P + r - c)Q(P, B)$.

I assume that the second-order sufficient conditions for program (10) hold true:

$$R_{11} < 0, \quad R_{22} < 0, \quad R_{11}R_{22} - R_{12}{}^2 > 0. \tag{11}$$

[9] For the detailed discussions, see Jeon and Wildman (2002).

FIG. 2. CABLE INDUSTRY STRUCTURE



Then, the optimal solutions in the above maximization program, which I will denote by $P(n)$ and $B(n)$, satisfy the following first-order conditions:

$$R_1(P(n), B(n))=0, \ nR_2(P(n), B(n))=1. \qquad (12)$$

Notice that the value of coalition $S=\{0, i_1, ..., i_k\}$ critically depends upon its network size $n=n_{i_1}+....+n_{i_k}$. That is, given $P(n)$ and $B(n)$ by (12), the value of a coalition with network size n can be expressed as:

$$v(S)=\pi(n)\equiv nR(P(n), B(n))-B(n). \qquad (13)$$

This may have the same interpretation of the aggregate profits of an industry with network size n as in (2)

Now I can show the convexity of $\pi(n)$, i.e., $\pi''(n)>0$. By the envelope theorem, we have: $\frac{d\pi}{dn}=R(P(n), B(n))$. Hence, with (12), we have: $\frac{d^2\pi}{dn^2}=R_1\frac{dP}{dn}+R_2\frac{dB}{dn}=\frac{1}{n}\frac{dB}{dn}$. Applying the implicit function rule into (12), we have: $\frac{dB}{dn}=\frac{R_{11}}{R_{11}R_{22}-R_{12}{}^2}$. Due to assumption (11), we have: $\frac{dB}{dn}>0$. This proves: $\frac{d^2\pi}{dn^2}>0$.

In the context of bargaining between the programming network and multi-system operators in the cable industry, I showed that the industry profit function features increasing returns to network size. Again, this suggests that operators do not benefit from mergers, either vertical or horizontal, in terms of the Shapley bargaining outcome. It predicts that as far as Shapley bargaining is concerned, firms tend to separate rather than integrate.

3. Wage Bargaining in the Firm

Stole and Zwiebel (1996) considered an intrafirm bargaining game where employees and the firm engage in wage negotiations. They showed that when contracts cannot bind employees to the firm, the resulting stable wage and profit profiles are equivalent to the Shapley values to a corresponding cooperative game. Since the firm possesses the essential capital assets with which employees to work, we can reinterpret the current model to address wage bargaining in the firm.

Suppose that firm 0 employs K workers, produces $\pi(K)$, and pays worker i wage $w_i(i=1, \ldots, K)$. Then, given Shapley values $Sh_i(i=0, 1, \ldots, K)$, profit and wages are determined by:

$$Sh_0=\pi(K)-\sum_{i=1}^{K} w_i \text{ and } Sh_i=w_i(i=1, 2, \ldots, K).$$

The characteristic value function of the corresponding cooperative game is:

$$v(S)=0 \quad \forall S\subseteq I \text{ such as } 0\notin S.$$

$$v(S)=\pi(k) \quad \forall S\subseteq I \text{ such as } S=\{0, i_1, \ldots., i_k\}.$$

Hence, this application reduces formally to the special case of the current model with:

$$n_i=1 \quad \forall i=1, \ldots.., K.$$

Applying Shapley values in (4) and (5) into this special case, we have:

$$\begin{aligned} Sh_0&=\sum_{k=1}^{K} {}_K C_k \frac{k!(K-k)!}{(K+1)!}\pi(k) \\ &=\frac{1}{K+1}\sum_{k=1}^{K}\pi(k) \qquad (14) \\ Sh_1&=\sum_{k=0}^{K-1} {}_{K-1}C_k \frac{(k+1)!(K-k-1)!}{(K+1)!}\{\pi(k+1)-\pi(k)\} \\ &=\frac{1}{(K+1)K}\sum_{k=0}^{K-1}(k+1)\{\pi(k+1)-\pi(k)\} \\ &=\frac{1}{(K+1)K}\sum_{k=1}^{K}k\{\pi(k)-\pi(k-1)\} \qquad (15) \end{aligned}$$

Notice that this characterization of the profit and wage profiles is equivalent to that of Stole and Zwiebel (1996, p.199). Especially, the wage in (15) depends upon the weighted average of marginal products, with increasing weight in the wage expression the closer to the margin.

Given the same total output, employees prefer a convex production technology, while the firm prefers a concave technology. It is because the more concave a production technology, the more front-loaded marginal products and the less the work's share (the greater the firm's share). Unionization has the effect of linearizing production technology by making two changing marginal products into a single constant marginal product. Therefore, given decreasing marginal products, employees can cash in on the linearizing effect of unionization. Stole and Zwiebel (1996, p.211) constructed a simple numerical example with $K=2$ to illustrate the relationship between unionization and the concavity of production technology.

The results of the current paper on incentives of horizontal mergers generalize their result on employees' preference over unionization. Moreover, we may interpret the incentives of vertical mergers as the firm's preference over partnership with a group of workers. Incidentally, employees' preference over unionization and the firm's preference over partnership with a group of workers coincide. That is, when a production technology exhibits decreasing returns, the firm and a union compete to coalesce with a non-unionized group of workers.

V. *Conclusion*

In the paper, adopting the Shapley bargaining approach, I addressed merger incentives in network industries with essential facilities. We expect that firms benefit from mergers when there exist increasing returns to scale or scope. However, I obtained seemingly contrary results whose main implication is that firms are worse off from mergers when aggregate profits of integrating parties exhibit increasing returns to network size. It is due to the nature of Shapley bargaining, where firms cooperate to exploit all the efficiencies regardless of merger decisions. In case of mergers, mergers reduce the total value of merging parties because all players share equally the grand coalition outcome in a fair-share consideration of the Shapley value. On the other hand, mergers will lift up the merged parties' influence, and hence increase their shares in a bargaining-power consideration of the Shapley value. The costs of mergers due to the fair-share consideration become more prominent than the benefits of mergers due to the bargaining-power consideration when there exist increasing returns to network size. This is because the fair-share consideration is based on the grand coalition outcome, while the bargaining-power consideration is based on partial coalition outcomes.

When the value of a network grows with the square of the number of users, as Metcalfe's Law states, the network industry exhibits increasing returns to network size. Moreover, I showed that the industry structure of the cable industry is well fit into the model, and that the cable industry has the feature of increasing returns. These facts imply that firms tend to separate rather than merge as far as Shapley bargaining is concerned. Therefore, if we observe mergers in these network industries with increasing returns, we may infer that there must be merger benefits due to the enhancement of efficiency or monopoly power other than the bargaining advantage considered in this work.

Admittedly, the current work has several limitations. First, there are other important determinants of the boundary of the firm, e.g., economies of scale or scope, monopoly power, and relation-specific investment. The paper, abstracting from all the other determinants, confines narrowly to bargaining benefits or costs as a determinant of the limit of the firm. The first line of extension is to incorporate other elements into the model. Second, the Shapley value is one of many bargaining solution concepts even though it has been given many justifications. Nonetheless, it is worthwhile to check whether the results in the paper are robust to other bargaining solution concepts, either cooperative or noncooperative. Third, the cable industry was taken as an example for the network industry with essential facilities. Many other industries, such as the Internet, telecommunications, gas, and electricity, have the features that are assumed in the model. Attempts to extend this work to capture other interesting elements in these industries also await future research.

Appendix

Proof of Theorem 1. We can expand (4), (5), and (7) as follows:

$$
\begin{aligned}
Sh_0 = {} & \frac{(K-1)!}{(K+1)!}\pi(n_1) + \sum_{i\in I\setminus\{0,\,1\}} \frac{(K-1)!}{(K+1)!}\pi(n_i) \\
& + \sum_{i\in I\setminus\{0,\,1\}} \frac{2!(K-2)!}{(K+1)!}\pi(n_1+n_i) + \sum_{\substack{i_1<i_2 \\ \{i_1,\,i_2\}\subseteq I\setminus\{0,\,1\}}} \frac{2!(K-2)!}{(K+1)!}\pi(n_{i_1}+n_{i_2}) \\
& + \ldots\ldots. \\
& + \sum_{\substack{i_1<\ldots<i_{k-1} \\ \{i_1,\,\ldots,\,i_{k-1}\}\subseteq I\setminus\{0,\,1\}}} \frac{k!(K-k)!}{(K+1)!}\pi(n_1+n_{i_1}+\ldots+n_{i_{k-1}}) \\
& + \sum_{\substack{i_1<\ldots<i_k \\ \{i_1,\,\ldots,\,i_k\}\subseteq I\setminus\{0,\,1\}}} \frac{k!(K-k)!}{(K+1)!}\pi(n_{i_1}+\ldots.+n_{i_k}) \\
& + \ldots\ldots\ldots \\
& + \sum_{\substack{i_1<\ldots<i_k \\ \{i_1,\,\ldots,\,i_k\}\subseteq I\setminus\{0,\,1\}}} \frac{(K-k)!k!}{(K+1)!}\pi(N-n_{i_1}-\ldots.-n_{i_k}) \\
& + \sum_{\substack{i_1<\ldots<i_{k-1} \\ \{i_1,\,\ldots,\,i_{k-1}\}\subseteq I\setminus\{0,\,1\}}} \frac{(K-k)!k!}{(K+1)!}\pi(N-n_1-n_{i_1}-\ldots.-n_{i_{k-1}}) \\
& + \ldots\ldots\ldots\ldots \\
& + \sum_{i\in I\setminus\{0,\,1\}} \frac{(K-1)!}{(K+1)!}\pi(N-n_i) + \frac{(K-1)!}{(K+1)!}\pi(N-n_1) \\
& + \frac{K!}{(K+1)!}\pi(N)
\end{aligned} \tag{16}
$$

$$
\begin{aligned}
Sh_1 = {} & \frac{(K-1)!}{(K+1)!}\pi(n_1) \\
& + \sum_{i\in I\setminus\{0,\,1\}} \frac{2!(K-2)!}{(K+1)!}\{\pi(n_1+n_i)-\pi(n_i)\} \\
& + \ldots\ldots\ldots\ldots
\end{aligned}
$$

$$+\sum_{\substack{i_1<\dots<i_k\\ \{i_1,\dots,i_k\}\subseteq I\setminus\{0,1\}}}\frac{(K+1)!(K-k-1)!}{(K+1)!}\{\pi(n_1+n_{i_1}+\dots+n_{i_k})-\pi(n_{i_1}+\dots+n_{i_k})\}$$

$$+\dots\dots\dots\dots$$

$$+\sum_{\substack{i_1<\dots<i_k\\ \{i_1,\dots,i_k\}\subseteq I\setminus\{0,1\}}}\frac{(K-k)!k!}{(K+1)!}\{\pi(N-n_{i_1}-\dots-n_{i_k})-\pi(N-n_1-n_{i_1}-\dots-n_{i_k})\}$$

$$+\dots\dots\dots\dots.$$

$$+\sum_{i\in I\setminus\{0,1\}}\frac{(K-1)!}{(K+1)!}\{\pi(N-n_i)-\pi(N-n_1-n_i)\}$$

$$+\frac{K!}{(K+1)!}\{\pi(N)-\pi(N-n_1)\} \tag{17}$$

$$Sh_{0+1}=\frac{(K-1)!}{K!}\pi(n_1)$$

$$+\sum_{i\in I\setminus\{0,1\}}\frac{(K-2)!}{K!}\pi(n_1+n_i)$$

$$+\dots\dots.$$

$$+\sum_{\substack{i_1<\dots<i_k\\ \{i_1,\dots,i_k\}\subseteq I\setminus\{0,1\}}}\frac{k!(K-k-1)!}{K!}\pi(n_1+n_{i_1}+\dots+n_{i_k})$$

$$+\dots\dots\dots$$

$$+\sum_{\substack{i_1<\dots<i_k\\ \{i_1,\dots,i_k\}\subseteq I\setminus\{0,1\}}}\frac{(K-k-1)!k!}{K!}\pi(N-n_{i_1}-\dots-n_{i_k})$$

$$+\dots\dots\dots.$$

$$+\sum_{i\in I\setminus\{0,1\}}\frac{(K-2)!}{K!}\pi(N-n_i)$$

$$+\frac{(K-1)!}{K!}\pi(N) \tag{18}$$

Denote $I\setminus\{0, 1\}$ $(=\{2, 3, \dots, \mathrm{K}\})\equiv I'$. From (16), (17) and (18), we have:

$$(Sh_0+Sh_1)-Sh_{0+1}=\left\{2\,\frac{(K-1)!}{(K+1)!}-\frac{(K-1)!}{K!}\right\}\pi(n_1) \tag{19.1}$$

$$+\left\{2\,\frac{2!(K-2)!}{(K+1)!}-\frac{(K-2)!}{K!}\right\}\sum_{i\in I'}\pi(n_1+n_i) \tag{19.2}$$

$$+\ldots\ldots\ldots\ldots\ldots$$

$$+\left\{2\frac{k!(K-k)!}{(K+1)!}-\frac{(k-1)!(K-k)!}{K!}\right\}\sum_{\substack{i_1<\ldots<i_{k-1}\\ \{i_1,\ldots,i_{k-1}\}\subseteq I'}}\pi(n_1+n_{i_1}+..+n_{i_{k-1}}) \tag{19.3}$$

$$+\ldots\ldots\ldots\ldots\ldots$$

$$+\left\{2\frac{(K-k+1)!(k-1)!}{(K+1)!}-\frac{(K-k)!(k-1)!}{K!}\right\}\sum_{\substack{i_1<\ldots<i_{k-1}\\ \{i_1,\ldots,i_{k-1}\}\subseteq I'}}\pi(N-n_{i_1}-\ldots-n_{i_{k-1}}) \tag{19.4}$$

$$+\ldots\ldots\ldots\ldots\ldots$$

$$+\left\{2\frac{(K-1)!}{(K+1)!}-\frac{(K-2)!}{K!}\right\}\sum_{i\in I'}\pi(N-n_i) \tag{19.5}$$

$$+\left\{\frac{(K-1)!}{(K+1)!}-\frac{2!(K-2)!}{(K+1)!}\right\}\sum_{i\in I'}\pi(n_i) \tag{19.6}$$

$$+\ldots\ldots\ldots\ldots\ldots$$

$$+\left\{\frac{(k-1)!(K-k+1)!}{(K+1)!}-\frac{k!(K-k)!}{(K+1)!}\right\}\sum_{\substack{i_1<\ldots<i_{k-1}\\ \{i_1,\ldots,i_{k-1}\}\subseteq I'}}\pi(n_{i_1}+\ldots.+n_{i_{k-1}}) \tag{19.7}$$

$$+\ldots\ldots\ldots\ldots\ldots$$

$$+\left\{\frac{(K-k)!k!}{(K+1)!}-\frac{(K-k+1)!(k-1)!}{(K+1)!}\right\}\sum_{\substack{i_1<\ldots<i_{k-1}\\ \{i_1,\ldots,i_{k-1}\}\subseteq I'}}\pi(N-n_1-n_{i_1}-\ldots.-n_{i_{k-1}}) \tag{19.8}$$

$$+\ldots\ldots\ldots\ldots$$

$$+\left\{\frac{(K-2)!2!}{(K+1)!}-\frac{(K-1)!}{(K+1)!}\right\}\sum_{i\in I'}\pi(N-n_1-n_i) \tag{19.9}$$

$$+\left\{\frac{(K-1)!}{(K+1)!}-\frac{K!}{(K+1)!}\right\}\pi(N-n_1) \tag{19.10}$$

$$+\left\{2\frac{K!}{(K+1)!}-\frac{(K-1)!}{K!}\right\}\pi(N) \tag{19.11}$$

Notice that (19.4) is just a reformulation of (19.3). In order to avoid duplication, $k<K-k+1$, i.e., $k<\dfrac{K+1}{2}$. The same holds true for (19.7) and (19.8)

We can check that the coefficients of (19.1) and (19.10) are equivalent. Similarly, notice that the coefficients of (19.2) and (19.9), those of (19.3) and (19.8), those of (19.4) and (19.7), and those of (19.5) and (19.6) are equivalent pair-wise. To confirm the equivalence in general terms, consider two sequences that are composed of terms in equation (19) — the first sequence of (19.1) through (19.5) and the second sequence of (19.6) through (19.10); each one is composed of (K-1) elements. The k-th term from the top of the first sequence is (19.3), while the k-th term from the bottom of the second sequence is (19.8). The coefficients of (19.3) and (19.8) are the same as $-(K+1-2k)\,\dfrac{(K-k)!(k-1)!}{(K+1)!}$. On the other hand, the $(k-1)$-th term from the bottom of the first sequence is (19.4), while the $(k-1)$-th term from the top of the second sequence is (19.7). Incidentally, the coefficients of (19.4) and (19.7) are the same as $(K+1-2k)\,\dfrac{(K-k)!(k-1)!}{(K+1)!}$.

Hence, equation (19) can be rearranged into:

$$Sh_0+Sh_1-Sh_{0+1}=-(K-1)\,\frac{(K-1)!}{(K+1)!}\{\pi(n_1)+\pi(N-n_1)\} \tag{20.1}$$

$$-(K-3)\,\frac{(K-2)!}{(K+1)!}\sum_{i\in I'}\{\pi(n_1+n_i)+\pi(N-n_1-n_i)\} \tag{20.2}$$

$$+\ldots\ldots\ldots\ldots\ldots$$

$$-(K+1-2k)\,\frac{(K-k)!(k-1)!}{(K+1)!}\sum_{\substack{i_1<\ldots<i_{k-1}\\ \{i_1,\,\ldots,\,i_{k-1}\}\subseteq I'}}\{\pi(n_1+n_{i_1}+..+n_{i_{k-1}})+\pi(N-n_1-n_{i_1}-..-n_{i_{k-1}})\} \tag{20.3}$$

$$+\ldots\ldots\ldots\ldots\ldots$$

$$+(K+1-2k)\,\frac{(k-1)!(K-k)!}{(K+1)!}\sum_{\substack{i_1<\ldots<i_{k-1}\\ \{i_1,\,\ldots,\,i_{k-1}\}\subseteq I'}}\{\pi(n_{i_1}+..+n_{i_{k-1}})+\pi(N-n_{i_1}-..-n_{i_{k-1}})\} \tag{20.4}$$

$$+\ldots\ldots\ldots\ldots\ldots$$

$$+(K-3)\,\frac{(K-2)!}{(K+1)!}\sum_{i\in I'}\{\pi(n_i)+\pi(N-n_i)\} \tag{20.5}$$

$$+(K-1)\,\frac{(K-1)!}{(K+1)!}\,\pi(N). \tag{20.6}$$

Notice again, in oder to avoid duplication, $k<K-k+1$, i.e., $k<\dfrac{K+1}{2}$ in (20.3) and (20.4). The last term of (20.6) can be expressed as $(K-1)\,\dfrac{(K-1)!}{(K+1)!}\{\pi(0)+\pi(N)\}$ since $\pi(0)=0$.

Finally, we can reexpress (20) compactly as:

$$Sh_0+Sh_1-Sh_{0+1}=$$

$$\sum_{1\le k<\frac{K+1}{2}}\ \sum_{\substack{i_1<\ldots<i_{k-1}\\ \{i_1,\,\ldots,\,i_{k-1}\}\subseteq I'}}(K+1-2k)\,\frac{(k-1)!(K-k)!}{(K+1)!}\,[\{\pi(n_{i_1}+..+n_{i_{k-1}})+\pi(N-n_{i_1}-..-n_{i_{k-1}})\}$$

$$-\{\pi(n_1+n_{i_1}+..+n_{i_{k-1}})+\pi(N-n_1-n_{i_1}-..-n_{k-1})\}] \tag{21}$$

Now we can show that if Assumption 1 holds true,

$$\{\pi(n_{i_1}+..+n_{i_{k-1}})+\pi(N-n_{i_1}-..-n_{i_{k-1}})\}$$
$$-\{\pi(n_1+n_{i_1}+..+n_{i_{k-1}})+\pi(N-n_1-n_{i_1}-..-n_{i_{k-1}})\}\lesseqgtr 0 \text{ if } \pi''(n)\lesseqgtr 0 \tag{22}$$

for any $k<\dfrac{K+1}{2}$ and for any $(i_1, \ldots, i_{k-1})$ such that $i_1<\ldots<i_{k-1}$ and $\{i_1, \ldots, i_{k-1}\}\subseteq I'$, and with strict inequalities for at least one k's. Denote $n_{i_1}+..+n_{i_{k-1}}\equiv M$. Then, we have:

$$\int_0^{N-2M-n_1}\int_0^{n_1}\pi''(M+x+y)dxdy$$
$$=\{\pi(N-M)-\pi(n_1+M)\}-\{\pi(N-n_1-M)-\pi(M)\}$$
$$=\{\pi(M)+\pi(N-M)\}-\{\pi(n_1+M)+\pi(N-n_1-M)\}. \tag{23}$$

Since the number of members in $S=\{1, 2, \ldots, K\}\backslash\{i_1, \ldots, i_{k-1}\}$ is $s=K-k+1$ and that in $S'=\{1, i_1, \ldots, i_{k-1}\}$ is $s'=k$, we know $s>s'$ for any $k<\dfrac{K+1}{2}$. Assumption 1 implies: $\sum_{i\in S} n_i=N-M\geq\sum_{i\in S'} n_i=n_1+M$, i.e., $N-2M-n_i\geq 0$.[10] Therefore, from expression (23), we know that (22) holds true for $k<\dfrac{K+1}{2}$. Moreover, it holds true with strict inequality for $k=1$. It is because $M=0$ and $N-2M-n_1>0$ when $k=1$. Therefore, $Sh_0+Sh_1-Sh_{0+1}$ in (21) is greater (less) than 0 when $\pi(n)$ is convex (concave). This proves Theorem 1. Q.E.D.

Proof of Theorem 2. For $K=2$, (21) is reduced to:

$$Sh_0+Sh_1-Sh_{0+1}=\frac{1}{6}\,[\pi(n_1+n_2)-\{\pi(n_1)+\pi(n_2)\}]$$
$$=\frac{1}{6}\int_0^{n_2}\int_0^{n_1}\pi''(x+y)dxdy.$$

For $K=3$, it is:

$$Sh_0+Sh_1-Sh_{0+1}=\frac{1}{6}\,[\pi(n_1+n_2+n_3)-\{\pi(n_1)+\pi(n_2+n_3)\}]$$
$$=\frac{1}{6}\int_0^{n_2+n_3}\int_0^{n_1}\pi''(x+y)dxdy.$$

This proves Theorem 2. Q.E.D.

Proof of Theorem 3. We can expand (5), (6), and (8) as follows:

$$Sh_1=\frac{(K-1)!}{(K+1)!}\pi(n_1)$$
$$+\frac{2!(K-2)!}{(K+1)!}\{\pi(n_1+n_2)-\pi(n_2)\}+\sum_{i\in I\backslash\{0,\,1,\,2\}}\frac{2!(K-2)!}{(K+1)!}\{\pi(n_1+n_i)-\pi(n_i)\}$$

[10] Since n_1 is cancelled out in $N-2M-n_1$, the proof holds true for any n_1. That is, as far as the size of n_1 is concerned, Assumption 1 should not be restrictive. However, Theorem 1 is not just for merging between 0 and 1. Hence, I state Assumption 1 in general terms.

$$+\,\ldots\ldots\ldots$$

$$+\sum_{\substack{i_1<\ldots<i_{k-1}\\ \{i_1,\ldots,i_{k-1}\}\subseteq I\setminus\{0,1,2\}}}\frac{(k+1)!(K-k-1)!}{(K+1)!}\{\pi(n_1+n_2+n_{i_1}+..+n_{i_{k-1}})-\pi(n_2+n_{i_1}+..+n_{i_{k-1}})\}$$

$$+\sum_{\substack{i_1<\ldots<i_k\\ \{i_1,\ldots,i_k\}\subseteq I\setminus\{0,1,2\}}}\frac{(k+1)!(K-k-1)!}{(K+1)!}\{\pi(n_1+n_{i_1}+\ldots.+n_{i_k})-\pi(n_{i_1}+\ldots.+n_{i_k})\}$$

$$+\,\ldots\ldots\ldots$$

$$+\sum_{i\in I\setminus\{0,1,2\}}\frac{(K-1)!}{(K+1)!}\{\pi(N-n_i)-\pi(N-n_1-n_i)\}$$

$$+\frac{(K-1)!}{(K+1)!}\{\pi(N-n_2)-\pi(N-n_1-n_2)\}$$

$$+\frac{K!}{(K+1)!}\{\pi(N)-\pi(N-n_1)\} \tag{24}$$

$$Sh_2=\frac{(K-1)!}{(K+1)!}\pi(n_2)$$

$$+\frac{2!(K-2)!}{(K+1)!}\{\pi(n_1+n_2)-\pi(n_1)\}+\sum_{i\in I\setminus\{0,1,2\}}\frac{2!(K-2)!}{(K+1)!}\{\pi(n_2+n_i)-\pi(n_i)\}$$

$$+\,\ldots\ldots\ldots$$

$$+\sum_{\substack{i_1<\ldots<i_{k-1}\\ \{i_1,\ldots,i_{k-1}\}\subseteq I\setminus\{0,1,2\}}}\frac{(k+1)!(K-k-1)!}{(K+1)!}\{\pi(n_1+n_2+n_{i_1}+..+n_{i_{k-1}})-\pi(n_1+n_{i_1}+..+n_{i_{k-1}})\}$$

$$+\sum_{\substack{i_1<\ldots<i_k\\ \{i_1,\ldots,i_k\}\subseteq I\setminus\{0,1,2\}}}\frac{(k+1)!(K-k-1)!}{(K+1)!}\{\pi(n_2+n_{i_1}+\ldots.+n_{i_k})-\pi(n_{i_1}+\ldots.+n_{i_k})\}$$

$$+\,\ldots\ldots\ldots$$

$$+\sum_{i\in I\setminus\{0,1,2\}}\frac{(K-1)!}{(K+1)!}\{\pi(N-n_i)-\pi(N-n_2-n_i)\}$$

$$+\frac{(K-1)!}{(K+1)!}\{\pi(N-n_1)-\pi(N-n_1-n_2)\}$$

$$+\frac{K!}{(K+1)!}\{\pi(N)-\pi(N-n_2)\} \tag{25}$$

$$Sh_{1+2}=\frac{(K-2)!}{K!}\pi(n_1+n_2)$$

$$+\sum_{i\in I\setminus\{0,1,2\}}\frac{2!(K-3)!}{K!}\{\pi(n_1+n_2+n_i)-\pi(n_i)\}$$

$$+\ldots\ldots\ldots\ldots$$

$$+\sum_{\substack{i_1<\ldots<i_k \\ \{i_1,\ldots,i_k\}\subseteq I\backslash\{0,1\}}} \frac{(k+1)!(K-k-1)!}{(K+1)!}\{\pi(n_1+n_2+n_{i_1}+\ldots.+n_{i_k})-\pi(n_{i_1}+\ldots.+n_{i_k})\}$$

$$+\ldots\ldots\ldots\ldots$$

$$+\sum_{i\in I\backslash\{0,1,2\}} \frac{(K-2)!}{K!}\{\pi(N-n_i)-\pi(N-n_1-n_2-n_i)\}$$

$$+\frac{(K-1)!}{K!}\{\pi(N)-\pi(N-n_1-n_2)\} \tag{26}$$

Denote $I\backslash\{0, 1, 2\}$ $(=\{3, \ldots, K\})\equiv I''$. Then, from (24), (25) and (26), we have:

$$(Sh_1+Sh_2)-Sh_{1+2}=\left\{2\frac{2!(K-2)!}{(K+1)!}-\frac{(K-2)!}{K!}\right\}\pi(n_1+n_2) \tag{27.1}$$

$$+\left\{\frac{(K-1)!}{(K+1)!}-\frac{2!(K-2)!}{(K+1)!}\right\}\{\pi(n_1)+(n_2)\} \tag{27.2}$$

$$+\left\{2\frac{3!(K-3)!}{(K+1)!}-\frac{3!(K-3)!}{K!}\right\}\sum_{i\in I''}\pi(n_1+n_2+n_i) \tag{27.3}$$

$$+\left\{\frac{2!(K-3)!}{K!}-2\frac{2!(K-2)!}{(K+1)!}\right\}\sum_{i\in I''}\pi(n_i) \tag{27.4}$$

$$+\left\{\frac{2!(K-2)!}{(K+1)!}-\frac{2!(K-3)!}{(K+1)!}\right\}\sum_{i\in I''}\{\pi(n_1+n_i)+\pi(n_2+n_i)\} \tag{27.5}$$

$$+\ldots\ldots\ldots\ldots\ldots$$

$$+\left\{2\frac{(k+2)!(K-k-2)!}{(K+1)!}-\frac{(k+1)!(K-k-2)!}{K!}\right\}\sum_{\substack{i_1<\ldots<i_k \\ \{i_1,\ldots,i_k\}\subseteq I''}}\pi(n_1+n_2+n_{i_1}+..+n_{i_k}) \tag{27.6}$$

$$+\left\{\frac{(k+1)!(K-k-2)!}{K!}-2\frac{(k+1)!(K-k-1)!}{(K+1)!}\right\}\sum_{\substack{i_1<\ldots<i_k \\ \{i_1,\ldots,i_k\}\subseteq I''}}\pi(n_{i_1}+..+n_{i_k}) \tag{27.7}$$

$$+\left\{\frac{(k+1)!(K-k-1)!}{(K+1)!}-\frac{(k+2)!(K-k-2)!}{(K+1)!}\right\}\sum_{\substack{i_1<\ldots<i_k \\ \{i_1,\ldots,i_k\}\subseteq I''}}\{\pi(n_1+n_{i_1}+..+n_{i_k})$$

$$+\pi(n_2+n_{i_1}+..+n_{i_k})\} \tag{27.8}$$

$$+\ldots\ldots\ldots\ldots\ldots$$

$$+\left\{2\frac{K!}{(K+1)!}-\frac{(K-1)!}{K!}\right\}\pi(N) \tag{27.9}$$

$$+\left\{\frac{(K-1)!}{K!}-2\frac{(K-1)!}{(K+1)!}\right\}\pi(N-n_1-n_2) \tag{27.10}$$

$$+\left\{\frac{(K-1)!}{(K+1)!}-\frac{K!}{(K+1)!}\right\}\{\pi(N-n_2)+\pi(N-n_1)\} \tag{27.11}$$

Incidentally, the coefficients of (27.1) and (27.2) are the same in terms of absolute value, and their signs are opposite. Moreover, the coefficients of (27.3), (27.4) and (27.5) are the same in terms of absolute value, and the signs of (27.3) and (27.4) are the opposite of (27.5). We can state similarly for (27.9), (27.10) and (27.11). We can check the conformity of coefficients with the general terms of (27.6), (27.8), and (28.9). The coeffients of (27.6) and (27.8) are the same as $-(K-2k-3)\dfrac{(k+1)!(K-k-2)!}{(K+1)!}$, while the coefficient of (27.9) is $(K-2k-3)$ $\dfrac{(k+1)!(K-k-2)!}{(K+1)!}$

Now define:

$$\theta_k(k)\equiv(K-2k-3)\frac{(k+1)!(K-k-2)!}{(K+1)!} \tag{28}$$

Then, we can express equation (27) with:

$$\begin{aligned}(Sh_1+Sh_2)-Sh_{1+2}=\sum_{k=0}^{K-2}\Bigg[-\theta_K(k)\sum_{\substack{i_1<\ldots<i_k\\ \{i_1,\ldots,i_k\}\subseteq I''}}&\{\pi(n_1+n_2+n_{i_1}+..+n_{i_k})+\pi(n_{i_1}+..+n_{i_k})\\ &-\pi(n_1+n_{i_1}+..+n_{i_k})-\pi(n_2+n_{i_1}+..+n_{i_k})\}\Bigg]\end{aligned} \tag{29}$$

Notice a symmetric relationship among coefficients $\theta_K(\cdot)$'s in (29), i.e.,

$$\theta_K(K-k-3)=-\theta_K(k)\ \ \forall k,\ 0\leq k<\frac{K-3}{2}. \tag{30}$$

Defineing

$$\begin{aligned}\Psi_K(k)\equiv\sum_{\substack{i_1<\ldots<i_{K-k-3}\\ \{i_1,\ldots,i_{K-k-3}\}\subseteq I''}}&\{\pi(n_1+n_2+n_{i_1}+..+n_{i_{K-k-3}})+\pi(n_{i_1}+..+n_{i_{K-k-3}})\\ &-\pi(n_1+n_{i_1}+..+n_{i_{K-k-3}})-\pi(n_2+n_{i_1}+..+n_{i_{K-k-3}})\}\\ -\sum_{\substack{j_1<\ldots<j_k\\ \{j_1,\ldots,j_k\}\subseteq I''}}&\{\pi(n_1+n_2+n_{j_1}+..+n_{j_k})+\pi(n_{j_1}+..+n_{j_k})\\ &-\pi(n_1+n_{j_1}+..+n_{j_k})-\pi(n_2+n_{j_1}+..+n_{j_k})\},\end{aligned} \tag{31}$$

and using the symmetric relationship of (30), we can reexpress (29) as:

$$\begin{aligned}(Sh_1+Sh_2)-Sh_{1+2}=&\sum_{0\leq k<\frac{K-3}{2}}\theta_K(k)\Psi_K(k)\\ &+\frac{K-1}{(K+1)K}\{\pi(n_1+n_2+n_3+\ldots+n_K)+\pi(n_3+\ldots+n_K)\end{aligned}$$

$$-\pi(n_1+n_3+\ldots+n_K)-\pi(n_2+n_3+\ldots+n_K)\} \tag{32}$$

where the summation in the above covers $k=1$ through $k=K-3$, and the last term is for $k=K-2$ in (29).

We know that the last term in (32) is positive (negative) if $\pi''(n)>0$ (<0) because

$$\int_0^{n_2}\int_0^{n_1}\pi''(x+y+n_3+\ldots+n_K)dxdy=\{\pi(n_1+n_2+n_3+\ldots+n_K)+\pi(n_3+\ldots+n_K)$$
$$-\pi(n_1+n_3+\ldots+n_K)-\pi(n_2+n_3+\ldots+n_K)\}.$$

Now, notice that if $k<\dfrac{K-3}{2}$, then $K-k-3>k$ and ${}_{K-2}C_{K-k-3}>{}_{K-2}C_k$. From these facts, we know that for any $\{j_1, \ldots., j_k\}\subseteq I''$ such that $j_1<\ldots.<j_k$, there exist at least one $\{i_1, \ldots., i_{K-k-3}\}\subseteq I''$ such that $i_1<\ldots.<i_{K-k-3}$ and $\{j_1, \ldots., j_k\}\subseteq\{i_1, \ldots., i_{K-k-3}\}$. This implies that $\Psi_K(k)$ in (31) is composed of two kinds of terms. The first is such as:

$$\phi_1=\{\pi(n_1+n_2+S)+\pi(S)-\pi(n_1+S)-\pi(n_2+S)\}$$
$$-\{\pi(n_1+n_2+T)+\pi(T)-\pi(n_1+T)-\pi(n_2+T)\}$$

where

$$S=n_{i_1}+\ldots\ldots\ldots+n_{i_{K-k-3}}$$
$$T=n_{j_1}+\ldots\ldots\ldots+n_{j_k}(<S).$$

The number of this kind of terms in $\Psi_K(k)$ of (31) is ${}_{K-2}C_k$. On the other hand, the second kind of terms is such as:

$$\phi_2=\{\pi(n_1+n_2+S)+\pi(S)-\pi(n_1+S)-\pi(n_2+S)\}$$

where

$$S=n_{i_1}+\ldots\ldots\ldots+n_{i_{K-k-3}}.$$

The number of this kind of terms in $\Psi_K(k)$ of (31) is ${}_{K-2}C_{K-k-3}-{}_{K-2}C_K$. Notice that we can transform ϕ_1 and ϕ_2 as follows:

$$\phi_1=\int_0^{n_1}\int_0^{n_2}\int_T^{S}\pi'''(x+y+z)dzdydx;$$

$$\phi_2=\int_0^{n_1}\int_0^{n_2}\pi''(x+y+S)dydx.$$

Hence, we know that $\phi_1\leqq 0$ if $\pi'''(n)\leqq 0$, and that $\phi_2\leqq 0$ if $\pi''(n)\leqq 0$. Therefore, for any $k<\dfrac{K-3}{2}$, $\Psi_K(k)\leqq 0$ if $\pi''(n)\leqq 0$ and $\pi'''(n)\leqq 0$. This proves Theorem 3. Q.E.D.

Proof of Theorem 4. For $K=2$, (32) is reduced to:

$$Sh_1+Sh_2-Sh_{1+2}=\frac{1}{6}\,[\pi(n_1+n_2)-\{\pi(n_1)+\pi(n_2)\}]$$

$$=\frac{1}{6}\int_0^{n_2}\int_0^{n_1}\pi''(x+y)dxdy.$$

For $K=3$, it is:

$$Sh_1+Sh_2-Sh_{1+2}=\frac{1}{6}\left[\{\pi(n_1+n_2+n_3)+\pi(n_3)\}-\{\pi(n_1+n_3)+\pi(n_2+n_3)\}\right]$$

$$=\frac{1}{6}\int_0^{n_2}\int_0^{n_1}\pi''(n_3+x+y)dxdy.$$

This proves Theorem 4. Q.E.D.

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