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HUMAN CAPITAL ACCUMULATION AND ENDOGENOUS GROWTH IN A DUAL ECONOMY*

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Abstract

This paper develops an endogenous growth model of a dual economy where human capital accumulation is the source of economic growth. The dualism between the rich sector and the poor sector exists in the mechanism of human capital accumulation. Individuals in the rich sector (region) allocate labour time not only for their own production and knowledge accumulation but also to train the individuals in the poor sector (region). External effects of human capital are considered not only in the production technology in the rich sector but also in the production technology and in the human capital accumulation in the poor sector. The model helps us to derive some important properties of the steady state growth path of a competitive household economy as well as that of a command economy. Steady-state growth equilibrium in the competitive economy may not be socially inefficient.

Keywords: Human Capital, Dualism, Economic growth, Rural, Urban, Competitive Equilibrium, Steady-state growth, Planned Economy.
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I. *Introduction*

With the emergence of the 'new' growth theory, human capital accumulation and its role on economic growth has become a major area of research in macroeconomics. The literature starts with the seminal paper of Lucas (1988) which shows that the growth rate of per capita income depends on the growth rate of human capital which again depends on the time allocation of the individuals for acquiring skill. Since then many eminent economists have dealt with the issue of human capital accumulation and endogenous growth.[1]

However, these endogenous growth models do not provide appropriate framework for analysing the problems of growth of less developed countries. Less developed economies are often characterized by the existence of opulence and poverty side by side. Rich individuals stay in contrast with the poor individuals who consume whatever they earn and thus do not have anything to save and invest to build up physical and human capital. This co-existence of the rich and the poor individuals leads to dualism in the less developed countries.

There exists a substantial theoretical literature dealing with the dualism and income inequalities in less developed countries.[2] However, none of the existing models focuses on the dualism in the mechanism of human capital formation of two different classes of people. In a less developed economy, the stock of human capital of the poor individuals is far lower than that of the rich individuals. Also there exists a difference in the mechanism of human capital accumulation of the rich and the poor individuals. On the one hand, there are rich families who can afford to spend a lot of time and resources for schooling of their children. On the other hand, there are poor families who have neither time nor resources to spend for education of their children. The opportunity cost of schooling of their children is very high because they can be alternatively employed as child labour. However they receive support from exogenous sources. Government sets up free public schools and introduces various schemes of paying book grants and scholarships to the meritorious students coming from the poor families. The rich individuals who are the owners of firms or industries open NGO s or give donations to them. These NGOs provide free or subsidized educational service to the poor. Government meets the cost of public education programme through taxes imposed on the rich individuals. So the efficiency enhancement mechanism for wealthier individuals and poor individuals are different. While the rich individuals can build up their human capital on their own, the poor individuals need the support of exogenous sources in accumulating their human capital.

There are substantial evidences that private individuals and firms provide voluntary services to education. Corporate giants like The Coca-Cola Company, American Express, General Electric Company, Bank of America, Nokia Corporation, Chevron Texaco Corporation are members of CECP (Committee to Encourage Corporate Philanthropy) and are providing various services including education to the underprivileged communities of both developing and developed countries. Timberland Co. reports that 95% of its employees have in total contributed some 300,000 service hours in 13 countries. 'Make a Connection' program

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[1] Some of these works are Rebelo(1991), Rebelo and Stokey (1995), Chamley (1993), Jones et. al. (1993) and of Alonso-Carrera and Freire Seren(2004), Greiner (1996), Mauro and Carmeci (2003)etc.

[2] This includes the works of e.g Lewis (1954), Ranis and Fei (1961), Jorgenson (1961), Sen (1966), Dixit (1969), Todaro (1969), Benabou (1994, 1996a, 1996b), Eicher, Penalosa (2001), Galor, Zeira (1993), Glomm, Ravikumar (1992); Gradstein, Justman (1996); Leach (1996); Nordblom (2003).

undertaken by Nokia is active in 19 countries including countries of South Africa and Latin America. This program focuses on developing non academic skills like co-operation, communication skills, conflict management etc.[3] Menchik and Weisbrod (1987) report that, according to a recent survey, over 80 million adults in the US volunteered 8.4 billion hours of labour to organization in 1980. Other estimates of the number of volunteer workers, relying on non-survey methods, place volunteer labour as high as 8 percent of the labour force. In India, Titan, Broadcom, Infosys Foundation, Asea Brown Boveri, Siemens Ltd, Yahoo.com are among the many corporates who are fulfilling part of their social responsibilities by linking up with Akshaya Patra Foundation, a Bangalore based non profit organisation, that provides mid day meals to the unprivileged children in the schools in and around Bangalore. ABB India has identified education as a key area for social and community development activities and helping the teachers of a govt. school of a village close to Peenya, Bangalore, to make their lessons more meaningful and effective.[4] Confederation of Indian Industries (CII) has initiated a programme in various parts of India under which training is imparted to the unskilled workers; and a certificate recognising the skill acquired by the worker is given. These are pure private sector initiatives.

In the present paper, we develop a growth model of an economy in which human capital accumulation is viewed as the source of economic growth and in which difference exists in the mechanism of human capital accumulation of the two types of individuals — the rich and the poor. The distinction between these two is made in terms of the difference in their initial human capital endowment. The poor individuals lag behind the rich individuals in terms of their initial knowledge and they need outside assistance to be educated. The rich individuals not only allocate their labour time between production and their own skill accumulation but also allocate a part of their labour time to the training of the poor people.[5] We have assumed the presence of external effect on production as well as on the human capital accumulation of the poor individuals.[6] We consider a two sector growth model of a dual economy. The rich sector (region) is similar to the one sector economy described in Lucas (1988). However the representative individual in the rich sector not only allocates its labour time between production and his (her) own skill accumulation but also allocates a part of the labour time in training the people in the poor sector. The individuals in the poor sector work not only in the poor sector but also in the rich sector. In the competitive economy, the labour time allocation among the different sectors is achieved through solving dynamic optimization problem by the agents. However, in the planned economy this allocation is directly controlled by the planner's dynamic optimization exercise. We also consider external effect of human capital accumulation in the production function of both the sectors and in the human capital accumulation function

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[3] Source: Various newsletters published by CECP

[4] Source: Various issues of Business India

[5] This voluntary allocation of labour time to the training of the poor individual can not be supported in a world where the contribution comes mainly in the form of tax payment. However, we have mentioned evidences of voluntary contributions too.

[6] There exists a large theoretical literature in both urban economics and macroeconomics that has considered external effects emanating from human capital in explaining growth of cities, religions and countries e.g. Glaeser and Mare (1994), Glaeser (1997), Peri (2002), Ciccone and Peri (2002). In some other literature, it is found that education generates very little externalities. e.g Rudd (2000), Acemoglu, Angrist (2000). Moretti (2003) rightly points out that the empirical literature on the subject is still very young and more work is needed before we can draw convincing conclusions about the size of human capital externalities.

of the poor sector; and thereby analyse the role of externality in the sector on the properties of the long-run growth rate of the economy.

We derive some important results from this model. First, externality parameters of both the sectors play an important role in determining the long run rate of growth of the different macro economic variables. Secondly, the rate of growth of human capital in the rich sector in the competitive economy is always less than that in the command economy if there is no externality of rich sector's human capital on the human capital accumulation in the poor sector. However, in the presence of that externality, we may get the opposite result. In Lucas (1988) rate of growth in the competitive economy is always less than that in the planned economy because Lucas (1988) and other extensions of this model do not consider human capital accumulation in the poor sector. Thirdly, if there is no externality in either sector, rates of growth are same in both the systems and are equal to that obtained in Lucas (1988) model. Lastly, the external effect of the poor sector's human capital accumulation is important only if there is external effect of the rich sector's human capital. If this externality comes from the human capital in the poor sector only and not from the human capital in the rich sector, then the steady-state rate of growth in the planned economy is higher than that obtained in the competitive equilibrium.

This paper is organized as follows. Section II discusses the assumptions of the model with specified focus on the nature of the dualism. In section III we present the steady state growth rates of the macroeconomic variables in the household (competitive) economy; and in section IV we do the same for the planned (command) economy. In section V, we consider an extension of the basic model introducing accumulation of the physical capital in the poor region too. Concluding remarks are made in section VI.

II. *The Dual Economy Model*

We consider a closed economy with two sectors — a rich sector and a poor sector. In both the regions (sectors) same and single commodity is produced. By human capital we mean the set of specialized skills or efficiency level of the workers that they can acquire by devoting time to an activity called schooling. This skill level (human capital stock) of the representative worker in either region accumulates over time. There are external effects of human capital on the production technology in both the regions and on the human capital accumulation function in the poor region. Total number of workers in each region is normalised to unity. All the individuals in a region are assumed to be identical. There is full employment of labour and capital and the factor markets are competitive.

1. Dualism in the Production Technology and Organization

Rich region undertakes the capitalist mode of production. Workers of the poor region are employed as wage labourers in the rich region. Physical capital is an essential input in producing commodity there and the individuals invest a part of their income to augment the stock of physical capital. The individuals (workers) from the rich region and the poor region are treated as two imperfectly substitute factors of production in the rich sector. A person of the rich region allocates 'a' fraction of the total non-leisure time in the production sector in that region. Labour originating from the poor region is perfectly mobile between the two regions. The representative worker from the poor region allocates 'u' fraction of his non-leisure time to work in the poor region, v fraction of time to acquire education and the remaining fraction to work in the rich region. Let H_R and H_P be the skill type of the representative individual (worker) of the rich region and the poor region respectively.

The production function in the rich region takes the form:

$$Y_R = (aH_R)^{\alpha}\{(1-u-v)H_P\}^{\beta} K^{1-\alpha-\beta} \bar{H}_R^{\epsilon_R} \bar{H}_P^{\epsilon_P} \tag{1}$$

where $0<\alpha<1$, $0<\beta<1$, $0<\alpha+\beta<1$, $0<\mathrm{a}<1$, $0\leq u<1$. Here $\epsilon_R>0$ and $\epsilon_P>0$ are the parameters representing the magnitude of the external effect of H_R and H_P on the production technology in the rich region. $\bar{H}_R$ and $\bar{H}_P$ are the average level of human capital of these two types of individuals from which the external effects come.[7] K is the stock of physical capital. Production function satisfies CRS in terms of the private inputs while it is subject to social IRS.

On the other hand there is family farming in the poor region and labour expressed in terms of human capital is the only input there.[8] Total product produced in the poor region is equally divided among the workers employed. The production function of the poor region is given by the following:

$$Y_P = (uH_P)\bar{H}_R^{\eta_R} \bar{H}_P^{\eta_P} \tag{2}$$

This also satisfies CRS at the private level and IRS at the social level. η_R and $\eta_P>0$ are the parameters representing the magnitude of the external effect of H_R and H_P respectively on the production technology in the poor region.

βY_R is the wage payment to the workers from the poor region employed in the rich region. So $(1-\beta)Y_R$ is the income of the individuals in the rich region. A part of $(1-\beta)Y_R$ is consumed and the other part is saved (invested). So the budget constraint of the household of the rich region is given by

$$\dot{K} = (1-\beta)Y_R - C_R \tag{3}$$

where C_R is the level of consumption of the representative household in the rich region. It is assumed that there is no depreciation of physical capital. The individual of the poor region consumes whatever he earns; and this assumption is borrowed from Lewis (1954). They earn the competitive wage share of rich region income (βY_R) and the entire income from the production in the poor region, Y_P. Hence, we have

$$Y_P + \beta Y_R = C_P \tag{4}$$

where C_P is the level of consumption of the representative worker in the poor region. However, the representative household (worker) in the rich sector allocates income between savings and consumption maximizing his discounted present value of utility over the infinite time horizon. The household of both the regions have instantaneous utility function given by

$$U(C_i) = \frac{C_i^{1-\sigma}}{1-\sigma},\ \sigma>0 \tag{5}$$

[7] We consider aggregate external effects, not sector specific external effects.

[8] It is a simplifying assumption. In next section, we introduce capital.

Here σ is the elasticity of marginal utility of consumption; and $i=R, P$.

2. Dualism in the Mechanism of Human Capital Accumulation

Mechanism of the human capital accumulation in the rich sector is assumed to be similar to that in Lucas (1988). The relative rate of human capital formation varies proportionately with the time or effort devoted to acquire skill. Hence

$$\dot{H}_R = mbH_R \tag{6}$$

where b is the fraction of the non-leisure time devoted to acquiring own skill. Here $0 \leq b \leq 1$. m is a positive constant, representing the productivity parameter of the human capital accumulation technology.

However, the mechanism of human capital formation in the two regions are different. The skill formation of a poor person takes place through a training programme conducted by the individuals in the rich region. The poors need outside assistance provided by the rich because the poors lag behind the rich individuals in terms of initial human capital endowment and the knowledge accumulation technology in such a way that the knowledge needs to trickle down from the more knowledgeable persons to the inferiors.[9] Each individual in the rich region spends $(1-a-b)$ fraction of its time in this training. The individuals of the rich region have incentive to train the workers of the poor region because they work as labourers in the rich sector.[10] Every worker in the poor region devotes v fraction of time for acquiring skill. We assume that there exists a positive external effect of the average skill level of the rich and of the poor individuals on the human capital accumulation in the poor region. Hence we have

$$\dot{H}_P = m_P\{(1-a-b)H_R\}^{\delta}(vH_P)^{1-\delta-\gamma}\bar{H}_R^{\mu\gamma}\bar{H}_P^{(1-\mu)\gamma} \tag{7}$$

Here $0<\delta<1$, $0<\mu<1$ and $\gamma>0$. Here γ is the parameter representing the magnitude of the external effect on the skill formation in the poor region and $m_P>0$ is the efficiency parameter of the education technology of the poor individuals. The human capital accumulation function of poor individuals follow DRS at private level and CRS at social level.

In the models of Tamura (1991), Eaton and Eckstein (1997), Lucas (2004) etc. the human capital accumulation technology is subject to external effects. In the models of Eaton and Eckstein (1997) and Tamura (1991) the average human capital is affecting human capital accumulation technology externally where as in the model of Lucas (2004) human capital of the leader throws the external effect on the human capital accumulation of all other individuals. Leader is the person with the highest skill level. In our model, the rich individuals have already attained high level of human capital and the poor individuals are lagging behind. The

[9] In reality, poors need assistance of the riches also due to credit market imperfection. This is not applicable here because the process of human capital accumulation does not require non labour input.

[10] This story is valid when the process of human capital accumulation refers to internal training provided by the employing firm. In the case of formal schooling, each rich individual may deviate unilaterally from contributing to educational services. However, this is not true in a situation where some kind of Folk Theorem holds. For example, all the rich individuals may co-operate among themselves and may come to an agreement that each of them would employ equal number of educated poor workers. In that case, equal distribution of benefit provided by formal schooling is ensured for the rich individuals. All the rich individuals are identical in terms of their preference, capital endowment, production technology and skill. Similarly all the poor individuals are identical in terms of skill. So equal allocation is the optimum allocation in this case.

rich individuals and the poor individuals are assumed to be identical among themselves. So it is justified that the human capital stock of the representative rich individual should have external effect on the poor individual's human capital accumulation technology; and it should not be the other way round.

We assume that the rich provides labour time to educate the poor and does not provide output or capital. Marginal productivity of labour of the rich individual is always positive in this model and so the sacrifice of labour time indirectly implies a sacrifice of income. In reality contributions are generally made in terms of non labour resources. Our objective is to reanalyse the results of Lucas (1988) model and so we follow the framework of Lucas (1988) which also solves a labour time allocation problem between production and education. It should also be noted that in many adult education programmes organized in India, teachers and organizers donate labour time and these are more important than monetary contributions.

III. *Growth in the Household Economy*

1. The Optimization Problem of the Rich

The objective of the representative individual of the rich region is to maximize the discounted present value of utility over the infinite time horizon given by:

$$J_H = \int_0^{\infty} U(C_R) e^{-\rho t} dt$$

This is to be maximized with respect to C_R, a and b subject to the equations of motion given by

$$\dot{K} = (1-\beta) Y_R - C_R;$$
$$\dot{H}_R = mbH_R;$$

and

$$\dot{H}_P = m_P \{(1-a-b) H_R\}^{\delta} (vH_P)^{1-\delta-\gamma} \bar{H}_R^{\mu\gamma} \bar{H}_P^{(1-\mu)\gamma}$$

and given the initial values of K, H_R and H_P. Here $U(C_R)$ is given by equation (5) and Y_R is given by equation (1). Here ρ is the positive discount parameter. The control variables are C_R, a and b, where $0 \le C_R < \infty$, $0 \le a \le 1$, $0 \le b \le 1$ and $0 \le a+b \le 1$. The state variables are K, H_R and H_P. The household can not internalise the external effects. If $a+b=1$, this optimization problem is identical to that in Lucas (1988).

2. The Optimization Problem of the Poor

The representative poor individual maximizes the objective functional given by:

$$J_{H_P} = \int_0^{\infty} U(C_P) e^{-\rho t} dt$$

with respect to the control variables u and v subject to the equation of motion given by

$$\dot{H}_P = m_P\{(1-a-b)H_R\}^{\delta}(vH_P)^{1-\delta-\gamma}\bar{H}_R^{\mu\gamma}\bar{H}_P^{(1-\mu)\gamma}$$

and given the initial values of H_R and H_P. Here H_P is the state variable and u and v are the control variables satisfying $0 \le u \le 1$, $0 \le v \le 1$, and $0 \le u+v \le 1$. Here C_P is given by equation (4) and Y_R and Y_P are given by equation (1) and (2). The household can not internalise the external effects.

3. Steady State Growth Path

Now, we analyze the steady state growth properties of the system. Along the steady state growth path (SGP), C_R, K, Y_R, H_R, H_P, Y_P grow at constant rates; and a, b and u are time independent. At this stage we assume the existence of the Steady State Growth Path (SGP). It can be shown that the movement along the steady state growth path is optimal because it satisfies the transversality conditions. The rates of growth of the major macroeconomic variables can be derived[11] as follows:

$$\frac{\dot{H}_P}{H_P} = \frac{\dot{H}_R}{H_R} = mb, \tag{8}$$

$$\frac{\dot{Y}_R}{Y_R} = \frac{\dot{C}_R}{C_R} = \frac{\dot{K}}{K} = \frac{(\alpha+\beta+\epsilon_P+\epsilon_R)}{(\alpha+\beta)}mb, \tag{9}$$

and

$$\frac{\dot{Y}_P}{Y_P} = (1+\eta_R+\eta_P)mb; \tag{10}$$

where

$$mb = \frac{m-\rho}{1-\dfrac{(1-\sigma)(\alpha+\beta+\epsilon_P+\epsilon_R)}{(\alpha+\beta)}}; \tag{11}$$

and

$$a = \frac{\alpha(1-b)\left[\dfrac{m}{mb}-(1-\delta-\gamma)\right]}{\delta\beta+\alpha\left[\dfrac{m}{mb}-(1-\delta-\gamma)\right]}. \tag{12}$$

From the above equation we find that $a=(1-b)$ when $\beta=0$. This implies that if the workers from the poor region are not required as input in the rich sector's production technology then the household of the rich region would not allocate any time to educate an individual in the poor region.

Here,

[11] The derivation in detail is given in Appendix (A).

$$v=\frac{(m-\rho)(1-\delta-\gamma)}{m\left\{1-\frac{(1-\sigma)(\alpha+\beta+\epsilon_P+\epsilon_R)}{(\alpha+\beta)}\right\}} \tag{13}$$

As the magnitude of external effect on the human capital accumulation of the poor individuals (γ) increases v falls. If $\sigma(<)>1$, v is (positively) negatively related with ϵ_P and ϵ_R.

Note that, if there is no externality, i.e., $\epsilon_R=\epsilon_P=\eta_R=\eta_P=\gamma=0$, then we have

$$mb=\frac{m-\rho}{\sigma}$$

In this case, consumption, income and human capital of both the regions and physical capital of the rich region grow at the common rate mb. This is also the growth rate obtained in Lucas (1988) model in the absence of external effect. We need to assume $m>\rho$ because b can not take non positive value.

If $\sigma=1$ i.e. if $U(C_R)=log_eC_R$, then we have

$$\frac{\dot{H}_R}{H_R}=mb=m-\rho$$

even if all the externality parameters take positive values. This is also the rate of human capital accumulation in Lucas (1988) with $\sigma=1$. However, all other macro-economic variables like K, C_P, C_R, Y_P, Y_R do not necessarily grow at this rate when $\sigma=1$ and when externalities exist.

In this case, the rate of human capital accumulation in the rich sector is independent of the degrees of various types of externalities. However, the common balanced growth rate of other macro-economic variables as shown by the equation (9) varies positively with the degree of externality in the production and/or in the human capital accumulation function in the poor (poor) region. Similarly equation (10) shows that the rate of growth of output in the poor sector varies positively with the degree of externality in the production technology of the poor sector. In Lucas (1988) there is no poor sector and hence the effect of externalities in the poor sector can not be analyzed there.

IV. *Command Economy*

In a command economy the social planner maximizes the discounted present value of the instantaneous social welfare function over the infinite time horizon. The instantaneous social welfare is assumed to be a positive function of the level of consumption of the representative individual in the rich region as well as of that in the poor region. This function is defined as

$$W=\frac{(C_R{}^{\theta}C_P{}^{1-\theta})^{1-\sigma}}{1-\sigma},\ 0\leq\theta\leq 1 \tag{14}$$

where θ and $(1-\theta)$ are the weights given to consumption of the representative individual in the rich region and in the poor region respectively. If $\theta=1(0)$, it is same as the utility function of the representative individual in the rich (poor) region which we have considered in section III.

1. The Optimisation Problem

The objective of the social planner is to maximize

$$J_P = \int_0^{\infty} W e^{-\rho t} dt$$

with respect to C_R, C_P, u, v, a and b subject to the constraints

$$\dot{K} = Y_R + Y_P - C_R - C_P,$$

$$\dot{H}_R = mbH_R,$$

and

$$\dot{H}_P = m_P\{(1-a-b)H_R\}^{\delta}(vH_P)^{1-\delta-\gamma} H_R^{\mu\gamma} H_P^{(1-\mu)\gamma}$$

Here Y_R and Y_P are given by equations (1) and (2);and W is given by (14). Here the control variables are C_R, C_P, a, b, u and v. The social planner can internalise the externalities what the household in the competitive economy can not do.

2. Steady State Growth Path

We define the steady state growth path following the same style adopted in section III. Along the SGP, the rates of growth of the major macroeconomic variables are derived[12] as follows:

$$\frac{\dot{C}_R}{C_R} = \frac{\dot{C}_P}{C_P} = \frac{\dot{Y}_R}{Y_R} = \frac{\dot{Y}_P}{Y_P} = \frac{\dot{K}}{K} = \frac{(\alpha+\beta+\epsilon_R+\epsilon_P)}{(\alpha+\beta)} mb^* \tag{15}$$

and

$$\frac{\dot{H}_R}{H_R} = \frac{\dot{H}_P}{H_P} = mb^* \tag{16}$$

where

$$mb^* = \frac{(m-\rho) + m\left[a^*\left\{\frac{\epsilon_R}{\alpha} + \frac{\beta\eta_R u}{\alpha(1-u-v)}\right\} + \frac{\mu\gamma}{\delta}(1-a^*)\right]}{\left[1 + \frac{\mu\gamma}{\delta} - \frac{(1-\sigma)(\alpha+\beta+\epsilon_R+\epsilon_P)}{(\alpha+\beta)}\right]} \tag{17}$$

and b^* and a^* are the optimum values of b and a in the command economy.

3. Planned Economy Vs Household Economy

The presence of externality creates divergence between the socially optimum growth rate in the command economy and the equilibrium growth rate in the household economy. If there is no externality then

[12] The derivtion in detail is given in the appendix (B).

$$mb^* = mb = \frac{(m-\rho)}{\sigma}$$

So the growth rate in the competitive economy is socially efficient in the absence of external effects. This result is similar to that obtained in Lucas (1988).

Comparing equations (11) and (17) we find that $(mb^* - mb)$ may take any sign. However if $\gamma = 0$, then

$$mb^* = \frac{(m-\rho) + ma^*\left\{\frac{\epsilon_R}{\alpha} + \frac{\beta\eta_R u}{\alpha(1-u-v)}\right\}}{\left[1 - \frac{(1-\sigma)(\alpha+\beta+\epsilon_R+\epsilon_P)}{(\alpha+\beta)}\right]}$$

From the equation (11) we find that mb is independent of γ. So, $mb^* > mb$ if $\gamma = 0$ and if either ϵ_R or η_R is positive.

Setting $\sigma = 1$ from the equation (11) and (17) we have,

$$mb = m - \rho;$$

and

$$mb^* = \frac{(m-\rho) + m\left[a^*\left\{\frac{\epsilon_R}{\alpha} + \frac{\beta\eta_R u}{\alpha(1-u-v)}\right\} + \frac{\mu\gamma}{\delta}(1-a^*)\right]}{\left(\frac{\mu\gamma}{\delta} + 1\right)}$$

Hence, with $\sigma = 1$, we have

$$mb^* - mb = \frac{\left[ma^*\left\{\frac{\epsilon_R}{\alpha} + \frac{\beta\eta_R u}{\alpha(1-u-v)} - \frac{\mu\gamma}{\delta}\right\} + \frac{\rho\mu\gamma}{\delta}\right]}{\left(\frac{\mu\gamma}{\delta} + 1\right)}$$

The above term may be positive or may be negative. If there does not exist any kind of external effect i.e. if $\epsilon_R = \eta_R = \gamma = 0$ then $mb^* = mb$. If $\gamma = 0$ but η_R or η_P is positive and if we have an interior solution such that $\frac{u}{(1-u-v)} > 0$ then $mb^* > mb$. It is negative if the following condition is satisfied:

$$a^* > \frac{\rho\mu\gamma}{m\delta\left\{\frac{\mu\gamma}{\delta} - \frac{\epsilon_R}{\alpha} - \frac{\beta\eta_R u}{\alpha(1-u-v)}\right\}} = \bar{a}$$

$\frac{\epsilon_R}{\alpha} > \frac{\mu\gamma}{\delta}$ is sufficient condition for mb^* to be greater than mb. If $\gamma = 0$ this condition is always satisfied.

This leads to the following proposition.

Proposition 1 *Suppose that* $\sigma = 1$. *(i) If* $\gamma = 0$ *then* $mb^* > mb$ *provided either* ϵ_R *or* η_R *or both are* *positive; (ii)* (mb^*-mb) *may take any sign with* $\mu>0$, $\gamma>0$; *and* $mb>mb^*$ *if* $a^*>\bar{a}$

So the socially efficient rate of growth of the rich sector's human capital is always higher than its competitive equilibrium growth rate if there is no externality in the human capital accumulation in the poor sector. This is the generalization of the result of Lucas (1988) model. Lucas (1988) has also shown that competitive equilibrium growth rate of human capital falls short of the socially efficient rate. However, his result was proved in the one sector (region) model with externality in the production function. The present paper shows that Lucas (1988) result is valid even in a dual economy with production externality in the rich sector as well as that in the poor sector provided that there is no externality on the human capital accumulation in the poor sector.

However, if there is externality on the human capital accumulation in the poor sector, then the result may be reversed. In the presence of positive externality on the human capital accumulation in the poor sector, the time allocation of the rich individual to the training of the poor region workers is higher in a command economy than that in the household economy because the command economy can internalise the externality. So the time allocated to acquiring his own skill of the individual in the rich region may be lower in the command economy than that in the competitive economy. So the socially optimum growth rate of human capital may be lower than its competitive equilibrium growth rate in this case.

If $\mu=0$, then from equation (17) we have

$$mb^*=\frac{(m-\rho)+m\left[a^*\left\{\frac{\epsilon}{\alpha}+\frac{\beta\eta_R u}{\alpha(1-u-v)}\right\}\right]}{\left[1-\frac{(1-\sigma)(\alpha+\beta+\epsilon_R+\epsilon_P)}{(\alpha+\beta)}\right]}$$

and comparing with mb given by the equation (11) we find that $mb^*>mb$ in this case. Here $\mu=0$ implies that there is no external effect of H_R on the accumulation of H_P. So we have the following proposition.

Proposition 2 *If* $\mu=0$ *then* $mb^*>mb$.

If the human capital of the rich sector does not create any externality on the human capital accumulation in the poor sector and the entire external effect comes from the human capital of the poor sector, then the rate of growth of the human capital in the household economy is less than that in the command economy. Here $\bar{H}_R^{\mu\gamma}\bar{H}_P^{(1-\mu)\gamma}$ represents the total external effect on the human capital accumulation in the poor sector. $\bar{H}_R^{\mu\gamma}$ is the external effect of teaching and $\bar{H}_P^{(1-\mu)\gamma}$ represents the external effect of learning. It is the external effect of teaching which matters in this case. $\mu=0$ implies the absence of the externalities of teaching.

V. *Capital Formation in the Rural Sector*

1. Household Economy

We now consider capital formation in the poor sector too which takes place through investment of the poor sector individuals. The representative individual in the poor sector maximizes $\int_0^\infty e^{-\rho t} U(C_P)dt$ subject to the equations of motion given by (7) and

$$\dot{K}_P = Y_P + \beta Y_R - C_P \tag{18}$$

where $U(C_P)$ is given by equation (4) and K_P represent the level of capital stock in the poor sector. Here C_P is the control variable and K_P and H_P are the state variables. Capital stock in the poor sector now enters as an input in the production function of that sector which is given by

$$Y_P = A_P(uH_P)^\phi K_P^{1-\phi} \bar{H}_R^{\eta_R} \bar{H}_P^{\eta_P} \tag{19}$$

The optimization problem of the representative household in the rich region remains same as in section III.1. Following the same style adopted in the earlier section we derive[13] the steady state rates of growth of the different macro economic variables. Here

$$\frac{\dot{Y}_P}{Y_P} = \frac{\dot{C}_P}{C_P} = \frac{\dot{K}_P}{K_P} = \frac{(\phi + \eta_P + \eta_R)}{\phi} mb \tag{20}$$

$$\frac{\dot{Y}_R}{Y_R} = \frac{\dot{C}_R}{C_R} = \frac{\dot{K}_R}{K_R} = \frac{(\alpha + \beta + \epsilon_P + \epsilon_R)}{(\alpha + \beta)} mb \tag{21}$$

where

$$mb = \frac{m - \rho}{1 - \dfrac{(1-\sigma)(\alpha + \beta + \epsilon_P + \epsilon_R)}{(\alpha + \beta)}} \tag{22}$$

Note that mb is independent of $(1-\phi)$ which represents the capital elasticity of output in the poor sector. This expression of mb is same as that given in the equation (11).

2. Command Economy

The social planner solves the same problem analysed in section IV. However, now the planner controls the capital allocation between the two sectors in addition to controlling the labour allocation and consumption-investment allocation. The optimization problem to be solved is given by the following: Maximize

$$J_P = \int_0^\infty We^{-\rho t} dt$$

subject to the constraints

$$\dot{K} = Y_P + Y_R - C_P - C_R,$$
$$\dot{H}_R = mbH_R,$$

and

$$\dot{H}_P = m_P\{(1-a-b)H_R\}^\delta vH_P^{1-\delta-\gamma} \bar{H}_R^{\mu\gamma} \bar{H}_P^{(1-\mu)\gamma}$$

[13] Derivation in detail is given in the Appendix (C).

with respect to the control variables which are a, b, C_R, C_P, u, v and x. Here

$$Y_R = A_R(aH_R)^{\alpha}\{(1-u-v)H_P\}^{\beta}\{xK\}^{1-\alpha-\beta}H_R{}^{\epsilon_R}H_P{}^{\epsilon_P} \tag{23}$$

and

$$Y_P = A_P(uH_P)^{\phi}((1-x)K)^{1-\phi}H_R{}^{\eta_R}H_P{}^{\eta_P} \tag{24}$$

Here x is the additional control variable with the property $0 \leq x \leq 1$. v represents the fraction of physical capital allocated to the rich sector (region).

In the steady-state growth equilibrium we can derive[14] the rates of growth of the macro economic variables as follows:

$$\frac{\dot{H}_P}{H_P} = \frac{\dot{H}_R}{H_R} = mb^* \tag{25}$$

$$\frac{\dot{Y}_R}{Y_R} = \frac{\dot{Y}_P}{Y_P} = \frac{\dot{K}}{K} = \frac{\dot{C}_R}{C_R} = \frac{\dot{C}_P}{C_P} = [\alpha+\beta+\epsilon_R+\epsilon_P]\,\frac{mb^*}{(\alpha+\beta)} \tag{26}$$

and

$$mb^* = \frac{(m-\rho)+m\left[a^*\left\{\frac{\epsilon_R}{\alpha}+\frac{\eta_R u\beta}{\alpha\phi(1-u-v)}\right\}+\frac{\mu\gamma}{\delta}(1-a^*)\right]}{\frac{\mu\gamma}{\delta}+\sigma-\frac{(\epsilon_R+\epsilon_P)}{(\alpha+\beta)}} \tag{27}$$

If we compare equations (17) and (27)we find that they are identical when $\phi=1$. Also equation (27) clearly shows that mb^* varies negatively with ϕ. However, in section V.1, we have found that mb is independent of ϕ. This leads to the following proposition.

Proposition 3 *The socially efficient rate of growth of human capital varies positively with the capital elasticity of output in the poor sector while the competitive equilibrium rate of growth is independent of that.*

Its explanation lies in the assumption of the model. In the household economy, entire surplus originating from a sector is invested to that sector itself; and there is no intersectoral capital mobility. So capital accumulation in the poor sector does not affect the labour-time allocation problem of the rich individual. However, in the planned economy, the planner allocates the total capital stock between the two sectors. Investment of the surplus of any sector is not sector specific. Planner controls total investment which is the sum of surplus originating from both the sectors.

Comparing equations (11) and (27) we find that $mb^* > mb$ when $\gamma=\mu=0$; and mb may be greater than mb^* when $\gamma>0$ and $\mu>0$. So the central points of the results summarized in propositions 1 and 2 remain unchanged in this extended model.

[14] Derivation in detail is given in the Appendix (D).

VI. *Conclusion*

Existing endogenous growth models have not considered dualism and the old dual economy models did not consider the aspect of human capital accumulation and endogenous growth. This paper tries to bridge the gap. In this paper we have analyzed the model of a dual economy in which growth stems from human capital accumulation and the dualism exists in the nature of human capital accumulation between the two sectors. Like Lucas (1988) we analyze the steady state growth properties of the model and put special emphasis on the role of externalities. Since we consider a dual economy we consider not only the role of externality on the rich sector's production but also its role on the production as well as on the human capital accumulation in the poor sector.

We have derived some interesting results from this model. First, externality parameters in the poor sector appear to be important determinants of the long run rate of growth of the different macro economic variables. Secondly the rate of growth of human capital in the competitive economy is always less than that in the command economy if there is no externality in the human capital accumulation in the poor sector. However, in the presence of that externality, we may get the opposite result. Competitive equilibrium growth rate may exceed the growth rate in a planned economy. Lucas (1988) and its extended models did not find this possibility; and so this is an important result. Lastly, if there is no externality in either sector, rates of growth are same in both the systems. In a command economy, the planner has the power of allocating poor region workers between the two sectors and of maximizing an welfare function which takes care of consumption of the people of both the sectors. However, this power does not help the planner to achieve a higher rate of growth than that obtained in the competitive equilibrium in the absence of externalities. Also the external effect on the poor sector's human capital accumulation is important only if this external effect comes from rich sector's human capital. If externalities come from the human capital in the poor sector only and not from the human capital in the rich sector, then the rate of growth in the planned economy exceeds that obtained in the competitive equilibrium.

These results have important implications in the context of educational subsidy policies. Lucas (1988) advocates for educational subsidy policy because the competitive equilibrium rate of growth of human capital in the Lucas (1988) model falls short of its socially efficient rate of growth. However this is not necessarily true in the present model when the rich gives training to the poor and the human capital accumulation of the poor people is subject to the external effects. So this model may question the necessity of subsidizing the higher education sector which generally benefits the rich and not the poor.

The model is highly abstract and fails to consider many important features of reality. Both the sectors produce the same commodity is a restrictive assumption. If the two sectors produce two different commodities and if there is competitive exchange then, in a closed economy model, terms of trade will be another endogenous variable. The present model does not consider the problems of marketable surplus of the rural sector, rural urban migration, unemployment etc. which the old dual economy models have dealt with. The role of various agrarian institutions like tenancy, money-lending etc. and the role of urban informal sector activities are also not analysed in this model. The modelling of the rich sector shares all the limitations common to Lucas(1988). The external effect is aggregative in nature where all the workers employed in various sector produce identical external effects. Sector specific external effect is not considered here. The human capital accumulation sector does not use physical capital as an input. Our purpose is to focus on the dualism in the human capital accumulation. In order to keep the analysis otherwise simple, we do all kinds of abstraction—a standard practice often followed in the theoretical literature.

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APPENDIX A

The optimality conditions of the dynamic optimization problem solved by rich household
(A) The first order conditions necessary for this optimization problem with respect to the control variables C_R, a, b are given by the following:

$$C_R^{-\sigma} - \lambda_K{}^R = 0; \tag{A.1}$$

$$\lambda_K{}^R \alpha(1-\beta)\frac{Y_R}{a} - \lambda_P{}^R \delta \frac{\dot{H}_P}{(1-a-b)} = 0; \tag{A.2}$$

and

$$\lambda_H{}^R mH_R - \lambda_P{}^R \delta \frac{\dot{H}_P}{(1-a-b)} = 0. \tag{A.3}$$

(B)Time derivatives of the co-state variables satisfying the optimum growth path are given by the following:

$$\dot{\lambda}_K{}^R = \rho\lambda_K{}^R - \lambda_K{}^R(1-\alpha-\beta)(1-\beta)\frac{Y_R}{K}; \tag{A.4}$$

$$\dot{\lambda}_H{}^R = \rho\lambda_H{}^R - \lambda_K{}^R \alpha(1-\beta)\frac{Y_R}{H_R} - \lambda_P{}^R \delta \frac{\dot{H}_P}{H_R} - \lambda_H{}^R mb; \tag{A.5}$$

and

$$\dot{\lambda}_P{}^R = \rho\lambda_P{}^R - \lambda_K{}^R \beta(1-\beta)\frac{Y_R}{H_P} - \lambda_P{}^R(1-\delta-\gamma)\frac{\dot{H}_P}{H_P} \tag{A.6}$$

The optimality conditions of the dynamic optimization problem solved by poor household
(A) The first order conditions necessary for this optimization problem with respect to the control variables u and v are given by the following:

$$(\beta Y_R + Y_P)^{-\sigma}\left[-\beta^2 \frac{Y_R}{(1-u-v)} + \frac{Y_P}{u}\right] = 0; \tag{A.7}$$

$$(\beta Y_R + Y_P)^{-\sigma}\left[-\beta^2 \frac{Y_R}{(1-u-v)}\right] + \lambda_H{}^P(1-\delta-\gamma)\frac{\dot{H}_P}{v} = 0 \tag{A.8}$$

(B) Time derivative of the co-state variable satisfying the optimum growth path are given by the following:

$$\dot{\lambda}_H{}^P = \rho\lambda_H{}^P - (\beta Y_R + Y_P)^{-\sigma}\left[\beta^2 \frac{Y_R}{H_P} + \frac{Y_P}{H_P}\right] - \lambda_H{}^P(1-\delta-\gamma)\frac{\dot{H}_P}{H_P}; \tag{A.9}$$

We define a new set of variables $z = \dfrac{H_R}{H_P}$ and $y = H_R{}^{\alpha+\beta+\epsilon_P+\epsilon_R} K^{-(\alpha+\beta)}$.

From equation (7) we find that the growth rate of the human capital of the poor region is given by

$$\frac{\dot{H}_P}{H_P} = m_P(1-a-b)^{\delta} H_R^{\delta+\mu\gamma} v^{1-\delta-\gamma} H_P^{-\delta-\gamma+(1-\mu)\gamma} \tag{A.10}$$

Since on SGP a, b, v and r are constant, the growth rate of H_P is given by

$$\frac{\dot{H}_P}{H_P} = \frac{\dot{H}_R}{H_R} = m_P(1-a-b)^{\delta} v^{1-\delta-\gamma} z^{\delta+\mu\gamma} = mb \tag{A.11}$$

Using equations (A.1) and (A.4) we have

$$\frac{\dot{\lambda}_K{}^R}{\lambda_K{}^R} = -\sigma \frac{\dot{C}_R}{C_R} = \rho - (1-\alpha-\beta)(1-\beta)\frac{Y_R}{K}. \tag{A.12}$$

Since $\frac{\dot{C}_R}{C_R}$ is constant along SGP $\frac{Y_R}{K}$ is also constant.

Using equations (3) and (A.12)we get the growth rate of physical capital stock given by

$$\frac{\dot{K}}{K} = (1-\beta)\frac{Y_R}{K} - \frac{C_R}{K},$$

Or,

$$\frac{\dot{K}}{K} = \frac{(\rho+\sigma\chi)}{(1-\alpha-\beta)} - \frac{C_R}{K}.$$

where $\chi = \frac{\dot{C}_R}{C_R}$

Since $\frac{\dot{K}}{K}$ and the first term in the RHS of the above equation are constant, $\frac{C_R}{K}$ is also constant.

Hence,

$$\frac{\dot{Y}_R}{Y_R} = \frac{\dot{C}_R}{C_R} = \frac{\dot{K}}{K} = \chi$$

Log differentiating both sides of equation (A.12) and using (A.11) we get the common rate at which the consumption of the rich region, physical capital and output of rich region would grow which is given by the equation (9).

From equation (A.7), in migration equilibrium,

$$\frac{\beta^2 Y_R}{(1-u-v)} = \frac{Y_P}{u} \tag{A.13}$$

From equation (A.13), we find that if Y_P grows at higher rate than Y_R then $(1-u-v)$ will tend to zero and if Y_R grows at higher rate than Y_P then u tends to zero. We get interior solution of u and $(1-u-v)$ if and only if the growth rate of Y_R and Y_P are equal. The condition for growth rate of Y_R and growth rate of Y_P to be equal is

$$(\eta_R+\eta_P)-\frac{(\epsilon_P+\epsilon_R)}{(\alpha+\beta)}=0 \tag{A.14}$$

So if equation (A.14) holds then u is constant and $0<u<1$ which means the incomplete specialization of labour of the poor region. This implies that in steady state growth equilibrium workers of the poor region work in both the sectors. From equations (A.2) and (A.3) we have,

$$\frac{\lambda_K{}^R}{\lambda_H{}^R}=\frac{mH_Ra}{\alpha(1-\beta)Y_R}$$

Log-differentiating the above equation and using equations (8), (9), (A.12) and (A.17) we have the solution of mb giben by the equation (11). From the equation (A.6) and using equations (A.2) we get

$$\frac{\dot{\lambda}_P{}^R}{\lambda_P{}^R}=\rho-\left[\frac{\delta\beta a}{\alpha(1-a-b)}+(1-\delta-\gamma)\right]\frac{\dot{H}_P}{H_P} \tag{A.15}$$

Differentiating the equation (A.3) with respect to t and using (8) we get

$$\frac{\dot{\lambda}_H{}^R}{\lambda_H{}^R}=\frac{\dot{\lambda}_P{}^R}{\lambda_P{}^R} \tag{A.16}$$

From the above equation the equation (12) follows.
From equations (A.5), (A.2) and (A.3) we have

$$\frac{\dot{\lambda}_H{}^R}{\lambda_H{}^R}=\rho-m. \tag{A.17}$$

From the above three equations (A.15), (A.16) and (A.17) and using equation (8) we can solve for a which is given by (12).

From equation (A.9) we have

$$\frac{\dot{\lambda}_H{}^P}{\lambda_H{}^P}=\rho-(1-\delta-\gamma)\left[\frac{(1-u-v)}{v\beta^2Y_R}(\beta^2Y_R+Y_P)+1\right]\frac{\dot{H}_P}{H_P}$$

Now using equation (A.13) we have

$$\frac{\dot{\lambda}_H{}^P}{\lambda_H{}^P}=\rho-(1-\delta-\gamma)\left[\frac{(1-v)}{v}+1\right]\frac{\dot{H}_P}{H_P}$$

Log-differentiating the equation (A.8) and using equations (9) and (8) we have

$$\frac{(1-\delta-\gamma)}{v}=\frac{\rho}{mb}-(1-\sigma)\frac{(\alpha+\beta+\epsilon_P+\epsilon_R)}{(\alpha+\beta)}+1 \tag{A.18}$$

Substituting mb from equation (11) in the above equation we have the solution of v which is given by the equation (13). From equation (A.13) and using equation (A.14) we have the expression of time allocated by poor individuals for the production of rich region

$$(1-u-v)^{\beta-1}=\frac{A_P}{\beta^2 A_R a^{\alpha}} z^{\beta+\epsilon_P-1-\eta_P} y^{\frac{(1-\alpha-\beta)}{(\alpha+\beta)}}$$

From the equations (A.12) using (9) we have,

$$y=\frac{\sigma z^{(\beta+\epsilon_P)}\left[\frac{(\alpha+\beta+\epsilon_P+\epsilon_R)}{(\alpha+\beta)} mb+\frac{\rho}{\sigma}\right]}{(1-\alpha-\beta)(1-\beta)A_R a^{\alpha}(1-u-v)^{\beta}}$$

where z can be derived from the condition that H_R and H_P grow at equal rate.

Appendix B

The optimality conditions derived from the dynamic optimization problem solved by the social planner

(A) The first order conditions of maximization with respect to C_R, C_P, a, b, u and v are as follows:

$$(C_R{}^{\theta}C_P{}^{1-\theta})^{-\sigma}\theta C_R{}^{\theta-1}C_P{}^{1-\theta}-\lambda_K=0; \tag{B.1}$$

$$(C_R{}^{\theta}C_P{}^{1-\theta})^{-\sigma}(1-\theta)C_R{}^{\theta}C_P{}^{-\theta}-\lambda_K=0; \tag{B.2}$$

$$\lambda_K\alpha\frac{Y_R}{a}-\lambda_P\delta\frac{\dot{H}_P}{(1-a-b)}=0; \tag{B.3}$$

$$\lambda_R mH_R-\lambda_P\delta\frac{\dot{H}_P}{(1-a-b)}=0; \tag{B.4}$$

$$\lambda_K\frac{Y_P}{u}-\lambda_K\beta\frac{Y_R}{(1-u-v)}=0. \tag{B.5}$$

and

$$-\lambda_K\frac{\beta Y_R}{(1-u-v)}+\lambda_P(1-\delta-\gamma)\frac{\dot{H}_P}{v}=0. \tag{B.6}$$

(B) Time derivative of the co-state variables which satisfy their time behaviour along the optimum growth path are given by the followings:

$$\dot{\lambda}_K=\rho\lambda_K-\lambda_K(1-\alpha-\beta)\frac{Y_R}{K}; \tag{B.7}$$

$$\dot{\lambda}_R=\rho\lambda_R-\lambda_K(\alpha+\epsilon_R)\frac{Y_R}{H_R}-\lambda_R mb-\lambda_P(\delta+\mu\gamma)\frac{\dot{H}_P}{H_R}-\lambda_K\eta_R\frac{Y_P}{H_R}; \tag{B.8}$$

and

$$\dot{\lambda}_P=\rho\lambda_P-\lambda_K(\beta+\epsilon_P)\frac{Y_R}{H_P}-\lambda_P\{(1-\delta-\gamma)+(1-\mu)\gamma\}\frac{\dot{H}_P}{H_P}-\lambda_K(1+\eta_P)\frac{Y_P}{H_P} \tag{B.9}$$

First we consider the case where $0<\theta<1$ From the equation (B.5)we have

$$\frac{(1-u-v)}{u}=\beta\frac{Y_R}{Y_P}. \tag{B.10}$$

Here also for $\frac{(1-u-v)}{u}$ to be constant Y_R and Y_P must grow at equal rate.

Since on SGP the growth rate of H_R, H_P, a, b and v are constant, the following equation holds true in this case also

$$\frac{\dot{H}_P}{H_P}=\frac{\dot{H}_R}{H_R}=m_P(1-a-b)^{\delta}v^{1-\delta-\gamma}z^{\delta+\mu\gamma}=mb \tag{B.11}$$

From equations (B.1) and (B.2) we have

$$\frac{C_R}{C_P}=\frac{\theta}{1-\theta} \tag{B.12}$$

We consider the case where $0<\theta<1$. Differentiating equation (B.1) with respect to time we have

$$\frac{\dot{\lambda}_K}{\lambda_K}=\{(1-\sigma)\theta-1\}\frac{\dot{C}_R}{C_R}+(1-\sigma)(1-\theta)\frac{\dot{C}_P}{C_P} \tag{B.13}$$

Equation (B.12) shows that $\frac{C_R}{C_P}$ is constant. Hence using equations (B.1), (B.2) and equation (B.7) we have,

$$\frac{\dot{\lambda}_K}{\lambda_K}=-\sigma\frac{\dot{C}_R}{C_R}=-\sigma\frac{\dot{C}_P}{C_P}=\rho-(1-\alpha-\beta)\frac{Y_R}{K} \tag{B.14}$$

Since the growth rate of C_R and C_P are constant along steady-state growth path, $\frac{Y_R}{K}$ is also constant. Now

$$\frac{\dot{K}}{K}=\frac{Y_R}{K}\left[1+\frac{Y_P}{Y_R}\right]-\frac{C_R}{K}\left[1+\frac{C_P}{C_R}\right]$$

Along the SGP $\frac{\dot{K}}{K}$, $\frac{Y_R}{K}$, $\frac{C_P}{C_R}$ are constants. We have assumed $\frac{Y_P}{Y_R}$ is constant. So $\frac{C_R}{K}$ must be constant. Hence along SGP

$$\frac{\dot{C}_R}{C_R}=\frac{\dot{C}_P}{C_P}=\frac{\dot{K}}{K}=\frac{\dot{Y}_R}{Y_R}=\frac{\dot{Y}_P}{Y_P}$$

Now using the above equation, equation (1) and equation (B.11) we get the growth rate as given by the equation (15).

From equation (B.8) we have

$$\frac{\dot{\lambda}_R}{\lambda_R}=\rho-\frac{ma(\alpha+\epsilon_R)}{\alpha}-mb-\frac{m(\delta+\mu\gamma)}{\delta}(1-a-b)-\frac{ma\eta_R Y_P}{\alpha Y_R} \tag{B.15}$$

From equations (B.3) and (B.4) we have

$$\frac{\lambda_K}{\lambda_R}=\frac{mH_R a}{\alpha Y_R}$$

Log-differentiating the above equation and using the equation (B.14), equation (B.15), equation (B.11) and equation (15) we get the expression for mb^* given by the equation (17).

From equation (B.3) and (B.6) we have

$$\frac{\delta a}{\alpha(1-a-b)}=\frac{(1-\delta-\gamma)(1-u-v)}{\beta v} \tag{B.16}$$

From equation (B.9) and using equation (B.3) we have

$$\frac{\dot{\lambda}_P}{\lambda_P}=\rho-\left[\frac{\delta a(\beta+\epsilon_P)}{\alpha(1-a-b)}+\frac{\delta a(1+\eta_P)Y_P}{(1-a-b)\alpha Y_R}+\{(1-\delta-\gamma)+(1-\mu)\gamma\}\right]\frac{\dot{H}_P}{H_P} \tag{B.17}$$

Differentiating (B.4) with respect to time t we have

$$\frac{\dot{\lambda}_R}{\lambda_R}=\frac{\dot{\lambda}_P}{\lambda_P} \tag{B.18}$$

This equation (B.18) is same as (A.16)

We now analyze how the optimum values of a, b and u are determined in the command economy. Using equations (B.17), (B.18), (B.15) and (B.16) we have

$$\left[\frac{(\beta+\epsilon_P)(1-\delta-\gamma)(1-u-v)}{\beta v}+\frac{(1-\delta-\gamma)(1+\eta_P)u}{v}+(1-\delta-\gamma)+(1-\mu)\gamma+\frac{\mu\gamma}{\delta}\right]mb$$
$$=ma\left[\frac{\epsilon_R}{\alpha}-\frac{\mu\gamma}{\delta}+\frac{\eta_R\beta u}{\alpha(1-u-v)}\right]+m\frac{(\delta+\mu\gamma)}{\delta}$$

From equation (17), b can be expressed in terms of a, u, v. Substituting that value of b in the above equation we get a in terms of u and v. Once a and b are determined in terms of u and v, z can be determined in terms of u and v by using the fact that H_R and H_P grow at equal rate.

Now using equations (B.14) and (B.7) we have

$$(1-\alpha-\beta)A_R a^{\alpha}(1-u-v)^{\beta}yz^{-(\beta+\epsilon_P)}=\rho+\sigma\frac{(\alpha+\beta+\epsilon_P+\epsilon_R)mb}{(\alpha+\beta)}.$$

From the above equation y can be determined in terms of u and v. Now from equations (B.16) and (B.10) u and v can be determined. Substituting Y_P and Y_R and using the notations we use equation (B.10) can be written as

$$(1-u-v)^{\beta-1}=\frac{A_P}{\beta A_R a^{\alpha}}z^{\beta+\epsilon_P-1-\eta_P}y^{\left(\frac{1-\alpha-\beta}{\alpha+\beta}\right)}$$

The above equation holds if Y_R and Y_P grow at equal rate and the condition for that is same as the equation given by (A.14).

Appendix C

The first order optimality conditions derived from the dynamic optimization problem solved by the poor household are given by the following:

$$C_P^{-\sigma}-\lambda_{K_P}=0; \tag{C.1}$$

$$\lambda_{H_P}(1-\delta-\gamma)\frac{\dot{H}_P}{v}-\lambda_{K_P}\beta^2\frac{Y_R}{(1-u-v)}=0 \tag{C.2}$$

$$\phi\frac{Y_P}{u}-\beta^2\frac{Y_R}{(1-u-v)}=0 \tag{C.3}$$

and

$$\dot{\lambda}_{K_P}=\rho\lambda_{K_P}-\lambda_{K_P}\frac{(1-\phi)Y_P}{K_P}; \tag{C.4}$$

$$\dot{\lambda}_{H_P}=\rho\lambda_{H_P}-\lambda_{H_P}(1-\delta-\gamma)\frac{\dot{H}_P}{H_P}-\lambda_{K_P}\phi\frac{Y_P}{H_P}-\lambda_{K_P}\beta^2\frac{Y_R}{H_P} \tag{C.5}$$

Since on SGP the growth rate of H_R, the growth rate of H_P, a, b and v are constant, the following equation holds true in this case also

$$\frac{\dot{H}_P}{H_P}=\frac{\dot{H}_R}{H_R}=m_P(1-a-b)^{\delta}v^{1-\delta-\gamma}z^{\delta+\mu\gamma}=mb \tag{C.6}$$

As the optimization problem of the representative household in the rich region remains unchanged, the optimality conditions given by equations (A.1)-(A.6) are also valid here. Hence the expressions for mb and the growth rate given by the equations (22) and (21) remain same. From the equation (C.3) migration equilibrium condition of the workers of the poor region is now given by

$$\frac{u}{1-u-v}=\frac{\phi Y_P}{\beta^2 Y_R} \tag{C.7}$$

So in order to obtain an interior solution for u and $(1-u-v)$, Y_P and Y_R must grow at equal rate. From equations (C.1) and (C.4) we have

$$\frac{\dot{\lambda}_K}{\lambda_K}=-\sigma\frac{\dot{C}_P}{C_P}=\rho-(1-\phi)\frac{Y_P}{K_P}$$

Since along SGP $\frac{\dot{C}_P}{C_P}$ is constant, $\frac{Y_P}{K_P}$ is also constant. Now

$$\frac{\dot{K}_P}{K_P}=\frac{Y_P}{K_P}\left[1+\beta\frac{Y_R}{Y_P}\right]-\frac{C_P}{K_P}$$

Since along SGP $\frac{\dot{K}_P}{K_P}$ and $\frac{Y_P}{K_P}$ are constant and we have assumed that $\frac{Y_R}{Y_P}$ is constant, so $\frac{C_P}{K_P}$ must be constant along SGP. Hence we have

$$\frac{\dot{C}_P}{C_P}=\frac{\dot{Y}_P}{Y_P}=\frac{\dot{K}_P}{K_P}$$

Using the above equation, equation (19) and equation (C.6) we get the common growth rate of Y_P, K_P and C_P given by the equation (20). Here the two sectors grow at the same rate if the following condition is satisfied.

$$(\alpha+\beta+\epsilon_P+\epsilon_R)\,\phi=(\alpha+\beta)(\phi+\eta_P+\eta_R) \tag{C.8}$$

If there does not exist any externalities, i.e., if $\epsilon_R=\epsilon_P=\eta_R=\eta_P=0$ then the above condition is always satisfied. If $\phi=1$, then production functions (2) and (19) are same and then the condition given by the equation (C.8) is same as condition (A.14) used in the basic model.

Appendix D

The first order optimality conditions derived from the dynamic optimization problem solved by the social planner are given by the following:

(A) The first order conditions of maximization with respect to C_R, C_P, a, b, u, v and x are as follows:

$$(C_R{}^{\theta}C_P{}^{1-\theta})^{-\sigma}\theta C_R{}^{\theta-1}C_P{}^{1-\theta}-\lambda_K=0; \tag{D.1}$$

$$(C_R{}^{\theta}C_P{}^{1-\theta})^{-\sigma}(1-\theta)C_R{}^{\theta}C_P{}^{-\theta}-\lambda_K=0; \tag{D.2}$$

$$\lambda_K\alpha\,\frac{Y_R}{a}-\lambda_P\delta\,\frac{\dot{H}_P}{(1-a-b)}=0; \tag{D.3}$$

$$\lambda_R mH_R-\lambda_P\delta\,\frac{\dot{H}_P}{(1-a-b)}=0; \tag{D.4}$$

$$\lambda_K\phi\,\frac{Y_P}{u}-\lambda_K\beta\,\frac{Y_R}{(1-u-v)}=0; \tag{D.5}$$

and

$$-\lambda_K\,\frac{\beta Y_R}{(1-u-v)}+\lambda_P(1-\delta-\gamma)\,\frac{\dot{H}_P}{v}=0; \tag{D.6}$$

and

$$\lambda_K(1-\alpha-\beta)\,\frac{Y_R}{x}-\lambda_K(1-\phi)\,\frac{Y_P}{(1-x)}=0. \tag{D.7}$$

(B) Time derivative of the co-state variables which satisfy their time behaviour along the optimum growth path are given by the followings:

$$\dot{\lambda}_K = \rho\lambda_K - \lambda_K(1-\alpha-\beta)\frac{Y_R}{K} - \lambda_K(1-\phi)\frac{Y_P}{K}; \tag{D.8}$$

$$\dot{\lambda}_R = \rho\lambda_R - \lambda_K(\alpha+\epsilon_R)\frac{Y_R}{H_R} - \lambda_R mb - \lambda_P(\delta+\mu\gamma)\frac{\dot{H}_P}{H_R} - \lambda_K\eta_R\frac{Y_P}{H_R}; \tag{D.9}$$

and

$$\dot{\lambda}_P = \rho\lambda_P - \lambda_K(\beta+\epsilon_P)\frac{Y_R}{H_P} - \lambda_P\{(1-\delta-\gamma)+(1-\mu)\gamma\}\frac{\dot{H}_P}{H_P} - \lambda_K\,(\phi+\eta_P)\frac{Y_P}{H_P} \tag{D.10}$$

In the steady-state growth equilibrium, following optimality conditions are obtained when $0<\theta<1$.

$$\frac{(1-u-v)}{u} = \frac{\beta Y_R}{\phi Y_P} \tag{D.11}.$$

$$\frac{C_R}{C_P} = \frac{\theta}{1-\theta} \tag{D.12}$$

and

$$\frac{x}{(1-x)} = \frac{(1-\alpha-\beta)}{(1-\phi)}\frac{Y_R}{Y_P} \tag{D.13}$$

To have an interior solution of u, $(1-u-v)$ and x, Y_R and Y_P must grow at equal rate. From equation (D.12) we get that $\frac{C_R}{C_P}$ is constant. Since on SGP the growth rate of H_R, growth rate of H_P, a, b and v are constant, the following equation holds true in this case also

$$\frac{\dot{H}_P}{H_P} = \frac{\dot{H}_R}{H_R} = m_P(1-a-b)^{\delta}v^{1-\delta-\gamma}z^{\delta+\mu\gamma} = mb \tag{D.14}$$

From the equation (D.8) we have

$$\frac{\dot{\lambda}_K}{\lambda_K} = \rho - \left[(1-\alpha-\beta)\frac{Y_R}{K} + (1-\phi)\frac{Y_P}{K}\right]$$

Now using equations (D.13), (D.1) and (D.2) we have

$$\frac{\dot{\lambda}_K}{\lambda_K} = \rho - \frac{(1-\alpha-\beta)}{x}\frac{Y_R}{K} = -\sigma\frac{\dot{C}_R}{C_R} = -\sigma\frac{\dot{C}_P}{C_P} \tag{D.15}$$

Since on SGP the growth rate of C_R, C_P, x are assumed to constant, $\frac{Y_R}{K}$ must be constant. Now using equation (D.15), equation (D.12), equation (D.14) and equation (23) we obtain the common growth rate of Y_R, Y_P, C_R, C_P, K given by the equation (26). From the equation (D. 9) we have

$$\frac{\dot{\lambda}_R}{\lambda_R}=\rho-\frac{ma(\alpha+\epsilon_R)}{\alpha}-mb-\frac{m(\delta+\mu\gamma)}{\delta}(1-a-b)-\frac{ma\eta_R Y_P}{\alpha Y_R} \tag{D.16}$$

From equations (D.3) and (D.4) we have

$$\lambda_K\alpha\frac{Y_R}{a}=\lambda_R m H_R$$

Log-differentiating the above equation and using the equations (D.15), (D.16), (25) and (26) we get the equation (27).