EQUILIBRIUM ENVIRONMENTAL TAXES ON INTERMEDIATE-GOOD PRODUCTION WHEN MARKETS ARE VERTICALLY RELATED IN OPEN ECONOMIES*

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Abstracts

The purpose of this research is to examine strategic incentives to distort the use of pollution taxes on intermediate-good production in a successively oligopoly model where both intermediate-good and final-good trade exist. Since the rent capture effects of the pollution tax, which depends on the trans-boundary externality(α), operate in opposite directions in the upstream and downstream sectors, the non-cooperative pollution tax level can be stricter or laxer than the cooperative tax level in accordance with the magnitude of the trans-boundary externality(α). If α is relatively small (resp. large), then the non-cooperative pollution tax is necessarily over-corrected (resp. under-corrected) in terms of world welfare. Moreover we also investigate the effect of trade liberalization on the equilibrium pollution tax.

Keywords: vertically related market, pollution tax on the intermediate-good, trans-boundary pollution, rent capture

JEL classification: F12; Q56

I. Introduction

There has been a lively research discussion regarding the interaction between international trade and environment. Considering negative externalities such as trans-boundary pollutions, trade liberalization cannot always give rise to welfare-improving results, because conventional gains from trade may be more than offset by increased pollution from a trading partner (Siebert, 1977; Baumol and Oates, 1988). Therefore, much of the popular opposition to trade liberalization is based on fears about its consequences for the environment.

In addition to these arguments of economic impact of trade liberalization on environment, many other researchers have paid attention to the possible strategic distortions of environmental policies for trade-related goals. According to the standard public finance

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analysis, the most appropriate intervention in a market characterized by negative externalities in production is a tax on the externality itself. And it is well known that the first order condition characterizing the market solution can be made to coincide with those for Pareto-optimality by pricing the externality at a rate equal to the sum of marginal damages (so called “Pigouvian tax”).

But if there are additional distortions such as imperfect competition in the global economy other than externalities, the optimal tax on polluting production will be deviated away from the standard Pigouvian level. In particular, when firms are competing with foreign rivals in the imperfectly competitive market, each government would have an incentive to relax their environmental taxes in order to gain a competitive position and to capture the rents from their rivals. It should be noted however that more complicated issues arise with respect to rent shifts when the taxes are levied on intermediate-good production in circumstances where firms are competing in successively oligopolistic markets in open economies. This is because the rent shift effects of environmental tax which are levied on intermediate-goods production could operate in opposite directions in the upstream and downstream sectors.

The purpose of this paper is to examine the strategic incentives to distort the use of pollution taxes on intermediate-goods production when markets are vertically related in the presence of trade, not only in the intermediate-good but also in the final-good. And we also investigate the effects of trade liberalization on the equilibrium pollution tax. Here, the strategic use of pollution tax means that the government imposes such a tax for reasons other than encouraging domestic environmental levels, or that the government imposes pollution taxes at the level more or less than the optimum in terms of world welfare. In order to incorporate the strategic use of pollution tax on intermediate-good, we build a partial equilibrium model which allows vertically related upstream and downstream markets with imperfect competition, the tax choose game between competing countries, and trans-boundary pollutions in the upstream sector.

A number of theoretical studies have been made on the use of environmental policy to control the trade-related goal. Of these, Markusen (1975a) and Baumol and Oates (1988) show how tariffs can improve welfare by targeting foreign pollution. If tariffs are not available, other instruments can be used as second best policies. Krutilla (1991) shows that net-exporting (resp. net-importing) large country will set the optimal tax levels on pollution production above (resp. under) the standard Pigouvian level due to terms of trade effects associated with the tax. Ludema and Wooton (1994) analyze the incentives of an importing country to use the trade policy to control foreign pollution. Using the asymmetric two countries model in which externality is produced in fixed proportion to output, they show that the specific tariff or the pollution tax can be used to exploit monopoly power in trade. Copeland (1996) also examines the incentive of trade policy when exists trans-boundary pollution, and shows that pollution content tariffs applied to imports may be part of the optimal response for the importing country. Kennedy (1994), which is most close to this study, examines the incentives of distorting the pollution taxes in a single oligopolistic market when negative externalities such as trans-boundary pollutions exist between countries.

The above studies confine their attention to the single commodity market, therefore, they
do not take into consideration the vertical connection between markets for the intermediate and final-goods. Noting that, however, many environmental taxes are levied on intermediate-good rather than final-good (see Poterba and Rotemberg, 1995) and that industries at each vertical stage may be imperfectly competitive, the appropriate set up for analyzing the strategic effects of environmental policies for trade-related goals is one of successive or vertically related oligopoly market model.

Since the 1990s, a considerable number of studies have been made on the strategic trade policy with vertically related markets (Spencer and Jones, 1991, 1992; Bernhofen, 1997; Ishikawa and Lee, 1997; Ishikawa and Spencer, 1999). These papers analyze various trade policies in the context of vertically related markets with particular attention to the effects of imperfect competition and vertical integration.²

Although the analysis of strategic trade policies with vertically related markets has had extensive development from the 1990s, so far only few studies have been made on the relation between trade and environment using this model. McCorriston and Sheldon (2005) analyze the effectiveness of border-tax adjustment to compensate exporters for domestic environmental taxes when the environmental taxes are imposed on intermediate-good. Adopting the concept of “back-shifting effect” in the vertically related oligopoly market, which is similar to backfiring effect in Ishikawa and Lee (1997), they show that border-tax adjustment rules currently allowed for in GATT/WTO are likely to be too low to maintain the competitiveness of exporters. Unlike McCorriston and Sheldon (2005) which focus on the imports of intermediate-goods, Poterba and Rotemberg (1995) deals with the taxation problem on the imports of final-goods in the context of a model with a vertical industry structure. They show that if there is no joint production, an import tariff equal to the tax on the intermediate-good times the amount of intermediate-good used in domestic production of the final-good will raise the marginal costs of domestic and foreign producers by the same amount.³

The rest of the paper is organized as follows. In Section II, we outline a model of vertically related market with oligopoly that will form the basis of the analysis. The industry equilibrium in the vertically related markets is then derived in Section III. Section IV derives the efficient taxes as the solution to a cooperative problem. This serves as a benchmark against the non-cooperative equilibrium taxes. In Section V, non-cooperative equilibrium tax rates are derived and we discuss the implication they have by comparison with the benchmark. In Section VI, we examine how trade liberalization in the upstream sector might affect the non-cooperative pollution tax rate. And in Section VII, we summarize the main results of the paper.

II. Model

Consider two identical countries: the home (domestic) country is denoted by H and the foreign country is denoted by F. The model introduced here is a two-country successive


³ When there is joint production, however, they argue that intermediate-good intensity may not be a reliable standard for choosing border-tax adjustments associated with domestic environmental taxes.
oligopoly model where both upstream (intermediate-good) and downstream (final-good) industries are imperfect competitive. In the upstream stage, a homogenous intermediate-good is produced, while in the downstream stage, a homogenous final-good is produced. There are \( n \) identical downstream firms and \( m \) identical upstream firms in each country. The numbers of firms are given and constant. The final-good markets are assumed to be integrated across the two countries, therefore, they can be considered as a single market.

Final-good output by a typical firm in country \( I \) is denoted by \( q_I \), and the total final-good output in country \( I \) is \( Q_I = nq_I \). It is assumed that final-good producers follow Cournot behavior in the integrated market. Intermediate-good output by a typical firm in country \( I \) is denoted by \( y_I \), therefore, the total intermediate-good output in country \( I \) is \( Y_I = my_I \). Intermediate-good output by a typical firm in country \( I \), \( y_I \), is divided between the sales to final-good producers in home country, \( y_{H}^{I} \), and the sales to final-good producers in foreign country, \( y_{F}^{I} \) (i.e., \( y_I = y_{H}^{I} + y_{F}^{I} \)). And we can denote as \( Y_{H}^{I} = my_{H}^{I} \) and \( Y_{F}^{I} = my_{F}^{I} \). We assume that firms in the upstream sector in both countries also follow Cournot behavior. Assuming that the upstream sector is characterized by the segmented market model of trade, each upstream firm perceives each country's downstream sector as being separate from one another.

By appropriate choice of units, there is no loss of generality in assuming that just one unit of the intermediate-good is required to produce one unit of the final-good. Other costs of producing the final good are normalized to zero. For the final-good producer, therefore, the only cost to produce the final-good is the cost of purchasing the intermediate-goods

Production of the intermediate-good in country \( I \) generates pollution, \( M_I = \theta Y_I \), where \( \theta \) stands for the amount of the pollution emission per unit of the intermediate-good in each country. The pollution crosses the border and also harms the other country. Environmental damages in country \( I \) are given by \( E_I = E(M_I + \alpha M_{-I}) \), where a “-I” subscript denotes the other country. It is assumed that \( E_I > 0 \) and \( E_{-I} \geq 0 \). A fraction \( \alpha \in [0, 1] \) of this pollution affects the other country. The linear inverse demand function for the final-good in the integrated market is given by \( p = a - b(X_H + X_F) \), where \( p \) is the price of the final-good and \( X_I \) is the consumption of this good in country \( I \) and obtained by \( X_I = (Q_H + Q_F)/2 \). Although the assumption of linear demand is restrictive, it leads to clear-cut results in the analysis.

The subgame perfect equilibrium of the model incorporates three stages of decision. In stage 0, each country’s government commits to the values of its pollution tax on intermediate-good production. In stage 1, the upstream firms in each country commit to the quantities of the intermediate-good supplied to country \( H \) and \( F \) on the basis of a Cournot-Nash equilibrium. In stage 2, downstream firms set their supplies in the downstream markets on the basis of Cournot competition taking the prices of the intermediate-good \( w_H \) and \( w_F \) as given.

III. Industry Equilibrium in the Vertically Related Markets

As is usual in these models, equilibrium at the downstream stage is derived first and then the upstream stage continues afterwards. In the second stage of the game the firms take the intermediate-good prices as given and choose their profit maximizing final-good supplies. The profit maximization problem for the typical final-good producer in country \( I \) is

\[
\max_{q_I} \pi_I = (p - w_I)q_I. \tag{1}
\]
Solving the problems given the intermediate-good prices \( (w_H, w_F) \), we obtain the first order condition:

\[
\frac{\partial \pi_I}{\partial q_I} = p'q_I + p - w_I = 0, \quad I = H, F. \tag{2}
\]

Assuming the linear demand function, the second order condition holds globally. Clearly the second stage Nash equilibrium quantities are functions of intermediate-good prices and number of firms in the downstream sector with

\[
q_I = \frac{a - (n + 1)w_I + nw_{-I}}{(2n + 1)b}, \quad Q_I = nq_I. \tag{3}
\]

Since final-good producers in both countries do a Cournot-Nash competition in the integrated downstream market, each firm’s optimal output is affected by the costs of its rival firms located in the other country. Hence, the input price chosen in the first stage will affect the behavior of downstream firms in both countries. The second stage equilibrium price of final-good is

\[
p = a - b(X_H + X_F) = \frac{a + n(w_H + w_F)}{(2n + 1)}, \tag{4}
\]

which shows that the final-good price is a function of the average price of intermediate-good. In equilibrium, from the assumption of identical countries, the intermediate-good prices are equal in both countries, that is \( w_H = w_F = w \). And in a perfectly competitive market in downstream sectors \( (n = \infty) \), (4) would reduce to \( p = w \).

**Lemma 1:** 1) The price of the final-good is a positive function of the average intermediate-good price \( (w_H + w_F) \). 2) In the competitive market of the downstream sector, the equilibrium price of final-good equals the average of intermediate-good price, \( \lim_{n \to \infty} p = w \).

Now considering the intermediate-good market, each firm makes a decision about how much to produce for the home country sales \( (y_H^I) \) and how much for the foreign country sales \( (y_F^I) \) regarding each country’s downstream market as being separated from another: i.e., \( y_I = y_H^I + y_F^I \). Recall that one unit of the intermediate good is transformed into one unit of the final good. Noting that, \( nq_H(w_H, w_F) = Y_H^I + Y_F^I, nq_F(w_H, w_F) = Y_F^I + Y_H^I \) and taking the inverse defines

\[
w_H = w_H(Y_H^I + Y_F^I, Y_F^I + Y_H^I) = a - b \left\{ \frac{n + 1}{n} \left( Y_H^I + Y_F^I \right) \right\}, \tag{5.1}
\]

\[
w_F = w_F(Y_F^I + Y_H^I, Y_H^I + Y_F^I) = a - b \left\{ \frac{n + 1}{n} \left( Y_F^I + Y_H^I \right) \right\}, \tag{5.2}
\]

which is the inverse derived demand function for the intermediate-good in country \( H \) and \( F \) respectively. From (5.1) and (5.2), the intermediate-good price in country \( I \) is affected not only by the intermediate-good demand in country \( I \) but also by the demand in the other country. And if downstream markets in both countries are perfectly competitive \( (n = \infty) \), the derived demand functions for the intermediate-good become identical with the final-good
demand functions, and hence the home and foreign country’s upstream markets are integrated and can be considered as a single market.

Lemma 2: If downstream markets in both countries are perfectly competitive \((n = \infty)\), then the intermediate-good prices between home and foreign country become equivalent regardless of market equilibrium, implying market integration in the upstream sector between two countries.

Let’s define the pollution tax on intermediate-good production before solving the profit maximization of upstream firms. Let \(e_I\) be country \(I\)’s pollution tax rate on an upstream firm’s production of externality, such that the upstream firm that produces output level \(y_I\) and emits \(q\) of the externality must pay \(e_Iq\) in pollution taxes. Thus \(B_I = (k + e_I\theta)\) is the marginal production costs including pollution tax to a typical upstream firm in country \(I\), where \(k\) is the marginal manufacturing cost of the intermediate-good and assumed constant, and equivalent between the two countries.

The upstream firms are assumed to play the Cournot-Nash game by selecting the profit-maximizing output levels, given its knowledge of how these output levels translate into market prices in the second stage game. And each upstream firm in each country will pay tariff, \(t\), to an importing country’s government on every export unit of intermediate-good. Since optimal trade policy is not the focus of this paper and one of our main interests is the effects of trade liberalization (which is represented by a co-reduction in \(t\)) on the equilibrium decision about the pollution taxes in both countries, we do not introduce the optimal decision of the tariff levels into the model. In this context, we treat the tariff level \(t\) as a parameter and assume same across the two countries. Pollution tax, thus, is the only policy instrument available in the model.

Considering these, a typical upstream firm’s profit function in country \(I\), \(G_I\), can be written as

\[
G_I = (w_I - B_I)y_I^H + (w_F - B_H - t)y_I^F, \quad (6.1)
\]

\[
G_F = (w_F - B_F)y_F^H + (w_H - B_F - t)y_F^F. \quad (6.2)
\]

The assumption of the constant marginal cost of the intermediate-good production implies that an upstream firm’s profit maximizing output choices are independent across markets, which simplifies the analysis. Cournot competition in intermediate-good production in each market then gives rise to the following first order conditions;

\[
\frac{\partial G_I}{\partial y_I} = \frac{\partial w_I}{\partial y_I} y_I^H + w_I - B_I + \frac{\partial w_I}{\partial y_I} y_I^F = 0, \quad (7.1)
\]

\[
\frac{\partial G_I}{\partial y_I^i} = \frac{\partial w_I}{\partial y_I^i} y_I^i + \frac{\partial w_I}{\partial y_I^i} y_I^i + w_I - B_I - t = 0, \quad (7.2)
\]

where \((I, i) \equiv (H, h)\) or \((F, f)\).

Solving these first order conditions simultaneously, we can obtain the sub-game perfect Nash equilibrium quantities for the intermediate and final-good as functions of \(e_H\) and \(e_F\).
\[ Y_i = m y_i = \frac{mn(a - (m + 1)B_i + mB_{i-1} + (2mn + m + n)t)}{b(2m + 1)(2n + 1)}, \quad (8.1) \]

\[ Y_i^* = m y_i^* = \frac{mn(a - (m + 1)B_i + mB_{i-1} - (2mn + m + n + 1)t)}{b(2m + 1)(2n + 1)}, \quad (8.2) \]

\[ Y_i = Y_i + Y_i^* = m(y_i + y_i^*) = \frac{mn(2a - 2(m + 1)B_i + 2mB_{i-1} - t)}{b(2m + 1)(2n + 1)}, \quad (8.3) \]

As we have normalized the input-output coefficient to unit one, we have \( Q_i = Y_i + Y_i^* = m(y_i + y_i^*). \) It is straightforward to obtain that

\[ Q_i = Q_i^* = Y_i + Y_i^* = m(y_i + y_i^*) = \frac{mn(2a - B_i - B_{i-1} - t)}{b(2m + 1)(2n + 1)}, \quad (8.4) \]

\[ X_i - X_i^* = \frac{Q_i + Q_i^*}{2} = \frac{mn(2a - B_i - B_{i-1} - t)}{b(2m + 1)(2n + 1)}. \quad (8.5) \]

The equilibrium prices of the intermediate-good and final-good are obtained by substituting (8.1)~(8.5) into (5.1), (5.2) and (4):

\[ w_i = w_{i-1} = \frac{a + m(B_i + B_{i-1} + t)}{2m + 1}, \quad (8.6) \]

\[ p = \frac{a(2m + 2n + 1) + 2mn(B_i + B_{i-1} + t)}{(2m + 1)(2n + 1)} = \frac{a + n(w_i + w_{i-1})}{2n + 1}. \quad (8.7) \]

Furthermore, since the difference between domestic production and domestic consumption of a good is reflected as net export of that good, the net exports of the intermediate-good and final-good in country \( I \) are

\[ Y_i - Q_i = -\frac{mn(B_i - B_{i-1})}{b(2n + 1)}, \quad \text{and} \quad Q_i - X_i = 0, \quad (9) \]

where \( Y_i - Q_i(= Y_i^* - Y_{i-1}) \) and \( Q_i - X_i \) represents the net exports of the intermediate and final-good, respectively. Equation (9) shows that \( Y_i - Q_i \) is inversely related to the value in \( B_i(=k + e_\theta) \) while \( Q_i - X_i \) always zero and hence is independent from the value in \( B_i \).

Lemma 3: A unilateral reduction (resp. augmentation) in the level of the pollution tax rate on the intermediate-good production increases (resp. decreases) the net exports of the intermediate-good but has no effects on the net exports of the final-good.

These results are very straightforward. A unilateral augmentation in the pollution tax rates on domestic intermediate-good production raises the upstream firms’ marginal costs. This would lower the domestic upstream firms’ relative competitiveness against the foreign country’s upstream firms, reducing domestic net exports of the intermediate-good. In case of the downstream sector, however, firms in both countries purchase the intermediate-good not only from the home country but also from the foreign country. This implies that downstream firms, whether they are located in the home country or not, are symmetrical with respect to purchasing the intermediate-good. In this context, an unilateral change in the pollution tax in
country I would give the same cost effects to downstream firms both in the home and foreign country, and hence the relative competitiveness of each downstream firm would not be affected by the tax in the upstream sector.

In order to examine the welfare implications of the policy choice in pollution taxes in the following sections, we define country I’s welfare, denoted \( SW_I \), as the sum of the consumers’ surplus, profits of upstream and downstream firms, and tax revenue less environmental damage:

\[
SW_I = \left( \frac{1}{2} \int_{X_U}^{X_D} p(\bar{X})d\bar{X} - pX_d \right) + (p - w_I)Q_I + (w_I - k)Y_I \\
+ (w_I - k - t)Y_I^i + tY_I^{d_i} - E[\theta(Y_I + \alpha Y_{-I})]
\]

(10)

IV. The Cooperative Equilibrium Taxes

In this section, we investigate the cooperative equilibrium as a benchmark in order to compare the non-cooperative equilibrium. Here, non-cooperative means that each government chooses the pollution tax to maximize its own welfare, while cooperative means that each government chooses the tax level to maximize world welfare. Under the assumption of symmetry between the two countries, the cooperative tax level on pollution, which is Pareto-efficient, is chosen to maximize the welfare of a representative country, given the equilibrium behavior of the firms.

To begin with, let’s characterize the industry equilibrium in the cooperative regime. Since all the upstream firms both in the home and foreign country face the same tax rates in this regime, we omit the subscript “I” from the variables representing pollution tax rates (i.e., \( e_I = e_{-I} = e, B_I = B_{-I} = B \)). Thus the industry equilibrium can be obtained by replacing \( B_I \) and \( B_{-I} \) with \( B \) in the equations from (8.1) to (8.7). Adopting the superscript “C” to denote the industry equilibrium in the cooperative regime, we obtain,

\[
Y^C_{H} = Y^C_{F} = \frac{mn[a - B + (2mn + m + n)t]}{b(2m + 1)(2n + 1)},
\]

(11.1)

\[
Y^C_{H} = Y^C_{F} = \frac{mn[a - B - (2mn + m + n + 1)t]}{b(2m + 1)(2n + 1)},
\]

(11.2)

\[
Y^C_{I}^i = Y^C_{I}^i = \frac{2mn[a - (B + t/2)]}{b(2m + 1)(2n + 1)},
\]

(11.3)

\[
Q^C_{I} = X^C_{I} = \frac{2mn[a - (B + t/2)]}{b(2m + 1)(2n + 1)},
\]

(11.4)

\[
w^C = \frac{a + 2m(B + t/2)}{b(2m + 1)(2n + 1)},
\]

(11.5)

\[
p^C = \frac{a(2m + 2n + 1) + 4mn(B + t/2)}{b(2m + 1)(2n + 1)} = \frac{a + 2nw^C}{b(2n + 1)}. \quad (11.6)
\]

Now consider the cooperative equilibrium in pollution tax. Since \( Q^C_{I} = X^C_{I} = Y^C_{I}^i, Y^C_{I}^i = \)}
Y^C_I, and \( w^T = w^C_I \) hold from the equations (11.1), (11.4) and (11.5), the social welfare of the representative country in the cooperative regime, \( SW^C_I \), can be written as the sum of the social surplus on domestic consumption less domestic environmental damage.

\[
SW^C_I = \left( \frac{1}{2} \int_0^{2X^C_I} p(\bar{X})d\bar{X} - kX^C_I \right) - E[\theta(1 + \alpha)Y^C_I] \tag{12}
\]

Since both \( X^C_I \) and \( Y^C_I \) are functions of \( B + \frac{t}{2}(=k + e\theta + \frac{t}{2}) \) from the (11.4), \( SW^C_I \) also is a function of \( B + \frac{t}{2} \). In the cooperative regime of an identical two-country model, \( e\theta \) and \( \frac{t}{2} \) have the same meaning in terms of world welfare. Hence we do not need to discriminate between the pollution tax \( e \) and the import tariff \( t \) except that \( t \) is regarded as a parameter while \( e \) is treated as the only available policy instrument in this model. If we differentiate (12) with regard to \( e \) and consider the relation \( \frac{\partial X^C_I}{\partial e} = \frac{\partial Y^C_I}{\partial e} \) in equation (11.4), then we get the following first order condition:

\[
\frac{\partial SW^C_I}{\partial e} = 0 \iff (p^C - k) \frac{\partial X^C_I}{\partial e} = E'[1 + \alpha] \frac{\partial Y^C_I}{\partial e} \tag{13}
\]

The RHS in the second equation of (13) is the reduction of marginal environmental damage and the LHS is the welfare cost of the reduced final-good output associated with the pollution tax on intermediate-good production. In a perfectly competitive market in both upstream and downstream markets, in which final-good price is equal to the tax-inclusive marginal cost, (13) would reduce to \( e\theta + \frac{t}{2} = E'[1 + \alpha] \). As \( \frac{t}{2} \) has the same meaning with \( e\theta \) in the cooperative regime, \( e\theta + \frac{t}{2} \) can be regarded as the price charged on the externality itself. In the competitive case both in upstream and downstream markets, thus, the price charged on externality equals to the marginal damage (i.e., Pigouvian rule holds).

We denote \( e^* \) as the optimal pollution tax rate on the production of intermediate-good when both countries cooperate to maximize world welfare. The cooperative equilibrium tax \( e^* \), which is given by equation (13), is a function of \( t \) and \( \alpha \): i.e., \( e^* = e^*(t, \alpha) \). \( m \) and \( n \) are omitted because they are not major focuses of this paper. Proposition 1 follows.

Proposition 1: The cooperative equilibrium in pollution tax imposed on the production of intermediate-good is a negative function with respect to import tariffs on intermediate-good \( t \), and a positive function with respect to the trans-boundary externality \( \alpha \): i.e.,

\[
\frac{de^*}{dt} = -\frac{1}{2\theta} < 0, \tag{14.1}
\]

\[
\frac{de^*}{d\alpha} = \frac{(2m + 1)(2n + 1)b[E' + (1 + \alpha)\partial Y^C_I \partial E^*]}{2mn(2b + (1 + \alpha)^2\partial^2 E^*)} > 0. \tag{14.2}
\]

Equation (14.1) implies that \( e^* \) can be a substitute for \( t \) in a proportional rate under the
cooperative regime. Equation (14.2) means that since as \( \alpha \) rises the marginal environmental damage also increases, therefore the equilibrium tax as the price on the negative externalities also should be set at the higher level in the context of global welfare maximization. Next we consider the game where each government chooses the rate of the pollution tax to maximize its own welfare.

V. Non-cooperative Pollution Taxes and Comparison with Efficient Pollution Taxes

In this section we examine the non-cooperative Nash equilibrium in pollution taxes in which each government chooses its pollution tax level to maximize its own welfare given the tax level of the other country. The governments in both countries will choose their tax rates knowing that the choice of the tax rate will affect the equilibrium of upstream and downstream sectors. Considering \( w = w_{-} = w \) from (8.6) and \( Y_{+} - Y_{-} = Y_{+} - Q_{t} \) from (9), the social welfare function in the non-cooperative regime, \( SW_{I}^{N} \), can be rewritten as follows from (10):

\[
SW_{I}^{N} = \left( \frac{1}{2} \int_{0}^{X_{+} - X_{-}} p(\bar{X})d\bar{X} - kX_{I} \right) + (p - k)(Q_{t} - X_{I}) + (w - k - t)(Y_{+} - Q_{t}) - E[\theta(Y_{+} + \alpha Y_{-})],
\]

where the first term of the RHS is the social surplus on domestic consumption, the second and third term are the social surplus earned on net export of the final-good and of the intermediate-good, and the last term is domestic environment damage. Given the pollution tax rate of the other country, we focus on the country \( I \)'s optimal tax rate. From (15), the first-order condition for the government of country \( I \) is given by

\[
\frac{\partial SW_{I}^{N}}{\partial e_{I}} = (p - k)\frac{\partial X_{I}}{\partial e_{I}} + \frac{\partial w}{\partial e_{I}} (Y_{+} - Q_{t}) + (w - k - t) \left( \frac{\partial Y_{I}}{\partial e_{I}} - \frac{\partial Q_{t}}{\partial e_{I}} \right) \]

\( - E^{\prime} \theta \left( \frac{\partial Y_{I}}{\partial e_{I}} + \alpha \frac{\partial Y_{-}}{\partial e_{I}} \right). \) (16)

The second-order condition is

\[
\frac{\partial^{2} SW_{I}^{N}}{\partial e_{I}^{2}} = \frac{\partial p}{\partial e_{I}} \frac{\partial X_{I}}{\partial e_{I}} + 2 \frac{\partial w}{\partial e_{I}} \left( \frac{\partial Y_{I}}{\partial e_{I}} - \frac{\partial Q_{t}}{\partial e_{I}} \right) - E^{\prime \prime} \theta^{2} \left( \frac{\partial Y_{I}}{\partial e_{I}} + \alpha \frac{\partial Y_{-}}{\partial e_{I}} \right)^{2} < 0. \] (17)

Since \( \frac{\partial X_{I}}{\partial e_{I}} < 0 \) and \( \frac{\partial Y_{I}}{\partial e_{I}} - \frac{\partial Q_{t}}{\partial e_{I}} < 0 \) hold from (8.5) and (9), and \( E^{\prime \prime} \geq 0 \) from the concavity condition for the function \( E \), the second-order condition is automatically satisfied. Furthermore, in order to introduce the incentive of pollution tax in both countries, it is assumed that the value of \( \frac{\partial SW_{I}^{N}}{\partial e_{I}} \) evaluated at \( e_{I} = e_{-} = 0 \) is positive.

The first order condition in (16) provides the usual reaction curve for country \( I \) on the policy space of \( (e_{I}, e_{-}) \) given the other country's choice in pollution tax. Considering the assumption of two identical countries, the non-cooperative Nash equilibrium is given by the
intersection of country $I$’s reaction curve with a 45° line. Solving equation (16) and $e_I = e_{-I}$ simultaneously, we obtain the non-cooperative equilibrium taxes on the intermediate-good production as functions of $\alpha$ and $t$. (For the convenience of analysis, $m$ and $n$ are omitted.): i.e.,

$$e_N^i = e_N^{-i} = e_N(\alpha, t),$$

(18)

where superscript “N” denotes the non-cooperative equilibrium. The effect of $\alpha$ on $e_N^i$ can be obtained by differentiating (16) with regard to $\alpha$.

$$\frac{de_N}{d\alpha} = -\frac{\partial^2 SW_N}{\partial \alpha_1}$$

(19)

where $\eta = E'' \cdot (M_1 + \alpha M_{-1}) / E'$ and $\Omega = \frac{\partial^2 SW_N}{E^{(1)}} + \frac{\partial^2 SW_N}{E^{(-1)}} < 0$. Here, $\eta$ represents the elasticity of the slope of the environmental damages function $E$, and it necessarily holds $\eta \geq 0$ from the assumption of the quasi-convexity of function $E$. In the expression (19), the sign of $\frac{de_N}{d\alpha}$ depends on the shape of $E$. From (19), we obtain the following proposition.

Proposition 2: If environmental damages function ($E$) is sufficiently convex (i.e., $\eta \geq \frac{m(1+\alpha)}{1+m(1-\alpha)}$), then $\frac{de_N}{d\alpha}$ is greater than zero. On the other hand, if $E$ is not too convex (including linear function) (i.e., $0 \leq \eta < \frac{m(1+\alpha)}{1+m(1-\alpha)}$), then $\frac{de_N}{d\alpha}$ is strictly negative.

Proposition 2 can be explained using the cross-partial derivative of country $I$’s social welfare function, i.e., the derivative of its marginal welfare of pollution tax with respect to the trans-boundary externality parameter ($\alpha$). Differentiating (16) with respect to $\alpha$, we obtain

$$\frac{\partial^2 SW_N}{\partial \alpha_1} = -E' \theta \left( \frac{\eta}{1+\alpha} \left( \frac{\partial Y_I}{\partial e_I} + \alpha \frac{\partial Y_{-I}}{\partial e_I} \right) + \frac{\partial Y_{-I}}{\partial e_I} \right) < 0.$$

(20)

Since $\frac{\partial Y_I}{\partial e_I} + \alpha \frac{\partial Y_{-I}}{\partial e_{-I}} < 0$ and $\frac{\partial Y_{-I}}{\partial e_I} > 0$ from (8.3), the first term of RHS is greater than zero with $\eta \geq 0$ by multiplying $-E' \theta$, while the second term is negative by multiplying the same term. The degree of the trans-boundary ($\alpha$) of the pollution has two welfare effects on country $I$’s community through affecting the environment level.

4 We have,

$$\Omega = 2 \left( \frac{\partial Y_I}{\partial e_I} + \alpha \frac{\partial Q_I}{\partial e_I} \right) E' \theta \left( \frac{\partial Y_I}{\partial e_I} + \alpha \frac{\partial Y_{-I}}{\partial e_I} \right) (1+\alpha).$$

Here, since $\frac{\partial Y_I}{\partial e_I} - \frac{\partial Q_I}{\partial e_I} < 0$ from (9) and $\frac{\partial Y_{-I}}{\partial e_I} + \alpha \frac{\partial Y_{-I}}{\partial e_I} < 0$ for all $\alpha \in [0, 1]$ from (8.3), the sign of $\Omega$ is strictly negative, i.e., $\Omega < 0.$
First, as shown at the first term of RHS of (20), it raises the size of the pollution level in country $I$, and hence raises the marginal environmental damage if $E$ is convex in its argument. In this case, the pollution taxes as the price charged on negative externalities would rise as $\alpha$ increases. Second, it changes the pollution-shifting effect of the tax, which is represented by the second term of (20). As we will see afterwards, a unilateral increase of pollution tax in the home has an effect of diverting the associated pollution to the other country through the decrease of intermediate-good production in the home country, and an increase of that in the foreign country. If $\alpha$ increases, however, the diverted pollution will flow back further to the home, lowering the effectiveness of the pollution-shifting effect of the tax and hence reducing the level of the pollution tax. And this second term does not depend on the convexity of function $E$.

Therefore, the larger (resp. smaller) value of $\eta$ implies that the former effect becomes bigger (resp. smaller) relative to the latter effect, making the total net effect positive (resp. negative). In an extreme case where $\eta=0$ ($E$ is linear function), the first term of RHS in (20) vanishes and hence an increase in $\alpha$ would necessarily lower the level of non-cooperative pollution tax.

Next, we compare the non-cooperative equilibrium with the cooperative equilibrium that is already derived as a benchmark in Section IV. To do this, as in Kennedy (1994), we examine each country’s unilateral incentive to deviate from the cooperative equilibrium taxes by evaluating country $I$’s first order condition, equation (16), at the cooperative equilibrium tax rates($e^*$). We then decompose the overall incentive into four separate effects. In order to help interpret equation (16) evaluated at $e^*$, we subtract the first order condition for cooperative equilibrium tax evaluated at $e^*$, $\left(\frac{\partial SW^c_I}{\partial e}\right)^c = 0$, from the RHS of (16) evaluated at $e^*$. And considering that $p^c(e^*) = p(e^*)$ holds, it can be transformed as follows:

$$
\begin{align*}
\left(\frac{\partial SW^c_I}{\partial e}\right)^c &= (p(e^*) - k) \left(\frac{\partial X^c_I}{\partial e} - \frac{\partial X^c_{-I}}{\partial e}\right) + (w(e^*) - k - t) \left(\frac{\partial Y^c_I}{\partial e} - \frac{\partial Q^c_{-I}}{\partial e}\right) \\
&- E'(e^*) \frac{\partial Y^c_I}{\partial e} + \alpha \frac{\partial Y^c_{-I}}{\partial e} \frac{\partial Y^c_I}{\partial e} + E'(e^*) \frac{\partial Y^c_I}{\partial e} \frac{\partial Y^c_{-I}}{\partial e}.
\end{align*}
$$  

Consider each of the RHS terms of (21) in turn. As already suggested in Kennedy (1994), the third term and the fourth term of the RHS capture the pollution-shifting effect (PSE) and the trans-boundary externality effect (TBE), respectively. And since the PSE is greater than zero, it tends to positively distort the equilibrium tax rates from their cooperative level for $\alpha < 1$, while the TBE, of which sign is clearly less than zero for any values in $\alpha$, distorts negatively.

As for the PSE, a unilateral increase in the pollution tax rates has an effect of shifting the associated pollution to the other country through the decrease of intermediate-good production in the domestic country and an increase of the production in the foreign country. If the domestic government maximizes its own welfare, it does not take into consideration the environmental damage experienced by the foreign citizens. In this context, each country has an incentive to set their tax rates higher at the non-cooperative Nash equilibrium than at the cooperative equilibrium.
And the larger value in $\alpha$ implies that the PSE becomes weaker. If pollution is perfectly trans-boundary then $\alpha=1$ and pollution-shifting effect vanishes. In order to confirm this, considering $\frac{\partial Y^c}{\partial e} = \frac{\partial Y^1}{\partial e} + \frac{\partial Y^{-1}}{\partial e}$ from (11.3) and (8.3), it holds that
\[
PSE = -E'(e^*)\theta \left( \frac{\partial Y^1}{\partial e} + \alpha \frac{\partial Y^{-1}}{\partial e} - \frac{\partial Y^c}{\partial e} \right) = E'(e^*)\theta(1-\alpha) \frac{\partial Y^{-1}}{\partial e} \geq 0. \tag{22}
\]
And differentiating PSE with respect to $\alpha$ yields
\[
\frac{dPSE}{d\alpha} = \theta \frac{\partial Y^{-1}}{\partial e} \frac{E'}{(1+\alpha)Y^c} \left[ Y^c(\eta(1-\alpha) - (1+\alpha)) + \eta(1-\alpha^2) \frac{\partial Y^c}{\partial e} \frac{de^*}{d\alpha} \right], \tag{23}
\]
which is strictly negative only if $\eta \leq 1 + \frac{2\alpha}{1+\alpha}$. We assume for the remainder of the paper that $\eta \leq 1$ to make $\frac{dPSE}{d\alpha} < 0$. As for the TBE, since $\frac{\partial Y^c}{\partial e} < 0$, it holds that TBE $= E'(e^*) \frac{\partial e^*}{\partial \alpha}$ is negative. If each government maximizes its own welfare, it does not take into consideration the effect of the pollution created within its boundaries on the environment of the other country. In this context regarding TBE, the pollution tax rate that is set non-cooperatively is lower than the cooperative level. If $\alpha=0$ then this term vanishes. The larger value of $\alpha$, the greater the effect of trans-boundary externality. It holds that
\[
\frac{dTBE}{d\alpha} = \frac{\partial Y^c}{\partial e} \frac{2b\theta}{Z} E'(e^*) \left( \frac{\eta\alpha}{1+\alpha} + 1 \right) < 0, \tag{24}
\]
where $Z = 2b + (1+\alpha^2)\theta^2 E^c > 0$. As noted above, PSE and TBE show positive and negative signs, respectively. Considering $\frac{\partial Y^c}{\partial e} = \frac{\partial Y^1}{\partial e} + \frac{\partial Y^{-1}}{\partial e}$ holds, we can get the net effect of PSE + TBE as follows:
\[
PSE + TBE = E'(e^*) \theta \left( \frac{\partial Y^{-1}}{\partial e} + \alpha \frac{\partial Y^1}{\partial e} \right) = E'(e^*) \left( \frac{2mn\theta^2(m-\alpha(m+1))}{b(2m+1)(2n+1)} \right). \tag{25}
\]
From the above discussion, the following proposition is established.

Proposition 3: 1) PSE is greater than zero for $\alpha \in [0, 1]$, and $\frac{dPSE}{d\alpha} < 0$. And PSE is strongest when $\alpha = 0$, and PSE $= 0$ when $\alpha = 1$. TBE is less than zero for $\alpha \in [0, 1]$, and $\frac{dTBE}{d\alpha} < 0$. And TBE $= 0$ when $\alpha = 0$, and negatively strongest when $\alpha = 0$. 2) PSE + TBE is positive for $\alpha \in [0, \hat{\alpha}]$, and negative for $\alpha \in [\hat{\alpha}, 1]$, and zero for $\alpha = \hat{\alpha}$, where $\hat{\alpha} = \frac{m}{(m+1)} \geq \frac{1}{2}$. If either $\alpha \leq \frac{1}{2}$ or $m = \infty$ holds, therefore, PSE + TBE $> 0$.

Figure 1 describes the relation of PSE, TBE, and PSE + TBE with $\alpha$ for the case of $E^c =$
0. PSE dominates TBE when $\alpha$ is relatively small (i.e., $\alpha \in [0, \hat{\alpha}]$), however, TBE dominates PSE when $\alpha$ is relatively large (i.e., $\alpha \in [\hat{\alpha}, 1]$).

Next consider the first and second terms of RHS in (21). These terms represent the rent captured from the production of the final-good (i.e., Downstream Rent Capture Effect: DRCE) and that from the net export of the intermediate-good (i.e., Upstream Rent Capture Effect: URCE), respectively. In the vertically related markets, the pollution tax on intermediate-good production raises the intermediate firms’ costs, which subsequently raises the downstream firms’ costs due to the price of the intermediate-good. And in circumstances of imperfect competition, this would affect the rents in those markets.

First, as for the URCE, it is clearly negative and hence tends to negatively distort the non-cooperative equilibrium tax rates from their cooperative equilibrium level. This can be explained as follows.

As can be seen in lemma 3, since a unilateral reduction in the pollution tax rate on the intermediate-good production has an effect in increasing the net exports of this good, the domestic government can capture the rent in the intermediate-good market from foreigners by reducing the pollution tax when market is imperfectly competitive. And if the domestic government maximizes its own welfare, it does not take into consideration the profit loss of the rival country’s firm. Thus, a reduction in the level of the pollution tax on intermediate-good production decreases the profit of the foreign upstream firms, and the domestic pollution tax rate that is set non-cooperatively is lower than the cooperative level.

In this context both countries reduce pollution taxes in an attempt to exploit monopoly power in trade. However, it is worth noting that no rents are actually captured in equilibrium because two countries act symmetrically in this model. This suggests that the distortions are harmful for the countries.

Noting that $\frac{de^*}{d\alpha} > 0$ from (14.2) and $\frac{\partial w}{\partial e} > 0$ from (11.5), an increase in $\alpha$ raises the price of the intermediate-good evaluated at $e^*$, extending the rent capture effect in the upstream markets.\(^5\)

\[
URCE = [w(e^*) - k - 1] \left( \frac{\partial Y_t}{\partial e_t} - \frac{\partial Q_t}{\partial e_t} \right) < 0, \quad (26)
\]

\[
\frac{dURCE}{d\alpha} = \frac{\partial w}{\partial e} \frac{de^*}{d\alpha} \left( \frac{\partial Y_t}{\partial e_t} - \frac{\partial Q_t}{\partial e_t} \right) < 0. \quad (27)
\]

As for the rent capture effect in the downstream sector (DRCE), this is clearly positive. Considering $\frac{\partial X^*_I}{\partial e} = 2 \frac{\partial X_I}{\partial e_I}$, and $2X_I = Q_I + Q_{-I}$, the DRCE can be rewritten as

\(^5\) In the first order condition of (7.2),

\[
w_{-I} - B_{-I} = - \frac{\partial w_{I}}{\partial y_{I}}y_{I} - \frac{\partial w_{-I}}{\partial y_{-I}}y_{-I} - b \left[ y_{I} + \left( \frac{n + 1}{n} \right)y_{I} \right].
\]

By evaluating the above equation at the cooperative equilibrium level of the pollution tax ($e^*$), we obtain:

\[
w(e^*) - k - t = e^* + b \left[ y_{I} + \left( \frac{n + 1}{n} \right)y_{I} \right] > 0.
\]
Let $\text{DRCE} = (p(e^*) - k) \left( \frac{\partial Q_I}{\partial e_I} - \frac{\partial X_I}{\partial e} \right) = -(p(e^*) - k) \frac{\partial Q_I}{\partial e_I} > 0. \quad (28)$

And differentiating DRCE with respect to $a$, we obtain

$$\frac{d\text{DRCE}}{da} = - \frac{\partial Q_I}{\partial e_I} \frac{\partial p}{\partial e^*} \frac{de^*}{da} > 0. \quad (29)$$

We assume in this model that a typical upstream firm supplies its products for the downstream firms located not only in the home country but also in the foreign country. Under such circumstances, for example, a unilateral increase in the domestic pollution tax on the intermediate-good production raises the domestic upstream firms' costs, which subsequently raises not only domestic but also foreign downstream firms' costs due to the price of the intermediate-good. Thus although an increase in the domestic pollution tax would have a direct effect of decreasing the domestic final-good production, at the same time it would have an indirect effect which would offset such a decrease in the domestic final-good production through raising the rival downstream firms' costs located in the foreign country.

And since the domestic government maximizes its own welfare, it does not take into consideration the decrease of final-good production and profit losses of the foreign downstream firms. Rather, domestic government would have an incentive to use the pollution tax on intermediate-good production as an instrument to raise the downstream firms' costs in the rival country when both countries behave non-cooperatively. Thus, the domestic pollution tax rate that is set non-cooperatively is higher than the cooperative level. Considering that the two countries act symmetrically in this model, raising rivals' costs do not actually exist in equilibrium.

The rent capture effect between the upstream and downstream sector operate in opposite directions, but as we see below, URCE always dominates DRCE as far as intermediate-good trade exists. From (26) and (28)

$$\text{DRCE} + \text{URCE} = -(p(e^*) - k) \frac{\partial Q_I}{\partial e_I} + \{w(e^*) - k - t\} \left( \frac{\partial Y_I}{\partial e_I} - \frac{\partial Q_I}{\partial e_I} \right)$$

$$= -\frac{mnq}{b(2m+1)^2(2n+1)^2} \left[ \Psi + (2m+1)(2n+1)(2me\theta + (2mn-m-n+1)t) \right] < 0,$$

where $\Psi = 4mn\{a - k - e\theta - (2mn + m + n + 1)t\}$ is greater than zero with $Y_I > 0$ from (8.2). And this result reconfirms the proposition in the Spencer and Jones (1992), where they derive the necessary and sufficient conditions for the existence of intermediate-good export by a vertically integrated firm to the rival firm in the downstream market. Spencer and Jones (1992) show that a vertically integrated firm would have a tendency to restrict exports of the intermediate-good to the rival downstream firms in the other country. This is because an integrated firm may gain rents in the market for the final-good (corresponds DRCE in this paper) from raising its rivals' costs even at some expense in the market for the intermediate-good in the form of foregone rents form sales of that good (corresponds URCE). And a vertically integrated firm engages in export of intermediate-good to its rival if and only if the rents captured in the upstream market from net exports of intermediate-good dominates the rents lost in the downstream market due to the weakened competitive edge in that market,
when evaluated at the foreclosure price of intermediate-good.  

If both countries impose prohibitively high tariffs on the imports of the intermediate-good, then the trade of these goods are forced to zero between countries. We assume for the remainder of the paper that \( \frac{t}{c^814c^t_P} \), where \( t_P \) denotes the prohibitive tariff and is implicitly defined by \( 4mn(a-k-e\theta -(2mn+m+n+1)t^p)=0 \).

Furthermore, considering that \( \frac{2n}{2n+1} \frac{\partial w}{\partial e} \) from (8.7), the \( \frac{dURCE}{d\alpha} + \frac{dDRCE}{d\alpha} \) can be rewritten as:

\[
\frac{dURCE}{d\alpha} + \frac{dDRCE}{d\alpha} = \left( \frac{\partial Y^{-1}}{\partial e^*_1} - \frac{\partial Q^{-1}}{\partial e^*_1} \right) \frac{\partial w}{\partial e^*_1} \frac{\partial e^*_1}{\partial e} \frac{d\alpha}{d\alpha} + \frac{\partial Q^{-1}}{\partial e^*_1} \frac{\partial w}{\partial e^*_1} \frac{d\alpha}{d\alpha} \\
= \left\{ - \frac{\partial Y^{-1}}{\partial e^*_1} + \left( \frac{1}{2n+1} \right) \frac{\partial Q^{-1}}{\partial e^*_1} \right\} \frac{\partial w}{\partial e^*_1} \frac{d\alpha}{d\alpha} < 0. \tag{31}
\]

The above analysis about rent capture effect in upstream and downstream sectors is summarized in the following proposition.

Proposition 4: 1) The rent capture effect in the upstream (resp. downstream) by the pollution tax on intermediate-good production is clearly negative (resp. positive) for \( \alpha \in [0, 1] \) and so tends to negatively (resp. positively) distort the equilibrium tax rates from their efficient level. 2) And the larger value of \( \alpha \) implies the greater magnitude of both URCE and DRCE, i.e., \( \frac{dDRCE}{d\alpha} > 0 \) and \( \frac{dURCE}{d\alpha} < 0 \), where URCE has the negative value. 3) URCE dominates DRCE for any value of \( \alpha \), thus the total net rent capture effect falls as \( \alpha \) rises: i.e., \( URCE + DRCE < 0 \) for \( t \leq t^p \) and \( \frac{dURCE}{d\alpha} + \frac{dDRCE}{d\alpha} < 0 \).

As seen above, we have investigated each country’s unilateral incentive to deviate from the cooperative level in the pollution tax when both countries choose their policy simultaneously. Out of these four effects, PSE and DRCE have the positive signs, while TBE and URCE have negative ones, so the net effect of these four effects could potentially be either positive or negative.

Next, we should examine the net results of these four effects. Considering (25) and (30), the first order condition (21) evaluated at the cooperative level of the pollution tax can be transformed as follows: 7

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6 But it should be noted that unlike Spencer and Jones (1992), which deals with the behavior of the vertically integrated firm, we deal with separated or independent firms in the context of vertical connection. However considering the government in this paper is concerned with the sum of URCE and DRCE and that the vertically integrated firm in the Spencer and Jones (1992) model is concerned with the joint profit of upstream and downstream sector, the government in this paper plays the same role as the vertically integrated producer in the Spencer and Jones (1992). Therefore it is not critical in the analysis whether the firm is vertically integrated or not.

7 Substituting the second expressions of (25) and (30) into the relevant terms of (21) yields:

\[
\left( \frac{dSW^*}{de^*_1} \right)_x = URCE + DRCE + PSE + TBE
\]
\[ \left( \frac{\partial SW^T}{\partial e_t} \right)_{e^*} = \left[ p(e^*) - w(e^*) + t - \left( \frac{2\alpha}{1 + \alpha} \right) \{ p(e^*) - k \} \right] \left( \frac{\partial Y_{-1}}{\partial e_t} - \frac{\partial Q_{-1}}{\partial e_t} \right). \] (32)

Since \( \frac{\partial Y_{-1}}{\partial e_t} - \frac{\partial Q_{-1}}{\partial e_t} > 0 \) holds from lemma 2, the sign of equation (32) would be coincident with the sign of the square bracket of RHS. Let's define function \( \Phi \) as the square bracket of equation (32):

\[ \Phi(\alpha; m, n, t) = \{ p(e^*) - w(e^*) + t \} - \left( \frac{2\alpha}{1 + \alpha} \right) \{ p(e^*) - k \}. \] (33)

Since \( p = w \) with \( n = \infty \), \( \Phi \) equals zero and hence \( \left( \frac{\partial SW^T}{\partial e_t} \right)_{e^*} = 0 \) if \( n = \infty \) and \( \alpha = t = 0 \) (i.e., \( \Phi(0; m, \infty, 0) = 0 \)). Proposition 5 follows immediately from (33).

Proposition 5: If \( \alpha = t = 0 \) under perfect competition in the upstream market, the non-cooperative pollution tax level on the intermediate-good production coincides with the cooperative level.

Next, consider the more general case where the assumptions in Proposition 5 do not hold. Suppose neither \( n = \infty \) nor \( \alpha = t = 0 \) holds. Evaluating \( \Phi \) in (33) at \( \alpha = 0 \) and \( t = 1 \) yields \( \Phi(0; m, n, t) = p - w + t > 0 \), \( \Phi(1; m, n, t) = - (w - k - t) < 0 \). Since \( \frac{d\Phi}{d\alpha} < 0 \) holds from (23), (24) and (31), there necessarily exists a unique value of \( \bar{\alpha} \) that satisfies \( \Phi(\bar{\alpha}; m, n, t) = 0 \) for any values of parameters.

Proposition 6: Suppose neither \( n = \infty \) nor \( \alpha = t = 0 \) holds. Then, there necessarily exists a unique value of \( \bar{\alpha}(m, n, t) \) that satisfies \( \Phi(\bar{\alpha}; m, n, t) = 0 \) for any values of parameters. If \( \alpha < \bar{\alpha} \) (resp. \( \alpha > \bar{\alpha} \)), the non-cooperative pollution tax is necessarily over-corrected (resp. under-corrected) in terms of world welfare. If \( \alpha = \bar{\alpha} \), then the non-cooperative tax level coincides with the cooperative level.

These results differ in two key respects from those of Kennedy (1994), who deals with a single market.

First, Kennedy suggests that the non-cooperative pollution tax is always laxer than the cooperative level for any values in \( \alpha \) that are not zero, hence there is no possibility of being over-corrected in the pollution tax in terms of world welfare. In contrast, we show that the non-cooperative pollution tax can be stricter, coincident or laxer than the cooperative level according to the value in \( \alpha \), and specify those conditions.

Second, in Kennedy, the non-cooperative pollution tax is coincident with the social optimum only when the relevant market is perfectly competitive with \( \alpha = 0 \). In this research,
however, we show that the non-cooperative pollution tax can be coincident with the cooperative level with $\alpha = t = 0$ regardless of the market structure of the tax imposed industry (here, upstream sector), only if the other vertically related market (here, downstream market) is perfectly competitive. Moreover, we also show that even if $\alpha = 0$ does not hold, there necessarily exists a unique value of $\alpha (>0)$ where the non-cooperative tax level coincides with the cooperative level.

These results strongly depend on the nature of the model adopted here. In our vertically related markets where both intermediate-good and final-good trade exist in imperfectly competitive environments, the rent capture effects of the pollution tax operate in opposite directions in both vertically related markets: i.e., the negative rent capture effect in the upstream market would be offset by the positive rent capture effect in the downstream market. In order to see this, suppose that the downstream market is perfectly competitive, thus there
is no rent capture effect in this market. In this case, (21) would be transformed to

$$
\left( \frac{\partial SW_I^N}{\partial e_I} \right)_{e^*, \alpha = \infty} = -\{w(e^*) - k\} \frac{\partial Y - I}{\partial e_I} + E'(e^*) \frac{\partial Y - I}{\partial e_I} + E'(e^*) \frac{\partial Y - I}{\partial e},
$$

where the first and second terms respectively represent the negative rent capture effect and positive pollution shift effect with relation to shifted production of the intermediate-good due to the pollution tax in the upstream market. And as can be seen in Kennedy (1994), the rent capture effect dominates the pollution shift effect. Therefore, considering that the transboundary externality effect (the third term of RHS in (34)) is less than zero, the overall net effect in (34) is necessarily negative for all $\alpha$ that is not zero. In this context, the arguments in Kennedy (1994) hold as one special limiting case in our model.

VI. The Effect of Trade Liberalization

In this section, we examine how trade liberalization in the upstream might affect the non-cooperative pollution tax on the intermediate-good production. Although we only discuss import tariffs on goods in this model, we interpret a decrease in their levels in both countries more broadly as a reduction in trade barriers. The effect of $t$ on $e_I^N$ can be obtained as follows by differentiating (16) with respect to $t$:

$$
\frac{de_I^N}{dt} = -\left( \frac{\partial^2 SW_I^N}{\partial t \partial e_I} \right) \bigg|\Omega
$$

Since $\Omega = \frac{\partial^2 SW_I^N}{\partial e_I^2} + \frac{\partial^2 SW_I^N}{\partial e_I \partial t} < 0$ from footnote 4, the sign of $\frac{de_I^N}{dt}$ in (35) is coincident with that of numerator, where

$$
\frac{\partial^2 SW_I^N}{\partial t \partial e_I} = \frac{\partial p}{\partial t} \frac{\partial X_I}{\partial e_I} + \left( \frac{\partial w}{\partial t} - 1 \right) \left( \frac{\partial Y_I}{\partial e_I} - \frac{\partial Q_I}{\partial e_I} \right) - E'^\prime \alpha (1 + \alpha) \frac{\partial Y - I}{\partial e} \left( \frac{\partial Y_I}{\partial e_I} + \alpha \frac{\partial Y - I}{\partial e_I} \right).
$$

The second term of (36) that is positive represents the effect of trade liberalization on the marginal social welfare of the pollution tax in terms of net exports in the intermediate-good. Trade liberalization through a co-reduction of the import tariffs on the intermediate-good decreases the price of the intermediate-good (i.e., $\frac{\partial w}{\partial (1 - t)} < 0$). However, this raises the rent per unit of the intermediate-good export (i.e., $\frac{\partial (w - k - t)}{\partial (1 - t)} > 0$), and ceteris paribus, it extends the magnitude of the rent capture effect of the pollution tax, resulting in a decline of the equilibrium pollution tax on intermediate-good production. This result confirms the arguments that trade liberalization may lead countries to reduce their environmental standards in an attempt to gain a competitive edge over their trading partners.

The first term, that is negative, captures the effect of trade liberalization on the marginal
social welfare of pollution tax in terms of domestic consumption (or production) of the final-good. A co-reduction of the tariffs on the import of the intermediate-good reduces downstream firms’ cost due to the price of the intermediate-good, which subsequently lowers the price of the final-good and hence the rent per unit of final-good production (i.e., \( \frac{\partial(p-k)}{\partial(-t)} \)). This would abate the marginal welfare loss which is caused by the decrease in the final-good production due to the pollution tax. Consequently, a co-reduction of tariffs would have an effect of raising the non-cooperative pollution tax in terms of domestic consumption of the final-good.

The third term represents the effect of trade liberalization on the marginal social welfare of pollution tax in terms of environmental damage, and the sign of this term is less than zero. Tariff reduction in both countries expands the production of the intermediate-good and in turn raises the marginal social benefit of the pollution tax as far as \( E^I \). Therefore, it works to raise the non-cooperative pollution tax on the intermediate-good production. From the above analysis, the following proposition is established.

Proposition 7: Suppose \( E^I > 0 \). Trade liberalization in terms of tariff reduction on intermediate-good imports in both countries necessarily lowers the non-cooperative pollution tax on intermediate-good production.

pf. Since the third term of RHS in (36) vanishes, if the absolute value of the second term exceeds the first term, it holds \( \frac{\partial SW^N t}{\partial t} > 0 \). In order to verify this, it is sufficient to show that

\[
\left| \frac{\partial w}{\partial t} - \frac{\partial p}{\partial t} \right| > 0 \quad \text{and} \quad \left| \frac{\partial Y_I}{\partial e_I} - \frac{\partial Q_I}{\partial e_I} \right| = \left| \frac{\partial X_I}{\partial e_I} \right| > 0.
\]

As for the former,

\[
\left| \frac{\partial w}{\partial t} - \frac{\partial p}{\partial t} \right| = 1 - \left( \frac{\partial w}{\partial t} + \frac{\partial p}{\partial t} \right) = 1 - \frac{4n+1}{2n+1} \frac{\partial w}{\partial t} = \frac{m+2n+1}{(2m+1)(2n+1)} > 0.
\]

And as for the latter, considering \( 2X_I = Q_I + Q_{-I} = Y_I + Y_{-I} \),

\[
\left| \frac{\partial Y_I}{\partial e_I} - \frac{\partial Q_I}{\partial e_I} \right| = \left| \frac{\partial X_I}{\partial e_I} + \frac{\partial Q_I}{\partial e_I} - \frac{\partial Y_I}{\partial e_I} \right| = 2 \left( \frac{\partial X_I}{\partial e_I} - \frac{\partial Y_I}{\partial e_I} \right) = \frac{\partial Y_{-I}}{\partial e_I} > 0.
\]

The trade liberalization in the upstream sector has a direct effect of reducing the pollution tax on the intermediate-good production in an attempt to capture the competitiveness edge in the upstream market, however, in the downstream sector, it has an indirect off-setting effect of raising the pollution tax. Since trade liberalization is applied to the trade of the intermediate-good, it is straightforward that the direct effect which takes place in the upstream market dominates the indirect effect which occurs in the downstream market.

When \( E^I > 0 \), however, the sign of \( \frac{de^N I}{dt} \) is ambiguous. Therefore, it can be potentially either positive or negative, especially according to the shape of function \( E \). If environmental damages function \( E \) is not too convex (including linear function), \( \frac{de^N I}{dt} \) is likely to be negative, because the third term of the effect on marginal environmental damages in (36) would be
relatively small. On the other hand, as the convexity of function $E$ increases, $\frac{d\theta}{dt}$ is more likely to be positive, because the third term dominates the net effects of the first and second term due to the increasing marginal environmental damages.

VII. Conclusion

We have analyzed the strategic incentive to distort the use of pollution tax on the intermediate-good production in the context of international oligopoly both in the upstream and downstream sector. And we also have investigated the effect of trade liberalization on the equilibrium pollution tax. The main results are as follows.

First, we have found that the non-cooperative pollution tax level can be stricter or laxer than the cooperative pollution tax level in accordance with the magnitude of the trans-boundary externality ($\alpha$): i.e., there exists critical value $\bar{\alpha}$, such that for $\alpha \leq \bar{\alpha}$, the non-cooperative pollution tax is necessarily over-corrected in terms of world welfare, while for $\alpha > \bar{\alpha}$, on the contrary, the non-cooperative pollution tax is necessarily under-corrected.

Of particular importance is the fact that the pollution tax on the production of the intermediate-good generates two conflicting effects with relation to rent capture effects: it gives each government an incentive to negatively distort the efficient pollution tax so as to capture the rent from the increasing net exports of the intermediate-good, while giving an incentive to positively distort the efficient pollution tax in the downstream sector. Although the former dominates the latter (therefore the sign of the net rent capture effect is still negative), the magnitude of the net rent capture effect (RCE) of pollution tax is reduced by the offsetting effect in the downstream sector. As a result, the pollution-shifting effect (PSE), by which the non-cooperative pollution tax is positively distorted from the efficient level, dominates the net rent capture effect when it is relatively small, resulting in the over-correction of pollution tax in terms of world welfare. However, as $\alpha$ rises, the PSE becomes weak. And thus the trans-boundary externality effect and net RCE dominates the PSE, resulting in the under-correction of pollution tax.

Second, it is related to the effect of trade liberalization. In the cooperative regime where both countries choose the tax level to maximize world welfare, the pollution tax and import tariff have the equivalent effect on welfare. This implies that the equilibrium pollution tax in this regime would rise in response to the tariff co-reduction in both countries. However, in the non-cooperative regime, each country would have an incentive to use the pollution tax strategically from the viewpoint of maximizing one’s own welfare.

This suggests that the trade liberalization in the upstream sector has a direct effect of reducing pollution tax on intermediate-good production in an attempt to capture the competitiveness positions in the upstream market, which dominates the indirect offsetting effect of raising the pollution tax that is induced in the downstream sector.

Thus if the marginal utility from the production of the externality is constant, trade liberalization in terms of co-reduction of tariffs on intermediate-good imports would necessarily lower the non-cooperative pollution tax. If the marginal disutility of pollution is sufficiently increasing, however, trade liberalization would tend to raise the pollution tax due to the
increasing marginal environmental damages.

Third, it is related to the condition under which the level of the non-cooperative Nash equilibrium pollution tax is equal to that of the efficient pollution tax. In Kennedy, non-cooperative pollution tax is coincident with the social optimum only when the relevant market is perfectly competitive with $\alpha=0$. In our research, however, we show that non-cooperative pollution tax can be coincident with the level in the cooperative regime with $\alpha=t=0$ regardless of the market structure of the tax imposed industry (here, upstream sector), only if the other vertically related industry (here, downstream sector) is perfectly competitive.

This result depends on the nature of the market structure that the upstream and downstream sectors are vertically related to, and that the downstream sector is assumed to be integrated between the two countries. From lemma 2, if final-good markets in both countries are perfectly competitive ($n=\infty$), then the prices charged for the intermediate-good become identical between the home and foreign country regardless of market equilibrium, implying market integration in the upstream sector. In this case where final-good markets are perfectly competitive, since both upstream and downstream markets are respectively integrated across the two countries, the model considered here is coincident with that of the closed economy. Thus the non-cooperative Nash equilibrium pollution tax would be same with the equilibrium pollution tax in the cooperative regime where the equilibrium tax rates are chosen in terms of world welfare maximization.

**REFERENCES**


