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Lawsuit as a Signaling Game under Asymmetric Information: A Continuum Types Model

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Lawsuit as a Signaling Game under Asymmetric Information: A Continuum Types Model

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Abstract
This paper analyzes the signaling nature of litigation selection under asymmetric information. For the robustness of the analysis, we separate the litigation selection process, where the signaling from the informed party plays the key role, from the actual settlement where a more general bargaining method than the usual ‘take-it-or-leave-it’ is adopted. With full characterization of a sequential equilibrium, our model provides not only a sound theoretic base, but richer testable hypotheses for the comparative static analyses regarding adjudication rate, plaintiff’s win rate, and the relationship between these two key indices, both under the defendant’s and the plaintiff’s private information.

JEL Classification: K41, D82
Keywords: asymmetric information, signaling, litigation, settlement, win rate

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1. Introduction

Asymmetric information is an inherent problem of daily life. Legal dispute is not an exception. In a lawsuit game where a defendant, a plaintiff, and the court are the major players, asymmetric information plays a key role in determining the final outcome. Naturally, the asymmetric information (hereafter AI) theory has emerged as a convincing answer to the litigation puzzle and also has provided testable hypotheses about the adjudication rate, the plaintiff’s win rate, and the relationship between them.

There is no doubt that the AI theory is still in the exploring stage. As such, it needs further improvement in order to become a standard by which legal disputes can be analyzed. For such a goal, the following three trends of the current literature should be reexamined. First, the uninformed party moves first. Second, the settlement follows the ‘take-it-or-leave-it’ bargaining method. Finally, the litigation/settlement selection process is mixed with the actual settlement bargaining process.

If the uninformed party makes the first move in the litigation/settlement selection, the information transmission through the action taken by the informed party, which is the key aspect of the AI analysis, cannot be appropriately analyzed. The better way would be to allow the informed party to move first so that his action could play the signaling role.1)

Second, even when the informed party is modeled to move first, the analysis is not robust if we assume the ‘take-it-or-leave-it’ method of bargaining. It seems that the ‘take-it-or-leave-it’ settlement offer has become a standard way to describe the settlement bargaining process in most current AI literature. Nonetheless, such an extreme bargaining process, which gives the absolute advantage to the first mover, affects the result not only quantitatively, but also qualitatively. If we were to assume a more general settlement process which does not give the whole pie to the first mover, many results of the

1) Who offers first is very important in a signaling game. It is somewhat disappointing that many studies on AI intentionally avoid the signaling analysis by modeling the uninformed party to move first. Shavell(1996) refers to such a trend by saying that it is now the standard model of ‘settlement versus trial’ that the side without information makes a single settlement offer. However, Bechuk(1984), which is Shavell’s reference model, admits that adopting such a sequence of moves was to avoid a complicated signaling nature. There are nonetheless a few studies, such as Png(1987), Reinganum and Wilde(1986), that appropriately allow the informed party to move first for the signaling analysis.
current AI literature would be erroneous.2)

Finally, in current AI literature, the litigation/settlement selection process is mixed with the settlement process. When the two processes are mixed, the size of the settlement demand naturally becomes the signaling device. Consequently, who offers the settlement demand first affects not only the decision to litigate or to settle, but also the settlement outcome. Although, who moves first matters in the litigation/settlement selection because of the signaling nature, it should play a more neutral role in determining the settlement outcome. It is because there is no \textit{ex-ante} reason why either the defendant or the plaintiff should offer the settlement demand first in the actual settling process. Separating settlement from the litigation/settlement selection, and adopting a more general bargaining method for the settlement, which is more neutral against who moves first, seems a better model specification.

This paper is an attempt to improve the AI theory in these three respects, and, by doing so, to enrich AI theory as the main framework in analyzing litigation and settlement.3) Before moving to the formal analysis, we will provide some further observations and criticisms of the existing literature regarding these three points.

Bebchuk (1984) is the pioneering study on the role of AI in litigation and settlement. Even though Bebchuk provides the basic intuition about the AI nature of the legal disputes, he does not examine the signaling role of settlement offers. He confesses in his concluding remarks that the reason why the uninformed party is modeled to move first is to avoid the issue of signaling, and adds that, to analyze the signaling aspect, we need a complete model of settlement decisions under imperfect information.

Reinganum and Wilde (1986) analyze a signaling model where the plaintiff has private information about the size of the stake in dispute, and show that AI explains the existence of costly litigation. Their main result is that the equilibrium, more specifically, the trial rate, is affected only by the total litigation cost, and not by its allocation. Yet, such a result is not robust because

2) In this regard, Reinganum and Wilde (1986) is a good example. Their main proposition holds only under the assumption that the first mover retains the whole pie. This implies that the analysis based on the ultimatum settlement bargaining is not robust compared to more general bargaining models where the first mover does not have the absolute advantage.

3) This paper, which assumes continuum types for the informed party, is an extension of the two-type model by Kim and Kim (2000).
it holds only when the following two conditions are satisfied: both parties share common beliefs about the likelihood of a judgement in favor of the plaintiff, and the plaintiff retains the entire settlement. The first condition is not satisfied if the source of AI is the plaintiff’s win rate per se, as in many models, including ours. Furthermore, the second condition is satisfied only under the 'take-it-or-leave-it’ type of bargaining. If the settlement process is not such an extreme ultimatum bargaining, their proposition cannot hold.\textsuperscript{4}

Png (1987) studies the hidden action problem and derives a sequential equilibrium in a signaling game setting. Png allows the informed party, the defendant, to offer the settlement demand first, so that the uninformed plaintiff can update his belief about the defendant’s type. Even though Png deals with the moral hazard problem in the care level by the defendant, the prior probability that the defendant is guilty is assumed to be fixed. It is clear that, in a model of moral hazard, the probability of being guilty should be dependent on the defendant’s choice in care. If the probability of being guilty determined is by the care level, his equilibrium might be totally changed.\textsuperscript{5}

Hylton (1993) also emphasizes the signaling role of the settlement demand to explain the litigation puzzle. Even though Hylton correctly points out that the strategic behavior of the parties, that is, the adverse selection problem under asymmetric information, is the key to litigation selection, she is somewhat inaccurate in explaining information transmission, because, we submit, she has not actually solved for a sequential equilibrium.

The structure of the paper is as follows. In Section 2, we will describe the lawsuit as a signaling game under AI, with details about the litigation and the settlement bargaining process. Section 3 characterizes the sequential equilibrium of the signaling lawsuit game. Several interesting comparative static analyses will be provided in Section 4, with additional implications to the divergent

\textsuperscript{4} Reinganum and Wilde also provide no further comparative static analyses other than the proposition on the aggregated litigation cost.

\textsuperscript{5} The separating equilibrium, which is Png’s main focus, might not even exist. Actually, in his paper, Png repeatedly hints that the care level and the probability of being guilty are correlated. Furthermore, Png derives a mixed strategy equilibrium, rather than the standard pure strategy equilibrium. Png’s model is a two-type one, where the defendant is either guilty (negligent) or innocent (careful). When there are only two the defendant types, without mixed strategy, there is no hybrid equilibrium where some portion of the guilty defendants goes to trial and the remainders settle out of the court. Nevertheless, it would be more desirable to analyze a pure strategy equilibrium, if it exists, than a somewhat unrealistic mixed strategy equilibrium.
expectations theory of litigation. Section 5 equivalently analyzes the situation under which the plaintiff (not the defendant) is the party who has private information, which is of great importance in utilizing our comparative statics for future empirical studies. Section 6 concludes the paper.

2. A Signaling Model

Consider a two-person two-stage game with incomplete information, as in &lt;Figure 1&gt;[6] The defendant (D) incurs harm to the plaintiff (P). A defendant is identified by \( \nu \), the probability that he is guilty. The defendant’s type is randomly determined by the Nature (N). We assume that \( \nu \) is uniformly distributed on the \([0, 1]\) interval.[7] We also assume that the defendant knows his own type, while the plaintiff knows only the distribution of \( \nu \).[8]

The game is a sequential one, with the defendant moving first, and the plaintiff moving after observing what the defendant has chosen. When a suit is filed, the defendant first chooses either ‘litigate’ (\( l \)) or ‘settle’ (\( s \)). If the defendant chooses \( l \), the game ends with a litigation. If the defendant chooses \( s \), the plaintiff chooses between \( s \) and \( l \) without knowing the defendant’s type.[9] If the plaintiff chooses \( s \), the settlement bargaining begins, and if he chooses \( l \), the litigation subgame starts.

Let \( U_d \) and \( U_p \) be the expected payoffs in the settlement subgame, and \( U_d^l \) and \( U_p^l \) be the expected payoffs from litigation, for the defendant and for the plaintiff, respectively. Note that the defendant’s payoff is the cost or burden to be minimized, while that of the plaintiff is measured by the usual return to be maximized.

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6) The game model in &lt;Figure 1&gt; is a reduced form where the subgames for the settlement bargaining and for the litigation are not explicitly represented. Each subgame will be explained in detail separately.

7) The uniform distribution is only for convenience, and is not crucial to our analysis.

8) Our model analyzes a hidden information (adverse selection) problem since we assume that \( \nu \) is exogenously given. However, a defendant can affect \( \nu \) through his effort or care. See Png(1987) for such a hidden action (moral hazard) problem.

9) This is represented by the dotted line, the information set, in &lt;Figure 1&gt;.
<Figure 1> A Signaling Game of Lawsuit

N: Nature  D: defendant  P: plaintiff
l: 'litigate'  s: 'settle'
L: litigation subgame  S: settlement subgame

Litigation Subgame

If litigation starts, then there is no strategic behavior by the two parties, and the payoffs are determined by the court. We assume that the court identifies the true type of the defendant subject to some minor random errors. Let $Q_1$ be the probability that a defendant who has violated the legal standard will be found not guilty (type I error), and $Q_2$ be the probability that an innocent defendant will be found liable (type II error). $Q_1$ and $Q_2$ are public information and satisfy $1 - Q_1 - Q_2 > 0$.\(^{10}\)

The expected payoffs from litigation are $U_d^L = P_d J + C_d$ for the defendant and $U_p^L = P_p J - C_p$ for the plaintiff, where $J$ is the size of the stake in dispute, $P_d$ and $P_p$ are the subject probabilities of the plaintiff’s win, and $C_d$ and $C_p$ are the litigation costs, of the defendant and the plaintiff, respectively. We assume $U_p^L \geq 0$ against all defendant types so that the litigation choice is not a non-credible threat.\(^{11}\) $C_d$, $C_p$, and $J$ are exogenously fixed, while $P_d$ and $P_p$ are endogenously determined by the information of the players as follows.

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10) See Hylton(1993) for the implications of this assumption.

11) If the plaintiff’s payoff is negative against some defendants’ types, then the credibility of the litigation choice by the plaintiff becomes an important issue. See Nalebuff(1987) and Spier(1992) for credibility in the pretrial negotiation.
\[ \begin{align*}
P_d &= \nu(1-Q_1) + (1 - \nu)Q_2, \\
P_p &= \int_V [\nu(1-Q_1) + (1 - \nu)Q_2] \, dF(V).
\end{align*} \]

Note that the plaintiff can only know that the defendant type belongs to some range \( V \), which is a subset of \([0, 1] \).\(^{12}\) \( F(V) \), which is derived from the underlying uniform distribution, is the cumulative distribution function for this interval \( V \). Note that if \( V = [0,1] \), then \( dF(V) = dv \).

**Settlement Bargaining Subgame**

Any settlement, if it exists, must make both parties better-off than under litigation, because the payoff from litigation is the opportunity cost of the bargaining. Lemma 1 refers to such a condition for the existence of a settlement.

**Lemma 1.** Assume that settlement is at no cost. Then, settlement occurs if and only if \( U^S_d > U^F_p \). Otherwise, litigation occurs.

<Figure 2> describes a settlement bargaining assuming \( U^S_d > U^F_p \). The settlement should be in the preferred set, which is to the west and north of the status quo, \( E(U^S_d, U^F_p) \). Furthermore, since the bargaining is a zero-sum game, the bargaining outcome should be on the bargain-feasible set \( B \), which is the 45° line from the origin. Let \( Z \) be the contract zone, which is the intersect of the preferred set and the feasible set. A settlement, if one exists, should be in \( Z \). However, the specific location in \( Z \) depends on the bargaining method and the bargaining powers of the two parties.

We model the settlement process as an infinitely repeated bargaining with alternating offers as follows. The defendant either quits bargaining or offers a settlement demand first. If the defendant stops bargaining, the case goes to trial. If he offers a settlement demand, the plaintiff either accepts or rejects it. If the plaintiff accepts it, the game ends with a settlement. Upon rejection, he then counter-offers another settlement demand or stops bargaining. If the plaintiff chooses to stop, the case goes to trial. If the plaintiff counter-offers to the

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\(^{12}\) This is because there are only two different signals in our model, while the defendant types are infinitely varying.
defendant, then it is defendant’s turn again. The game is repeated until one party stops bargaining or accepts the other’s offer.

**Figure 2** Bargaining as a Settlement Process

Rubinstein(1982) shows that there exists a unique subgame perfect Nash equilibrium in such a noncooperative bargaining game which depends on the bargaining powers, or the discount factors of the two parties. Assume that the two parties have the same discount factor, denoted by \( \delta = [0, 1] \).

**Lemma 2.** There exists a unique subgame perfect Nash equilibrium in a settlement bargaining game where the settlement is

\[
(U_d, U_p) = \left( U_p - \frac{U_p - U_p'}{1 + \delta}, U_p + \frac{\delta(U_p - U_p')}{1 + \delta} \right).
\]

13) If discounter factors are different such that \( \delta_1 \) is for the defendant and \( \delta_2 \) for the plaintiff, then the pie \( U_d - U_p \) will be divided in the ratio of \( \frac{1 - \delta_2}{1 - \delta_1 \delta_2} : \frac{\delta_1 (1 - \delta_1)}{1 - \delta_1 \delta_2} \). Lemma 2 is the case of \( \delta_1 = \delta_2 = \delta \).

<Proof> Refer to Rubinstein(1982).

Note that there is a first mover advantage in Lemma 2, since the dividing rule is more favorable to the first mover: the defendant in our case. Even though our analysis of the litigation selection does not depend on asymmetry in the settlement subgame, for the tractability of the analysis, and for the neutrality of the settlement process against the sequence of moves, we will further assume that the discount factor converges to 1, so that the first mover...
advantage in the settlement disappears.

**Lemma 2'**. Assume that the common discount factor converges to 1. Then, the settlement, which is the subgame perfect Nash equilibrium outcome of the settlement bargaining, is symmetric such as \( (U_d, U_p) = \left( \frac{U_d^0 + U_p^0}{2}, \frac{U_d^0 + U_p^0}{2} \right) \).

3. Equilibrium

Now, let us find an equilibrium of the lawsuit signaling game. The equilibrium concept in this paper is the sequential equilibrium by Kreps and Wilson (1982).

**Definition.** A sequential equilibrium of the signaling game is the defendant’s strategy, the plaintiff’s strategy, and the plaintiff’s belief about the defendant’s type, such that the strategies are the best responses to each other given plaintiff’s belief, and that the plaintiff’s belief is consistent with the defendant’s strategy.

The equilibrium, if it exists, is either separating, pooling, or hybrid. In a (pure) separating equilibrium, each defendant sends a different signal so that the true type can be predicted by observing the defendant’s behavior. In a (pure) pooling equilibrium, all the defendants act in the same way; that is, they send the same signal, so that observing a signal does not provide any additional information about the defendant’s type to the uninformed plaintiff. Finally, in a hybrid equilibrium, defendants are divided into two groups, and each group sends a different signal so that observing a signal gives information regarding to which group the defendant belongs. In a hybrid equilibrium, however, the true type of a defendant cannot be identified within the same group.\(^\text{14}\)

The situation on which we will particularly focus is the hybrid equilibrium where those defendants with low \( v \) choose litigation, while those with high \( v \)

\(^{14}\) The terminology of a hybrid equilibrium might be somewhat confusing. It is because we sometimes call an equilibrium ‘hybrid’ when the same types send different signals so that an equilibrium is characterized by both separating and pooling natures at the same time. However, in our model, since there are no identical defendants, and there are only two available signals, the description of a hybrid equilibrium is natural.
choose settlement. Before moving to a formal analysis, some justifications for restricting our attention to such a hybrid equilibrium may be necessary.

First, consider pooling equilibria. A pooling equilibrium where all types of defendants choose settlement, although it might exist, is not in our interest because it has nothing to say about litigation. If all defendant types choose settlement, the plaintiff cannot distinguish their types. The plaintiff will then choose settlement as an optimal strategy to avoid unnecessary litigation costs.

On the other hand, a pooling equilibrium where all types of defendants choose litigation cannot exist. This is because the defendant with $v=1$, who is without doubt guilty, will always deviate from litigation to settlement. By extension, the defendants who are almost surely guilty will act in the same way.\footnote{15}

Furthermore, it is clear that a pure separating equilibrium where every defendant type is identified cannot exist. This is because there are only two signals while there are infinitely many types of defendants. It follows that the only meaningful equilibrium is the hybrid as described above.

In a hybrid equilibrium, there must be a marginal type of defendant who is indifferent to sending signal $s$ or $l$. By extension, all defendants who are located between this marginal type and $v=1$ will choose $s$, while the others will choose $l$. Since not just the purely innocent type goes to trial, and because those with some positive probability of guilt also choose $l$, we can now perform a much richer analysis of the plaintiff’s win rate.

To summarize, a hybrid equilibrium, if it exists, must be such that (relatively) innocent defendants reveal true types, meaning that they signal that they are of low $v$’s, by choosing litigation, which will not be followed by the (relatively) guilty defendants.

**Proposition 1.** There exists an equilibrium in the lawsuit signaling game where the defendant has private information such that:

- defendant’s strategy: $l$ for $v < v^*$, and $s$ for $v \geq v^*$,
- plaintiff’s belief: if $l$, then $v < v^*$, and if $s$, then $v \geq v^*$,
- plaintiff’s strategy: $s$,

where $v^* = 1 - \left( \frac{C_{ul} + C_p}{1 - Q_1 - Q_2} \right)$ is the marginal defendant type who is indifferent to choosing between signals $l$ or $s$.

\footnote{15} This is the same as Proposition 2 in Hylton (1993).
<Proof> First, it is clear that the plaintiff’s belief is consistent with the defendant’s strategy. Let us prove the optimality of the two parties’ strategies.

**Optimality of the Defendant’s Strategy**

Consider first a defendant whose type belongs to \( v \leq v^* \). If he chooses \( l \), his expected payoff from \( l \) will be:

\[
U_l(v) = [v(1-Q_1)+(1-v)Q_2] + C_d \quad \text{--- (1)}
\]

On the other hand, if he chooses \( s \), the case is settled out of court since the plaintiff’s strategy is also \( s \). The defendant’s payoff from the settlement depends on the status quo of the bargaining, which are the payoffs when the disputes were resolved through litigation. What would have been the payoff if the case had gone to trial? Since the plaintiff’s belief is that the defendant, who has sent signal \( s \), belongs to \( v \geq v^* \), the bargaining should be based on the payoffs estimated by the plaintiff, not on the true payoffs expected by the defendant. This is because the defendant, who has already sent signal \( s \), cannot make the plaintiff believe that his type is low \( v \).

Therefore, from Lemma 2’, the defendant’s payoff from settlement will be:

\[
U_s(v^*) = P_s(v^*) J + (C_d - C_p)/2, \quad \text{--- (2)}
\]

where the plaintiff’s estimate of his winning probability \( P_s(v^*) \) is

\[
P_s(v^*) = \int_{v^*}^1 \frac{v(1-Q_1)+(1-v)Q_2}{1-v^*} \, dv
= \left[ (1+v^*)(1-Q_1)+(1-v^*)Q_2 \right]/2. \quad \text{--- (3)}
\]

The defendant with \( v \leq v^* \) will choose \( l \) if and only if \( (1) \leq (2) \); that is, \( U_l(v) \leq U_s(v^*) \) (the equality holds when \( v = v^* \), since the marginal defendant

---

16) If the defendant wanted to inform the plaintiff of his true type, that is, the fact that he is a more innocent type, then he should have sent signal \( l \). However, there would then have been no settlement bargaining. We can assume that the plaintiff’s belief is again updated during the settlement process, which is beyond the scope of this paper.
with \( v^* \) is indifferent between the two signals).\(^{17}\) Since \( U_d(v) \) in equation (1) is increasing in \( v \), this condition will be satisfied if \( U_d(v^*) = U_d(v^*) \).

Rewriting \( U_d(v^*) = U_d(v^*) \), using equation (3), we obtain

\[
(1-v^*)(1-Q_1-Q_2)J = C_d + C_p \quad --- \quad (4)
\]

From equation (4), we find a marginal defendant type \( v^* \).

\[
v^* = 1 - \frac{C_d + C_p}{J(1-Q_1-Q_2)} \quad --- \quad (5)\(^{18}\)
\]

If \( v^* \) is given as in equation (5), the defendants with \( v \leq v^* \) will choose \( l \) as the optimal strategy.

Now, consider a defendant whose type belongs to \( v > v^* \). If he chooses \( l \), his payoff from litigation \( U_d(v) \) is the same as in equation (1). If he chooses \( s \), he again receives the same payoff \( U_d(v^*) \) from the settlement bargaining, as in equation (2). Since \( U_d(v) \) is increasing in \( v \), \( U_d(v^*) = U_d(v^*) \) is sufficient for \( U_d(v) \geq U_d(v^*) \) for all \( v \geq v^* \) (again the equality holds for \( v = v^* \)), which is already proven to hold at \( v^* \) in equation (5). It follows that a defendant with \( v > v^* \) will choose settlement as his optimal strategy.

In sum, the defendant’s strategy is optimal against the plaintiff’s belief and strategy, and the marginal defendant type \( v^* \) is defined as in equation (5).

**Optimality of Plaintiff’s Strategy**

The plaintiff moves only when the defendant has chosen settlement. When the signal \( s \) is observed, the plaintiff believes that the defendant is type \( v > v^* \). If the plaintiff chooses \( s \), his payoff will be:

\[
U_p(v^*) = P_p(v^*)J + (C_d - C_p)/2. \quad --- \quad (6)
\]

\(^{17}\) Recall that the defendant’s payoff is the cost, and thus needs to be minimized.

\(^{18}\) We assume that \( v^* \) is not a boundary point in \([0, 1]\) without loss of generality.
If he chooses \( l \), his payoff from litigation will be:

\[
U_l(v^*) = P_l(v^*)J - C_{pl} \quad \text{(7)}
\]

where \( P_l(v^*) \) is the plaintiff’s winning probability defined in equation (3). The plaintiff’s strategy of \( s \) is optimal, since it always holds that (6)>(7), that is, \( U_p(v^*) > U_l(v^*) \). Q.E.D.

4. Comparative Static Analysis

Recent studies on litigation selection show special interests in the adjudication rate, the plaintiff’s win rate (among adjudicated cases), and the relationship between these two indices. Our analysis with continuum types of defendants who have private information provides rich implications regarding such issues.\(^{19}\)

(1) Comparative Statics for the Adjudication Rate

Let \( T \) be the adjudication rate, the fraction of cases for trial, which is equal to \( v^* \) in our analysis. Now let us undertake some comparative static analysis. Since the impacts of \( C_d \) and \( C_p \) and also those of \( Q_1 \) and \( Q_2 \) on \( T \) are symmetric, let \( C = C_d + C_p \) and \( Q = Q_1 + Q_2 \). As such, the marginal defendant type, or the adjudication rate, \( v^* \) can be rewritten as:

\[
v^* = 1 - \frac{C}{J(1 - Q)} \quad \text{(8)}
\]

From equation (8) the following comparative statics can be obtained.

---

19) Bebchuk (1984) also attempts comparative static analyses regarding the settlement amount. He predicts that the settlement amount increases with the size of the stake, and decreases with the plaintiff’s litigation cost, but that it can increase, decrease, or do neither when the defendant’s litigation cost increases. Our analysis mostly supports Bebchuk’s prediction. The settlement amount is given in equation (2), where \( P_l(v^*) \) is as in equation (3) and \( v^* \) is as in equation (5). We can easily see that the settlement amount in our analysis increases in \( J \), decreases in \( C_{pl} \) and does not change with \( C_d \).
\[ \frac{\partial v^*}{\partial J} > 0. \quad \text{--- (9)} \quad \frac{\partial v^*}{\partial C} < 0. \quad \text{--- (10)} \quad \frac{\partial v^*}{\partial Q} < 0. \quad \text{--- (11)} \]

**Proposition 2.** The adjudication rate increases with the size of the stake \( J \), but decreases with the litigation costs \( C \) and court errors \( Q \).

**Effect of the Stake Size**

Equation (9) says that the adjudication rate is positively related to the size of the stake.\(^{20}\) First, consider how the marginal type defendant, who was indifferent to litigation or settlement, will change his behavior when the stake becomes more important. From equations (1) and (2), we can determine the impact of the change in the stake size on the payoffs of the marginal defendant type \( v^* \) from both litigation and settlement. From equation (1), the change in the payoff from litigation is \( \partial U_d/\partial J = v^*(1-Q_1)+(1-v^*)Q_2 \), which is the plaintiff’s win rate when the defendant type is \( v^* \). On the other hand, from equation (2), the change in the payoff from settlement is \( \partial U_d/\partial J = P_f(v^*) \), which is the plaintiff’s average win rate for \( v \geq v^* \). Therefore, it is clear that \( P_f(v^*) > v^*(1-Q_1)+(1-v^*)Q_2 \), that is, \( \partial U_d/\partial J / \partial U_d/\partial J \). As the stake increases, the expected payment by the marginal defendant also increase both in litigation and in settlement. However, the payment increase is less significant in litigation than in settlement, so the marginal defendant will now choose litigation. The new \( v^* \) under the increased \( J \) will be closer to \( v=1 \) than before, which of course means a higher frequency of litigation.

Now check plaintiff’s incentive. From equations (6) and (7), \( \partial U_d/\partial J = \partial U_d/\partial J \). This means that the change in the stake size does not affect the plaintiff’s incentive to litigate. This is because, as we can see from equations (6) and (7), to the plaintiff, the only difference between litigation and settlement is whether he pays the whole litigation cost, or gets some reimbursement in the settlement bargaining.

In conclusion, there will be more litigation as the stake becomes higher. Furthermore, such a change is initiated by the defendant, while there is no

\(^{20}\) This is consistent with the prediction by Bebchuk(1984, 1988) and Waldfogel(1998). However, Png(1987) argues that the effect of stake size on the trial frequency is ambiguous.
change in plaintiff’s incentive.

**Effect of Litigation Costs**

Equation (10) implies that there will be less trials under a higher litigation cost.\(^{21}\) Consider first the change in the defendant’s litigation cost. We will again focus on the incentive of the marginal defendant type. From equations (1) and (2), \(\partial U_d^p / \partial C_d = 1\), and \(\partial U_d^d / \partial C_d = 1/2\). When his litigation cost increases, the marginal defendant \(\nu^*\) is no longer indifferent to choosing between litigation and settlement. He now prefers settlement.

We can also confirm that the plaintiff also desires settlement, since \(\partial U_p^p / \partial C_d = 1/2 > \partial U_p^p / \partial C_d = 0\) from equations (6) and (7). The increase in the defendant’s litigation cost is more advantageous to the plaintiff in settlement, though it does not change the payoff resulting from litigation. Thus, the plaintiff will prefer settlement more. To summarize, the increase in \(C_d\) makes both the defendant and the plaintiff choose settlement more.

Now consider the change in the plaintiff’s litigation cost. From equations (1) and (2), \(\partial U_p^p / \partial U_p = 0\), and \(\partial U_p^d / \partial C_p = -1/2\). The increase in the plaintiff’s litigation cost has no effect on the defendant’s payoff from litigation. Nonetheless, it lowers the defendant’s payment in settlement. The defendant will thus be more inclined to opt for settlement. Furthermore, from equations (6) and (7), \(| \partial U_p^p / \partial C_p | = | -1/2 | < | \partial U_p^d / \partial C_p | = | -1 | \). The plaintiff’s loss from the increased litigation cost is then less significant in settlement. He will thus more often choose settlement. Again, an increase in the plaintiff’s litigation cost consistently changes the defendant’s and plaintiff’s incentives toward settlement.\(^{22}\)

\(^{21}\) Our result that the litigation selection depends only on the aggregated litigation cost, and does not depend on its allocation between the two parties is consistent with the main proposition of Reinganum and Wilde(1986). However, such a result is obtained in our analysis without assuming the two critical conditions needed in Reinganum and Wilde, as mentioned in the Introduction.

\(^{22}\) This result is consistent with Bebchuk(1984, 1988), Eisenberg and Farber(1997), and Waldfogel(1998). However, Pang(1987) argues that while the defendant’s litigation cost has a negative effect on the litigation rate, the effect of the plaintiff’s litigation cost can be either positive or negative. Furthermore, our results (9) and (10) are also consistent with many studies following the divergent expectations model originally claimed by Priest and Klein(1984). In fact, our overall comparative statics results are, in spirit, consistent with their so called ‘selective litigation hypothesis,’ that the set of cases that go to trial is not a random sample of all cases filed. See Priest and Klein(1984), Wittman(1985), Eisenberg(1990), Waldfogel(1995), Siegelman and Waldfogel(1999), etc.
Effect of Court Errors

Let us investigate the impact of the change in court errors, which is summarized in equation (11). Consider the type I error first. From equation (1), the reduction in the marginal defendant’s payoff from litigation due to the increased type I error is $|\partial U_d/\partial Q_1| = |1 - v^*|$. Also, from equation (2), the payoff change from settlement is $|\partial U_d/\partial Q_1| = |1 - (1 + v^*)/2|$. Since $v^* \leq (1 + v^*)/2$, $|\partial U_d/\partial Q_1| \leq |\partial U_d/\partial Q_1|$. The marginal defendant now changes his behavior and chooses settlement, since he pays less in settlement. This implies less litigations.\(^{23}\) Moreover, we can easily confirm that the plaintiff’s incentive does not change with the increase in $Q_1$. Accordingly, when the court makes more type I errors, it is the defendant’s incentive which induces more settlements.

Now consider the change in $Q_2$. The marginal type defendant will more often opt for settlement, since the increased burden from litigation is larger than that from settlement: $\partial U_d/\partial Q_2 = (1 - v^*)/2$; $\partial U_d/\partial Q_2 = (1 - v^*)/2$. Nor does the plaintiff’s behavior change with a higher type II error, since $\partial U_d/\partial Q_2 = \partial U_d/\partial Q_2$. For that reason, there will be more settlements, initiated by the defendant, when the court makes more type II errors.

Hylton(1993) provides an interesting interpretation of court errors. She postulates that court errors will decline over time, for example, due to some error-reducing technology such as the stock of refined legal doctrines. Accepting Hylton’s postulation, we expect that the adjudication rate will increase over time, ceteris paribus.

(2) Comparative Statics for the Plaintiff’s Win Rate

\(^{23}\) One might think that defendants would prefer litigation as the type I error increases. Yet, such a conjecture is incorrect, since it only considers the litigation side, and ignores the settlement side. When type I error increases, both the plaintiff’s and the defendant’s estimates about plaintiff’s win rate change. The plaintiff, who does not know the true type of the defendant in trial, will have a lower winning estimate against all defendant types in $[v^*, 1]$. This implies that the plaintiff’s position in settlement becomes much weaker, or equivalently, that the defendant can retain a much larger portion from the settlement. The decrease in the plaintiff’s subjective win rate is in absolute terms larger than that estimated by the marginal type defendant. The marginal defendant benefits more from settlement, since the plaintiff’s subjective win rate affects the settlement outcome, while the litigation outcome is based only on the defendant’s objective estimate.
Now define $P_1$ to be the potential win rate of plaintiffs, which is the fraction of the plaintiff’s winning cases among all disputes including both those settled and litigated. In other words, $P_1$ is the plaintiff’s win rate from litigation when all types of defendants would actually go to trial.

$$P_1 = \int_0^1 \left[ v(1-Q_2) + (1-v)Q_2 \right] dv$$
$$= \frac{(1-Q_1) + Q_2}{2}. \quad (12)$$

Next, define $P_2$ to be the actual win rate of plaintiffs among the litigated cases.

$$P_2 = \int_0^v \left[ v(1-Q_1) + (1-v)Q_2 \right] \frac{1}{v} dv$$
$$= \frac{1}{2} (1-Q_1-Q_2) v^* + Q_2$$
$$= \frac{(1-Q_1) + Q_2}{2} - \frac{C_d + C_k}{2J}. \quad (13)$$

From equations (12) and (13), we can make two important predictions regarding the plaintiff’s win rate. The first prediction is that $P_2 < P_1$, which implies that the litigated cases are more skewed toward ‘defendant’s innocence’ than would be the case for the whole population. This is intuitively plausible and also consistent with the predictions of the existing literature regarding AI.

**Proposition 3.** Litigated cases are more skewed toward ‘defendant’s innocence’ than would be the case for the whole population of the disputes, that is, $P_2 < P_1$.

The second prediction is about the effects of the exogenous variables on the win rate. Based on equation (13), Proposition 4 summarizes them.

**Proposition 4.** The plaintiff’s win rate $P_2$ increases with the size of the stake $J$, and with the court’s type II error $Q_2$. However, it decreases with the litigation costs $C$, and with the court’s type I error $Q_1$. 

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The effects of the litigation costs and of the stake size on the win rate are straightforward, since they affect $P_2$ only through $\nu^*$ in equation (13). In contrast, the effects of the court errors are not so simple to understand, since they affect not only the adjudication rate, but also the ex-post win rate directly. Consider the type I error first. If $Q_1$ increases, the adjudication rate will decrease, and the defendants in trial become more ‘innocent,’ such that the plaintiff’s win rate decreases. Furthermore, a higher $Q_4$ implies that the court will identify a guilty defendant as innocent more often than before. This clearly reinforces the fall in the plaintiff’s win rate.

Next, consider the type II error. If $Q_2$ increases, there will again be fewer litigations, and defendants in trial become more innocent. This will lower the plaintiff’s win rate. However, there is a counter-effect. Since the chance that an innocent defendant will be identified as guilty increases, the plaintiff’s win rate will increase. Even though we have two opposite effects, it is not difficult to see that the positive effect is more pronounced than the negative effect. The negative effect is marginal in the sense that it affects only those defendants in the neighborhood of the marginal type $\nu^*$. Meanwhile, the positive effect is global in the sense that it affects all defendants $\nu \leq \nu^*$. Surely, the global effect would dominate the marginal effect.

Finally, if we accept Hylton’s (1993) assumption that court errors converge to zero over time, equation (13) predicts that the plaintiff’s win rate will converge to some number which is significantly less than 50%.

(3) Relationship between Adjudication Rate and Win Rate

Whether there is any systematic relationship between the adjudication rate and the win rate is also a main issue in the literature. Even though the current literature tries to deduce some specific correlation between these two indices (whether it is positive, negative, or even independent), we argue that such an attempt is incorrect. Specifically, the underlying assumption that the adjudication rate is the cause and the win rate is the effect is amiss.

The relationship between the two indices cannot be defined in a simple way. It is true that the win rate changes as the adjudication rate changes.

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24) Waldfogel (1998) also argues that the plaintiff’s win rate increases with $J$ and decreases with $C$, even though he does not provide any predictions about the effect of court errors.
However, the adjudication rate cannot be changed by itself. If there has been any change in the adjudication rate, it must be due to changes in the more fundamental exogenous factors such as $J$, $C_\delta$, $C_\nu$, $Q_1$, and $Q_2$. To put it differently, the adjudication rate is not an independent variable in the causal relationship, but rather a dependant variable per se, which is, just like the plaintiff’s win rate, affected by exogenous factors. It is not surprising that the same factor can affect the adjudication rate and the plaintiff’s win rate either in the same, or in opposite directions. Combining Propositions 2 and 4, we get the relationship between the two indices, as in Proposition 5.

**Proposition 5.** If the change is caused by type II error $Q_2$, the plaintiff’s win rate $P_2$ is negatively correlated with the adjudication rate $T$. To the contrary, if the change is caused by $J$, $C$, or $Q_1$, the plaintiff’s win rate is positively correlated with the adjudication rate.

(4) Implications of the ’50% Win Rate’ Hypothesis

Our analysis of the plaintiff’s win rate enables us to make some predictions about the famous ’50% win rate’ hypothesis of Priest and Klein (1984), which is considered as the major tenet of divergent expectations theory. If we can, from the AI model, derive some implications regarding this 50% win rate hypothesis, it should be another important step toward a reconciliation (Waldfogel, 1998) between the two competing theories.

From equation (13), it is clear that the win rate can be lower than, higher than, or equal to 50%, depending on the litigation costs, stake size, and court errors. If we make some reasonable assumptions about the court errors, more specific predictions are feasible.

First, if the court errors are small, the win rate will be lower than 50%, even though how much below 50% will depend again on the litigation costs and the size of the stake. With high litigation costs, more defendants will opt for settlement. Since the switch to settlement starts with 'more guilty' defendants,

25) Shavell(1996) also points out that the win rate can be any number, depending on the underlying probability distribution about the type of the party with private information. In our model, even under a given distribution function, the win rate can be any value, depending on the exogenous parameters.

26) Following Hylton(1993), such predictions can be realized in the long run.
the plaintiff’s win rate will be much lower than 50%. As litigation costs converge to zero, the plaintiff’s win rate also converges to 50% (under the assumption of negligible court errors). To the extent that the stake becomes higher, there will be more litigations with ‘less innocent’ defendants, driving the win rate up towards 50%.

Second, if the court makes type I and type II errors with an equal chance, the plaintiff’s win rate will be lower than 50%. If the court adopts a stricter rule to detect illegal cases so that type I errors decrease and type II errors (relatively) increase, the win rate will go up. Under this circumstance, a win rate higher than 50% is even possible. By the same token, if the court’s standards become more relaxed, the win rate will be much lower than 50%.

One final comment on the win rate in association with the 50% win rate hypothesis is inevitable. As Shavell (1996) correctly emphasizes, the actual win rate critically depends on the underlying distribution function for the defendant’s type. If we assume any distribution functions other than the uniform one, we might have different implications about the 50% win rate hypothesis. Accordingly, the most important message regarding the win rate should be that any win rate is possible as the title of Shavell’s paper suggests.

5. When the Plaintiff Has Private Information

What if the plaintiff has private information? Hylton (1993) makes an opposite prediction regarding the plaintiff’s win rate in such a case. She argues that, if the plaintiff has the informational advantage, a high win rate may be observed.\(^{27}\) Even though such a prediction seems quite intuitive, it is worth confirming its accuracy in our signaling game model.

To avoid unnecessary duplication of the calculations, we provide only intuitive predictions about the results.\(^{28}\) Except for two things the model is basically the same as the case in which the defendant has private information. First, since it is the plaintiff who has private information, the plaintiff moves first and the defendant moves later, after observing a signal from the plaintiff. Second, we assume a continuum types model of plaintiffs, not of defendants. A

\(^{27}\) A similar prediction is made in Shavell (1996).

\(^{28}\) The details about the equilibrium and comparative statics can be provided from the authors upon request.
plaintiff’s type is identified by the probability that he does not prevail, which is again denoted by $v$. Of course, $v$ is the private information of the plaintiff when the defendant only knows that $v$ is uniformly distributed in the range of $[0, 1]$.

We also keep the notations about court errors. $Q_1$ is the type I error such that the plaintiff prevails when he should not, and $Q_2$ is the type II error such that the plaintiff does not prevail when he should.

As in the case of continuum defendants, we will focus on the hybrid equilibrium where those with low $v$ choose litigation and those with high $v$ choose settlement. A hybrid equilibrium, if it exists, must be such that the plaintiffs with low $v$ reveal that they are low $v$ types by choosing litigation. This course is not imitated by the high $v$ plaintiffs.

**Proposition 6.** In a signaling lawsuit model where the plaintiff has private information about his own type, there exists an equilibrium such that:

- plaintiff’s strategy: $l$ for $v \leq v^*$, and $s$ for $v > v^*$,
- defendant’s belief: if $l$, then $v \leq v^*$, and if $s$ then $v > v^*$,
- defendant’s strategy: $s$

where $v^* = 1 - \frac{C_d + C_p}{f(1 - Q_1 - Q_2)}$ is the marginal plaintiff type who is indifferent between $s$ and $l$.

**Proposition 7.** When the plaintiff has private information, the adjudication rate $T$, the potential win rate $P_1$, and the actual win rate $P_2$ are as follows:

$$T = v^*.$$  

$$P_1 = \int_0^1 v Q_1 + (1 - v) (1 - Q_2) dv = \frac{Q_1 + (1 - Q_2)}{2}. \quad \quad (14)$$

$$P_2 = \int_0^{v^*} v Q_1 + (1 - v) (1 - Q_2) \frac{1}{v} dv$$

$$= (1 - Q_2) - (1 - Q_1 - Q_2) v^*/2$$

$$= \frac{Q_1 + (1 - Q_2)}{2} + \frac{C_d + C_p}{2f}. \quad \quad (15)$$

What predictions can we make about the adjudication rate and the plaintiff’s win rate? Since we already have some understanding about the selection mechanism for litigation, we can easily derive the following
observations for the case in which the plaintiff has private information.

**Proposition 8.** When the plaintiff has private information,

(i) $T$ increases with the size of the stake, $J$, and decreases with the
    litigation cost, $C$, and the court error, $Q$,
(ii) $P_2 > P_1$,
(iii) $P_2$ is positively related to $C_d$, $C_p$, and $Q_1$, and negatively related to $J$ and $Q_2$,
(iv) $P_2$ and $T$ move in the same direction when the change is initiated by $Q_2$, but move in the opposite direction when the change is initiated by $C_d$, $C_p$, $J$, and/or $Q_1$.

The case in which the plaintiff has private information renders the same predictions about the adjudication rate, but the exact opposite predictions about the plaintiff’s win rate and about the relationship between the two indices. Table 1 summarizes them. The final observation is that the plaintiff’s win rate might well be over 50%, if we assume either low court errors, or the equal chance of the two types of court errors.

<table>
<thead>
<tr>
<th>Defendant’s Private Information</th>
<th>Plaintiff’s Private Information</th>
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<tbody>
<tr>
<td>i) $T$↑ as $J$↑, $C$↓, $Q$↓</td>
<td>$T$↑ as $J$↑, $C$↓, $Q$↓</td>
</tr>
<tr>
<td>ii) $P_1 &gt; P_2$</td>
<td>$P_1 &lt; P_2$</td>
</tr>
<tr>
<td>iii) $P_2$↑ as $J$↑, $C$↓, $Q_1$↓, $Q_2$↑</td>
<td>$P_2$↑ as $J$↓, $C$↑, $Q_1$↑, $Q_2$↓</td>
</tr>
<tr>
<td>iv) $P_2$↑ as $T$↓ if $Q_2$ changes; $P_2$↑ as $T$↑ if $J$, $C$, $Q_1$ change; $P_2$↑ as $T$↑ if $Q_2$ changes</td>
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6. Concluding Remarks

In this paper we have attempted to provide a more comprehensive analysis for the AI nature of a legal dispute. Firstly, we explicitly solved a signaling game between the informed and the uninformed, both for the defendant and the
plaintiff, and found out the critical characteristics of the sequential equilibrium. We hope this will help in understanding the signaling nature of the lawsuit game under AI. Secondly, after separating settlement process from the litigation/settlement selection process, we introduced a more general bargaining process rather than the extreme ‘take-it-or-leave-it’ method, which, we posit, has made the analysis more robust against any arbitrary assumption about who moves first in a settlement/litigation selection game. Finally, based on the detailed equilibrium analysis, we were able to provide extensive comparative static predictions about the trial rate, plaintiff’s win rate, and the relationship between these two key indices. This will hopefully invoke many interesting empirical works for the AI theory.

We conclude the paper with two comments on the implications of our work on the divergent expectations theory. First, AI theory can be reconciled with the divergent expectations theory. Two major propositions of the divergent expectations theory might be the non–random selection of the litigated cases, and the famous 50% win rate hypothesis. Our analysis, just like all other AI models, is in accord with the non–random selection proposition. The underlying reasons for, or the characteristics of, the non–randomly selected samples in litigation might be significantly different between the divergent expectations theory and ours. Nonetheless, such differences are by no means impasses, but rather more viable motivations for further studies.

Second, the 50% win rate hypothesis of the divergent expectations theory has relevance to our model in two ways. In terms of the theoretic arena, AI theory justifies the hypothesis under certain conditions as discussed in Section 6. Furthermore, in terms of empirical observation, AI studies can also be pertinent to the 50% win rate with data aggregation increasingly pursued. To put it differently, if we assume that informational asymmetries between defendants and plaintiffs are randomly (symmetrically) distributed among all different types of legal cases, the plaintiff’s win rate can be predicted to converge to 50% as the aggregation level increases. The empirical validation of this last claim itself will be very intriguing, or obligatory at the least.
References


