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Incentive Monopoly Regulation with Entry

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Abstract
This paper analyzes the non-dichotomy nature of the entry and the price regulations under asymmetric information. When current market is a monopoly and there is a potential entrant, the government should make decisions both on the monopoly (price) regulation and on the entry regulation simultaneously. In this case, if information is incomplete, entry and price regulations should be incentive compatible not only individually but also jointly against each other. An integrated incentive regulation which incorporates entry and price regulations at the same time is derived for the comprehensive analysis of the monopoly regulation under asymmetric information.

JEL Classification: D82, L51
Keywords: asymmetric information, price regulation, entry regulation, incentive compatibility
1. Introduction.

A monopoly, or more broadly speaking a concentrated market, is always a main subject of the government regulation. The inefficiency generated by a monopoly justifies the behavioral regulation, for example the price regulation, by the benevolent government. At the same time, monopoly usually implies supernormal profit and so new firms would like to enter the market as long as entry is profitable. The benevolent government again intervenes in the entry decision since free entry may generate socially too many firms. The possibility of excess entry is the logical basis of the structural regulation, or simply the entry regulation, by the government.

Even though it is common in reality that price regulation and entry regulation are implemented against a (natural) monopoly market at the same time, they are always analyzed separately in the theoretic studies. Theories on the monopoly regulation usually assume no entry either due to the natural entry barrier or due to the entry regulation, without any further analysis regarding mutual impacts between price and entry regulations. On the other hand, entry regulation, even though it does not have rich analyses like price regulation, is usually analyzed without much reference to the price regulation before entry or after entry is regulated.

However, such a dichotomy between entry regulation and price regulation can be a major limit in the study of government regulations. If they are considered together, many existing theories on monopoly regulation might have to be retested. Let us consider two important points of the non–dichotomy between the two types of regulation: price regulation as a standard of entry regulation, and the joint incentive compatibility of price and entry regulations under asymmetric information.

First, the decision on entry regulation depends on the nature of the monopoly regulation. To implement an entry regulation, the government should compare the social welfare of the pre–entry and post–entry markets. However, if the pre–entry market is monopolistic, then there must have been a

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1) Refer to Braeutigam(1989) for the optimal regulation of a natural monopoly under complete information, and Baron(1989) for the incentive regulation under incomplete information.

2) This is a well–known fact for the natural monopoly. For the non–natural monopoly markets, or oligopoly markets, refer to Mankiw and Whinston(1986), Perry(1984), and Suzumura and Kiyono(1987) for the excess entry theorem.

3) In this paper, 'monopoly regulation' and 'price regulation' will be used interchangeably unless we need to distinguish them explicitly. Since we assume no price regulation on the post–entry oligopoly in this paper, there will be no confusion.
complementary monopoly regulation. This implies that the social welfare in the pre-entry (or entry-regulated) monopoly market should be evaluated by the regulated price, not by the unregulated monopoly price. Entry regulation is thus critically dependent on the nature of the monopoly regulation.

This seemingly natural logic, however, makes things more difficult. If monopoly is regulated by some optimal measures, then the social welfare under regulated monopoly can easily be higher than that under unregulated imperfect (for example, post-entry duopoly) market. Then should the government always regulate entry because it can make regulated monopoly more efficient than free entry markets? If the answer can hardly be affirmative, then what should be the correct standard to initiate entry regulation in connection with the price regulation on the monopoly market?

Second, if information is incomplete, the incentive compatibility of price and entry regulations should be satisfied simultaneously. Consider a case of government’s incomplete information about firm’s production cost. Assuming the same technology both to the incumbent and to the new entrant, a new entry, which incurs some fixed entry cost, is socially desirable when the production cost is low and it will be welfare decreasing under high production cost. Then, the incumbent will have an incentive to over-report the production cost to the regulator to induce an entry regulation and maintain its monopoly position.

Such an incentive problem should be taken into considerations in designing an optimal entry regulation, just as in optimal monopoly regulation under asymmetric information. Economists have accumulated abundant theories on the optimal monopoly regulation under asymmetric information. However, at least to my knowledge, there is no work on the incentive entry regulation even though the same incentive problem as under price regulation exists under entry regulation.

Furthermore, if the government implements entry regulation and price regulation at the same time, the incumbent’s incentive problem against both

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4) We can think of different model specifications to overcome such an inconsistency between optimal monopoly regulation and entry regulation, and justify the decision of entry allowance. For example, the government is not an ideal social welfare maximizer, or the imperfectly competitive markets are also regulated, and etc.
5) If only the incumbent’s cost is unknown to the government, while the cost of a new entrant is public information, the incumbent’s incentive will be under-reporting true cost to induce entry regulation. This is because, for given entrant’s cost, entry will be excessive when the incumbent has low cost. However, even in this case, it is all the same that the government should take double incentive problems into considerations when it designs entry and price regulations together.
types of regulation should be also handled simultaneously. Especially, it is 
important to note that the standard optimal price regulation under asymmetric 
information will not be incentive compatible any more if entry regulation is 
considered together. This problem is clearly because the incentive monopoly 
regulation in the current literature is derived without considering entry 
regulation simultaneously.

This paper is to find a solution to such non-dichotomy problems between 
monopoly (price) and entry regulations. More specifically, we will find an 
optimal monopoly regulation under asymmetric information which incorporates 
both entry regulation and price regulation simultaneously such that they are 
incentive compatible not only individually but also jointly with each other.

The structure of the paper is as follows. Section 2 provides a basic model 
with some justifications. The concept and the conditions of an incentive entry 
regulation under asymmetric information are given in Section 3, and the 
standard incentive monopoly regulation is derived in Section 4. Section 5 
combines entry and monopoly regulations and proposes an integrated incentive 
regulation under asymmetric information against a monopoly confronting a new 
entrant. Section 6 concludes the paper with some remarks.

2. Model.

Consider a homogeneous product market where firm 1 is the incumbent 
monopolist and firm 2 is a potential entrant. The inverse demand function is 
\( p = 1 - X \), where \( p \) is the market price and \( X \) is the total output level. Both 
firms have the same cost function \( C(x) = cx \) for a positive output level \( x > 0 \), 
and firm 2 must pay additional cost \( F > 0 \) when it enters the market.\(^7\) Assume 
that the unit variable cost \( c \) is a private information of the firms, and the 
benefvolent government, or the regulator, only knows the distribution of the true 
cost such that \( c \in [c_L, c_H] \) with \( f(c) > 0 \) for all \( c \) in this range.

A natural model to analyze a monopoly with a potential entrant and a 
regulator may be a three-person two-period game as described in <Figure 1>. 
In the first period \( t_1 \), firm 1 is a monopolist and reports its production cost to 
the government.\(^8\) Let \( \hat{c} \) be the reported cost. It is assumed that there is no

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\(^7\) We can also assume different costs, either correlated or independent, to the incumbent and 
to the entrant.

\(^8\) This is to follow the standard incentive mechanism design technique which is based on
production in the first period. This is to avoid the complicated issue of the incumbent’s strategic entry deterrence and concentrate on the optimality of the government regulations.\(^9\)

In the second period \(t_2\), firm 2 decides on entry, and if it decides to enter the market (IN), the benevolent government \(G\) implements an entry regulation to allow (Y) or disallow (N) firm 2’s entry. If entry is allowed, then firm 1 and firm 2 compete with each other \(\alpha\) la Cournot producing \(x_1\) and \(x_2\) respectively. If either firm 2 decides not to enter (OUT) or the government disallows entry, then the government implements a regulation (R) on the monopolist firm 1.

<Figure 1> Monopoly regulation game I

![Diagram](image)

In the monopoly subgames, firm 1’s payoff is the regulated monopoly profit \(\pi^{RM}\), and the payoff of the government is \(W^{RM}\), which is the social welfare of the regulated monopoly.\(^{10}\) In the post-entry duopoly subgame, firm 1 and firm 2 obtain symmetric profit \(\pi^D=\frac{(1-c)^2}{9}\), net of entry cost for firm 2, and the government’s payoff is \(W^D-F=4(1-c)^2/9-F\), which is the social welfare in the duopoly market at \(t_2\) net of firm 2’s entry cost. Note that since there is no regulation on the duopoly market, information is complete and therefore profits and social welfare are represented as functions of the true cost. The payoffs of firm 1, firm 2, and the government are shown in this sequence at the end of the game.

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\(^9\) If we allow production in the pre-entry stage, the government should take incumbent’s strategic behavior in the pre-entry stage into considerations when it designs an optimal regulation for a monopoly. See Kim (1997, 2000) for more on this issue.

\(^{10}\) The social welfare is measured as the sum of consumer surplus and firms’ profit.
Firm 2 will enter the market if and only if \( \pi_D - F = \frac{(1-c)D}{9-F} \geq 0 \). Since \( \pi_D \) is a decreasing function of \( c \) in the relevant range, there is a critical value \( c^0 \), as a function of \( F \), such that firm 2 will enter if and only if \( c \leq c^0 \). Assume \( c_H \leq c^0 \) so that firm 2 will always decide to enter regardless of \( c \) given \( F \).\(^{11}\)

Since there is no strategic entry deterrence by the incumbent monopolist at \( t_1 \), and firm 2 always wants to enter the market, the market structure at \( t_2 \) will be determined solely by the government’s entry regulation. The natural standard of an optimal entry regulation will be \( Y \) if \( W^P - F \geq W^{RM} \) and \( N \) otherwise. However, once we accept such a standard for an entry regulation, we have to face a fundamental problem regarding the relationship between monopoly and entry regulations, which was briefly mentioned in the introduction section.

If a monopoly is regulated by the benevolent government, and if some optimal regulation is implemented, and furthermore if the post-entry oligopoly market is not regulated even though competition is not perfect yet, then the regulated monopoly would easily be more efficient than the unregulated oligopoly. The first-best marginal cost price regulation will definitely be the case. However, even if the regulation is not the ideal first-best, the second-best one, or the optimal monopoly regulation under asymmetric information, will also generate such a situation.

This problem highlights the discrepancy between the optimal regulation theory and the regulatory practice, or between behavioral regulation (monopoly regulation) and structural regulation (entry regulation). Economists have continuously developed optimal regulations of the monopoly under various situations. If what they suggest is correct, then the regulated monopoly would be at least constrained optimal and so presumably more efficient than the unregulated imperfect market, for example, a post-entry duopoly market. If this is true, then the optimal entry regulation would be always limiting entry. Should the government always limit entry because it can make the regulated monopoly market more efficient than the (imperfectly) competitive market with entry? It is no doubt that the answer can hardly be affirmative.

How can we justify the inconsistency between entry and monopoly regulations? Even though we do not yet have a clear standard for an entry

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11) Since \( c^0 \) is a function of \( F \), the assumption \( c_H \leq c^0 \) cannot be true for all relevant values of \( F \). However, such an assumption is not harmful since the main focus of this paper is on the incentive entry and price regulations under asymmetric information when a new firm wants to enter the currently monopolistic market.
regulation in this regard, one possible solution is the separation of the regulatory authorities. Let’s modify the basic model in <Figure 1> such that entry regulation and monopoly regulation are implemented separately by two independent government authorities. Even though such a modification may be somewhat ad-hoc in terms of theory, it is closer to the regulatory practice.

For example, it is a common regulatory practice in many countries that while an independent regulatory authority is in charge of the monopoly regulation, entry is controlled at a different level of government under the more broad name of industrial policy or competition policy. In such a situation, the inconsistency between the entry decision and the following monopoly regulation can be a natural phenomenon.

<Figure 2> describes a modified version of the regulation game which will be adopted in this paper. \( G_E \) represents the government authority that controls entry into the market, and \( G_M \) refers to the standard monopoly regulation authority, and the two government authorities are assumed to be independent. We assume that \( G_E \) makes a decision on entry based on whether entry raises social welfare compared to the unregulated monopoly, and \( G_M \) implements an optimal monopoly regulation which maximizes expected social welfare under monopoly.

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12) Consider telecommunications industry as an example. In UK, entry is controlled by DTI (Department of Trade and Industry) while regulations on the dominant carriers are under the control of another independent regulatory authority OFTEL. The same practice can be found in other markets and in other countries.
The payoffs in <Figure 2> represent those for firm 1, firm 2, the government authority that controls entry \( (G_E) \), and the monopoly regulation authority \( (G_M) \) in this sequence. \( G_E \) makes a decision on entry regulation based on the relative value of \( W^P - F \) and \( W^M \), where both welfare indices are measured by unregulated market prices, and \( G_M \) chooses an optimal \( R \) which maximizes \( W^{PM} \), and thus the inconsistency between entry and price regulations in <Figure 1> can be overcome.

3. Incentive Entry Regulation.

Once we accept the modified version of the monopoly regulation game, the remaining task is to find out an optimal regulation of the monopoly which not only is incentive compatible by itself but induces proper incentive from the incumbent against entry regulation, too.

Define \( c^w \) such that \( \Delta W = W^P - W^M = 5(1-c^w)^2/\tau_2 = F \), then the optimal entry regulation under complete information will be \( Y \) if \( c \leq c^w \) and \( N \) otherwise. However, under asymmetric information, such an optimal entry regulation will collapse due to the strategic behavior of the incumbent. This is because \( \Delta W \) the welfare increment due to entry, is a decreasing function of the unit variable cost, and so the incumbent monopolist can maintain its monopoly position by reporting a high cost and inducing entry regulation. Then, how can we make the optimal entry regulation free of incentive distortion under asymmetric information?

**Definition.** An optimal entry regulation that allows entry for \( c \leq c^w \) and disallows otherwise is incentive compatible if the regulated firm with \( c \leq c^w \) reports \( c \leq c^w \), and that with \( c > c^w \) reports \( c > c^w \).\(^{13}\)

Incentive compatibility of the optimal entry regulation implies that the incumbent monopolist doesn't have any incentive to distort the entry regulation. If entry regulation is incentive compatible, then the government’s decision on

\(^{13}\) Actually, incentive compatibility of an entry regulation can be defined not only with an optimal entry regulation as in this definition but with any entry regulations.
entry control, even though it is subject to the asymmetric information, is the
first-best in the sense that it induces the same market structure as under
complete information.

Note that the incentive compatibility condition for an entry regulation is
discrete, or binary, contrary to the continuous nature of the incentive
compatibility for a price regulation. Under incentive compatible entry regulation,
the regulated firm may report other value than the true cost, however, there will
be no incentive problem because it does not distort the regulator’s entry
decision.

**Lemma 1.** The entry regulation that allows entry for \( \hat{c} \leq c^W \) and disallows
otherwise is incentive compatible if \( \pi^{RM}(c) \in [\pi^D(c), \pi^D(c^W)] \) for all \( c \in (c^W, c_H] \).

**<Proof>** Refer to <Figure 3> for the proof.\(^{14}\) First assume that the true cost
is such as \( c \leq c^W \). If \( \hat{c} = c \), then entry is allowed and firm 1’s profit becomes
\( \pi^D(c) \). On the other hand, if firm 1 reports \( \hat{c} > c^W \) and induces entry regulation,
it obtains a regulated profit \( \pi^{RM}(\hat{c}) \). Since \( \pi^{RM}(\hat{c}) \leq \pi^D(c^W) \leq \pi^D(c) \), firm 1 does
not have an incentive to report a higher cost than the truth to induce entry
regulation. Second, assume that \( c > c^W \). Truth-telling brings \( \pi^{RM}(c) \) to the
incumbent. If firm 1 under-reports enough to distort entry regulation such that
\( \hat{c} \leq c^W \), then there is an entry and firm 1 obtains \( \pi^D(c) \) which is less than
\( \pi^{RM}(c) \). Firm 1 has no incentive to under-report and distort entry regulation.
To sum, the regulated firm does not have any incentive to distort entry
regulation by reporting other value than the true cost. Q.E.D.

When the condition in the Lemma 1 is satisfied, firm 1 will not report
other value than \( c \) in the range of \( \hat{c} \leq c^W \) since the profit is the same for both
\( c \) and \( \hat{c} \). However, <Figure 1> shows that firm 1 will under-report in the
range of \( c > c^W \) since the regulated monopoly profit is based on the reported
cost. The continuity of the incentive compatibility under the entry regulation,
even though it is not required when only entry regulation is considered, will
also be satisfied by the complementary monopoly regulation as we can see later
in the paper.

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14) <Figure 3> is drawn assuming \( \pi^D(c_H) = F \) without any loss of generality.
Lemma 1 shows that we can design an entry regulation which gives a right incentive to the incumbent monopolist by imposing an additional restriction on the regulated monopoly profit, or equivalently on the monopoly regulation.\(^{15}\) Note that the standard incentive monopoly regulation, which does not take entry regulation into considerations together, cannot satisfy the condition in Lemma 1. Therefore, we need to design a new optimal monopoly regulation which satisfies the additional condition in Lemma 1 and so induces right incentive from the incumbent monopolist not only against monopoly regulation itself but against entry regulation at the same time.

4. Incentive Monopoly Regulation without Entry.

Does Lemma 1 mean that the government can achieve the first-best entry regulation even under asymmetric information? This is an important question since we know that, in general, the first-best outcome cannot be obtained under asymmetric information. Especially in regulation theory, it is well-known that the first-best is not feasible under incomplete information because some rent must be given up to induce proper incentive from the regulated firm.\(^{16}\)

\(^{15}\) 'Additional restriction' implies that the optimal monopoly regulation should satisfy the condition in Lemma 1 in addition to the usual constraints such as incentive compatibility and individual rationality.

\(^{16}\) See Laffont and Tirole(1993) for the trade-off between rent extraction and incentive
It should be clear that the standard trade-off between rent extraction and incentive provision still exists in our incentive entry regulation. Even though there is no loss of optimality in entry regulation under asymmetric information if the condition in Lemma 1 is satisfied, it is because the informational burden shifts to the monopoly regulation without disappearing. It is obvious that, with an additional constraint, the monopoly regulation becomes less strict to extract monopoly rent.

The optimal monopoly regulation with entry should now satisfy two incentive compatibility constraints: one for the entry regulation and another for the traditional monopoly regulation without entry. Let’s find out the conditions which satisfy the double incentive constraints embodied in the profit function.

**Lemma 2.** If a monopoly regulation without entry is incentive compatible with profit function $\pi(c)$, then it is also incentive compatible with a new profit function $\pi'(c) = a\pi(c) + \beta$, where $a \geq 0$ and $\beta$ are constants.

**Proof** Pick any $c$, $c' \in [c_L, c_H]$, $c \neq c'$. Incentive compatibility under $\pi'$ implies that $\pi'(c' \mid c) = a\pi(c' \mid c) + \beta \geq a\pi(c \mid c) + \beta = \pi'(c \mid c)$, and this holds if and only if $\pi(c' \mid c) \geq \pi(c \mid c)$ for $a > 0$, which is the incentive compatibility condition under $\pi$. Q.E.D.

Lemma 2, even though it is simple and intuitively straightforward, is very helpful in deriving incentive compatibility conditions both for the entry and for the monopoly regulation simultaneously. This can be done in two steps. First, find an incentive compatible monopoly regulation without considering entry following the standard technique in the current literature. Second, modify the profit function and find $a$ and $\beta$ which satisfy the incentive condition in Lemma 1.

Let $(\mu(c), s(c))$ be the monopoly regulation without entry, where $\mu(c)$ is the regulated price and $s(c)$ is the subsidy, both as functions of firm’s cost $c$. Lemma 3, which is from Baron and Myerson(1982), describes the optimal monopoly regulation under asymmetric information.

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(provision under incomplete information.

17) Since the optimal monopoly regulation is incentive compatible, we will use the true cost instead of reported cost.)
\textbf{Lemma 3.} The optimal monopoly regulation \((p^*(c), s^*(c))\) under asymmetric information, which maximizes expected social welfare subject to the incentive compatibility and the voluntary participation constraints, ignoring the incentive problem against entry regulation, is \(p^*(c) = \Phi(c)\) and 
\[s^*(c) = \int_c^{c^*} (1 - \Phi(r)) dr - (\Phi(c) - c)(1 - \Phi(c)),\] for all \(c \in [c_L, c_R]\), where 
\[
\Phi(c) = c + (1 - \lambda) \frac{F(c)}{f(c)} \text{ for } \lambda \in [0, 1] \]
is the adjusted marginal cost of the regulated firm, which is assumed to be non-decreasing in \(c\). The monopoly profit under such a regulation is \(\pi^*(c) = \int_c^{c^*} (1 - \Phi(r)) dr\).\(^{18}\)

<Proof> See Appendix.

5. Incentive Monopoly Regulation with Entry.

Now, let’s combine Lemma 1, 2, and 3 to design an integrated incentive regulation for a monopoly market with entry. The remaining question is whether we can find \(a\) and \(\beta\) which transform \(\pi^*(c)\) derived in Lemma 3 to \(\pi^{RM}(c)\) which satisfies the incentive constraint in Lemma 1. If we can find such \(a\) and \(\beta\), then the monopoly regulation in Lemma 3 and the entry regulation in Lemma 1 together give right incentive to the incumbent monopolist against entry as well as against monopoly regulations under asymmetric information.

\textbf{Proposition 1.} An optimal incentive regulation for a monopoly market with entry under asymmetric information consists of an entry regulation, a price regulation, and a tax/subsidy as follows.
(1) entry regulation : allow entry if \(c \leq c^W\) and disallow entry if \(c > c^W\),
(2) monopoly price regulation : \(p^*(c) = \Phi(c)\), where \(\Phi(c) = c + (1 - \lambda) \frac{F(c)}{f(c)},\) \(\lambda \in [0, 1],\)
(3) tax/subsidy to the monopoly : \((1 - a^*)\%\) tax on \([p^*(c) - c]x^*(c)\) and

\(^{18}\) Since the monopoly regulation without entry is implemented only in case of \(c > c^W\), the cost distribution can be redefined such as \(c_L = c^W\), with corresponding redefinitions of \(f\) and \(F\).
\(a^*s^*(c)+\beta^*\) lump-sum subsidy, where 
\[a^* = \frac{2(1-c^W)}{9(1-\Phi(c^W))},\]
\[\beta^* = \pi^D(c^W) - a^*\pi^*(c^W),\]
and 
\[s^*(c) = \int_c^{c^W} (1-\Phi(r))dr - (\Phi(c) - c)(1-\Phi(c)).\]

<Proof> Refer to <Figure 3> again for the proof. From Lemma 1, in order for the incumbent monopolist to have a proper incentive against the optimal entry regulation, the regulated monopoly profit \(\pi_{RM}(c) = a^*\pi^*(c) + \beta\) should satisfy the condition \(\pi^D(c) \leq \pi_{RM}(c) \leq \pi^*(c)\) for all \(c \in (c^W, c_H]\). First note that this condition requires \(\pi_{RM}(c^W) = a^*\pi^*(c^W) + \beta = \pi^D(c^W)\) at \(c = c^W\). Furthermore, since both \(\pi_{RM}(c)\) and \(\pi^D(c)\) are decreasing and convex, the slope of \(\pi_{RM}(c)\) must be greater than that of \(\pi^D(c)\) at \(c = c^W\) for the condition \(\pi^D(c) \leq \pi_{RM}(c)\) to be satisfied for all \(c \in (c^W, c_H]\). That is,
\[
\frac{d\pi_{RM}(c^W)}{dc} = -a(1-\Phi(c^W)) \geq \frac{d\pi^D(c^W)}{dc} = -\frac{2}{9}(1-c^W),
\]
or equivalently,
\[
a \leq \frac{2(1-c^W)}{9(1-\Phi(c^W))).
\]
However, since \(\pi_{RM}(c)\) becomes more deviating from \(\pi^D(c)\) as \(a\) decreases, the optimal \(a\) which minimizes rent to the regulated monopolist will be
\[
a^* = \frac{2(1-c^W)}{9(1-\Phi(c^W))}.
\]
Then from \(\pi_{RM}(c^W) = a^*\pi^*(c^W) + \beta = \pi^D(c^W)\), we can find
\[
\beta^* = \pi^D(c^W) - a^*\pi^*(c^W) = \pi^*(c^W) - \frac{2(1-c^W)}{9(1-\Phi(c^W))}\pi^*(c^W).
\]
Q.E.D.

The implementation of the optimal monopoly regulation is as follows: the government offers \((p^*, s^*, a^*, \beta^*)\) to the incumbent monopolist, and then the monopolist will accept such an offer, reporting true cost to the government. If \(c \leq c^W\), there will be an entry in \(t_2\), and the Cournot–Nash equilibrium will be the post-entry market equilibrium. If \(c > c^W\), entry will be disallowed by the government. In this case, the monopoly price is regulated at \(p^*(c)\), and the monopoly firm in \(t_2\) will pay \((1-a^*)%\) of the net revenue \([p^*(c) - c]\pi^*(c)\) as tax and receives \(a^*s^*(c) + \beta^*\) as a lump-sum subsidy from the government.

Under \((p^*, s^*, a^*, \beta^*)\), social welfare is maximized under asymmetric information, and the regulated monopolist in case of no entry obtains
\[
\pi_{RM}(c) = a^*\pi^*(c) + \beta^* = a^*\left[p^*(c) - c\right][1 - p^*(c)] + [a^*s^*(c) + \beta^*].
\]
**Corollary 1.** If \( \lambda=1 \), then \( p^*(c)=c \), \( s^*(c)=\int_c^{c_H} (1-r)dr \), \( a^*=\frac{2}{3} \), \( \beta^*=(\frac{1-c_H}{9})^2 \), and \( \pi^{RM}(c) \) is the same as \( \pi^D(c) \).

If \( \lambda=1 \), \( \varphi(c)=c \), and \( p^*(c)=c \). In this case, since the regulated monopolist’s revenue is equal to cost, there will be no tax payment ex-post, and the regulated profit is simply equal to the subsidy from the government, which is \( s^*(c)=\int_c^{c_H} (1-r)dr \). Furthermore, if \( \lambda=1 \), then \( a^*=\frac{2}{3} \), \( \beta^*=(\frac{1-c_H}{9})^2 = \pi^D(c_H) \), and \( \pi^{RM}(c) \) becomes identical to \( \pi^D(c) \). The monopoly firm is receiving the same profit as under duopoly with entry.

Note that even the most inefficient type of the monopoly firm obtains strictly positive rent under the integrated incentive regulation in Proposition 1. This is clear with \( \lambda<1 \) since \( \pi^{RM}(c) \) is strictly above \( \pi^D(c) \). Even if \( \lambda=1 \), this holds again since \( \pi^{RM}(c)=\pi^D(c) \) and \( \pi^D(c_H)\geq F>0 \). The fact that the most inefficient type of the monopoly firm obtains positive profit under regulation implies that the trade-off between rent extraction and incentive provision in our model is more skewed away from the rent extraction than under the standard incentive price regulation. As we already know, this is because the government is subject to double incentive constraints; one for the standard monopoly regulation and another for the entry regulation.

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6. **Concluding Remarks.**

This paper is only the first step toward an integrated theory of monopoly regulation. The current studies in the field of government regulation are mostly partial approaches which focus on some specific elements of the regulated markets. Price regulation under complete and incomplete information, which is the most blossomed subject in regulation theory, excludes other related aspects in the monopoly market such as entry regulation and the incumbent’s strategic entry deterrence against potential entrants.

On the other hand, entry regulation does not provide any clear description about complementary regulations (for example, a price regulation) on the concentrated markets. Furthermore, we also do not know much about the impact
of various forms of government regulations on the strategic behavior of the incumbent confronting new entrants. An integrated theory is necessary to better understand the strategic interdependence among the involved players such as the incumbent, potential entrants, and the regulator, and the mutual impacts between the structural and the behavioral regulations. This is the motivation of this paper.

Several questions are brought up for the next step. First, what if a single regulatory authority is in charge of both entry regulation and monopoly regulation? This is the question about the consistency between the structural and the behavioral regulations that is raised in Section 2 of this paper. It seems somewhat difficult to compromise the success in the theory of optimal regulations for a monopoly and the practical need to encourage entries into the monopoly market.

Second, what if tax/subsidy is distortionary? Many economists recognize that public fund through tax and subsidy distorts efficiency and take this into considerations when they set up the regulator’s objective function. Since the solution of this paper includes tax and subsidy which are assumed to be non-distortionary, it needs to be checked against distortionary case.\textsuperscript{19}

Third, a more general model which includes incumbent’s strategic entry deterrence should be interesting. As Kim(1997, 2000) shows that entry regulation changes incumbent’s incentive toward entry–deterring which would not have been an optimal strategy if there were no entry regulation, the price regulation, or more broadly the monopoly regulation might also change entry–deterring incentive of the incumbent monopolist.

Finally, we need a new technique to find out an incentive compatible mechanism when the underlying utility function of the agent is not continuous. The analysis of an incentive entry regulation under asymmetric information reveals that the current technique of incentive mechanism design cannot be directly applied to the entry regulation. It is because the underlying profit function of the regulated firm is discontinuous depending on entry and no–entry. Even though an incentive compatible entry regulation is obtained in this paper, it is due to the joint analysis of the monopoly regulation. If we have to handle some discontinuous cases without resorting to any other continuous cases as complementarities, a new approach is needed to derive an optimal incentive mechanism.

\textsuperscript{19} See Laffont and Tirole(1986, 1993) for the analysis of the distortionary public fund.
<Appendix> Proof of Lemma 3.

The proof is based on the standard technique of the incentive mechanism design in Baron and Myerson (1982). Consumer surplus is
\[ CS(c) = \int_0^{x(c)} (1-r)d\bar{r} - \rho(c)x(c) - s(c), \]
and the social welfare is defined as
\[ W(c) = CS(c) + \lambda \pi(c), \quad \lambda \in [0, 1]. \]

For any \( c \) and \( c' \) in \([c_L, c_H]\), \( c \neq c' \), incentive compatibility conditions imply

\[ \begin{align*}
    \pi(c) &= [\rho(c) - c]x(c) + s(c) \\
    \pi(c') &= [\rho(c') - c']x(c') + s(c')
\end{align*} \]

which can be rewritten as

\[ \begin{align*}
    \pi(c) &\geq \pi(c') - (c - c')x(c') \\
    \pi(c') &\geq \pi(c) - (c' - c)x(c).
\end{align*} \]

Combining the two conditions, we obtain the following condition.

\[-(c - c')x(c') \leq \pi(c) - \pi(c') \leq -(c - c')x(c)\]

Dividing both sides by \((c - c')\) and taking a limit, we can find the condition for the incentive compatibility.

\[ \lim_{c \to c_H} -x(c') \leq \lim_{c \to c_H} \frac{\pi(c) - \pi(c')}{c - c'} \leq \lim_{c \to c_H} -x(c) \]

\[ \pi'(c) = -x(c) \] \(20\)

Integrating both sides, the profit of the regulated firm becomes as follows.

\[ \int_c^{c_H} \pi(r)dr = - \int_c^{c_H} x(r)dr \]

\(20\) We can derive another condition for the incentive compatibility such that \( x(c) \) is decreasing in \( c \). This condition will be confirmed to hold after optimal regulation is derived.
\[
\pi(c_H) - \pi(c) = \int_c^{c_H} x(r)dr
\]
\[
\pi(c) = \pi(c_H) + \int_c^{c_H} x(r)dr.
\]

Since \( \pi(c) \) is decreasing in \( c \), the voluntary participation condition can be simplified as \( \pi(c_H) = 0 \) with a normalization, and then the regulated monopoly profit becomes \( \pi(c) = \int_c^{c_H} x(r)dr \) with properties of \( \pi'(c) < 0 \) and \( \pi(c_H) = 0 \).

Finally, the government finds a price which maximizes expected social welfare.

\[
EW = \int_c^{c_H} \left[ \int_0^{\pi(c)} [\pi'(c)dx - \pi(c)x(c) - s(c) + \lambda \pi(c)]f(c)dc \right]dc
\]
\[
= \int_c^{c_H} \left[ \int_0^{\pi(c)} [\pi'(c)dx - \alpha(c) - (1 - \lambda) \pi(c)]f(c)dc \right]dc
\]
\[
= \int_c^{c_H} \left[ \int_0^{\pi(c)} [\pi'(c)dx - \alpha(c)]f(c)dc - (1 - \lambda) \int_c^{c_H} \pi(c)f(c)dc \right]dc
\]
\[
= \int_c^{c_H} \left[ \int_0^{\pi(c)} [\pi'(c)dx - \alpha(c)]f(c)dc - (1 - \lambda) \int_c^{c_H} x(r)dr f(c)dc \right]dc
\]
\[
= \int_c^{c_H} \left[ \int_0^{\pi(c)} [\pi'(c)dx - \alpha(c)]f(c)dc - (1 - \lambda) \int_c^{c_H} x(c)f(c)dc \right]dc
\]
\[
= \int_c^{c_H} \left[ \int_0^{\pi(c)} [\pi'(c)dx - \alpha(c)]f(c)dc - (1 - \lambda) \int_0^{\pi(c)} x(c)dF(c)dc \right]dc
\]

Define \( \Phi(c) = c + (1 - \lambda) \frac{F(c)}{f(c)} \) and assume \( \Phi(c) \) is nondecreasing in \( c \).

Then \( \int_c^{c_H} \left[ \int_0^{\pi(c)} [\pi'(c)dx - \Phi(c)x(c)]f(c)dc \right]dc \) is maximized when the integrand is maximized at \( \rho^*(c) = \Phi(c) \). The monopoly profit is

\[
\pi^*(c) = \int_c^{c_H} (1 - \Phi(r))dr = (\Phi(c) - c)(1 - \Phi(c)) + s^*(c).
\]

Therefore, the optimal subsidy is \( s^*(c) = \int_c^{c_H} (1 - \Phi(r))dr - (\Phi(c) - c)(1 - \Phi(c)) \).

Q.E.D.

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21) \( F \) is the cumulative distribution function of \( f \).

22) This assumption is for the optimal output level to be decreasing in \( c \). If \( \Phi(c) \) is not non-decreasing for some \( f \), we can redefine \( \Phi(c) \) following Baron and Myerson (1982).
References


