Delegated Contracting and Corporate Hierarchies

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Abstract

In a typical corporate hierarchy, the manager is delegated the authority to make decisions that set directions for the organization, employ subordinates and contract with external suppliers. This paper explains when such delegation of authority can be optimal, using a model of a firm with three parties: the principal, the manager and the worker. In centralization with two two-tier hierarchies, the principal designs contracts for both agents. In delegation with a three-tier hierarchy, the principal directly contracts with a delegated agent who, in turn, contracts with the other agent. We identify an environment where the principal can benefit from delegating authority to the manager, but not to the worker. Beneficial delegation arises endogenously when delegation motivates the manager to acquire valuable information, which is used for better decision-making and more efficient incentive provision to the worker. We also show how total surplus is distributed in delegation vis-à-vis centralization, document comparative statics results regarding the benefits of delegation and the distribution of total surplus, and discuss when delegation is more likely to dominate centralization.

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JEL Classification: C72, D21, D82, L22.

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1. Introduction

The so-called separation of ownership and control (Berle and Means, 1932; Fama and Jensen, 1983) refers to the fact that the nominal owners of corporations - shareholders - delegate authority to managers. The authority is vested in several important dimensions for the top managers of corporations. They make executive decisions that set directions for corporations, employ subordinates and contract with external suppliers. This multiple dimension of authority is a deciding factor for the organizational form of corporations. Rather than a set of two-tier hierarchies in which owners are at the top of each two-tier hierarchy, modern corporations are often organized as multi-tier hierarchies.\(^{(1)}\) Chandler (1977, 1990) attributes such a transformation of family-oriented “personal capitalism” to “managerial capitalism” in the US to a sharp increase in demand for, and supply of professional, qualified managers as corporations become larger with increasingly sophisticated operations. The resulting modern business enterprise, according to Chandler, is an organization with many distinct operating units that are managed by a hierarchy of professional, salaried executives. In such organizations, shareholders hire top managers - through boards - and managers, in turn, hire subordinates or contract with external suppliers. Why are such multi-tier hierarchies, rather than multiple two-tier hierarchies, often the norm? Why are managers, instead of other stakeholders, at the center of the multi-tier hierarchy? This study attempts to provide answers to these questions from an incentive perspective.

A typical explanation for delegation in corporations is based on managers’ expertise and the ensuing benefits of specialization. Jensen and Murphy (1990, p. 251) put it aptly: “Managers often have better information than shareholders and boards in identifying investment opportunities and assessing the profitability of potential projects; indeed, the expectation that managers will make superior investment decisions explains why shareholders relinquish decision rights over their assets by purchasing common stocks.” Underlying this explanation is the assumption that communicating managers’ information is costly, or that shareholders or boards do not have necessary expertise to process the information for decision-making even if communicating the information is costless.\(^{(2)}\) For, otherwise, shareholders or boards will be able to make decisions based on the information that managers have, which is the central insight from the revelation principle.

We take Jensen and Murphy’s explanation as a starting point, but go a step further by assuming that managers need to incur private costs to acquire and process information. The incentive problem becomes relatively easier without such costs. Our basic model is thus em-

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\(^{(1)}\) Separation of ownership and control in this sense, although not universally the case, is most prevalent in the Anglo-American system of corporate governance. See La Porta, Lopez-De-Silanes and Shleifer (1999).

\(^{(2)}\) The benefits of hierarchies facing the costs of communicating and processing information have been put forward by Arrow (1974) and Williamson (1983) among many others.
bedded in an environment where managers can, at some costs, acquire information necessary for investment decisions, which cannot be used by shareholders in designing incentive contracts for managers. Several authors have resorted to such contractual incompleteness either implicitly or explicitly to explain why a multi-tier hierarchy with delegation can be superior to a centralized mechanism. In the context of general revelation mechanisms, Melumad, Mookherjee and Reichelstein (1995) show that the outcome of an optimal revelation mechanism can be achieved using decentralized contracts and proper sequencing of the contracts. Thus their main point is that, when various contracting costs such as those of communicating information necessary for the revelation mechanism are taken into account, there may be benefits to delegation. Laffont and Martimort (1998) show that delegation can dominate centralized contracts when the possibility of collusion down the hierarchy is combined with limits on communication. The limits on communication, according to these authors, require the centralized contracts be anonymous, and different agents be treated symmetrically. This facilitates collusion. With decentralization, such a problem disappears.

Our paper is similar in spirit to the above studies, but has more concrete objectives. Specifically we describe what we believe is a realistic, but tractable model of a corporate hierarchy, and show when and why putting managers at the center of the multi-tier hierarchy can benefit shareholders. The main point of this paper can be explained using a simple scenario. Consider a firm that consists of three parties, whom we call the principal, the manager and the worker. The firm has two investment projects, for which the principal provides necessary funds. The manager can acquire private information at some costs, which can be used in choosing a right project to undertake. The worker can exert effort that can increase the likelihood that the chosen project is successful. Neither the manager’s private information nor the worker’s effort can be used for contracting purpose. A centralized mechanism in this setup has the principal offering contracts to both agents. A hierarchical mechanism puts either the manager or the worker at the center of the three-tier contracting relationships: the principal designs a contract for the agent at the center, who, in turn, designs a contract for the other agent.

Our main point is that a hierarchical mechanism with proper delegation can dominate centralized contracting. The intuition is as follows. When the manager’s private information cannot be used for centralized contracting purpose, there is a limit on the types of contracts the principal can offer the manager and the worker. This constrains the principal’s ability in inducing desired actions from both agents, which may result in efficiency losses. On the other hand, in a hierarchical mechanism where the manager is at the center, the manager can design the worker’s contract after learning his private information. While he cannot condition the worker’s contract on the private information, he can signal the private information through the contract offered. This can alleviate the asymmetry of information between the manager and the worker, thereby enabling the manager to design a contract that can provide work incentives
at lower costs than the one designed by the principal. However, it does not automatically follow that such efficiency gains will flow back to the principal. This is because the manager needs to be compensated for in order to realize the efficiency gains. If the required rent to the manager is too large, then delegation could even hurt the principal despite overall efficiency gains. We identify factors that affect the size of the manager’s rent and characterize conditions under which the principal is better off by delegating to the manager. Our analysis also shows why delegating to the worker cannot benefit the principal, since the worker is in no better position than the principal when offering contracts to the manager.

Other studies on delegation in a hierarchy include, among others, Baron and Besanko (1992), Gilbert and Riordan (1995), McAfee and McMillan (1995), Baliga and Sjöström (1998), Macho-Stadler and Pérez-Castrillo (1998), Mookherjee and Reichelstein (2001), and Faure-Grimaud, Laffont and Martimort (2003). Baron and Besanko (1992), and Gilbert and Riordan (1995) establish equivalence between centralized and decentralized mechanisms when risk-neutral agents provide complementary inputs to production. McAfee and McMillan (1995) consider a three-tier hierarchy subject to limited liability constraints, showing losses involved in a three-tier hierarchy relative to centralized contracting. Equivalence of a decentralized mechanism and a centralized mechanism subject to the possibility of side-contracting is established in a moral hazard environment by Baliga and Sjöström (1998), and Macho-Stadler and Pérez-Castrillo (1998), in an environment with additional coordination problems by Mookherjee and Reichelstein (2001), and in a principal-supervisor-agent setup by Faure-Grimaud, Laffont and Martimort (2003). An additional conclusion of Baliga and Sjöström (1998) relates to the pattern of delegation: the agent with superior information is more likely to be delegated. While not directly concerned with delegation, Itoh (1992, 1993) studies a multiple-agent moral hazard environment to show when the principal can benefit by allowing coalition of agents, when agents can monitor each other. With the equivalence result described above, his findings can be regarded as supportive of delegation over centralization when agents have informational advantages over the principal.

Our work differs from, but complements these and afore-mentioned studies on hierarchy at least in two important ways. In our model, the manager, or the intermediate agent, is not endowed with private information. Rather, he needs to incur private costs to acquire information. Because of this information acquisition, there are benefits from delegating authority to the manager to represent shareholders in dealing with other stakeholders. In the above studies on hierarchy, there is no a priori reason why a particular agent should be at the center of the

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(3) In a similar vein but in a costly verification environment, Choe (1998) shows that the contract designed by the informed party can reduce the verification cost compared to the one designed by the uninformed party. The reason is that the informed party, in an attempt to maximize the value of his residual claim, designs the contract to give himself truth-telling incentives.
multi-tier hierarchy.\(^{(4)}\) It could be any of the agents supplying inputs. In our model, beneficial delegation occurs only when the manager, not the worker, assumes the role. Second and related, the managerial input and the worker’s input are quite distinct. We believe that the manager’s information acquisition and subsequent decision making are what distinguish managerial inputs from those of other employees in corporations. Roughly speaking, the manager’s decision making can be identified with the choice of a particular distribution of profits, while other employees’ inputs affect the likelihood of profit realization given the chosen distribution. It is in this sense that one of the manager’s roles can be described as that of direction setting. We thus expect optimal incentive schemes for the manager to be quite different from those for other employees. Indeed we show that the manager, when delegated authority, can actively affect his own payoff through the choice of project and the design of contract for the worker. Thus incentives and authority are strongly complementary for the party who is delegated authority. For the worker, the scope of such influence upon his own payoff is less significant, as is the case for employees lower in the corporate hierarchy: the worker in our paper is paid an efficiency wage under manager delegation.\(^{(5)}\)

The rest of the paper is organized as follows. Section 2 describes the basic model. Section 3 studies the centralized mechanism. Section 4 analyzes the case of manager delegation, which is then compared with centralization in Section 5. Section 6 discusses other relevant issues that are left out: worker delegation and the possibility of using additional information for mechanism design. Section 7 concludes the paper. The appendix contains the proofs of the results that are not central to the exposition of our main ideas.

## 2. The Model

There are three parties, whom we call the principal, the manager and the worker.\(^{(6)}\) The principal has two projects, denoted by \(\psi_1\) and \(\psi_2\), whose return has the same support: \(x > 0\) (success) or 0 (failure). The return is publicly observable and can be used for contracting purpose.\(^{(7)}\) The manager can privately observe a signal \(\theta \in \{\theta_1, \theta_2\}\) at a monetary cost of \(c > 0\), which we will call information gathering. We can also think of a signal as a perfect predictor of a ‘state’. It is common knowledge that \(\theta_1\) will be observed with probability

\(^{(4)}\) As mentioned above, Baliga and Sjöström (1998) is a notable exception.

\(^{(5)}\) One could take this as an incentives-based explanation of why stock options have been the single most important incentive for CEOs in Anglo-American corporations (Murphy, 1999). While the use of stock options for non-executive employees was also growing in the late 1990s (Core and Guay, 2001), the proportion of incentives provided through stock options is eclipsed compared to that for CEOs (Economist, 2003, p. 9.).

\(^{(6)}\) We will use the female gender pronoun for the principal and the male gender pronoun for the manager and the worker.

\(^{(7)}\) If the return has a different support, then contracts can be written effectively on the identity of project as well. We rule out this possibility for most part of the paper. In Section 6.2, we discuss the case where project choice can be contracted upon.
\( \pi \in (0, 1) \). The manager’s decision of information gathering is denoted by \( d_m \in \{0, 1\} \). If \( d_m = 0 \), then the manager observes nothing and we denote this null signal by \( \emptyset \), and the set of all possible signals by \( \Theta = \{\theta_1, \theta_2, \emptyset\} \). The worker privately chooses “work” or “shirk”, respectively denoted by \( d_w = 1 \) and \( d_w = 0 \). The monetary cost of work is \( \ell \) and that of shirk is normalized to 0. Given \( \theta_i \), the success probability for \( \psi_1 \) (\( \psi_2 \), respectively) is \( p_i \) \( (q_i \), respectively) if \( d_w = 1 \).\(^8\) If \( d_w = 0 \), then the success probability is \( r \) for either project and state.\(^9\) We assume that all the players are risk neutral, limited liability sets a lower bound of 0 for payments to the manager and the worker, and that reservation utilities for both agents are zero.

The principal wishes to hire the manager to use his information for project choice, and the worker to exert effort for the chosen project. As the manager is the only player who can contribute to project choice through his private information, we will assume that, once hired, the manager is delegated the project choice decision. The manager’s project choice decision is denoted by a mapping \( C : \{\theta_1, \theta_2\} \rightarrow \{\psi_1, \psi_2\} \) if \( d_m = 1 \) and by \( C(\emptyset) \in \{\theta_1, \theta_2\} \) if \( d_m = 0 \). One can also imagine a message game where the principal asks the manager to report his signal, based on which she makes a project choice decision. For this revelation game to be meaningful, the principal needs to commit to a rule that details how the manager’s report will be used for project choice, which is known to the manager. Since the principal does not have an opportunity to gather information herself, real authority of project choice resides with the manager while the principal’s role is reduced to that of rubber-stamping. It is thus without loss of generality to assume that the manager has both formal and real authority of project choice (Aghion and Tirole, 1997). Moreover, we rule out the possibility of such message games in line with the Grossman-Hart-Moore models (Grossman and Hart (1986), Hart and Moore (1990)) and focus on the case where the project return is the only variable available for contracting purpose.

Define \( \Delta p_i \equiv p_i - r, \Delta q_i \equiv q_i - r, i = 1, 2 \). These are improvements in success probabilities due to the worker’s contribution. We maintain the following assumptions.

**Assumption 1**: \( \pi p_1 + (1 - \pi) p_2 > \pi q_1 + (1 - \pi) q_2 \).

**Assumption 2**: \( p_1 > p_2 > r, q_2 > q_1 > r, \Delta p_1 > \frac{\ell}{x} > \Delta q_2 \).

**Assumption 3**: \( (1 - \pi)(q_2 - p_2) > \frac{\Delta}{x} \).

**Assumption 4**: \( [\pi \Delta p_1 + (1 - \pi) \Delta p_2]x > \ell \).

Assumption 1 states that \( \psi_1 \) is better than \( \psi_2 \) given the prior belief. To understand the implications of the remaining assumptions, consider the following project choice decision:

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\(^8\) Projects are identified with \( p \) and \( q \) and, states, with the subscripts.

\(^9\) Our main qualitative results are robust to different success probabilities depending on the state and project when \( d_w = 0 \), as long as they are sufficiently small relative to \( p_i \) and \( q_i \).

\(^10\) \( p_1 > p_2 \) is implied by the rest of Assumption 2 and Assumption 3, but we listed it for completeness.
$C(\theta_i) = \psi_i$ for $i = 1, 2$. Assumption 2 implies that such a decision is necessarily optimal and that the worker’s contribution is valuable only in $\psi_1$ given this decision.\(^{(11)}\) Assumption 3 says that the above project choice decision leads to a larger expected return than when $\psi_1$ is chosen all the time, given that the worker’s action is the same in both cases. Assumptions 1 and 3 jointly lead to $\pi(p_1 - q_1) > \frac{c}{x}$. That is, the expected return from the above project choice decision is also larger than that from always choosing $\psi_2$, given the same action by the worker. Finally, Assumption 4 states that the expected return (less the worker’s cost of work) from always choosing $\psi_1$ and engaging the worker is larger than that from not engaging either agent at all, the latter being $rx$. This assumption is intended to make the trivial option of engaging neither agent a less likely outcome to be chosen by the principal. These assumptions lead to

**Lemma 1:** If the principal can observe both the manager’s signal and the worker’s action, and enforce desired actions, then the resulting first-best outcome involves the following: the manager gathers information ($d_m = 1$) and chooses project $i$ if and only if $\theta_i$ is observed ($C(\theta_i) = \psi_i$ for $i = 1, 2$), and the worker exerts effort only in state 1 ($d_w = 1$ only in $\theta_1$).\(^{(12)}\)

**Proof:** See the appendix.

We will call the project choice decision in the first-best outcome optimal. Assumptions 1 to 4 portray a natural but economical environment where a meaningful comparison of delegation vis-à-vis centralization can be made. That is, delegation to a proper agent (the manager, as we are about to establish) can implement a larger set of outcomes than centralization, including the first-best one. It is because delegation can motivate the agent to acquire valuable information and the revelation game is precluded. On the other hand, the delegated agent will have to be given incentives to implement the outcome that the principal desires but cannot implement under centralization. Providing such incentives may require leaving a larger rent to the delegated agent than centralization. Delegation therefore entails both costs and benefits to the principal. The central aim of this paper is to investigate when such benefits outweigh costs, in which case meaningful delegation will emerge endogenously. Given that the principal has a

\(^{(11)}\) If the worker’s input is always valuable, then the manager’s information is valuable only for project choice, but not for incentive provision to the worker. In this case, the principal can provide separate incentives to the manager and the worker, thereby implementing the desired outcome through centralization. Therefore delegation and the accompanied interlocking incentives do not have much bite. If the worker’s optimal input is state-dependent, however, then the manager’s information has additional value: it can be used for providing efficient work incentives to the worker. The latter part of the manager’s information can only be utilized under delegation if the manager’s information cannot be used for contracting purpose.

\(^{(12)}\) When $d_w = 0$, project choice becomes irrelevant since success probability is the same in both projects and states. The first-best outcome, to be more precise, is thus: $d_m = 1, C(\theta_1) = \psi_1, C(\theta_2) \in \{\psi_1, \psi_2\}, d_w = 1$ only in $\theta_1$. Since this outcome yields the same expected return as the outcome in Lemma 1, we focus on the latter outcome only.
final say in the choice of mechanism, we could view such endogenous delegation as an incentive-based explanation of transition from “personal capitalism” to “managerial capitalism”.

3. Centralization

Under centralization, the principal designs contracts for both agents. Because contracts are based on the final return only, and limited liability sets a lower bound for payments, centralized contracts are a payment $s \geq 0$ to the manager and $w \geq 0$ to the worker if $x$ is realized, and 0 otherwise. We will focus on the principal’s problem of implementing a desired outcome as a Nash equilibrium of the game between the two agents. Since the principal does not have access to the manager’s information, she cannot induce different work decisions from the worker in different states: the worker is induced to work in both states or shirk in both states. Note also that the principal is never better off if the manager, after gathering information, makes a project choice other than the optimal one. Moreover, the principal is (weakly) better off with $\psi_1$ than $\psi_2$ if the manager does not gather information. The last two observations are due to the assumptions we made above. Finally, if the worker chooses to shirk in both states, then the manager’s information gathering and project choice do not have any value. Therefore, there are essentially only three different outcomes for the principal to consider.

**LEMMA 2**: In equilibrium under centralization, one of the following outcomes takes place:

- **(C1)** The manager gathers information ($d_m = 1$), makes an optimal project choice ($C(\theta_i) = \psi_i$ for $i = 1, 2$), and the worker chooses to work in both states ($d_w = 1$ in $\theta_1$ and $\theta_2$);
- **(C2)** The manager does not gather information ($d_m = 0$), always chooses $\psi_1$ ($C(\emptyset) = \psi_1$), and the worker chooses to work in both states ($d_w = 1$ in $\theta_1$ and $\theta_2$);
- **(C3)** The worker chooses to shirk in both states regardless of the manager’s decisions ($d_w = 0$ in $\theta_1$ and $\theta_2$).

We solve below for optimal centralized contracts implementing each of the above outcomes.

**3.1. Centralization implementing outcome (C1)**

Let $U(d_m, d_w)$ be the worker’s expected payoff given $(d_m, d_w)$. Note that, if $d_m = 1$, then the manager will always make the optimal project choice since his contract is monotonic in the

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(13) This is because we rule out the possibility of side contracting between the two agents. If side contracting is allowed, then the principal is able to implement different work decisions from the worker, similar in spirit to Itoh (1993) that allowing coalition of agents can benefit the principal when agents have superior information. Since such a collusive outcome under centralization can be replicated by proper decentralization, this brings us back to the equivalence principle: centralization with the possibility of collusion and proper decentralization lead to the same outcome.
final return. Given \( d_m = 1 \) and the optimal project choice, the worker’s incentive compatibility constraint is:

\[
U(1, 1) \geq U(1, 0) \iff [\pi p_1 + (1 - \pi) q_2] w - \ell \geq [\pi r + (1 - \pi) r] w
\]

\[
\iff w \geq \frac{\ell}{\pi \Delta p_1 + (1 - \pi) \Delta q_2}.
\]

Thus the optimal contract for the worker in this case is \( w_C = \frac{\ell}{\pi \Delta p_1 + (1 - \pi) \Delta q_2} \).

Let \( V(d_m, d_w) \) be the manager’s expected payoff given \( (d_m, d_w) \) and the optimal project choice when \( d_m = 1 \). That is, \( V(1, 1) = [\pi p_1 + (1 - \pi) q_2] s - c \). When \( d_m = 0 \), denote \( V_i(0, d_w) \) to be the manager’s expected payoff when he always chooses \( \psi_i \), \( i = 1, 2 \). That is, \( V_i(0, 1) = [\pi p_1 + (1 - \pi) p_2] s \) and \( V_2(0, 1) = [\pi q_1 + (1 - \pi) q_2] s \). The manager could also randomize, but we can ignore this since randomization will be dominated by either of the above two. Given \( d_w = 1 \), then the manager’s incentive compatibility constraint is:

\[
V(1, 1) \geq \max\{V_1(0, 1), V_2(0, 1)\} \iff s \geq \max\left\{ \frac{c}{(1 - \pi)(q_2 - p_2)} \frac{c}{\pi(p_1 - q_1)} \right\}.
\]

Due to Assumption 1, \( (1 - \pi)(q_2 - p_2) < \pi(p_1 - q_1) \). Thus the optimal contract for the manager is given by \( s_C = \frac{c}{(1 - \pi)(q_2 - p_2)} \). The expected payoff for the principal is then

\[
Z_{C1} = [\pi p_1 + (1 - \pi) q_2] \left( x - \frac{c}{(1 - \pi)(q_2 - p_2)} - \frac{\ell}{\pi \Delta p_1 + (1 - \pi) \Delta q_2} \right).
\]

3.2. Centralization implementing outcome (C2)

In this case, the payment to the manager will be always 0 and we assume that the manager, out of indifference, makes a project choice decision that is desired by the principal. If the manager always chooses \( \psi_1 \), then the incentive compatibility constraint for the worker to choose \( d_w = 1 \) is given by \( [\pi p_1 + (1 - \pi) p_2] w - \ell \geq r w \), or \( w \geq \frac{\ell}{\pi \Delta p_1 + (1 - \pi) \Delta p_2} \). If the manager always chooses \( \psi_2 \), then the incentive compatibility constraint for the worker is \( [\pi q_1 + (1 - \pi) q_2] w - \ell \geq r w \), or \( w \geq \frac{\ell}{\pi \Delta q_1 + (1 - \pi) \Delta q_2} \). Due to Assumption 1, the optimal contract for the worker in this case is \( w_C = \frac{\ell}{\pi \Delta p_1 + (1 - \pi) \Delta p_2} \). The expected payoff for the principal is then

\[
Z_{C2} = [\pi p_1 + (1 - \pi) p_2] \left( x - \frac{\ell}{\pi \Delta p_1 + (1 - \pi) \Delta p_2} \right).
\]

\( ^{(14)} \) Participation constraints are satisfied for both agents as long as incentive compatibility constraints are, so we ignore participation constraints throughout.
3.3. Centralization implementing outcome (C3)

In this case, $w_C = s_C = 0$ and the expected payoff for the principal is

$$Z_{C3} = rx.$$  \hfill (5)

Combining these three cases, we have

PROPOSITION 1: The optimal contracts under centralization are one of the following:

(a) If $Z_{C1} \geq \max\{Z_{C2}, Z_{C3}\}$, then $s_C = \frac{C}{(1-\pi)(q_2-p_2)}$ in case of success, 0 otherwise, and
$$w_C = \frac{\ell \pi d_1 + (1-\pi) d_2}{\Delta p_1 + (1-\pi) \Delta p_2}$$ in case of success, 0 otherwise, which implement outcome (C1);
(b) If $Z_{C2} \geq \max\{Z_{C1}, Z_{C3}\}$, then $s_C = 0$, $w_C = \frac{\ell \pi d_1 + (1-\pi) d_2}{\Delta p_1 + (1-\pi) \Delta p_2}$ in case of success, 0 otherwise, which implement outcome (C2);
(c) If $Z_{C3} \geq \max\{Z_{C1}, Z_{C2}\}$, then $s_C = w_C = 0$, which implement outcome (C3).

4. Manager Delegation

The principal continues to design contracts for the manager, who is now delegated the authority to design a contract for the worker. To distinguish notation from the previous section, we now use $\sigma$ and $\omega$ to denote the payment to the manager and the worker, respectively, in case of success. Therefore the manager’s net income in case of success is $\sigma - \omega$. Since the manager has, or may have acquired, private information when offering a contract to the worker, we focus on a sequential equilibrium of the subcontracting game between the two agents. In the sequential equilibrium of the subcontracting game, the manager designs $\omega$ given his information gathering and project choice decisions. Given $\omega$, the worker forms a belief regarding the manager’s information gathering ($d_m$), project choice ($\psi \in \{\psi_1, \psi_2\}$), and the signal observed ($\theta \in \{\theta_1, \theta_2, \emptyset\}$). Denote this belief by $\mu(d_m, \psi, \theta | \omega)$.

Our focus is whether the principal can benefit from delegation. Intuitively a necessary condition for this is that the manager should be able to use his private information to provide better work incentives to the worker: the worker is induced to work at a minimal cost only when his work is valuable. In other words, the first-best outcome we looked at previously seems to be a candidate equilibrium outcome of the subcontracting game that can potentially benefit the principal compared to centralization. Of course, such benefits will not materialize if the principal has to pay too high a rent to the manager to induce the first-best outcome. In this section, we delineate when such benefits indeed materialize.

We start by identifying possible equilibria in the subcontracting game between the manager and the worker. We can divide all possible outcomes into several cases. First, the manager gathers information and makes an optimal project choice. Given this, there are four possibilities: $d_w = 1$ only in state 1, which is the first-best outcome; $d_w = 1$ in both states, which is
the same as outcome (C1) under centralization; $d_w = 1$ only in state 2; $d_w = 0$ in both states. Clearly the last outcome cannot be implemented since the manager will not have incentives to work if $d_w = 0$ in both states. The third outcome will be dominated by the first one due to Assumption 2. The only meaningful cases are thus the first two: the first-best outcome and (C1). Next are the outcomes where the manager gathers information but makes a suboptimal project choice. But each of the outcomes in this case will be dominated by the corresponding outcome of the previous case where the project choice is an optimal one. Finally there are outcomes where the manager does not gather information and hence cannot induce different work decisions from the worker in different states. Here we only need to consider the case where $\psi_1$ is chosen always since $\psi_2$ is dominated by $\psi_1$. One possible outcome from this is where the worker is induced to work in both states, which corresponds to outcome (C2) under centralization. The other possibility is where the worker is induced to shirk in both states, which is the same as outcome (C3) under centralization. Summarizing, we have

**LEMMA 3:** In equilibrium under manager delegation, one of the following takes place:

- **(MD1)** The manager gathers information ($d_m = 1$), makes an optimal project choice ($C(\theta_i) = \psi_i$ for $i = 1, 2$), and induces work from the worker only in state 1 ($d_w = 1$ only in $\theta_1$);
- **(MD2 = C1)** The manager gathers information ($d_m = 1$), makes an optimal project choice, and induces work from the worker in both states ($d_w = 1$ in $\theta_1, \theta_2$);
- **(MD3 = C2)** The manager does not gather information ($d_m = 0$), chooses $\psi_1$ ($C(\emptyset) = \psi_1$), and induces work from the worker in both states ($d_w = 1$ in $\theta_1, \theta_2$);
- **(MD4 = C3)** The manager does not gather information, chooses either project and induces shirk from the worker in both states ($d_w = 0$ in $\theta_1, \theta_2$).

We argued above that the principal may benefit from manager delegation if (MD1) ensues. In Section 4.1, we first show that the principal is never better off with manager delegation when delegation implements outcomes other than (MD1). This leads us to focus on outcome (MD1) for the case of beneficial delegation, which is discussed in Section 4.2.

### 4.1. Manager delegation implementing outcomes other than (MD1)

From Lemma 3, we know that outcomes other than (MD1) that can be implemented under manager delegation are exactly those that are implementable under centralization. Intuition tells us, then, that the principal may not be better off by delegating to the manager to implement the same outcome that she could implement under centralization. With centralization, the principal directly controls both agents’ incentives. With delegation, the principal directly controls only the manager’s incentives while the worker’s incentives are indirectly controlled through the manager’s contract. This creates the problem of double incentivization akin to
that of double marginalization, which could increase the total payment to the two agents compared to centralization. Therefore, if the same outcome is implemented under both regimes and so the total surplus remains the same, the principal would be generally worse off with delegation.

PROPOSITION 2: The principal is never better off under manager delegation than under centralization when delegation implements outcomes other than (MD1).

PROOF: See the appendix.

4.2. Manager delegation implementing (MD1)

The equilibrium leading to outcome (MD1) is described in more detail as follows: the principal contracts with the manager paying $\sigma \geq 0$ in case of success; the manager accepts the contract and incurs $c$ to gather information; if $\theta_1$ is observed, he chooses $\psi_1$ and offers the worker $\omega_1 \geq 0$ in case of success, which the worker accepts and exerts effort (i.e., $d_w(\omega_1) = 1$); if $\theta_2$ is observed, the manager chooses $\psi_2$ and offers the worker $\omega_2 \geq 0$ in case of success, which the worker accepts and does not exert effort (i.e., $d_w(\omega_2) = 0$).\(^{(15)}\)

Below we check the conditions for such a strategy profile to indeed constitute an equilibrium. First, note that $\omega_1 \neq \omega_2$ so that the worker can respond differently to the two equilibrium wage offers. Since the manager can ensure inducing $d_w = 0$ from the worker with an arbitrarily low but nonnegative wage offer, $\omega_2 = 0$ in equilibrium. When $\omega_1$ is offered, the worker infers correctly that the success probability is $p_1$ if he works and $r$ if not, hence he would work in equilibrium as long as

$$\omega_1 \geq \frac{\ell}{\Delta p_1}. \quad (6)$$

Since the worker observes $\omega_1$ but neither $\theta$ nor the project choice by the manager, the worker’s strategy is a function of wage offer only. Given $\omega_1$ satisfying (6), the worker’s equilibrium strategy is thus $d_w(\omega) = 0$ for all $\omega < \omega_1$, which is supported by the worker’s belief, $\mu(d_m = 1, \psi_1, \theta_1 \mid \omega \geq \omega_1) = \mu(d_m = 1, \psi_2, \theta_2 \mid \omega < \omega_1) = 1$.\(^{(16)}\) The manager’s equilibrium expected payoff is then

$$V_1 = \pi p_1 (\sigma - \omega_1) + (1 - \pi) r \sigma - c. \quad (7)$$

To check the manager’s incentive compatibility, we now consider possible deviations by the manager. Suppose first the manager deviates by inducing different work decisions from the

\(^{(15)}\) If $\theta_2$ is observed and $d_w = 0$ is induced, it does not matter which project is chosen. We assume in this case that the manager chooses $\psi_2$.

\(^{(16)}\) We focus on pure strategies of the worker only since the worker would accept any positive wage offer because nonnegative payoff is guaranteed by shirking.
worker, while adhering to the rest of the equilibrium strategy. That is, \( d_m = 1 \) and \( C(\theta_i) = \psi_i \) for \( i = 1, 2 \). First, the manager can implement \( d_w = 1 \) in both states by offering \( \omega_1 \) in both states. The resulting expected payoff for the manager is

\[
V_2 = [\pi p_1 + (1 - \pi) q_2](\sigma - \omega_1) - c. \quad (8)
\]

Second, the manager can implement \( d_w = 0 \) by offering \( \omega = 0 \) in both states, and securing himself \( r\sigma - c \). However this will be worse for the manager than choosing \( d_m = 0 \) and inducing \( d_w = 0 \), which leads to the expected payoff of \( r\sigma \). In other words, \( r\sigma \) is the minimum payoff the manager can secure himself. So we ignore this deviation. Third, the manager can induce \( d_w = 1 \) only in state 2 by offering \( \omega_1 \) only in state 2. The manager’s expected payoff in this case is

\[
V_3 = \pi rs + (1 - \pi) q_2(\sigma - \omega_1) - c. \quad (9)
\]

Next we can consider the manager’s deviations, \( d_m = 1 \) but \( C(\theta_i) = \psi_j, i \neq j \). Regardless of subsequent wage offers and \( d_w \), the manager will not benefit from this compared to the above cases. This is because of Assumptions 1 and 2, which imply that \( \psi_1 \) dominates \( \psi_2 \), and that \( \theta_i \) is good news for project \( i \). So we can ignore this case.

The remaining cases involve \( d_m = 0 \). First, the manager can choose \( \psi_1 \) and induce \( d_w = 1 \) in both states by offering \( \omega_1 \) in both states. The expected payoff in this case is

\[
V_4 = [\pi p_1 + (1 - \pi)p_2](\sigma - \omega_1). \quad (10)
\]

Second, the manager can choose \( \psi_2 \) and induce \( d_w = 1 \) in both states by offering \( \omega_1 \) in both states. But this will be dominated by the above deviation due to Assumption 1.\(^{(17)}\) Finally, the manager can secure himself \( r\sigma \) by offering \( \omega = 0 \) in both states regardless of project choice. The minimum expected payoff the manager can secure himself is

\[
V_5 = r\sigma. \quad (11)
\]

For \( V_1 \) to be the equilibrium expected payoff for the manager, we need \( V_1 \geq V_k \), for \( k = 2, ..., 5 \). Note that \( V_1 \geq V_5 \) implies that \( \pi p_1 (\sigma - \omega_1) - c \geq \pi r \sigma \), hence \( V_2 \geq V_3 \). Therefore, \( V_1 \geq V_3 \) is implied by \( V_1 \geq V_2 \) and \( V_1 \geq V_5 \), so the manager’s incentive compatibility is satisfied if and only if the following inequalities hold:

\[
V_1 \geq V_2 \iff \sigma \leq \frac{q_2}{\Delta q_2} \omega_1; \quad (12)
\]

\(^{(17)}\) When \( d_m = 0 \), the manager cannot offer separating contracts. The manager can also randomize, but this will be dominated by either of the two pure strategies.
\[ V_1 \geq V_4 \iff \sigma \leq \frac{p_2}{\Delta p_2} \omega_1 - \frac{c}{(1-\pi)\Delta p_2}; \quad (13) \]

\[ V_1 \geq V_5 \iff \sigma \geq \frac{p_1}{\Delta p_1} \omega_1 + \frac{c}{\pi \Delta p_1}. \quad (14) \]

Summarizing the discussions so far, we can conclude that the principal can implement (MD1) if the set of \((\sigma, \omega_1)\) satisfying (6), (12), (13) and (14) is not empty. In that case, the principal can maximize her expected payoff by offering \(\sigma^* = \frac{p_1}{\Delta p_1} \omega_1 + \frac{c}{\pi \Delta p_1}\) to the manager. The following proposition describes the optimal contracts under manager delegation that implement (MD1).

**PROPOSITION 3:** The principal can implement outcome (MD1) under manager delegation by offering the manager \(\sigma^* = \frac{p_1}{\Delta p_1} \omega_1 + \frac{c}{\pi \Delta p_1}\) in case of success, where \(\omega_1\) is the optimal subcontract that the manager offers to the worker in case of success when \(\theta = \theta_1\), as specified below:

(a) If \(\pi > \frac{q_2 - p_2}{p_1 - p_2}\) and \(\ell < \frac{c \Delta p_1 \pi \Delta p_1 + (1-\pi)\Delta p_2}{\pi (1-\pi)(p_1 - p_2)^r} \equiv \ell_1(\pi)\), then \(\omega_1 = \frac{c \pi \Delta p_1 + (1-\pi)\Delta p_2}{\pi (1-\pi)(p_1 - p_2)^r} \equiv \ell_1(\pi)\);

(b) If \(\pi > \frac{q_2 - p_2}{p_1 - p_2}\) and \(\ell \geq \frac{c \Delta p_1 \pi \Delta p_1 + (1-\pi)\Delta p_2}{\pi (1-\pi)(p_1 - p_2)^r} \equiv \ell_1(\pi)\), then \(\omega_1 = \frac{\ell}{\Delta p_1}\);

(c) If \(\pi \leq \frac{q_2 - p_2}{p_1 - p_2}\) and \(\ell < \frac{c \Delta p_1 \pi \Delta q_2}{\pi (p_1 - q_2)^r} \equiv \ell_2(\pi)\), then \(\omega_1 = \frac{c \Delta q_2}{\pi (p_1 - q_2)^r} \equiv \ell_2(\pi)\);

(d) If \(\pi \leq \frac{q_2 - p_2}{p_1 - p_2}\) and \(\ell \geq \frac{c \Delta p_1 \pi \Delta q_2}{\pi (p_1 - q_2)^r} \equiv \ell_2(\pi)\), then \(\omega_1 = \frac{\ell}{\Delta p_1}\).

**PROOF:** See the appendix.

To understand the intuition behind Proposition 3, let us denote \(\omega^* \equiv \frac{\ell}{\Delta p_1}\), \(\omega_{12} \equiv \frac{c \Delta q_2}{\pi (p_1 - q_2)^r}\), and \(\omega_{13} \equiv \frac{c \pi \Delta p_1 + (1-\pi)\Delta p_2}{\pi (1-\pi)(p_1 - p_2)^r}\). Note that \(\omega^*\) is the minimum wage that satisfies the worker’s incentive constraint (6), and \(\omega^* \leq \min\{\omega_{12}, \omega_{13}\}\) for each of the corresponding cases. Since the manager has residual claim in the subcontracting stage with the worker, the principal is more likely to benefit from manager delegation and (MD1) if \(\omega_1 = \omega^*\). Figure 1 describes the optimal subcontract for the worker. It is easy to verify that \(\pi \geq \frac{q_2 - p_2}{p_1 - p_2}\) if and only if \(\ell_1(\pi) \geq \ell_2(\pi)\). As shown in the figure, the optimal subcontract for the worker is \(\omega^*\) in either of the following two situations. First, if \(\ell\) is large enough, then a wide range of \(\pi\) admits \(\omega^*\) as an optimal contract for the worker. Suppose, for example, that \(\pi\) is sufficiently large or small, which implies that the manager’s information does not have much value. For large \(\pi\), the principal could be better off by asking the manager to choose \(\psi_1\) without information gathering, thus implementing (C2) under centralization. Similarly, for sufficiently small \(\pi\), the principal could be better off with outcome (C3). Thus, for the principal to benefit from manager delegation when the manager’s information is not valuable enough, \(\ell\) should be large enough, which can be saved with manager delegation when \(\theta_2\) is observed. Second, \(\pi\) is in the intermediate range if \(\ell\) is not large enough. That \(\pi\) is in the intermediate range implies that the manager’s information and the optimal project choice are sufficiently valuable. Therefore, if \(\ell\) is not large enough, then the manager’s information has to be valuable enough for the principal to benefit from manager delegation.
In the next section, we compare centralization with manager delegation when delegation implements (MD1) and the equilibrium subcontract for the worker is $\omega^*$. The principal’s expected payoff from manager delegation is then

$$Z_D = [\pi p_1 + (1 - \pi)r] \left( x - \frac{c}{\pi \Delta p_1} - \frac{p_1 \ell}{(\Delta p_1)^2} \right).$$

(15)

5. Comparing Centralization with Manager Delegation

Let us summarize what we have obtained so far. There are three possible equilibrium outcomes under centralization, (C1), (C2) and (C3), each leading to the principal’s expected payoff given by (3), (4) and (5). Due to Proposition 2, we know that the only possible way manager delegation can dominate any of the above is when delegation implements (MD1), which leads to the principal’s equilibrium expected payoff given by (15) if the optimal contract for the worker is $\omega^*$. We start with a numerical example.

We set parameter values: $p_1 = 0.9$, $p_2 = 0.3$, $q_1 = 0.4$, $q_2 = 0.45$, $\pi = 0.63$, $r = 0.25$, $\ell = 4$, $c = 0.1$, $x = 20$. These values satisfy Assumptions 1 to 4 and the conditions under which $\omega^*$ is the optimal contract for the worker under manager delegation. The principal’s equilibrium expected payoffs are then $Z_{C1} = 7.28$, $Z_{C2} = 7.22$, $Z_{C3} = 5$, $Z_D = 7.41$, verifying that the principal is better off under manager delegation when outcome (MD1) is implemented. To see how manager delegation performs relative to centralization, we plot how the principal’s expected payoffs change as $c$, $\ell$ and $\pi$ change. Figure 2.1 shows how the principal’s equilibrium expected payoffs change when $c$ changes from 0.01 to 0.39. As $c$ increases, $Z_{C1}$, $Z_{C2}$ and $Z_D$ all decrease, but $Z_{C2}$ decreases at the smallest rate, with the optimal outcome changing from (C1) to (MD1), and then to (C2). In Figure 2.2, the principal’s equilibrium expected payoffs are plotted against $\ell$ as $\ell$ changes from 4 to 7.8.(18) As $\ell$ increases, all of them decrease but $Z_D$ decreases at the smallest rate, making (MD1) dominate (C1) and (C2) until eventually (C3) becomes optimal. Finally, Figure 2.3 shows how the principal’s equilibrium expected payoffs change as $\pi$ changes from 0.3 to 0.68. As explained before, small or large $\pi$ implies that the manager’s information does not have much value. This is shown in the figure: (MD1) is initially dominated by (C3) (up to $\pi = 0.31$), becomes optimal for intermediate values of $\pi$, but eventually dominated by (C2) as $\pi$ becomes larger (for $\pi$ larger than 0.65). This exercise seems to suggest the following as necessary conditions for beneficial manager delegation. First, neither the manager’s cost of information gathering nor the worker’s cost of work should be

(18) We set $c = 0.08$ so that $Z_{C1}$ is larger than $Z_D$ at the start.
extreme. Second, the manager’s information should be reasonably valuable. In what follows, we will formally compare centralization with manager delegation, identify factors that favor manager delegation over centralization, and provide interpretations. In doing so, our discussion will be focused on comparing (C1) and (MD1) since other outcomes, namely (C2) and (C3), do not involve active inputs from one or both of the agents.

— Figure 2 goes about here. —

From the above example, we know that there exist suitable parameter values for which manager delegation with (MD1) dominates any other outcomes under centralization. Let \( R_{C1}, U_{C1}, \) and \( V_{C1} \) denote, respectively, the gross expected return (net of the manager’s and the worker’s costs), the worker’s equilibrium expected payoff, and the manager’s equilibrium expected payoff under centralization and outcome (C1). Similarly, let \( R_D, U_D \) and \( V_D \) denote those under manager delegation and (MD1). Then the principal’s equilibrium expected payoffs are \( Z_{C1} = R_{C1} - U_{C1} - V_{C1} \) and \( Z_D = R_D - U_D - V_D \), hence \( Z_D - Z_{C1} = (R_D - R_{C1}) - (U_D - U_{C1}) - (V_D - V_{C1}) \). Decomposing the change in the principal’s expected payoffs this way helps us identify the costs and benefits of delegation.

Note first that \( R_{C1} \equiv [\pi p_1 + (1 - \pi)q_2]x - (c + \ell) \) and \( R_D \equiv [\pi p_1 + (1 - \pi)r]x - (c + \pi \ell) \). Due to Assumption 2, we have \( R_D - R_{C1} = (1 - \pi)(\ell - \Delta q_2 x) > 0 \). This is one source of possible benefits from delegation. That is, delegation can lead to a larger expected return by inducing an optimal effort decision from the worker. Next let us compare the worker’s equilibrium expected payoffs: \( U_D = \pi(p_1\omega^* - \ell) = \frac{\pi r}{\Delta p_1} \) and \( U_{C1} = [\pi p_1 + (1 - \pi)q_2]w_C - \ell = \frac{\pi (p_1 - q_2)\omega^*}{\pi \Delta p_1 + (1 - \pi)\Delta q_2} \) where we have used \( \omega^* = \frac{\ell}{\Delta p_1} \) and \( w_C = \frac{\ell}{\pi \Delta p_1 + (1 - \pi)\Delta q_2} \). Since \( \Delta p_1 > \Delta q_2 \), it follows that \( U_{C1} > U_D \). This is another source of possible benefits from delegation. Under centralization, the worker exerts effort in both projects even when putting in effort in \( \psi_2 \) is suboptimal. Because of limited liability and the principal’s inability to offer state-dependent contracts, suboptimal effort in \( \psi_2 \) is not penalized and the worker enjoys a rent larger than is necessary. Since manager delegation can eliminate this inefficiency, the worker would be strictly worse off under manager delegation. On the other hand, the manager may or may not be better off under delegation. Comparing the manager’s equilibrium expected payoffs, we have \( V_D - V_{C1} = \pi p_1 (\sigma^* - \omega^* - s_C) + (1 - \pi)(r\sigma^* - q_2 s_C) \) where \( s_C = \frac{c}{(1 - \pi)(q_2 - p_2)} \) and \( \sigma^* = \frac{c}{\pi \Delta p_1} + \frac{p_1 \ell}{(\Delta p_1)^2} \). The first term is the change in the manager’s payoff from \( \psi_1 \): the manager receives \( \sigma^* - \omega^* \) under delegation and \( s_C \) under centralization. The second term is that from \( \psi_2 \): the manager does not need to pay \( \omega^* \) to the worker and receives \( \sigma^* \) under delegation instead of \( s_C \). However, the probability of receiving \( \sigma^* \) is lower since the worker is induced not to exert effort in \( \psi_2 \). One cannot say unambiguously if either of these two terms is positive or negative. Therefore, the change in the manager’s expected payoff could be either another source of benefits from delegation, or its costs. Needless to say, delegation would increase
the principal’s expected payoff if and only if \((R_D - R_{C1}) + (U_{C1} - U_D) \geq V_D - V_{C1}\).(19) The following proposition shows how the equilibrium expected payoffs for the manager and the worker change from centralization to delegation.

PROPOSITION 4: Suppose centralization implements (C1) and manager delegation implements (MD1) with \(\omega^*\) as the equilibrium contract for the worker. Then,

(a) \(U_D < U_{C1}\);
(b) If \(\pi > \frac{q_2 - p_2}{p_1 - p_2}\) and \(\ell \geq \ell_1(\pi)\), then there exists \(\hat{\ell}(\pi) > \ell_1(\pi)\) such that \(V_D \geq V_{C1}\) if and only if \(\ell \geq \hat{\ell}(\pi)\) where \(\ell_1(\pi)\) is as defined in Proposition 3;
(c) If \(\pi \leq \frac{q_2 - p_2}{p_1 - p_2}\) and \(\ell \geq \ell_2(\pi)\), then \(V_D \geq V_{C1}\) where \(\ell_2(\pi)\) is as defined in Proposition 3.

PROOF: See the appendix.

The worker is strictly worse off under manager delegation for reasons explained earlier. Then, should the manager be always better off when delegated authority? The above proposition says it is not necessarily the case. In particular, if \(\pi > \frac{q_2 - p_2}{p_1 - p_2}\) and \(\ell_1(\pi) \leq \ell < \hat{\ell}(\pi)\), then the manager is worse off with delegation. From the proof of Proposition 4, one can show that \(\hat{\ell}(\pi)\) is increasing in \(\pi\), \(\hat{\ell}(\pi) \geq \ell_1(\pi)\) if \(\pi \geq \frac{q_2 - p_2}{p_1 - p_2}\), and \(\hat{\ell}(\pi) \leq \ell_2(\pi)\) if \(\pi \leq \frac{q_2 - p_2}{p_1 - p_2}\). Since \(\ell_1(\pi) = \ell_2(\pi)\) if \(\pi = \frac{q_2 - p_2}{p_1 - p_2}\), we must have \(\hat{\ell}(\pi) = \hat{\ell}_1(\pi) = \ell_2(\pi)\) for the same value of \(\pi\). Based on this, Figure 3 describes the set of \((\pi, \ell)\) for which delegation makes the manager worse off, the area labeled as \(V_D \leq V_{C1}\).

— Figure 3 goes about here. —

To see why delegation could make the manager worse off, suppose \(\pi = \frac{q_2 - p_2}{p_1 - p_2}\). Then, from Proposition 3, the minimum value of \(\ell\) that supports the worker’s optimal wage at its minimum incentive-compatible level (i.e., \(\omega^*\)) is where \(\hat{\ell}(\pi)\), \(\ell_1(\pi)\) and \(\ell_2(\pi)\) all intersect, as shown in Figure 3. Given such \((\pi, \ell)\), we know \(V_D = V_{C1}\) from Proposition 4. Consider now changes in \((\pi, \ell)\) while keeping \(\omega^*\) as the optimal wage for the worker. Suppose that \(\pi\) increases. This is bad news for the manager in (MD1) since larger \(\pi\) implies more frequent payment of \(\omega^*\), which is independent of \(\pi\). This can be confirmed by straightforward calculation:

\[
\frac{\partial V_D}{\partial \pi} = -\frac{c r}{\pi^2 \Delta p_1} < 0.
\]

However, an increase in \(\pi\) benefits the manager in (C1) since larger \(\pi\)

(19) In an environment of team production with mutual monitoring, Itoh (1993) identifies conditions under which allowing side contracting between agents can strictly improve the principal’s welfare compared to when side contracting is not allowed. Since the outcome from a centralized mechanism with side contracting can be replicated by suitable delegation (i.e., the equivalence result mentioned earlier in the paper), his findings can be recast in our setting. One of Itoh’s conditions is that the agents’ preferences are restricted so that their participation constraints are binding in equilibrium. Therefore, the benefits from better effort coordination through mutual monitoring (when side contracting is allowed) come at no additional cost. This would mean, in our setting, that only the first source of benefits from delegation remains and the next two terms will vanish, increasing the principal’s welfare unambiguously. In our model, however, participation constraints are not binding because of limited liability.
implies more chance of success: \[ \frac{\partial V_{C1}}{\partial \pi} = \frac{cp_1}{(1-\pi)^2(q_2-p_2)} > 0. \] On the other hand, an increase in \( \pi \) needs to be accompanied by an increase in \( \ell \) to keep \( \omega^* \) as the optimal wage for the worker. In (MD1), an increase in \( \ell \) benefits the manager since the principal can compensate the worker for the increase only indirectly by increasing the gross payment to the manager. Under centralization, the worker’s and the manager’s incentives are separated and, therefore, the manager’s expected payoff does not depend on \( \ell \). Since the manager controls the worker’s incentives under manager delegation, an increase in \( \ell \) leads to not only an increase in the worker’s payoff but also an increase in the manager’s payoff. This is the problem of double incentivization. It is easy to verify that \[ \frac{\partial V_D}{\partial \ell} = \frac{p_1 r}{\Delta p_1^2} > 0. \] Putting all these together, we can conclude that, when \( \pi \) increases from \( \frac{q_2-p_2}{p_1-p_2} \), the manager will be worse off in (MD1) if the corresponding increase in \( \ell \) is not large enough. (20)

An alternative explanation of the above can be offered by re-interpreting the benefits of delegated contracting authority to the manager. The manager benefits from delegated contracting authority through its option value: when \( \theta_2 \) is observed, the manager can realize the full value of his residual claim by inducing an efficient effort level \( (d_w = 0) \) from the worker. The value of his residual claim is smaller if \( \ell \) is smaller since, as \( \ell \) becomes smaller, the principal’s gross payment to the manager becomes smaller. At the same time, the option value of delegated contracting authority decreases as \( \pi \) increases. Therefore, delegation can make the manager worse off relative to centralized contracting if the option value of delegated contracting authority is smaller. In this case, the principal would prefer delegation to centralization. But this is only one sufficient case for beneficial delegation. If the first two benefits of delegation previously discussed are large enough, then manager delegation could benefit both the principal and the manager. To understand better when manager delegation is likely to dominate centralization, we now move onto direct comparison of the principal’s expected payoffs in response to changes in \( c, \ell \) and \( \pi \).

Consider first changes in \( \ell \) and \( \pi \). We consider simultaneous changes in \( \ell \) and \( \pi \) since, as discussed in Proposition 4, changes in both may be necessary to support \( \omega^* \) as the optimal contract for the worker in (MD1). Suppose \( \ell \) increases. Then the principal’s expected payoff in (C1) decreases primarily due to a decrease in the gross expected return \( (\frac{\partial R_{C1}}{\partial \ell} = -1) \), and an increase in the worker’s expected payoff \( (\frac{\partial U_{C1}}{\partial \ell} = \frac{r}{\pi \Delta p_1 + (1-\pi) \Delta q_2}) \). As explained before, the manager’s incentives do not depend on \( \ell \) under centralization and, therefore, changes in \( \ell \) do not affect the manager’s expected payoff in (C1). On the other hand, the principal’s expected payoff in (MD1) decreases in \( \ell \) because of the above two factors as well as an increase in the manager’s expected payoff. As shown above, \[ \frac{\partial V_D}{\partial \ell} = \frac{p_1 r}{\Delta p_1^2} > 0. \] Suppose now \( \pi \) increases at the same time as \( \ell \) increases. In (C1), this affects only the change in the worker’s expected

\(^{(20)}\) In the other case where \( \pi \) decreases from \( \frac{q_2-p_2}{p_1-p_2} \) while \( \ell \) increases to support \( \omega^* \), the manager is better off unambiguously in (MD1) since both changes in \( \pi \) and \( \ell \) benefit the manager in (MD1).
payoff. As $\pi$ increases, the increase in $U_{C1}$ in response to an increase in $\ell$ becomes smaller: $\frac{\partial}{\partial \pi} \left( \frac{\partial U_{C1}}{\partial \ell} \right) < 0$. Therefore, the rate of decrease in $Z_{C1}$ with respect to $\ell$ becomes smaller when $\pi$ increases. In (MD1), however, a simultaneous increase in $\pi$ escalates the decrease in $Z_D$. First, larger $\pi$ leads to a faster decrease in $R_D$: $\frac{\partial}{\partial \pi} \left( \frac{\partial R_D}{\partial \ell} \right) = -1$. This is because, in (MD1), the worker exerts effort only in $\psi_1$, which is chosen more often as $\pi$ increases. Second, larger $\pi$ leads to a faster increase in $U_D$: $\frac{\partial}{\partial \pi} \left( \frac{\partial U_D}{\partial \ell} \right) > 0$. Unlike in (C1), the worker benefits in (MD1) when $\psi_1$ is chosen, which is more likely, the larger $\pi$ becomes. To summarize, when $\pi$ increases, the rate of decrease in $Z_D$ with respect to $\ell$ becomes larger. Based on this, we can establish the following proposition.

**PROPOSITION 5:** There exists $\tilde{\pi} \in [0, 1)$ such that $\frac{\partial Z_{C1}}{\partial \ell} \geq \frac{\partial Z_D}{\partial \ell}$ for all $\pi \geq \tilde{\pi}$. Moreover, if $r$ is small in the sense that $p_1 \Delta q_2 r - (\Delta p_1)^2 q_2 < 0$, then $\tilde{\pi}$ is strictly positive and $\frac{\partial Z_{C1}}{\partial \ell} < \frac{\partial Z_D}{\partial \ell}$ for all $\pi < \tilde{\pi}$.

**PROOF:** See the appendix.

The above proposition implies that (MD1) is more likely to dominate (C1) as $\ell$ becomes smaller (larger, respectively) if $\pi$ is large (small, respectively). To understand why, let us note that we can measure the value of the manager’s information as the difference between the gross expected return in (MD1) and that from the next best alternative without the manager’s information. In manager delegation, the latter is given by $[\pi p_1 + (1 - \pi) p_2] x - \ell$, which follows from Assumptions 1, 2 and 4. Thus the value of the manager’s information is $(1 - \pi)(\ell - \Delta p_2 x) - c$. The manager’s information is more valuable, the smaller $\pi$ is, or the larger $\ell$ is.$^{(21)}$

This is because the manager’s information can be used to correct inefficiency in centralization, which is due to suboptimal effort incentives for the worker when $\psi_2$ is chosen. Inefficiency from suboptimal effort incentives (i.e., the worker’s moral hazard problem) becomes more serious if $\ell$ is larger. The incidence of such inefficiency is more likely if $\pi$ is smaller. While the worker’s moral hazard problem and corresponding inefficiency could be solved through delegation and, therefore, larger $\ell$ implies larger efficiency gains from delegation, it also implies larger costs of double incentivization in delegation. Thus the choice between delegation and centralization depends primarily on two kinds of considerations: efficiency gains and the costs of double incentivization. This is the main implication of Proposition 5. The first part of the proposition says that, if $\pi$ is large, then centralization is more likely to dominate delegation when $\ell$ is larger. That is, if the manager’s information is less valuable, then centralization is more likely to dominate delegation when the worker’s moral hazard problem becomes more serious. The costs of double incentivization in this case are more likely to outweigh the benefits of efficiency.

$^{(21)}$ Obviously, neither $\pi$ nor $\ell$ can take extreme values since, then, (MD1) will be dominated by (C2) or (C3). Our discussion is therefore confined to the range of parameter values where (MD1) is optimal under manager delegation.
gains if the manager’s information is less valuable. By a similar reasoning, if the manager’s information is more valuable, then delegation is more likely to dominate centralization when the worker’s moral hazard problem becomes more serious. The efficiency benefits of delegation then could be large despite corresponding increases in the costs of double incentivization.

Next let us now look at changes in $c$. As $c$ increases, the manager’s information gathering cost increases, and so does the cost of motivating the manager. The latter cost is larger in (C1) than in (MD1) as the rise in $c$ is partly compensated for by the optimal incentives given to the worker in (MD1). Since the manager has residual claim in the subcontracting stage, the optimal work incentives to the worker in (MD1) imply that the manager does not need to be compensated for an increase in $c$ as much in (MD1) as in (C1):

$$\frac{\partial V_{C1}}{\partial c} = \frac{\pi_{p1} + (1-\pi)p_2}{(1-\pi)(q_2 - p_2)} > \frac{\partial V_{D}}{\partial c} = \frac{r}{\pi \Delta p_1}.$$  

Note also that an increase in $c$ decreases the gross expected return by the same amount in (C1) and (MD1):

$$\frac{\partial R_{C1}}{\partial c} = \frac{\partial R_{D}}{\partial c} = -1.$$  

Finally the worker’s equilibrium expected payoff does not depend on $c$ in (C1) nor in (MD1). In sum, we have $\frac{\partial Z_{C1}}{\partial c} < \frac{\partial Z_{D}}{\partial c} < 0$. Therefore, as $c$ increases, we would expect the principal to prefer (MD1) to (C1) if she was initially indifferent between the two. Moreover, if (C1) is optimal under centralization and $Z_D = Z_{C1}$ initially, then (MD1) is optimal under manager delegation, which follows from Proposition 2. Finally, the range of $c$ in which (C1) is optimal under centralization is a convex set since $Z_{C1}$ decreases linearly in $c$ while $Z_{C2}$ and $Z_{C3}$ are independent of $c$.

Summarizing, we have

**PROPOSITION 6:** Suppose (C1) is optimal under centralization for all $c \in [c, \bar{c}]$, and $Z_D = Z_{C1}$ for some $\tilde{c} \in [c, \bar{c}]$. Then, for all $c \in [\tilde{c}, \bar{c}]$, (MD1) is optimal under manager delegation and $Z_D \geq Z_{C1}$.

The main implication of Proposition 6 is that manager delegation is more likely to benefit the principal if the manager’s information gathering cost becomes larger.\(^{22}\) One could think of this cost as a proxy for the degree of managerial moral hazard. As $c$ increases, it becomes more costly to motivate the manager to gather information and make the right investment decision. Compared to centralization, delegation can reduce this cost by making the manager a residual claimant in the subcontracting stage. The manager can increase the value of his residual claim by inducing the efficient effort level from the worker, of which the prerequisite is information gathering and the optimal investment decision. It is in this sense that the delegated decision-making authority and the delegated contracting authority are complementary with each other.

### 6. Further Discussions

#### 6.1. Worker delegation

\(^{22}\) Again $c$ cannot be too large since, if it were, (C2) or (C3) will eventually dominate (MD1).
The analyses of the previous sections indicate that manager delegation can benefit the principal only when delegation implements the outcome that the principal could not implement under centralization. The corollary is then that the principal would never benefit from delegating authority to the worker. The reason is that the principal and the worker share the same prior information as regards the realization of states, nor do they have access to the manager’s information. Therefore worker delegation can at best implement the same outcome as that under centralization. Moreover, the worker, if delegated the authority to contract with the manager, could have incentives not to induce the manager’s action that the principal desires. Controlling such incentives could impose further costs on the principal. Therefore we can conclude

PROPOSITION 7: The principal is never better off under worker delegation than under centralization.

PROOF: See the appendix.

6.2. The case of verifiable project choice

Throughout the paper we have assumed that the return from the project is the only variable upon which contracts can be written. This assumption was motivated on two grounds. First, if the project choice can be also used for contracting purpose, then the principal is weakly better off under centralization than any forms of delegation. This is because of the binary nature of our model where the revelation principle applies: the principal can design the manager’s contract in such a way that the manager truthfully reveals his private information through project choice. In such an environment, it is well known that a centralized mechanism where agents cannot side-contract is weakly preferred by the principal to any other mechanisms, and a centralized mechanism where agents can side-contract can be replicated by suitable delegation. To account for the prevalence of delegation, we are thus led to an environment where centralized contracts can only be incomplete or there are additional costs of centralized contracting. Second, due possibly to the complex nature of managerial decision-making, one rarely observes in reality project-dependent compensation contracts for top management.

In this paper we have introduced contractual incompleteness through nonverifiability of project choice. As alluded to in the previous paragraph, removing such incompleteness will lead to the dominance of centralization over any other mechanisms, including different forms of delegation. The intuition is straightforward. If the project choice is verifiable, then the principal can implement outcomes that were not possible to implement previously under centralization. In particular, the principal can implement the first-best outcome by using project-dependent contracts. Compared to centralization, manager delegation does not provide additional signaling benefits since centralized contracts can also elicit the manager’s private information, based
on which efficient incentives can be provided to the worker. Consequently, the principal can implement the desired outcome with centralization without having to delegate, which could otherwise require leaving a larger rent to the delegated agent than under centralization.

PROPOSITION 8: If the project choice can be used for contracting purpose, then the principal is weakly better off under centralization than under delegation.

PROOF: See the appendix.

7. Conclusion

In a model with a principal and two agents, this paper has shown when delegation to a suitable agent can improve the principal’s welfare compared to centralized contracting. Identifying delegation with conferral of the authority to make investment decisions and design contracts for the other agent in the hierarchy, we have found conditions for beneficial delegation. First, a necessary condition for beneficial delegation is intrinsic incompleteness in centralized contracting when agents’ private information cannot be used for contracting purpose. Second, beneficial delegation obtains only when the agent who has access to private information is delegated both types of authority. Third, the delegated agent should be motivated to gather information, use it for efficient decision-making and for the provision of better incentives to the other agent. These are the benefits of delegation, which stem mainly from efficiency gains. The potential costs of delegation are that it may be necessary to reward the delegated agent more than is necessary compared to centralized contracting, which we dubbed the costs of double incentivization. The costs of delegation are therefore mainly distributional. Beneficial delegation obtains when the benefits exceed the costs, which is more likely if the managerial moral hazard problem is more serious, the worker’s moral hazard problem is less serious when the manager’s information is less valuable, and the worker’s moral hazard problem is more serious when the manager’s information is more valuable.

In our paper, delegation has both efficiency and distributional consequences. Beneficial delegation, while necessarily correcting inefficiency in centralized contracting, changes the distribution of rent among all the involved parties. As a result, the agent at the bottom end of the hierarchy is strictly worse off as the hierarchy becomes deeper. Although the delegated agent may or may not be better off, we have identified a situation when the delegated agent is worse off as well. In this case, the principal would benefit from delegation through efficiency gains as well as distributional changes. It is thus conceivable that beneficial delegation may be possible even in the absence of efficiency gains. Exploring into such a possibility will be an extension of the current work.

While delegation in this paper combines the authority to make decisions and the authority to contract with other agents, our analysis also suggests some interesting implications regarding
internal organization of a firm when these two types of authority are decoupled. Consider an organization where different agents have, or need to acquire and process, sufficiently distinct and valuable information, and the cost of communicating the information is high. For example, organizations with high human-capital intensity (such as computer industry firms in Silicon Valley) would fit this description. Delegated decision-making authority would be valuable in this case. However, delegated contracting is unlikely to lead to more efficient incentive provision compared to centralization. The problem of double incentivization could loom large relative to the benefits of delegated contracting. Should this be the case, we would expect an organization with flatter hierarchies and decentralized decision making.\footnote{Depending on the extent to which organizational rents are expropriable, Rajan and Zingales (2001) provide an explanation as to why firms in human-capital intensive industries will have flatter hierarchies compared to those in physical-capital intensive industries. Indeed Rajan and Wulf (2003) report evidence in support of this: the intensity of physical capital as measured by the real value of fixed assets per employee is positively and significantly correlated with the depth of hierarchy in an organization.}

An additional conclusion from this paper is that the delegated agent has more influence upon his own compensation than the other agent does, since the delegated agent assumes residual claim in the subcontracting stage. This, combined with the decision-making authority, can be viewed as a reasonable portrayal of a corporate hierarchy where top managers, not other stakeholders, are delegated authority, whose key role is that of direction-setting, and who are often motivated through stocks and stock options. An extension of the current model that can more fruitfully elucidate the nature of incentive pay in a hierarchy seems to be an exciting avenue for future research.

Appendix

PROOF OF LEMMA 1: Any implementable outcome can be denoted by a triple, \((d_m, C, d_w)\). We can divide all possible outcomes to two groups: \(d_m = 1\) and \(d_m = 0\). If \(d_m = 1\) in the first-best outcome, then we should necessarily have \(C(\theta_i) = \psi_i, i = 1, 2\) regardless of \(d_w\). This follows from Assumptions 1 to 3. Moreover, Assumption 2 implies that the worker’s input is valuable only in \(\psi_1\). The first-best outcome, if involving the manager’s input, should thus be: \(d_m = 1, C(\theta_1) = \psi_i\) for \(i = 1, 2\), and \(d_w = 1\) only in \(\theta_1\). The gross expected return (net of the manager’s and the worker’s costs) from this outcome is \(R_1 = [\pi p_1 + (1 - \pi) r] x - (c + \pi \ell)\).

In case \(d_m = 0\) in the first-best outcome, we should necessarily have \(C(\emptyset) = \psi_1\) regardless of \(d_w\). This is because of Assumption 1. Therefore we are left with two possibilities: \(d_w = 1\) or \(d_w = 0\). If \(d_w = 1\), then the gross expected return is \(R_2 = [\pi p_1 + (1 - \pi) p_2] x - \ell\). If \(d_w = 0\), then the gross expected return is \(r x\), which is smaller than \(R_2\) because of Assumption 4. To prove our claim, it is therefore sufficient to show \(R_1 \geq R_2\). But \(R_1 - R_2 = (1 - \pi) (\ell - \Delta p_2 x) - c > 0\) since \(\Delta p_2 < \frac{\ell}{x} - \frac{c}{(1 - \pi) x}\) due to Assumptions 2 and 3. \(\blacksquare\)
PROOF OF PROPOSITION 2: Consider the case where delegation implements (MD2 = C1). Let us start with the subcontracting game between the manager and the worker. Let \( \omega(\theta_i) \geq 0 \) denote the equilibrium wage for the worker in case of success. Given \( d_m = 1 \) and \( C(\theta_i) = \psi, \) the minimum value of \( \omega(\theta_i) \) that induces \( d_w = 1 \) in both states is \( \omega(\theta_1) = \omega(\theta_2) = \frac{\ell}{\pi \Delta p_1 + (1 - \pi) \Delta q_2}. \) Note that this is the same equilibrium wage that the principal would offer under centralization and (C1). For consistency, let us denote this by \( w_C. \) Thus the equilibrium wage does not signal the manager’s information and, therefore, the worker’s equilibrium belief is the same as the prior one: \( \mu(d_m = 1, \psi_1, \omega \geq w_C) = \pi, \mu(d_m = 1, \psi_2, \omega \geq w_C) = 1 - \pi. \) The manager’s equilibrium expected payoff is then \( V_1 = [\pi p_1 + (1 - \pi) q_2](\sigma - w_C) - c. \) We now consider the manager’s deviations.

Suppose the manager deviates by inducing different work decisions from the worker, while adhering to the rest of the equilibrium strategy. First, the manager can induce \( d_w = 1 \) only in state 1 by offering \( w_C \) in \( \theta_1 \) only. The resulting expected payoff for the manager is \( V_2 = \pi p_1(\sigma - w_C) + (1 - \pi) r \sigma - c. \) Second, the manager can induce \( d_w = 1 \) only in state 2 by offering \( w_C \) only in \( \theta_2, \) realizing the expected payoff \( V_3 = \pi r \sigma + (1 - \pi) q_2(\sigma - w_C) - c. \) Third, the manager can induce \( d_w = 0 \) always by offering 0 always, securing himself \( r \sigma - c. \) However this will be worse for the manager than choosing \( d_m = 0 \) and inducing \( d_w = 0, \) which leads to the expected payoff of \( V_4 = r \sigma. \)

Next we can consider the manager’s deviations, \( C(\theta_i) = \psi_j, \ i \neq j, \) while \( d_m = 1 \) still. Regardless of subsequent wage offers and \( d_w, \) the manager will not benefit from this compared to the above cases due to assumptions 1 to 3. The remaining cases involve \( d_m = 0. \) First, the manager can choose \( \psi_1 \) and induce \( d_w = 1 \) always by offering \( w_C \) always. The expected payoff in this case is \( V_5 = [\pi p_1 + (1 - \pi) p_2](\sigma - w_C). \) Second, the manager can choose \( \psi_2 \) and induce \( d_w = 1 \) always by offering \( w_C \) always. But this will be dominated by the above deviation due to Assumption 1. Finally, the manager can secure himself \( V_4 = r \sigma \) by offering \( \omega = 0 \) always regardless of project choice.

For outcome (MD2 = C1) to be implementable under delegation, we need \( V_1 \geq V_k, \) for \( k = 2, \ldots, 5. \) These conditions are: \( V_1 \geq V_2 \iff \sigma \geq \frac{d w_C}{\Delta q_2} \equiv \sigma_1; \) \( V_1 \geq V_3 \iff \sigma \geq \frac{p_1 w_C}{\Delta p_1} \equiv \sigma_2; \) \( V_1 \geq V_4 \iff \sigma \geq \frac{\pi p_1 + (1 - \pi) q_2 w_C + c}{\pi \Delta p_1 + (1 - \pi) \Delta q_2} \equiv \sigma_3; \) \( V_1 \geq V_5 \iff \sigma \geq \frac{c}{(1 - \pi)(q_2 - p_2)} + w_C \equiv \sigma_4. \) Recall that, under the centralized equilibrium implementing the same outcome as the current one, the equilibrium payment to the manager was \( s_C = \frac{c}{(1 - \pi)(q_2 - p_2)}, \) hence \( \sigma_4 = s_C + w_C. \) Since the equilibrium payment to the manager is minimum \( \sigma \) that satisfies \( \sigma \geq \max\{\sigma_1, \sigma_2, \sigma_3, \sigma_4\}, \) the cost of implementing the same outcome is not smaller under delegation than under centralization. Therefore the principal is never better off under delegation. This completes the proof for the case (MD2 = C1) is implemented.

Remaining cases are (MD3 = C2) and (MD4 = C3). The second case can be implemented under delegation trivially by setting \( \sigma = 0, \) leading to the same expected payoff for the principal.
as centralization. To implement the first case under delegation, it is easy to see that $\sigma > \tilde{\omega}$ where $\tilde{\omega} = \frac{\ell}{\pi \Delta p_1 + (1-\pi)\Delta p_2}$ is the equilibrium wage for the worker, which, along with $s_c = 0$, implements the same outcome under centralization. Since the same outcome is implemented at a lower cost under centralization, the principal is worse off under delegation.

**PROOF OF PROPOSITION 3:** An optimal contract is minimum $(\sigma, \omega_1)$ satisfying (6), (12), (13) and (14). Clearly any optimal contract satisfies $\sigma = \frac{p_1}{\Delta p_1} \omega_1 + \frac{\rho}{\pi \Delta p_1}$. That is, the manager’s optimal contract is determined by constraint (14), once the worker’s optimal contract is determined. Note first that the minimum value of $\omega_1$ satisfying the manager’s incentive constraint (6) is $\omega^* \equiv \frac{\rho}{\Delta p_1}$. This $\omega^*$ is the minimum wage for the worker that supports (MD1) as the equilibrium outcome, which will be compared with the minimum value of $\omega_1$ that satisfies the manager’s incentive constraints (12), (13) and (14). The set of $(\sigma, \omega_1)$ satisfying (12), (13) and (14) is determined by the intersection of three half spaces. Since $\frac{p_1}{\Delta p_2} > \frac{q_2}{\Delta q_2} > \frac{p_1}{\Delta p_1} > 1$, this intersection is nonempty, bounded below and unbounded above. By setting (12) and (13) as equalities and solving for $\omega_1$, we obtain $\omega_{11} \equiv \frac{c q_2}{\pi(p_1 - q_2) r}$. Similarly, from (12) and (14), we have $\omega_{12} \equiv \frac{c q_2}{\pi(p_1 - q_2) r}$ and, from (13) and (14), we have $\omega_{13} \equiv \frac{c [r \Delta p_1 + (1-\pi) \Delta q_2]}{\pi(1-\pi)(p_1 - p_2) r}$. We divide the discussion into two cases: $\omega_{11} > \omega_{12}$ or $\omega_{11} \leq \omega_{12}$.

Suppose $\omega_{11} > \omega_{12}$ or, equivalently, $p_1 (1-\pi) p_2 > q_2 \Leftrightarrow \pi > \frac{q_2 - p_2}{p_1 - p_2}$. In this case, $\omega_{13}$ is the minimum value of $\omega_1$ satisfying constraints (12), (13) and (14), with (12) being a nonbinding constraint. Since the optimal contract for the worker is $\min \{ \omega^*, \omega_{13} \}$, we have: (a) If $\omega_{13} > \omega^*$ or $\ell < \frac{c p_1 [\pi \Delta p_1 + (1-\pi) \Delta q_2]}{\pi(1-\pi)(p_1 - q_2) r} \equiv \ell_1(\pi)$, then $\omega_1 = \omega_{13}$; (b) If $\omega_{13} \leq \omega^*$ or $\ell \geq \ell_1(\pi)$, then $\omega_1 = \omega^*$. Suppose next $\omega_{11} \leq \omega_{12}$ or, equivalently, $p_1 (1-\pi) p_2 < q_2 \Leftrightarrow \pi < \frac{q_2 - p_2}{p_1 - p_2}$. In this case, $\omega_{12}$ is the minimum value of $\omega_1$ satisfying constraints (12), (13) and (14), with (13) being a nonbinding constraint. Since the optimal contract for the worker is $\min \{ \omega^*, \omega_{12} \}$, we have: (c) If $\omega_{12} > \omega^*$ or $\ell < \frac{c p_1 \Delta q_2}{\pi(p_1 - q_2) r} \equiv \ell_2(\pi)$, then $\omega_1 = \omega_{12}$; (d) If $\omega_{12} \leq \omega^*$ or $\ell \geq \ell_2(\pi)$, then $\omega_1 = \omega^*$.

**PROOF OF PROPOSITION 4:** Consider first the worker’s expected payoffs: $U_D = \frac{\rho \ell}{\Delta p_1}$, $U_C_1 = \frac{\rho \ell}{\pi \Delta p_1 + (1-\pi) \Delta q_2}$. Since $\Delta p_1 > \Delta q_2$, we have $U_D < U_C_1$. For the manager, $V_D - V_C_1 = \pi p_1 (\sigma^* - \omega^* - s_c) + (1-\pi)(r \sigma^* - q_2 s_c)$ where $\sigma^* = \frac{p_1}{\pi \Delta p_1} + \frac{\rho \ell}{(\Delta p_1)^2}$, $\omega^* = \frac{\ell}{\Delta p_1}$ and $s_c = \frac{c}{(1-\pi)(p_1 - q_2)}$. Substituting $\sigma^*$, $\omega^*$ and $s_c$ into $V_D - V_C_1$ leads to $V_D - V_C_1 = \frac{p_1 \ell}{(\Delta p_1)^2} (\ell - \hat{\ell}(\pi))$ where $\hat{\ell}(\pi) \equiv \left( \frac{p_1 (1-\pi) q_2}{(1-\pi)(p_1 - q_2) r} - \frac{p_1 (1-\pi) r}{\pi \Delta p_1} \right) \frac{(\Delta p_1)^2 \ell}{p_1 \ell r}$. Therefore, $V_D \geq V_C_1$ if and only if $\ell \geq \hat{\ell}(\pi)$. From Proposition 3, we know that $V_D$ is valid if $\pi > \frac{q_2 - p_2}{p_1 - p_2}$ and $\ell \geq \frac{c \Delta p_1 [\pi \Delta p_1 + (1-\pi) \Delta q_2]}{\pi(1-\pi)(p_1 - q_2) r} \equiv \ell_1(\pi)$, or $\pi \leq \frac{q_2 - p_2}{p_1 - p_2}$ and $\ell \geq \frac{c \Delta p_1 \Delta q_2}{\pi(p_1 - q_2) r} \equiv \ell_2(\pi)$. Suppose first $\pi > \frac{q_2 - p_2}{p_1 - p_2}$ and $\ell \geq \ell_1(\pi)$. It can be shown that $\pi > \frac{q_2 - p_2}{p_1 - p_2}$ implies $\hat{\ell}(\pi) > \ell_1(\pi)$, from which (b) follows. Suppose next $\pi \leq \frac{q_2 - p_2}{p_1 - p_2}$ and $\ell \geq \ell_2(\pi)$. Proceeding similarly as before, it can be shown that $\hat{\ell}(\pi) \leq \ell_2(\pi)$ if $\pi \leq \frac{q_2 - p_2}{p_1 - p_2}$. Therefore, if $\ell \geq \ell_2(\pi)$, we must have $V_D \geq V_C_1$.  


PROOF OF PROPOSITION 5: One can show that \( \frac{\partial Z_{C_1}}{\partial t} = -\frac{\pi p_1 + (1-\pi)q_2}{\pi \Delta p_1 + (1-\pi)\Delta q_2} < 0 \) and \( \frac{\partial Z_D}{\partial t} = -\frac{[\pi p_1 + (1-\pi)r]p_1}{(\Delta p_1)^2} < 0 \). Thus both \( Z_{C_1} \) and \( Z_D \) decrease linearly in \( t \). Differentiating with respect to \( \pi \), we have \( \frac{\partial}{\partial \pi} (\frac{\partial Z_{C_1}}{\partial t} - \frac{\partial Z_D}{\partial t}) = \frac{(q_1 - q_2)r}{(\pi \Delta p_1 + (1-\pi)\Delta q_2)^2} + \frac{p_1}{\Delta p_1} > 0 \). Since \( \frac{\partial Z_{C_1}}{\partial t} - \frac{\partial Z_D}{\partial t} > 0 \) when \( \pi = 1 \), there must be \( \tilde{\pi} < 1 \) such that \( \frac{\partial Z_{C_1}}{\partial t} \geq \frac{\partial Z_D}{\partial t} \) for all \( \pi \geq \tilde{\pi} \). If \( p_1\Delta q_2 r - (\Delta p_1)^2 q_2 < 0 \), then \( \frac{\partial Z_{C_1}}{\partial t} - \frac{\partial Z_D}{\partial t} < 0 \) when \( \pi = 0 \), from which the second part follows. \[\square\]

PROOF OF PROPOSITION 7: Since the worker has the same information as the principal when offering contracts to the manager, the set of outcomes that can be implemented under worker delegation is the same as that under centralization, (C1), (C2) and (C3). Let \( \omega_D \) be the principal’s payment to the worker and \( \sigma_D \) be the worker’s payment to the manager in case of success. Outcome (C3) can be trivially implemented with \( \omega_D = \sigma_D = 0 \), hence worker delegation and centralization are equivalent in this case.

Consider outcome (C1). Recall that the equilibrium contracts implementing outcome (C1) under centralization are \( s_C = \frac{c}{(1-\pi)(q_2 - p_2)} \) and \( w_C = \frac{\ell}{\pi \Delta p_1 + (1-\pi)\Delta q_2} \). Given \( \omega_D \) and delegated authority, the worker faces three options. First, he can indeed implement outcome (C1) by paying the manager \( \sigma_D = s_C = \frac{c}{(1-\pi)(q_2 - p_2)} \) for success, leaving himself \( \omega_D - s_C \). His expected payoff in this case is \( U_1 = [\pi p_1 + (1-\pi)q_2](\omega_D - s_C) - \ell \). Second, he can implement outcome (C2) by offering 0 to the manager, securing himself the expected payoff of \( U_2 = [\pi p_1 + (1 - \pi)p_2]w_D - \ell \). Third, the worker can implement outcome (C3) again offering 0 to the manager, and the resulting expected payoff is \( U_3 = r\omega_D \). For the equilibrium of the subcontracting game to implement outcome (C1), we must have \( U_1 \geq max\{U_2, U_3\} \). This leads to \( \omega_D \geq \left( \frac{\pi p_1 + (1-\pi)q_2}{\pi \Delta p_1 + (1-\pi)\Delta q_2} \right) \sigma_D + \frac{\ell}{\pi \Delta p_1 + (1-\pi)\Delta q_2} > s_C + w_C \). Since the same outcome is implemented at larger costs under worker delegation, the principal is strictly worse off.

A similar argument shows that the principal can implement outcome (C2) under worker delegation with \( \omega_D = \frac{\ell}{\pi \Delta p_1 + (1-\pi)\Delta p_2} \), the same wage that implements outcome (C2) under centralization. Thus worker delegation and centralization are equivalent in this case. \[\square\]

PROOF OF PROPOSITION 8: We will show this only for the case of manager delegation. The case of worker delegation is similar. To show that the principal is weakly better off under centralization, it is sufficient to show that any outcome that can be implemented under manager delegation can be implemented under centralization at the same expected costs. Let \((s_1^*, s_2^*)\) be the optimal contract for the manager under delegation where \( s_i^* \) is the payment to the manager when project \( i \) is chosen and successful. Let \((w_1^*, w_2^*)\) be the optimal contract for the worker that the manager offers in the subcontracting stage. Again \( w_i^* \) denotes the payment to the worker when project \( i \) is chosen and successful. Given the outcome \((d_m, C, d_w)\) and contracts \{\((w_1, w_2), (s_1, s_2)\)\}, let \( EU(d_m, C, d_w; w_1, w_2) \) and \( EV(d_m, C, d_w; s_1 - w_1, s_2 - w_2) \) be the expected payoff for the worker and the manager, respectively. Suppose the equilibrium outcome under delegation is \((d_m^*, C^*, d_w^*)\). Then we have \( d_w^* \in argmax(d_w)EU(d_m^*, C^*, d_w; w_1^*, w_2^*) \) and
\[(d^*_m, C^*, w_1^*, w_2^*) \in \text{argmax}_{(d_m, C, w_1, w_2)} EV(d_m, C, d^*_m; s_1^* - w_1, s_2^* - w_2).\] It is straightforward to see that the principal can implement the same outcome under centralization with contracts \((s_1^* - w_1^*, s_2^* - w_2^*)\) for the manager and \((w_1^*, w_2^*)\) for the worker.

References


Figure 1: Optimal Wage Contract Implementing (MD1)
Figure 2: Comparative Statics of the Principal’s Expected Payoffs
Figure 3: Comparison of the Manager’s Expected Payoffs