Moral Hazard and Renegotiation with Multiple Agents*

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Abstract

We investigate the effects of contract renegotiation in multi-agent situations where risk averse agents negotiate a contract offer to the principal after the agents observe a common, unverifiable perfect signal about their actions. We show that renegotiation with multiple agents reduces the cost of implementing any implementable action profile down to the first-best level, even though the principal cannot observe the agents' actions. Moreover, it is sufficient for the principal to use a "simple" initial contract, in the sense that it consists of no more than a single sharing scheme for each agent and the total payments to the agents are the same regardless of the realised state. An important implication is that decentralization, in the sense of delegated negotiation and proposals from the agents, can be as effective as centralized schemes that utilize revelation mechanisms in unrestricted ways.

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1. INTRODUCTION

In this paper we extend the analysis of renegotiation under moral hazard to a multi-agent setting. Existing literature exclusively studies renegotiation between a principal and a single agent (Fudenberg and Tirole, 1990; Hermalin and Katz, 1991; Ma, 1994; and Matthews, 1995).[1] The general lesson from these studies is that the information available to the principal concerning the agent's action matters for the effects of renegotiation. If the principal cannot observe the agent's action, the optimal (second-best) outcome under commitment cannot be improved upon by renegotiation: The principal is worse off when she makes renegotiation offers (Fudenberg and Tirole) while she can attain the second-best efficiency in all equilibria when the agent makes offers, provided that some belief restrictions are imposed (Ma, Matthews). On the other hand, if the principal can receive a perfect signal, although unverifiable, about the agent's action, she can raise her welfare by reducing the costs of inducing any implementable action down to the first-best level, when the principal makes take-it-or-leave-it offers (Hermalin and Katz).[2]

Following Fudenberg and Tirole, Ma, and Matthews, we also maintain the standard assumption that the principal can observe no agent's action. We then show that when there are multiple agents and they make renegotiation offers, the second-best outcome can be improved upon: In fact, we show that renegotiation with multiple agents reduces the cost of inducing any implementable action pair down to the first-best level. The conclusion thus turns out to be similar to the one in Hermalin and Katz, although the principal's position in renegotiation is weaker in our model than in theirs: The principal in our model is informationally disadvantaged, and has no bargaining power in renegotiation.

Three key features of the model account for the result. First, there is a *verifiable* and *independent* performance signal for each agent, and hence no use is made of comparative performance evaluation when renegotiation is not allowed. Second, the agents can observe a common, *unverifiable* signal that reveals their actions perfectly. One way the principal utilizes the shared information among the agents is to set up a direct revelation mechanism in which the agents report their actions to the principal. It is well known that when actions are mutually observable among agents, there exists a mechanism that induces a given vector of actions as an equilibrium, and the resulting expected cost to the principal is at the first-best level (Ma, 1988). In this paper, as Hermalin and Katz (1991) do, we take an alternative approach that does not rely on complicated centralized mechanisms. Furthermore, we assume as in

[1]An exception is Osano (1998) who extends some of the results from the literature to a multi-agent situation. We discuss differences between his model and ours in Subsection 4.4.

[2]Hermalin and Katz (1991) also show that when the agent makes renegotiation offers, the first-best outcome is attainable.

Matthews (1995) that menu contracts are infeasible: Both the initial contract and the renegotiation contract consist of a single payment scheme for each agent. We thus confine our attention to relatively "simple" contracts. Although this sort of restriction typically leads to inefficiency, we show that it is not necessarily the case.

Third, we suppose that to incorporate the unverifiable information into the agents' payment schemes, the principal lets the agents negotiate with each other and propose a new contract. More precisely, we consider the following renegotiation structure, assuming there are two agents. After learning each other's action, one of the agents proposes a new contract to the other agent in a take-it-or-leave-it fashion. If the latter agent accepts it, the agreement becomes verifiable and they propose it as a new contract to the principal. The principal revises her beliefs about the agents' actions based on the proposal, and then determines whether to accept or reject the new contract (in the latter case, the initial contract offered by the principal is enforced). Examples of such a decentralized renegotiation structure abound in practice. For example, imagine that the principal is a firm and the agents are workers who are members of the labor union. The firm makes an initial offer, and then the workers may negotiate with each other to propose a counteroffer. Another example is the transaction between an automobile assembler as a principal and parts manufacturers as agents who form a supplier association and negotiate with the assembler.

In this setting, we show that if a vector of actions is implementable without renegotiation, there exists an initial contract accepted by the agents such that an equilibrium of the subgame defined by the initial contract has the following features.[3] (a) The agents choose the given actions. (b) Renegotiation occurs: The agents offer a new contact and the principal accepts it. (c) The renegotiation contract insures the agents perfectly, and hence the cost of implementing the given action vector is at the first-best level. Although each of the agents is paid a fixed wage on the equilibrium path of the subgame, they do not shirk because the principal can design an initial contract so that choosing a more costly action improves his bargaining position vis-a-vis the other agent in the renegotiation process.

The proof is constructive, and the initial contract in the constructed equilibrium has an interesting property. It is a specific form of relative performance evaluation, and we call it a "constant-budget" contract because the total wage payments to the agents are the same across performance outcomes: If $\boldsymbol{y}$ is a vector of verifiable outcomes and $w_A(\boldsymbol{y})$ and $w_B(\boldsymbol{y})$ are the payments to agents A and B, respectively, in the initial contract, then $w_A(\boldsymbol{y})+w_B(\boldsymbol{y})$ is constant for all feasible $\boldsymbol{y}$. Our result thus continues to hold under an alternative (and more plausible) assumption that individual performance measures are unverifiable (but the total payments to the agents

[3] This is a weak implementation result: there may be other equilibria that do not have all these properties.

are verifiable): The principal cannot benefit from misrepresenting the performance of the agents, just as she cannot under rank-order tournaments (Carmichael, 1983; Malcomson, 1984).

The remaining sections are organized as follows. We present the model in the next section. In Section 3 we offer a preliminary analysis of the benchmark case in which renegotiation is not allowed. The main results are presented in Section 4. In Section 5 we discuss a few applications of our results, to centralization versus decentralization and efficiency versus equity issues.

2. THE MODEL

A principal hires two agents indexed by $n = A, B$. Agent n has a von Neumann and Morgenstern utility function $U_n(z_n) - G_n(e_n)$ over his payment made by the principal $z_n \in (-\infty, +\infty)$ and his action $e_n \in E_n$, where E_n is the set of possible actions for agent n. We assume E_n is a finite set with $L_n + 1$ elements ($L_n \geq 1$). $U_n(\cdot)$ is assumed to be strictly increasing, strictly concave (the agents are risk averse), and unbounded. Under these assumptions the inverse function of $U_n(\cdot)$ is well defined, and is denoted by $\phi_n(\cdot)$.

The principal can write an initial contract that is contingent upon verifiable performance signals y_A and y_B. We assume that y_n takes one of the K_n possible values $y_n^1, y_n^2, \ldots, y_n^{K_n}$ where $K_n \geq 2$. To simplify the notations we hereafter assume $K_A = K_B$ and $L_A = L_B$, and drop the subscripts. We denote by $P_{ij}(\boldsymbol{e})$ the joint probability that $y_A = y_A^i$ and $y_B = y_B^j$ are realised, given action pair $\boldsymbol{e} = (e_A, e_B)$. The initial contract consists of a pair of vectors $\boldsymbol{w}^A = (w_{ij}^A)$ and $\boldsymbol{w}^B = (w_{ij}^B)$ where w_{ij}^n is the payment to agent n when performance pair (y_A^i, y_B^j) results. We often use the corresponding utility payment instead of monetary payment, and hence initial contract can alternatively be written as $(\boldsymbol{u}^A, \boldsymbol{u}^B)$ where $\boldsymbol{u}^n = (u_{ij}^n)$ and $u_{ij}^n = U_n(w_{ij}^n)$ for $i, j = 1, \ldots, K$ and $n = A, B$.

The principal obtains revenue $R(\boldsymbol{e})$ that may be a deterministic function, or the expected value of a stochastic function, of $\boldsymbol{e}$. For example, in the standard agency model, y_n would be the financial return from agent n and hence $R(\boldsymbol{e}) = \mathcal{E}[y_A + y_B | \boldsymbol{e}]$, where $\mathcal{E}$ is the expectation operator. However, our model allows the possibility that performance measures do not enter the principal's revenue directly, as well as the possibility of technological externalities across the agents' actions in production. The principal is risk neutral and maximizes the expected revenue minus the sum of the expected payments to each agent.

The timing of the game is as follows:

1. The principal offers the agents an initial contract.

2. The agents decide whether to accept the contract or not. If the contract is accepted, the game goes to the third stage. If either agent rejects the offer, the game ends and all the parties obtain their reservation payoffs. Denote by V_n^0 agent n's reservation utility. The principal's reservation utility is zero.

3. The agents choose actions simultaneously and independently. They are unobservable to the principal.

4. The agents observe a common unverifiable signal that reveals their actions perfectly to each other. The principal observes nothing.

5. The agents engage in bargaining to determine a new contract to be proposed to the principal. We assume that agent A makes a take-it-or-leave-it offer to agent B. If agent B rejects the offer, the initial contract is in force. If agent B accepts it, the new contract becomes verifiable. That is, the agents sign the new contract and each of them keeps a copy. They hence cannot falsify another contract. We call the new contract a *renegotiation contract*.

6. The agents offer the renegotiation contract to the principal. The principal then decides whether to accept or reject the renegotiation contract. If she accepts (rejects) it, the renegotiation (initial, respectively) contract will be enforced.

7. The verifiable outcomes are realised and the payments are made according to the contract in force.

3. PRELIMINARY ANALYSIS

3.1. *The first-best outcome*

As the first benchmark, suppose that the agents' action choice were observable and verifiable. In this case, the principal could force the agents to choose an arbitrary action pair $\boldsymbol{e}$. The principal could implement $\boldsymbol{e}$ by guaranteeing each agent exactly his reservation utility. Denote the minimum cost to induce $\boldsymbol{e}$ by $C_{FB}(\boldsymbol{e})$, which is equal to

$$C_{FB}(\boldsymbol{e}) = \phi_A(V_A^0 + G_A(e_A)) + \phi_B(V_B^0 + G_B(e_B)).$$

We refer to the action pair that maximizes the principal's expected net benefit $R(\boldsymbol{e}) - C_{FB}(\boldsymbol{e})$ as the first-best action pair, and denote it by $\boldsymbol{e}^{FB} = (e_A^{FB}, e_B^{FB})$.[4] The first-best outcome is the one in which the principal implements $\boldsymbol{e}^{FB}$ at the first-best cost $C_{FB}(\boldsymbol{e}^{FB})$. To avoid the trivial case, we assume that (i) e_n^{FB} is not the least costly

[4]We make an inconsequential assumption that the first-best action pair is unique.

action, that is, $G_n(e_n^{FB}) > \min_{e \in E_n} G_n(e)$ for $n = A, B$; and (ii) the principal's expected net benefit is positive in the first-best outcome.

3.2. *The commitment case*

We next consider the standard case in which the agents' actions are unobservable to the principal, while the principal and the agents can commit themselves to an initial contract. Following Grossman and Hart (1983) and Mookherjee (1984) we decompose the principal's problem into two subproblems: (a) Given an action pair, find the contract that implements the action pair with minimum expected cost; (b) Find the action pair that maximizes the principal's expected profit.

Implementable actions

Suppose the principal wants to implement the action pair $\hat{\boldsymbol{e}} = (\hat{e}_A, \hat{e}_B)$. An initial contract $(\boldsymbol{u}^A, \boldsymbol{u}^B)$ implements action pair $\hat{\boldsymbol{e}}$ *without renegotiation*, if and only if it is incentive compatible (IC) and individually rational (IR):

$$\sum\sum_{i,j=1}^{K} P_{ij}(\hat{\boldsymbol{e}})u_{ij}^A - G_A(\hat{e}_A) \geq \sum\sum_{i,j=1}^{K} P_{ij}(e, \hat{e}_B)u_{ij}^A - G_A(e) \quad \text{for all } e \in E_A,$$

$$\sum\sum_{i,j=1}^{K} P_{ij}(\hat{\boldsymbol{e}})u_{ij}^B - G_B(\hat{e}_B) \geq \sum\sum_{i,j=1}^{K} P_{ij}(\hat{e}_A, e)u_{ij}^B - G_B(e) \quad \text{for all } e \in E_B, \tag{IC}$$

and

$$\sum\sum_{i,j=1}^{K} P_{ij}(\hat{\boldsymbol{e}})u_{ij}^n - G_n(\hat{e}_n) \geq V_n^0 \quad \text{for } n = A, B. \tag{IR}$$

Action pair $\hat{\boldsymbol{e}}$ is said to be *implementable without renegotiation* if there exists $(\boldsymbol{u}^A, \boldsymbol{u}^B)$ that satisfies (IC). Note that (IR) can be ignored because we can always adjust $\boldsymbol{u}^n$ so as to satisfy (IR), without violating (IC), by adding some constant term to each u_{ij}^n.

To simplify the analysis of the subsequent renegotiation case, hereafter we maintain the following two assumptions on the probability distribution of (y_A, y_B). First, we exclude technological externalities (in performance signals) by assuming that the probability distribution of y_n depends only on e_n. Let $P_k^n(e_n)$ be the probability that $y_n = y_n^k$ is realised, which satisfies $P_k^n(e_n) > 0$ for all $e_n \in E_n$, $k = 1, \ldots, K$, and $n = A, B$ (no moving support). The second assumption is that the agents' performance signals are statistically independent of each other, and hence $P_{ij}(\boldsymbol{e}) = P_i^A(e_A)P_j^B(e_B)$ holds for all $\boldsymbol{e} = (e_A, e_B)$ and $i, j = 1, \ldots, K$.

We call a contract *individual-based* if the payment to agent n is contingent only on performance signal y_n for $n = A, B$. An individual-based contract $(\boldsymbol{u}^A, \boldsymbol{u}^B)$, in which $\boldsymbol{u}^n = (u_1^n, \ldots, u_K^n)$ for $n = A, B$, implements actions $\hat{\boldsymbol{e}}$ without renegotiation, if and only if for $n = A, B$,

$$\sum_{k=1}^{K} P_k^n(\hat{e}_n)u_k^n - G_n(\hat{e}_n) \geq \sum_{k=1}^{K} P_k^n(e)u_k^n - G_n(e) \quad \text{for all } e \in E_n, \tag{IIC}$$

and

$$\sum_{k=1}^{K} P_k^n(\hat{e}_n)u_k^n - G_n(\hat{e}_n) \geq V_n^0. \tag{IIR}$$

Under the assumptions of independent performance signals, for each implementable action pair, there exists an individual-based contract that implements the same action pair: The set of implementable actions does not change as initial contracts are restricted to be individual-based. Furthermore, since individual-based contracts can remove gratuitous risk from the agents' income, it is without loss of generality to restrict our attention to such initial contracts *in the commitment case* (Holmstrom, 1982). In the next section we will use this fact to analyze the case in which renegotiation is possible[5]. From now on, when we refer to an action pair that is implementable without renegotiation, we only consider individual-based contracts that implement it.[6]

The following lemma, due to Hermalin and Katz (1991), provides the necessary and sufficient condition for a given action pair $\hat{\boldsymbol{e}}$ to be implementable without renegotiation. Let $\ell = 1, \ldots, L$ index the actions other than $\hat{e}_n$ for $n = A, B$, and $\mathbf{0}$ be the L-dimensional vector of zeros.

Lemma 1. *Action pair $(\hat{e}_A, \hat{e}_B)$ is implementable without renegotiation if and only if for $n = A, B$, there exists no vector $(\theta_n^1, \ldots, \theta_n^L) \geq \mathbf{0}$ that satisfies the following two conditions simultaneously:*

$$\sum_{\ell=1}^{L} \theta_n^\ell (P_k^n(e_n^\ell) - P_k^n(\hat{e}_n)) = 0 \quad \textit{for } k = 1, \ldots, K, \tag{1}$$

and

$$\sum_{\ell=1}^{L} \theta_n^\ell (G_n(e_n^\ell) - G_n(\hat{e}_n)) < 0. \tag{2}$$

Proof. See Proposition 2 of Hermalin and Katz (1991) ||

Since the above inequality system is homogeneous, $(\theta_n^1, \ldots, \theta_n^L)$ can be interpreted as a mixed strategy of agent n over actions $e_n^1, \ldots, e_n^L$, and hence can be normalized

[5]We will discuss the effects of relaxing the assumptions of independent signals in Subsection 4.4.

[6]Remember, however, that the possibility of technological externalities in production are incorporated into the model through $R(\boldsymbol{e})$. A key ingredient of the model is thus a personalized performance signal available for each agent, which, for example, may be obtained from a direct but noisy observation of the agent's action. Note in this respect that as we will argue in Subsection 4.3, the personalized signals need not be verifiable, although we assume they are verifiable for most of the analysis.

as $\sum_\ell \theta_n^\ell = 1$. What the above conditions imply is that there is no mixed strategy of agent n that induces the same density over y_n as $\hat{e}_n$ and which yields a smaller expected cost than $\hat{e}_n$. Hermalin and Katz introduce the following "convex hull" condition that is sufficient for no vector to satisfy (1).

$$(P_k^n(\hat{e}_n)) \text{ is not an element of the convex hull of } \{(P_k^n(e))|e \neq \hat{e}_n\} \tag{3}$$

where $(P_k^n(\cdot)) = (P_1^n(\cdot), \ldots, P_K^n(\cdot))$. They argue that it is a weak condition and is satisfied under various specifications of the distribution functions.

The implementation problem

For a given implementable action pair $\hat{\boldsymbol{e}}$, the principal chooses a contract that solves

$$\min_{(\boldsymbol{u}^A, \boldsymbol{u}^B)} \sum \sum\nolimits_{i,j} P_{ij}(\hat{\boldsymbol{e}})(\phi_A(u_{ij}^A) + \phi_B(u_{ij}^B)) \quad \text{subject to (IC) and (IR).} \tag{P}$$

We call the solution to program (P) the optimal second-best contract for $\hat{\boldsymbol{e}}$. The argument given above implies that we can treat the problem for each agent separately by solving an optimal individual-based contract. Under our assumptions, the solution exists and is unique (Grossman and Hart, 1983; Mookherjee, 1984).

The optimal second-best actions

Suppose $(\hat{\boldsymbol{u}}^A, \hat{\boldsymbol{u}}^B)$ is the optimal second-best contract for $\hat{\boldsymbol{e}}$ where $\hat{\boldsymbol{u}}^A$ ($\hat{\boldsymbol{u}}^B$) depends only on y_A^i (y_B^j, respectively). Denote the value to problem (P) by

$$C_{SB}(\hat{\boldsymbol{e}}) \equiv \sum\nolimits_i P_i^A(\hat{e}_A)\phi_A(\hat{u}_i^A) + \sum\nolimits_j P_j^B(\hat{e}_B)\phi_B(\hat{u}_j^B),$$

which is additively separable in $\hat{e}_A$ and $\hat{e}_B$. The principal solves for $\hat{\boldsymbol{e}}$ that maximizes $R(\hat{\boldsymbol{e}}) - C_{SB}(\hat{\boldsymbol{e}})$. The solution can be called the second-best action pair, but it is not important for our analysis. An essential point to note is that unless both $\hat{e}_A$ and $\hat{e}_B$ are the least costly actions for the agents, $C_{SB}(\hat{\boldsymbol{e}}) > C_{FB}(\hat{\boldsymbol{e}})$ holds (Grossman and Hart, 1993): To induce the agents to choose actions other than the least costly ones, the principal must allocate some risk to them, a feature that increases the expected cost of implementation.

4. IMPLEMENTATION VIA RENEGOTIATION

We now analyze our model in which the agents can propose a renegotiation contract to the principal. In the model, there is a subgame defined by the principal's choice of an initial contract $\boldsymbol{w} = (\boldsymbol{w}^A, \boldsymbol{w}^B)$ and the agents' decision to accept it. We call it a *renegotiation subgame* and denote it by $\Gamma(\boldsymbol{w})$. In this subgame, each agent chooses his action, and after observing the actions, agent A offers a new contract $\boldsymbol{x} = (\boldsymbol{x}^A, \boldsymbol{x}^B)$ to agent B. Agent B decides whether to accept or reject it, and upon acceptance, the renegotiation contract $\boldsymbol{x}$ is proposed to the principal. The principal decides which to choose between $\boldsymbol{w}$ and $\boldsymbol{x}$, according to a belief function $\pi(\cdot|\boldsymbol{w}, \boldsymbol{x})$, which is a probability distribution over $E_A \times E_B$.

The principal's strategy is an initial contract $\boldsymbol{w}$ and the probability of accepting the renegotiation contract $p(\boldsymbol{w}, \boldsymbol{x})$. Agent A's strategy is the probability of accepting the initial contract $a(\boldsymbol{w})$, action $e_A(\boldsymbol{w})$, and a contract offer $\boldsymbol{x}(\boldsymbol{w}, \boldsymbol{e})$. Agent B's strategy is the probability of acceptance $b_1(\boldsymbol{w})$, action $e_B(\boldsymbol{w})$, and the probability $b_2(\boldsymbol{w}, \boldsymbol{x}, \boldsymbol{e})$ of accepting the offer from agent A. Note that once renegotiation is allowed, there is no a priori reason to exclude inter-dependent initial contracts, and hence $\boldsymbol{w}^n$ and $\boldsymbol{x}^n$ may be dependent on both y_A and y_B.

The solution concept to be used is a perfect Bayesian equilibrium (PBE). It is a profile of strategies and a belief function satisfying the following conditions: (i) It induces a Nash equilibrium in each subgame; (ii) the belief function is consistent with Bayes' rule and the agents' strategies whenever possible; and (iii) the principal chooses an acceptance rule that minimizes the expected total payments to the agents according to her beliefs.

4.1. *Renegotiation contracts*

Denote by $C(\boldsymbol{w}, \boldsymbol{e})$ the expected total payments to the agents under initial contract $\boldsymbol{w}$ when the principal believes that the agents' actions are $\boldsymbol{e}$:

$$C(\boldsymbol{w}, \boldsymbol{e}) = \sum\sum\nolimits_{i,j} P_{ij}(\boldsymbol{e})(w_{ij}^A + w_{ij}^B).$$

We first consider the following program, given an initial contract $\boldsymbol{w}$ and the agents' choice of actions $\boldsymbol{e}$:

$$\max_{(\boldsymbol{x}^A, \boldsymbol{x}^B)} \sum\sum\nolimits_{i,j} P_{ij}(\boldsymbol{e}) u_A(x_{ij}^A) \tag{AA}$$

subject to

$$\sum\sum\nolimits_{i,j} P_{ij}(\boldsymbol{e}) u_B(w_{ij}^B) \le \sum\sum\nolimits_{i,j} P_{ij}(\boldsymbol{e}) u_B(x_{ij}^B) \tag{AB}$$

and

$$\sum\sum\nolimits_{i,j} P_{ij}(\boldsymbol{e})(x_{ij}^A + x_{ij}^B) \le C(\boldsymbol{w}, \boldsymbol{e}). \tag{AP}$$

The motivation behind the program is as follows. Suppose that the agents have chosen $\boldsymbol{e}$ with probability one. Program (AA) solves for the renegotiation contract that gives agent A a maximum payoff subject to constraints that both agent B and the principal accept the contract, *given that the principal has correct beliefs about their actions.*

We call a contract $\boldsymbol{x}$ a *wage contract* if there exists a constant $\overline{x}^n$ for $n = A, B$ such that $x_{ij}^n = \overline{x}^n$ for all i, j. We denote a wage contract as $\overline{\boldsymbol{x}} = (\overline{x}^A, \overline{x}^B)$. The following lemma shows that the solution to program (AA) is a wage contract.

Lemma 2. *For each $\boldsymbol{w}$ and $\boldsymbol{e}$, program (AA) has a unique solution. It is the wage contract $\overline{\boldsymbol{x}}(\boldsymbol{w}, \boldsymbol{e}) = (\overline{x}^A(\boldsymbol{w}, \boldsymbol{e}), \overline{x}^B(\boldsymbol{w}, \boldsymbol{e}))$ that is defined by the following conditions:*

$$u_B(\overline{x}^B(\boldsymbol{w}, \boldsymbol{e})) = \sum\sum\nolimits_{i,j} P_{ij}(\boldsymbol{e}) u_B(w_{ij}^B) \tag{4}$$

$$\overline{x}^A(\boldsymbol{w}, \boldsymbol{e}) = C(\boldsymbol{w}, \boldsymbol{e}) - \overline{x}^B(\boldsymbol{w}, \boldsymbol{e}) \tag{5}$$

Proof. Problem (AA) is concave and hence the Kuhn-Tucker conditions are necessary and sufficient. Denote by λ and ξ multipliers for (AB) and (AP), respectively. The Kuhn-Tucker conditions are given as follows: For $i, j = 1, \ldots, K$,

$$u'_A(x_{ij}^A) = \xi \qquad \text{and} \qquad \lambda u'_B(x_{ij}^B) = \xi.$$

These equations imply that for each n there exist a constant $\overline{x}^n$ such that $x_{ij}^n = \overline{x}^n$ for all i, j. And $\lambda > 0$ and $\xi > 0$, and hence both (AB) and (AP) bind. Conditions (4) and (5) follow. ||

Lemma 2 shows that the agents, after choosing actions, want to engage in mutual insurance and propose a contract that perfectly insures themselves against risk, subject to the principal's acceptance constraint, given that the principal has correct beliefs as she does in equilibrium. The lemma ensures that such a contract exists for each combination of initial contract and action pair.

4.2. *Providing the appropriate incentives*

Although the existence of a renegotiation wage contract that may be accepted by the principal is necessary for the first-best implementation, one might argue that if such a renegotiation contract is to be the final contract in force, the agents would not be interested in choosing costly actions. In fact, in the case of a single agent, the agent proposing such a riskless scheme would choose the least costly action if he expects the principal to accept it, whether or not the initial contract is a wage contract. The uninformed principal hence would not accept such a renegotiation contract.

However, when there is more than one agent, they can be provided with incentives to choose appropriate actions. To see this, we first analyze the following hypothetical static game, given an initial contract $\boldsymbol{w}$, in which the agents choose their actions independently and simultaneously, and the payoff function is defined by $u_n(\overline{x}^n(\boldsymbol{w}, \boldsymbol{e})) - G_n(e_n)$ where $\overline{x}^n(\boldsymbol{w}, \boldsymbol{e})$ is defined as in Lemma 2. We denote this game by $\Delta(\boldsymbol{w})$. Action pair $\hat{\boldsymbol{e}} = (\hat{e}_A, \hat{e}_B)$ is a Nash equilibrium of $\Delta(\boldsymbol{w})$ if and only if the following two conditions hold:

$$\hat{e}_A \in \max\nolimits_e u_A(\overline{x}^A(\boldsymbol{w}, e, \hat{e}_B)) - G_A(e) \tag{6}$$

$$\hat{e}_B \in \max\nolimits_e u_B(\overline{x}^B(\boldsymbol{w}, \hat{e}_A, e)) - G_B(e) \tag{7}$$

We now show that for a given implementable action pair $\hat{\boldsymbol{e}}$, there exists an initial contract $\boldsymbol{w}$ such that (i) $\hat{\boldsymbol{e}}$ is a Nash equilibrium of game $\Delta(\boldsymbol{w})$, (ii) the resulting equilibrium payoff to each agent is equal to his reservation utility level, and (iii) the total expected payments to the agents coincide with the first-best cost of implementing $\hat{\boldsymbol{e}}$. We in particular find such a contract from the set of "constant-budget" payment schemes in which the total payments to the agents are the same across outcomes: there exists a constant $\overline{W}$ such that $w_{ij}^A + w_{ij}^B = \overline{W}$ for all i, j. The initial contract consisting of such schemes is "simple," in the sense that the expected total payments to the agents do not depend on the principal's beliefs about the agents' actions. Constraint (AP) in program (AA) thus becomes

$$\sum\sum\nolimits_{i,j} P_{ij}(\boldsymbol{e})(x_{ij}^A + x_{ij}^B) \leq \overline{W}. \tag{8}$$

and by (4) and (5) the solution to program (AA) is given as follows:

$$\overline{x}^A(\boldsymbol{w}, \boldsymbol{e}) = \overline{W} - \phi_B\Big(\sum\sum\nolimits_{i,j} P_{ij}(\boldsymbol{e}) u_B(w_{ij}^B)\Big); \tag{9}$$

$$\overline{x}^B(\boldsymbol{w}, \boldsymbol{e}) = \phi_B\Big(\sum\sum\nolimits_{i,j} P_{ij}(\boldsymbol{e}) u_B(w_{ij}^B)\Big). \tag{10}$$

Suppose $\hat{\boldsymbol{e}}$ is implementable without renegotiation, and $\hat{\boldsymbol{w}} = (\hat{\boldsymbol{w}}^A, \hat{\boldsymbol{w}}^B)$, or $\hat{\boldsymbol{u}} = (\hat{\boldsymbol{u}}^A, \hat{\boldsymbol{u}}^B)$ in utility terms, is an arbitrary (individual-based) contract that implements $\hat{\boldsymbol{e}}$ without renegotiation and gives each agent exactly his reservation utility.[7] Using a vector $(f_1, \ldots, f_K)$, to be appropriately specified later, we define the new payment scheme for agent B as follows: For $i, j = 1, \ldots, K$,

$$u_{ij}^B \equiv u_B(w_{ij}^B) = -f_i + \hat{u}_j^B + g \tag{11}$$

where the fixed part g is defined by $g = \sum_k P_k^A(\hat{e}_A) f_k$. The new scheme for agent A is defined by $w_{ij}^A = \overline{W} - w_{ij}^B$ for all i, j, with

$$\overline{W} = C_{FB}(\hat{\boldsymbol{e}}) = \sum\nolimits_{n=A,B} \phi_n(V_n^0 + G_n(\hat{e}_n)). \tag{12}$$

[7]Actually we only use $\hat{\boldsymbol{w}}^B$ (or $\hat{\boldsymbol{u}}^B$) in what follows. Note that $\hat{\boldsymbol{w}}$ is not necessarily the optimal second-best contract. All we need is an individual-based pay scheme for agent B that implements $\hat{e}_B$.

Our problem is to find a vector $(f_1, \ldots, f_K)$ that makes $\hat{\boldsymbol{e}}$ a Nash equilibrium of game $\Delta(\boldsymbol{w})$ and gives each agent exactly his reservation utility. Once the vector is fixed, there exists w^B_{ij} that satisfies (11) for all i, j, since $u_B(\cdot)$ is unbounded.

First consider agent B. Since $\hat{\boldsymbol{u}}^B$ satisfies (IIR) with equality, (4) and (11) yield

$$\begin{aligned} u_B(\overline{x}^B(\boldsymbol{w}, \hat{\boldsymbol{e}})) - G_B(\hat{e}_B) &= \sum\sum\nolimits_{i,j} P_{ij}(\hat{\boldsymbol{e}}) u^B_{ij} - G_B(\hat{e}_B) \\ &= -\sum\nolimits_i P^A_i(\hat{e}_A) f_i + \sum\nolimits_j P^B_j(\hat{e}_B)\hat{u}^B_j + g - G_B(\hat{e}_B) \\ &= V^0_B, \end{aligned}$$

and hence agent B obtains his reservation utility under the new scheme $\boldsymbol{u}^B = (u^B_{ij})$. Furthermore, since $\hat{\boldsymbol{u}}^B$ implements $\hat{e}_B$ without renegotiation, $\hat{e}_B$ is an optimal choice for agent B under the new scheme as well, and hence satisfies (7).

Turning to agent A, we can calculate his expected utility under the new scheme and action pair $\hat{\boldsymbol{e}}$ as follows.

$$\begin{aligned} &u_A\Big(\overline{W} - \phi_B\Big(\sum\sum\nolimits_{i,j} P_{ij}(\hat{\boldsymbol{e}}) u^B_{ij}\Big)\Big) - G_A(\hat{e}_A) \\ &= u_A(\overline{W} - \phi_B(V^0_B + G_B(\hat{e}_B))) - G_A(\hat{e}_A) \\ &= u_A(\phi_A(V^0_A + G_A(\hat{e}_A))) - G_A(\hat{e}_A) = V^0_A. \end{aligned}$$

The remaining problem therefore is to show that it is optimal for agent A to choose action $\hat{e}_A$, provided that agent B chooses $\hat{e}_B$. To this end, we must show

$$V^0_A \geq u_A\Big(\overline{W} - \phi_B\Big(\sum\sum\nolimits_{i,j} P_{ij}(e, \hat{e}_B) u^B_{ij}\Big)\Big) - G_A(e) \quad \text{for all } e \neq \hat{e}_A. \tag{13}$$

We can rewrite the right hand side of the above condition, using (11), as follows:

$$u_A\Big(\overline{W} - \phi_B\Big(\sum\nolimits_i (P^A_i(\hat{e}_A) - P^A_i(e)) f_i + \sum\nolimits_j P^B_j(\hat{e}_B)\hat{u}^B_j\Big)\Big) - G_A(e). \tag{14}$$

Inequalities (13) are hence rewritten as

$$\phi_A(V^0_A + G_A(e)) \geq \overline{W} - \phi_B\Big(\sum\nolimits_i (P^A_i(\hat{e}_A) - P^A_i(e)) f_i + \sum\nolimits_j P^B_j(\hat{e}_B)\hat{u}^B_j\Big) \quad \text{for all } e \neq \hat{e}_A.$$

By using the fact that $\hat{\boldsymbol{u}}^B$ satisfies (IIR) with equality, we find that inequalities (13) are equivalent to

$$\sum\nolimits_i (P^A_i(e^\ell_A) - P^A_i(\hat{e}_A)) f_i \leq V^0_B + G_B(\hat{e}_B) - u_B(\overline{W} - \phi_A(V^0_A + G_A(e^\ell_A))) \quad \text{for } \ell = 1, \ldots, L \tag{15}$$

where ℓ indexes actions other than $\hat{e}_n$. If we can find a vector $(f_1, \cdots, f_K)$ which solves the inequality system (15), the corresponding payment scheme defined by (11) induces agent A to choose $\hat{e}_A$. The following lemma, which is the key to the main result, shows that we can find such a vector if $\hat{\boldsymbol{e}}$ is implementable *without renegotiation*.

Lemma 3. *Suppose* $\hat{\boldsymbol{e}}$ *is implementable without renegotiation. Then there exists an initial contract* $\boldsymbol{w}$ *within the class of constant-budget schemes such that* (i) $\hat{\boldsymbol{e}}$ *is a Nash equilibrium of game* $\Delta(\boldsymbol{w})$*;* (ii) *the equilibrium payoff to each agent is equal to his reservation utility level; and* (iii) *the total expected payments to the agents coincide with the first-best cost of implementing* $\hat{\boldsymbol{e}}$*.*

Proof. From the definition of $\boldsymbol{w}$ and the discussion given above, we only need to show that the system of inequalities (15) has a solution. By the same argument as that in Lemma 1, the necessary and sufficient condition is that there exists no vector $(\mu^1, \cdots, \mu^L) \geq \mathbf{0}$ that satisfies the following two conditions simultaneously:

$$\sum_{\ell=1}^{L} \mu^\ell (P_k^A(e_A^\ell) - P_k^A(\hat{e}_A)) = 0 \qquad \text{for } k = 1, \ldots, K, \tag{16}$$

and

$$\sum_{\ell=1}^{L} \mu^\ell \Big\{ V_B^0 + G_B(\hat{e}_B) - u_B\Big(\overline{W} - \phi_A(V_A^0 + G_A(e_A^\ell))\Big)\Big\} < 0. \tag{17}$$

Since the above linear system is homogeneous, it is without loss of generality to normalize (μ^ℓ) as $\sum_\ell \mu^\ell = 1$, which we assume hereafter.

The last condition (17) reduces to

$$\begin{aligned} V_B^0 + G_B(\hat{e}_B) &< \sum_\ell \mu^\ell u_B\Big(\sum_{n=A,B} \phi_n(V_n^0 + G_n(\hat{e}_n)) - \phi_A(V_A^0 + G_A(e_A^\ell))\Big) \\ &\leq u_B\Big(\sum_{n=A,B} \phi_n(V_n^0 + G_n(\hat{e}_n)) - \sum_\ell \mu^\ell \phi_A(V_A^0 + G_A(e_A^\ell))\Big) \\ &\leq u_B\Big(\sum_{n=A,B} \phi_n(V_n^0 + G_n(\hat{e}_n)) - \phi_A\Big(V_A^0 + \sum_\ell \mu^\ell G_A(e_A^\ell)\Big)\Big), \end{aligned}$$

where the second and last inequalities follow from concavity of $u_B(\cdot)$ and convexity of $\phi_A(\cdot)$, respectively. Condition (17) thus implies

$$G_A(\hat{e}_A) > \sum_{\ell=1}^{L} \mu^\ell G_A(e_A^\ell), \tag{18}$$

which is in turn equivalent to (2) in Lemma 1 for $n = A$ and $\theta_A^\ell = \mu^\ell$. Since $\hat{\boldsymbol{e}}$ is implementable without renegotiation, $\hat{e}_A$ satisfies (1) and (2), and hence (16) and (17). The result follows. ||

Intuition

Since the proof of this key result is rather mechanical, we develop intuition in detail. To understand how the proper incentives are provided for the agents, suppose one of the agents is "degenerate" in the sense that his action does not affect the principal's payoff (but his participation is required for production). First consider the case in which agent A is degenerate. Agent B is then provided with the appropriate incentives in the same way as the agent in the one-agent case in which the principal makes a take-it-or-leave-it offer at the renegotiation stage (Hermalin and Katz, 1991, Proposition 1). In this degenerate case the initial payment scheme for agent B defined by (11) is simplified as $u_j^B = \hat{u}_j^B$ for all j. At the renegotiation stage, agent A, observing action e_B, offers the certainty equivalent of agent B's expected payoff under the initial contract $\phi_B(\sum_j P_j^B(e_B)\hat{u}_j^B)$. Then although agent B correctly expects his final income to be constant, his optimal choice is $\hat{e}_B$. In the degenerate case, although the principal cannot observe agent B's action, agent A plays exactly the same role as the principal in Hermalin and Katz, by offering a riskless renegotiation contract after observing his action.

Next suppose agent B is degenerate. Since agent A has all the bargaining power at the renegotiation stage, his final income does *not* depend on his initial payment scheme. However, it is affected by agent B's initial payment scheme, the sole role of which is hence to provide the proper incentives for agent A (in addition to ensuring participation). To simplify the notation, suppose $V_B^0 + G_B \equiv 0$. Then setting $\hat{u}_j^B = 0$ for all j yields $u_i^B = -f_i + g$ for all i as the initial scheme for agent B. The initial scheme for agent A is defined by $w_i^A = \overline{W} - \phi_B(u_i^B)$. Then agent A, anticipating renegotiation, chooses $\hat{e}_A$ if and only if

$$\hat{e}_A \in \arg\max_e u_A\Big(\overline{W} - \phi_B\Big(\sum\nolimits_i (P_i^A(\hat{e}_A) - P_i^A(e_A))f_i\Big)\Big) - G_A(e). \qquad (19)$$

Define

$$\Psi(e) = u_A\Big(\overline{W} - \phi_B(0)\Big) - u_A\Big(\overline{W} - \phi_B\Big(\sum\nolimits_i (P_i^A(\hat{e}_A) - P_i^A(e_A))f_i\Big)\Big).$$

$\Psi(e)$ represents the marginal benefit to changing action from e to $\hat{e}_A$. The decrease in agent B's expected utility resulting from the action change is translated into the increase in agent A's constant income, and hence his final utility. Condition (19) is then rewritten as

$$\Psi(e) \geq G_A(\hat{e}_A) - G_A(e) \quad \text{for all } e \in E_A.$$

The right-hand side is the marginal cost of action $\hat{e}_A$ with respect to e. If for each e the marginal cost does not exceed the marginal benefit, it is optimal for agent A to choose $\hat{e}_A$ under the initial contingent contract $(\boldsymbol{u}^A, \boldsymbol{u}^B)$ despite his correct anticipation of a constant final income.[8]

[8]These degenerate cases look similar to the single-agent case studied by Ma (1994) and Matthews (1995) in which only the second-best efficiency is attained in equilibrium. However, the principal in our model can achieve the first-best efficiency because the degenerate agent plays an important role of bearing risk with the other agent. Note that there is no "discontinuity" between the single-agent case and the multi-agent case: The mere existence of multiple agents does not change the result of the single-agent case if each agent renegotiates separately with the principal (see Subsection 4.4 for more on this).

Relation with Lemma 1

Lemmas 2 and 3 indicates the outcome of the subgame we are interested in: For a given action pair, if the principal offers the initial contract identified in Lemma 3, the agents can be induced to choose that action pair, provided that agent A proposes the new contract characterized in Lemma 2, which is accepted by agent B as well as by the principal. Lemma 3 presents a sufficient condition for such an outcome to be possible: $\hat{\boldsymbol{e}}$ is implementable *without renegotiation*. Since conditions (1) in Lemma 1 and (16) in Lemma 3 are identical, if $\hat{e}_B$ is implementable without renegotiation and $\hat{e}_A$ satisfies the "convex hull" condition (3) for $n = A$, then the conclusion of Lemma 3 follows immediately.

If (3) is violated and there are other strategies that induce the same density as $\hat{e}_A$, we must turn to the other condition (17), which is rewritten as follows.

$$V_B^0 + G_B(\hat{e}_B) < \sum_{\ell=1}^{L} \mu^\ell u_B(\overline{W} - \phi_A(V_A^0 + G_A(e_A^\ell))). \tag{20}$$

The left-hand side of (20) is the reservation utility of agent B (gross of the disutility of action). The right-hand side can be understood as follows. $V_A^0 + G_A(e_A^\ell)$ is the payment to agent A evaluated by $u_A(\cdot)$ such that when he chooses e_A^ℓ his final utility is equal to his reservation utility. The right-hand side is therefore agent B's final "expected" utility at A's mixed strategy (μ^ℓ). Condition (20) implies that this value exceeds agent B's reservation utility. If such a strategy existed, agent A would be better off by choosing the mixed strategy and proposing a renegotiation contract that reduces agent B's utility to the reservation level than choosing $\hat{e}_A$.

Note that "mixed strategy" (μ^ℓ) is costly for agent A in the following sense: It introduces additional risk to agent B, and to ensure agent B's participation, agent A has to leave more to agent B, which in turn reduces agent A's final income. This risk issue is absent in the commitment case. As a result, if (μ^ℓ) satisfies (18), then the same mixed strategy can be used to upset the implementation of $\hat{e}_A$ *without renegotiation*: condition (2) in Lemma 1 holds.[9] The converse is not generally true, however, as the following example shows.

Example Suppose that agent B is "degenerate," $V_B^0 + G_B \equiv 0$, and $V_A^0 = 0$. The set of agent A's feasible actions is $E_A = \{e_1, e_2, e_3\}$, and define $g_i = G_A(e_i)$ with $g_1 < g_2 < g_3$. Suppose there exists the unique vector (θ^1, θ^3) with $\theta^1 > 0$, $\theta^3 > 0$, and $\theta^1 + \theta^3 = 1$, that satisfies the conditions in Lemma 1: $\theta^1 P_k^A(e_1) + \theta^3 P_k^A(e_3) = P_k^A(e_2)$ for all k, and $g_2 - (\theta^1 g_1 + \theta^3 g_3) = \epsilon > 0$. Action e_2 is thus not implementable when

[9] Condition (18) in the renegotiation case actually reduces to (2) in the commitment case if (μ^ℓ) is a "pure strategy" (only one component of the vector is nonzero), or if the agents are risk neutral.

renegotiation is not possible. Condition (17) in the proof of Lemma 3 is given by

$$0 < \theta^1 u_B(\phi_A(g_2) + \phi_B(0) - \phi_A(g_1)) + \theta^3 u_B(\phi_A(g_2) + \phi_B(0) - \phi_A(g_3)).$$

By Jensen's inequality, the right-hand side is strictly smaller than $u_B(\phi_A(g_2) + \phi_B(0) - \phi_A(\theta^1 g_1 + \theta^3 g_3))$, which goes to zero as $\epsilon \downarrow 0$. We can thus conclude that for ϵ sufficiently small, agent A can be induced to choose e_2 in the renegotiation case.

4.3. *The main results*

Let us now turn back to the original game. What remains to be done is to show that there are the principal's beliefs that support the outcome of the subgame in which an implementable action pair is implemented at the first-best cost via renegotiation. It is the aim of the next proposition, which is our first main result. The proof is in Appendix.

Proposition 1. *Suppose $\hat{\mathbf{e}}$ is implementable without renegotiation. Then there exists an initial contract $\boldsymbol{w}$ within the class of constant-budget schemes such that, in subgame $\Gamma(\boldsymbol{w})$, there exists a perfect Bayesian Nash equilibrium in which* (i) *the agents choose $\hat{\mathbf{e}}$, and then agent A proposes the wage contract $\overline{\boldsymbol{x}}(\boldsymbol{w}, \hat{\mathbf{e}})$, which is surely accepted first by agent B and then by the principal;* (ii) *the equilibrium payoff to each agent is equal to his reservation utility level; and* (iii) *the principal's total expected payments to the agents coincide with the first-best cost of implementing $\hat{\mathbf{e}}$.*

The proposition establishes the existence of a constant-budget initial contract for each implementable action pair such that the agents are induced to select the action pair with the first-best cost. Out of the equilibrium path, the principal uses her beliefs on the agents' actions to calculate the expected total payments to the agents in a renegotiation contract.[10] In the proof of Proposition 1, we specify the principal's beliefs as follows: A renegotiation contract being proposed to the principal implies that agent A proposed and agent B accepted it. The principal infers from this fact that the actions selected make both agents better off under the proposed renegotiation contract than under either the initial contract or the optimal wage contract. Knowing the actions will be inferred in this way, the agents choose the intended action pair and offer the optimal wage contract.[11]

An equilibrium of the whole game can be identified if the principal can induce the agents to choose the first-best action pair, since it provides her with the highest expected payoff. Lemma 1 provides a sufficient condition for the existence of an equilibrium of the whole game in which the first-best outcome (the principal implements

[10]Her beliefs are irrelevant on the equilibrium path since the initial contract is constant-budget and the agents propose a riskless renegotiation contract.

[11]We thank the referees for turning our attention to the principal's beliefs.

the first-best action pair at the first-best cost) is achieved. We state the result in the following corollary.

Corollary 1. *There exists an equilibrium of the whole game in which the principal attains the first-best outcome by offering a constant-budget initial contract if the first-best action pair* $\mathbf{e}^{FB}$ *is implementable without renegotiation.*

If $\boldsymbol{e}^{FB}$ satisfies the weak conditions in Lemma 1 (e.g., the "convex hull" condition), the principal can find a constant-budget contract that defines the subgame in which it is an equilibrium for the agents to choose $\boldsymbol{e}^{FB}$ and the principal incurs the first-best cost $C_{FB}(\boldsymbol{e}^{FB})$.

Now suppose $\boldsymbol{e}^{FB}$ does not satisfies the condition. The principal may still be able to attain the first-best outcome, since the set of implementable actions expand as renegotiation is allowed. The example at the end of the previous subsection shows that renegotiation allows the first-best implementation of actions that are not implementable without renegotiation.

When the principal fails to attain the first-best outcome by constant-budget contracts, we cannot exclude the possibility that there exists an initial contract outside the set of constant-budget contracts that attains the first-best outcome via renegotiation.[12] However, there is at least one reason we prefer continuing to focus on constant-budget initial contracts: They are robust to the following modification of the model. Suppose that performance signals are observable by the principal but *unverifiable*. Then if payments to each agent are only contingent on his performance signals, the principal is tempted to reduce payments by insisting that his performance is low, even though the true performance is high, and hence the agents, expecting such a moral hazard behavior from the principal, will not exert costly actions. Under constant-budget schemes, the principal cannot benefit from such an opportunistic

[12] Hermalin and Katz (1991) and Matthews (1995) show in their one-agent models that when the agent makes a renegotiation offer, the "selling the store" (sales) contract, which transfers the random revenues from the principal to the agents, attains efficiency (first-best or second-best, depending on whether or not the principal can observe the agent's action). To examine the performance of the sales contract in our model, suppose that the verifiable performance signals are revenues: $R(\boldsymbol{e}) = \mathcal{E}[y_A + y_B|\boldsymbol{e}]$. If one of the agents, say agent A in our model, were risk neutral, the principal could simply sell the project to agent A with the price equal to the first-best expected benefits $R(\boldsymbol{e}^{FB}) - C_{FB}(\boldsymbol{e}^{FB})$. Agent A would then be provided with the incentives to select the first-best action, and the relationship between agent A and agent B would be reduced to the principal-agent relationship in which the new principal (agent A) can observe agent B's action. The result of Hermalin and Katz (1991, Proposition 3) would then apply. This arrangement does not work if both agents are risk averse as in our model, since they cannot eliminate all the risk between themselves. They must ask the principal to buy the project back from them. However, the principal in our model cannot know the agents' actions, and hence cannot buy it for a price depending on their action choice.

behavior as long as the total payments to the agents are verifiable. This is a well known benefit from rank-order tournaments (Carmichael, 1983; and Malcomson, 1984). Constant-budget schemes are more general contracts that share the same feature, and our analysis continues to be valid under this more plausible assumption.

If we confine our attention to constant-budget schemes and the first-best action pair cannot be implemented with renegotiation, the principal would choose an action pair that maximizes $R(\boldsymbol{e}) - C_{FB}(\boldsymbol{e})$ over the set of implementable action pairs, and the corresponding initial contract characterized by Lemma 3.

Proposition 1 shows that for each implementable action pair, the existence of just *an* equilibrium with desirable features in the subgame defined by an appropriately designed initial contract. It is a sort of *weak* implementation result. It does not exclude the possibility that other undesirable equilibria coexist in our subgame. Since the principal cannot observe the agents' actions, their proposal of a renegotiation contract serves as a signal of the actions. There hence exist many equilibria in subgame $\Gamma(\boldsymbol{w})$. However, we can establish what happens on the equilibrium path of *every* equilibrium once the actions have been chosen, as the following proposition shows. The proof is provided in Appendix.

Proposition 2. (a) *Suppose that initial contract $\boldsymbol{w}$ is a wage contract. In every equilibrium of subgame $\Gamma(\boldsymbol{w})$, if $\boldsymbol{x} \neq \boldsymbol{w}$ is proposed with positive probability, then the principal's equilibrium response is to surely reject $\boldsymbol{x}$.* (b) *Suppose that initial contract $\boldsymbol{w}$ consists of constant-budget schemes, but it is not a wage contract. Then in every equilibrium of subgame $\Gamma(\boldsymbol{w})$ in which $\mathbf{e}$ is chosen with positive probability, agent A proposes $\overline{\boldsymbol{x}}(\boldsymbol{w}, \boldsymbol{e})$ following the choice of action pair $\boldsymbol{e}$, and the principal as well as agent B surely accepts it.*

If the initial contract does not impose any risk on the agents, there is no gain from renegotiation, and the principal and the agents know some party must lose. They hence cannot reach an agreement.[13] In particular, the principal, although she cannot observe the actions, infers them correctly and rejects renegotiation contracts on the equilibrium path, and the initial wage contract goes through. The agents then obviously have no incentive to choose costly actions. This result is actually a multi-agent version of a result obtained from the analysis of the one-agent renegotiation model by Matthews (1995, Lemma 4).

The second result in Proposition 2 distinguishes our analysis from that of the one-agent model. Matthews (1995) shows that the second-best outcome cannot be improved, and it is actually attained in all the equilibria. In our multi-agent model, if initial contract is not a wage contract (but belongs to the set of constant-budget

[13]The idea is similar to the no-trade theorem by Milgrom and Stokey (1982).

contracts), on the equilibrium path agent A proposes a wage contract as a renegotiation contract and it is certainly accepted by agent B, and then by the principal.[14] In other words, in every equilibrium of the subgame defined by a constant-budget initial contract, renegotiation occurs and the final contract in force shields the agents completely from the risk. There is hence no "problem" of multiplicity at the renegotiation stage.[15]

4.4. *Discussions*

There are three key features of the model that derive our main results. First, we assume that performance signals are technologically and stochastically independent. It is then without loss of generality to limit our attention to individual-based contracts when we refer to initial contracts that implement a given action pair without renegotiation. And in the proof of Lemma 3, we have used an arbitrary individual-based contract that implements a given action $\hat{e}_B$ without renegotiation in order to construct an appropriate initial contract. Since we focus on initial contracts with a specific form of relative performance evaluation, technological externalities on performance measures would make the analysis harder, and the first-best implementation would no longer be sustained in equilibrium we construct. On the other hand, even if performance signals were correlated, the main insight of our analysis would be valid for actions pairs that are implementable without renegotiation by *individual-based contracts*. Proposition 1 would still holds with this slight modification of the condition, although it does not cover action pairs that are implementable without renegotiation *only* by interdependent contracts.

The second feature of our model is mutual monitoring between the agents. It is

[14] Note that the proposition does not tell us whether agent A proposes $\overline{\boldsymbol{x}}(\boldsymbol{w}, \boldsymbol{e})$ following *out-of-equilibrium* action choice $\boldsymbol{e}$. We will require imposing some belief restriction to derive such a strong result.

[15] There also may be a problem due to multiplicity of the agents' equilibrium action choice, even if we fix particular beliefs such as those specified in the proof of the existence result (Proposition 1). The beliefs in the proof induce the agents to choose actions as if they were to play simultaneous-move game $\Delta(\boldsymbol{w})$. Although $\hat{\boldsymbol{e}}$ is a Nash equilibrium of this game, there may be another action pair $\boldsymbol{e}' \neq \hat{\boldsymbol{e}}$ that satisfies (6) and (7). And if both agents weakly prefer (and at least one of them strictly prefers) $\boldsymbol{e}'$ to $\hat{\boldsymbol{e}}$, they may coordinate their equilibrium choice to deviate from the one the principal wishes to implement. It is pointed out that this kind of multiple-equilibria problems is common in multi-agent situations (Mookherjee, 1984).

In a working paper version of the paper (Ishiguro and Itoh, 1998), however, we show the following result: There is no equilibrium that is preferred by both agents (strictly by at least one of them) to the one given in Proposition 1, if the action pair satisfies a condition slightly stronger than that in Lemma 1. Note that the multiple-equilibria problem can also arise when the principal takes an alternative, centralized approach and designs a direct revelation mechanism. Ma (1988) shows that the designed actions can be implemented as the unique equilibrium, although *more complex multistage communication mechanisms must be allowed*.

crucial for our results. To see this, suppose that only one of the agents can observe the action of the other agent.[16] First, suppose that agent A cannot know the action chosen by agent B. Since it is the uninformed agent who makes offers, the information about agent B's action cannot be incorporated into renegotiation contracts. Agent A would have to give up information rent to agent B, and the uninformed principal would not be able to extract the rent unless some direct communication is allowed.

Next consider the case in which agent B cannot observe agent A's action. Suppose that action pair $\hat{\boldsymbol{e}}$ can be implemented at the first-best cost, via the renegotiation contract $\overline{\boldsymbol{x}}(\boldsymbol{w}, \hat{\boldsymbol{e}})$. Agent B would then accept the offer on the equilibrium path, believing that agent A has chosen $\hat{e}_A$. The equilibrium payoff to agent A would be $u_A(\overline{x}^A(\boldsymbol{w}, \hat{\boldsymbol{e}})) - G_A(\hat{e}_A)$. Now suppose agent A chooses e'_A instead of $\hat{e}_A$, which satisfies $G_A(e'_A) < G_A(\hat{e}_A)$, and offers $\overline{\boldsymbol{x}}(\boldsymbol{w}, \hat{\boldsymbol{e}})$. Since agent B would assign probability one to $\hat{e}_A$ following the observation of offer $\overline{\boldsymbol{x}}(\boldsymbol{w}, \hat{\boldsymbol{e}})$ and accept it, agent A would be able to enjoy $u_A(\overline{x}^A(\boldsymbol{w}, \hat{\boldsymbol{e}})) - G_A(e'_A) > u_A(\overline{x}^A(\boldsymbol{w}, \hat{\boldsymbol{e}})) - G_A(\hat{e}_A)$: Agent A is tempted to deviate from $\hat{e}_A$, and the principal cannot prevent him from deviation. The first-best implementation via renegotiation of a simple contract cannot therefore be sustained if only one of the agents is informed of the action of the other.[17]

Third, we make two assumptions concerning the bargaining protocol among the relevant parties: (i) one of the agents has full bargaining power; and (ii) renegotiation is decentralized in the sense that the principal lets the agents negotiate between them and propose a renegotiation contract after they reach an agreement. Although the proof of Lemma 3 relies on (i), we believe its main insight (each agent can be provided with appropriate incentives even though he knows he will not bear any risk) does not depend crucially on this assumption. In more general bargaining procedures whose outcome depends on both agents' threat points, the initial payment schemes for both agents can be used to provide the proper incentives for them. Implementation under the take-it-or-leave-it bargaining we have studied is in this respect more difficult since the initial scheme for just *one* agent must serve the dual role of providing the incentives for *both* agents.[18]

[16] We thank a referee for suggesting this issue to us.

[17] It will probably not be possible even if the informed agent can send a report to the principal, because there is no mechanism that can stop him from cheating. Mutual monitoring and reports from both agents are likely to be needed whether or not direct communication to the principal is possible. Miller (1997) shows that in the deterministic partnership model (no principal), if one of the agents can observe the action of at least one other agent, the first-best efficiency is sustained as an equilibrium by a contract that satisfies budget balancing and limited liability, allowing the informed agent to announce a public report concerning his observation. His model does not contain any stochastic element and the agents are risk neutral, and hence there is no risk concern.

[18] In Ishiguro and Itoh (1998), we study the standard CARA Normal model in which the agents propose a renegotiation contract following the generalized Nash bargaining solution. We show that

On the other hand, (ii) is crucial. Osano (1998) studies renegotiation with multiple agents in an alternative renegotiation procedure in which the agents independently and simultaneously propose new contracts to the principal. He shows that the principal attains the second-best efficiency in all equilibria under some belief restriction, and hence extends the results of Ma (1994) and Matthews (1995).[19]

5. CONCLUDING REMARKS

In this paper we show that when there are multiple agents and they can monitor their actions perfectly, the principal, who cannot observe the agents' actions, can utilize the information held by the agents and reduce the costs of implementing given actions down to the first-best level. She can do this by first proposing a "simple" constant-budget initial contract and then letting the agents negotiate and propose a new contract.[20]

An important implication from our analysis is that decentralization can be as effective as centralization under mild conditions. Most of the previous literature that identifies the advantages of decentralization over centralization in organizations assume either revelation mechanisms are infeasible or they are costly in terms of communication.[21] Under such assumptions, a centralized organization mode is short of efficiency, and hence room for improvement emerges, even though decentralization as

for each feasible action pair (in particular, for the first-best action pair), renegotiation of an initial contract with linear sharing schemes can implement the action pair with the first-best cost, even though performance signals may be correlated.

[19]Another relevant issue is endogenous bargaining power between the principal and the agents. If it is the principal who makes an offer at the renegotiation stage, there is no way she can incorporate the information possessed by the agents, and the first-best implementation is not attainable. Thus if the principal could choose and commit herself to a renegotiation process, she would choose to give the right to make renegotiation proposals to the agents.

[20]In our analysis we implicitly assume that the agents cannot write enforceable side contracts: The renegotiation contract they propose is not enforceable unless accepted by the principal. Our results continue to hold under the following alternative (not so compelling) setting. Suppose that after observing their actions but before the true values of publicly observable performance signals (y_A, y_B) are realised, two agents can write and *enforce* side contracts contingent *only* on the performance signals. In this "collusion" setting, the agents do not need approval from the principal to enforce side transfers. Given the initial contract, the agents simply play their game expecting that the final allocation is determined by the initial contract and the side transfers they agreed upon. The analysis is simpler because the principal is not involved in renegotiation and hence we do not need to concern about the signaling effects of renegotiation proposals. See Ishiguro and Itoh (1996) for the details of the analysis of the model from the perspective of collusion and the difference from the existing literature on collusion.

[21]See, for example, Bolton and Farrell (1990), Laffont and Martimort (1998), Macho-Stadler and Pérez-Castrillo (1998), McAfee and McMillan (1995), and Melumad, Mookherjee, and Reichelstein (1995)

well is less than efficient. Although the current paper does not aim at studying the centralization/decentralization issues, our result suggests that decentralization, in the sense of delegated negotiation and proposals from the agents, can be as effective as centralized schemes that utilize revelation mechanisms in unrestricted manners. Decentralized renegotiation of a simple contract may thus perform better than centralized complex mechanisms if (even a small amount of) bounded rationality is taken into account.[22]

Our analysis may also shed light on the issue of efficiency versus equality. The equality-efficiency tradeoff is often pointed out.[23] In particular, sharing labor income risks suffers from the loss of incentives. Labor union may be more interested in egalitarian policies than management, while the latter pursues productive efficiency by introducing differential treatments among workers. Our analysis suggests that the tradeoff can be resolved. For example, suppose, for some exogenous reasons, that management, as a principal, designs a rank-order tournament scheme to motivate homogeneous agents to work hard. The optimal tournament scheme is often criticized on the ground that it generates an extreme differential treatment between winners and losers who are determined only by luck. Our analysis can be applied to this situation, and can show that management can resolve the problem by allowing the labor union to propose an alternative scheme in which winners and losers receive the same payments. The workers are still motivated to work hard because the rank-order tournament scheme in the initial contract can be designed to provide the appropriate incentives for them to improve their bargaining positions against each other.

[22]Our approach may thus fall within the "complete contracting approach to simple institutions" in Maskin and Tirole (1999): "simple" contracts can perform as well as more complicated schemes do.

[23]See Putterman, Roemer, and Silvestre (1998) for a survey.

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Appendix

Proof of Proposition 1.

For a given action pair $\hat{\boldsymbol{e}}$ that is implementable without renegotiation, define $\boldsymbol{w}^B$ as in (11) with (f_i) solving (15), and $\boldsymbol{w}^A$ by $w_{ij}^A = \overline{W} - w_{ij}^B$ for $i, j = 1, \ldots, K$, where $\overline{W}$ is defined by (12). Lemma 3 ensures the existence of such a contract. We construct an equilibrium of game $\Gamma(\boldsymbol{w})$ with the properties in the proposition.

Strategy profiles

1. Agent A's strategy: Choose $\hat{e}_A$. Propose $\overline{\boldsymbol{x}}(\boldsymbol{w}, \boldsymbol{e})$ when action pair $\boldsymbol{e}$ is observed.

2. Agent B's strategy: Choose $\hat{e}_B$. Accept agent A's proposal $\boldsymbol{x}$ with probability one if and only if it satisfies (AB) when $\boldsymbol{e}$ is observed. Otherwise, reject $\boldsymbol{x}$ with probability one.

3. Principal's strategy: Accept the renegotiation contract $\boldsymbol{x}$ with probability one if and only if $\boldsymbol{x}$ satisfies

$$\mathcal{E}_\pi\Big[\sum\sum\nolimits_{i,j} P_{ij}(\boldsymbol{e})(x_{ij}^A + x_{ij}^B)\Big] \leq \overline{W} \tag{A1}$$

   where $\mathcal{E}_\pi[\cdot]$ is the expectation operator over action pairs based on the principal's beliefs $\pi(\cdot|\boldsymbol{w}, \boldsymbol{x})$ over $E_A \times E_B$. Otherwise, reject $\boldsymbol{x}$ with probability one.

Principal's beliefs Define $E_1(\boldsymbol{w}, \boldsymbol{x})$, $E_2(\boldsymbol{w}, \boldsymbol{x})$, and $E(\boldsymbol{w}, \boldsymbol{x})$, each of which is a subset of $E_A \times E_B$, as follows.

$$E_1(\boldsymbol{w}, \boldsymbol{x}) = \Big\{\boldsymbol{e} \;\Big|\; \sum\sum\nolimits_{i,j} P_{ij}(\boldsymbol{e}) u_A(x_{ij}^A) \geq u_A(\overline{x}^A(\boldsymbol{w}, \boldsymbol{e}))\Big\} \tag{A2}$$

$$E_2(\boldsymbol{w}, \boldsymbol{x}) = \Big\{\boldsymbol{e} \;\Big|\; \sum\sum\nolimits_{i,j} P_{ij}(\boldsymbol{e}) u_B(x_{ij}^B) \geq \sum\sum\nolimits_{i,j} P_{ij}(\boldsymbol{e}) u_B(w_{ij}^B)\Big\} \tag{A3}$$

$$E(\boldsymbol{w}, \boldsymbol{x}) = E_1(\boldsymbol{w}, \boldsymbol{x}) \cap E_2(\boldsymbol{w}, \boldsymbol{x}) \tag{A4}$$

The principal's beliefs at the information set $(\boldsymbol{w}, \boldsymbol{x})$ are specified as follows. If the renegotiation contract is $\boldsymbol{x} = \overline{\boldsymbol{x}}(\boldsymbol{w}, \boldsymbol{e})$ for some $\boldsymbol{e}$, then $\pi(\boldsymbol{e}|\boldsymbol{w}, \boldsymbol{x}) = 1$. If $\boldsymbol{x} \neq \overline{\boldsymbol{x}}(\boldsymbol{w}, \boldsymbol{e})$ for all $\boldsymbol{e} \in E_A \times E_B$, then $\pi(E(\boldsymbol{w}, \boldsymbol{x})|\boldsymbol{w}, \boldsymbol{x}) = 1$ unless $E(\boldsymbol{w}, \boldsymbol{x}) = \emptyset$, in which case her beliefs are arbitrary.

Principal's best response The principal's strategy specified above is obviously optimal. In particular, if $\boldsymbol{x} = \overline{\boldsymbol{x}}(\boldsymbol{w}, \boldsymbol{e})$ for some $\boldsymbol{e}$, accepting it with probability one is optimal. To see this, since $\overline{\boldsymbol{x}}(\boldsymbol{w}, \boldsymbol{e})$ is a wage contract and solves (AA), (8) holds with equality for all action pairs. (A1) hence holds with equality irrespective of the principal's beliefs, and the principal is indifferent between accepting and rejecting $\boldsymbol{x}$. It is thus optimal to accept it with probability one.

Agent A's best response The major part of the proof is to show that it is optimal for agent A to propose the wage contract $\overline{\boldsymbol{x}}(\boldsymbol{w}, \boldsymbol{e})$ for each action pair $\boldsymbol{e}$ selected and observed.

Step 1: If $\boldsymbol{x} = \overline{\boldsymbol{x}}(\boldsymbol{w}, \boldsymbol{e})$, then agent B and the principal surely accept it, and hence agent A obtains $u_A(\overline{x}^A(\boldsymbol{w}, \boldsymbol{e})) \geq \sum\sum_{i,j} P_{ij}(\boldsymbol{e}) u_A(w^A_{ij})$.

Step 2: Suppose $\boldsymbol{x}$ is another wage contract and $\boldsymbol{x} = \overline{\boldsymbol{x}}(\boldsymbol{w}, \boldsymbol{e}')$ for some $\boldsymbol{e}' \neq \boldsymbol{e}$. We consider two cases separately. *Case 1*: (AB) is satisfied and hence agent B surely accepts it. Since the principal surely accept it, agent A obtains $u_A(\overline{x}^A(\boldsymbol{w}, \boldsymbol{e}'))$. Suppose $u_A(\overline{x}^A(\boldsymbol{w}, \boldsymbol{e}')) > u_A(\overline{x}^A(\boldsymbol{w}, \boldsymbol{e}))$. Since $\overline{x}^A(\boldsymbol{w}, \boldsymbol{e}) + \overline{x}^B(\boldsymbol{w}, \boldsymbol{e}) = \overline{x}^A(\boldsymbol{w}, \boldsymbol{e}') + \overline{x}^B(\boldsymbol{w}, \boldsymbol{e}') = \overline{W}$, $\overline{x}^B(\boldsymbol{w}, \boldsymbol{e}) > \overline{x}^B(\boldsymbol{w}, \boldsymbol{e}')$ must hold. Then since

$$u_B(\overline{x}^B(\boldsymbol{w}, \boldsymbol{e}')) < u_B(\overline{x}^B(\boldsymbol{w}, \boldsymbol{e})) = \sum\sum\nolimits_{i,j} P_{ij}(\boldsymbol{e}) u_B(w^B_{ij}),$$

(AB) cannot hold, which is a contradiction. $u_A(\overline{x}^A(\boldsymbol{w}, \boldsymbol{e}')) \leq u_A(\overline{x}^A(\boldsymbol{w}, \boldsymbol{e}))$ thus must hold. *Case 2*: (AB) is violated and hence agent B surely rejects it. In this case, agent A's payoff is $\sum\sum_{i,j} P_{ij}(\boldsymbol{e}) u_A(w^A_{ij})$: Agent A cannot be better off by proposing $\overline{\boldsymbol{x}}(\boldsymbol{w}, \boldsymbol{e}')$ instead of $\overline{\boldsymbol{x}}(\boldsymbol{w}, \boldsymbol{e})$.

Step 3: Suppose $\boldsymbol{x} = (\overline{x}^A, \overline{x}^B)$ (i.e., a wage contract) but $\boldsymbol{x} \neq \overline{\boldsymbol{x}}(\boldsymbol{w}, \boldsymbol{e})$ for all $\boldsymbol{e}$. If either the principal or agent B rejects it, agent A's payoff is $\sum\sum_{i,j} P_{ij}(\boldsymbol{e}) u_A(w^A_{ij})$. If both the principal and agent B accept it, it must satisfy $\overline{x}^A + \overline{x}^B \leq \overline{W}$ and $u_B(\overline{x}^B) \geq \sum\sum_{i,j} P_{ij}(\boldsymbol{e}) u_B(w^B_{ij})$, and hence $\boldsymbol{x}$ is feasible for program (AA). Since $\overline{\boldsymbol{x}}(\boldsymbol{w}, \boldsymbol{e})$ is the solution to (AA), $u_A(\overline{x}^A) \leq u_A(\overline{x}^A(\boldsymbol{w}, \boldsymbol{e}))$. Now if $\boldsymbol{e} \in E_1(\boldsymbol{w}, \boldsymbol{x})$, then $u_A(\overline{x}^A) \geq u_A(\overline{x}^A(\boldsymbol{w}, \boldsymbol{e}))$, and hence $\boldsymbol{x} = \overline{\boldsymbol{x}}(\boldsymbol{w}, \boldsymbol{e})$ must hold. Contradiction. If $\boldsymbol{e} \notin E_1(\boldsymbol{w}, \boldsymbol{x})$, $u_A(\overline{x}^A) < u_A(\overline{x}^A(\boldsymbol{w}, \boldsymbol{e}))$ and hence agent A cannot be better off by proposing $\boldsymbol{x}$ than $\overline{\boldsymbol{x}}(\boldsymbol{w}, \boldsymbol{e})$.

Step 4: Suppose agent A proposes $\boldsymbol{x}$ which is not a wage contract. Suppose both the principal and agent B accept it. It is then feasible for program (AA), and hence $u_A(\overline{x}^A(\boldsymbol{w}, \boldsymbol{e})) > \sum\sum_{i,j} P_{ij}(\boldsymbol{e}) u_A(x^A_{ij})$ must hold. Agent A is thus worse off by proposing $\boldsymbol{x}$ than $\overline{\boldsymbol{x}}(\boldsymbol{w}, \boldsymbol{e})$.

Step 5: Suppose agent A proposes $\boldsymbol{x}$ such that $E(\boldsymbol{w}, \boldsymbol{x}) = \emptyset$. It implies either $\boldsymbol{e} \notin E_1(\boldsymbol{w}, \boldsymbol{x})$ or $\boldsymbol{e} \notin E_2(\boldsymbol{w}, \boldsymbol{x})$. If the former holds, agent A is better off by offering $\overline{\boldsymbol{x}}(\boldsymbol{w}, \boldsymbol{e})$ instead, which is surely accepted. If the latter holds, agent B rejects $\boldsymbol{x}$, and hence agent A cannot increase his payoff by proposing $\boldsymbol{x}$.

The five steps given above prove that wage contract $\overline{\boldsymbol{x}}(\boldsymbol{w}, \boldsymbol{e})$ is an optimal contract offer for agent A when $\boldsymbol{e}$ is observed.

Finally, the argument given above implies that agent A's payoff is given by $u_A(\overline{x}^A(\boldsymbol{w}, e_A, \hat{e}_B)) - G_A(e_A)$ when he chooses e_A and agent B follows the strategy specified above. Lemma 3 ensures that it is optimal for agent A to choose $\hat{e}_A$.

Agent B's best responses Agent B's acceptance decision specified above is obviously optimal. And given that agent A and the principal follow the strategies specified above, agent B's payoff, when he chooses e_B, is $u_B(\overline{x}^B(\boldsymbol{w}, \hat{e}_A, e_B)) - G_B(e_B)$. Lemma 3 shows that it is in fact optimal for agent B to choose $\hat{e}_B$.

Payoffs Lemmas 2 and 3 ensure that on the equilibrium path, their payoffs satisfy conditions (ii) and (iii) in the proposition. ||

Proof of Proposition 2.

(a) Suppose $\boldsymbol{w} = \overline{\boldsymbol{w}}$ (wage contract) and consider an equilibrium of subgame $\Gamma(\boldsymbol{w})$. Let $\boldsymbol{x}$ be an equilibrium renegotiation contract proposed by agent A, which is accepted by agent B with probability $b > 0$. Now suppose the principal accepts $\boldsymbol{x}$ with probability $p > 0$. Then there must be an action pair $\boldsymbol{e}$, chosen with a positive probability on the equilibrium path, such that

$$\sum\sum\nolimits_{i,j} P_{ij}(\boldsymbol{e}) u_B(x_{ij}^B) \geq u_B(\overline{w}^B) \tag{A5}$$

$$\sum\sum\nolimits_{i,j} P_{ij}(\boldsymbol{e})(x_{ij}^A + x_{ij}^B) \leq \overline{w}^A + \overline{w}^B. \tag{A6}$$

Since agent A could have proposed $\boldsymbol{w}$ instead of $\boldsymbol{x}$,

$$pb\sum\sum\nolimits_{i,j} P_{ij}(\boldsymbol{e}) u_A(x_{ij}^A) + (1 - pb) u_A(\overline{w}^A) \geq u_A(\overline{w}^A).$$

Since $pb > 0$, it implies

$$u_A(\overline{w}^A) \leq \sum\sum\nolimits_{i,j} P_{ij}(\boldsymbol{e}) u_A(x_{ij}^A) \leq u_A\Big(\sum\nolimits_i \sum\nolimits_j P_{ij}(\boldsymbol{e}) x_{ij}^A\Big)$$

or $\overline{w}^A \leq \sum_i \sum_j P_{ij}(\boldsymbol{e}) x_{ij}^A$ with strict inequality unless $x_{ij}^A = \overline{w}^A$ for all i, j. This and (A6) result in $\overline{w}^B \geq \sum\sum_{i,j} P_{ij}(\boldsymbol{e}) x_{ij}^B$, and hence

$$u_B(\overline{w}^B) \geq u_B\Big(\sum\sum\nolimits_{i,j} P_{ij}(\boldsymbol{e}) x_{ij}^B\Big) \geq \sum\nolimits_i \sum\nolimits_j P_{ij}(\boldsymbol{e}) u_B(x_{ij}^B)$$

with strict inequality unless $x_{ij}^B = \overline{w}^B$ as well as $x_{ij}^A = \overline{w}^A$ for all i, j. However, the strict inequality cannot hold because of (A5). Hence $\boldsymbol{x} = \boldsymbol{w}$. ||

(b) Let initial contract $\boldsymbol{w}$ satisfy $w_{ij}^A + w_{ij}^B = \overline{W}$ for all i, j and consider an equilibrium of subgame $\Gamma(\boldsymbol{w})$ in which $\boldsymbol{x}$ is an equilibrium proposal by agent A, which is accepted by agent B with probability $b(\boldsymbol{e})$ and by the principal with probability p. Let $E^*(\boldsymbol{w}, \boldsymbol{x})$ be the set of action pairs that are chosen with positive probabilities and are followed by the renegotiation proposal $\boldsymbol{x}$ on the equilibrium path. For each $\boldsymbol{e} \in E^*(\boldsymbol{w}, \boldsymbol{x})$, agent A's payoff, gross of disutility of action, is

$$U_A(\boldsymbol{e}) \equiv pb(\boldsymbol{e}) \sum\sum_{i,j} P_{ij}(\boldsymbol{e}) u_A(x_{ij}^A) + (1 - pb(\boldsymbol{e})) \sum\sum_{i,j} P_{ij}(\boldsymbol{e}) u_A(w_{ij}^A).$$

Note that since agent A could propose $\boldsymbol{w}$ instead, $U_A(\boldsymbol{e}) \geq \sum\sum_{i,j} P_{ij}(\boldsymbol{e}) u_A(w_{ij}^A)$ or

$$\sum\sum_{i,j} P_{ij}(\boldsymbol{e}) u_A(x_{ij}^A) \geq \sum\sum_{i,j} P_{ij}(\boldsymbol{e}) u_A(w_{ij}^A) \tag{A7}$$

must hold.

Step 1: For each $\boldsymbol{e} \in E^*(\boldsymbol{w}, \boldsymbol{x})$, agent A could always offer $(\overline{x}^A(\boldsymbol{w}, \boldsymbol{e}) - \epsilon, \overline{x}^B(\boldsymbol{w}, \boldsymbol{e}) + \delta)$ instead, which is surely accepted by agent B and the principal for arbitrary $\epsilon > \delta > 0$, and could enjoy $u_A(\overline{x}^A(\boldsymbol{w}, \boldsymbol{e}) - \epsilon) > \sum\sum_{i,j} P_{ij}(\boldsymbol{e}) u_A(w_{ij}^A)$ for sufficiently small ϵ since $\boldsymbol{w}$ is not a wage contract. Therefore $U_A(\boldsymbol{e}) \geq u_A(\overline{x}^A(\boldsymbol{w}, \boldsymbol{e}))$ must hold for all $\boldsymbol{e} \in E^*(\boldsymbol{w}, \boldsymbol{x})$, for $\boldsymbol{x}$ to be an equilibrium renegotiation contract. This also implies $pb(\boldsymbol{e}) > 0$ must hold for all $\boldsymbol{e} \in E^*(\boldsymbol{w}, \boldsymbol{x})$.

Step 2: Since $\boldsymbol{x}$ is accepted by the principal with a positive probability, there must exist $\hat{\boldsymbol{e}} \in E^*(\boldsymbol{w}, \boldsymbol{x})$ such that

$$\sum\sum_{i,j} P_{ij}(\hat{\boldsymbol{e}})(x_{ij}^A + x_{ij}^B) \leq \overline{W} = \overline{x}^A(\boldsymbol{w}, \hat{\boldsymbol{e}}) + \overline{x}^B(\boldsymbol{w}, \hat{\boldsymbol{e}}). \tag{A8}$$

Now suppose $U_A(\hat{\boldsymbol{e}}) > u_A(\overline{x}^A(\boldsymbol{w}, \hat{\boldsymbol{e}}))$. Then by (A7) and Jensen's inequality,

$$u_A\Big(\sum\sum_{i,j} P_{ij}(\hat{\boldsymbol{e}}) x_{ij}^A\Big) \geq \sum\sum_{i,j} P_{ij}(\hat{\boldsymbol{e}}) u_A(x_{ij}^A) \geq U_A(\hat{\boldsymbol{e}}) > u_A(\overline{x}^A(\boldsymbol{w}, \hat{\boldsymbol{e}}))$$

or $\sum\sum_{i,j} P_{ij}(\hat{\boldsymbol{e}}) x_{ij}^A > \overline{x}^A(\boldsymbol{w}, \hat{\boldsymbol{e}})$ holds. Then by (A8), $\sum\sum_{i,j} P_{ij}(\hat{\boldsymbol{e}}) x_{ij}^B < \overline{x}^B(\boldsymbol{w}, \hat{\boldsymbol{e}})$ must hold. Using Jensen's inequality and (10) yields

$$\sum\sum_{i,j} P_{ij}(\hat{\boldsymbol{e}}) u_B(x_{ij}^B) < \sum\sum_{i,j} P_{ij}(\hat{\boldsymbol{e}}) u_B(w_{ij}^B).$$

It contradicts the fact that $\boldsymbol{x}$ is accepted by agent B with positive probability following action choice $\hat{\boldsymbol{e}}$. $U_A(\hat{\boldsymbol{e}}) \leq u_A(\overline{x}^A(\boldsymbol{w}, \hat{\boldsymbol{e}}))$ thus must hold.

Step 3: Two steps given above yields $U_A(\hat{\boldsymbol{e}}) = u_A(\overline{x}^A(\boldsymbol{w}, \hat{\boldsymbol{e}}))$. Then Step 2 can be repeated to show that $u_A\Big(\sum\sum_{i,j} P_{ij}(\hat{\boldsymbol{e}}) x_{ij}^A\Big) = \sum\sum_{i,j} P_{ij}(\hat{\boldsymbol{e}}) u_A(x_{ij}^A) = U_A(\hat{\boldsymbol{e}})$, that is, $\boldsymbol{x} = \overline{\boldsymbol{x}}(\boldsymbol{w}, \hat{\boldsymbol{e}})$, $p = 1$, and $b(\hat{\boldsymbol{e}}) = 1$.

Step 4: Since $\boldsymbol{x}$ is a wage contract, (A8) must hold for all $\boldsymbol{e} \in E^*(\boldsymbol{w}, \boldsymbol{x})$, and repeating Steps 2 and 4 yields $\boldsymbol{x} = \overline{\boldsymbol{x}}(\boldsymbol{w}, \boldsymbol{e})$ and $b(\boldsymbol{e}) = 1$ must hold for all $\boldsymbol{e} \in E^*(\boldsymbol{w}, \boldsymbol{x})$. ||