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The post-war growth experiences of developing countries lead to the idea that income equality may accelerate economic growth. In this paper, a theoretical model showed the possibility that equality makes a country human-capital abundant, which enables industrialization and higher economic growth. On the other hand, in unequal developing countries where majority of people manage to survive at minimum consumption level, human capital investment such as schooling cannot be done. Such countries become unskilled labor abundant and suffer further from low economic growth. In addition, the two-good framework showed the possibility that protecting infant industry with dynamic externality enhances economic growth.

JEL Classification Numbers: O15, F11, O41
Keywords: income distribution; economic growth; human capital; comparative advantage; learning-by-doing.

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1. Introduction

Recently, it is often advocated that equal income distribution accelerates economic growth of developing countries. This argument is derived by empirical researches (Alesina and Rodrik, 1991; Persson and Tabellini, 1992, 1994) and rapid economic growth of the relatively equal East Asian Newly Industrializing Economies (World Bank, 1993). Much research effort has been devoted to explain this fact.


This paper examines the effect of distribution on growth through human capital accumulation on the lines of Galor and Zeira (1993). This paper, however, differs from other researches that the employed model is a two-good growth model. By using a two-good framework, this model contributes to clarify the following two points in addition to the effect of distribution on growth.

First, this model can explain that an economy generally experiences industrialization from agricultural economy when it grows rapidly. Secondly, this model showed that protecting an infant industry with dynamic externality can enhance
economic growth. Even when an economy does not have comparative advantage in manufacturing sector, protecting it can accelerate growth rate and raise the welfare of the economy.

The logic to support the above arguments is as follows. In an equal developing country, large share of the people can receive education and the country becomes human-capital abundant. Therefore, such a country has comparative advantage in the production of human-capital intensive good such as manufactured good. As production of manufactured good exhibits externality and raises the general productivity through learning-by-doing, such equal country can experience the increase of productivity and average income. Higher income raises the educational level in the next period, which further accelerates industrialization and economic growth (virtuous circle). On the other hand, in an unequal economy, only a small number of rich people can afford education, which makes the economy unskilled labor abundant. As a result, such an economy specializes in the production of agricultural good, productivity stagnates, and growth rate becomes lower (vicious circle). Therefore, an economy becomes industrialized as it grows and protecting manufacturing sector enhances economic growth. In addition, the model also examines how income distribution changes as the economy grows and shows the possibility that distribution changes as the inverted-U hypothesis of Kuznets (1955).

The considered model extends a basic trade model of a small open economy with two goods and two factors in two aspects. First, the factor endowments are determined endogenously, depending on distribution of income. Secondly, overlapping generations model and endogenous growth theory are used to make the model dynamic.
in order to examine growth rate. The static equilibrium of the model is examined in section 2, and the dynamic equilibrium in section 3. Section 4 examines the policy implications, and the final section is summary and conclusion.

2. The static equilibrium

A small country that trades two goods at exogenously given world prices is studied. The goods are Z (agricultural goods) and M (manufactured goods), which are produced using two factors: \( A_tL \) (unskilled labor) and \( A_tH \) (human capital). The factors are not traded, and \( A_t \) denotes the productivity level of factors. Production technology exhibits constant returns to scale and time-invariant. Manufactured goods are assumed to be relatively human-capital intensive, while agricultural goods be unskilled labor-intensive. The economy produces both goods or specializes in the production of one of the goods, depending on the state of its comparative advantage.

Let the upper bound of the incomplete specialization cone be denoted by \( \left( \frac{A_tH}{A_tL} \right) \), and the lower bound by \( \left( \frac{A_tH}{A_tL} \right) \). Then, incomplete specialization results when factor endowment is in \( \left( \frac{A_tH}{A_tL}, \frac{A_tH}{A_tL} \right) \). Otherwise, the economy specializes in manufactured good if factor endowment is larger than \( \left( \frac{A_tH}{A_tL} \right) \), and specializes in agricultural good if it is smaller than \( \left( \frac{A_tH}{A_tL} \right) \). These equilibria are examined in the following separate sections.
2.1. The case of incomplete specialization

In the case of incomplete specialization, the equilibrium can be shown in the following three stages.

2.1.1 The determination of wage rates and input coefficients

In the first stage, incomplete specialization implies that unit cost of each good must be equal to its world price. Namely,

\[ w_H a_{HM}(w_H, w_L) + w_L a_{LM}(w_H, w_L) = P_M \]  \hspace{1cm} (1)

and

\[ w_H a_{HZ}(w_H, w_L) + w_L a_{LZ}(w_H, w_L) = P_Z, \]  \hspace{1cm} (2)

where \( P_M \) and \( P_Z \) are respectively the world price of good M and Z, \( w_H \) and \( w_L \) are the rewards to human capital and low-skilled labor, and \( a_{j'}(w_H, w_L) \) denotes the unit input coefficient of factor j for good j’. Note that \( w_H, w_L \) and \( a_{j'}(w_H, w_L) \) are measured with efficiency unit of inputs, namely, \( \frac{1}{A_i} \). The production technology is described by these unit input coefficients. Given \( P_M \) and \( P_Z \), these equations give the equilibrium \( w_H, w_L \) and \( a_{j'}(w_H, w_L) \).

2.1.2 Utility maximization and determination of factor supplies

In the second stage, given the wage rates determined in the first stage, altruistic individuals maximize their utility by choosing levels of their consumption and their children’s education, which subsequently determines the aggregate supply of human
capital. In each period $t = 0, 1, 2, \cdots \infty$, agents are born and live for two periods. Each agent gains one child at the beginning of the second period, and therefore the population is constant.

In the first period, individuals have no endowment of labor or goods. They receive education financed by their parents and gain human capital. In the second period, they are endowed with one unit of unskilled labor and work by supplying their labor inelastically. They spend their wage to consume and educate their children. Some of the adults give education to their children as bequest, because agents are assumed to be altruistic and care also about their children’s income. Agents born in period $t$ and receive $h_t$ units of education gain $\phi(h_t)$ units of human capital. When they work in period $t+1$, their productivity is $A_{t+1}$ and they receive $A_{t+1}w_L$ for their unskilled labor and $A_{t+1}w_H\phi(h_t)$ for their human capital. Therefore, the income of individuals who are born at period $t$ and work at period $t+1$ is given by

$$A_{t+1}y_{t+1} = A_{t+1}(w_L + w_H\phi(h_t)).$$

(3)

Note that all individuals have same potential ability and differ only in their levels of education.

Given the above income, agents choose the levels of their consumption of each good and children’s education. First, consider the optimization of the share of $c_{M_{t+1}}$ and $c_{Z_{t+1}}$ for given amount spent on consumption. The utility maximization problem is given by

$$\max_{c_{M_{t+1}}, c_{Z_{t+1}}} u(c_{M_{t+1}}, c_{Z_{t+1}})$$

(4)
\[
\begin{align*}
s.t. \quad c_{t+1} &= P_M c_{M\{t+1\}} + P_Z c_{Z\{t+1\}}. \\
\end{align*}
\]

Denote the Marshallian demand functions of this problem as \( c_{M\{t+1\}}^* (P_M, P_Z, c_{t+1}) \) and \( c_{Z\{t+1\}}^* (P_M, P_Z, c_{t+1}) \). As \( P_M \) and \( P_Z \) are constant, \( c_{M\{t+1\}}^* \) and \( c_{Z\{t+1\}}^* \) depend only on \( c_{t+1} \). Therefore, a Hicks’ composite good can be defined as

\[
P_M c_{M\{t+1\}}^* (c_{t+1}) + P_Z c_{Z\{t+1\}}^* (c_{t+1}) = c_{t+1},
\]

which is called consumption thereafter and its price is one.

Secondly, agents choose \( c_{t+1} \) and \( h_{t+1} \). They solve

\[
\begin{align*}
\max_{c_{t+1}, h_{t+1}} u_{t+1} &= u(c_{t+1}) + v(A_{t+2} (w_L + w_H \phi(h_{t+1}))) \\
\text{s.t.} \quad & A_{t+1} (w_L + w_H \phi(h_{t+1})) = c_{t+1} + \alpha h_{t+1} \\
& h_{t+1} \geq 0, \quad c_{t+1} \geq 0,
\end{align*}
\]

where \( u \) is the utility from the adults’ consumption, \( v \) is the utility the altruistic parents gain from their children’s income, and \( \alpha \) denotes the unit cost of education. Perfect foresight is assumed concerning the level of \( A_{t+2} \) and individuals treat \( A_{t+2} \) as given.

The first constraint is an ordinary budget constraint, and the second and third are the non-negativity constraints on \( h_{t+1} \) and \( c_{t+1} \). It is assumed that \( u' > 0, u'' < 0, v'' < 0, v' > 0, v' < 0, \phi' > 0, \phi'' < 0 \).

Using Lagrange multiplier \( \lambda \) and Kuhn-Tucker multipliers \( \mu \) and \( \eta \), the first-order conditions of the above problem are given by the following equations and (8):

\[
\begin{align*}
u'(c_{t+1}) &= \lambda_{t+1} - \eta_{t+1} \\
v'(.)A_{t+2} w_H \phi'(h_{t+1}) &= \lambda_{t+1} \alpha + \mu_{t+1}
\end{align*}
\]
\[
\begin{align*}
\mu_{t+1} & \geq 0, \quad h_{t+1} \geq 0, \quad \mu_{t+1} h_{t+1} = 0 \\
\eta_{t+1} & \geq 0, \quad c_{t+1} \geq 0, \quad \eta_{c_t} c_{t+1} = 0 \quad \square
\end{align*}
\]

As there are Kuhn-Tucker conditions, the solutions can be divided into some cases. Assume, however, that the non-negativity constraint on consumption does not become binding as long as agents have positive income, because agents need to consume something to survive. Therefore it is not necessary to examine the case with \( c_{t+1} = 0 \) and \( \mu_{t+1} > 0 \), and \( c_{t+1} > 0 \) and \( \mu_{t+1} = 0 \) is assumed in the rest of the paper.

On the other hand, the non-negativity constraint on educational level sometimes becomes binding and some very poor agents do not give any education to their children. Thus, solutions are divided into the two cases with \( h_{t+1} > 0 \) and \( h_{t+1} = 0 \). Figure 2 gives the income-expansion path with such utility function. When income is lower than some level, the optimum choice becomes a corner solution with \( h_{t+1} = 0 \).

When \( \mu_{t+1} = 0 \) and \( h_{t+1} = 0 \), the first-order conditions can be rewritten as

\[
\frac{u'(A_{t+1} \tilde{y}_{t+1})}{v'(A_{t+2} w_1) A_{t+1} w_2 \phi'(0)} = \frac{1}{\alpha}.
\]

The level of income, \( A_{t+1} \tilde{y}_{t+1} \), which divides the two cases is given by the above equation for given \( A_{t+1} \). Then, when \( y_{t+1} > \tilde{y}_{t+1} \), the non-negativity constraint on education is not binding and \( h_{t+1} > 0 \), when \( y_{t+1} \leq \tilde{y}_{t+1} \), it is binding and \( h_{t+1} = 0 \). Note that \( \tilde{y}_{t+1} \) rises if \( A_{t+1} \) increases.

In the first case with \( h_{t+1} > 0 \) and \( \mu_{t+1} = 0 \), \( y_{t+1} > \tilde{y}_{t+1} \)

The first-order conditions become

\[
\frac{u'(c_{t+1})}{v'(.) A_{t+2} w_2 \phi'(h_{t+1})} = \frac{1}{\alpha}, \quad \left( \frac{\text{MU of } c_{t+1}}{\text{MU of } h_{t+1}} = \frac{\text{MC of } c_{t+1}}{\text{MC of } h_{t+1}} \right). \quad \square
\]
By differentiating the above equation, the comparative statics give the following results.

When the parents’ income increases, their consumption and their children’s education and income change such that

\[
\frac{dh_{t+1}}{dy_{t+1}} = \frac{\alpha u''A_{t+1}}{\alpha u'' + v'A_{t+2}w_H^2(\phi'')^2} > 0, \quad (16)
\]

\[
\frac{d^2h_{t+1}}{dy_{t+1}^2} = \frac{\alpha u'''A_{t+2}[-v'A_{t+2}w_H^2(\phi'')^2 - v''A_{t+2}^2w_H^2(\phi'')^2]}{[\alpha u'' + v'A_{t+2}w_H^2(\phi'')^2 + v''A_{t+2}^2w_H^2(\phi'')^2]^2} < 0. \quad (\because u''' < 0) \quad (17)
\]

To exclude an unrealistic solution that educational level of children becomes infinite as the income level of parents rises, \( u''' \) is assumed to be negative.

In the second case with \( h_{t+1} = 0 \) and \( \mu_{t+1} \geq 0 \) (\( y_{t+1} \leq \tilde{y}_{t+1} \)), the first-order conditions become

\[
\frac{u'(c_{t+1})}{v'(A_{t+2}w_H\phi'(h_{t+1}))} = \frac{\lambda_{t+1}}{\alpha \lambda_{t+1} - \mu_{t+1}}. \quad (18)
\]

The levels of consumption and education are given by

\[
c_{t+1} = A_{t+1}y_{t+1}, \quad (19)
\]

\[
h_{t+1} = 0. \quad (20)
\]

In this case, the parents are too poor to educate their children, and spend all their income on their consumption. In the developing countries where the average income level is low, such households consist considerable part of the economy.

These optimum choices of the children’s education which satisfy the above first-order conditions are shown in figure 3 for given \( A_{t+1} \). Notice that education reaches zero level at positive \( y_{t+1} \).

As shown above, the level of education is a function of the level of their parents’
income. Thus, in the whole economy, the pattern of income distribution determines the aggregate level of education and human capital. As income is approximately distributed lognormally, the three density functions of lognormal distribution with different variance and the same mean are shown in figure 4. Figure 3 is put on figure 4 in figure 5a and 5b. Figure 5a shows an example that the larger the inequality (variance \( \sigma^2 \)) is, the less people can receive education and the lower the economy’s level of human capital, if average income is moderately low. In other words, in an equal developing country large share of people can receive education and the country becomes human-capital abundant. On the other hand, in an unequal country only a small number of people can receive education and therefore it becomes low-skilled labor abundant.

In a country where average income is extremely low, however, the opposite is true (Figure 5b). If income is equally distributed, everyone is equally poor and unable to afford education. If distribution is unequal, at least some of the agents can educate their children and therefore the country gains some aggregate human capital.

Using the above relationship between income distribution and factor endowment, the relationship between distribution of income and the pattern of production can be described. As the considered economy is a developing country, assume that it either specializes in the production of agricultural good (Z) or produces both agricultural goods (Z) and manufacturing goods (M).

Consider, first, the case of a moderately poor country. If distribution is unequal with \( \sigma^2 > \bar{\sigma}^2 \), \( H_L \) is scarce and \( \frac{A_L H_L}{A_L} < \frac{A_L H_L}{A_L} \). This economy
completely specializes in agriculture \((z)\). In a relatively equal country with \(\sigma^2 \leq \overline{\sigma^2}\), \(\frac{A_iH_i}{A_iL} \in \left[\frac{A_iH_i}{A_iL}, \frac{A_iH_i}{A_iL}\right]\) and it incompletely specializes in production. Namely, industrialization occurs in addition to agriculture. Therefore, the case currently we are considering is a moderately equal country.

In an extremely poor country, the opposite relationship between inequality and factor ratio exists. Therefore, unequal country has a better chance to succeed in industrialization.

### 2.1.3 The factor market equilibrium

In the third stage, given the factor supplies examined in the second stage, the amount of production of goods are determined in the equilibrium of the factor markets. Market clearing implies that

\[
a_{HM} M_i + a_{HZ} Z_i = A_i H_i \tag{21}
\]

\[
a_{LM} M_i + a_{LZ} Z_i = A_i L \tag{22}
\]

Solving these two equation gives the equilibrium amount of production of each good as

\[
Z_i = \frac{1}{|a|} (a_{HM} A_i L - a_{LM} A_i H_i), \tag{23}
\]

\[
M_i = \frac{1}{|a|} (a_{HZ} A_i L - a_{LZ} A_i H_i), \quad \text{where} \quad |a| = \left| \begin{array}{c} a_{HM} \\ a_{LM} \\ a_{HZ} \\ a_{LZ} \end{array} \right| > 0 \tag{24}
\]

Therefore, the GNP at period \(t\), \(Q_t\), is given by

\[
Q_t = P_Z Z_t + P_M M_t
\]

\[
= P_Z \left\{ \frac{1}{|a|} (a_{HM} A_i L - a_{LM} A_i H_i) \right\} + P_M \left\{ \frac{1}{|a|} (-a_{HZ} A_i L + a_{LZ} A_i H_i) \right\}. \tag{25}
\]
Notice that the level of GNP indicates the level of welfare, because the considered economy is a small country.

2.2. The case of complete specialization

If only small number of people can receive education, the economy becomes unskilled labor abundant. Such country completely specializes in the production of the agricultural goods. The equilibrium $w_H, w_L, H_t, L_t, Z_t$ are simultaneously determined by the following equations:

\[ w_H a_{HZ} + w_L a_{LZ} = P_Z \]  \hspace{1cm} (26)

\[
\max_{c_{t+1}, h_{t+1}} u(c_{t+1}) + V(A_{t+2}(w_L + w_H \phi(h_{t+1})))
\]

\[ s.t. \quad A_{t+1}(w_L + w_H \phi(h_t)) = c_{t+1} + \alpha h_{t+1} \]  \hspace{1cm} (27)

\[ h_{t+1} \geq 0, \quad c_{t+1} \geq 0 \]

\[ a_{HZ}Z = A_t H_t \]  \hspace{1cm} (28)

\[ a_{LZ}Z = A_t L_t \]  \hspace{1cm} (29)

The GNP is given by

\[ Q_t = P_Z Z_t. \]  \hspace{1cm} (30)

3. Dynamic Equilibrium

Now consider how the economy evolves dynamically. In dynamic equilibrium, the increase of $H_t$ and the learning-by-doing of manufacturing good production cause
3.1 The increase of $H_t$

First, examine the effects of the increase of aggregate human capital, $H_t$. As shown in the previous section, adults determine their children’s educational level and income for given $A_{t+1}$ and $A_{t+2}$. As a result, the educational level in each dynasty changes and as does the aggregate human capital. In order to illustrate the dynamic evolution of education and income through time, the dynamics of $y_t$ based on figure 3 is presented in figure 6 for given $A_{t+1}$. As the educational level corresponds to income level by one-to-one in this model, this figure represents the evolution of educational level $h_t$ as well as $y_t$.

The figure 6 depicts the case where the dynamics of $y_t$ intersects with the 45° line at two points. In this case, the descendants of rich individuals with income more than $\hat{y}$ receive more and more education and converge to the high-level equilibrium with income $y^*$. On the other hand, the descendants of poor agents with income less than $\hat{y}$ may receive some education but converge to the low level equilibrium with zero education and low income. In other words, all the dynasties are concentrated in two groups, depending on the level of the initial income.

3.2 The increase of productivity through learning-by-doing

Secondly, consider the effect of learning-by-doing. In the country where
industrialization occurs, learning-by-doing raises the factor productivity $A_t$ and accelerate economic growth. Namely, the cumulative amount of produced manufacturing goods raise the factor productivity $A_t$ as described by the following functions:

$$A_t = f(\kappa_t), \quad \text{where} \quad \kappa_t = \sum_{s=1}^{t-1} M_s, \quad (31)$$

$$f' > 0, \quad f'' < 0, \quad \lim_{\kappa_t \to \infty} f' = 0, \quad \lim_{\kappa_t \to \infty} f = \bar{A}, \quad w_L \geq \frac{c}{\bar{A}}. \quad (32)$$

For the sake of discussion, assume that knowledge accumulation of manufacturing sector completely spillovers to the agricultural sector and raises the factor productivity of agricultural sector at the same rate of the manufacturing sector. This case is analytically interesting and mainly considered in this paper. The other case with incomplete knowledge spillover is briefly examined in the last section.

In a relatively equal country, industrialization occurs and it raises $A_t$. The effects of increase of $A_t$ can be examined by differentiating the first-order-conditions. The following assumption is imposed to analyze this effect.

**Assumption 1**

$$-v''(\cdot)A_t \pi \left[ w_L + w_L \phi(h_{t-1}) \right] v'(\cdot) < 1$$

This means that the measure of comparative risk aversion is small enough and intertemporal substitution is large. Therefore, when $A_{t+2}$ rises and the return to education increases, the optimal educational level of children $h_t$ increases under this
assumption.

In case 1, for given \( y_t \),

\[
\frac{dh_{t+1}}{dA_{t+1}} = \frac{cau^*(w_L + w_H \phi(h_t))}{\alpha^2 u''(\cdot) + v^*(\cdot) [A_{t+2}w_H \phi'(\cdot)]^2 + v'(\cdot)A_{t+2}w_H \phi''(\cdot)} > 0,
\]

(33)

\[
\frac{dh_{t+1}}{dA_{t+2}} = \frac{-[v'^*(\cdot)w_L + w_H \phi(\cdot)]A_{t+2}w_H \phi'(\cdot) + v'(\cdot)w_H \phi'(h_t)]}{\alpha^2 u''(\cdot) + v^*(\cdot) [A_{t+2}w_H \phi'(\cdot)]^2 + v'(\cdot)A_{t+2}w_H \phi''(\cdot)} > 0,
\]

(34)

under assumption 1. Therefore,

\[
\frac{dh_{t+1}}{dA_{t+1}} + \frac{dh_{t+1}}{dA_{t+2}} > 0
\]

(35)

and educational level rises.

In case 2, educational level remains zero. As for the income level which divides the two cases,

\[
\frac{d\tilde{y}_{t+1}}{dA_{t+1}} = \frac{\tilde{y}_{t+1}}{A_{t+1}} < 0,
\]

(36)

\[
\frac{d\tilde{y}_{t+1}}{dA_{t+2}} = \frac{\nu^*(\cdot)w_L A_{t+2}w_H \phi'(0) + v'(\cdot)w_H \phi'(0)}{\alpha^2 u^*(\cdot)A_{t+1}} < 0,
\]

(37)

under assumption 1. Therefore,

\[
\frac{d\tilde{y}_{t+1}}{dA_{t+1}} + \frac{d\tilde{y}_{t+1}}{dA_{t+2}} < 0
\]

(38)

and increase of productivity enables more individuals to receive education.

Therefore, this effects can be shown in figures by the upward shift of the dynamics of \( y_t \) as \( A_t \) approaches the upper bound \( A \) (Figure 7). Non-negativity constraint on consumption become unbinding for more individuals, and more and more dynasties approach the high-level equilibrium. As \( A \) assumed to satisfy \( w_L \geq \frac{c}{A} \), all
the dynasties start to converge to the high-level equilibrium as infinite time passes.

In an unequal country where no manufacturing goods are produced, no learning-by-doing occurs and the dynamics are completely described by figure 6. In this case, the growth occurs only by the increase of $H_t$. Therefore, growth rate is lower than that in the equal country where industrialization takes place.

Next, examine how the income distribution changes as the economy grows. In an unequal economy which completely specializes in agriculture, individuals become polarized into the rich and the poor as shown in figure 6. Therefore, an originally unequal country become unequal and poor.

In an equal economy, at first polarization takes place. Some of the agents approaches the high-level equilibrium, while the rest moves toward the low-level equilibrium. Next, as $\frac{c}{A_t}$ declines, more and more people become richer and educate their children. This further increase their income and accelerate industrialization until all people reach high-education and high-income equilibrium. In this process, inequality first rises and then declines. Therefore, there is a possibility that income distribution changes as the inverted-U hypothesis by Kuznets (Kuznets 1955).

4. The Effects of Government Policies

In this section, the implications of three government policies on economic growth and welfare are considered. The policies are income redistribution, subsidy on education and import tariffs.
4.1. The Optimal Income Redistribution Policy

It is interesting to analyze what kind of income redistribution favors economic growth. For analytical purpose, assume that income follows a uniform distribution $U[\mu - \sigma, \mu + \sigma]$. Then, the variance of income is $\sigma^2/3$ and the ratio of the agents who can receive education is $\mu + \sigma - \hat{y}/2\sigma$. Consider the welfare implications of a redistribution policy where government alters the variance.

$$\frac{\partial}{\partial \sigma} (\mu + \sigma - \hat{y} \frac{\sigma}{2\sigma^2}) = \frac{\hat{y} - \mu}{2\sigma^2}$$

(39)

indicate the following results.

In countries which are not extremely poor with $\mu > \hat{y}$, the more equally income is redistributed, the more agents receive education. It raises the level of human capital and the growth rate. This result is consistent with the findings that equal East Asian countries grew faster than unequal Latin American countries.

In the very poor countries with $\mu < \hat{y}$, on the other hand, the more unequally income is redistributed, the more agents can receive education. This is because everyone is too poor to educate his/her child if distribution is equal, but some rich can afford education if distribution is unequal. Therefore, unequal redistribution raise the aggregate human capital, growth rate and steady-state income level. This effect is particularly clear when the country become incompletely specialized from complete specialization.

4.2. Subsidy to education

When government gives subsidy to education and lowers the cost of education
from $\alpha$ to $(1 - \psi)\alpha$, its effects on the optimal choice of agents are as follows.

Assuming incomplete specialization, the effects can be shown by comparative statics with differentiating equations (14), (15) and (20) and evaluating the derivative at $\psi = 0$.

In Case 1, from equation (15),

$$\frac{dh_{r+1}}{d\psi} \bigg|_{\psi=0} = \frac{-\alpha u'(c_{r+1}) + \alpha^2 u''(c_{r+1})h_{r+1}}{\alpha^2 u''(c_{r+1}) + v^*(A_{r+2}y_{r+2})A_{r+2}w_H\varphi'(h_{r+1})^2 + u'(A_{r+2}y_{r+2})A_{r+2}w_H\varphi''(h_{r+1})} > 0$$

(40)

indicates that agents gives more education to their children with the subsidy to education.

In case 2, from equation (20),

$$h_{r+1} = 0$$

indicates that very poor agents still cannot give any education to their children, even if education is subsidized.

As for $\tilde{y}_{r+1}$, from equation (14),

$$\frac{d\tilde{y}_{r+1}}{d\psi} \bigg|_{\psi=0} = \frac{u'(\cdot)}{u''(\cdot)A_{r+1}} < 0$$

(41)

shows that ratio of individuals who can receive education increases.

This effect can be shown as the upward shift of $h_{r+1}(y_r)$. Subsidy enables the agents who are originally receive any education to gain more education, and increases the number of agents who can receive education. As a whole economy, level of education always rises, which accelerate industrialization and raise the steady-state level of income.
4.3. Import Tariff on Manufactured goods

What are the welfare implications of trade policy? To clarify the results, examine this policy with specific Cobb-Douglas utility and production functions.

\[ u_{t+1} = C^\beta_t \left( A_{t+2} y_{t+2} - \bar{y} \right)^{1-\beta} , \quad \phi (h_{t+1}) = h_{t+1}^\theta, \]

\[ M_{t} = B_{M} H_{t}^{\delta} L_{t}^{1-\delta}, \quad Z_{t} = B_{Z} H_{t}^{\gamma} L_{t}^{1-\gamma} \]

Assuming incomplete specialization, factor prices changes as the domestic prices of the manufactured good changes with the import tariff.

\[ \frac{\partial w_L}{w_L} = \frac{-\gamma}{\delta - \gamma} \frac{dP_M}{P_M}, \quad \frac{\partial w_H}{w_H} = \frac{1-\gamma}{\delta - \gamma} \frac{dP_M}{P_M}. \] (42)

When the tariff rate is denoted by \( T_M \), the assumption of a small country ensures \( dP_M = dT_M \). These conditions enables to examine the effects of import tariff on the optimal choice of education\(^1\).

In case 1, the first-order-condition of utility maximization is given by

\[ \frac{\beta}{y_{t+1} - \alpha h_{t+1}} \alpha = (1-\beta) w_H \theta h_{t+1}^{\theta-1}. \] (43)

Total differentiation of this condition implies

\[ \frac{\partial h_{t+1}}{\partial w_L} = \frac{\alpha \beta}{\alpha w_H \theta h_{t+1}^{\theta-1}} > 0, \quad \frac{\partial h_{t+1}}{\partial w_H} = \frac{\beta w_L}{w_H \theta h_{t+1}^{\theta-1}} > 0. \] (44)

Therefore, combining equations (40) and (41), the effects of import tariff on education is given by

\[ \frac{dh_{t+1}}{dT_M} = \frac{\beta w_L (1-2\gamma)}{w_H \theta h_{t+1}^{\theta-1} (\delta - \gamma) P_M} \]

which is positive if \( \gamma < 1/2 \) (\( \because \delta - \gamma > 0 \)). This condition has the following implications.

When domestic \( P_M \) rises due to import tariff, \( w_H \) increases and \( w_L \) decreases as unit cost curve of good M shifts to the right in figure 1 (Stolper-Samuelson Theorem). If \( \gamma \) is small enough and good Z is very unskilled labor intensive, \( w_H \) increases largely and \( w_L \) decreases only slightly. In this case, parent’s income increases and it
raises the children’s educational level and income, because children’s income is assumed to be a normal good. Therefore, if $\gamma < 1/2$, import tariff raises the educational level and human capital. This argument shows that protecting an industry with externality such as manufacturing can accelerate economic growth, even if the country currently does not have comparative advantage in such an industry (Infant industry).

The case with incomplete knowledge spillover from learning by doing can be analyzed in the present context. If productivity in the manufacturing sector rises more than that in the agricultural sector, the unit cost curve of good M in figure 1 shifts to the right more than good Z. Therefore, $w_H$ increases and $w_L$ decreases, which changes the educational level of children. This effect on human capital and growth are the same as the case of import tariff on good M. Thus, in this case of incomplete knowledge spillover, learning-by-doing in manufacturing sector raises growth rates only when good Z is very unskilled labor intensive.

**Conclusion**

The post-war growth experiences of developing countries lead to the idea that income equality may accelerate economic growth. In this paper, a theoretical model showed the possibility that equality makes a country human-capital abundant, which enables industrialization and higher economic growth. In addition, the two-good framework showed the possibility that protecting infant industry with dynamic externality enhances economic growth.

Two additional extensions can be addressed in future work. First, unequal
distribution may lower economic growth rate through higher population growth. As shown in Barro and Becker (1989) and Becker, Murphy and Tamura (1990), if quantity and “quality” of children are substitutes, poor households tend to have many children with low education. Therefore, in an unequal economy with large number of the poor, population growth rate is higher and the economy becomes more unskilled labor abundant, which further deters industrialization. Secondly, whether the comparative advantage explanations will pass a proper econometric investigation should be examined in future work.

**Footnote**

1 When domestic $P_M$ changes due to import tariff, the price of the composite good $C_t = P_M C_w + P_Z C_z$ changes. In this case, however, Cobb-Douglas utility function ensures the constant expenditure share for $C_t$ and $A_{t+2}y_{t+2} - \bar{y}$, and it is unnecessary to examine the effect of the change of $P_M$ on education.

**References**


Figure 1
Figure 3
Figure 4
Figure 5a