JJIE IN PREPARATION ???, 1–44 (2000)

ARTICLE NO. 0123456789

Does International Diversi¯cation Really Diversify Risks?

Tokuo Iwaisako*

Institute of Policy and Planning Sciences, University of Tsukuba
Tsukuba, Ibaraki 305-8573, JAPAN
and
Institute of Economic Research, Hitotsubashi University
2-1 Naka, Kunitachi City, Tokyo 186-8603, JAPAN
E-mail: iwaisako@ier.hit-u.ac.jp

Communicated by Someone

Received ????; revised ????; accepted ????

Thispaper demonstrates that large adverse shocks are more highly correlated with one another than positive shocks across national stock markets of industrialized economies. This …nding is robust if we allow for an ARCH process or if we exclude the data of October 1987. It is shown that the negative skewness of the world market portfolio is primarily responsible for such time-varying correlations of national stock markets. We propose to model the world market portfolio return by using the extended QGARCH model of Campbell and Hentschel (1992). The …nding suggests that the U.S. investors' bene…t from international portfolio diversi…cation could be far more limited than is commonly thought.

Key Words: keywords: international diversi¯cation; GARCH models; QGARCH;

* This paper is a revised version of Essay I from the author's Ph.D. thesis (Iwaisako 1997) submitted to Harvard University. I am particularly grateful to John Campbell for suggesting much of what is good in this paper, and to Sanjiv Das and Takatoshi Ito for detailed comments on earlier drafts. I also thank Nick Barberis, Brian Hall, Andrew Metrick, Arun Muralidhar, Jim Stock, Hitoshi Takehara, Akiko Tamura, Cong-Minh Trinh, Luis Viceira, and seminar participants at Harvard, Hitotsubashi University, International Monetary Fund, Tokyo-Metropolitan University, University of Tokyo, and

During the last twenty years, the world has witnessed substantial deregulation of national …nancial markets and the liberalization of international capital ‡ows. Investors in major industrialized countries now have more opportunities for international diversi…cation than in earlier decades. The bene…ts from international diversi…cation have been emphasized by both academic economists and practitioners.[1] However, only limited fractions of investors' portfolios are invested in foreign equities. Recently, French and Poterba (1991) have highlighted this home-country bias in stock investments.[2]

It has been informally suggested that investors cannot really diversify risks by investing in foreign stocks because stock prices in di¤erent national markets covary more closely when large adverse shocks hit. An obvious example of such a negative shock is the October Crash of 1987. If such a large negative shock can not be hedged, risk-averse investors will not bene…t much by international diversi…cation. In this paper, the correlations of stock markets in developed economies are carefully examined. Strong evidence is found for higher correlations in large negative returns, even if Black Monday is excluded from the data or if the ARCH e¤ect in asset

University of Tsukuba for comments and discussions; Lee Branstetter, Yuki A. Honjo, and Tomoko Wakabayashi-Takai for their assistance in improving the style of English used in this paper; and the Murata Overseas Scholarship for …nancial support.

[1]For example, Ibbotoson and Brison(1993), Obstfeld(1994), and Siegel(1994).

[2]See Frankel (1994) and Lewis (1995) for the good survey of research on this and related topics.

returns is taken into account. We focus on the statistical aspects and the robustness of this …nding; other independent studies also suggested the fact that international correlations are higher during bear markets.[3] In particular, this paper shows that the distribution of the world market portfolio is signi…cantly negatively skewed, that is, the undiversi…able world market risk is fat-tailed at the down side. In order to capture such a statistical property of the world market risk, this paper proposes to use Campbell and Hentschel's (1992) extension of the QGARCH model which was originally developed by Sentana (1995). The extended QGARCH model allows us to directly parameterize the negative skewness of the world market risk, and it is shown that this model successfully captures various aspects of the international stock market data. Following the application of the QGARCH model to the international environment in the …rst half, in the second half of the paper, we focus on the implications of the statistical …nding to international diversi…cation. The bene…ts from international diversi…cation is calculated in various ways paying particular attention to the common down side risk in the world capital markets. It is shown that the conventional mean-variance framework, which completely ignores the e¤ects of higher moments, could signi…cantly overestimate the bene…ts from international diversi…cation.

The remainder of the paper is organized as follows. Section I presents statistical evidence that large negative innovations in the stock returns of national markets are more highly correlated than positive innovations. It

[3]See Das (1994), and particularly De Santis and Gerard (1997).

is shown that an ARCH e¤ect and the negatively skewed distribution of the world market portfolio return are responsible for this …nding. Section II proposes a bivariate GARCH model of international asset pricing in which the world market risk is modeled as a quadratic GARCH (QGARCH) process and country speci…c risk is modeled by GARCH(1,1). It is shown that this model captures the pattern of the time-varying international correlations very well, compared with ordinary GARCH models. In section III, the bene…ts from international diversi…cation are calculated, paying attention to the down side risk and changing volatility. Section IV concludes the paper.

1. CORRELATIONS BETWEEN NATIONAL STOCK MARKETS: THE DATA AND THE SETUP

The data used in this paper are monthly international stock indices tabulated by Morgan Stanley Capital International (MSCI data). The sample period is from January 1970 through Feburary 1998. For the sample of individual countries, I chose the G7 countries (Canada, France, Germany, Italy, Japan, United Kingdom, and USA), and the biggest non-G7 market, Switzerland. The total value of these eight countries is more than 60 percent of the value covered by the MSCI data. All indices are computed with dividends reinvested so that returns include both capital gains and dividend yields.

This paper takes the viewpoint of the absolute long-run passive investor: she holds her portfolio for a certain period without rebalancing it, calculates her return in the domestic currency term, and does not hedge exchange rate risks. In some previous studies, such as French and Poterba (1991) and Tesar and Werner (1994, 1995), the researchers emphasized the existence of home-bias and used currency risk-hedged data. On the other hand, Engel (1994) tested the international CAPM in unhedged national currency terms, and Siegel (1994) recommended the currency risk-unhedged investment to U.S. investors who care mainly about the long-run. Using exchange-risk-unhedged data, this paper might underestimate the bene…t from international diversi…cation when it is actually calculated in section III. However, previous studies using hedged data do not explicitly include the cost of hedging in their calculations. Thus there are shortcomings in both approaches. Moreover, the statistical properties of the international stock market correlations found in this paper are robust whether the hedged or unhedged data are used.

Table I reports descriptive statistics of the excess returns over the U.S. Treasury bill rate with one month left to expiration. In addition to individual stock markets, the statistics of the simple average of eight countries (eight) and the value-weighted world index of all capitalization covered by the MSCI data (world) are reported.

[Table I about here]

Let us set up the framework to evaluate the bene…t from international diversi…cation and see why the higher moments of the international stock returns potentially have large effects on an investor's expected utility. Consider an investor in country p who has a portfolio constructed by the domestic stock market index and a weighted basket of the other seven countries' indices. We denote the weight of the domestic asset in her portfolio by w_p. Then the return of the portfolio, R_p, is

$$R_p = w_p \cdot r_p + (1 - w_p) \cdot R_{seven} \tag{1}$$

where r_p is the return of the domestic market index, and R_{seven} is the average return of seven other countries. We assume that the investor's utility depends on her terminal wealth a, and she has a power utility function.

$$u(a) = \frac{1}{1-\beta} a^{1-\beta} \tag{2}$$

where β is the coefficient of the constant relative risk aversion (CRRA). The CRRA utility is invariant to scale so that we can normalize the investor's initial wealth equal to one. Thus her terminal wealth is equal to the gross return of her portfolio, that is $a = 1 + R_p$. Let us define the expected gross return of the normalized portfolio by $\mu \equiv E[1 + R_p]$, then u(a) can be expanded in a Taylor's series about its expected value, μ

$$u(a) = u(\mu) + \frac{u'(\mu)}{1!}(a-\mu) + \frac{u''(\mu)}{2!}(a-\mu)^2 + \frac{u'''(\mu)}{3!}(a-\mu)^3 + \cdots \tag{3}$$

Taking expectations, we have,

$$E[u(a)] = u(\mu) + \frac{u''(\mu)}{2!}E[(a - \mu)^2] + \frac{u'''(\mu)}{3!}E[(a - \mu)^3] + \cdots \quad (4)$$

If the absolute values of the moments higher than the second are negligible, the mean-variance analysis is a good approximation of the more general case of the CRRA utility in (4). If not, the third and the fourth moments might have significant effects on the utility of a risk averse investor. The higher moments have certain economic interpretations as well. The third moment would become significant if the return of her portfolio is negatively skewed, that is, if there is a significant down side risk. The fourth moment captures the effect of the variation of conditional volatility, which is captured by the class of ARCH models.

Taking U.S. and Japanese investors for examples, Table II shows how the standard error and the third and fourth moments of the return of the investor's portfolio changes as her position becomes more internationally diversified. The standard errors and the fourth moments decrease as the weight of foreign assets in her portfolio increases. So international diversification reduces the unconditional variance of her portfolio and the volatility of the conditional variance. On the other hand, the third moments increase as the weight of foreign assets increases, so as w_p decreases in all cases considered here. Especially, in the case of the U.S. data excluding Black Monday, the third moment increases by an order of magnitude, from -0.009 to -0.086 to -0.188, as the share of foreign assets goes up from 0% to 5% to 47%. This suggests that there are large common negative shocks in the national stock markets, and thus, the bene…t from international diversi…cation is somewhat o¤set. Going back to equation (4), this e¤ect appears as an increase of the third moment in a Taylor approximation of the investor's expected utility.

[Table II about here]

2. THE EFFECTS OF EXTREME OBSERVATIONS TO THE INTERNATIONAL CORRELATIONS OF STOCK MARKETS

Next, we examine if international stock returns really covary more closely in responding to large negative shocks by examining the sensitivity of their correlations without extreme observations. In particular, we focus on the correlation between the U.S. stock market (USA) and the average of the remaining seven countries (seven). The strategy is as follows. First, the observations of USA and seven from the same date are paired. These pairs are then sorted according to the values of USA. Thus the pair consists of the maximum observation of USA and the observation of seven on the same date is ranked at the top. Next, the largest X percent pairs according to the values of USA are excluded from the sample, and the correlation is calculated for the remaining sub sample. In the same manner, the correlations are calculated for the sub samples excluding the lowest X percent of the pairs according to the values of USA. Finally, the whole exercise is repeated for the case when the observations are sorted according to the values of seven.

The results of this exercise are shown in Table III. The full sample correlation coefficient is 0.613, and if the observation of October 1987 is excluded from the sample, it is 0.586. In Panel (B) and (C), correlations are calculated for the sub samples excluding extreme observations. It is obvious that eliminations of the largest and the smallest observations produce asymmetric effects on the correlations of USA and seven. Excluding the lower percentiles has the effect of lowering the correlation more significantly. In every column, the entry value in the second low (Lower X%) is smaller than in the first low (Upper X%). This pattern is equally observed for when the series were sorted according to USA or seven. Thus, the statistical evidence confirms that the U.S. market and other seven markets move more closely when they face large adverse shocks.

[Table III is about here]

3. POSSIBLE SOURCES OF THE TIME-VARYING INTERNATIONAL CORRELATIONS

If a bivariate nonlinear relationship exists with the returns of the national stock market indices, it is easy to explain why the markets covary more closely when they are going down. Let us denote the function which exhibits such nonlinearity as $\Phi(.)$, and the world market risk at time t as $R_{world,t}$. Then the individual country return of country p would be written as

$$r_{p,t} = \Phi(R_{world,t}) + \varepsilon_{p,t} \qquad (5)$$

In addition, we assume that $R_{world;t}$ and $\varepsilon_{p;t}$ follow some symmetric distributions, although they do not have to be unconditionally normal distributions. For example, usual ARCH models maintaining a conditional normality assumption or t-GARCH model satisfy this restriction.

In Figure 1, the existence of such contemporaneous nonlinearity is examined using non-parametric kernel regressions, plotting the USA series against the seven series. The solid line in the graph is the fitted OLS regression, and the dotted lines are the results from applying the normal kernel-type smoother. The curves bend downward in the region that excess returns are negative, so apparently there is contemporaneous nonlinearity. However, if we exclude October 1987 from the sample, we do not find significant nonlinearity in this graph. In Table IV, the existence of nonlinearity in the sample excluding October 1987 is statistically examined, using the following parametric regression:

$$r_{p;t} = \alpha + \beta_1 \cdot R_{eight;t} + \beta_2 \cdot R^2_{eight;t} \cdot I_t \qquad (6)$$

$$\begin{aligned} I_t &= \text{dummy variable} = 1 \quad \text{if} \quad R_{eight;t} < 0 \\ &= 0 \qquad\qquad\qquad \text{otherwise} \end{aligned}$$

where $r_{p;t}$ is the excess return of each country and $R_{eight;t}$ is the excess return of eight. For all countries, the nonlinear terms (β_2) are found to be insignificant. Using other specifications for the nonlinear term or using the robust regression yielded similar results. Overall, it is safe to conclude that if the observation of October 1987 is excluded from the sample, there is very little evidence of the contemporaneous nonlinearity.

[Figure 1 and Table IV about here]

Thus, it is not contemporaneous nonlinearity causing higher correlations in response to large negative shocks. However, if the world market portfolio follows a negatively skewed distribution, it is possible that the time-varying international correlation between $R_{world;t}$ and $r_{p;t}$ can be observed even if $\Phi(\cdot)$ is a linear function.

$$r_{p;t} = \alpha + \beta_p \cdot R_{eight;t} + \varepsilon_{p;t} \tag{7}$$

(7) is a special case in which $\Phi(\cdot)$ in (6) is a linear function. However, the symmetric distribution assumption in (6) for $r_{p;t}$ and $R_{eight;t}$ is relaxed.

In table I, we have already seen that $R_{eight;t}$ is more negatively skewed than $r_{p;t}$ (see also column (i) in Table V). Except in the case of Canada, this result is found to be robust even if we excluded October 1987, and the results for this case are reported in column (iii) of Table V.

[Table V about here]

The same problem can be examined in a different way. The right hand side of equation (7), the return of the country p's index, $r_{p;t}$, is the sum of the negatively skewed variable ($\beta_p \cdot R_{eight;t}$) and the orthogonal noise term ($\varepsilon_{p;t}$). Thus, it is expected that the noise term $\varepsilon_{p;t}$ is less negatively (or more positively) skewed than both $r_{p;t}$ and $R_{eight;t}$. This conjecture can be examined by comparing the skewness of $r_{p;t}$ and $\hat{\varepsilon}_{p;t}$ in columns (i) and (ii) in Table V. Except for Japan and the UK, the negative skewness of $r_{p;t}$ is more pronounced than of $\hat{\varepsilon}_{p;t}$. In column (iii) and (iv), October 1987 is excluded from the sample, and it lowers the significance of this difference. However, it is still true that the negative skewness is less pronounced for $\hat{\varepsilon}_{p;t}$ in the same six countries.

The possible cause of the inconsistent results for Japan and U.K. is the instability of the country betas, β_p. In Figure 2, we plotted the conditional betas for Japan, Switzerland, U.K., and the U.S. They were estimated using rolling regressions with a sixty-month window. From this graph, the constancy of beta appears invalid for the case of Japan and U.K. According to the structural break test,[4] the country betas (β_p) are found to be unstable for Canada, Japan and the UK. Thus the same regressions in (7) are estimated for these three countries, using the longest possible sub samples with stable betas. In the columns (v) and (vi), the skewness of $r_{p;t}$ and $\hat{\varepsilon}_{p;t}$ for the sub samples are reported. In this case, $\hat{\varepsilon}_{p;t}$ are found to be less negatively skewed for Japan, the UK, and Canada too, and the overall results support our conjecture.

[Figure 2 about here]

[4] The test used here is based on the Quandt likelihood ratio staistic using asymptotic critical values reported in Andrews (1993).

4. THE BIVARIATE GARCH MODEL FOR THE INTERNATIONAL STOCK MARKET RETURNS

In the previous section we have seen that large negative returns are more closely correlated across national stock markets than positive ones: the fat tail at the lower end of the world market risk's distribution explains such time-varying international correlations. In this section, we suggest a particular type of the bivariate GARCH model to capture this statistical property of the international stock market correlations.

The framework considered in this section is the single market factor model in which ARCH processes are allowed for both the world market risk and the country specific risk. The world market risk (eight) is assumed to follow some type of the univariate ARCH process.

$$R_{eight;t} = \mu_{eight} + \varepsilon_{eight;t} = \mu_{eight;t} + \sigma_{eight;t} \cdot z_{eight;t} \qquad (8)$$

The conditional expected return of a country p's index is written as the following

$$r_{p;t} = \mu_p + \beta_{p;t} \cdot R_{eight;t} + \varepsilon_{p;t} \qquad (9)$$

$$= \mu_p + \beta_{p;t} \cdot \mu_{eight;t} + \beta_p \cdot \sigma_{eight;t} \cdot z_{eight;t} + \sigma_{p;t} \cdot z_{p;t}$$

where $\{z_{eight;t}, z_{p;t}\}$ is assumed to be i.i.d. with zero mean (0,0) and an identity covariance matrix. $¾^2_{eight;t}$ and $¾^2_{p;t}$ are, respectively, the conditional variances of $R_{eight;t}$ and the country speci…c (non-market) risk.

In addition to the general framework described by (8) and (9), three key assumptions are made : (i) the world market risk is modeled by Campbell and Hentschel's (1992) extended QGARCH model; (ii) the country speci…c risk is modeled by GARCH(1,1); (iii) the country beta is assumed to be constant over time.

Next, we brie‡y discuss these features one by one.

4.1. Modelling the World Market Risk

In order to capture the negative skewness found in the distribution of the world market portfolio, we employ Campbell and Hentschel's (1992) extended quadratic ARCH model which was originally developed by Sentana (1991, 1995). The Campbell-Hentschel-Sentana framework allows us to directly parameterize the skewed and/or fat tailed distribution, and as we will see in the following, this approach turns out to be an excellent modelling strategy to describe the time-varying correlations between international stock markets.

Following Campbell and Hentschel's identi…cation for monthly U.S. stock return data, we employ QGARCH(1,1) model (or GQARCH in Sentana's terminology) for the world market risk.

$$R_{eight;t} = \mu_{eight;t} + \gamma \cdot \sigma^2_{eight;t-1} + \cdot \eta_t - \lambda(\eta_t^2 - \sigma^2_{eight;t-1}) \qquad (10)$$

$$\sigma^2_{eight;t} = \omega + \alpha \cdot (\eta_t - b)^2 + \beta \cdot \sigma^2_{eight;t-1}$$

The first equation means that the observed return of the asset, $R_{eight;t}$, is a quadratic function of the underlying variable η_t which follows a QGARCH process described in the second equation. The variable η_t would be interpreted as the dividend payout process or the stochastic process of some more general fundamental variable. Because of the quadratic term in the last term of the first equation, a negative shock in η_t the fundamental variable is amplified and a positive shock is dampened, which allows the distribution of the asset to be asymmetric. It will be negatively (positively) skewed if λ ¸ 0 (λ 0). If $\lambda = 0$, the model reduces to the original QGARCH model. The second term in the second equation, $\alpha \cdot (\eta_t - b)^2 = \alpha(\eta_t^2 - 2\eta_t b + b^2)$, implies that, depending on the parameter of b, the innovations of different signs in today's asset return have different impacts on the expected volatility tomorrow. If $b = 0$, the second equation reduces to the simple GARCH(1,1) model.

Two types of estimations of QGARCH for the world market risk is reported in Table VI. The first one, labeled as the unrestircted model, is QGARCH-M of Campbell and Hentschel (1992) and corresponds to their "λ free" version model. I also repotrted the restircted model in which is γ in

(10) was set to be zero. This is the simpler version of the QGARCH model in which there is no volatility feedback effect. Unfortunately, the estimate of γ in our unrestricted version is found to be insignificant and has a wrong sign. Although Campbell and Hentschel's original paper mainly focused on volatility feedback to the expected stock return, our main focus here is the third moment of the world market risk's distribution. Preliminary examinations suggest Monte Carlo simulations.reported later in this paper will not be affected by the choice of un/restricted version of the model. So the expected returns of the portfolios are assumed to be constant to avoid unnecessary complications and we will stick to the restricted version of the modelin the remaining of this paper.

[Table VI about here]

The use of the extended QGARCH process as the world market risk might appear to be completely ad hoc. We suggest two possible economic interpretations of it, though a complete investigation of the source of the observed negative skewness in the world market risk will be left to as a subject for future research due to the limitations of the space. First, it is conceivable that the distribution of the world stock market risk embodies the stochastic processes of some underlying economic factors that are common to different national stock markets, and the distributions of such factors themselves might follow the negatively skewed distribution. A possible candidate of such a factor is the oil price related factor. The oil price appreciation has a much stronger impact than its depreciation, so that the oil price factor might follow the negatively skewed distribution even if the distribution of oil price growth rate itself follows some symmetric distribution. Second, the negative skewness of the world market portfolio return can be interpreted as the revelation of the potential world wide systemic risk. The existence of systemic risk would generate the fat-tail at the lower end of market returns even if the observed economic factors follow the symmetric distributions.

4.2. Constancy of the Country Beta

Since β_p is assumed to be constant in our specification in (9), the estimate can be obtained by simple OLS. The first two columns of Table VII summarize the estimates of country betas. As we saw in Figure 2, country betas are unstable for the countries such as Japan and the U.K. However, the country beta for the U.S. seems very stable over the sample period. Thus we maintain the constancy assumption for β_p, and stick to the U.S. case in the following.

4.3. Modelling the Country Specific Risk

The remaining task is to find the appropriate specification for the country specific risk, which are the OLS residuals ($\varepsilon_{p;t}$) from the beta regressions (9) in this case. The third column of Table VII reports the $T \cdot R^2$ statistics of Engle (1984) for the country specific risks to test the existence of the ARCH effect. For all eight countries, the null in which there is no ARCH in the country specific risks are rejected. Thus it is natural to adopt some kind of ARCH process in modeling country specific risk. What about skewness? In Section II, we saw that the OLS residuals are less skewed than either market risk or the country risk. In Table V, none of the entries in the skewness column of the country specific risks were statistically significant. Thus the conditional normality assumption for the country specific risk should be maintained. Higher order GARCH models and complicated specifications — for example, a model which allows innovations in world market risk to affect the conditional variance of the non-market risk — were also examined. However, none of them significantly outperforms the simple GARCH(1,1) specification for the country specific risks employed here, that is

$$\sigma^2_{p;t} = c_0 + c_1\sigma^2_{p;t-1} + f_1\varepsilon^2_{p;t-1} \qquad (11)$$

The estimation results for (11) is reported in Table VII.

[Table VII about here]

4.4. The Performance Comparison of Different Models for the World Market Risk

Next we turn our attention to the performance of the extended QGARCH + GARCH(1,1) model of the international stock market return (referred as QGARCH hereafter). The constant country beta and the country specific risk estimated from the data are reported in Table VII. Keeping these estimates unchanged, we compare the performance of the QGARCH market risk model with other alternatives.

For this purpose, the GARCH model maintaining the conditional normality assumption is estimated as the benchmark. The specification employed here is the one proposed by Glosten, Jagannathan and Runkle (1993) which allow the response of the conditional volatility to the shock with different signs to be asymmetric (hence Glosten et.al.). According to Engle and Ng (1991) who examined alternative ARCH specifications using Japanese stock returns, Glosten et.al.'s modification of usual GARCH or Nelson's EGARCH specification (1991) best describes the asymmetric response of the conditional volatility to the innovations with different signs. The particular model estimated here is,

$$\sigma^2_{eight;t} = a_0 + a_1\sigma^2_{eight;t-1} + g_1\varepsilon^2_{eight;t-1} + g_2\varepsilon^2_{eight;t-1} \cdot I_{eight;t-1} \qquad (12)$$

$$\sigma^2_{p;t} = c_0 + c_1\sigma^2_{p;t-1} + f_1\varepsilon^2_{p;t-1}$$

$$I_{eight;t} = \text{dummyvariable} = 1 \quad \text{if} \quad \varepsilon_{eight;t} \geq 0$$

$$= 0 \qquad \text{otherwise}$$

If g_1 is positive and g_2 is negative, innovation in returns in the present will increase the conditional volatility in the future, and the effect is more pronounced if the shock is negative. Again, we assume a constant expected return, and ignore the possibility of the GARCH-in-Mean effect to avoid unnecessary complication.

In order to compare the different models of the world market portfolio return, we have to introduce some statistical measure of their performances. In section II, as such a measure, we used the correlations of the full sample and of the sub sample from which extreme observations according to the percentiles of domestic or foreign portfolio were excluded. Since our main interest in this section is the relationship between large negative shocks and the international cross-market correlations as in section II, it is natural to adopt a similar measure here. Thus, we mainly consider the statistical measure defined by the difference between the full sample correlation and the sub sample correlation without extreme observations in the lower X percentiles.

$$\xi_{x\%} \equiv \rho_{full} - \rho_{x\%} \tag{13}$$

where

ρ_{full} = the estimated correlation for the full sample.

$\rho_{x\%}$ = the sub sample correlation without lower x percentiles.

This measure is very straightforward, although it is neither the only nor the best measure for our interest. Statistics $\xi_{5\%}$ and $\xi_{10\%}$ are used repeatedly in the remainder of this section. The strategy is as follows. First, the model under the null hypothesis is specified and estimated. Then using the parameter estimates, critical values for »$_{x\%}$ under the null hypothesis is tabulated by Monte Carlo simulations. If the sample »$_{x\%}$ is greater — that is, the exclusion of extreme observations causes a stronger e¤ect than the critical values calculated assuming the particular model —, then that model is diagnosed to be unacceptable as the "true model."

The estimation result of the QGARCH model for the market return in (10) is reported in Table VI , and Glosten et.al.'s modi…ed GARCH in (12) is shown in the same table. For both Glosten et.al.'s modi…ed GARCH and Campbell-Hentschel's QGARCH models, the estimated parameters bear the expected signs. As the additional benchmark, we also use the actual data (Empirical Distribution) for the world market portfolio returns. We use three di¤erent stochastic processes for the world market risk (QGARCH, Glosten et.al., and Empirical Distribution) with the same country beta and the same GARCH(1,1) country speci…c risk to calculate the critical values for »$_{5\%}$ and »$_{10\%}$ by Monte Carlo simulations, and compare the performances of the alternative models for the world market risk. We also calculate the bootstrap percentiles of »$_{x\%}$ statistics using the full sample and the sub sample excluding October 1987.

These results are reported in Table VIII. The simulation results, especially those based on data that excludes the October Crash of 1987, suggest the following: …rst, Glosten et.al.'s modi…ed GARCH generates high correlations in the lowest returns. It is consistent with the raw data according to the percentiles based on the world market risk. However, according to the percentiles based on the U.S. returns, this model is rejected by onesided tests of both »$_{5\%}$ and »$_{10\%}$ statistics at the …ve percent level. Also the medians of the simulated »$_{5\%}$ and »$_{10\%}$ are too low compared with the sample values and the medians produced by the bootstrap. They even have a minus sign if the data is sorted according to USA. Thus this speci…cation cannot explain the asymmetry in the correlations. It implies that the fat-tailed (but symmetric) distributions of the asset returns alone cannot explain the observed pattern of the time-varying international correlation.

[Table VIII about here]

Empirical Distribution and QGARCH are both successful in generating the asymmetry of the correlations found in the raw data. In order to further investigate the validity of QGARCH modeling the world market risk, we also report the Monte Carlo simulations of the basic descriptive statistics of eight and USA in Table IX. Admittedly, there are some shortcomings in the QGARCH speci…cation of the world market risk. The QGARCH model seems to produce slightly lower »$_{x\%}$ statistics than the sample value and the bootstrap percentiles when the data is sorted according to eight. It also tends to generate a smaller kurtosis for the world market risk. However, except for these points, Campbell and Hentschel's QGARCH model successfully captures most of the statistical aspects of the data. If one is only interested in getting the con…dence sets for unconditional moments of the portfolio returns, bootstrapping might be the best way. However, as the parametric ARCH model which allows us to calculate and forecast the conditional volatility, the bivariate "QGARCH market risk - GARCH country specific risk" modelling strategy is quite successful in capturing the important aspects of the actual data.

[Table IX is about here]

5. THE IMPLICATIONS FOR INTERNATIONAL DIVERSIFICATION

In this section, we consider the implications of the findings of the previous section for international portfolio allocation.

Let us go back to the basic framework in (1) and (2), and extend it to the lifetime optimal portfolio selection problem. We assume the lifetime expected utility of the investor, V_t, is the sum of the utility from the terminal wealth at each period. Thus it is written as

$$V_t = E_t \sum_{i=0}^{\infty} \delta^i u_{t+i} \tag{14}$$

where δ is the discount factor. If the returns of the individual portfolios follow a multivariate normal distribution, and if the expected returns are constant over time, then the maximization problem of (14) will be reduced to the standard mean-variance optimization problem. However, if the conditional covariance matrix changes over time, optimal portfolio allocation can be quite different. For example, West, Edison, and Cho (1993) considered the significance of the time-variation of volatility on the investor's utility in the univariate framework of exchange rate forecasting.

If the true asset return process follows an ARCH process but the investor uses unconditional variance in assessing the conditional volatility, then she will experience periods of volatility lower and higher than she expected. For any risk averse investor, the cost of underestimation of the volatility is larger than the windfall from overestimation. Therefore, the portfolio choice problem when asset returns follow an ARCH process could be quite di¤erent from the case of the unconditional normality. Negative skewness will have a similar implication. The e¤ect of negative extreme outcomes on utility is more signi…cant than a positive outcome of the same magnitude. Thus there is a necessity to evaluate the e¤ect of the third and the fourth moments of the asset returns in the appropriate framework.

As a more realistic example, let us consider a …nancial institution that wants to assess and quantify the sensitivity of its trading position to the market risk. One example of such a risk management system is the "Value at Risk (VAR)."[5] The …nancial institution would like to construct a con…-dence interval of its portfolio value within a predetermined length of time, so that it calculates the risk in a way such that "with the probability of x%, we could lose more than $y million." If the portfolio return follows a conditionally normal distribution, the con…dence interval can be calculated by N times its standard error.[6] However, if the market risk follows a nega-

[5]See Jorion (1997) and JP Morgan (1995) for more details about Value at Risk.

[6]If N=1, this calculation constructs the 84.13% con…dence interval. If N=2, 97.73%, if N=3, 99.87%, and so on.

tively skewed distribution, more careful consideration of the downside risk is required.

In order to examine the importance of the findings in the previous section in such environments, the gains from international diversification are quantified under different criteria. As in the previous section, we limit our attention here to the case of international stock investment without hedging exchange-rate risk. For more general cases including bonds and the consideration of various currency exposure positions, see Grauer and Hakansson (1987).

Let us denote the investor's utility from investing in the global minimum variance portfolio as $u(a_p^{min})$, and her utility with 95% domestic portfolio as $u(a_p^{domestic})$. To measure the benefit from international diversification, we calculate the monetary equivalents (m) that compensate an investor who owns a 95% domestic portfolio so that her utility level is indifferent from when she invests in the global minimum variance portfolio. More precisely, m is defined as the difference between the certainty equivalents of the two portfolios so that:

$$m \equiv u^{-1}(E[u(a_p^{min})]) - u^{-1}(E[u(a_p^{domestic})]) \tag{15}$$

where $u^{-1}(.)$ is the inverse function of $u(.)$.

In calculating m, the following simplifying assumptions are made. First, it is known that the first moment of Brownian motion is estimated inaccurately compared with the estimation of its second moment (Merton, 1980).

Thus, using ex post mean returns will not necessarily provide good proxies of ex ante expected returns. Therefore, it is simply assumed that the expected returns are the same across different portfolios regardless of their international exposure. Second, initial wealth is set to $100,000, and the investment horizon is set to one month, and we force the investor to hold all her wealth in equities. Finally, the power utility function represents the preference of the investor who cares about large negative shocks. The utility level of the investor with the power utility function goes to minus infinity as her terminal wealth approaches to zero. Unlike the mean-variance utility function, the power utility function penalizes a large negative deviation from the mean far more severely than a positive deviation of the same magnitude.

In undertaking the actual calculation of m, the expected utility of the investor is tabulated in five different ways. The first three ways use Taylor approximations of the expected utility as demonstrated in (4):

(1) Second: Use only the first two moments:

$$E[u(a)] = u(\mu) + \frac{u''(\mu)}{2!}E[(a-\mu)^2]$$

(2) Third: Use up to the third moment:

$$E[u(a)] = u(\mu) + \frac{u''(\mu)}{2!}E[(a-\mu)^2] + \frac{u'''(\mu)}{3!}E[(a-\mu)^3]$$

(3) Fourth: Use up to the fourth moment:

$$E[u(a)] = u(\mu) + \frac{u''(\mu)}{2!}E[(a-\mu)^2] + \frac{u'''(\mu)}{3!}E[(a-\mu)^3] + \frac{u''''(\mu)}{4!}E[(a-\mu)^4]$$

Unfortunately, as discussed in details in the appendix , if the investor is very risk averse, there is a serious problem with these three Taylor approximation based calculations. The accuracy of the approximation by a Taylor expansion depends on the curvature of the underlying (utility) function and the variance of the asset return. Therefore, the larger the risk aversion coefficient β and/or the longer the investment horizon (because the variance of terminal wealth increases in proportion of the holding period), the worse the approximation. For monthly data, the Taylor approximation will become too inaccurate if β exceeds twenty. We report the calculations based on the above three ways later, but readers are advised not to take the levels of these values too literally, especially if β is greater than 20.[7]

The other two ways use ex-post utilities from the data to calculate the average of the investor's expected utility. The first one uses the actual data from the sample.

[7] This is the same reason why using Taylor approximations in risk managment of derivertive securities could be very dangerous. In calculating the delta of the portfolio, an easy shortcut is to use Taylor approximations. However, the approximation easily breaks down in practice, if the nonlinearity is strong.

(4) Data Oriented:

$$E[u(a)] = \sum_t u(a_t)/T$$

The final method uses simulated data from the extended QGARCH model of Campbell and Henstchel in the last section. Thus this calculation is only applicable to U.S. data excluding Black Monday, for which the QGARCH parameters are estimated in section II.

(5) QGARCH:

$$E[u(a)] = \sum_t u(a_t)/T$$

In Table X, various calculations of m are reported. We present the case in which the expected monthly return, μ, is equal to 0.85%, but these results would not be affected very much as long as μ takes on a plausible value. Figure 3 plots m for U.S. data without Black Monday, for the range of β from 0 to 100.

[Table X and Figure 3 about here]

In all the panels in Table X, up to around $\beta = 10$, all five methods yield similar results. For $\beta > 10$, among the three Taylor approximation calculations, Third yields the smallest figures as compared with Second and Fourth. As discussed in the beginning of section II, as the portfolio is more internationally diversified, the variance of its return decreases. At the same time, the distribution of its return gets more fat-tailed in its lower tail, but it becomes less fat-tailed as a whole. Including the third moment term decreases the gain from international diversification, but including the fourth moment term increases it again. In fact, the values of m calculated by Fourth are even greater than those calculated by Second in the Japanese case. Overall, when β is large, the calculations based on Taylor approximations yield values that are too high for the U.S. and values that are too low for Japan when compared to the Data Oriented calculations of m.

In the U.S. full sample case, as the investor gets more risk averse, the effect of the sharp decline on Black Monday gradually becomes dominant in the calculation by Data Oriented. On the other hand, in the case of the U.S. excluding Black Monday, m could be negative based on the calculations by Third, Data Oriented, and QGARCH. Thus, an investor who is very risk averse would prefer to hold a domestic portfolio. The values of m obtained by QGARCH are always greater than the means provided by Data Oriented, though they are always in the 90% intervals according to the bootstrap. By any calculation, the benefits from international diversification will hardly exceed 0.3% of the initial investment position, if we ignore the observation corresponding to Black Monday. At the same time it is decreasing as β increases, and can be negative. International diversification could cost as much as 0.75% according to the Data Oriented calculation. Among previous studies, Hiraki and Takehara (1995) applied the

Grauer-Hakansson methodology, which is very close to the Data Oriented approach, to investment opportunities including the U.S. and Japanese equities and bonds. Their results are similar to the current paper, although they did not provide any explanation for the seemingly contradictory result. They found the risk-return performance improvement from international diversification was not significant but was somewhat worsened, especially for conservative investors.[8]

In the Japanese case, the potential benefits from international diversification are much larger than in the case of the U.S. and could be more than 1.5% of the investment position. On the other hand, we set the expected returns to be equal among the domestic and internationally diversified portfolios, despite the strong upward trend of the yen's value during our sample period. So this calculation perhaps overestimates the benefit from international diversification.

It is hard to come to a comprehensive conclusion about the effect of Black Monday on international portfolio diversification. In Figure 4, the conditional variance of the world market portfolio implied by the QGARCH model is shown. In our sample period, there are three notable periods of persistent high volatility in the world capital markets. Seemingly, these periods correspond to the first and the second oil crises and the Gulf war, all caused by factors related to oil price movements.

[8] I thank Prof.Takehara for calling my attention to these studies.

[Figure 4 about here]

On the other hand, the October Crash had originated solely in the U.S., and was a one-shot extreme innovation. Usually, the value of the dollar against other currencies appreciates when a large international crisis occurs. In the case of the October Crash of 1987, other national stock markets fell together with the U.S. market, but the dollar value fell against other currencies too. In addition, the subsequent persistence of volatility after Black Monday was very limited compared to the magnitude of its initial shock.[9] One way to think about Black Monday is to treat it as an outlier and draw implications from the data excluding October 1987. However, the argument can be made that we have to include it as something we really would like to take into account. In this case, international diversi…cation is worth its cost for U.S. investors.

We conclude this section by discussing the issue of the investor's time horizon. Suppose the investor maximizes her wealth over T periods instead of one. Then as T gets larger, the e¤ect of higher moments on the investor's utility diminish, and the bene…ts from international diversi…cation measured by m will increase. Suppose that the one month return of the stock market is truly generated by a QGARCH process as described in the previous section. If the investor optimizes her portfolio over T months,

[9] This is the reason that the GARCH model estimation in Section II was not very a¤ected by the exclusion of October 1987.

then as $T \to \infty$, both unconditional and conditional distributions of asset returns approach normal distributions. Intuitively, this means that as the investor's time horizon gets longer, temporary large negative shocks (i.e., negative skewness) and temporary large conditional variance will become irrelevant to her. So, asymptotically, the differences made by excluding the third and the fourth derivatives in a Taylor approximation will vanish, but the speed that the effects of the higher moments will disappear has to be considered empirically. Preliminary simulation results suggest that when a investment horizon of more than twenty months is considered, the returns generated from a monthly QGARCH model are not distinguishable from those generated by a normal distribution. However, this argument is based on the assumption that stock returns are well described by the class of ARCH models with constant expected return. For example, if the stock return exhibits the mean reversion in the long-run, the simulation result can be dramatically altered.

6. CONCLUSIONS

In this paper, we empirically investigated the comovement of national stock markets in the global economy paying particular attention to the higher moments of their returns. It is found that the correlations among national stock markets are high when large adverse shocks hit the market. The primary reasons for this asymmetry in correlations are the ARCH effect and the negative skewness of the world market portfolio return distribution. This paper proposed to model the negative skewness by using Campbell and

Henstchel's extended QGARCH model. When the effect of time-varying international correlations is taken into the account, investors' potential benefits from international diversification could be much more limited than previously thought and might even be negative if she is extremely risk averse.

Serious consideration of the transaction costs incurred through international diversification would result in even smaller gains from international diversification. In their recent papers, Tesar and Werner (1994, 1995) examined transaction volume data and found that the turnover rate on the component of portfolios allocated to international equities is substantially larger than the turnover rate on national equity markets. From this finding, they concluded that transaction costs are unlikely to be the main cause for home country bias. However, a more plausible explanation would be that the observed large turnover rate on international equities simply suggests that a dynamic asset allocating strategy is more dominant in international equity transactions. Thus, Tesar and Werner's result could mean that transaction costs are very crucial considerations for passive investors in international stock investments. Many recent studies also argue that transaction costs and taxes are extremely important factors in securities trading (Bertsimas and Lo 1995, Campbell and Froot 1994).

What about exchange-rate risk? Many papers emphasize the existence of the home bias puzzle based on exchange-rate-risk-hedged data using forward contracts. However, such calculations typically completely ignore the costs of hedging. First, currency-hedging is obviously costly. Second, at the time that investments are made, an investor does not know exactly how much money should be covered since the payo¤ of equity is stochastic. Third, di¤erent investors have di¤erent investment horizons. Using the data based on one-month forward contract alone might give misleading results. In fact, there is recent evidence that in the long-run, exchange rate risk hedges might increase the volatility of dollar returns from international investments (Froot 1993).

Combining these considerations of transaction costs and the arguments presented in this paper, the home-country bias may not be as large as commonly thought or may not exist at all. As brie‡y discussed in the section II, one important question left unanswered in this paper is why the world market portfolio return has a negatively skewed distribution. In this paper, we suggest that the negative skewness might be directly explained by the empirical distribution of some factors in the framework of the multi-factor model. Another possibility is that the observed negative skewness is the result of the existence of the world systemic risk. Further research is required, but the author and Minh Trinh's preliminary results suggest that the distributions of underlying macroeconomic factors cannot fully explain the negative skewness of the world market portfolio. Some nonlinear relations may exist between the international stock returns and economic factors, such as in the case of the oil price index (Iwaisako-Trinh, 1995).

At the practical level, the …ndings of this paper might have important implications for risk management and for the regulation of internationally diversi…ed …nancial institutions. The models and rules used to evaluate the risk of …nancial institutions' portfolios should be constructed in a way that takes su¢cient account of the fact that the source of a large negative shock in one market tends to be shared in the other markets as well. Such models will suggest a more conservative portfolio management policy for …nancial institutions, compared with models using only covariance matrices.

APPENDIX: APPENDIX

In section III, Taylor approximations of the power utility function are used to calculate the bene…ts from international diversi…cation:

$$E[u(a)] = E[\frac{1}{1-\beta}a^{1-\beta}] \simeq u(\mu) + \frac{u''(\mu)}{2!}E[(a-\mu)^2] + \cdots \tag{A.1}$$

Unfortunately, this approximation does not work properly if β, the coef…cient of relative risk aversion, is large. In this appendix, the behavior of a Taylor approximation of the power utility is numerically examined in a simple setting.

We begin by restating Taylor's original theorem.

Theorem (Taylor's theorem for functions from R^1 to R^1, Protter and Morrey, 1991, p.184).

Suppose that f: R^1 to $R^1$and all derivatives of f up to and including order $n+1$ are continuous on an interval $I = \{x : |x-\mu| < r\}$. Then for each x on I, there is a number ξ on the open interval between a and x such that

$$f(x) = \sum_{k=0} \frac{1}{k!} f^{(k)}(\mu)(x - \mu)^k + \frac{1}{(n+1)!} f^{(n+1)}(\xi)(x - \mu)^{n+1} \qquad \text{(A.2)}$$

Thus, the accuracy of a Taylor approximation is valid only if x is concentrated in the interval I around μ, i.e., if the variance of x is relatively small. As the variance of the underlying stochastic variable becomes larger, approximation gets worse. In our case, since the variance of the asset return increases linearly with the investment horizon, a Taylor approximation will get worse as the investment horizon gets longer. On the other hand, r, the upper bound for $|x - \mu|$, will be smaller as the curvature of the function $f(\cdot)$ increases, that is, a larger β in our case.

The actual effects of the longer investment horizon, T, and the higher risk aversion, β, can only be considered numerically. So we conducted the Monte Carlo simulations in the following way. As in section III, we assume that the investor has a power utility function and the gross return of her portfolio (thus, her terminal wealth) follows the log-normal distribution. We artificially generate N sample asset return paths each consisting of L observations. For each path of sample returns, we calculate the expected utilities in two ways.

(1) By a Taylor expansion, using sample moments.

(Denoted by subscript AX)

(2) Direct calculation by the average of (ex post) realized utility.

(Denoted by subscript DI)

Next, we calculate the certainty equivalences of (1) and (2), m_{AX} and m_{DI}. Using m_{AX} and m_{DI}, I define the "goodness" of a Taylor approximation by,

$$\Delta = 100 \cdot (m_{AX}/m_{DI} - 1) \tag{A.3}$$

I assume the portfolio yields on average a 10% return per year, and its standard error on a monthly basis is 4.6%. The value for the standard error is taken from the U.S. data used in this paper. We calculate Δ for one week, one month, and one year investment horizons, and for the range of the risk aversion coefficient from 1 (the log utility) to 100.

Table A.I shows the underlying parameter values and the simulation results. For every investment horizon, the deviation of a Taylor approximation from the direct calculation becomes larger as the risk aversion coefficient β gets higher. And for the same Δ, the corresponding β becomes smaller for the longer investment horizon. These outcomes are exactly what the theory predicts. The value β takes here covers a much wider range than is commonly considered to be plausible. So for weekly data, the use of a Taylor expansion (thus, the mean-variance utility) closely approximates the results from a power utility function. On the other hand, a Taylor expansion is less than satisfactory for annual data unless one has a strong prior that β is well below 10. In the case of monthly data, it is hard to make a judgement, but this casts doubt on the accuracy of a Taylor approximation.

[Table A.I is about here]

The result here cannot be easily generalized for more complicated situations. With caveats, the tentative conclusion here is that approximation by a Taylor expansion will work satisfactorily only for weekly data and monthly data with ¯ smaller than 10. If we observe a “strange” result from calculations using a Taylor approximation, and if the data frequency is low or ¯ is large, then we should doubt such a result because the approximation does not work properly in these situation.

REFERENCES

Andrews, D.W.K.. (1993) Tests for Parameter Instability and Structural Change with Unknown Change Point. Econometrica 61, 821-856.

Bertimas, D. and Lo A.W.. (1995) Optimal Control of Execution Costs. Working paper, Sloan School of Management, MIT.

Braun, P.A., Nelson, D.B., and Sunier, A.M.. (1995) Good News, Bad News, Volatility, and Betas. Journal of Finance 50, 1575-1603.

Campbell, J.Y., and Froot, K.A.. (1994) International Experiences with Securities Transaction Taxes. In The Internationalization of Equity Markets, ed. J.A. Frankel, University of Chicago Press.

Campbell, J.Y. and Hentschel, L.. (1992) No News is Good News: An Asymmetric Model of Changing Volatility in Stock Returns. Journal of Financial Economics 31, 281-318.

Campbell, J.Y., Lo, A.W., and MacKinlay, A.C.. (1997) The Econometrics of Financial Markets, Princeton University Press.

Das, S. R.. (1994) Exchange Rate Shocks & Risk Diversi…cation. Working Paper, Department of Finance, Harvard Business School.

De Santis, G. and Gerard, B.. (1997) International Asset Pricing and Portfolio Diversi…cation with Time-Varying Risk. Journal of Finance 52, 1881-1912.

Engel, C.M.. (1994) Tests of CAPM on an International Portfolio of Bonds and Stocks. In The Internationalization of Equity Markets, ed. J.A. Frankel, University of Chicago Press.

Engle, R.F.. (1984) Wald, Likelihood Ratio, and Lagrange Multiplier Tests in Econometrics. In Handbook of Econometrics Volume 2, eds. Z. Griliches and M.D. Intriligator, North-Holland.

Engle, R.F. and Ng., V.. (1993) Measuring and Testing the Impact of News on Volatility. Journal of Finance 48, 1749-1778.

Frankel, J.A.. (1995) Introduction. In The Internationalization of Equity Markets, ed. J.A. Frankel, University of Chicago Press.

French, K.R. and Poterba, J.M.. (1991) Investors Diversi…cation and International Equity Markets. American Economic Review 81, 222-226.

Froot, K.A.. (1993) Currency Hedging over Long Horizons. NBER working paper no.4355.

Grauer, R.R. and Hakansson, N.H. . (1987) Gains from International Diversi…cation 1968-85: Returns on Portfolio of Stocks and Bonds. Journal of Finance 42, 721-739.

Glosten, L.R., Jagannathan, R., and Runkle, D.. (1993) On the Relation between the Expected Value and the Volatility of the Nominal Excess Return on Stocks. Journal of Finance 48, 1779-1801.

Hiraki, T. and Takehara, H.. (1995) International Diversi…cation When Small Stocks are Treated as Separate Investment Assets: An Application of the Multiperiod Model. In Global Portfolio Diversi…cation: Risk Management, Market Microstructure, and Implementation Issues, eds. Raj Aggarwal and David C. Schirm, Academic Press.

Ibbotson, R.E. and Brinson, G.P.. (1993) Global Investing: The Professional's Guide to the World Capital Markets, McGraw-Hill.

Iwaisako, T.. (1997) Essays on Financial Integration, Ph.D. thesis, Department of Economics, Harvard University.

Iwaisako, T. and Trinh, C. M.. (1995) Macroeconomic Factors and U.S.-Japan Comovements of Stock Market Returns. Mimeo., Department of Economics, Harvard University.

Jorion, P.. (1997) Value at Risk: The New Benchmark for Controlling Derivative Risk, Irwin Professional Publishing.

JP Morgan,. (1995) Introduction to RiskMetrics, Fourth ed. JP Morgan.

Lewis, K.K.,. (1995) Puzzles in International Financial Markets. In Handbook of International Economics, Volume 3, eds. G.M.Grossman and K.Rogo¤, North-Holland.

Merton, R.C.. (1980) On Estimating the Expected Return on the Market: An Exploratory Investigation. Journal of Financial Economics 8, 323-61.

Nelson, D.. (1991) Conditional Heteroskedasticity in Asset Returns: A New Approach. Econometrica 59, 347-370.

Obstfeld, M.. (1994) Risk-Taking, Global Diversi…cation, and Growth. American Economic Review 84, 1310-1329.

Protter, M.H. and Morrey, C.B.. (1991) A First Course in Real Analysis, Second Edition. Springer-Verlag.

Sentana, E.. (1991) Quadratic ARCH models: A Potential Re-interpretation of ARCH Models. LSE Financial Markets Group Discussion Paper no.122.

Sentana, E.. (1995) Quadratic ARCH models. Review of Economic Studies 62, 639-661.

Siegel, J.J.. (1994) Stock for the Long Run: A Guide to Selecting Markets for Long-Term Growth. Irwin Professional Publishing.

Tesar, L.L. and Werner, I.M.. (1994) International Equity Transactions and U.S. Portfolio Choice. In The Internationalization of Equity Markets, ed. J.A. Frankel, University of Chicago Press.

Tesar, L.L. and Werner, I.M.. (1995) Home Bias and High Turnover, Journal of International Money and Finance 14. 467-492.

West, K. D., Edison, H. J., and Cho, D.. (1993) A Utility-based Comparison of Some Models of Exchange Rate Volatility. Journal of International Economics 35, 23-45

Table I
Summary Statistics of Monthly Excess Returns

Series	Mean(%)	S.E.(%)	Min.(%)	Max.(%)	skewness	kurtosis
Canada	0.2693	5.3886	-26.0940	15.6977	-0.7029 (0.0000)	3.0823 (0.0000)
France	0.4454	6.7024	-27.4364	23.2292	-0.3528 (0.0084)	1.6778 (0.0000)
Germany	0.4451	5.8652	-19.9952	17.8312	-0.3978 (0.0030)	1.2451 (0.0000)
Italy	0.0421	7.5491	-24.7535	26.4680	-0.0129 (0.9235)	0.6470 (0.0163)
Japan	0.5160	6.5441	-22.1833	21.0647	-0.0390 (0.7705)	0.6105 (0.0234)
Switzer-land	0.6135	5.4637	-20.0086	21.4405	-0.3064 (0.0219)	1.3883 (0.0000)
U.K.	0.5500	6.8369	-24.8544	44.1951	0.4506 (0.0008)	5.4498 (0.0000)
USA	0.4485	4.3892	-24.4586	15.8490	-0.5811 (0.0000)	3.2561 (0.0000)

Series	Mean(%)	S.E.(%)	Min.(%)	Max.(%)	skewness	kurtosis
World	0.4137	4.1153	-19.1925	13.1880	-0.6076 (0.0000)	2.1043 (0.0000)
eight	0.4162	4.3333	-19.6976	17.4670	-0.6053 (0.0000)	2.5417 (0.0000)

Sample period: 1970:01-1998:02, The number of observations: 338.

Log excess returns over a one-month U.S. Treasury bill return, calculated in dollar terms using spot exchange rates. World is the value weighted index of all capitalization covered by Morgan Stanley Capital International (MSCI) Data. eight is the simple average of the eight individual countries' return each period. Signi…cance levels are in parentheses for skewness and kurtosis.

Table II

Domestic Portfolio v.s. Global Minimum Variance Portfolio: Standard Errors and Higher Moments

USA(Japan) x% portfolio:

$\omega = x/100$, $R_p = \omega \cdot r_p + (1 - \omega) \cdot R_{seven}$

r_p = Domestic portfolio, R_{seven} = Average of other seven countries

Standard Error(S.E.) $= \sigma = \sqrt{\sum (R_p - \mu)^2 / N}$,

Third Moments (Third)$= \sum (R_p - \mu)^3 / N$, Skewness=Third/σ^3

Fourth Moments (Fourth)$= \sum (R_p - \mu)^4 / N$, Kurtosis=Fourth/$\sigma^4 - 3$

(i) USA (Full Sample)

Series	S. E.	Skewness	Third($\times$10000)	Kurtosis	Fourth ($\times$10000)
USA 55%	3.985	-0.785	-0.496	3.701	0.169
(Min. Variance)		(0.00)		(0.00)	
USA 95%	4.308	-0.619	-0.495	3.371	0.220
		(0.00)		(0.00)	
USA 100%	4.389	-0.581	-0.491	3.256	0.232
		(0.00)		(0.00)	

(ii) USA (Excluding Black Monday)

Series	Std. Err.	Skewness	Third($\times$10000)	Kurtosis	Fourth ($\times$10000)
USA 55%	3.797	-0.344	-0.188	1.479	0.093
(Min. Variance)		(0.01)		(0.00)	
USA 95%	4.099	-0.125	-0.086	0.857	0.109
		(0.35)		(0.01)	
USA 100%	4.180	-0.095	-0.009	0.821	0.117
		(0.48)		(0.00)	

(iii) Japan

Series	Std. Err.	Skewness	Third($\times$10000)	Kurtosis	Fourth ($\times$10000)
Japan 42%	3.892	-0.654	-0.691	2.179	0.198
(Min. Variance)		(0.00)		(0.00)	
Japan 95%	5.183	-0.356	-0.561	1.756	0.350
		(0.02)		(0.00)	
Japan 100%	5.372	-0.323	-0.565	1.706	0.395
		(0.03)		(0.00)	

Note: Significance levels are in parentheses. Domestic 95% portfolios roughly correspond to the actual investment positions of the U.S. and Japanese representative investors.

Table III
Effects of Extreme Observations on Correlations

Panel(A): Volatility and Correlation for the Full Sample.

USA: Excess return of US index over one-month U.S. T-bill rate.
Seven: Simple average of other seven countries excess returns.

Covariance/Correlation matrix

	USA	Seven
USA	21.32	**0.613**
Seven	11.98	17.92

Note: Correlation is (bold face) above the diagonal.

Panel (B) : Pecentiles According to USA

	Max/Min	1%	5%	10%	25%	50%	75%	90%
Upper	0.64	0.63	0.62	0.62	0.65	0.65	0.66	0.59
Lower	0.59	0.56	0.50	0.45	0.39	0.32	0.21	-0.00

Panel (C): Percentiles According to Seven

	Max/Min	1%	5%	10%	25%	50%	75%	90%
Upper	0.62	0.61	0.61	0.59	0.56	0.56	0.63	0.73
Lower	0.59	0.57	0.49	0.47	0.44	0.42	-0.08	0.21

The observations of USA and Seven from the same date are paired and treated as one observation, such as (USA_t, $Seven_t$). These pairs were sorted according to the values of one of two series. Then the pairs ranked at the top/bottom were excluded, and the correlations were calculated for the remaininng subsamples. For example, for the pairs sorted according to the values of series USA, the results are shown in Panel (B). The correlation after excluding the pairs in the upper five percentile is reported in the first rows (Upper), the third column (5%), and the value is 0.6057. The second rows, Lower, show the correlations after eliminating pairs in the lower X percentile. In Panel (C), the pairs were sorted according to Seven.

Table IV

Testing Contemporaneous Nonlinearity by Parametric Regression

Data: Monthly Return 1970:1-1998:02 excluding 1987:10

Regression: $r_{p;t} = \alpha + \beta_1 \cdot R_{eight;t} + \beta_2 \cdot R^2_{eight;t} \cdot I_t$

if $R_{eight;t}$ 0; $I_t = 1$; otherwise $I_t = 0$

$r_{p;t}$	β_1	β_2	$\bar{R}^2$
Canada	0.7091	-0.0238	0.432
	[9.0138]	[-1.9772]	
France	1.2726	0.0018	0.614
	[15.2094]	[0.1421]	
Germany	0.9323	-0.0166	0.523
	[11.6013]	[-1.3543]	
Italy	1.1667	0.0101	0.409
	[10.4052]	[0.5884]	
Japan	1.0328	0.01653	0.403
	[10.6201]	[1.1122]	
Switz.	0.9127	-0.0063	0.591
	[13.9699]	[-0.6280]	
U.K.	1.3542	0.0232	0.567
	[14.8157]	[1.6613]	
USA	0.6196	-0.0050	0.405
	[9.5612]	[-0.5008]	

Table V

The Skewness of Excess Returns ($r_{p,t}$) and Country Specific Risks ($\hat{\varepsilon}_{p,t}$)

$\gamma(r)$ = "Skewness of Series r" $\equiv E[(r - \mu_r)^3]/\sigma_r^3$;

$r_{p,t}$ = Excess Return of the MSCI Index of Country p (in Dollar Terms).

$\hat{\varepsilon}_{p,t}$ = Country Specific Risk of Country p

= The Residuals from the Regression: $r_{p,t} = \alpha_p + \beta_p \cdot R_{eight,t}$;

	Full	Sample	Excluding	Oct.87
	(i) $\gamma(r_{p,t})$ return	(ii) $\gamma(\hat{\varepsilon}_{p,t})$ residuals	(iii) $\gamma(r_{p,t})$ return	(iv) $\gamma(\hat{\varepsilon}_{p,t})$ residuals
eight	-0.608 [0.00]	N/A	-0.381 [0.004]	N/A
Canada	-0.703 [0.000]	-0.123 [0.358]	-0.463 [0.001]	-0.123 [0.358]
France	-0.353 [0.008]	0.062 [0.643]	-0.297 [0.027]	0.074 [0.582]
Germany	-0.398 [0.003]	-0.052 [0.699]	-0.319 [0.017]	-0.052 [0.697]
Italy	-0.013 [0.924]	0.248 [0.063]	-0.012 [0.931]	0.256 [0.056]
Japan	-0.039 [0.771]	-0.090 [0.500]	-0.044 [0.743]	-0.101 [0.453]
Switzer-land	-0.3064 [0.022]	0.024 [0.859]	-0.190 [0.155]	0.023 [0.862]
U.K.	0.451 [0.001]	0.239 [0.074]	0.608 [0.000]	0.241 [0.072]
USA	-0.581 [0.000]	0.147 [0.273]	-0.095 [0.480]	0.246 [0.066]

Note: Significance levels are in parentheses.

Table V (continued)

	Sub	Samples:	Period
	(v) °($r_{p;t}$) return	(vi) °($\mathbf{b}_{p;t}$) residuals	
Canada	-0.580 [0.002]	-0.127 [0.500]	70:1-84:4
France	N/A		
Germany	N/A		
Italy	N/A		
Japan	-0.032 [0.846]	0.011 [0.945]	70:1-88:12
Switzer-land	N/A		
U.K.	0.136 [0.535]	0.175 [0.427]	82:3-92:10
USA	N/A		

Note: Signi…cance levels are in parentheses.

Table VI

Univariate GARCH Models of the World Market Portfolio

Sample period: January 1970 to Feburary 1998; number of observations: 338

(1) Campbell-Hentschel's Extended QGARCH(1,1) Model

Campbell and Hentschel (1992). Equation (10) is in the text.

$$R_{eight,t} = \mu_{eight,t} + \gamma \cdot \sigma^2_{eight,t-1} + \quad \cdot \eta_t - \lambda(\eta_t^2 - \sigma^2_{eight,t-1}) \qquad (10)$$

$$\sigma^2_{eight,t} = \omega + \alpha \cdot (\eta_t - b)^2 + \beta \cdot \sigma^2_{eight,t-1}$$

where $\quad = 1 + 2\lambda b$.

(1-a) Unrestricted Model

$\omega(\times 1000)$ [SE]	α [SE]	$b(\times 100)$ [SE]	β [SE]	$\mu(\times 100)$ [SE]	γ [SE]	λ [SE]	
0.921	0.121	3.560	0.713	0.348	-0.387	0.734	1.052
[0.546]	[0.058]	[1.869]	[0.119]	[0.283]	[1.024]	[0.383]	

(1-b) Restricted Model: $\gamma = 0$

$\omega(\times 1000)$ [SE]	α [SE]	$b(\times 100)$ [SE]	β [SE]	$\mu(\times 100)$ [SE]	λ [SE]	
0.818	0.120	2.483	0.750	0.291	0.795	1.039
[0.451]	[0.044]	[1.599]	[0.092]	[0.264]	[0.360]	

(2) Glosten, Jagannathan, and Runkle's GARCH(1,1) Model

Glosten, Jagannathan, and Runkle (1993). Equation (12) is in the text.

$$R_{eight,t} = \mu + \varepsilon_t \qquad (12)$$

$$\sigma_t^2 = a_0 + a_1 \cdot \sigma^2_{t-1} + g_1 \cdot \varepsilon^2_{t-1} + g_2 \cdot \varepsilon^2_{t-1} \cdot I_{t-1}$$

$$I_{t-1} = 0 \text{ (if } \varepsilon_{t-1} < 0\text{)};$$
$$= 1 \text{ (if } \varepsilon_{t-1} \geq 0\text{)}$$

Variable	$\mu(\times 100)$	$a_0(\times 100)$	a_1	g_1	g_2
Coefficients	0.4314	0.0249	0.8160	0.0654	-0.0316
	[02584]	[0.0200]	[0.1270]	[0.0415]	[0.0475]

Note: Asymptotic standard errors are in parentheses.

Table VII

Estimations of Country Betas and Country Specific Risks

The Regression for the Country Beta: $r_{p;t} = \alpha_p + \beta_p \cdot R_{eight;t} + u_t$
GARCH(1,1) for Country Specific Risk:
$E[u_t^2|\Omega_{t-1}] \equiv h_t^2 = \alpha_0 + \alpha_1 \cdot h_{t-1}^2 + \gamma_1 \cdot u_{t-1}^2$

	β_p	$\bar{R}^2$	$\alpha_0(\times 100)$	α_1	γ_1	Log likelihood
Canada	0.842	0.46	0.094	0.291	0.109	911.3
	[0.066]		[0.068]	[0.611]	[0.063]	
France	1.228	0.63	0.034*	0.538*	0.231*	913.6
	[0.056]		[0.015]	[0.124]	[0.066]	
Germany	0.995	0.54	0.021	0.804*	0.065	909.6
	[0.058]		[0.000]	[0.111]	[0.046]	
Italy	1.110	0.40	0.083	0.611	0.155	943.903
	[0.077]		[0.195]	[0.739]	[0.178]	
Japan	0.937	0.38	0.040	0.731*	0.125*	826.1
	[0.077]		[0.025]	[0.121]	[0.050]	
Switz.	1.012	0.64	0.026	0.641*	0.120	976.2
	[0.040]		[0.021]	[0.233]	[0.072]	
U.K.	1.197	0.57	0.005	0.892*	0.083*	890.1
	[0.081]		[0.004]	[0.042]	[0.025]	
USA	0.678	0.45	0.056	0.327	0.141*	980.0
	[0.052]		[0.028]	[0.297]	[0.064]	

Notes:
* Asymptotic standard errors are in parentheses.

* $T \cdot R^2 \equiv$ "Sample Size$\times$uncentered R^2" from the regression,

$$\hat{u}_t^2 = \delta_0 + \delta_1 \cdot \hat{u}_{t-1}^2 + e_t$$

Under H_0: u_t^2 = i.i.d., $T \cdot R^2$ converges in the distribution to χ^2 variable with a degree of freedom of one. See Engle (1984).

Table VIII
Performance Comparison Based on Correlations between the World Market Risk and the U.S. Data

The Model under the null hypothesis: r_{us} = ¯ ¢ R_{eight} + "$_t$

? ¯ is constant and the country speci…c risk ("$_t$) follows GARCH(1,1). These are common across the di¤erent models of the world market risk.

? The Bootstrap Percentiles : Calculated by the bootstrap with 5,000 replications.

? Alternative models of the world market risk (R_{eight}).

QGARCH: Campbell-Hentschel's QGARCH(1,1) model from Table VI (1).
GARCH: Glosten et.al.'s (1993) GARCH(1,1) from Table VII (2).
Empirical Distribution: The world market risk is the raw data.

? Critical values: »$_{x\%}$ = ½$_{full}$ i ½$_{x\%}$ (13)

½$_{full}$ = The full sample correlation between eight and USA.
½$_{x\%}$ = The sub sample correlation excluding the observations in lower x percntile. See the note in Table III for the construction of the percentiles.

Using the parameter estimates of the models of the world market risks, critical values are calculated by the Monte Carlo simulations with 5,000 sample paths each consists of 275 observations. If the sample »$_{x\%}$ statistics is greater than the critical value, the null hypothesis is rejected, i.e., we conclude the model cannot explain the time-varying correlation observed in the data. (**) indicates that the model, for the data both including and excluding October 1987, cannot be rejected. (*) indicates that the model cannot be rejected only for the data October 1987 excluded. In generating the arti…cial sample using GARCH/QGARCH processes, the initial conditional variances are set to the estimated unconditional variances. Then 375 observations are generated for each sample, and the last 275 observations are used as an arti…cial sample process.

(1) Percentiles according to eight
Sample: »$_{5\%}$ = :121(Full Sample); »$_{5\%}$ = :0909 (Excluding October 1987)

The Bootstrap Percentiles

	5%	Median	95%
Full Sample	0.1684	0.1109	0.0617
No Oct.87	0.1284	0.0879	0.0469

Critical Values

	1%	5%	10%	Median
Glosten et.al.	0.1996**	0.1301**	0.0993*	0.0005
Empirical	0.1119*	0.0984*	0.0914	0.0665
QGARCH	0.1110*	0.1068*	0.0894	0.0546

Table VIII (continued)

Sample: »$_{10\%}$ = :1217(Full Sample); »$_{10\%}$ = :0916(Excluding October 1987)

The Bootstrap Percentiles

	5%	Median	95%
Full Sample	0.1916	0.1163	0.0472
No Oct.87	0.1455	0.0872	0.0302

Critical Values

	1%	5%	10%	Median
Glosten et.al.	0.2019**	0.1327**	0.1012*	0.0006
Empirical	0.1601**	0.1412**	0.1320**	0.0982*
QGARCH	0.1110*	0.1068*	0.0888	0.0536

(2) Percentiles according to USA

Sample: »$_{5\%}$ = :1113(Full Sample); »$_{5\%}$ = :0812 (Excluding October 1987)

The Bootstrap Percentiles

	5%	Median	95%
Full Sample	0.1596	0.1017	0.0507
No Oct.87	0.1182	0.0774	0.0380

Critical Values

	1%	5%	10%	Median
Glosten et.al.	0.1393*	0.0399	0.0165	-0.0022
Empirical	0.1071*	0.0943*	0.0866*	0.0618
QGARCH	0.1461**	0.1241**	0.1133**	0.0797

Sample: »$_{10\%}$ = :1336(Full Sample); »$_{10\%}$ = :1035(Excluding October 1987)

The Bootstrap Percentiles

	5%	Median	95%
Full Sample	0.2029	0.1321	0.0678
No Oct.87	0.1592	0.1051	0.0534

Critical Values

	1%	5%	10%	Median
Glosten et.al.	0.1586*	0.0738	0.0355	-0.0046
Empirical	0.1484**	0.1320*	0.1227*	0.0901
QGARCH	0.1446**	0.1228*	0.1118*	0.0776

Table IX

The Monte Carlo Simulations of the Extended QGARCH Model as the World Market Portfolio: Standard Errors, Higher Moments, and Correlations with the U.S. Domestic Portfolio

The Monte Carlo simulations with 5000 replications.

Definitions of the Variables

$\hat{\sigma}$ = Standard Error

$\hat{\rho}$ = $\hat{\rho}$(eight; USA) = correlation

$\hat{S}$ = Skewness

$\hat{K}$ = Kurtosis

	Data		Monte Carlo Simulations				
	Full Sample	No Oct.87	1%	5%	Median	95%	99%
$\hat{\sigma}$ (eight)	4.523	4.366	5.059	4.844	4.377	3.963	3.825
$\hat{\sigma}$ (USA)	4.626	4.385	4.895	4.724	4.368	4.026	3.899
$\hat{\rho}$	0.670	0.641	0.729	0.707	0.644	0.572	0.537
$\hat{S}$ (eight)	-0.566*	-0.329	0.178	0.077	-0.172	-0.454	-0.585
$\hat{S}$ (USA)	-0.513	-0.045	0.309	0.199	-0.043	-0.292	-0.412
$\hat{K}$ (eight)	2.426**	1.657*	2.297	1.400	0.332	-0.145	-0.288
$\hat{K}$ (USA)	3.041**	0.373	1.023	0.611	0.008	-0.398	-0.535

Note:
(*) denotes actual data is outside of the 90% interval, but within the 98% interval.
(**) denotes actual data is outside of the 98% interval.
For the parameters used in simulations, see Tables VI (1) and VII.

Table X

Different Calculations of Benefits from International Diversification

$$u(a) = \frac{1}{1-\beta} a^{1-\beta}; \quad m \equiv u^{-1}(E[u(a_{x\%})]) - u^{-1}(E[u(a_{y\%})])$$

$$a = (1 + R) \times (\$100{,}000); \quad E[R_{x\%}] = E[R_{y\%}] = \mu = 0.85\%$$

(1) Second: $E[u(a)] = u(\mu) + \frac{u''(\mu)}{2!} E[(a - \mu)^2]$

By a Taylor approximation using the sample first and second moments.

(2) Third: $E[u(a)] = u(\mu) + \frac{u''(\mu)}{2!} E[(a - \mu)^2] + \frac{u'''(\mu)}{3!} E[(a - \mu)^3]$

By a Taylorapproximation using up to the third moments.

(3)Fourth: $E[u(a)] = u(\mu) + \frac{u''(\mu)}{2!} E[(a - \mu)^2] + \frac{u'''(\mu)}{3!} E[(a - \mu)^3] + \frac{u''''(\mu)}{4!} E[(a - \mu)^4]$

By a Taylorapproximation using up to the fourth moments.

(4) Data Oriented: $E[u(a)] = \sum_t u(a_t)/T$

Direct calculation from the ex post utility using the realized sample returns.

(5) QGARCH: $E[u(a)] = \sum_t u(a_t)/T$

Direct calculation by the simulated data using QGARCH model in Table VIII (3000 sample pathes, each containing 275 observations).

(i) USA (Full Sample) USA (55%-95%)

β	(1) Second	(2) Third	(3) Fourth	(4) Data
1	15.58	15.33	15.33	15.64
2	31.07	30.91	31.50	31.52
3	46.38	46.07	47.53	47.77
5	76.17	75.33	80.34	82.22
10	142.53	138.78	166.63	195.96
20	226.44	207.82	327.37	850.53
30	252.76	212.39	404.43	1979.55
40	248.02	188.84	404.40	2366.15
50	231.40	159.78	373.99	2400.04
75	184.57	102.02	290.21	2337.67
100	148.70	67.72	231.27	2294.40

Table X (continued)

(ii) USA (Excluding Black Monday) USA (55%-95%)

‾	(1) Second	(2) Third	(3) Fourth	(4) Data	(5) QGARCH
1	13.74	13.23	13.23	12.67	19.47
2	27.40	26.08	26.28	24.87	38.14
3	40.93	38.31	38.79	36.58	56.04
5	67.32	60.84	62.51	58.35	89.67
10	126.81	104.30	113.69	101.20	162.07
20	205.62	135.12	179.93	115.81	260.61
30	234.30	115.07	203.18	7.24	298.02
40	233.64	76.791	200.28	-190.67	274.40
50	220.51	37.90	187.31	-392.68	195.36
75	178.73	-34.76	151.95	-691.16	-131.97
100	145.04	-76.70	125.68	-794.32	-423.10

(iii) Japan (Full Sample) Japan(42%-95%)

‾	(1) Second	(2) Third	(3) Fourth	(4) Data
1	49.14	48.25	48.25	49.14
2	97.97	96.81	98.30	98.31
3	146.15	143.82	147.51	147.57
5	239.43	233.48	245.98	246.40
10	443.38	420.10	487.46	494.27
20	683.88	592.16	856.00	929.17
30	742.78	569.59	965.28	1127.37
40	714.56	480.33	914.26	1226.39
50	657.82	387.89	821.09	1326.86
75	515.54	222.71	619.50	1478.28
100	412.24	132.28	489.96	1524.39

Figure 1

Nonparametric Regression: **USA** on **Seven**

Sample: January 1970 to February 1998

Kernel: normal, Bandwidth: 10%

(1) Full sample kernel regression

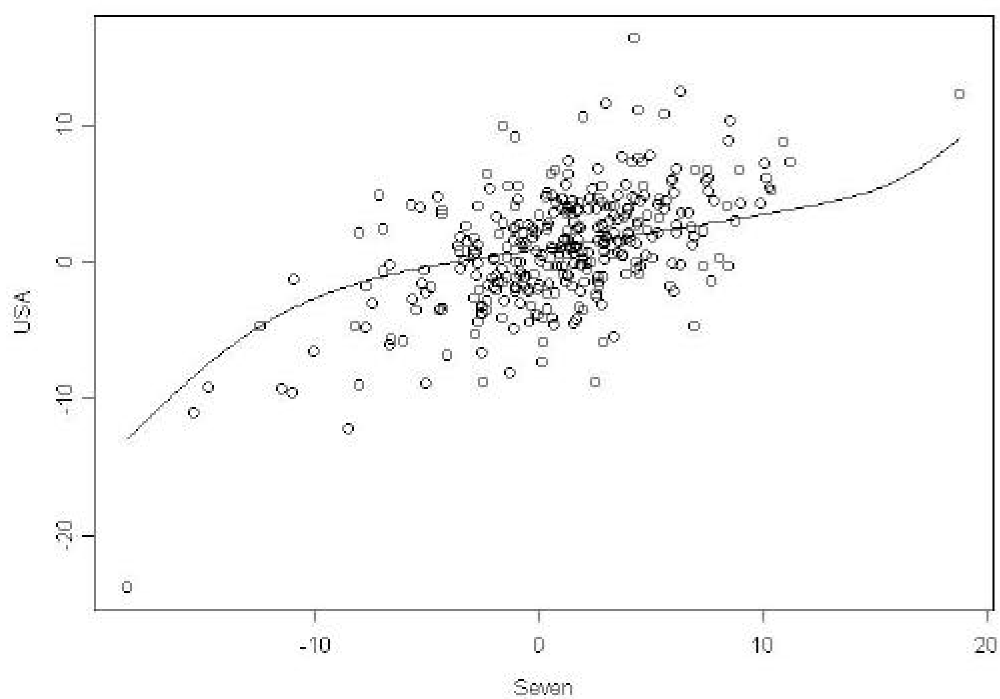


Figure 1 (continued)

(2) Excluding October 1987

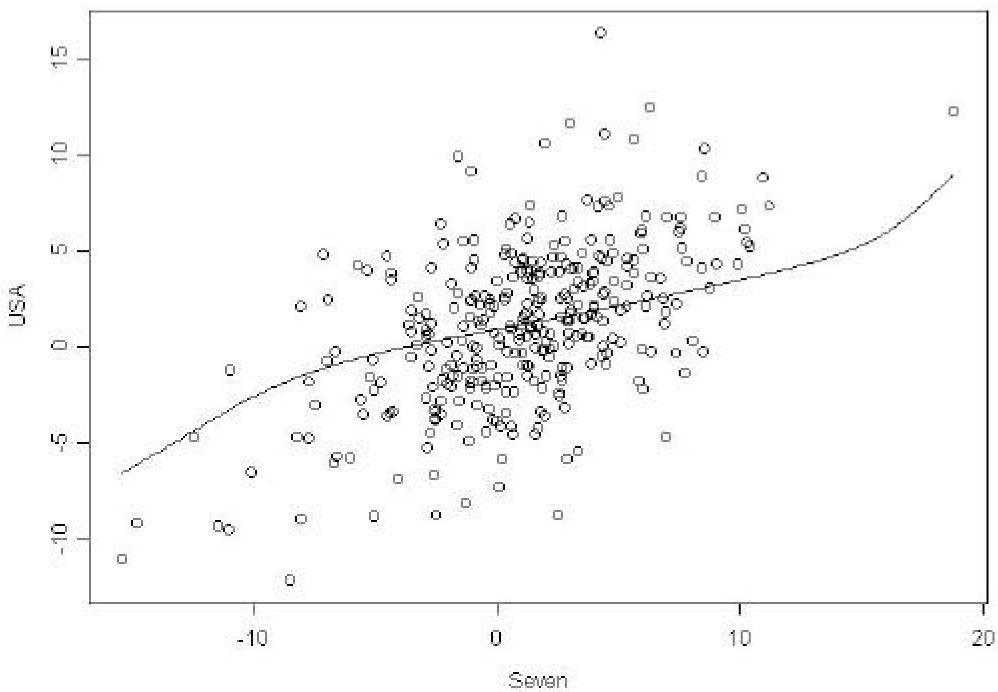
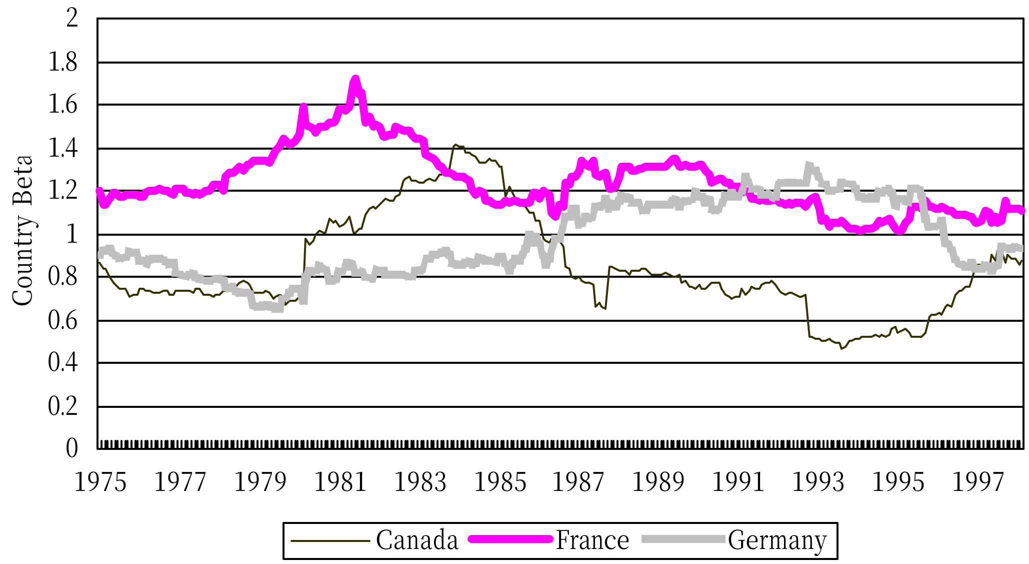


Figure 2

Country Betas: Rolling Beta Regression

Sample: January 1975 to February 1998

(1) Canada, France and Germany



(2) Italy, Japan and Switzerland

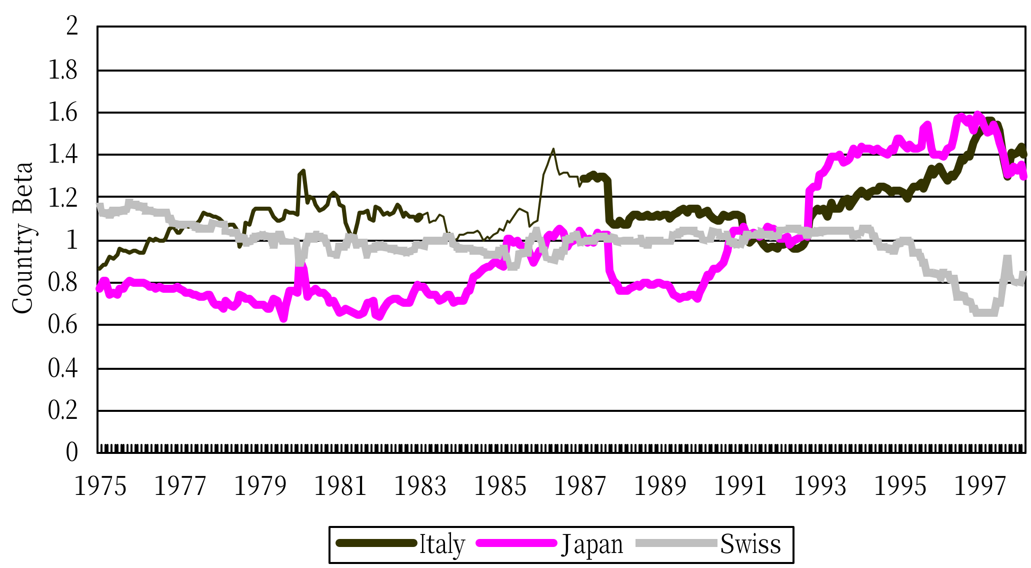


Figure 2 (continued)

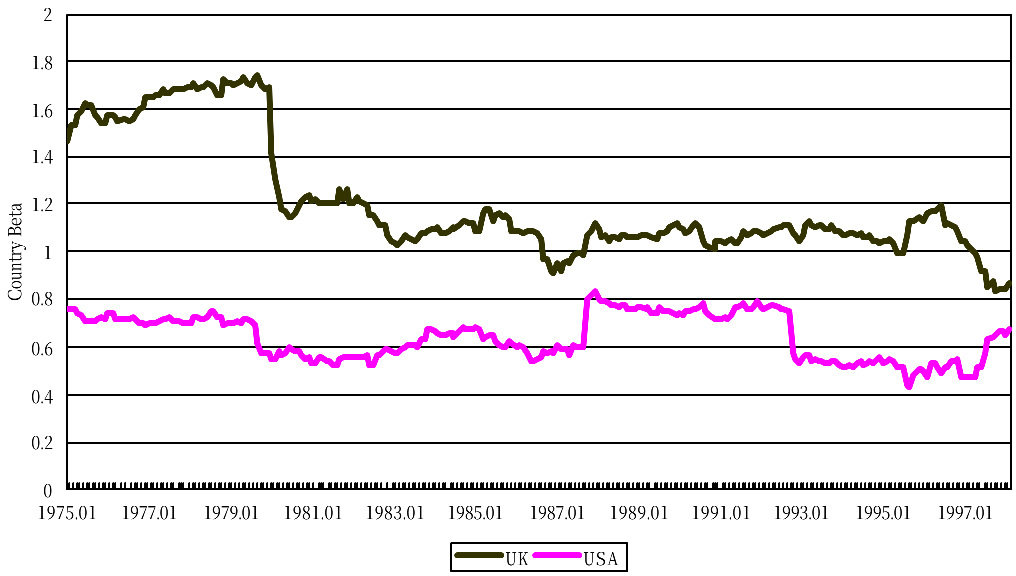
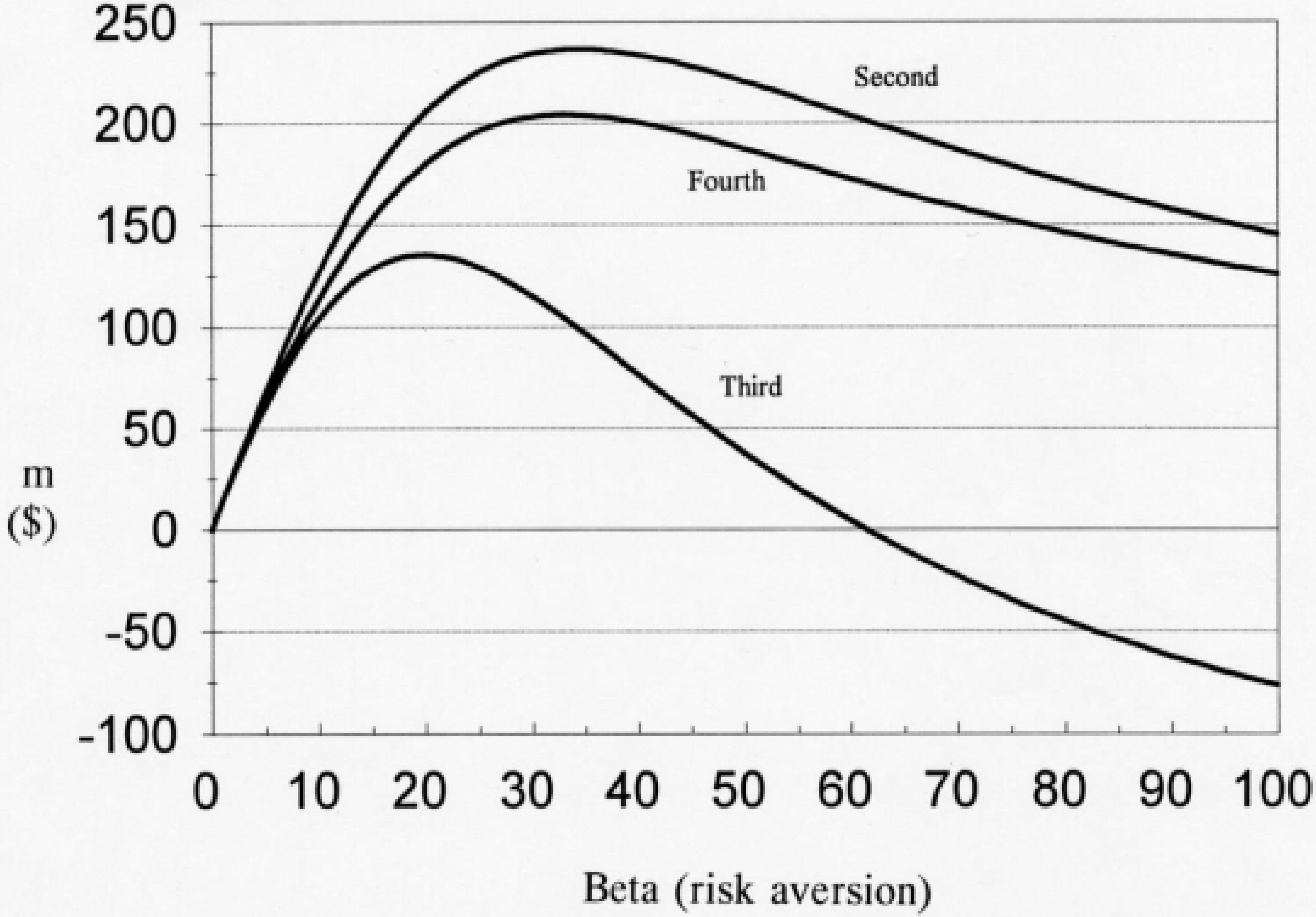


Figure 4. The Benefit from International Diversification

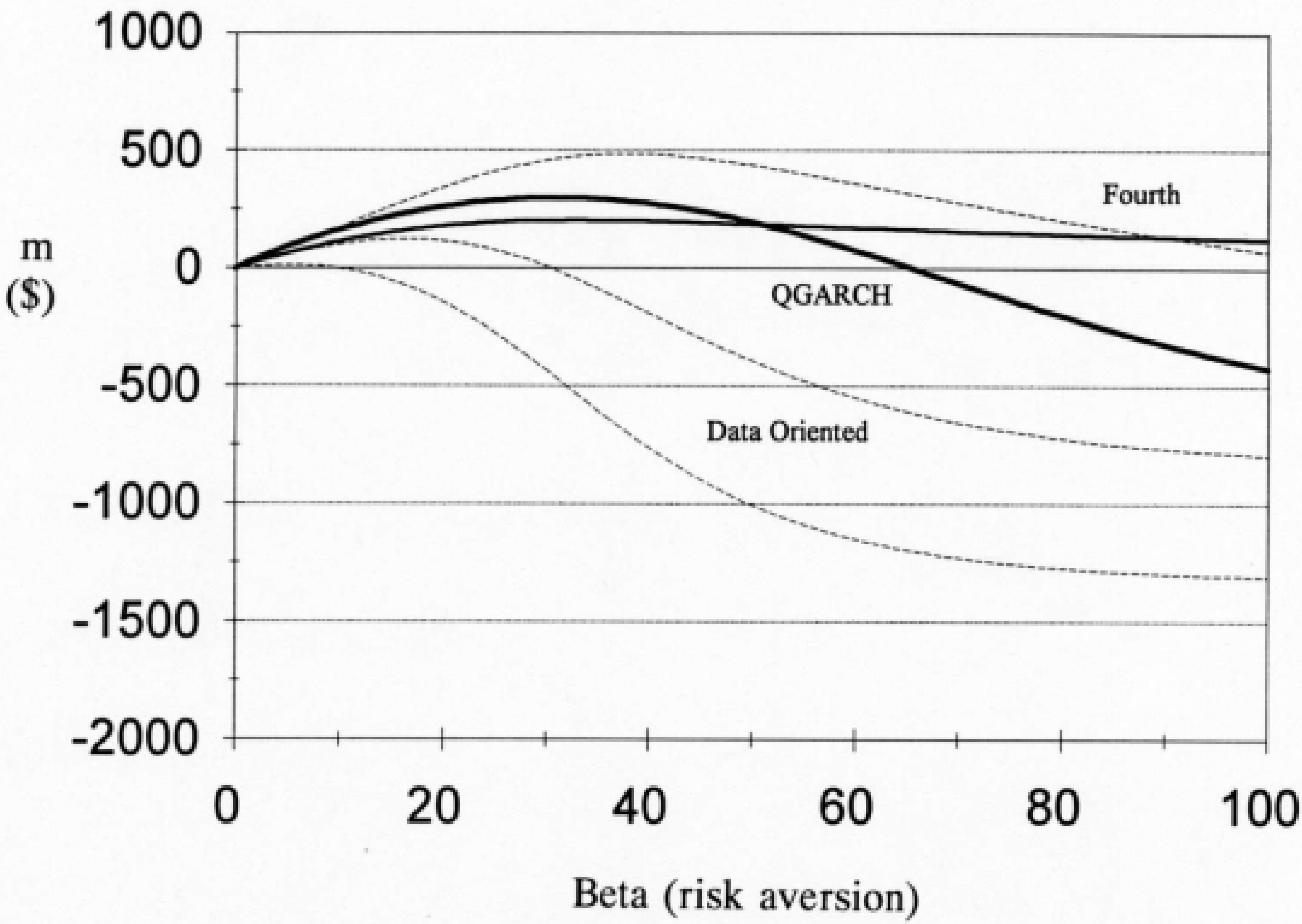
(A) Calculations by Taylor Approximations



m is the difference between the certainty equivalents (measured in the dollar-term) of the internationally diversified and the 100%-domestic portfolios, assuming the initial investment is equal to $100,000. For example, if the investments in Portfolio A attains the utility level equal to receiving $101,000 for sure, and in Portfolio B attains equal to $100,900, m = $100 (= $101,000-$100,900). **Beta** is the coefficient of relative risk aversion of the investor's utility function. In panel (A), **m** is calculated by Taylor approximations using up to the second, third, and fourth moments of the sample. In panel (B), the utility level is directly calculated from the sample data, and the artificial data generated by QGARCH model.

Figure 4. The Benefit from International Diversification

(B) Fourth, QGARCH, and Data Oriented



Note: The upper and lower dotted lines are the 90% bootstrap confidence interval of the Data Oriented caclculation.

Figure 4

Conditional Volatility of the World Market Risk

Conditional standard error implied by QGARCH model

Sample: June 1970 - February 1998