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Inefficiency and social exclusion in a coalition formation game: experimental evidence

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Abstract

This paper experimentally investigates the impact of reciprocal behavior in multilateral bargaining and coalition formation. Our results show that reciprocal fairness strongly affects the efficiency and equity of coalition formation. In a large majority of cases, inefficient and unfair coalitions are chosen when their coalition values are relatively high. Up to one third of the experimental population is excluded from bargaining and earns nothing. In monetary terms economically significant efficiency losses occur. We find that the interplay of selfish and reciprocal behavior unavoidably leads to this undesirable consequences. We also compare the predictions of recently developed models of social preferences with our experimental results. We find that some of these models capture the empirical regularities surprisingly well.

Key words: Coalition formation, inefficiency, reciprocity, social exclusion \\
JEL classification: A13, C91, D61, D63

1 Introduction

Bargaining is one of the central aspects in economic activity. Some bargaining is bilateral and negotiations take place only between two economic agents such as a buyer and a seller. There are, however, many multilateral bargaining situations in which agents are free to form coalitions. Examples are abundant:

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URL: www.fee.uva.nl/creed/people/arno.htm (Arno Riedl).
private-ownership firms, cartel of firms, labor unions, clubs, networks, international trading blocks, coalitions of political parties, etc. An important aspect in multilateral bargaining is the possible conflict in coalition formation. In particular, it may play an important role for the efficiency and equity of agreements. Standard literature on bargaining theory analyses this conflict under the assumption of narrow selfishness of bargainers. The literature on bargaining experiments, however, leaves little doubt that behavior of people is also influenced by considerations of fairness and reciprocity. The main purpose of this paper is to experimentally investigate the impact of reciprocal fairness on multilateral bargaining.

To experimentally study coalition-forming behavior in the most clear-cut way, we introduce a simple non-cooperative bargaining procedure of a three-person super-additive game in coalition form. In the game, a group benefit is assigned to each possible coalition while any single player produces zero benefit. In the experiment a ‘proposer’ has to choose between the efficient three- and an inefficient and unfair two-person coalition. Thereafter, the proposer makes a proposal about the division of the coalition value. Only if all members of the chosen coalition accept the proposal the allocation is implemented. Otherwise the surplus is destroyed and nobody earns anything. Subjects who are not members of a coalition earn nothing for sure. This set-up not only allows us to investigate (ultimatum) bargaining behavior in two- and three-person coalitions within one setting but also whether people are ready to forego resources and increase inequality simultaneously.

To investigate the effect of different gains from cooperation on coalition choices and payoff distributions we systematically varied the value of an inefficient coalition, keeping the value of the efficient coalition unchanged. We implemented four different values of two-person coalitions within four experimental conditions. The conditions differed only with respect to the value of the two-person coalition. The grand coalition was always worth 3000 points and the two-person coalition values varied between 2800, 2500, 2100, and 1200 points.

Our results indicate a clear link between two-person coalition values and coalition formation. In the two conditions with efficiency losses of 7 and 17 per-

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1 When using the terms reciprocity, reciprocal behavior, or reciprocal action we do not only mean reciprocity in the narrow sense of responding (un)kindly to (un)kind behavior but also behavior that may be interpreted as reciprocal though it is not driven by intentions. The reason is that some purely outcome-based models of social preferences predict behavior that is not distinguishable from intentional rewarding or punishment. We discuss these models in Section 4.2.

2 As an anonymous referee rightly remarked one has to be careful when talking about inefficiency in the presence of social or other regarding preferences. Whenever we are using this term it should be interpreted in the material or monetary sense, which we believe is in terms of forgone resources an economically important measure.
cent, respectively, an overwhelming majority of up to 95 percent of ‘proposers’ take up the inefficient two-person coalition. They thereby exclude almost one third of the population from participation. In the condition where the two-person coalition induces an efficiency loss of 40 percent still about 40 percent of the proposers choose this small coalition. The actual behaviorally induced efficiency losses are economically significant and vary between 6 and 15 percent. We provide evidence that these economically and socially undesirable results are an unavoidable consequence of reciprocal behavior of responders and (seemingly) selfish behavior of proposers.

While reciprocity is often identified as a force which leads to more even income distributions (like in standard ultimatum games\textsuperscript{3}) and/or increases efficiency (like in gift-exchange and trust games\textsuperscript{4}) our study shows that the same behavioral predisposition can have the exact opposite consequences in other institutional environments.

Our main result that an overwhelming majority of proposers choose inefficient and unfair two-person coalitions when efficiency losses due to such a choice are relatively low stands in stark contrast to the prediction of standard game theory. When players are motivated only by their own monetary payoff, the unique subgame perfect equilibrium of our game is that the proposer chooses the grand coalition thereby demanding the total surplus and that such a proposal is accepted. To account for observed behavior we compare the predictions of recently developed models of social preferences due to Fehr and Schmidt (1999) and Bolton and Ockenfels (2000) with our experimental results. It turns out that these models can capture our empirical regularities surprisingly well.

In the model of inequity aversion by Fehr and Schmidt (1999) players suffer utility loss from both advantageous and disadvantageous inequality. It is assumed that the marginal utility loss from advantageous inequality is less than one. In our game this implies that responders accept any offer larger than the equal share of the coalition value. If the offer is less than the equal share responders suffer from both the low monetary payoff and disadvantageous inequality. This leads responders to reject very small offers and to accept offers if and only if they are not below a certain threshold. In this way, the Fehr and Schmidt model can capture reciprocal behavior of responders.

Anticipating responders’ reciprocal behavior the proposer knows that she can receive at least the equal share of the coalition value. Whether the proposer demands more than the equal share depends on her marginal utility loss from

\textsuperscript{3} See the seminal paper of Guth et al. (1982), and for overviews Roth (1995) and Camerer (2003).

\textsuperscript{4} See e.g. Berg et al. (1995), Fehr et al. (1993), Fehr et al. (1997), and Fehr et al. (1998). For one of the first accounts of efficiency increasing reciprocal behavior, see the seminal work of Axelrod (1984).
advantageous inequality. If this loss is sufficiently large she suffers from the unequal outcome when choosing the two-person coalition since an excluded player receives zero payoff. Such a proposer will choose the grand coalition with equal payoff distribution.

If, however, the proposer’s utility loss from advantageous inequality is relatively small she prefers to increase her monetary payoff in both the grand coalition and two-person coalitions at the expense of the responders. That is, in a given coalition the proposer maximizes her monetary payoff, subject to responders’ reciprocal response. A two-person coalition is chosen if and only if its value relative to the value of the grand coalition is greater than a particular threshold. For reasonable values of the inequity aversion parameters, the Fehr and Schmidt model allows for two-person coalitions in equilibrium.

In the model of social preferences by Bolton and Ockenfels (2000) a similar mechanism as in the Fehr and Schmidt model is at work. Specifically, it can be shown that given the coalition value responders accept a proposal if and only if their relative share is not less than some threshold. In this sense, the model can capture reciprocal behavior of responders. A proposer maximizing her motivation function is then more likely to choose a two-person coalition the higher the coalition value of the small coalition. In this sense the model of Bolton and Ockenfels allows for two-person coalitions in equilibrium, too. In summary, the predictions of these models of social preferences are consistent with social exclusion and material inefficiencies as observed in our experiment.

For the first time, our study experimentally investigates how potential efficiency losses - due to inefficient subcoalitions - relate to coalition formation in a systematic way. Earlier experimental studies - shortly discussed below - on multilateral bargaining either focus on three-person ultimatum bargaining with an inactive player and no coalition formation or do not vary the efficiency loss of an inefficient coalition decision. None of these studies relates coalition formation to reciprocal behavior and its consequences for allocative efficiency and distributional concerns.

The experimental work closest to ours are the studies by Bolton and Chatterjee (1996), Bolton et al. (2003), Guth and van Damme (1998), and Riedl and Vyrastekova (2003). The latter two studies are three-person ultimatum games without coalition decision. Guth and van Damme (1998) investigate proposer and responder behavior in an ultimatum game setting where the third player is inactive. In particular, they examine how different information about the proposal affects rejection behavior. In the experiment of Riedl and Vyrastekova (2003) all players are active. The authors are primarily interested in acceptance behavior when the rejection consequences are varied. Bolton and Chatterjee (1996) and Bolton et al. (2003) conducted experiments with a three-person coalition-form game similar to our game. They investigated how
different bargaining procedures and communication structures affect coalition formation.\(^5\)

In the next section we describe the design of our experiment, including a portray of our three-person coalition-form game and a description of the experimental procedures. In section 3 our experimental results are presented. In section 4 we discuss them shortly in the light of some recently developed models of social preferences. Section 5 summarizes and concludes. Proofs are given in the Appendix.

2 Experimental Setup

2.1 A Non-Cooperative Coalition Formation Game

The game implemented in the laboratory is a non-cooperative three-person coalition formation game with an ultimatum bargaining stage. The three players involved are called proposer, responder 1, and responder 2. The sequence of the play is the following (see also Figure 1):

(1) The proposer \(P\) chooses either a two-person (small) coalition or the three-person (grand) coalition. The grand coalition has a value of \(V(P, R_1, R_2)\), where \(R_1\) and \(R_2\) stands for responder 1 and responder 2, respectively. The value of the two-person coalition, denoted \(V(P, R_i)\) \((i = 1, 2)\), is strictly smaller than the value of the grand coalition.

(2) After \(P\) has chosen her coalition, she proposes how to divide the coalition value between her and the chosen bargaining partner(s).

(a) If she has chosen the grand coalition, she proposes \((x_P, x_{R_1}, x_{R_2})\) with 
\[x_P + x_{R_1} + x_{R_2} = V(P, R_1, R_2)\]
to both responders.

(b) If she has opted for a small coalition, she proposes \((x_P, x_{R_i})\) with 
\[x_P + x_{R_i} = V(P, R_i)\]
only to the chosen responder \(R_i\).

(3) If \(R_1\) has been chosen as a member of either the three- or two-person coalition he has to decide whether to accept or reject the proposal. If he has not been chosen he has nothing to decide on.

(4) If the grand coalition was chosen and \(R_1\) has accepted the proposal, \(R_2\) decides whether to accept or reject the proposal. Otherwise, for \(R_2\) the

\(^5\) There exist some other ultimatum-like bargaining studies involving three players which are less close to our work. Guth et al. (1996) and Guth and Huck (1997) investigate proposer behavior in two-stage ultimatum games with uncertainty about the pie size; Knez and Camerer (1995) run experiments where a proposer plays two ultimatum games simultaneously and responders have asymmetric outside options; Kagel and Wolfe (2001) investigate how acceptance behavior changes if upon rejection an inactive third player receives different consolidation prizes.
same holds as for $R_1$.

![Diagram of a Non-Cooperative Three-Person Coalition Formation Game](image)

**Fig. 1. A Non-Cooperative Three-Person Coalition Formation Game**

The payoffs are allocated as follows: (i) If $P$ has chosen the grand coalition and both responders *accept* the proposal then all players receive their shares according to the proposal. If any of the responders *rejects* nobody earns anything. (ii) If $P$ has opted for a two-person coalition and the chosen responder *accepts* the proposal then these two players receive their shares according to the proposal. If he *rejects* both earn nothing. The responder who has not been chosen always has a payoff of zero.

All this is known by all players, and all players are informed about the decisions of all other players in previous moves. Assuming for the moment that there is no smallest money unit, then it can be easily seen that this game has a unique subgame perfect equilibrium (payoff). In the subgame starting after the proposer has opted for the three-person coalition there exists a unique subgame perfect equilibrium where $P$ demands the whole pie for herself and both responders accept. In the subgame after $P$ has chosen the two-person coalition with $R_i$ ($i = 1, 2$) the unique subgame perfect equilibrium implies that the proposer demands the whole pie $V(P, R_i)$ for herself, leaving $R_i$ a payoff of zero which he will accept. Since the value of the two-person coalition is strictly smaller than the value of the grand coalition, the unique best decision for the proposer is to opt for the grand coalition. Hence, standard game theory predicts that $P$ chooses the efficient coalition and makes the proposal $(x^*_P, x^*_{R1}, x^*_{R2}) = (V(P, R1, R2), 0, 0)$ which is accepted by both responders.

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6. In our experiment there is a smallest money unit. This destroys the uniqueness of the equilibrium. It can be shown, however, that in any subgame perfect equilibrium proposers always choose the grand coalition if the difference $V(P, R1, R2) - V(P, R_i)$ is larger than twice the smallest money unit, and that any proposal which gives each responder at least the smallest money unit is accepted.
We conducted ten experimental sessions involving 240 subjects. In each session we implemented one of two treatments (called T1 and T2). Both treatments consisted of two phases (or conditions). In a session the 24 subjects were divided into two separate groups of twelve. Within these groups in each phase eight rounds of the above described game were played with random matching in each round. These created 10 statistically independent observations per treatment on the group level. If not mentioned otherwise we will base our tests and estimates on these independent units of observation.

The two phases within a treatment differed only with respect to the value of the two-person coalition. All values were described in points. The value \( V(P, R_1, R_2) \) of the grand coalition was always 3000 points. In T1 the value \( V(P, R_i) \) of the two-person coalition was 2800 points in phase 1 and 1200 points in phase 2. In treatment T2 the value of the two-person coalition was only 2100 points in phase 1 and 2500 points in phase 2. Note, that our experimental procedures (see Box 1 for details) ensured the following: (i) during phase 1 subjects did not know that there would be a second phase, and (ii) they only heard about their actual earnings at the end of both phases. Furthermore, the use of a within-subject design in each treatment enabled us to examine whether the same persons make different choices, depending only on the value of the two-person coalition.

In the following we shall refer to the different conditions by T1-2800, T1-1200, T2-2100, and T2-2500. All ten sessions were computerized and run in English language at the CREED laboratory at the Faculty of Economics and Econometrics of the University of Amsterdam in March 2003.\(^7\) Proposals had to be made in steps of 10 points. The exchange rate from points to euro was 250 points = €1,–. (At the time of the experiment €1,– was worth approximately $1,–.) Hence, the grand coalition was worth approximately $12,–.

### 3 Experimental Results

Subjects’ average earnings (net of a show-up fee of €5,–) were €12,15 in T1 and €11,75 in T2. Sessions lasted between 75 and 100 minutes. In the following

\(^7\) About two-thirds of the participants were undergraduate students in economics, econometrics, business administration or management studies. The remainder came from various fields. Slightly above 60 percent of the students held the Dutch nationality. The rest came from different countries, mostly within Europe. None of the subjects had participated in a similar experiment before.
**Box 1: Experimental procedures**

*Phase 1:* After arriving in the laboratory subjects were randomly assigned letters “R’s”, “M’s”, and “L’s”, which assigned them the different roles in the experiment. The “R’s” were the proposers, the “M’s” the first responders and the “L’s” the second responders. A bargaining group consisted of one “R”-, one “M”-, and one “L”-subject. Subjects had the same role throughout the whole experiment. Subjects were seated in cubicles with sight shields. Any form of communication other than via the computer net was made impossible. Neither during nor after the experiment was the identity of bargaining partners revealed. Subjects received the instructions on-screen and they were also read aloud. One practice round was conducted. Thereafter, eight real rounds were carried out with random re-matching in each round. In the instructions subjects were informed that - in addition to the showup fee - they would be paid in cash the sum of their earnings in two out of the eight rounds after the experiment. These two rounds were randomly selected at the end of the experiment and subjects were aware of this procedure.

*After* the last round of *Phase 1* subjects were informed that there will be another experiment. Subsequently *Phase 2* started. The instructions were again given on-screen and read aloud. Participants were informed that there will be another eight rounds and that, thereafter, the experiment would definitely be finished. Furthermore, they were informed that they would be paid in cash the sum of their earnings in two randomly chosen rounds. The earnings of the first phase were unaffected by those of the second phase. The matching procedure was the same as in phase 1 and subjects were informed about that.

we present first the results concerning the coalition decisions. Thereafter, we analyze bargaining behavior of responders and proposers within the chosen two- and three-person coalitions.

### 3.1 Coalition Decisions

The following result reports the coalition decisions.

**Result 1**

(i) *If the value of the two-person coalition is 2800 or 2500 an overwhelming majority of proposers opts for the two-person coalition.*

(ii) *If the value of the two-person coalition is 2100 still about 40 percent of the proposers chooses the two-person coalition.*

(iii) *Only for a value of the two-person coalition of 1200 the grand coalition is almost always formed.*

(iv) *The frequency of two-person coalitions does not decrease over time.*

Evidence for this result is provided by Figure 2, which shows the percentage of chosen two-person coalitions per condition and round. The figure clearly indicates that in T1-2800 and T2-2500 most proposers choose the two-person coalition right from the beginning. In the first round of T1-2800 82.5 percent
(33 out of 40 proposers) choose the two-person coalition. In the first round of T2-2500 even 90.0 percent (36/40) opt for the small coalition. Hence, social exclusion takes place right from the beginning. The frequency of two-person coalitions does not decrease over rounds. For T1-2800 the Spearman rank-

order correlation coefficient of the average number (across all proposers) of two-person coalitions on rounds yields a value of $r_s = 0.64$ ($p = 0.044$, one-sided test). For T2-2500 this correlation coefficient is zero.

If the value of the two-person coalition is only 2100, about one third of the proposers (13/40) choose the small coalition in the first round. This number increases to 45 percent (18/40) in round eight leading to 38.1 percent across all rounds. The Spearman rank order coefficient indicates a significantly increasing trend in the frequency of two-person coalitions ($r_s = 0.97; p < 0.001$, one-sided test). Only for the very low coalition value of 1200 (almost) no small coalitions are observed. Over all rounds, in only 9 cases (out of 320 decisions) the two-person coalition is chosen.

**Result 2** *There is no difference in the frequency of two-person coalitions between the values of 2800 and 2500 of the small coalition. If the value of the small coalition is 2100 the frequency of two-person coalitions is smaller than for the higher values but higher than for the small value of 1200.*

Statistical support for this result comes from round by round comparisons with the frequency of two-person coalitions per round and group as unit of observation. When comparing T1-2800 with T2-2500 the Mann-Whitney rank-sum test does not reject the null hypothesis of no difference for any round ($p \geq 0.282$; 2-sided tests). When testing T2-2100 against T1-2800 and T2-2500, respectively, the difference in frequencies of two-person coalitions
turns out to be statistically significant in each round ($p \leq 0.003$, 2-sided Mann-Whitney test, and $p \leq 0.006$, 2-sided Wilcoxon signed-ranks test, respectively). A comparison of T2-2100 with T1-1200 also reveals significant differences in each round ($p \leq 0.005$, 2-sided Mann-Whitney test).

3.2 Bargaining Behavior in Dividing Coalition Values

3.2.0.1 Behavior in Two-Person Coalitions. Table 1 summarizes the behavior of responders and proposers in two-person coalitions. Although relative offers are a bit on the low side, on average, observed behavior is within keeping earlier results in stand-alone ultimatum games.\(^8\) In the following we

<table>
<thead>
<tr>
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<th>Means and Medians of Offers to Chosen Responder and Disagreement Rates(^a)</th>
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<tr>
<td></td>
<td>T1-2800</td>
</tr>
<tr>
<td></td>
<td>T2-2100</td>
</tr>
<tr>
<td># of 2-PC in % (in %)</td>
<td># of 2-PC in % (in %)</td>
</tr>
<tr>
<td>281 18.2 865 800</td>
<td>122 9.8 797 800</td>
</tr>
<tr>
<td>(30.9) (28.6)</td>
<td>(38.0) (38.1)</td>
</tr>
</tbody>
</table>

|                        | T2-2500                                                                 |
|                        | # of 2-PC in % (in %)                                                      |
| 284 8.8 818 800        | (32.7) (32.0)                                                             |

Note: \(^a\)... the number of two-person coalitions (2-PC) is the total across all triads and rounds; means, medians, and disagreement rates are calculated across two-person coalitions and rounds. Due to the lack of two-person coalitions in T1-1200 no statistics for this condition are presented.

\(^8\) In this paper we do not investigate how responder and proposer behavior in two-person coalitions compares to behavior in standard stand-alone ultimatum game experiments. In another paper we examined this issue by comparing data from experiments we have run in Japan and Austria with those from Slonim and Roth (1998). The main result there is that for a comparable pie size responders in our study seem to be significantly ‘softer’. For details we refer the interested reader to Okada and Riedl (2001).
analyze responder and proposer behavior in more detail.

**Responder behavior in two-person coalitions:** Figure 3 depicts the rejection rates by offer ranges (in points) and condition. The figure shows that in all conditions responders behave reciprocally, in the sense that lower offers are rejected with a higher probability. The figure also suggests that in all conditions relatively low offers are accepted with a relatively high frequency. In T2-2100 any offer above 40.5 percent (850 points) is accepted (42 offers), in T2-2500 any offer above 38 percent (950 points) is accepted (153 offers), and in T1-2800 all offers above 37.5 percent (1050 points) are accepted for sure (56 offers).

With the help of probit estimates we also investigate whether there is a difference in responder behavior across the different conditions. The obtained estimation results clearly indicate that reciprocal behavior is prevalent in all conditions. We do, however, not find strong differences across the different coalition values.\(^9\)

\(^9\) Specifically, we estimate two probit models with robust standard errors and allowing for observations being not independent within groups. The specification is as follows: \(\text{Accept} = f(\alpha + \beta_{\text{rel}} \times \text{rel} + \beta_{v} \times v + \beta_{\text{int}} \times \text{int} \times \text{round})\), where ‘*’ stands for the coalition values 2100 and 2500, respectively. \(\text{Accept} = 1\) if the offer was accepted and 0 otherwise, \(f(x)\) denotes the probit function, and \(\text{rel}\) is the offer measured relative to the value of the respective two-person coalition.
Proposer behavior in two-person coalitions: Inspection of Table 1 reveals that the average relative offer is monotonically decreasing with the coalition value. Statistical tests only partly corroborate this visual impression. Using the average relative offer per group as unit of observation neither Mann-Whitney tests nor t-tests detect significant differences in offers between T1-2800 and T1-2500 and T1-2800 and T1-2100, respectively. However, when comparing relative offers in T2-2100 with those made in T2-2500 a Wilcoxon signed-rank test and a t-test reject the null hypothesis of equal offers at the 5 percent significance level (two-sided). Hence, statistically proposers are only slightly more demanding in two-person coalitions with higher values.

Result 3 Responder behavior in two-person coalitions does not substantially differ across different coalition values. Consistent with responder behavior proposers show only a slight tendency towards lower relative offers in coalitions with higher values.

3.2.0.2 Behavior in Three-Person Coalitions. Table 2 summarizes responder and proposer behavior in three-person coalitions for T1-1200 and T2-2100.

Responder behavior in three-person coalitions: Table 2 shows the disagreement rates (that is, the frequency with which at least one responder rejected the proposal) across three-person coalitions and rounds. For both conditions disagreement rates are higher than in two-person coalitions. In particular, in T2-2100 rejections are quite frequent. Interestingly, the higher disagreement rates in T2-2100 are probably due to the fact that the values of the two-person coalitions are higher. In the model comparing T1-2800 with T2-2500, v2500 is significantly positive at the 5 percent confidence level. The interaction variables and the dummy v2100 are not significantly different from zero (p ≥ 0.106) We also run random effects probit estimates. The results do not differ from those reported above.

Round-by-round comparisons lead to similar conclusions. When comparing relative offers in T1-2800 with relative offers in T2-2100 (T2-2500) only rounds 2 to 5 turn out to be significantly different (in no round significant differences are found); α = 0.05, 2-sided Mann-Whitney tests. For T2-2100 versus T2-2500 Wilcoxon signed rank tests find significant differences in round 4 to 7 (α = 0.05; 2-sided tests). Since in T1-2800 and T2-2500 almost no three-person coalitions have been chosen no meaningful statistics can be presented for these subgames.
Table 2
Summary of Behavior in Three-Person Coalitions
Means and Medians of Offers
(in % of the Grand Coalition Value)
to Responder 1 and Responder 2
and Disagreement Rates$^a$

<table>
<thead>
<tr>
<th></th>
<th>T1-1200</th>
<th></th>
<th>T2-2100</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Resp. 1</td>
<td>Resp. 2</td>
<td>Resp. 1</td>
</tr>
<tr>
<td># of</td>
<td>Dis. in %</td>
<td>Mean (in %)</td>
<td>Med. (in %)</td>
</tr>
<tr>
<td>3-PC</td>
<td>311</td>
<td>24.8</td>
<td>748</td>
</tr>
<tr>
<td></td>
<td>(24.9)</td>
<td>(25.0)</td>
<td>(24.7)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td># of</td>
<td>Dis. in %</td>
<td>Mean (in %)</td>
<td>Med. (in %)</td>
</tr>
<tr>
<td>3-PC</td>
<td>198</td>
<td>37.9</td>
<td>702</td>
</tr>
<tr>
<td></td>
<td>(23.4)</td>
<td>(25.0)</td>
<td>(23.2)</td>
</tr>
</tbody>
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*Note:* $^a$... the number of three-person coalitions (3-PC) is the total across all triads; means, medians, and disagreement rates are calculated across three-person coalitions and rounds.

The rate in T2-2100 seems mainly due to the second responders’ decisions. Given an acceptance by the first responder, in T2-2100 the proposal was declined by responder 2 in 22 percent of the cases whereas this was the case in only 13 percent of the cases in T1-1200. Similar numbers are obtained when we control for unequal offers.

Figures 4(a) and (b) show the (unconditional) rejection rates of first and second responders by offer range and condition. They indicate that, generally, both responders accept lower offers less often than higher offers. Furthermore, offers in the neighborhood and above one third of the grand coalition value are (almost) always accepted. Hence, reciprocal fairness considerations seem to be at work in three-person coalitions, too.

An interesting feature of the presence of two instead of only one active responder is that it allows us to examine whether acceptance behavior is influenced by the offer made to the other responder. We run probit regressions with accep-
In the figure empty squares indicate that no offers in the respective range have been made and bars with zero height indicate that all offers have been accepted.

(a) Responder 1  

(b) Responder 2

Fig. 4. Rejection Rates in Three-Person Coalitions

tance behavior of responder \(i (i = 1, 2)\) as dependent variable and the relative offer, the difference in relative offers, and the round number as explanatory variables. The estimates for the second responder take only those observations into account where the first responder accepts the proposal. In all regressions robust standard errors are calculated also allowing for observations that are not independent within groups.  

Table 3 depicts the regression results for three-person coalitions in T1-1200 and T2-2100. The coefficients \(\beta_{\text{relof}R1}\) and \(\beta_{\text{relof}R2}\) are all significantly greater than zero \((p < 0.001)\), corroborating the visual impression of reciprocal behavior from Figure 4.

In T1-1200, for both responders the coefficients measuring the impact of the

\[ \text{Accept} R1 = f(\alpha + \beta_{\text{relof}R1} \ast \text{relof} R1 + \beta_{R1-R2} \ast (\text{relof} R1 - \text{relof} R2) + \beta_{\text{round}} \ast \text{round}) \]

for the first responder, and

\[ \text{CondAccept} R2 = f(\alpha + \beta_{\text{relof}R2} \ast \text{relof} R2 + \beta_{R2-R1} \ast (\text{relof} R2 - \text{relof} R1) + \beta_{\text{round}} \ast \text{round}) \]

for the second responder. \(\text{Accept} R1 = 1 \) \((\text{CondAccept} R2 = 1)\) if the offer is accepted by the first (second) responder; 0 otherwise. \(f(x)\) denotes the probit function, and \(\text{relof} R1\) (\(\text{relof} R2\)) is the relative offer - as share of the value of the grand coalition - made to the first (second) responder. The coefficient \(\beta_{R2-R1}\) measures the influence of the relative standing with respect to the other responder. Since, in three-person coalitions an agreement is reached only if both responders accept, the analysis of the second responder’s behavior is restricted to the cases where the first responder accepts the proposal \((\text{CondAccept} R2)\).
Table 3
Probit Regressions: Responder Behavior in Three-Person Coalitions

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>T1-1200</th>
<th>T1-2100</th>
<th>T1-2100</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>-1.147</td>
<td>-1.390*</td>
<td>-1.526**</td>
</tr>
<tr>
<td>(p = 0.163)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta_{rel\text{of}Ri}$</td>
<td>9.485***</td>
<td>10.94***</td>
<td>9.355***</td>
</tr>
<tr>
<td>$\beta_{Ri-Rj}$</td>
<td>9.687**</td>
<td>7.828</td>
<td>3.703</td>
</tr>
<tr>
<td>(p = 0.065)</td>
<td>(p = 0.105)</td>
<td>(p = 0.420)</td>
<td></td>
</tr>
<tr>
<td>round</td>
<td>0.006</td>
<td>-0.001</td>
<td>0.062</td>
</tr>
<tr>
<td>(p = 0.912)</td>
<td>(p = 0.978)</td>
<td>(p = 0.288)</td>
<td>(p = 0.889)</td>
</tr>
</tbody>
</table>

Observations 311 268 198 157
Log Likelihood -107.5 -81.34 -87.84 -62.97
Wald $\chi^2$ 36.90 29.00 71.15 56.47

Note: *** $p \leq 0.001$, ** $p \leq 0.01$, * $p \leq 0.05$; two-sided tests. All estimates are with robust standard errors also allowing for observations that are not independent within groups. In regression for responder 1 (responder 2), i = 1 and j = 2 (i = 2 and j = 1).

difference in offers are significantly positive, at least at the 10 percent level (2-sided tests). In T2-2100 the coefficient is positive for responder 1 and negative for responder 2 (for both responders they are not significantly different from zero, however). This indicates that, deviations from equal treatment - with respect to the other responder - increases a responder’s acceptance likelihood when treated favorable and decreases it when treated unfavorable.

Proposer behavior: From Table 2 we see that, on average, proposers treat both responders more or less equally. In all cases the median offer is 25 percent of the grand coalition value. Based on groups as unit of observations, neither a non-parametric Wilcoxon signed-rank test nor a t-test detects a significant difference in offers made to the responders. This holds across all eight rounds as well as for each round separately. We also do not detect any significant difference in offers between the two reported conditions. We have also no

---

13 We also run another specification with dummy variables indicating whether a responder is strictly better or strictly worse off than the other responder as explanatory variables. It turns out that an offer where a responder is strictly better off than the other responder is always accepted (with one exception for responder 1 in T2-2100). The impact of being worse off turns out to be significantly negative in T1-1200 for both responders. For T2-2100 we do not find a significant effect.
indications that proposers change behavior across rounds within a condition.

**Result 4** (i) Responders in three-person coalitions behave reciprocally with respect to their own received offer. There is also evidence that the acceptance likelihood decreases with disadvantageous treatment compared to the other responder.

(ii) In three-person coalitions proposers treat the two responders equally. They offer them on average about 25 percent of the grand coalition value. This holds independently of the value of the two-person coalition and the experience level.

3.3 Inefficiency

Contrary to stand-alone ultimatum game experiments observed material efficiency losses in our experiment are not (only) the direct result of rejections of unfair offers. Rather, they are the consequence of proposers’ inefficient choices because of anticipated reciprocal responses by responders. Recall that the frequency of two-person coalitions in T2-2500 is as high as in T1-2800. Thus, the increase of the material efficiency loss from 6.67 to 16.67 percent does not retain proposers from choosing the inefficient and unfair allocation. Furthermore, even if the value of the two-person coalition is only 2100 points still about two-fifth of the proposers choose the inefficient allocation thereby inducing an efficiency loss of 30 percent.

![Fig. 5. Behaviorally Induced Material Efficiency Losses across Rounds](image)

In Figure 5 these inefficient coalition decisions are reflected by the actually induced material efficiency losses of 5.9 percent in T1-2800, 11.4 percent in T2-2100, and 14.8 percent in T2-2500. These material efficiency losses are
economically not negligible. As we will argue in the following, in coalitional bargaining situations as in our experiment, inefficient small coalitions are unavoidable as long as responders behave reciprocally and proposers act as (as if) income maximizers. Additionally, we show that, for a wide range of social preferences, this result is also predicted by outcome oriented models of reciprocal fairness.

4 Discussion

In this section we first provide arguments that reciprocal fairness considerations on the responders side and income maximization on the side of proposers are consistent with the observed inefficient coalitions and its adverse distributional consequences. Thereafter, we relate our empirical outcomes to the predictions of recently developed models of social preferences.

4.1 Reciprocal Behavior and Income Maximization

**Result 5** Income maximization of proposers dictates the choice of the two-person coalition whenever its coalition value is sufficiently high.

First support for this result is provided by Figure 6. It shows the average earnings (in points) of proposers by condition and coalition across rounds. The three leftmost bars depict the average earnings in two-person coalitions

![Fig. 6. Average Actual Earnings of Proposers across Rounds](image-url)
in T1-2800, T2-2500, and T2-2100, respectively. The four rightmost bars show the average earnings in three-person coalitions for all four values of the small coalition. The figure clearly indicates that in T1-2800 and T2-2500 a proposer earns considerably more when choosing a two-person coalition than when choosing the grand coalition; and even for the low value of 2100 proposers serve better in two- than in three person coalitions. Based on this information it seems obvious that an income maximizing proposer should choose the two-person coalition if its coalition value is high. One might argue, however, that only fair-minded proposers choose the three-person coalition leading to a downward bias on proposer earnings in these coalitions. We have therefore calculated the income maximizing offer ranges using the empirically observed acceptance rates. The results of this exercise are given in Table 4. The table depicts the income maximizing offer ranges $x^*$, and the maximal expected income $\max E\pi$ (in points) belonging to the maximizing offers, for two- and three-person coalitions in the different conditions. For the sake of comparison the table also contains the average offers across the last two periods in the respective condition and coalition.

Table 4
Proposer’s Actual and Income Maximizing Offers

<table>
<thead>
<tr>
<th>Offers</th>
<th>Mean</th>
<th>$x^*$</th>
<th>$\max E\pi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2-Pers. Coal.:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>T1-2800</td>
<td>0.303</td>
<td>[0.25, 0.30]</td>
<td>[1715, 1835]</td>
</tr>
<tr>
<td>T1-2500</td>
<td>0.310</td>
<td>[0.30, 0.35]</td>
<td>[1543, 1660]</td>
</tr>
<tr>
<td>T2-2100</td>
<td>0.345</td>
<td>[0.35, 0.40]</td>
<td>[1213, 1311]</td>
</tr>
</tbody>
</table>

| 3-Pers. Coal.: |       |             |             |
| T2-2100    | 0.232 | [0.25, 0.30] | [1030, 1282]|
| T1-1200    | 0.252 | [0.25, 0.30] | [961, 1196]  |

Note: Actual mean offers are based on the last two rounds. For all calculations the independent groups are used as units of observation. For the calculation of $x^*$ in three-person coalitions equal treatment of both responders is assumed. $x^*$ denotes therefore an offer made to both responders.

---

14 T1-1200 is not shown since we have observed too little two-person coalitions in this condition.

15 For the calculation of the maximizing offers in three-person coalitions we assumed for simplicity equal offers to both responders. In T1-2800 and T2-2500 we observe too few three-person coalitions to obtain any statistically reliable results.
The values of \( \max E\pi \) clearly show that given the responders’ behavior expected money income for proposers is maximized in two-person coalitions with high coalition value. Even for the intermediate coalition value the maximal proposer income in a three-person coalition is smaller than in a two-person coalition. Note that, on average, proposers make offers that come surprisingly close to the optimal offers. In our view, these observations together with the other results provide ample evidence that allows us to conclude that proposers act as if they are expected money maximizers under the constraint of negatively reciprocally behaving responders.\(^{16}\)

**Result 6** Together, reciprocal behavior of responders in two- and three-person coalitions and (seemingly) selfish behavior of proposers lead to social exclusion and (materially) inefficient coalition formation if the value of the small coalition is sufficiently high.

### 4.2 Models of Social Preferences

Fehr and Schmidt (1999), Bolton and Ockenfels (2000), and Charness and Rabin (2002) recently developed models of social preferences, which assume that people are not only motivated by their own money income but also by a taste for equity.\(^{17}\) Can these models explain the results obtained in our experiment? In the following we present theoretical predictions of the model by Fehr and Schmidt (1999) (hereafter FS) and Bolton and Ockenfels (2000) (hereafter BO). Thereby we confine ourself to those predictions we can relate to the results obtained in our experiment. Thereafter we shortly discuss the model of Charness and Rabin (2002). The formal proofs of the statements are relegated to Appendix A.

#### 4.2.0.3 The Fehr and Schmidt model.

In the FS model, every player \( i(=1,\cdots,n) \) is assumed to be endowed with a utility function of the following form: for a monetary payoff distribution \( x = (x_1,\cdots,x_n) \) among \( n \) players the

\(^{16}\) This result is in line with the theoretical model of Bolton (1991) and also matches experimental evidence from standard ultimatum game experiments conducted by Roth et al. (1991) and Prasnikar and Roth (1992). Charness and Rabin (2002), however, find in a different context no evidence that supports the view that first mover behavior is an optimizing response to the fear of rejection.

\(^{17}\) We are focusing on these outcome oriented models, which - as will turn out - predict the behavior in our game quite well. For models taking intentions into account see Rabin (1993), Dufwenberg and Kirchsteiger (1998), Falk and Fischbacher (1998), Levin (1998).
utility function of player $i$ is given by

$$U_i(x) = x_i - \alpha_i \frac{1}{n-1} \sum_{j \neq i} |x_j - x_i|^+ - \beta_i \frac{1}{n-1} \sum_{j \neq i} |x_i - x_j|^+$$  \quad (1)$$

where $z^+ = \max(z, 0)$. The two parameters $\alpha_i$ and $\beta_i$ ($\beta_i \leq \alpha_i$ and $0 \leq \beta_i < 1$) are considered to measure player $i$’s utility loss from disadvantageous and advantageous inequality, respectively. In the context of our game $n = 3$ and $i = P, 1, 2$ for the proposer and the two responders.

**Theorem 1 Responder behavior**

The subgame perfect equilibrium point of the three-person ultimatum coalition formation game prescribes the following behavior of responder $i = 1, 2$ in two- and three-person coalitions:

(1) Suppose that the proposer $P$ chooses the grand coalition and proposes a monetary distribution $x = (V - x_1 - x_2, x_1, x_2)$ of the coalition value $V$, where $x_i, 0 \leq x_i \leq V$, is the offer to responder $R_i$.

(a) If $(V - x_j)/2 \leq x_i$ and $x_j \leq x_i$ then accept the proposal.

(b) If $(V - x_j)/2 \leq x_i$ and $x_i < x_j$ then accept the proposal if and only if

$$x_i \geq \frac{(\alpha_i + \beta_i)x_j - \beta_i V}{\alpha_i + 2(1 - \beta_i)}.$$

(c) If $x_i < (V - x_j)/2$ and $x_j \leq x_i$ then accept the proposal if and only if

$$x_i \geq \frac{(\alpha_i + \beta_i)x_j - \alpha_i V}{2(1 + \alpha_i) - \beta_i}.$$

(d) If $x_i < (V - x_j)/2$ and $x_i < x_j$ then accept the proposal if and only if

$$x_i \geq \frac{\alpha_i}{2 + 3\alpha_i} V.$$

(2) Suppose that the proposer $P$ chooses a two-person coalition and proposes a monetary distribution $x = (v - x_i, x_i, 0)$ of the coalition value $v$, where $x_i, 0 \leq x_i \leq v$ is the offer to the chosen responder $R_i$, then $R_i$ accepts the offer if and only if

$$x_i \geq \frac{\alpha_i}{2(1 + \alpha_i) - \beta_i} v.$$

The first part of Theorem 1 prescribes the behavior of a responder in the grand coalition. The responder’s behavior depends on whether he is faced with advantageous or disadvantageous inequality with respect to the other members in the coalition. In case (a) of Theorem 1, responder $R_i$ faces an offer that treats him better than the proposer and the other responder. Since
the marginal utility loss from advantageous inequality is less than one ($\beta_i < 1$), it is in utility terms costly and thus not optimal for responder $R_i$ to destroy the entire surplus $V$ by rejection. This result corresponds to the known fact that in the standard two-person ultimatum game accepting any offer not less than the equal payoff is the dominant action for the responder; see Proposition 1 in Fehr and Schmidt (1999). In case (d) of Theorem 1, responder $R_i$ is confronted with an offer that puts him in a worse position than both the proposer and the other responder. In this situation a decrease of the offer to responder $R_i$ reduces his monetary payoff and also increases the utility loss from disadvantageous inequality. Hence, $R_i$'s total utility decreases as an offer to him is decreased. At some point an offer will give less utility than the utility from nobody receiving anything. This implies that there exists a threshold of an offer below which it is optimal for responder $R_i$ to reject. In the intermediate cases (b) and (c), responder $R_i$ faces offers making him worse off than only one of the other group members. Similarly to case (d), the responder rejects any offers below certain threshold levels.

Figure 7 illustrates the acceptance region (the shaded area) for Responder $R_2$. It nicely shows that the responder’s behavior generally depends not only on his own offer but also on the offer to the other responder (and thus on the proposer’s demand, too). If we assume that the inequality parameters are randomly distributed, then it is easy to deduce from the figure that, given the offer to the other responder, the likelihood that an offer is rejected decreases with the offer. Note that a responder’s rejection region is much larger when he is treated worse compared to his fellow responder than when he is better off. Hence, we can also say that the rejection likelihood is larger when a responder receives a lower offer than the other responder. All this is in accordance with our Result 4.

The second part of Theorem 1 prescribes responders’ behavior in two-person coalitions. Arguments similar to the case of the grand coalition can be applied, keeping in mind that the monetary payoff of the not chosen player is zero. The theorem shows that acceptance behavior in the two-person coalition depends only on the own relative share. That is, with respect to the relative offer responder behavior should be constant across two-person coalitions with different values. We have only very weak indications that responders’ behavior may not be constant across treatments. We therefore conclude that the prediction of the FS model is consistent with our Result 3.

**Theorem 2** PROPOSER BEHAVIOR

Suppose that $\alpha_1 \geq \alpha_2$ and $v < V$. Let $K_i := K_i(\alpha_1, \alpha_2, \beta_P, \beta_1, \beta_i) (i = 1, 2)$ be given. The subgame perfect equilibrium point of the three-person ultimatum coalition formation game prescribes the following behavior of the proposer:

(1) If $2/3 \leq \beta_P < 1$ then the proposer $P$ chooses the three-person coali-
Fig. 7. Acceptance Region of $R_2$ in the Three-Person Coalition

tion and proposes the equity distribution $(V/3, V/3, V/3)$, regardless of the two-person coalition value $v < V$.

(2) If $0 \leq \beta_P < 2/3$ then proposer behavior depends on the two-person coalitional value $v$ as follows:

(a) If $V K_i \geq v$ ($i = 1, 2$) then the proposer chooses the three-person coalition and proposes the payoff distribution $x = (x_P, x_1, x_2)$ satisfying

\[
1/3 < x_P = V - x_1 - x_2,
\]

\[
x_2 = \frac{\alpha_2}{2 + 3\alpha_2} V \leq x_1 = \frac{(\alpha_1 + \beta_1)x_2 - \alpha_1 V}{2(1 + \alpha_1) - \beta_1} < 1/3.
\]

(b) If $v > V K_i$ (for at least one $i$) then the proposer chooses the two-person coalition with the responder $R_i$ ($i = 1, 2$) satisfying $v > V K_i$. If both $v > V K_1$ and $v > V K_2$ hold then the proposer chooses the responder $R_i$ ($i = 1, 2$) satisfying $\alpha_i/(2 - \beta_i) < \alpha_j/(2 - \beta_j)$ ($i \neq j$). The proposer offers the payoff distribution $x = (x_P, x_i, x_j)$ satisfying

\[
x_P = v - x_i, \ x_i = \frac{\alpha_i}{2(1 + \alpha_i) - \beta_i} v < v/2, \ x_j = 0.
\]

The proposer chooses a coalition by comparing her best outcome in the grand coalition with those in two-person coalitions, subject to responders’ behavior.
described in Theorem 1. Consider first the best outcome for the proposer in
the grand coalition. If the equal distribution \( x_P = x_1 = x_2 = V/3 \) is proposed
it is certainly accepted. Anticipating this, the proposer never proposes a payoff
distribution which makes her worse off than both responders. Can the proposer
increase her utility by demanding a higher monetary payoff? The answer de-
dpends on how much utility loss she suffers from advantageous inequality. If
\( \beta_P > 2/3 \), the proposer’s utility is strictly decreasing in her own monetary
payoff, given that she suffers advantageous inequality relative to both respon-
ders. Hence, if this is the case the proposer prefers to share the total surplus
\( V \) equally rather than to maximize her monetary payoff. If \( \beta_P = 2/3 \), the pro-
poser is indifferent between giving half a dollar to each responder and keeping
one dollar to herself. If \( \beta_P < 2/3 \), the proposer strictly prefers to increase
her monetary payoff at the expense of the responders’ monetary payoffs. In
this case, her optimal action is to maximize monetary payoff subject to the
acceptance of the proposal.

Proposer behavior in a two-person coalition can be explained in a similar
way as in the case of the grand coalition, keeping in mind that the not
chosen player receives zero material payoff. If the proposer offers the equal
split, \( v/2 \), the chosen responder accepts with certainty. By the same reasons
as in the case of the grand coalition, the proposer never offers \( x_i > v/2 \).
Furthermore, if \( \beta_P > 2/3 \), then the proposer prefers to share the coalition
surplus \( v \) equally with the responder rather than to maximize her monetary
payoff. If \( \beta_P < 2/3 \), then the proposer wants to maximize her monetary payoff,
subject to acceptance of the proposal.

Concerning the coalition decision the above arguments imply that, if \( \beta_P \geq 2/3 \),
the proposer shares the coalition value \( V \) equally in the grand coalition and
receives utility \( V/3 \). In the two-person coalition the proposer also shares the
coalition value \( v \) equally, but suffers from the zero payoff of the not chosen
responder. The proposer’s utility in this case is \((1 - \beta_P/2)v/2\), which is less
than \( V/3 \). Therefore, a proposer with \( \beta_P \geq 2/3 \) chooses the grand coalition
and offers the equal split. In view of our Result 1, the Fehr-Schmidt model is
consistent with observed behavior only if the advantageous inequality param-
eter is smaller than \( 2/3 \) for basically all proposers in our experiment.

If \( \beta_P < 2/3 \), the proposer maximizes her monetary payoff in both the grand
and two-person coalitions subject to acceptance of the proposal. The coal-
ition choice depends on the relative value, \( v/V \), of the two-person coalition.
A proposer chooses a two-person coalition if and only if this relative value is
greater than a particular threshold, \( K_i \), that depends on the utility loss pa-
rameters for inequality of all involved players. Since this threshold is not too
large for reasonable values of the inequality parameters the FS model allows
for two-person coalitions in equilibrium. In this sense, the model’s prediction
is consistent with our Result 1. Additionally, if we assume that the parameters
\(\alpha_i\) and \(\beta_i\) are randomly distributed, the likelihood that condition \(VK_i < v\) is satisfied for some \(i = 1, 2\) increases as the two-person coalition value \(v\) becomes larger. In this sense, the model’s prediction is also consistent with our Result 2.

When \(\alpha_1 = \alpha_2 = \alpha\), the proposer treats the responders equally in the three-person coalition and offers them a relative share that is independent of the small coalition value \(v\). Furthermore, a proposer demands more than one-third of the pie. All these are consistent with our Result 4(ii). Notice also that the relative share offered by the proposer in a two-person coalition is independent of the value of this coalition (Theorem 2 2.(b)). We only find weak evidence that proposers are more greedy in coalitions with higher values. Hence, the model’s prediction can also be regarded as consistent with our Result 3.

Observe that, due to the ‘self-centered’ fairness notion implicit in the FS model (when \(\beta < 2/3\)), a proposer choosing the grand coalition offers the less (disadvantageous) inequity averse responder, \(R2 (\alpha_2 \leq \alpha_1)\), the smaller part of the pie. Similarly, a proposer choosing a two-person coalition chooses the ‘more selfish’ responder with a lower acceptance threshold as bargaining partner. The reason for this is simply that such responders accept lower material offers and, hence, give the proposer the opportunity of higher monetary earnings.

One might say that such proposers behave as if they would be selfish money maximizers and they will choose the two-person coalition with high coalition value for this reason. This squares nicely with our Results 5 and 6.

### 4.2.0.4 The Bolton and Ockenfels model.

The BO model also presumes that subjects are motivated by both their pecuniary payoff and their relative material standing. Specifically, every player \(i = 1, \ldots, n\) is assumed to maximize the expected value of her motivation function \(\nu_i = \nu_i (y_i, \sigma_i)\) where \(y_i \geq 0\) is player \(i\)’s monetary payoff, and \(\sigma_i\) is player \(i\)’s relative share of the payoff which is defined by

\[
\sigma_i = \sigma_i (y_i, c, n) = \begin{cases} 
  \frac{y_i}{c} & \text{if } c > 0 \\
  \frac{1}{n} & \text{if } c = 0
\end{cases} \quad (\text{where } c = \sum_{j=1}^{n} y_j)
\]

BO make the following assumptions (\(\nu_{ij}\) denotes the partial derivative of \(\nu_i\) with respect to the \(j\)-th variable):

**Assumption 1** The function \(\nu_i\) is continuous and twice differentiable on the domain of \((y_i, \sigma_i)\).

**Assumption 2** \(\nu_{1i}(y_i, \sigma_i) \geq 0\) and \(\nu_{1ii}(y_i, \sigma_i) \leq 0\). Also, fixing \(\sigma\) and given two choices where \(\nu_i (y^1_i, \sigma) = \nu_i (y^2_i, \sigma)\) and \(y^1_i > y^2_i\), player \(i\) chooses \((y^1_i, \sigma)\).
Assumption 3 \( \nu_{i_2}(y_i, \sigma_i) = 0 \) for \( \sigma_i(c, y_i, n) = 1/n \), and \( \nu_{i_2}(y_i, \sigma_i) < 0 \).

Assumption 4 Fixing \( c \), let \( \nu_i^c(\sigma_i) \) denote \( \nu_i(c\sigma_i, \sigma_i) \). \( \nu_i^c(\sigma_i) \) is strictly concave in \( \sigma_i \) for all \( c > 0 \), and \( \nu_i^c(1) > \nu_i(0, 1/n) \).\(^{18}\)

Together these assumptions guarantee that, for each \( c > 0 \), there exists a unique \( r_i = r_i(c) \in [1/n, 1] \) such that

\[
   r_i = \arg \max_{0 \leq \sigma_i \leq 1} \nu_i(c\sigma_i, \sigma_i). \tag{2}
\]

and a unique \( s_i = s_i(c) \in [0, 1/n] \) such that

\[
   \nu_i(cs_i, s_i) = \nu(0, 1/n). \tag{3}
\]

These threshold functions \( r_i(c) \) and \( s_i(c) \) characterize players’ types and are assumed to be random variables with density functions \( f^r \) and \( f^s \). It is assumed that for all \( c > 0 \), \( f^r(r|c) > 0 \) for \( r \in [1/n, 1] \) and \( f^s(s|c) > 0 \) for \( s \in ]0, 1/n] \).

Following BO we analyze players’ behavior in a perfect Bayesian equilibrium of the coalition formation game where each player’s thresholds \( r \) and \( s \) are private information but the densities \( f^r \) and \( f^s \) are common knowledge.

Theorem 3 Responder behavior

In the perfect Bayesian equilibrium of the three-person ultimatum coalition formation game behavior of responder Ri \( (i = 1, 2) \) in two- and three-person coalitions is characterized as follows:

1. Suppose that the proposer \( P \) chooses the grand coalition and proposes a monetary distribution \( x = (V - x_1 - x_2, x_1, x_2) \) of the coalition value \( V \), where \( x_i \), \( 0 \leq x_i \leq V \) \((i = 1, 2)\), is the offer to responder Ri. Let \( p_i = p_i(V, \sigma_i) \) denote the probability that a randomly selected responder Ri \( (i = 1, 2) \) rejects such a proposal, then
   a. \( p_i(V, 0) = 1 \) and \( p_i(V, \sigma_i) = 0 \) for all \( \sigma_i \in [1/3, 1] \); furthermore, \( p_i(V, \sigma_i) \) is strictly decreasing in \( \sigma_i \) on the interval \( ]0, 1/3[ \).
   b. Given the offer to Ri, the rejection probability \( p_i \) is independent of the offer to the other responder.

2. Suppose that the proposer \( P \) chooses the small coalition and proposes a monetary distribution \( x = (v - x_i, x_i, 0) \) of the coalition value \( v \), where \( x_i \), \( 0 \leq x_i \leq v \) \((i = 1, 2)\), is the offer to the chosen responder Ri. Let \( p_i = p_i(v, \sigma_i) \) denote the probability that a randomly selected responder Ri \( (i = 1, 2) \) rejects such a proposal, then

\(^{18}\) We need this inequality, which is not part of BO’s original set of assumptions, to guarantee that \( \nu_i(c\sigma_i, \sigma_i) > \nu_i(0, 1/n) \) for any \( \sigma_i > s_i(c) \).
(a) \( p_i(v, 0) = 1 \) and \( p_i(v, \sigma_i) = 0 \) for all \( \sigma_i \in [1/3, 1] \); furthermore, \( p_i(v, \sigma_i) \) is strictly decreasing in \( \sigma_i \) on the interval \([0, 1/3] \).
(b) Fixing \( \sigma_i \in [0, 1/3] \), \( p_i(v, \sigma_i) \) is non-increasing in \( v \).

Responder behavior is critically governed by his motivation function \( \nu_i(c\sigma_i, \sigma_i) \). If he rejects a proposal, then the responder receives the motivation value \( \nu_i(0, 1/n) \). Clearly, the responder accepts the proposal if and only if his motivation to accept is not less than that to reject. Thus, the acceptance threshold of responder \( R_i \) is \( s_i(c) \), defined by (4.3). The BO model assumes that given his relative share, a responder weakly prefers more monetary payoff and, given the monetary payoff, a responder likes the equal share \( \sigma_i = 1/3 \) most. These assumptions imply that the motivation function \( \nu_i(c\sigma_i, \sigma_i) \) is strictly increasing in \( \sigma_i \) for \( 0 < \sigma_i < 1/3 \). That is, given the surplus \( c \), the responder prefers the equal share to any relative share lower than it. In the BO model a responder prefers rejection over the acceptance of an almost zero monetary payoff. (This is implied by \( \nu_i(0, 0) < \nu_i(0, 1/n) \).) Together, these properties guarantee \( 0 < s_i(c) \leq 1/3 \). The other important property of the acceptance threshold behind the theorem is that \( s_i(c) \) is (weakly) decreasing in the surplus \( c \).

Part 1 of Theorem 3 states that responders behave reciprocally in three-person coalitions and, specifically, predicts that offers above one-third of the grand coalition value are always accepted. Both these predictions are in accordance with the behavior observed in our experiment (Result 4 and Figure 4). The theorem also tells us that responder’s acceptance behavior should be independent of the offer to the other responder, given his own offer. Theoretically this is the case, because the BO model assumes that people only take their relative standing with respect to the whole group into account. This implies that, given an offer, redistributing money between the proposer and the other responder does not affect the value of the motivation function of the responder and, thus, also not his behavior. Since, we have evidence that the relative standing influences rejection behavior in three-person coalitions this prediction is not consistent with this part of our Result 4.

The second part of Theorem 3 states that also in two-person coalitions responders behave reciprocally, which is clearly consistent with our observations. Specifically, it also predicts that all offers above one-third of the small coalition value are accepted for sure. We actually observe relatively high acceptance rates for relatively low offers. For \( v = 2100 \), \( v = 2500 \), and \( v = 2800 \) the rejection rates for offers above \( v/3 \) are 8.2, 1.2, 3.5 percent. In our view, the model’s prediction approximates real behavior quite well. In this sense, the prediction of the BO model is in accordance with our observations (Result 3 and Figure 3).

The last part of Theorem 3 states that the rejection rate in two-person coalitions are at least not increasing in the two-person coalition value \( v \). Since we
do not have strong indications that responder behavior changes with \( v \) we regard this prediction as in accordance with our Result 3.

**Theorem 4 Proposer behavior**

*In the perfect Bayesian equilibrium of the three-person ultimatum coalition formation game proposer behavior in two- and three-person coalitions with values \( v \) and \( V \) is characterized by the maximization of the expected motivation functions*

\[
E_{\nu_P}(v, \sigma_P) = (1 - p_i(v, \sigma_i)) \cdot \nu_P(v \sigma_P, \sigma_P), \quad \sigma_i = 1 - \sigma_P, \ i = 1, 2, \\
E_{\nu_P}(V, \sigma_P) = (1 - p_1(V, \sigma_1)) \cdot (1 - p_2(V, \sigma_2)) \cdot \nu_P(V \sigma_P, \sigma_P), \\
\sigma_i = 1 - \sigma_j - \sigma_P, \ i = 1, 2, i \neq j.
\]

*The maximum value \( E_{\nu_P}^*(v) \) of the proposer’s expected motivation function in a two-person coalition is nondecreasing in the value \( v \).*

The proposer chooses a two-person coalition if and only if her optimal expected motivation value in the two-person coalition is greater than that in the three-person coalition. The above theorem shows that the former is non-decreasing in the two-person coalition value \( v \), while the latter is independent of \( v \). This implies that the likelihood of two-person coalitions increases with the two-person coalition value \( v \). In this sense the prediction of the BO model is consistent with our Results 1 and 2.

Note, that if two responders are drawn from the same pool of subjects, which means that they have the same rejection probabilities, then the proposer’s optimal behavior is to treat them equally. Furthermore, since offers above one-third of the coalition value are always accepted the assumptions made by BO imply that offers in three-person coalitions will not exceed \( V/3 \). A similar argument holds for two-person coalitions. Hence, in this respect, proposer behavior predicted by the BO model is consistent with our observations reported in Results 3 and 4.

In summary we can conclude that in qualitative terms both, the FS model and the BO model, predict real behavior of proposers and responders in our three-person ultimatum coalition formation game surprisingly well.

In another recent model Charness and Rabin (2002) argue that experimental subjects are more concerned with increasing social welfare and helping worst-off people rather than by self-centered inequality aversion. They show that in a number of predominantly two-person games their model indeed performs better than other models of (non-)social preferences. They admit, however, that their (basic) model cannot explain rejections in ultimatum games. Similarly, the evidence observed in our experiment stays in stark contrast to the predictions of their basic model. The choice of a two-person coalition decreases
social welfare and makes one person as worse-off as possible. We do, of course, not deny the existence of people who take the welfare of the worst-off into account. What our results, however, suggest is that this disposition is not strong enough to overcome the anticipated increased rejection probability, and thus the decreased expected material welfare, in case of a three-person coalition.\(^{19}\)

The inefficient outcomes observed in our experiment may also be compared with theoretical results in the literature on non-cooperative sequential bargaining models of coalition formation, initiated by Selten (1981). Chatterjee et al. (1993), Okada (1996) and Okada (2000) have shown that inefficient subcoalitions may be formed in equilibrium in Rubinstein (1982) type sequential bargaining models of coalition formation even under complete information about coalition values.\(^{20}\) In these models the reason of inefficient outcomes is that the minimum acceptance levels of responders become larger than zero, being equal to the (discounted) value of their continuation payoffs in future negotiations. With rational expectations about responders’ continuation payoffs it may be optimal for proposers to choose inefficient allocations. We point out, however, that inefficiency in these sequential models is very different from that induced by (anticipated) rejections of low offers as observed in our experiments. The anticipation of further negotiations can not play any role in our experiments by definition.

5 Conclusions

In this paper we provide experimental evidence that anticipated and actual reciprocal actions strongly affect coalition formation in multilateral bargaining. In particular, we observe that an overwhelming majority of subjects choose inefficient subcoalitions. They are ready to forego resources and to increase distributional inequality, by excluding other subjects from bargaining. We argue that the undesirable result of inefficiency and social exclusion is unavoidable when responders behave reciprocally and proposers act as (as if) income maximizers. We also compare the predictions of recently developed models of social preferences with our experimental results. We find that the models of Fehr and Schmidt (1999) and Bolton and Ockenfels (2000) capture the empirical regularities surprisingly well, at least in a qualitative sense.

Other experimental studies have shown that under some important institu-
ions reciprocal fairness can be a powerful force that leads to quite efficient and rather fair outcomes. Our study provides the complementary evidence that the same force may lead to precisely opposite consequences, under another important institutional environment. The evidence given in this paper may also shed some new light on the on-going debate about efficiency, inequality-aversion, and reciprocity, in particular, in the context of coalition formation. Needless to say that much more work is necessary for a better understanding of the interaction of reciprocal behavior and economic institutions and its likely consequences.

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A Formal proofs of the predictions of the models of social preferences (Theorems 1-4)

A.1 The Fehr and Schmidt Model (Theorems 1-2)

We now prove Theorems 1 and 2.

A.1.0.1 Responders’ behavior in the three-person coalition. Suppose that $P$ proposes a payoff distribution $x = (V - x_1 - x_2, x_1, x_2)$ of $V$
where \( x_i, 0 \leq x_i \leq V, (i = 1, 2) \) is the material payoff for responder \( R_i \). \( R_1 \) and \( R_2 \) either accept or reject sequentially. We apply backward induction and first analyze the optimal response of \( R_2 \). \( R_2 \)'s utility when accepting \( x = (V - x_1 - x_2, x_1, x_2) \) is

\[
\begin{align*}
    u_2(x) &= x_2 - \frac{\alpha_2}{2} \{|V - x_1 - 2x_2| + |x_1 - x_2|\} \\
    &\quad - \frac{\beta_2}{2} \{|x_1 + 2x_2 - V| + |x_2 - x_1|\}
\end{align*}
\]

and zero when rejecting.

We have to consider four cases with which we deal in turn:

(a) \( (V - x_1)/2 \leq x_2 \) and \( x_1 \leq x_2 \), (b) \( (V - x_1)/2 \leq x_2 \) and \( x_2 < x_1 \), (c) \( x_2 < (V - x_1)/2 \) and \( x_1 \leq x_2 \), (d) \( x_2 < (V - x_1)/2 \) and \( x_2 < x_1 \).

Case (a): \( R_2 \)'s utility reduces to \( u_2(x) = x_2 - (\beta_2/2)(3x_2 - V) \), which is greater than zero for all \( \beta_2 \in [0, 1] \) \((0 \leq x_2 \leq V)\). Hence, the optimal response is to accept the proposal \( x \).

Case (b): \( R_2 \)'s utility reduces to \( u_2(x) = x_2 - (\alpha_2/2)(x_1 - x_2) - (\beta_2/2)(x_1 + 2x_2 - V) \) and the proposal is accepted if and only if \( u_2(x) \geq 0 \), which is equivalent to

\[
x_2 \geq \frac{(\alpha_2 + \beta_2)x_1 - \beta_2 V}{\alpha_2 + 2(1 - \beta_2)}.
\]

Case (c): \( R_2 \)'s utility reduces to \( u_2(x) = x_2 - (\alpha_2/2)(V - x_1 - 2x_2) - (\beta_2/2)(x_2 - x_1) \), and analogously to case (b), the proposal is accepted if and only if

\[
x_2 \geq \frac{-(\alpha_2 + \beta_2)x_1 - \alpha_2 V}{2(1 + \alpha_2) - \beta_2}.
\]

Case (d): \( R_2 \)'s utility \( u_2(x) \) reduces to \( u_2(x) = x_2 - (\alpha_2/2)(V - 3x_2) \), and \( R_2 \) accepts the proposal if and only if \( u_2(x) \geq 0 \) or equivalently

\[
x_2 \geq \frac{\alpha_2}{2 + 3\alpha_2} V.
\]

Since the optimal response of \( R_1 \) can be proved in an equivalent way this proves Part 1 of Theorem 1.

\( R_2 \) is indifferent between accepting and rejecting the proposal when the equality holds, and the proposer’s equilibrium condition induces the acceptance of \( R_2 \) in the case of equality.
A.1.0.2 Responder’s behavior in a two-person coalition. Consider a two-person coalition, with coalition value $v_i$, of $P$ and an $R_i$ ($i = 1, 2$). $R_i$’s behavior can be analyzed by setting $V = v_i$ and $x_j = 0$ ($j \neq i$) in his optimal response in cases (a) and (c) above. It follows then immediately that the optimal response to a proposal $x = (x_P, x_i, x_j)$ ($x_P + x_i = v_i$, $x_j = 0$, $0 \leq x_i \leq v_i$) in the two-person coalition is to accept it if and only if

$$x_i \geq \frac{\alpha_i}{2(1 + \alpha_i)} - \beta_i v_i.$$ 

This proves Part 2 of Theorem 1.

A.1.0.3 Proposer’s behavior in the three-person coalition. Let the optimal responses of $R_1$ and $R_2$ be given and assume without loss of generality $\alpha_1 \geq \alpha_2$. Note that the region of all proposals $x = (V - x_1 - x_2, x_1, x_2)$ accepted by both responders can be divided into six subregions (for a graphical illustration see Figure A.1):

1. $x_P \leq x_1 \leq x_2$, $\frac{(\alpha_1 + \beta_1)x_2 - \beta_1 V}{\alpha_1 + 2(1 - \beta_1)} \leq x_1$,
2. $x_1 \leq x_P \leq x_2$, $\frac{\alpha_2}{2 + 3\alpha_2} \leq x_1$,
3. $x_1 \leq x_2 \leq x_P$, $\frac{\alpha_1}{2 + 3\alpha_1} \leq x_1$,
4. $x_2 \leq x_1 \leq x_P$, $\frac{\alpha_2}{2 + 3\alpha_2} \leq x_2$,
5. $x_2 \leq x_P \leq x_1$, $\frac{2 + 3\alpha_2}{\alpha_2} \leq x_2$,
6. $x_P \leq x_2 \leq x_1$, $\frac{2 + \beta_2}{\alpha_2} \leq x_2$.

The proposer $P$’s utility for $x = (x_P, x_1, x_2)$, $x_P = V - x_1 - x_2$, $0 \leq x_1$, $x_2 \leq V$, is given by

$$u_P(x) = x_P - \frac{\alpha_P}{2} \left\{ |x_1 - x_P| + |x_2 - x_P| \right\} - \frac{\beta_P}{2} \left\{ |x_P - x_1| + |x_P - x_2| \right\}.$$ 

We next characterize the optimal proposal in each subregion.

Regions (1) and (6): In these cases, the proposer’s utility $u_P(x)$ from an accepted proposal $x = (x_P, x_1, x_2)$ reduces to

$$u_P(x) = x_P - \frac{\alpha_P}{2} (x_1 - x_P + x_2 - x_P)$$
Fig. A.1. Regions of Acceptance in Three-Person Coalition ($\alpha_1 \geq \alpha_2$)

$$= (1 + \alpha_P) V - \left(1 + \frac{3\alpha_P}{2}\right)(x_1 + x_2).$$

Since $u_P(x)$ is decreasing in both $x_1$ and $x_2$ it follows that the optimal proposal in these regions is given by $x_P = x_1 = x_2 = V/3$ leading to the utility $u_P = V/3$.

Region (2): In this case the proposer’s utility is given by

$$u_P(x) = x_P - \frac{\alpha_P}{2} (x_2 - x_P) - \frac{\beta_P}{2} (x_P - x_1)$$

$$= \left(1 + \frac{\alpha_P}{2} - \frac{\beta_P}{2}\right) V - A_1 x_1 - A_2 x_2,$$

with $A_1 := 1 + \alpha_P/2 - \beta_P > 0$ and $A_2 := 1 + \alpha_P - \beta_P/2 > 0$. Since $A_1, A_2 > 0$ the optimal choice lies on the line $x_2 = x_P$ and depends the values of $A_1, A_2$. It follows that the optimal proposal is given by

$$x_P = x_1 = x_2 = V/3 \quad \text{if} \quad 2A_1 \leq A_2, \text{ i.e. } 2/3 \leq \beta_P < 1$$

$$x_P = \frac{1 + \alpha_1}{2 + 3\alpha_1} V = x_2, \quad x_1 = \frac{\alpha_1}{2 + 3\alpha_1} V, \quad \text{if} \quad 2A_1 > A_2, \text{ i.e.}, \ 0 \leq \beta_P < 2/3,$$
leading to a utility of

\[ u_P(x) = \begin{cases} \frac{V}{3} & \text{if } 2/3 \leq \beta_P < 1 \\ x_P - \frac{\beta_P}{2} (x_P - x_1) = \frac{2 + 2\alpha_1 - \beta_P}{2(2 + 3\alpha_1)} V & \text{if } 0 < \beta_P < 2/3. \end{cases} \]

For the latter \( u_P(x) > V/3 \) holds.

Region (5): Similar to the above case the proposer’s utility is given by

\[ u_P(x) = x_P - \frac{\alpha_P}{2} (x_1 - x_P) - \frac{\beta_P}{2} (x_P - x_2) = \left(1 + \frac{\alpha_P}{2} - \frac{\beta_P}{2}\right) V - A_2 x_1 - A_1 x_2, \]

with \( A_1 \) and \( A_2 \) as above. Now the optimal proposal lies on the line where \( x_1 = x_P \) and is given by

\[ x_P = x_1 = x_2 = \frac{V}{3} \quad \text{if } 2A_1 \leq A_2, \text{i.e., } 2/3 \leq \beta_P < 1, \]

\[ x_P = x_1 = \frac{1 + \alpha_2}{2 + 3\alpha_2} V, x_2 = \frac{\alpha_2}{2 + 3\alpha_2} V \quad \text{if } 2A_1 > A_2, \text{i.e., } 0 \leq \beta_P < 2/3, \]

giving a utility

\[ u_P(x) = \begin{cases} \frac{V}{3} & \text{if } 2/3 \leq \beta_P < 1, \\ x_P - \frac{\beta_P}{2} (x_P - x_2) = \frac{2 + 2\alpha_2 - \beta_P}{2(2 + 3\alpha_2)} V & \text{if } 0 < \beta_P < 2/3. \end{cases} \]

Regions (3) and (4): In these cases, the proposer’s utility reduces to

\[ u_P(x) = x_P - \frac{\beta_P}{2} (2x_P - x_1 - x_2) = (1 - \beta_P)V + \left(\frac{3\beta_P}{2} - 1\right) (x_1 + x_2). \]

If \( \beta_P > 2/3 \), it follows that the equity proposal is optimal, and if \( \beta_P < 2/3 \) the optimal proposal lies on the south-west boundary of region (4). Using this it follows that the optimal proposal is

\[ x_P = x_1 = x_2 = \frac{V}{3} \quad \text{if } 2/3 \leq \beta_P < 1, \]

\[ x_P = V - x_1 - x_2, \]

\[ x_1 = -\frac{\alpha_1 + \beta_1}{2(1 + \alpha_1)} x_2 - \alpha_1 V, x_2 = \frac{\alpha_2}{2 + 3\alpha_2} V \quad \text{if } 0 \leq \beta_P < 2/3, \]
with the following corresponding utilities:

if $2/3 \leq \beta_p < 1$ then

$$u_P(x) = V/3, \quad (A.1)$$

if $0 \leq \beta_p < 2/3$ then

$$u_P(x) = \left\{1 - \beta_p - \frac{2 - 3\beta_p}{2 + 3\alpha_2} \left(\alpha_2 + \frac{\alpha_1 - \alpha_2}{2 + 2\alpha_1 - \beta_1}\right)\right\} V. \quad (A.2)$$

Comparing utility from the optimal proposals across the six acceptance regions implies that the equilibrium proposal $(x_P, x_1, x_2)$ in the three-person coalition is characterized as follows (see also points $P_1$ and $P_2$ in Figure A.1):

1. If $2/3 \leq \beta_p < 1$, then the equilibrium proposal satisfies $x_P = x_1 = x_2 = V/3$.
2. If $0 \leq \beta_p < 2/3$, then the equilibrium proposal satisfies

$$x_1 = -\frac{(\alpha_1 + \beta_1)x_2 - \alpha_1 V}{2(1 + \alpha_1) - \beta_1}, x_2 = \frac{\alpha_2}{2 + 3\alpha_2} V,$$

with $x_2 \leq x_1$, $x_2 < x_P$ ($x_1 = x_2$ when $\alpha_1 = \alpha_2$).

**A.1.0.4 Proposer’s behavior in a two-person coalition.** Without loss of generality, consider a two-person coalition of proposer $P$ and responder $R2$. The proposer’s utility for a payoff distribution $x = (v_2 - x_2, 0, x_2)$ is given by

$$u_P(x) = v_2 - x_2 - \frac{\alpha_P}{2} |2x_2 - v_2|^+ - \frac{\beta_P}{2} \left\{v_2 - x_2 + |v_2 - 2x_2|^+\right\}.$$ 

We have to consider the following two cases:
(i) $v_2/2 \leq x_2 \leq v_2$ and (ii) $(\alpha_2 v_2)/(2(1 + \alpha_2) - \beta_2) \leq x_2 \leq v_2/2$.

Case (i): In this case the proposer’s utility is given by

$$u_P(x) = \left(1 + \frac{\alpha_P}{2} - \frac{\beta_P}{2}\right) v_2 - \left(1 + \alpha_P - \frac{\beta_P}{2}\right) x_2.$$ 

Since $1 + \alpha_P - \beta_P/2 > 0$ this implies an optimal proposal $x_P = x_2 = v_2/2$ and the proposer can obtain the utility $u_P(x) = (2 - \beta_P)(v_2/4)$.

Case (ii): Here the proposer’s utility is

$$u_P(x) = (1 - \beta_P)v_2 + \left(\frac{3\beta_P}{2} - 1\right) x_2.$$
In view of this, the proposer’s optimal choice is

\[ x_P = x_2 = \frac{v_2}{2} \quad \text{if} \quad \frac{2}{3} \leq \beta_P < 1, \]

\[ x_P = \frac{2 + \alpha_2 - \beta_2}{2(1 + \alpha_2) - \beta_2} v_2, \quad x_2 = \frac{\alpha_2}{2(1 + \alpha_2) - \beta_2} v_2 \quad \text{if} \quad 0 \leq \beta_P < \frac{2}{3}, \]

with the following corresponding utilities:

if \( 2/3 \leq \beta_P < 1 \) then

\[ u_P(x) = \frac{2 - \beta_P}{4} v_2, \quad \text{(A.3)} \]

if \( 0 \leq \beta_P < 2/3 \) then

\[ u_P(x) = \left\{ 1 - \beta_P - \frac{2 - 3\beta_P}{2} \frac{\alpha_2}{2(1 + \alpha_2) - \beta_2} \right\} v_2. \quad \text{(A.4)} \]

A.1.0.5 The proposer’s coalitional choice.

Consider first the case \( 2/3 \leq \beta_P < 1 \). Comparing the proposer’s maximal utility in the three-person coalition (A.1) with the maximum utility in attainable in the two-person coalition (A.3) implies that the optimal choice is the three-person coalition in this case, independent of the two-person coalitional values \( v_i \) \( (i = 1, 2) \).

Now consider the case \( 0 \leq \beta_P < 2/3 \). In what follows we assume \( v_1 = v_2 \), denoted by \( v \). With the help of (A.4) (replacing the index 2 with \( i \)) it is easy to show that the utility \( u^2_P \) a proposer can maximally attain when choosing \( R2 \) as bargaining partner in a two-person coalition is at least as large as her utility \( u^1_P \) with \( R1 \) as partner, i.e. \( u^1_P \leq u^2_P \) if and only if

\[ \frac{\alpha_1}{2 - \beta_1} \geq \frac{\alpha_2}{2 - \beta_2}. \]

In the subgame perfect equilibrium point, the proposer chooses the three-person coalition if and only if her maximal utility in the three-person coalition is at least as large as her maximal utility \( u^i_P \) \( (i = 1, 2) \) attainable in the two-person coalition. By comparing (A.2) with (A.4) one can show that the three-person coalition is chosen if and only if

\[ VK_1 \geq v, \quad \text{where} \quad K_i = \frac{1 - \beta_P - \frac{2 - 3\beta_P}{2 + \alpha_2 (\alpha_2 + \frac{\alpha_1 - \alpha_2}{2 + 2\alpha_1 - \beta_1})}}{1 - \beta_P - \frac{2 - 3\beta_P}{2} \frac{\alpha_i}{2 + 2\alpha_i - \beta_i}}, \quad i = 1, 2. \]

This proves Theorem 2.
A.2 The Bolton and Ockenfels Model (Theorems 3-4)

We now prove Theorems 3 and 4.

A.2.0.6 Responder’s behavior. Consider the three-person coalition and suppose that $P$ proposes a payoff distribution $x = (V - x_1 - x_2, x_1, x_2)$, $0 \leq x_i \leq V$; $(i = 1, 2)$, where $x_i$ is the offer to responder $R_i$. By backward induction, we first investigate the optimal response of $R_2$. Let $\sigma_2 = x_2/V$ be the relative share of $R_2$. The responder accepts the proposal if and only if the motivation value from accepting is at least as high as the motivation value from a rejection, that is, if and only if

$$\nu_{R_2}(V \sigma_2, \sigma_2) \geq \nu_{R_2}(0, 1/3).$$

(A.5)

Similarly, responder $R_1$ accepts the proposal if both (A.5) and

$$\nu_{R_1}(V \sigma_1, \sigma_1) \geq \nu_{R_1}(0, 1/3).$$

(A.6)

hold.\(^{22}\)

Let

$$p_i(V, \sigma_i) := \text{Prob}\{s_i | s_i(V) \geq \sigma_i\}$$

(A.7)

denote the probability that a randomly selected responder $R_i$ $(i = 1, 2)$ rejects a proposal $x = (V - x_1 - x_2, x_1, x_2)$.

We are now ready to prove statement 1.(a) of Theorem 3. From Assumptions 2 and 3 it follows that $\nu_i(V \sigma_i, \sigma_i)$ is strictly increasing in $\sigma_i$ on the interval $[0, 1/3[$ since $\partial \nu_i/\partial \sigma_i = V \cdot \nu_{i1}(V \sigma_i, \sigma_i) + \nu_{i2}(V \sigma_i, \sigma_i) > 0$.

For any $s_i > 0$, responder $R_i$ rejects the proposal yielding $\sigma_i = 0$ since $\nu_i(0, 0) < \nu_i(V s_i, s_i) = \nu_i(0, 1/3)$. Thus, $p_i(V, 0) = 1$.

Assumption 2 implies $\nu_i(V/3, 1/3) \geq \nu_i(0, 1/3)$. Therefore, responder $R_i$ accepts such a proposal. Thus, $p_i(V, 1/3) = 0$.\(^{23}\) Assumptions 3 and 4 imply $\nu_i(V \sigma_i, \sigma_i) \geq \nu_i(0, 1/3)$ for all $\sigma_i \in [1/3, 1]$.

\(^{22}\)Remark that $R_1$ is indifferent between accepting and rejecting when $R_2$ is expected to reject the proposal. That is, when (A.5) does not hold. However, if (A.5) indeed does not hold, $R_1$’s response is inessential in the sense that the proposal is rejected by $R_2$, even if $R_1$ accepts it. Since we are primarily interested in the case when a proposal is agreed by both responders, we assume that $R_1$ accepts the proposal if and only if (A.6) holds.

\(^{23}\)Note that, even if the equality holds, Assumption 2 guarantees the acceptance.
That $p_i(V, \sigma_i)$ strictly decreases in $\sigma_i$ on $]0, 1/3[$ follows directly from (A.7).
That the rejection probability (given an offer to the responder) does not depend on the offer to the other responder (Theorem 3.1(b)) is clear from the definition of the motivation function.

Next consider two-person coalitions. Similarly to the three-person coalition, let $p(v, \sigma_i)$ denote the probability that a randomly selected responder $R_i$ rejects a proposal $x = (v - x_i, x_i)$. Statement 2(a) of Theorem 3 can be proved in the same way as the corresponding statement for the three-person coalition.

We now show that, when fixing $\sigma_i \in ]0, 1/3[$, $p_i(v, \sigma_i)$ is non-increasing in $v$. To see this, differentiate the threshold function (3) with respect to $v$ to get

$$s_i'(c) = -s_i \nu_i(c s_i, s_i) + \nu_i(1) \leq 0. \quad (A.8)$$

Together with the assumptions on the threshold value function and (A.7) this implies that $p_i(v, \sigma_i)$ is non-increasing in $v$. This proves Theorem 3.

A.2.0.7 Proposer’s behavior. The formulations

$$E \nu_P(v, \sigma_P) = (1 - p_i(v, \sigma_i)) \cdot \nu_P(v \sigma_P, \sigma_P), \quad \sigma_i = 1 - \sigma_P, i = 1, 2,$n$$
$$E \nu_P(V, \sigma_P) = (1 - p_1(V, \sigma_1)) \cdot (1 - p_2(V, \sigma_2)) \cdot \nu_P(V \sigma_P, \sigma_P),$$

$$\sigma_i = 1 - \sigma_j - \sigma_P, \quad i = 1, 2, i \neq j$$

of the expected motivation values follow directly from the definition of the motivation function and the normalization $\nu_P(0, 1/3) = 0$, where $\sigma_P$ is the proposer’s relative share and $p_i(v, \sigma_i)$ is the probability that $R_i$ rejects the proposal.

Assuming an interior solution and denoting the maximum value of the proposer’s expected motivation function in a two-person coalition by $E \nu_P^*(v)$, gives

$$E \nu_P^*(v) = (1 - p_i(v, \sigma_i^*)) \cdot \nu_P(v \sigma_P^*, \sigma_P^*), \quad \sigma_i^* = 1 - \sigma_P^*.$$

(A.9)

By the envelope theorem we have

$$\frac{dE \nu_P^*(v)}{dv} = \partial E \nu_P(v, \sigma_P) = -\partial p_i(v, \sigma_P) \cdot \sigma_P \cdot \nu_P^1 \geq 0,$$

where the inequality follows from Assumption 1 and the monotonicity of $p_i$ in $v$. This proves Theorem 4.
References

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