

Increasing Marginal Costs and Satiation in the Private Provision of Public Goods: Group Size and Optimality Revisited

Masayoshi Hayashi^{*}, and Hiroshi Ohta^{}**

ABSTRACT

It is widely recognized that the degree of inefficiency in the voluntary provision of a public good increases with the group size of an economy. However, we find that only a slight modification in the conventional assumptions gives rise to a profound difference in outcome. In particular, we show that there is a case where the Nash equilibrium provision and the efficient provision will converge as the size of an economy grows. To show this we assume individuals face increasing marginal cost of voluntary provision and their preference function has a finite satiation point.

Key Words: voluntary provision of public goods, group size, increasing MC, satiation

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^{*} Associate Professor, Department of Economics and School of International and Public Policy
Hitotsubashi University, Tokyo, Japan

^{**} Professor of Economics, School of International Politics, Economics and Communication, Aoyama
Gakuin University, Tokyo, Japan.

1 Introduction

One of the main concerns in the literature on the voluntary provision of public goods is the impact the size of an economy may have on the provision level of a public good. In particular, it has been widely recognized since the seminal work by Olson (1965) that the size of a voluntary economy is inversely related to the degree of efficiency in provision of public goods. For example, Chamberlin (1974) and McGuire (1974) showed that, as the group size increases in an economy with identical individuals, the total provision of a public good increases, but per capita contributions fall. In a more general setting of a heterogeneous economy, Andreoni (1988) demonstrated that, as the size of the economy increases to infinity, the proportion of contributors decreases to zero while total provision approaches a finite level. These findings may imply that an inverse relation holds between group size and efficiency inasmuch as the summand of individual marginal rate of substitution of the public good explodes as the size grows to infinity. In fact, the inverse relation can indeed be established, for example, in a homogeneous economy with Cobb-Douglas preferences (Mueller 1989) or with quasi-linear preferences (Cornes and Sandler 1985, 1986).

In these studies, the marginal costs of the voluntary contributions are assumed to be uniform among individuals and independent of the contribution levels.¹ Moreover, the results are predicated on the standard assumption of non-satiation. In this paper, however, we relax both of the assumptions. We instead assume that individuals get satiated at some level of public good consumption and their marginal costs are increasing in their individual contributions. With these alternations, we will show in an identical economy that the Nash equilibrium of a public good provision converges upon an optimal level, as the group size *increases*. The intuition behind this is rather simple. We will show later in the paper that, as group size increases, (a) the optimal provision of a public good increases, and (b) individual contributions in a Nash equilibrium decrease. Then, marginal costs (and marginal rate of substitutions) individuals face in both allocations will become negligible as group size increases. If one does not get satiated, this makes both Nash and optimal provision levels infinite in quantities. But if it does, these two values are bounded below the satiation level, and an increase in group size

¹ Another strand of the literature does allow for different prices among economic agents (i.e., Schulyer 1982, Ihori 1996, Boadway and Hayashi 1999). However, since these models are developed in the context of international public goods, the group size (i.e., the number of countries) is assumed to be fixed.

locates the two allocations closer to the level since the two marginal values will converge in the neighborhood of the satiation level. As such, while an optimal provision is always larger than a Nash provision, the two will converge upon the satiation provision level as group size increases. We then see that Nash and optimal provisions will be located closer in a larger group, and moreover individual cost burdens approaching minimal.

The structure of the paper is as follows. Section 2 sets up the model, defends our assumptions of utility satiation and increasing marginal costs, and derives the results with a general class of preferences. Section 3 works on an example by specifying the model, provides intuitions, and presents some numerical results. Section 4 addresses the applicability of our model by elaborating on the merits and limitations of our key assumptions. Section 5 concludes the paper.

2 Group size and optimality

2.1. The Setup

2.1.1. Preferences

Suppose that an economy consists of n identical individuals with preferences:

$$U = U(x, G) \tag{1}$$

where x is a private good (numeraire) and G is a public good. The standard assumptions hold *except* that there exists a level of public good $G = \hat{G} > 0$ such that

$$\frac{\partial U(x, G)}{\partial G} = 0 \quad \text{if} \quad G \geq \hat{G} \tag{2}$$

We assume that the satiation level \hat{G} is independent of the consumption of x for convenience.² We denote the marginal rate of substitution $(\partial U / \partial G) / (\partial U / \partial x)$ by $\pi(x, G)$, which implies that $\pi(x, G) = 0$ for $G \geq \hat{G}$.

There should be no *a priori* reason to assume that there will never be satiation from public goods.³ For example, consider a shopping area defaced by hundreds of ugly

²In fact, this assumption may not be necessary. First, if the satiation level is decreasing in x , it is evident from Figure 1 in Section 2.3 that our results still hold. Second, if it is increasing in x , the optimal level of G may hit the varying satiation level as a corner solution, well before the Nash counterpart does so as group size increases. However, in the limits, both converge to the satiation point realized at $x = w$.

³While the implication of the assumption of satiation seems to be rarely discussed in the literature, Ohta (1993) utilizes it for purposes not directly related to the present paper.

graffiti. If shopkeepers want the area restored, erasing the graffiti (i.e., restoring a clean environment) is a public good. Once the clean environment is restored (and new graffiti prevented), no further cleaning would be needed if what shopkeepers care about is restoring a once clean area to the state in which it was originally. The analogous argument should apply to a number of environmental issues. Another example is the case of altruistic giving. Voluntary donations to the poor are a public good among altruists who would ‘feel better’ when the poor consume more. But the altruism may disappear, for example, if the ‘poor’ were given so much and consume more than altruists do. In other words, there is a level of donation (i.e., a public good) at which donors would not get additional benefits out of giving. We then see that utilities of voluntary contributors tend to be satiated in typical cases of the private provision of public goods (e.g., environmental problems and altruistic donation).

2.1.2. Technology

We now turn to the constraints of contributors. An individual voluntarily contributes g of a public good. Since public goods are non-excludable and non-rival, everyone can consume the sum of individual contributions $G = \sum g$. One’s contribution g eats up his/her endowment w by $c(g)$ and the rest is set aside for private consumption x . Thus, the individual constraint is

$$w = x + c(g) \tag{3}$$

While the standard assumption on $c(g)$ is that its marginal variations $c'(g) > 0$ are constant, we assume that they are increasing in g ($c''(g) > 0$) and approach zero as $g \rightarrow 0$ ($c'(0) = 0$).

Our assumption is natural when the good in question is a ‘household product’ such as a spring-cleaning, neighborhood security, even at a flight. Examples may abound. If we continue with the said example of graffiti in a shopping area, we see that erasing the graffiti (i.e., providing a clean environment) is a public good produced by shopkeepers in exchange for their private resources they could have used for other private purposes. Each storekeeper can erase the graffiti on his own store. But it becomes increasingly hard for him to erase all the graffiti in the area by himself since he is likely to find it increasingly boring and get exhausted soon enough. Similar examples would be found in the private contributions of materials to communal infrastructure

projects in developing countries. Such cases include the irrigation projects in the Philippines (Kikuchi et al. 1978) and the Harambee movement in Kenya (Wilson 1992).

2.2. Group size and contributions

2.2.1. Nash equilibrium contributions

We assume that there are n homogeneous individuals. Each individual chooses x and g to maximize (1) subject to (3), taking contributions by other members as given. With this Nash behavior, the following condition must hold in equilibrium

$$\pi(w - c(\bar{G}/n), \bar{G}) = c'(\bar{G}/n) \quad (4)$$

if solutions are interior. Variables with upper bars refer to equilibrium values. The expression takes advantage of the homogeneity assumption that yields constraints $ng = G$ and $nw = nx + nc(g)$. We can then characterize the effect of group size n on equilibrium provision \bar{G} , by totally differentiating (4) with respect to \bar{G} and n . After proper arrangement, we obtain the group size-elasticity of equilibrium provision

$$\eta \equiv \frac{d\bar{G}}{dn} \frac{n}{\bar{G}} = \frac{\bar{c}'' + \bar{c}'(\partial\bar{\pi}/\partial x)}{\bar{c}'' + \bar{c}'\partial\bar{\pi}/\partial x - n\partial\bar{\pi}/\partial G} \quad (5)$$

where bars indicate equilibrium values. Since $\partial\pi/\partial G < 0$ and $\partial\pi/\partial x > 0$ if both goods are normal and the equilibrium is interior, (5) along with our assumptions ($c' > 0$, $c'' > 0$) shows that $0 < \eta < 1$, i.e., equilibrium provision \bar{G} is increasing in group size n and the elasticity is less than unity. With $\bar{g} = \bar{G}/n$, \bar{g} will then be characterized by

$$\xi \equiv \frac{d\bar{g}}{dn} \frac{n}{\bar{g}} = \frac{d(\bar{G}/n)}{dn} \frac{n}{\bar{g}} = \frac{n d\bar{G}/dn - \bar{G}}{n^2} \frac{n}{\bar{g}} = \eta - 1 \quad (6)$$

Since $0 < \eta < 1$, $\xi < 0$. Individual contributions decrease as group size increases.

2.2.2. Optimal provisions

The optimal allocation is obtained as a solution to

$$\max_{x, G} \{U(x, G) \mid w = x + c(G/n)\}$$

which yields a variant of the Samuelson condition:

$$\pi(w - c(G^*/n), G^*) = \frac{c'(G^*/n)}{n} \quad (7)$$

where asterisks indicates optimal values. We can relate group size n to the optimal level of public good G^* , by totally differentiating (7). After proper arrangements, we obtain

the group size elasticity of optimal provision

$$\eta^* \equiv \frac{dG^*}{dn} \frac{n}{G^*} = \frac{c''^*/n + c'^* \partial \pi^* / \partial x + \pi^* / G^*}{c''^*/n + c'^* \partial \pi^* / \partial x - n \partial \pi^* / \partial G} \quad (8)$$

where asterisks indicates optimal values. This result shows $\eta^* > 0$: optimal provision G^* is increasing in group size n .

2.3. Group size and discrepancies between Nash and optimal provisions

Given the results above, we will show that a sufficiently large group size alleviates the suboptimality of the private provision of a public good. Figure 1 illustrates choice sets of identical contributors for a given value of n , with boundary $x = w - c(G/n)$. The figure shows five choice sets with different group sizes: $n_0 < n_1 < n_2 < n_3 < n_4$ where $n_0 = 1$ and $n_4 \rightarrow \infty$. The choice set, which is depicted on per capita basis, expands as n increases, since an increase in n makes the slope of the boundary $[c'(G/n)/n]$ flatter for a given value of G .

While it is trivial to see that both Nash and optimal allocation coincides for $n=1$, we need explain why Nash allocations (shaded dots) and optimal allocations (black dots) are positioned as illustrated in Figure 1 for $n \geq 2$. First, since the two allocations share the same choice set, they are located somewhere on the same boundary $x = w - c(G/n)$. Second, their locations are different since we know $G^* > \bar{G}$ for $n \geq 2$. Noticing these two properties, we then see that Nash allocations are located to the upper left of corresponding optimal allocations.

Notice also that marginal costs in optimal allocations $[c'(G/n)/n]$ are lesser than those in corresponding Nash allocations $[c'(G/n)]$, since indifference curves are tangent to the boundary of the choice set in optimal allocations [see Equation (7)] and the marginal rate of substitution equals $c'(G/n)$ in a Nash provision where the public good is under-provided ($G^* > \bar{G}$). As the relative positions of indifference curves in Figure 1 illustrate, this result is consistent with the facts that the Nash allocations are suboptimal.

Two trajectories of Nash and optimal allocations are drawn for varying values of n such that they satisfy the following analytical results: (i) $d\bar{G}/dn > 0$, (ii) $dG^*/dn > 0$, (iii) $G^* > \bar{G}$ and (iv) $\pi(w - c(\bar{G}/n), \bar{G}) = c'(\bar{G}/n)/n < \pi(w - c(G^*/n), G^*) = c'(G^*/n)$. Note that the two trajectories coincide as $n \rightarrow \infty$. In other words, the suboptimality of Nash equilibrium will be alleviated as group size becomes large enough. The logic behind this is explained as follows.

First, consider optimal allocations. As $n \rightarrow \infty$, the boundary of the choice set $[x=w-c(G/n)]$ approaches a horizontal line emanating from w . If one does *not* get satiated, the marginal rate of substitutions, albeit decreasing in G , will never be zero. As such, the flattening of the constraint makes G^* an infinite quantity. On the other hand, if utility gets maximized at \hat{G} , indifference curves are also now horizontal for $G \geq \hat{G}$. Optimal G^* and equilibrium \hat{G} then converge as the constraint becomes flatter.

Second, turn to Nash allocations. Individuals face marginal costs $c'(g)$ when contributing to a public good. We have shown that $d\bar{g}/dn < 0$ which implies that an increase in group size leads to a reduction in marginal costs. A larger group size thus again makes individuals' budget lines flatter. In addition, since we assume that $c'(0)=0$, the budget line will approach the horizontal line as $n \rightarrow \infty$. Then, if utility gets maximized at \hat{G} , Nash provision level \bar{G} also approaches \hat{G} .

We then see that two trajectories coincide at (w, \hat{G}) in Figure 1. While \bar{G} never exceeds G^* for a given level of n , the difference between the two will become negligible as group size increases. This is because a large group makes marginal costs in the two allocations very close in the neighborhood of \hat{G} . We then see that a sufficiently large group size makes the voluntary provision less suboptimal.⁴

– Figure 1 –

3. An example

Let us examine a more specific case of what we have discussed in the previous section. Here, we specify preferences quadratic in G :

$$U(x, G) = x + aG - \frac{b}{2}G^2 \quad (9)$$

which satiates at $G = \hat{G} \equiv a/b$ where a and b are positive parameters.⁵ On the other hand,

⁴ While 'group size' refers to the number of individuals in a group (n) in the current analysis, it may also refer to the amount of total endowment (nw). The effect of the total endowment is equivalent to the effect of per capita endowment (w) in our homogeneous case, since here we consider the effect when n is held constant. Of course, an increase in w does not yield any effects analogous to what n yields, because Figure 1 can in principle be replicated for an arbitrary value of w . For the concept of size, see Boadway and Hayashi (1999) and Shrestha and Feehan (2003).

⁵ This specification makes utility to decrease in G for $G > a/b$. While this is innocuous for our discussion, we may also additionally assume that $U = x + aG - bG^2/2$ for $G < a/b$ and $U = x + a^2/(2b)$ for $G \geq a/b$ so that

the cost for the voluntary contribution is

$$c(g) = \frac{\theta}{2} g^2 \quad (10)$$

which is consistent with our assumptions: $c'(g) = \theta g > 0$, $c'' = \theta > 0$, $c'(0) = 0$. The budget constraint is then $w = x + \theta g^2/2$.

With marginal rate of substitutions $a - bG$ and marginal costs θg , this setup yields the first order condition $a - bG = \theta g$. With symmetric assumption $ng = G$, we then obtain

$$\bar{g} = \frac{a}{nb + \theta} \quad (11)$$

$$\bar{G} = n\bar{g} = \frac{a}{b + \theta/n} \quad (12)$$

These conform to the analysis in the previous section that (a) $d\bar{g}/dn < 0$, (b) $d\bar{G}/dn < 0$, and (c) $\bar{G} \rightarrow a/b$ as $n \rightarrow \infty$. On the other hand, the optimal public good provision G^* is given as a solution to $a - bG = G\theta/n^2$, an analogue of (7) with (9) and (10):

$$G^* = \frac{a}{b + \theta/n^2} \quad (13)$$

This is also consistent with the results in the previous section that (a) $dG^*/dn > 0$ and (b) $G^* \rightarrow a/b = \hat{G}$ as $n \rightarrow \infty$. We then use (12) and (13) to obtain

$$\frac{G^*}{\bar{G}} = 1 + \frac{n-1}{(b/\theta)n^2 + 1}. \quad (14)$$

This ratio exceeds unity when there are more than one individual, implying equilibrium contributions are suboptimal. But it also shows that the ratio converges to unity as n becomes large enough, substantiating our analysis in the previous section. In other words, the discrepancies between the Nash and optimal provisions [i.e., the second term in the right-hand-side expression of (14)] will be minimal when group size is large enough.

This specific example helps simplify the mechanism of our analysis in the previous section. Equation (14) shows for a given level of n that the discrepancy will be reduced as ratio b/θ becomes larger. In other words, the ratio can be associated with ‘closeness’ between Nash and optimal allocations. First, because marginal utility decreases faster to the extent that b is larger, coefficient b is associated with the speed

utility will not decrease after the satiation.

toward satiation. As we have seen in the previous section, Nash and optimal allocations are closer to the extent that they are closer to the satiation point, which allows us to see that a large value of b also implies ‘closeness’ between the two allocations. Second, we have seen that such ‘closeness’ is also affected by the difference between marginal costs in the two allocations. Since the difference is $\theta(\bar{G}/n - G^*/n^2)$, a lower value of θ makes its value smaller if the other values are held constant. This allows us to associate a smaller value of θ with the ‘closeness.’

Equation (14) also implies that the discrepancy initially increases, and soon to be followed by a reversal when the size hits critical value $n'=1+(1+\theta/b)^{1/2}$.⁶ This formula for the critical value shows that a larger value of b/θ makes its value smaller, a result consistent with our interpretation of b/θ .

Figure 2 shows numerical examples for the second term in (14) calculated with $b/\theta = \{1/1000, 1/500, 1/100, 1/50, 1/10, 1.0, 1.5\}$. Note that since those values reflect the total effect of the speed to satiation b and closeness between the two marginal costs $1/\theta$, we cannot independently associate either of the two with the value of the ratio b/θ . The numerical results are consistent with the discussion above. With the smallest ratio of 1/1000, the decreasing discrepancy starts at a size of $n=33$ and its speed is slow. The highest discrepancy ratio $(G^*/\bar{G} - 1)$ is around 15, which is quite large. Still, its degree becomes smaller as the ratio gets larger, albeit slowly. If the ratio is relatively larger, say, more than 0.1, the difference becomes small at a relatively smaller number of the group size. For example, the discrepancies become less than 10% (i.e., 0.10) at $n=66$ for $b/\theta=0.1$, $n=50$ for $b/\theta=0.2$ and $n=9$ for $b/\theta=1.0$, and $n=6$ for $b/\theta=1.5$.

– Figure 2 –

4. Reservations

There are two key factors for our analysis. The first key is the assumption of satiation, which holds the optimal level of public good consumption *below* some finite

⁶ Since population is discrete, we approximate the critical value by treating n as if it were continuous. The value is given as a solution to a quadratic equation $-(b/\theta)n^2 + 2(b/\theta)n + 1 = 0$, which in fact yields two solutions $n'=1-(1+\theta/b)^{1/2}$ and $n'=1+(1+\theta/b)^{1/2}$. However, since the former is always less than unity, only the latter applies to our example.

value even when its marginal costs approach nil. Otherwise, our result will not hold. A concrete example may help. Assume that $U=x+aG^\alpha/\alpha$ and $c=cg^\gamma/\gamma$ where marginal utility $aG^{\alpha-1}$ is decreasing but remains strictly positive with $0<\alpha<1$, and marginal cost $cg^{\gamma-1}$ is increasing in contribution with $\gamma>1$. The optimal and Nash ratio will then be obtained as $G^*/\bar{G}=n^{1/(\alpha-\gamma)}$. This shows that however large the degree of increasing marginal costs (γ) may be, the ratio never approaches unity even if $n\rightarrow\infty$. Rather, as in the standard case, the optimal quantity deviates from the equilibrium quantity.

The second key factor is that ‘slopes’ for the Nash and optimal cases approach zero as $n\rightarrow\infty$. The crucial assumption here is that marginal costs are increasing. Furthermore, it is also required that marginal costs depend *only* on *individual* contributions g [$c=c(g)$] and approach zero as $g\rightarrow 0$ [$c'(0)=0$]. Otherwise, a larger group size may not make the ‘slope’ flat enough for the Nash allocations to attain satiation. While the optimal case only requires increasing marginal costs, the Nash case additionally requires both $c=c(g)$ and $c'(0)=0$ for the satiety condition. The former [$c=c(g)$] is needed for $dg/dn<0$ and so is the latter [$c'(0)=0$] for the budget line to become horizontal in the limit.

These assumptions on marginal costs are plausible when voluntary contributions are self-produced out of individual endowments. However, they may not be so when contributions are bought out of a market and/or are made in cash. First, consider the case where the good to be contributed is bought with a unit price p in a market. Then, unit price may well be dependent on G , the aggregate amount of a public good bought, $p=p(G)$ so that the cost of individual contributions may be given as $c=p(G)g$. When a contributor recognizes the effect of his contribution on the unit price, the marginal costs will be $p(G)+p'(G)g$, which may not yield $dg/dn<0$. This should be the case, if the suppliers of G are different from the contributors of g (i.e., G is bought in an *outer* market). On the other hand, if g is contributed and G consumed by the very same group of identical individuals (i.e., g is provided individually and G consumed collectively *within* the community), the case will be tantamount to ours with increasing marginal costs. The result may then depend on how a public good G is provided, either bought and sold at a market or provided and consumed directly in kind, which should constitute a topic for further research.

Second, consider the case of contributions in cash. If funds for donation are raised in a distortionary manner, the marginal costs of individual contributions are increasing.

This may apply to the case for individuals who borrows funds in imperfect credit markets to contribute to a collective enterprise (Esteban and Ray 2001), or to the case of marginal cost of public funds (MCPF) for national [local] governments which use distortionary taxes to contribute to an international [national] public goods (Sandler 1992, Boadway et al., 1993). While these marginal costs are increasing, thereby reflecting marginal distortions, they will not be less than unity since the marginal costs of raising one cent is at least one cent, violating our assumption that $c'(0)=0$.

5. Concluding remarks

It has been generally accepted in the literature that the sub-optimality of the voluntary provision of a public good aggravates as group size increases. We have however specified assumptions to challenge the common sense, accepted view. The assumptions are limited in some cases as exemplified above, but they should still be plausible in typical cases of voluntary public good contributions. As we have discussed, good candidates may include the cases of voluntary environmental restoration and those of in-kind contributions to development projects. In these cases, several studies identify successful cases of voluntary provisions and attribute the success to cooperative behavior of contributors (e.g., Kikuchi et al 1978, Wilson 1992). While this may be so, it might also be enlightening to see these cases from a different perspective. If our assumptions are relevant, successful provision of public goods does not require cooperative behavior. It only requires a large group size.

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Figure 1. The effect of group size

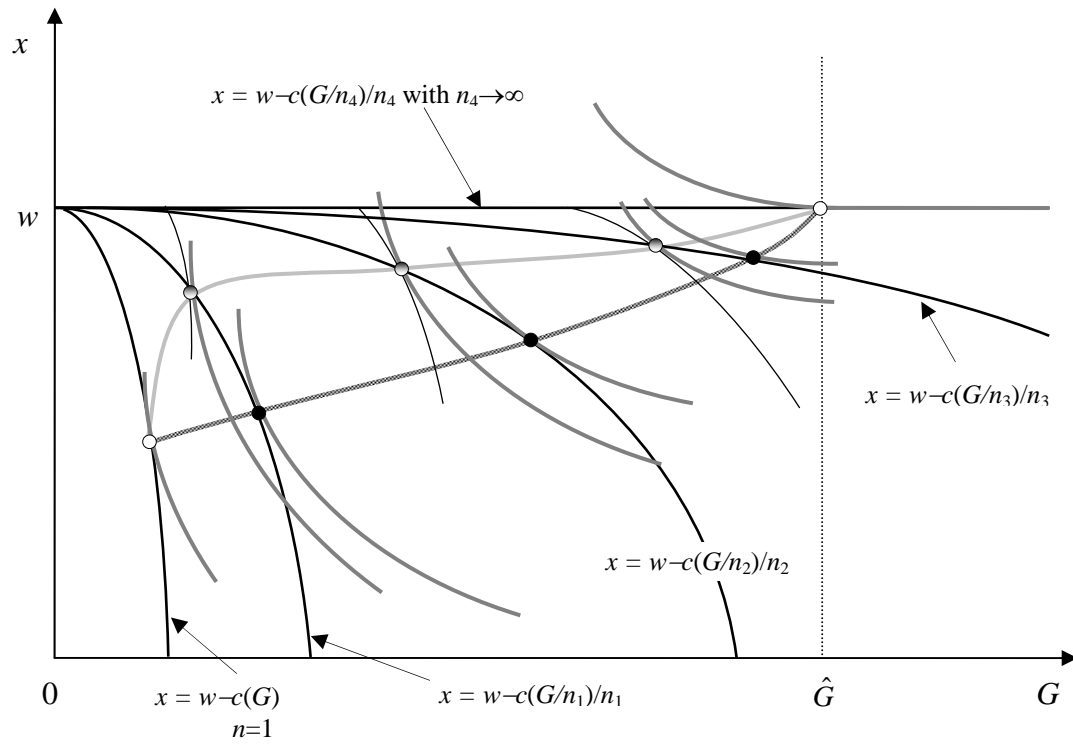


Figure 2. Group size and suboptimality

