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Differentiated Standards and Patent Pools*

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Abstract

We consider patent pool formation by owners of essential patents for differentiated standards that may be complements or substitutes in use. Pooling improves coordination in terms of royalty setting within a standard but provokes a strategic response from licensors in the competing standard. We characterise the incentives to form and defect from pools within standards and show how pool formation and stability depend on competition between standards. We also examine strategic patent pool formation by consortium standards and show that policies promoting compatibility of standards may increase or decrease welfare depending on the effects on the incentives to form pools.

Keywords: Patent pools, competing standards, consortium standards.

JEL: L15, L24, O34.

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1 Introduction

In high-technology industries, standards are often defined by a number of essential patents. Examples include the MPEG2 (video compression), DVD and 3G mobile telephony standards (Aoki & Nagaoka, 2005). The essential patents are by definition perfect complements in production of goods based on the standard. In such a situation, the incentives for the patent owners to form a pool, the welfare benefits of a pool, and the incentives for pool members to defect from a pool are well known (Lerner & Tirole, 2004, Shapiro, 2001, Aoki & Nagaoka, 2005).

The existing literature on the economics of patent pools for standards considers pool formation by a single standard in isolation. In this paper we examine formation of patent pools between differentiated standards that compete for licensees in a downstream market. Examples include competition between the HD-DVD and Blu-ray second-generation DVD standards, and GSM and CDMA mobile telephony standards, among others. We examine the implications of the downstream strategic interaction between competing standards for the upstream formation of pools within standards.

Our analysis is similar to Beggs (1994) who considers firms selling products that are complementary within certain groups but substitutes between groups, such as the suppliers of components in competing computer systems.\(^1\) He considers the incentives of these firms to price jointly within a group, which is analogous to formation of a patent pool. The tradeoff underlying the incentive to price jointly is that doing so benefits the firms within a group by improving coordination, but also provokes a strategic reaction from the competing system that makes firms worse off. A similar issue occurs with patent pools. Pooling reduces the total royalty charged by a standard as it eliminates the ‘anticommons’ problem. However, this induces licensors in the competing standard to set lower royalties when the standards are substitutes or higher royalties when they are complements. These inter-standard effects offset the intra-standard gains from pooling.

Our analysis extends Beggs’s in a number of ways that provide additional understanding of the economics of patent pool formation among competing standards. In particular, Beggs’s analysis is confined to the case where there are two firms in each system, competing systems are substitutes in use, and focuses on the incentives to price jointly within competing systems. We allow for standards to be complementary or compatible in downstream use and allow many essential patents in each standard. We also consider not only the incentive to form a patent pool, but the stability of pools and the incentive to defect from a pool when standards compete. We show how the strategic effects between standards affect the incentive for pool members to defect in such a way that makes the success of patent pools likely to

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\(^1\)See also the discussion of supermarkets in Armstrong (2006).
be correlated when standards are substitutes but inversely correlated when they are complements or highly compatible.

Beggs considers a simultaneous decision between groups to price jointly. We also consider pool formation games among consortium standards. We extend his analysis by showing how the incentives to pool depend on the number of patents in each pool and compatibility of standards. We also add a sequential version of the game where one consortium chooses whether to form a patent pool before the other. In this case, we show that the consortium which moves first has a greater incentive to form a pool when standards are complements or highly compatible, but a weaker incentive when they are substitutes. Intuitively, not pooling is a way for a consortium to commit to high royalties, which is generally desirable for the patent owners when the standards are substitutes, but not desirable when they are complements.

We also present a policy analysis which interprets the degree of substitutability between standards as a policy variable reflecting, for example, policies that influence the compatibility of competing consortium standards. Increased compatibility of standards has similar effects to making standards less substitutable through increased differentiation. In our model, as in other models with network effects, compatibility is desirable (when it is costless) although it leads to higher equilibrium prices. However, at certain levels of compatibility, we show that the strategic behaviour of the consortia with respect to patent pool formation can tip from forming pools to not forming pools, or vice versa. Since whether or not the consortia form pools affects equilibrium royalties, at certain points the gains from increased compatibility can be outweighed by a sudden increase in royalties due to the change in strategic behaviour. This means that policies which promote compatibility between standards are not always welfare enhancing, once the strategies of consortia with respect to pool formation are taken into account.

The remainder of this paper is as follows. The next section sets up the model and characterises the equilibrium. Section 3 then considers incentives for standards to form pools, incentives to defect from a pool, strategic formation of pools between consortium standards, and analyses policies designed to promote compatibility of standards. Section 4 offers concluding remarks.

2 The model and equilibrium

There are two competing standards, 1 and 2. Standard $i$ depends on $n_i$ essential patents, and each patent is specific to one standard. Some or all of the patent holders for standard $i$ may belong to a patent pool. Let $x_i \in \{1, 2, \ldots, n_i\}$ be the number of independent licensors for standard $i$. If all $n_i$ patents of standard $i$ are in a pool then $x_i = 1$ while if no patents belong to a pool then $x_i = n_i$. In general, if there are $m_i \leq n_i$ members of
a pool in standard $i$ then $x_i = n_i - m_i + 1$.

Each independent licensor in each standard sets a per-unit royalty simultaneously and non-cooperatively. Let $r^k_i$ be the royalty charged by licensor $k = 1, 2, \ldots, x_i$ in standard $i$ and let $\rho_i = \sum_{k=1}^{x_i} r^k_i$ be the total per-unit royalty payable by downstream licensees of standard $i$. The two standards are differentiated from the point of view of downstream licensees and demand for licenses from standard $i$ is $q_i (\rho_i, \rho_j)$. Each independent licensor sets $r^k_i$ to maximise their total royalty revenues, which are $\pi^k_i = q_i (\rho_i, \rho_j) r^k_i$.

Before making specific assumptions about the demand for licenses, it is useful to examine the general determinants of equilibrium royalty revenues. Suppose a symmetric equilibrium exists where $r^k_i = r^*_i (x_i, x_j)$ for all $k$, so that the total equilibrium royalty for standard $i$ is $\rho^*_i = x_i r^*_i (x_i, x_j)$ and the equilibrium revenues of a licensor in standard $i$ are $\pi^*_i (x_i, x_j)$. For the sake of illustration, suppose that $x_i$ and $x_j$ are continuous. Then, by the envelope theorem, the effects on $\pi^*_i$ of a change in $x_i$ are:

$$\frac{\partial \pi^*_i}{\partial x_i} = (x_i - 1) \frac{\partial q_i}{\partial \rho_i} \frac{\partial r^*_i}{\partial x_i} + x_j \frac{\partial q_i}{\partial \rho_j} \frac{\partial r^*_i}{\partial x_i} + \frac{\partial q_i}{\partial x_i} r^*_i.$$  

This shows that the effect on $\pi^*_i$ of a change in $x_i$ can be decomposed into three things. The first term is an intra-standard strategic effect where the other licensors in standard $i$ change their royalties, which affects demand for licenses from standard $i$. Second is an inter-standard strategic effect where licensors in standard $j$ change their royalties in response to the change in royalties for standard $i$, which also affects the demand for licenses from standard $i$. Finally there is the direct effect that the number of license payments per unit of standard $i$ increases, which reduces demand for licenses. Similarly, a change in $x_j$ has a direct effect and two strategic effects on $\pi^*_i$. These three effects underlie much of the analysis that follows.

For the remainder of this paper, we assume a representative downstream user of the standards has a simple quadratic gross welfare function of the form

$$W(q_1, q_2) = q_1 + q_2 - \frac{1}{2} (q_1^2 + q_2^2) - \gamma q_1 q_2$$

where $\gamma \in (-1, 1)$.

This type of function is commonly used to describe differentiated products, where $\gamma$ measures the extent to which the products are complements or substitutes. In general, two standards could be complements or substitutes in downstream use. The value of a standard to a user may also exhibit network effects, where the value increases with the number of users of the standard. With network effects, compatibility between standards becomes important. If the standards are compatible, then the network benefits experienced by users of one standard will depend on the usage of both standards.

$^2$The same basic setup is used by Beggs (1994) with the restriction that $\gamma > 0$. 

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In the case of network effects between standards, product differentiation and compatibility between competing standards have similar effects for determining the demand for a standard at given prices (see, for example, Doganoglu & Wright, 2006). This is because compatibility reduces the sensitivity of a network’s demand to changes in its own price, as when it raises prices for example, it loses some customers to the other network, but the remaining customers are still able to ‘communicate’ with those who switched networks, while that would not be possible if the networks were incompatible. Thus compatibility generally augments product differentiation, softens competition between networks, and leads to higher prices. Put another way, compatibility makes standards less substitutable, everything else equal. This does not necessarily harm consumers or welfare, however, as there are benefits associated with having access to a larger network.

In general, compatibility between networks can take varying degrees. In our model, we interpret $\gamma$ as summarising the extent to which the two standards are complements or substitutes in downstream use, and the extent to which they are compatible. If standards are close substitutes in downstream use, then $\gamma$ will be larger, while if they are complements it will be smaller. A high degree of compatibility between standards also implies a smaller value of $\gamma$. Later we will interpret $\gamma$ as a parameter that can be influenced by policy towards compatibility of standards.

Since a downstream user must pay $\rho_i$ per unit to use standard $i$, the representative user chooses $q_1$ and $q_2$ to maximise $W(q_1, q_2) = \rho_1 q_1 - \rho_1 q_1 - \rho_2 q_2$. Solving this problem gives the demand for licenses from standard $i$ as

$$q_i(\rho_i, \rho_j) = \frac{1 - \gamma - \rho_i + \gamma \rho_j}{1 - \gamma^2}. \quad (1)$$

Aside from the royalties, we assume for simplicity there are no other downstream or upstream costs of production. The first-order condition for an individual licensor $k = 1, 2, ..., x_i$ in standard $i$ is therefore

$$\frac{\partial \pi_k^i}{\partial r_k^i} = q_i(\rho_i, \rho_j) - r_k^i = 0. \quad (2)$$

We consider equilibria where all independent licensors in standard $i$ set a symmetric royalty, so that $r_k^i = r_1^i$ for all $k$ and for $i = 1, 2$. In this case, $\rho_i = x_i r_1^i$ and from (1) and (2) the equilibrium is given by

$$\frac{1 - \gamma - x_i r_1^i + \gamma x_j r_2^j}{1 - \gamma^2} - \frac{r_1^i}{1 - \gamma^2} = 0 \quad \text{for } i, j = 1, 2 \text{ and } i \neq j.$$

Solving these two equations simultaneously gives the equilibrium royalty $r_1^i$ set by licensors in standard $i$. The total equilibrium royalty per-unit payable by a user of standard $i$ is therefore

$$\rho_i^* (x_i, x_j) = \frac{(1 - \gamma) x_i (1 + x_j (1 + \gamma))}{1 + x_i + x_j + x_i x_j (1 - \gamma^2)}. \quad (3)$$
Proposition 1 The total equilibrium royalty of standard $i$ is increasing in $x_i$ and increasing (decreasing) in $x_j$ when $\gamma > 0$ ($\gamma < 0$).

Proof. From (3), some algebraic manipulation reveals that $\rho^*_{i} (x_i + 1, x_j) > \rho^*_{i} (x_i, x_j)$ when $x_j + 1 > 0$ which is always true. Similarly, $\rho^*_{i} (x_i, x_j + 1) \ge \rho^*_{i} (x_i, x_j)$ when $\gamma (1 + x_i (1 + \gamma)) \ge 0$ which is true when $\gamma \ge 0$ and false when $\gamma < 0$.

The fact that $\rho^*_{i}$ increases in $x_i$ is the standard ‘tragedy of the anticommons’ result (Buchanan & Yoon, 2000, Heller & Eisenberg, 1998). Licenses from the $x_i$ independent licensors of standard $i$ are perfect complements in its downstream use. As a result, the greater is $x_i$, the greater is the total royalty for that standard, because each individual licensor does not take account of the negative effect that raising her royalty has on demand for licenses from the other licensors within the same standard.

The equilibrium total royalty of standard $i$ also increases with $x_j$ if downstream users view the standards as substitutes ($\gamma > 0$). In this case, an increase in $x_j$ raises $\rho^*_{j}$ due to the anticommons effect among standard $j$ licensors and since total royalties of the two standards are strategic complements this also induces an increase in equilibrium royalties by licensors in standard $i$. If the standards are complements or compatibility is sufficiently important ($\gamma < 0$), the two standards are strategic substitutes, and a higher $\rho^*_{j}$ due to an increase in $x_j$ causes standard $i$ licensors to respond by lowering their royalties.

The equilibrium revenues of a licensor in standard $i$ are $\pi^*_{i} (x_i, x_j) = q_i (\rho^*_{i}, \rho^*_{j}) r^*_{i}$. From (1) and (3) we have

$$\pi^*_{i} (x_i, x_j) = \frac{(1 - \gamma) (1 + x_j (1 + \gamma))^2}{(1 + \gamma) (1 + x_i + x_j + x_i x_j (1 - \gamma^2))^2}.$$  \hspace{1cm} (4)

Proposition 2 Equilibrium revenues of a licensor in standard $i$ are always decreasing in $x_i$, and are increasing (decreasing) in $x_j$ when $\gamma > 0$ ($\gamma < 0$).

Proof. From (4), algebraic manipulation reveals that $\pi^*_{i} (x_i + 1, x_j) < \pi^*_{i} (x_i, x_j)$ when $1 + x_j (1 - \gamma^2) > 0$ which is always true. Similarly, we have $\pi^*_{i} (x_i, x_j + 1) \ge \pi^*_{i} (x_i, x_j)$ when $\gamma (1 + x_i (1 + \gamma)) \ge 0$ which is true when $\gamma \ge 0$ and false when $\gamma < 0$.

Intuitively, as discussed above, an increase in $x_i$ makes the tragedy of the anticommons worse within standard $i$, forcing up equilibrium royalties to the detriment of licensors of that standard. In addition, an increase in $x_j$ causes royalties for standard $j$ to increase, and if the standards are substitutes then this benefits licensors in standard $i$, but harms them if standards are complements or highly compatible.
3 Analysis of patent pools

So far, we have not specified who the licensors within each standard are, only that they act independently. Now suppose that an individual licensor in standard may either be a patent pool or an independent patent holder. An independent patent holder’s equilibrium revenues are simply $\pi^* (x_i, x_j)$. We assume that pools distribute their revenue equally to the patent holders that are members. The number of pool members in standard $i$ is $n_i - x_i + 1$, thus a pool member receives $\pi^* (x_i, x_j) / (n_i - x_i + 1)$.

3.1 Incentives to form and maintain pools

First, consider the effects of pool formation on the patent holders in standard $i$, taking $x_j$ as given. We are interested in the incentive to form a pool, and the incentive for members of a pool to break out and become independent licensors, when in competition with another standard. We consider the formation of a complete pool ($x_i = 1$) versus independent royalty setting by all members of standard $i$ ($x_i = n_i$). In this case, the gains to an individual patent holder from pool formation in standard $i$ given $x_j$ independent licensors in standard $j$ are

$$\Delta P = \pi^* (1, x_j) / n_i - \pi^* (n_i, x_j).$$

(5)

From (4), $\Delta P \geq 0$ if

$$\gamma^2 \leq \frac{(x_j + 1)(n_i + 1 - 2\sqrt{n_i})}{x_j (n_i - \sqrt{n_i})}. $$

(6)

It is easy to verify that the right-hand side of (6) is always positive for $n_i \geq 1$ and $x_j \geq 1$, and is always less than 1. So for some range of $\gamma$ around zero, forming a pool is beneficial for standard $i$ patent holders, for any given $x_j$. However, for sufficiently extreme values of $\gamma$, the standard $i$ patent holders are better off remaining as individual licensors. Beggs (1994) finds similar results for suppliers of systems. These results come from a basic tradeoff in pool formation when standards compete. Pooling allows patent holders within a standard to internalise the externalities that result in the tragedy of the anticommons, which is beneficial for them. Thus pooling is always worthwhile when the standards are independent ($\gamma = 0$). However, if standards are substitutes, for example, then the reduction in standard $i$’s royalty caused by forming a pool induces a strategic reaction of lower royalties set by the licensors in standard $j$. When standards are substitutes, this hurts the patent holders in standard $i$ and offsets the gains from pooling. Similarly, if standards are complements or compatibility is high, when standard $i$’s royalty reduces from pooling it induces standard $j$ to set higher royalties, which also hurts the patent holders in standard $i$. 

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Therefore, the decision to form a pool depends on the balance of the positive *intra*-standard effects from eliminating the tragedy of the anticommons versus the negative *inter*-standard strategic effects. The inequality (6) summarises the factors that influence this tradeoff. The right-hand side is increasing in $n_i$, as the greater the number of independent licensors that exist in standard $i$ without a pool, the greater are the benefits from overcoming the tragedy of the anticommons by pooling. Thus pooling becomes more likely when $n_i$ is high. The inequality is also more likely to be satisfied if $x_j$ is low or $\gamma$ is small in absolute value. Either of these means that the strategic reaction from standard $j$ in terms of its change in royalties in response to standard $i$ forming a pool is smaller. As this inter-standard strategic reaction always offsets the gains from pooling, thus a smaller reaction makes pooling more worthwhile. The following proposition summarises these results.

**Proposition 3** Pool formation is always beneficial for patent holders in a standard for values of $\gamma$ sufficiently close to zero that satisfy (6). In addition, for given $\gamma$, pool formation is more likely to be beneficial for patent holders in a standard if the standard involves many patents ($n_i$ is large) or there are fewer independent licensors in the competing standard ($x_j$ is small).

In general, patent pools are inherently unstable because each member can often do better as an independent licensor due to the complementarity of patents. If standard $i$ has formed a complete pool ($x_i = 1$) the gains to an individual patent holder from unilaterally defecting are

\[
\Delta_D = \pi^*_i(2, x_j) - \pi^*_i(1, x_j) / n_i.
\]

**Proposition 4** The gains for a patent owner in standard $i$ from defecting from a complete pool are always positive when $n_i \geq 3$ and are positive for $n_i = 2$ if $\gamma^2$ and/or $x_j$ are sufficiently large. In addition, when $n_i \geq 3$, $\Delta_D$ increases (decreases) with $x_j$ when $\gamma > 0$ ($\gamma < 0$).

**Proof.** In the appendix. ■

Intuitively, defecting from the pool causes the standard’s royalty to rise beyond the joint optimum. This reduces the per-licensor profit in standard $i$, but since the defector gets to keep all of the profit rather than sharing it with the other pool members, the basic incentive to defect (if $\gamma = 0$ and the standards were independent) is always positive in this model unless $n_i = 2$. Defection also induces a strategic response of higher royalties from standard $j$ when $\gamma > 0$ and lower royalties when $\gamma < 0$, and these strategic effects can make the net gain from defection positive.

From proposition 2, both $\pi^*_i(2, x_j)$ and $\pi^*_i(1, x_j) / n_i$ increase with $x_j$ if $\gamma > 0$ and decrease if $\gamma < 0$. If $n_i$ is large enough ($n_i \geq 3$) then the effect of higher $x_j$ on $\pi^*_i(2, x_j)$ always outweighs the effect on $\pi^*_i(1, x_j) / n_i$. 

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and the change in the incentive to defect when $x_j$ increases has the same sign as $\gamma$. Otherwise, if $n_i = 2$ the effect on the incentive to defect is ambiguous and may increase or decrease depending on the values of $\gamma$ and $x_j$. Overall, proposition 4 suggests that, with sufficiently many patents involved in each standard, if standards are substitutes then an increased number of independent licensors in standard $j$ makes it more difficult to maintain a pool in standard $i$. In other words, with substitute standards the success of patent pools are likely to be correlated across standards. If one standard is able to maintain a low number of independent licensors due to pooling, this reduces the incentive for members of a pool in the other standard to defect from the pool, due to the strategic interaction between pools. On the other hand, if standards are complements in downstream use (or compatibility is high), there should be an inverse correlation between the success of the pools.

3.2 Strategic pool formation

Proposition 3 indicates that patent pool formation may be beneficial for the IP owners in a standard, but this depends on whether or not the competing standard also forms a pool, the intensity of competition between the pools, whether or not the standards are substitutes or complements in downstream use and the degree of compatibility between standards. To address the pool formation issue in more detail, we consider a multi-stage game of pool formation between consortium standards, where two consortia choose whether or not to pool the essential patents within each standard before competing in the downstream market for licenses. For simplicity, we assume that the number of patents is the same in each standard, so that $n_1 = n_2 = n$. We consider the cases where the consortia choose whether to form pools simultaneously, and where one consortium moves first. In either case, we assume the two consortia choose between two strategies of forming a complete pool ($x_i = 1$) or licensing all patents independently ($x_i = n$).

When consortia choose whether to form pools simultaneously, we look for a subgame perfect equilibrium of this game by solving the first stage assuming that each consortium maximises the total royalty revenues of the patent owners for that standard in the second stage of competition between standards. Thus, for example, if standard 1 forms a pool and standard 2 does not, the payoff of standard 1 is $\pi^*(1,n)$ and the payoff of standard 2 is $n\pi^*(n,1)$. The normal form of this game is as follows:

<table>
<thead>
<tr>
<th>Std. 1 Pool</th>
<th>No Pool</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pool</td>
<td></td>
</tr>
<tr>
<td>$\pi^<em>(1,1)$, $\pi^</em>(1,1)$</td>
<td></td>
</tr>
<tr>
<td>$n\pi^<em>(n,1)$, $n\pi^</em>(n,1)$</td>
<td></td>
</tr>
<tr>
<td>No Pool</td>
<td></td>
</tr>
<tr>
<td>$n\pi^<em>(n,1)$, $\pi^</em>(1,n)$</td>
<td></td>
</tr>
<tr>
<td>$n\pi^<em>(n,n)$, $n\pi^</em>(n,n)$</td>
<td></td>
</tr>
</tbody>
</table>

Forming a pool is a best reply to the other consortium pooling when
\( \pi^* (1, 1) \geq n \pi^* (n, 1) \). From (4), after some manipulation, this is true for 
\( \gamma \in [\max \{-1, -\gamma (n)\}, \min \{1, \gamma (n)\}] \), where

\[
\gamma (n) = \sqrt{\frac{2 (1 + n - 2 \sqrt{n})}{n \sqrt{n}}}.
\]

Similarly, forming a pool is a best reply to the other consortium not pooling when 
\( \gamma \in [-\gamma (n), \gamma (n)] \), where

\[
\gamma (n) = \sqrt{\frac{(1 + n - 2 \sqrt{n}) (n + 1)}{n (n \sqrt{n})}}.
\]

Figure 1 sketches these inequalities, assuming that \( n \) is continuous for 
simplicity. If \( \gamma \) is low in absolute value, the inter-standard strategic response in terms of higher royalties from consortium \( j \) in response to consortium \( i \) forming a pool is relatively small. In this case, the gains from reducing the anticommons effect within standard \( i \) by forming a pool outweigh the strategic effects between standards, regardless of whether the other standard pools or not. Thus pooling is a dominant strategy and the only equilibrium is for both consortia to form pools. Similarly, if \( \gamma \) is relatively high (and \( n < 4 \)), it is a dominant strategy for both consortia not to pool as the negative effects of the inter-standard strategic response always outweigh any coordination gains within a standard.\(^3\)

If \( \gamma \) takes intermediate values given \( n < 4 \) or high values for \( n \geq 4 \), the two consortia wish to coordinate their actions with regard to pooling. In this case there are two pure strategy equilibria, where both consortia pool or both do not pool, as well as a mixed strategy equilibrium. This arises because the number of independent licensors in standard \( j \) affects the size of the inter-standard strategic response to pool formation by consortium \( i \). If consortium \( j \) does not pool, it contains more independent licensors and the inter-standard strategic response to consortium \( i \) forming a pool is larger compared to when consortium \( j \) also pools. This means that, for such values of \( \gamma \), whether or not the inter-standard strategic effect outweighs the intra-standard coordination gains within a standard depends on whether the competing standard is a pool. Proposition 5 summarises the equilibria when consortia choose whether or not to form pools simultaneously.

**Proposition 5** In the simultaneous move pool formation game, the equilibrium probability that a consortium standard forms a patent pool is given by \( \sigma \) where

\[
\sigma = \begin{cases} 
0 & \text{for } \gamma \in (\min \{-\gamma (n), 1\}, 1) \cup (\max \{-\gamma (n), -1\}, -1) \\
\{0, \hat{\sigma}, 1\} & \text{for } \gamma \in [\gamma (n), \min \{\gamma (n), 1\}] \cup [-\gamma (n), \max \{-\gamma (n), -1\}] \\
1 & \text{for } \gamma \in [-\gamma (n), \gamma (n)]
\end{cases}
\]

\(^3\)Beggs (1994) also considers the simultaneous version of a similar game and finds similar results for the case where \( n = 2 \) and \( \gamma > 0 \).
Figure 1: Illustration of the best reply regions in the simultaneous pool formation game between two consortium standards.

with

\[ \hat{\sigma} = \frac{n\pi^*(n,n) - \pi^*(1,n)}{\pi^*(1,1) - n\pi^*(n,1) + n\pi^*(n,n) - \pi^*(1,n)}. \]

If the timing of the first stage of the game is sequential rather than simultaneous then the consortium that moves first can influence the outcome in the case when there are multiple equilibria. The pure strategy equilibria of both standards pooling or both not pooling exist for the same parameter values where these are dominant strategies in the simultaneous move game. For the mixed strategy parameter regions of the simultaneous move game, in a sequential game the consortium that moves second will mimic the strategy with regards to pooling of the consortium that moved first, due to the desirability of coordinating the strategies of the standards towards forming pools. In this case, the consortium that moves first can effectively decide whether both pool or not, depending on which outcome has a higher payoff. Thus in the mixed strategy region, the first mover will choose to pool if \( \pi^*(1,1) \geq n\pi^*(n,n) \). From (4), this occurs if \( \gamma \leq \gamma(n) \) where

\[ \gamma(n) = \frac{1 + n - 2\sqrt{n}}{n - \sqrt{n}}. \]

It is easy to verify that \( \gamma(n) \in [0, \gamma(n)] \) for all \( n \geq 1 \). Figure 2 illustrates the equilibrium strategies as a function of \( \gamma \), for a given \( n \) in the sequential pool formation game. This shows that there is an asymmetry around zero in the range of \( \gamma \) for which consortia form patent pools in equilibrium. The consortia are more likely to pool in a subgame perfect equilibrium if
Both standards do not pool

Both standards pool

$-1 \leq \gamma(n) \leq -1$  
$N(n) \leq \gamma(n) \leq N(n) \leq N(n)$

$\gamma(n) = 0$  
$\gamma(n) < 1$

**Figure 2:** Equilibria in the sequential move version of the game, for $\gamma(n) < 1$.

Both standards pool

Both standards do not pool

$-1 \leq \gamma(n) \leq -1$  
$N(n) \leq \gamma(n) \leq N(n) \leq N(n)$

$\gamma(n) = 0$  
$\gamma(n) < 1$

**Figure 3:** Equilibrium probabilities of forming patent pools in the simultaneous (left) and sequential (right) versions of the game, for $n = 3$.

they are complements or if compatibility is high ($\gamma < 0$) compared to if they are substitutes ($\gamma > 0$). This is because, from proposition 2, pooling by consortium $j$ benefits consortium $i$ when the standards are complements (or highly compatible), but not when they are substitutes. This effect tips the balance in favour of pooling by the standard that moves first when $\gamma \in [-\gamma(n), -\gamma(n)]$. Put another way, not forming a pool is a way for a consortium to commit to setting a high total royalty for its standard. If standards are substitutes this softens competition between them and increases royalty revenues, and thus the standard that moves first has a tendency towards not pooling unless $\gamma$ is relatively close to zero so that strategic effects between standards are relatively small. However, if the standards are complements then a commitment to lower royalties by pooling is preferred when inter-standard effects are strong. Proposition 6 summarises the subgame perfect equilibria in the sequential move game.

**Proposition 6** In a subgame perfect equilibrium of the sequential move pool formation game, both consortia form pools for $\gamma \in \left[\max\{-\gamma(n), -1\}, \gamma(n)\right]$ and both do not pool for $\gamma \in (-1, \max\{-\gamma(n), -1\}) \cup (\gamma(n), 1)$.

Figure 3 graphs the equilibrium probabilities of forming patent pools in the simultaneous and sequential versions of the game, using the equilibrium completely mixed strategy when there are multiple equilibria in the simultaneous version, for $n = 3$. To interpret the results in the simultaneous case, consider what happens starting from $\gamma = 0$. At that point, the standards are independent and each consortium prefers to form a patent
pool regardless of what the other standard does. As $\gamma$ increases, say, the inter-standard strategic reactions increase in importance. Since the size of the strategic reaction also depends on whether or not the competing standard forms a pool, at some value of $\gamma$, each standard prefers to pool if and only if the other standard also pools. In terms of the payoffs, as $\gamma$ crosses $\gamma(n)$, $\pi^* (1, 1)$ remains greater than $n\pi^* (n, 1)$ while $n\pi^* (n, n)$ slightly exceeds $\pi^* (1, n)$. This pushes the equilibrium probability of pooling, given by $\hat{\sigma}$, close to zero. As $\gamma$ increases further, the increasing importance of the inter-standard strategic effects causes the difference between $\pi^* (1, 1)$ and $n\pi^* (n, 1)$ to fall, while the difference between $n\pi^* (n, n)$ and $\pi^* (1, n)$ increases, so the probability of pooling in a mixed strategy equilibrium must rise. Finally, as $\gamma$ crosses $\gamma(n)$, the inter-standard strategic effects become so important that not pooling becomes a dominant strategy, and the equilibrium probability of pooling falls to zero. Within the mixed strategy region, strategic effects between standards are strong enough that coordination of the decision to pool or not becomes the most important factor. Outside this region, inter-standard effects are either so small that they do not affect the decision to form a pool, or are so great that they dominate the decision, leading to dominant strategy equilibria.

### 3.3 Policy analysis

One of the important policy issues with competing consortium standards is the extent to which standards should be encouraged to be compatible in downstream use. As discussed earlier, we can interpret $\gamma$ as the combined effect on the welfare of downstream users of the degree of differentiation of the standards and the extent to which they are compatible. Policies that promote compatibility thus imply lower values of $\gamma$, everything else equal. To illustrate the effects of such policies, figure 4 graphs the equilibrium royalties as functions of $\gamma$ in the simultaneous and sequential versions of the game for $n = 3$, using the mixed strategy equilibrium and calculating the expected royalty in the case where there are multiple equilibria in the simultaneous game.

In general, increased differentiation or increased compatibility of standards causes equilibrium royalties to rise, due to the softening of competition between the two standards. In addition, at some values of $\gamma$ the nature of the strategic interactions between standards means that equilibrium royalties jump as the strategic pooling behaviour of the standards switches from one regime to another. Although royalties generally increase when $\gamma$ reduces, this does not necessarily mean that welfare decreases, since increased compatibility of standards yields additional benefits to downstream users even if royalties are higher. However, the sudden increases in royalties due to changing strategic behaviour of the standards can outweigh any gains from increased compatibility and cause welfare to fall. To illustrate, figure
Figure 4: Equilibrium royalties in the simultaneous (left) and sequential (right) patent pool formation games, for \( n = 3 \).

Figure 5: Equilibrium expected welfare in the simultaneous pool formation game, for \( n = 3 \) and \( \gamma < 0 \) (left) or \( \gamma > 0 \) (right).

5 shows the equilibrium expected welfare in the simultaneous version of the pool formation game for negative and positive values of \( \gamma \) when \( n = 3 \).

This model thus suggests that any analysis of policies designed to make standards more compatible must take account of the strategic behaviour of standards with regard to patent pool formation. The model also suggests when such regime switches are likely to occur and affect policy analysis. Dominant strategy equilibria occur when inter-standard strategic effects are either relatively strong or relatively weak, that is, for relatively low or high (absolute) values of \( \gamma \), given \( n \). In particular, from figure 1, a regime switch will not occur for small changes of \( \gamma \) between \( -\gamma(n) \) and \( \gamma(n) \), or \( \gamma \) outside of \( \pm \gamma(n) \) (when this is less than \( \pm 1 \) in absolute value). Thus if the inter-standard demand linkages are relatively weak or relatively strong, small changes in \( \gamma \) will likely not affect the strategic pooling behaviour of standards. The region of stable dominant strategy equilibria also increases with \( n \) as this increases the importance of inter-standard strategic effects.
4 Conclusion

Existing models of patent pool formation by standards ignore competition between standards. Our analysis shows that the strategic interaction between standards adds an extra dimension to the effects of pooling. For example, not pooling is a way of committing to high royalties between competing consortium standards and this may outweigh the gains to patent owners from pooling within a standard when standards are substitutes. This is a possible explanation why socially beneficial pools may not form. However, if standards are complementary or highly compatible in downstream use, there are gains to coordinating on lower royalties through pooling. Our analysis also implies that success of pools are likely to be positively correlated when standards are substitutes and negatively correlated when they are complements or highly compatible.

Consortium standards may also form pools for strategic reasons. If inter-standard effects are weak or strong, the strategic behaviour of each consortium with respect to pooling is independent of the other. However, for intermediate parameter regions, coordination effects between standards become relatively important. If one standard moves first with respect to pool formation, this may be an attempt to encourage or discourage a competing standard from pooling, depending on whether the standards are substitutes or complements in downstream use. These strategic effects also have implications for policies that promote compatibility of standards, and while compatibility is broadly desirable in our model, the benefits can be outweighed by increases in royalties if higher compatibility causes consortia change their behaviour with respect to forming patent pools.

5 Appendix

Proof of proposition 4

From (4), $\Delta_D \geq 0$ if

$$x_j (2 - \sqrt{n_i}) \gamma^2 + (x_j + 1)(2\sqrt{n_i} - 3) \geq 0$$

The left-hand side is a quadratic in $\gamma$. For $n_i = 3$ or $n_i = 4$ this inequality always hold since the coefficient on $\gamma^2$ is weakly positive and the constant term is positive. Otherwise, the roots of the quadratic in $\gamma$ are

$$\gamma = \pm \sqrt{\frac{(x_j + 1)(2\sqrt{n_i} - 3)}{x_j (\sqrt{n_i} - 2)}}$$

For $n_i > 4$ the coefficient on $\gamma^2$ is negative and these roots lie outside the interval $(-1, 1)$ since $(x_j + 1)/x_j > 1$ and $(2\sqrt{n_i} - 3)/(\sqrt{n_i} - 2) > 1$, thus the inequality holds for all $\gamma \in (-1, 1)$. For $n_i = 2$, the coefficient on
\( \gamma^2 \) is positive and the roots lie within \((-1,1)\). Furthermore, the roots are decreasing in \( x_j \).

Finally, to show that \( \Delta_D \) increases in \( x_j \) when \( \gamma > 0 \) and decreases when \( \gamma < 0 \), it is easier to assume that \( x_j \) is continuous. From (4), after some rearrangement it can be verified the sign of \( \partial \Delta_D / \partial x_j \) is the same as the sign of

\[
Z = \gamma \left( \frac{2\gamma + 3}{(3 + x_j + 2x_j (1 - \gamma^2))^3} - \frac{\gamma + 2}{n_i (2 + x_j + x_j (1 - \gamma^2))^3} \right)
\]

The term in brackets is positive if

\[
n_i \geq \frac{(\gamma + 2) (3 + x_j + 2x_j (1 - \gamma^2))^3}{(2\gamma + 3) (2 + x_j + x_j (1 - \gamma^2))^3}
\]

and it is straightforward to verify that the right-hand side is always less than 3 for \( x_j \geq 1 \) and \( \gamma \in (-1,1) \). Thus the sign of \( \partial \Delta_D / \partial x_j \) is the same as the sign of \( \gamma \).

References


