Welfare Analysis of Economic Systems from the Viewpoints of Distributive Justice and Incentive Compatibility

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INTRODUCTION

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Welfare Analysis of Economic Systems from the Viewpoints of Distributive Justice and Incentive Compatibility

INTRODUCTION

1. Welfaristic and Nonwelfaristic Criteria on Economic Systems

Welfare economic theory aims to investigate various economic systems, some of which may be really observed while others may be ideal ones hypothetically constructed as a first best referential criterion, from various viewpoints on social desirability. An economic system consists of individuals, several environmental structures such as production technology, a preference profile of individuals, initial endowments of wealth, systems of individual rights, etc., and a stylized process or pattern of resource allocations. The social desirability represents societal goals which individuals should or wish to pursue. The typical examples are social efficiency or fairness of allocational consequence such as Pareto optimality or fairness as no envy, efficiency or justice of allocational methods (or procedures), or incentive compatibility of the allocation process.

1.1. Welfaristic Analysis of Economic Systems

A traditional welfare economics in a narrow sense has mainly focused on the social consequence of the pattern of resource allocation, given the environmental structures. The fundamental theorem of welfare economics is one of the most brilliant success in such a traditional welfare economics. That theorem treats only one economic system, the private ownership economy with competitive markets, and focus on welfare performance of allocational consequence through the competitive mechanism from the viewpoint of social efficiency. Although the result of the theorem is very excellent, we should note that the implication of the theorem does not necessarily lead to justification of real capitalist economies, because the model of market economies, such as the Arrow-Debreu type, that the welfare theorem assumed seems to capture no basic characteristics of the capitalist economy.

In contrast to the fundamental welfare theorem, there have been several attempts to analyze allocational consequence of one and the other economic systems not only from the viewpoint of social efficiency but also from viewpoints of distributive justice. Theories of fair allocation such as "Fairness as No Envy" (Foley (1967)) and/or "Egalitarian equivalent allocations" (Pazner and Schmeidler (1978)) argued what concept is a reasonable criterion to capture economic equity, and show the existence and the characteristic of resource allocation rules which satisfies such an equity criterion.

These theories of fair allocation intended not to analyze equitable properties of some realistic economic system but rather to hypothetically construct the resource allocation systems regarded as the equitable, to which policy makers in real economic systems should mention as a first best criterion. The reason is that these theories do not refer to property relations in economic systems, right structures of economic systems or the existence of nontransferable resources, with which persons are arbitrary endowed, such as "talent" or "handicap", those of which seem to be very important environmental structures when we characterize the realistic economic systems. These fair allocation rules only concern with a preference profile, and at most aggregate initial endowments or the structure of production technology. This implies that the theories of fair allocations such as the no envy and/or the egalitarian equivalent are on the standpoint of welfarism. Welfarism requires that resource allocation rules should only take account of the preference profile in the economy as long as the aggregate endowment or the production technology is fixed. However, as many authors have already pointed out (for example, Sen (1979), Dworkin (1981a) or Roemer (1986b)), the welfaristic approach is inadequate to analyze realistic economic systems from the viewpoint of distributive justice.

In fact, we can easily show the case that a welfaristic criterion on fair allocation is incompatible with a rule respecting for individuals' right spheres or paying attention to arbitrariness of distribution of nontransferable resources. For example, in private ownership economies, person 1 has the right to claim all profits produced and person 2 has no rights to claim profit revenues, and it is assumed that such a right structure used to be assigned through a justifiable procedure. Then, even if person 2 envies person 1, the allocation rule respecting that right assignment distributes all produced profits to person 1, while for example, the no envy and Pareto efficient allocation rule may assign a competitive equilibrium allocation from equal division. Next, consider some institution compensating the handicapped, when person 1 is handicapped to work for his minimum standard life while person 2 is completely healthy man. Then, let us suppose that person 2 feels envy of person 1 at his guaranteed minimum standard life without working. Thus, the economic system adopting the no envy allocation rule must prohibit such a compensating institution.

There are some arguments, which have interests in states of environmental structures such as the distribution of initial endowments including nontransferable resources, property relations and the system of individual right, from viewpoints of nonwelfaristic desirability like "justice as fairness" (Rawls (1971)), liberty (Mill (1859), Sen (1970, 1992), Gaertner et al. (1992)), equality of opportunity (Dworkin (1981b), Sen (1980, 1985), Cohen (1989, 1993)). For example, Sen's argument on "Paretian Liberal Paradox" (Sen (1970, 1992)) indicates that a representative welfaristic criterion, the Pareto principle, is not compatible with a typical nonwelfaristic criterion on liberty of individuals. This also implies that the nonwelfaristic allocation rule respecting for individuals' rights is incompatible not only with the welfaristic criterion on fairness but also with that representative welfaristic criterion. As an example of the Paretian Liberal Paradox, we can find out the voluntary contribution scheme of public goods in public-good economy. The voluntary contribution scheme of public goods guarantees individuals' rights to choose freely how much pay for providing public goods. It is well known that the allocation induced by the scheme is not Pareto efficient.

1.2. Nonwelfaristic Theories of Distributive Justice

With regard to nonwelfaristic theories of distributive justice, we can also refer to many arguments— Rawls (1971), Nozick (1974), Sen (1980, 1985), Dworkin (1981a,b), Cohen (1985, 1986) and Roemer (1986b). *The difference principle*, that is the theory of Rawlsian distributive justice, represents, according to Rawls (1971), an agreement to regard the distribution of natural talents as a common asset and to share in the benefits of this distribution, so that the naturally advantaged are not to gain merely because they are more gifted, but only to cover the costs of training and education and for using their endowments in ways that help the less fortune as well. Thus, according to the difference principle, to distribute *primary goods* such as income and wealth in such a way that no one gains or losses from his arbitrary place in the distribution of nontransferable endowments, economic inequality in the distribution of primary goods would be permissible only if such an inequality contributes to the greatest rewards of the least advantaged. In his theory (1981a,b) of "Equality of Resource", Dworkin maintains that the bundle of the resources to be "equalized" should include not only all transferable goods but also nontransferable ones such as talents, handicaps and propensities of various kinds. As well as Rawls, Dworkin's theory also regards personal skills or talents as arbitrary distributed. He advocated a insurance mechanism to allocate transferable goods to implement the resource egalitarian in a manner that compensates people for the morally arbitrary distribution of talents that results from the birth lottery.

In contrast to Rawls (1971) and Dworkin (1981 a,b), Nozick's argument (1974) on *appropriation* maintains that in the state of nature, personal skills or talents were under self-ownership; each person is entitled to his own skills or talents. On the contrary, external resources such as land and natural resources were unowned. Nozick thinks but also that people can be entitled to appropriate the unowned objects as a result of proper exercise of their own and/or other's self-owned personal powers — that is justified by virtue of self-ownership, as long as others will not be rendered worse off than when the land was held in common — Lockean proviso (Locke (1690)). According to Nozick (1974), the free operation of the market system involving private appropriation of the external resources will not actually run afoul of the Lockean proviso, though it is not so clear why he can believe so. Thus, although a union of self-ownership and unequal appropriation of external resources readily leads to indefinitely great inequality of private property in external goods, Nozickian entitlement theory justifies such a inequality as well as the highly unequal income distribution in real capitalist economies.

While, critiquing the Rawlsian and the Dworkinian theories of distributive justice, Nozickian justifies inequality in real capitalism by declaring protection of self-ownership, it is the Marxian theory of exploitation that directly critiques unjustifiable inequality in real capitalism without necessarily denying self-ownership. The Marxian theory of exploitation rather maintains that the realistic capitalist economy itself is certainly denying self-ownership of the working class through the reproduction of itself. The reason is that, according to Marx (1867), as a result of the capitalist production process, the amount of labor embodied in the wage basket the worker received is less than the amount of labor he expended to earn that wage. This phenomenon implies that the workers are forced to expend their some time in working for the capitalists' profit revenues, that is surely a denial of self-ownership Nozick (1974) condemned for. Such a phenomenon is referred to, by the Marxian theorist, as the *existence of exploitation.* If a person command with his income more labor embodied in goods than his expended labor, he is an exploiter, and if he is in the inverse case, he is exploited. Note that the Marxian critique on capitalism seems to be the most serious one for its supporter in the sense that the Marxian critique seizes upon the inconsistency of the argument justifying the inequality in real capitalist economies, and by doing so, it shows that the capitalistic economic system is unjust with respect to its distributive performance, while each of the Rawlsian and the Dworkinian only argues that how can one justify the degree of inequality in distribution from a particular normative point of view, without arguing whether or not is justified the distributive performance of the realistic capitalist economy. Thus, when we see the distributive performance of the realistic capitalist economy, it seems to be meaningful for us to follow the Marxian argument of exploitation.

2. The Marxian Theories to Analyze the Realistic Capitalist Economic Systems

Before introducing Marxian arguments about the capitalist system, it seems to be beneficial to explain what model of the capitalist economy is realistic. As above mentioned, the Arrow-Debreu model, I think, captures no basic characteristics of the capitalist economy. In my opinion, at least the three components stated in the following constitute of basic characteristics of the capitalist economy. The first is the existence of the competitive market mechanism for implementing resource allocations. However, this component is not necessarily inherent to capitalism only, because there were several economic systems in which market mechanisms functioned such as simple commodity production economies. The Arrow-Debreu model surely captures this characteristic. The second component is private ownership system with unequal distribution of the means of production. Here, an important property of that unequal distribution is the mass existence of persons with no material productive assets. Without such persons whom we call "proletarian", it could not be established the capitalist system, which is indicated by much historical evidence such as the Enclosure in England. The third characteristic is of production activities in firms. In firms, there is a fundamental conflict of interests between the manager of the firm and the employee. While the manager would like to work the employee with as low wage as possible to attain as high profitability as possible, the employee is looking for a chance to shirk his work for a given payment. Such a conflict is not described in the Arrow-Debreu world, since in that world the manager-employee relationship is treated as in the block-box.

2.1. The Marxian Arguments of Exploitation in Mathematical Marxian Economics

Among various types of the Marxian school, I am mainly indebted to mathematical Marxian economics which, by adopting the mathematical tools developed in the neoclassical economics, tries to investigate the capitalistic economic system from the point of Marxian view. With regard to the Marxian exploitation theory, one of the most famous contribution of the mathematical Marxian is the fundamental Marxian theorem (FMT) originally discussed by Okishio (1963) and Morishima (1973). The FMT shows that the necessary and sufficient condition of positive profit rates in the capitalist economy is the existence of exploitation. Thus, according to this theorem, high profitability in the capitalist economy, which Nozick (1974) praised as the success of that system by using the means of production most efficiently, has the suppressed source in the unjustly distributive performance of the economy. After the seminal work by Morishima (1973), there were many generalizations and discussions of the FMT. While the original FMT is discussed in the simple Leontief economy with homogeneous labor, the generalization of the FMT to the Leontief economy with heterogeneous labor is discussed by Fujimori (1982), Krause (1982), e.t.c. The problem in generalizing the FMT to the von Neumann economy is discussed by Steedman (1977) and one resolution is proposed by Morishima (1978). Furthermore, Roemer (1980) generalized the theorem to a convex cone economy. These arguments may reflect the robustness of the FMT. However, this theorem has a crucial problem: it does not follow from the FMT that the exploitation of labor is the source of positive profits. The reason is that every commodity can be shown as exploited in a system with positive profits whenever the exploitation of labor exists. This observation was pointed out by Bowles and Gintis (1981), Samuelson (1982), and was named the "Generalized Commodity Exploitation Theorem (GCET)" by Roemer (1982).

After the arguments of GCET, one of the noticeable researches in the Marxian school is of Bowles and Gintis (1988, 1990) about "Contested exchange". Bowles and Gintis (1981) gave attention to the fundamental difference between labor-power and other commodities. In contrast to the usual exchange contract of material commodities, labor contract is not perfectly delineable and not costlessly enforceable: the manager cannot delineate exactly the tasks he will want the employee to perform, and it is costly to supervise the employee. Thus, labor of the employee must be extracted from labor-power of the employee by whatever system of control the manager may devise. When the exchange process has such a property, it is called *contested.* Then, if there exist mass industrial reserved armies both in the capitalist and non-capitalist sector, the manager has power over employees to make them perform along with his interest by threatening to impose sanctions on them. In such cases the ex post level of the contested attribute is determined by sanctioning mechanisms. Bowles and Gintis (1990) stressed one extremely important sanctioning mechanism: *contingent renewal labor contract.* This obtains when the manager elicits labor performance from the employee by promising to renew the contract in future periods if his performance is satisfactory and to terminate the contract if not. Notice that since monitoring perfectly whether the labor performance of each employee is satisfactory or not is quitely expensive, that monitoring is feasible only stochastically. Thus, in the contested exchange of labor, the equilibrium wage will involve paying the employee a premium over what he could get if he is unemployed, and this premium and the contingent renewal are to induce him to perform well for avoiding the possibility of being caught shirking even if he is not being watched. Such the equilibrium is characterized as a Stackelberg equilibrium, and the equilibrium wage is characterized as an *efficiency wage*. Bowles and Gintis (1990) stressed upon that when the labor exchange is modeled explicitly as contested one, the well-known theorem about the efficiency of market economies will no longer be established. I think this result is more plausible as a characteristic of real capitalist economies than the well-known welfare theorem, since the Bowles-Gintis model captures the production relationship in firms while the Arrow-Debreu not.

The argument of contested exchange seems to be not directly related to exploitation. However, as Marx (1867) discussed as the cause of surplus value the power relationship between capital and labor concerning the extraction of labor from labor-power, I also think it is a fundamental issue to argue the power relationship between the manager and employees when it is discussed the existence of exploitation in the capitalist economy. I think that a main contribution of the contested exchange theory is to show *the existence of a power relationship* between the manager and employees behind the unjustly distributive performance of the capitalist economy, although Bowles and Gintis themselves might not intend so. However, their argument does not explicitly analyze whether or not the strength of the manager's power over its employees can guarantee profitability in the capitalist economy. As far as this problem is concerned, it implies that there seems to be *an effect of distributive inequality of wealth in tightening the power of the employer over the employee*, although Bowles and Gintis (1990) themselves did not refer to this problem.

The other noticeable researches in the Marxian school is of Roemer (1982, 1986a) about the corresponding relationship between the existence of exploitation and the unequal distribution of wealth. By adopting a standard general equilibrium model of capitalist economies, Roemer (1982, 1986a) showed that how capitalist societies can generate endogenously in their economic systems the structure of class and exploitation that Marxian postulates for capitalism. In the capitalist economic model Roemer (1982, 1986a) put forth, it is assumed that there are no contested exchanges—in particular, that the labor contract is perfectly delineable and costly enforceable, while there exists the unequal distribution of privately owned wealth. Furthermore, it is assumed that all persons have the same utility function. In the economy's equilibrium, every person is optimizing against a constraint of his endowment, and so that the whole society is divided into four disjoint and exhaustive classes: that is, a class of capitalists, a class of petty bourgeoisie, a class of semiproletarians, and a class of proletarians. In the equilibrium, if a person can optimize by hiring other persons as workers to operate his capital, he belongs to the capitalist class, if a person can optimize by operating his capital with self-employed only, he belongs to the petty bourgeoisie class, if a person can optimize by operating his capital with self-employed only and selling his labor to others, he belongs to the class of semiproletarians, and if a person can optimize only by selling his labor to others, he belongs to the class of proletarians. Then, if the labor supplied by any person is inelastic with respect to his wealth, what class each person belongs to is determined corresponding to his status of wealth: Differential ownership of the means of production is responsible for differential class position. That was summarized up by Roemer (1982, 1986a) as Wealth-class correspondence principle (WCCP). Moreover, Roemer (1982, 1986a) showed that every person in the capitalist class is exploiter, while every person in either the class of semiproletarians or of proletarians is exploited. This theorem is named the "Class-exploitation correspondence principle" (CECP). By both WCCP and CECP, it is followed that if the labor supplied by any person is inelastic with respect to his wealth, every exploiter is wealthier than every exploited: that is, Wealth-exploitation correspondence principle. Thus, Roemer (1982, 1986a) showed that the Marxian kind of class structure and exploitation can emerge endogenously as a result of the existence of unequal wealth distribution, even if the labor market is not contested.

We should mention to some points about the above arguments. First, the Roemer's theorems (1982, 1986a) depend upon the inelasticity condition of labor supplied. As Roemer (1986a) said, it may be reasonable from the point of historical evidence to assume that condition. However, even if that condition is historically reasonable, the assumption of neoclassical labor market itself is not so. This is, furthermore, related to the following problem: that is, under a given economic environment the inelasticity condition of labor supply is not necessarily ensured in the contested exchange labor market even if it is ensured in the neoclassical labor market. Thus, replacing the neoclassical labor market in Roemer (1982, 1986a) by the contested exchange labor market, the robustness of the Wealth-exploitation correspondence is not necessarily guaranteed. This implies that it is necessary to show the robustness of the Wealth-exploitation correspondence in a more realistic capitalist economic model with a contested exchange of labor than the Roemer's model (1982, 1986a) with a neoclassical labor market.

2.2. A Theory of Wealth, Exploitation and Power Relationship

Succeeding to the above two noticeable Marxian researches, in Chapter I of this thesis, I try to analyze the distributive performance of the capitalistic economic system by synthesizing, in one Leontief economic model, the analysis of wealth distribution and exploitation by Roemer (1982) and the analysis of the contested exchange of labor by Bowles and Gintis (1988, 1990). In the capitalist economic model I put forth, it is assumed that labor exchange is contested, and also that the labor contract is organized as a sequential contingent renewal one, while there exists the unequal distribution of privately owned wealth. Furthermore, it is assumed that all agents have the same utility function as well as Roemer (1982) and Bowles and Gintis (1988, 1990). However, it is also assumed that all agents are either risk-neutral or non-increasing risk averse. These settings seem to be sufficiently reasonable to describe a realistic capitalist economy under uncertainty. In this paper, I propose *a method for measuring the level of the agent's labor-discipline.* I define *the level of an agent's discipline* as *the supply of labor effort per unit of the agent's received real wage.* So, the more labor effort some agent supplies per unit of his received real wage rate, the higher the level of his labor-discipline is. The level of the agent's labor-discipline is an index for expressing the strength of the manager's power over that agent. By introducing such the index and investigating the relationship between the unequal distribution of wealth and the problem of labor discipline, I try to solve whether or not there exists an effect of distributive inequality of wealth in tightening the power of the manager over the employee. Moreover, I try to analyze the relationship between the level of the agent's labor-discipline and the exploitation status of the agent, as well as to check the robustness of the Wealth-exploitation correspondence in this model. My main results in Chapter I are as follows. First, the less wealthy agent has a higher level of labor-discipline than the wealthier agent if agents are risk averse (The correspondence of wealth and the level of labor-discipline), and second, as a consequence of capitalist production, the income gap between the wealthy and the poor widens more and more (A poverty law in capital accumulation). Third, it is guaranteed that the robustness of the Wealth-Exploitation Correspondence in the Capitalist economy with the contested exchange labor market if agents are either risk-neutral or non-increasing risk averse. Finally, the less wealthy exploited agents are more labor-disciplined than the wealthier exploiters if agents are risk averse (The correspondence of exploitation and the level of labor-discipline).

The results obtained indicate the essential importance of the unequal distribution of wealth in understanding the contemporary capitalist economy. Especially, as well as Roemer's assertion, one of my results implies that the call for the abolition of exploitation is a call for an egalitarian distribution of material productive assets as long as all agents have the same risk-neutral or non-increasing risk averse preference. Such an implication is not true when preferences are different among agents. However, an essential critique of Marxism to capitalism is rather directed to the facts that even if there are no differential traits among agents such as preferences, which seem to be matters they should be responsible for (and/or labor endowments if we see so), nevertheless, in capitalist societies, there exist various injustices such as inequalities in income distribution or in opportunities to access to material productive assets, and as unfair power relationship between capital and labor in decision process of production. As well, my results also indicate that only as a consequence of differential wealth ownership which is an objective structure of the capitalist society every agent should not be responsible for, the distributive inequality like exploitation can emerge in the realistic capitalist economy. Hence, I think that my paper's critical power to capitalism is not lost essentially by the facts that my results are not robust in the case of preference-differential.

Notice that the implication of the correspondence of wealth and the level of labor-discipline is not the behavioral difference between the rich and the poor, but rather the importance of the unequal distribution of wealth in explaining *the modest contestedness in labor markets for the capitalist economy to be sufficiently profitable*. By this result, it can be seen that the labor market entered by the less wealthy suppliers alone is more moderately contested than the one by the wealthier suppliers alone. In real capitalism, it is usual that most of the employed workers are the agents with no or only a few productive assets. Thus, the contestedness in labor markets would be moderated to maintain enough profitability in real capitalist economies.

3. Theories of Public Ownership

In the above section, as an effective counter-argument to Nozick's justification of the highly unequal distribution of income in capitalist societies, I introduced the classical Marxian theory of exploitation and its recent developments in mathematical Marxian economics including the research contribution of myself. However, the classical Marxian theories of exploitation have some insufficient points as the counter-proposal to Nozick: first, the scopes of the classical theories of exploitation are essentially restricted to the world in which there are no differential traits among persons including preferences and internal talents, and second, in the classical theories of exploitation including mathematical Marxian approaches, there are no analytical arguments about desirable alternative economic systems. These two points have been often criticized by both non-Marxist and Marxist itself. In contrast, the arguments of Rawls (1971) and Dworkin (1981a,b) are free from these two criticisms. But, as mentioned above, their counter-proposals to Nozick's have denied the premise that people should morally be viewed as the owners of their talents: that is, denial of self-ownership. Hence, their counter-proposals seem to be supported only by the standpoints approving of denying self-ownership.

3.1. The Analytical Marxist Critique of Nozickian Entitlement Theory

Here is the alternative, third counter-proposal to Nozick's, of Analytical Marxists, Cohen (1985, 1986) and Roemer (1986b, 1988). Cohen (1985) argued that Nozickian revision of Lock's proviso cannot clearly give legitimacy for appropriation of things. Nozickian revision of Lock's proviso says that one person's appropriation of a thing is justified as long as no person is *left worse off* after the appropriation than he was before, had the thing is remained as unowned. However, according to Cohen (1985), there exist at least three cases Nozick's condition does not regard but important for judging whether an appropriation is legitimate or not.

Consider an economy with two people, A and B, and land commonly used by them. In this economy, A gathers m bushels of wheat and B gathers n bushels. Then A appropriates the land and designs a division of labor between himself and B, with the result that A ends up with $m+p$ bushels and B with $n+q$ bushels. Suppose $p > q > 0$. Since this appropriation passes Nozick's proviso, it is legitimate according to Nozick. However, if B could have appropriated the land as well and designed the same division of labor and the same distribution of the additional product, and so that B received $n+p$ bushels and A received $m+q$ bushels, B would have been better off than the actual situation. When one judged the actual situation by the above hypothetical situation, then A's appropriation giving rise to the actual situation would be unjustified. Or, consider if B could have appropriated the land and designed a more superior program of resource allocation than the one A imposes, and so that B received $n+p+r$ bushels and A received $m+q+r$ bushels. If such a case is possible, is it reasonably justified A's appropriation in the actual situation ? Third, consider the case that A and B would have agreed to a resource allocation program without either of them privately appropriating the land. Such an agreement institute a form of joint ownership. When the agreed program would be productively superior to the ones both of A's appropriation and of B's, it is not plausible nevertheless to justify A's appropriation. Thus, although A's appropriation is clearly legitimate according to Nozick, there are other counterfactual situations, at least three possibilities stated above, that are as relevant for judging the moral legitimacy of the appropriation.

Moreover, Cohen (1986) questioned why in the state of nature the external world such as land or lake should be considered to have been unowned as Nozickian supposed. In stead of such a supposition, Cohen (1986) viewed the external world in the initial state as having been jointly or publicly owned by its initial inhabitants, and explored the possibility to unite self-ownership with equality of external resources and preserve equality of condition. Consider an economy with two people, A and B, and land jointly owned by them. Each owns himself, and be rational and exclusively self-interested. A can produce life-sustaining and life-enhancing goods, but B has no productive power at all. Since the land is under joint ownership, no one may use it until all agree to a decision of resource allocation. As well, even if they own themselves, no one have any rights to appropriate parts of it as private property unless such appropriation is approved by its joint owners. It is only interesting to consider the case that A can produce not only what is needed to sustain both people but also a surplus. Then A and B will bargain over how much will be produced, and over who gets how much of the produced, against the constraint that threat point is no production, and therefore death for both. In such a situation, A gets nothing extra just because he has productive skill. If the exercise of A's talent is irksome to him, then he will indeed get additional compensation, but only because he is irked, not because it is he, and not B, who does the producing. Then, joint ownership of external world prevents self-ownership from generating an inequality to which egalitarian objects. Thus, according to Cohen, without denying self-ownership, one may move towards a form of equality in distribution by insisting on joint ownership of external world. For the inequality that Nozick defends depends on adjoining to self-ownership an inegalitarian principle of external resource distribution, which need not to be accepted.

3.2. A Proposal of Public Ownership — The Constant Returns Equivalent Solution

Roemer (1986b, 1988), succeeding to Cohen (1985, 1986), tries to a remaining Cohen's problem that what distribution of income is justifiable in the economy with public ownership of external resource and private ownership of self. This problem is very important but also from another angle, a resolution of "tragedy of commons", not only for the counter-proposal to Nozick. Consider a society of fisherfolks. In the society, there are n fisherfolks endowed with labor-power and a lake commonly used by all fisherfolks. Every fisherfolk expends his labor in fishing on the lake. Fishing on the lake is represented by a production function, which describes the conversion of labor into fish. Suppose the production function exhibits decreasing returns to labor. While each has the right to free access to the lake, that is, each can fish as much as he wishes, there is no coordination of their fishing among them. Thus, as a result of each fisherfolk's unilateral optimizing, the equilibrium allocation of fish and labor under this laissez-faire economy is Pareto inefficient, because each inflicts a negative externality on the others under the decreasing returns of the common resource. It is called "tragedy of commons". There seem to be two methods of resolution of the tragedy. The first one is privatization of the lake by appropriation of a competitive entrepreneurial fisherfolk. Then the entrepreneurial operates the lake competitively, hiring the other fisherfolks and selling the fish, so that every other fisherfolk is worse off than he was under the laissez-faire (Weitzman (1974)). This gives a typical counter-example to Nozick's belief that the free operation of market system involving private appropriation will not run afoul of Nozickian proviso. The second one is to institute public ownership of the lake, and so bring about Pareto efficient solutions. Then the problem is what allocations among Pareto efficient ones the public committee should assign as justifiable solutions respecting for public ownership. That is the same problem as Cohen and Roemer tried to.

By adopting an axiomatic method, Roemer (1988) and Moulin and Roemer (1989) characterize the class of resource allocation rules that satisfy the axioms describing the requirements of self-ownership and public ownership of external resource. Moulin and Roemer (1989) assumed production economies with publicly owned production technology and two persons having the same preference but different labor skills. It was imposed four axioms, that is, *Pareto optimality*, *Technological monotonicity*, *Limited self-ownership* and *Protection of low skill.* Technological monotonicity says that if two economies are identical except that the technology is unambiguously better in one than in the other, then neither agent should be rendered worse off under the allocation rule in the economy with the better technology. It implies that whatever public ownership requires, each agent at least has a right not to be harmed by an improvement in the publicly owned asset. Limited self-ownership requires that the more skilled agent should end up at least as well off as the less skilled one. This represents a minimum condition self-ownership might require necessarily. Protection of low skill says that the less skilled agent should be no worse off than he would be in an economy in which the other agent was as low skilled as he. This axiom is an individual rational condition. Moulin and Roemer (1989) showed that there exists a unique allocation rule satisfying the above four axioms, which assigns an allocation each agent gains the same utility. This result implies that the distribution of income recommended under public ownership is much more egalitarian than what exists in a capitalist economy, although its model is very specific.

A more general argument is discussed in Roemer (1988). Similar ones are in Moulin (1987, 1990a) and Roemer and Silvestre (1989). Let us follow its argument formally. Let us represent an economy as a vector $\xi = (f;\ s_1,\cdots,\ s_n;\ u_1,\cdots,\ u_n)$, where $f(\ell) = y$ is a production function describing the production of a single output from a single input (labor), by using a publicly owned technology. Let f be continuous, nondecreasing, and $f(0) = 0$. Each agent i owns one unit of labor-power with skill level s_i, and has a utility function, $u_i(\ell_i,\ y_i)$, which is continuous, decreasing in ℓ_i, and increasing in y_i. Let be the class of economies described as E. Let denote the set of agents by I. A feasible allocation for ξ is a list of vector $(\ell_i,\ y_i)_{i\in I}$ satisfying that for all $i \in I$, $\ell_i \leq 1$, and $f(\sum_{i\in I} s_i\ell_i) = \sum_{i\in I} y_i$. Then, an allocation rule defined on E is a function S that assigns to any economy $\xi \in$ E a feasible allocation.

Required axioms are described as follows.

Axiom PO (Pareto Optimality): The allocation rule S should assign a Pareto optimal allocation for each $\xi \in$ E.

Axiom TMON (Technological Monotonicity): Let $\xi^1 = (f;\ s_1, \cdots, s_i;\ u_1, \cdots, u_n)$ and $\xi^2 = (g;\ s_1, \cdots, s_i;\ u_1, \cdots, u_n)$ be two economies in E, and for all ℓ, $f(\ell) \geq g(\ell)$. Then for all $i \in I$, $u_i(S_i(\xi^1)) \geq u_i(S_i(\xi^2))$.

Axiom FALE (Free Access on Linear Economies): If $\xi = (f;\ s_1, \cdots, s_n;\ u_1, \cdots, u_n)$ and f is a linear function $[f(\ell) = \alpha\ell$ for some $\alpha > 0$, for all $\ell]$ then the allocation rule S should assign the allocation achieved under free access, where each agent chooses her optimal bundle (ℓ_i, y_i).

The axiom of FALE implies that whatever public ownership of the technology in conjunction with private ownership of labor consists in, it should at least require that free access solution which allowing each agent to use her private endowment as she chooses, when there are no positive and negative externalities from joint use of the technology.

Definition: The allocation rule S is the *Constant Returns Equivalent Solution* (CRE) if the following is satisfied: for each $\xi \in$ E, $(\ell_i^*, y_i^*)_{i \in I} = CRE(\xi)$ if and only if $(\ell_i^*, y_i^*)_{i \in I}$ is a Pareto efficient at ξ, and there exists a $\alpha > 0$ such that for all $i \in I$, there exists a bundle $d_i(\alpha)$ satisfying i) $d_i(\alpha) = \arg\max u_i(\ell_i, \alpha s_i \ell_i)$ where $\ell_i \leq 1$, and ii) $u_i(\ell_i^*, y_i^*) = d_i(\alpha)$.

Moulin (1987, 1990a) and Roemer and Silvestre (1989) showed that CRE is a unique allocation rule satisfying PO, TMON and FALE. Notice that CRE cannot be generalized in multi-imput and multi-output economies (Roemer and Silvestre (1989)), since there is no allocation rule satisfying PO, TMON and FALE in economies with many inputs. In the case of one-input and one-output economies, since any Pareto optimal allocation rule satisfies FALE, CRE is a unique Pareto optimal allocation rule satisfying TMON.

3.3. Another Proposals of Public Ownership — Equal Benefit Solutions and Proportional Solutions

There are another allocational rules respecting for public ownership of external resources in conjunction with private ownership of self, proposed by Roemer and Silvestre (1989): that is, the *Equal Benefit Solution* (EB) and the *Proportional Solution* (PR), both of which are well defined in multi-input and multi-output economies. The proportional solution (PR) assigns the Pareto efficient allocations such that each person receives his dividend according to his input contribution. The equal benefit solution (EB) assigns Pareto efficient allocations such that every person receives the same surplus (profit revenues). These solutions are intuitively natural equity solutions, because each of them satisfies one of the two representative principles of distributive justice — "One receives according to one's contribution", and "All ones receive equal distribution of social benefit". I think that PR and EB are more plausible proposals in the context of resolving the tragedy of commons than CRE.

Roemer and Silvestre (1989, 1993) showed that the existence of PR and EB is guaranteed as long as economies have convex structure — that is, each person has a convex preference and the production possibility set is convex. Remember that the existence of Walrasian solutions that assign competitive equilibrium allocations to private ownership economies is also guaranteed as long as economies have convex structure (Debreu (1959), Arrow-Hahn (1970)). This implies that the well-definedness of each of EB and PR is as robust as that of Walrasian solutions.

Following the proposal of Roemer and Silvestre (1989), by adopting the axiomatic method, Moulin (1990a,b) characterized PR and EB in convex production economies with one-input and one-output. Moulin (1990a,b) introduced new axioms: *Free Access Upper Bound* (FAUB) and *Lower Bound Egalitarian* (LBE). Each of FAUB and LBE represents a desirability from a viewpoint of equity in the context of cooperative production with convex technology.

FAUB expresses an upper bound of welfare a person would be able to gain in the public ownership economy. This axiom requires allocation rules to prevent one particular person from monopolizing most dividends of social benefit. FAUB defines such an upper bound of a person by the maximal welfare level he can attain in one-person economies composed only of himself. It also has a following implication: in commonly owned convex production economies, each member has free access to the production technology. However, in such a case, if all members behave to pursue their individually maximal welfare, it is well known that "the tragedy of the common" is consequential, since joint utilization of convex technology brings a negative externality. This implies that to avoid such a socially inefficient state, each member should bear some share of this negative externality instead of pursuing his individually maximal welfare. FAUB regards that in public ownership economies, it is natural that such a requirement is imposed.

In contrast to FAUB, LBE represents a lower bound of welfare level every person is equally guaranteed to gain. This axiom says that no person should be worse off than at the allocation that would be chosen if all other persons had preferences identical to him, under the requirements of efficiency and equal treatment of equals. In public ownership economies, identical members should be treated equally, and differences of the surplus opportunities should be caused only by the differences in preferences due to personal responsibility. In other words, all members should be guaranteed at least some minimal equality of welfare level whenever all members have identical preferences. Then, such a minimal equality of welfare imposes a lower bound on all members' welfare. Such a lower bound is constituted by the welfare that each agent is reachable by utilizing an equal share of the production set.

Moulin (1990a) showed that PR satisfies FAUB while EB satisfies LBE. Moulin (1990b) also showed that EB is a subselection of no envy and Pareto efficient solutions. Since PR and EB satisfy FALE, it is followed that both solutions do not satisfy TMON. PR and EB also satisfy Independence of Irrelevant Alternatives (IIA) and Maskin Monotonicity (Maskin (1977)). IIA is regarded as reminiscent of Nash's Independence of Irrelevant Alternatives (Nash (1950)). It relates to contractions of the production set: if an allocation chosen with the initial production function remains feasible with the contracted function, IIA requires that it remains in the solution. Maskin Monotonicity relates to Nash implementation problem, that is referred in the next section.

There is another axiom representing a desirability from a viewpoint of equity in the context of cooperative production: that is, *Population Monotonicity* (PMON). This axiom requires that when additional agents arrive, and the profile of welfare levels chosen by the solution for the initial group remains feasible only by "ignoring the newcomers", then none of the agents initially present gains. That is expressed formally as follows: for all $u_1,\cdots, u_n, u_{n+1}$ and $s_1,\cdots, s_n, s_{n+1}$, and any $i = 1,\dots, n$,

$$S_i(u_1,\cdots,u_n;s_1,\cdots,s_n;f) \geq S_i(u_1,\cdots, u_n, u_{n+1};s_1,\cdots, s_n, s_{n+1};f).$$

Notice that repeated applications of PMON yields FAUB: PMON is a stronger form of FAUB. Moulin (1990b) showed that in convex economies with one-input and one-output, CRE satisfies PMON. How about EB and PR ? Moulin (1990b) showed that there is no Pareto optimal solution that is no envy and meets FAUB. This implies that EB does not satisfy PMON. As well, PR does not satisfy PMON. Consider an economy with two persons, *1* and *2*. Let for both *1* and *2*, $u_1 = u_2 = y - \frac{1}{2}\ell$, and $s_1 = s_2 = 1$. The production technology is $f(\ell) = \begin{cases} 2\ell & 0 \leq \ell \leq 1 \\ \frac{1}{2}\ell + \frac{3}{2} & 1 \leq \ell \end{cases}$. Then $((\ell_1, y_1), (\ell_2, y_2)) = ((\frac{1}{2}, 1), (\frac{1}{2}, 1))$ is an allocation of PR. Let us add person *3* with $u_3 = y - \frac{1}{2}\ell$ and $s_3 = 1$. Then $((\ell_1, y_1), (\ell_2, y_2), (\ell_3, y_3)) = ((1, 1.5), (0.5, 0.75), (0.5, 0.75))$ is an allocation of PR in the new economy with three persons. This example violates the requirement of PMON, because person *1*'s welfare level is better off in the three persons economies than in the original one. Notice that on the domain of economies such that the marginal rate of substitution between the input and the output increases along all rays from the origin, PR is single-valued, so that satisfies PMON.

3.4. Full Axiomatizations of Equal Benefit Solutions and Proportional Solutions

While CRE is axiomatically full-characterized by Moulin (1987, 1990a) and Roemer and Silvestre (1989), Moulin's axiomatic characterizations of EB and PR are not complete: that is, the EB solution is included in any solution set satisfying PO, LBE and IIA, but the inverse relation is not true. Also, the PR solution is included in any solution set satisfying PO, FAUB, IIA and Maskin Monotonicity (Maskin (1977)), but the inverse relation is not true whenever preferences are weakly (not strictly) convex. On the contrary, it is Chapter II of my thesis that gives full axiomatic characterizations of EB and PR.

In Chapter II of my thesis, I introduce new axioms, *Pareto Independence* (PI) and *Support Price Independence* (SPI), the requirements of both of which are from another new angle: that is, *informational efficiency* of allocation rules. To assign a desirable allocation of a given solution, the public committee must collect information on the current economic environment to calculate the allocation. It is costly to collect information of all member's preferences and/or of characteristic of publicly owned production technology. So, it is more desirable for the committee to be able to assign allocations by collecting as less information as possible. Here informational efficiency has the implication as in the following statements: when some economy changed its characteristic to the other one, it is necessary for assigning a new allocation as the solution to the new economy to collect information on its new characteristic (the new profile of all members' preferences and the new production set). Then, if by collecting only local information on the new economy's characteristic, the new allocation is assignable as the solution, such the solution is referred to as meeting informational efficiency.

PI says that for any economy in domain, any allocation in the solution set, if the economy is changed such that the current allocation becomes Pareto efficient, then the current allocation remains in the solution set. SPI says that for any economy in domain, any allocation in the solution, some price which is supporting the current allocation as the solution, if the economy is changed such that the current price supports the current allocation as Pareto efficiency, then the current allocation remains in the solution set.

We can refer PI as representing a criterion of informational efficiency. The reason is as follows: If some current feasible allocation is equitable solution in some economic environment, whether the current allocation is also equitable solution or not in the other economic environment is verified by only checking whether or not this allocation is Pareto efficient in the other economic environment. Checking whether or not some allocation is Pareto efficient is very easy in convex economic environments, since it is enough to collect only local information on members' indifference curves at this allocation. In other words, it is sufficient to check whether or not all members weakly prefer this allocation to the other feasible allocation in some neighborhood of this allocation whenever all possible economies have convex properties. As well as PI, we can also refer SPI as representing a criterion of informational efficiency. The reason is that checking whether some price which supports the current allocation in the solution set becomes an efficiency price or not in some new economy is enough to collect members' preference information on some feasible allocations in some neighborhood of the current allocation.

In Chapter II of my thesis, I show that in convex technology production economies with one input and one output, PR is a unique solution satisfying PO, FAUB and SPI even if preferences are weakly (not strictly) convex. Also, I show that EB is a unique solution satisfying PO, LBE and SPI. Moreover, as a corollary of these results, I show that PR is a unique solution satisfying PO, PI and Individual Rationality (IR) even if preferences are weakly (not strictly) convex.

Next, I discuss on the case of convex technology production economies with multi-input and multi-output. In such economies, PR no longer satisfies FAUB. So I define a new axiom, "Upper Bound by Stand Alone Income (UBSAI)", that is a weaker version of FAUB in multi-input and multi-output economies. I show, in addition, that PR no longer is a unique solution satisfying PO, UBSAI and SPI when the economy has a positive commodity vector of publicly owned initial endowments. In contrast, EB is shown to be a unique solution satisfying PO, LBE and SPI even when there exists a positive commodity vector of publicly owned initial endowments.

I also discuss on *the Walrasian solution* (*W*) in private ownership production economies with multi-input and multi-output. By adopting the axioms of Full Individual Rationality (FIR) (Gevers (1986)), PO and SPI, I show that W is fully characterized. Some axiomatic characterizations of W were argued by Gevers (1986) and Nagahisa (1991, 1994). Nagahisa (1991) fully characterized W in the case of differentiable pure exchange economies. In contrast, with respect to the case of W in production economies, although Gevers (1986) and Nagahisa (1994) argued, their axiomatic characterizations were not complete. A difference between my result and their results is that I succeed in fully characterizing *W* by adopting SPI while they did not by adopting other axioms.

By those results, we can induce the following implication: with respect to informational efficiency of allocation rules, two public ownership solutions, EB and PR, and a representative private ownership solution, W, reveal the same good performance.

4. The Analysis of Economic Systems from the Viewpoint of Incentive Compatibility

In the above sections, as counter-arguments of Nozickian defending of greatly unequal income distribution in real capitalist economies from the standpoint of self-ownership, we discussed, first, that, according to the classical Marxian theory of exploitation, the realistic capitalist economy itself denies the self-ownership of the working class by the exploitation relationship between capital and labor, and that such the exploitation is caused by the unequal distribution of the material means of production in the realistic capitalist economy. Second, we discussed that there are alternative economic systems which institute public ownership of the material means of production compatible with respecting for self-ownership, and that in those economies, the distribution of income recommended by those allocation rules is much more egalitarian than what exists in the realistic capitalist economy. Those arguments seem to indicate that the realistic capitalist economy does not necessarily reveal good performance from the viewpoint of distributive justice, while there can be constructed alternative economic systems revealing much better distributive performance. However, we should mention another viewpoint in evaluating economic systems: that is, incentive problem of economic systems.

4.1. Incentive Problem of Economic Systems

The incentive problem of economic systems is related to realizability of resource allocations recommended by the allocation rule that the economic system choiced. So, if an economic system reveals bad performance from the point of incentive problem, then we cannot necessarily regard that system as desirable one even if its characteristic of distribution is enough to be justified.

Consider this problem concretely by taking up the example of the "tragedy of commons". In the initial state of fisherfolks' society, each fisherfolk unilaterally fishes as much as he wishes in freely accessing the commonly owned lake. Under such a laissez-faire economy, the equilibrium allocation of fish and labor is Pareto inefficiency. To resolve this tragedy, we can give the proposal to institute public ownership of the lake and to make the public committee assign a Pareto efficient allocation. The members of the committee are supposed to be elected by fisherfolks, and fisherfolks appoint the committee empowered to assign allocations in the interests of all. The committee proposes an allocation rule — for example, the proportional solution —, and that is supposed to be approved by all. Then, the following problem remains: that is, how to assign allocations consistent with true characteristics of the current economy. To assign an allocation of a given solution consistent with true characteristics of the current economy, the public committee must collect true information on the current economic environment to calculate the allocation. First, as mentioned above, it is costly to collect information of all member's preferences and/or of characteristic of publicly owned production technology. Second, collecting true information of all member's preferences is necessary to calculate a true allocation in the solution, but such information is originally known only to each fisherfolk himself: that is, all members' preferences are under their private information. It is a difficult problem to assign allocations consistent with true characteristics of the current economy when true characteristics of the current economy are under members' private information, because each fisherfolk does not necessarily reveal his true private information if he can gain by misrepresenting his information. When such a misrepresentation is beneficial to some member, the current allocation rule is said to be "*manipulable*". That is the incentive problem of economic systems we mentioned above.

The incentive problem has been discussed by many authors. Gibbard (1973) and Satterthwaite (1975) showed that in the social choice environment, there is no nondictatorial social choice rule satisfying non-manipulability. In contrast, on the case of economic environments, Hurwicz (1972) showed that any allocation rule satisfying Pareto optimality (PO) and Individual rationality (IR) is manipulable. Thus, by the theorem of Hurwicz (1972), we can understand that most of allocation rules we regard as desirable in the above are manipulable. The Walrasian solution in private ownership economies is manipulable. As well, the three public ownership solutions we mentioned above are also manipulable. Well, how to resolve this manipulation problem ?

Now, we can see the above process of misrepresenting private information as a game of revelation, in which each fisherfolk chooses a strategy concerning what information on himself to be revealed to maximize his outcome, taking account of what information the others reveal. So, we can see that the manipulability of the solution implies that the equilibrium allocations of such a revelation game do not coincide with the allocations of the solution under the true information on the current economy. Then we can understand that as long as the revelation game where the strategy set of each member consists of the possible class of his preference is played, the incentive problem could not be resolved. Hence, let us now consider whether the problem is resolved by instituting another noncooperative game. One noncooperative game consists of a profile of members' true preferences and a *game form* (Gibbard (1973)). A game form is a pair of members' strategy sets and an outcome function which assigns to each strategy combination that members take a unique allocation. Since the committee does not know a profile of members' true preferences, instituting a noncooperative game implies instituting a game form. We call such a game form a resource allocation mechanism (or a mechanism). Under a noncooperative game defined by a mechanism, each member takes some strategy, and so an allocation is assigned by the outcome function of the mechanism and the combination of members' strategies. If the equilibrium allocations of a mechanism coincide with the allocations of a given solution under a given equilibrium concept, then the mechanism implements the solution. It can be looked upon that the incentive problem of an economic system is resolvable when there exists a mechanism implementing the solution the system choiced in a reasonable equilibrium concept.

It is desirable that the equilibrium concept under which the mechanism implements the solution is the one of dominant strategy. However, it is impossible by *the revelation principle* (Gibbard (1973)), which says that if a solution is manipulable, there is no mechanism implementing the solution in dominant strategy equilibria. Moreover, by Dasgupta, Hammond and Maskin (1979), if a solution is manipulable, there is no mechanism implementing the solution in truth-telling Nash equilibria. Truth-telling Nash equilibrium consists of the Nash equilibrium strategy in which every person reveals truth information. So, the next problem to be explored is that whether or not there is a nonrevelation mechanism implementing solutions in Nash equilibria.

4.2. The Theory of Nash Implementation

It was Maskin (1977) that identified the class of Nash-implementable solutions in social choice environments, and that constructed mechanisms implementing them. Let us see Maskin's work formally in the context of our economic models.

Suppose that a list of labor skill, $s = (s_1, \cdots, s_n)$, and production technology f are fixed. Then an economy is specified by a list $u = (u_1, \cdots, u_n) \in U \equiv U_1 \times \cdots \times U_n$. Let denote an allocation by $z = (z_1, \cdots, z_n) = (\ell_i, y_i)_{i \in I}$. A *solution* is a mapping S associating with every economy $u \in U$ a non-empty subset $S(u)$ of feasible allocations. A *mechanism* (or game form) is a pair $\Gamma = (M, g)$ where $M = M_1 \times \cdots \times M_n$, M_i is the *strategy space of agent i*, and the outcome function, $g : M \rightarrow R_+^{2n}$, assigns to every $m \in M$ a unique element of R_+^{2n}. Denote the i-th component of $g(m)$ by $g_i(m)$. The list $m \in M$ will be written as (m_i, m_{-i}), where $m_{-i} = (m_1, \cdots, m_{i-1}, m_{i+1}, \cdots, m_n) \in M_{-i} \equiv \times_{j \neq i} M_j$. Given $m \in M$ and $m_i' \in M_i$, (m_i', m_{-i}) is obtained by the replacement of m_i by m_i'. Let $g(M_i, m_{-i})$ is the attainable set of member i at m_{-i}, i.e., the set of consumption bundles that member i can induce when the other members select m_{-i}. For $i \in I$, $u_i \in U_i$, and $z_i \in [0, 1] \times R_+$, let $L(z_i, u_i) := \{ z_i' \in [0, 1] \times R_+ \mid u_i(z_i) \geq u_i(z_i') \}$ be the *lower contour set for* u_i *at* z_i. Given a feasible mechanism $\Gamma = (M, g)$ and a profile of utility functions $u \in U$, the strategy profile $m \in M$ is a *Nash equilibrium of* Γ *at* u if for all $i \in I$, $g_i(M_i, m_{-i}) \subseteq L(g_i(m), u_i)$. Let $NE(\Gamma, u)$ be the set of Nash equilibria of Γ at u. Let $g(NE(\Gamma, u))$ be the set of Nash equilibrium allocations of Γ at u. The mechanism $\Gamma = (M, g)$ *implements the solution S in Nash equilibria* if for all $u \in U$, $S(u) = g(NE(\Gamma, u))$. The solution S is Nash-implementable if there exists a mechanism which implements S in Nash equilibria.

Maskin (1977) introduced the following monotonicity condition:

Maskin Monotonicity (Maskin (1977)): For all u, $u^* \in U$ and all $z \in S(u)$, if for all $i \in I$, $L(z_i, u_i) \subseteq L(z_i, u_i^*)$, then $z \in S(u^*)$.

Maskin (1977) showed that Maskin Monotonicity is necessary for Nash-implementability of solutions, and it is also sufficient for Nash implementation when there exist at least three members in economies. To prove the sufficiency of Nash implementability, Maskin (1977) concretely constructed a mechanism which implements the solutions satisfying Maskin Monotonicity in Nash equilibria. This mechanism has the following strategy space: for all $i \in I$, $M_i = U \times A \times N$ where A denotes the set of feasible allocations and N denotes the set of integers. For the case of two members, Moore and Repullo (1990) and Dutta and Sen (1991a) showed that a solution S is Nash-implementable if it satisfies Maskin Monotonicity and Non-empty lower intersection — viz., for any u, $u^* \in U$ and any pair of $z \in S(u)$ and $z^* \in S(u^*)$, there exists some feasible allocation $z^{**} \in A$ such that $u_1(z) > u_1(z^{**})$ and $u_2^*(z^*) > u_2^*(z^{**})$. After the seminal work of Maskin (1977) on Nash implementation, many mechanisms were designed in different equilibrium concepts; for example, subgame perfect implementation in Moore and Repullo (1988) and Abreu and Sen (1990), undominated Nash implementation in Palfrey and Srivastava (1988), undominated implementation in Jackson (1992), and strong Nash implementation in Dutta and Sen (1991b). For about the survey of these works, see Moore (1992) and Dutta (1993).

It is easy to show that each of PR and EB satisfies Maskin Monotonicity, so that they are Nash-implementable. In contrast, CRE does not satisfy Maskin Monotonicity so that not be Nash-implementable, though it is easily shown that CRE is implementable in subgame-perfect equilibria. By pointing out these facts, Roemer (1989) and Moulin (1990a) showed that PR and EB are Nash-implementable by the Maskin-type mechanism. It seems to be looked upon that the incentive problem of the public ownership economic system is resolved by constructing the Maskin-type mechanism as long as the selected solution is either PR or EB. However, is the

Maskin-type mechanism reasonable one as the realistic institution for proceeding resource allocations ? It seems not to be so, because the Maskin-type mechanism is excessively complicated — its strategy spaces are extremely large. This implies that information transmission in playing the game defined by the mechanism is costly and very complicated. Second, the Maskin-type mechanism requires each member to announce the characteristics of other members as his strategy. It seems not to be defended in democratic societies. Thus, we cannot refer the Maskin-type mechanism as a reasonable institution for proceeding resource allocations. Hence, if PR and EB are Nash-implementable only by the Maskin-type mechanism, and there is not a more reasonable mechanism Nash-implementing either PR or EB, we cannot regard that the incentive problem of PR and EB is resolved in a realistic sense.

4.3. Nash Implementation of A Specific Solution by Reasonable Mechanisms

In contrast to the above works, there was the other works in Nash implementation theory, which tried to construct a reasonable mechanism implementing a *specific* solution such as the Walrasian and Lindahl solutions in economic environments. For example, there are several reasonable mechanisms which implement the Walrasian solution or the Lindahl solution in Nash equilibria (Schmeidler (1980), Hurwicz (1979), Walker (1981), Postlewaite and Wettstein (1989), Tian (1989) (1992), and Hong (1995)).

Schmeidler (1980) constructed a *balanced* mechanism implementing the Walrasian solution in pure exchange private ownership economies. The balanced mechanism assigns to every strategy combination an allocation where the total demand is equal to the total supply. In the Schmeidler (1980) mechanism, a strategy for a member consists of a pair of a price and a net trade. Hurwicz (1979) then constructed *balanced* and *continuous* mechanisms implementing the Walrasian and the Lindahl solutions. The continuous mechanism implies that its outcome function is continuous. The continuity of the outcome function means a slight change in one's strategy will result in a slight change in the outcome. The strategy sets in the Hurwicz (1979) mechanism are the same as in the Schmeidler (1980) one. Walker (1981) presents an alternative balanced and continuous mechanism implementing the Lindahl solution in public goods economies. In the Walker (1981) mechanism, a strategy for a member consists of announcing a quantity. Hence, the strategy space of the Walker (1981) mechanism is smaller than that of the Hurwicz (1979) one. These three mechanisms have the same characteristic: nonequilibrium strategies may lead to individually nonfeasible allocations. An individually feasible allocation is the allocation in which every member's allocated bundle is in his consumption set.

Hurwicz, Maskin and Postlewaite (1984) constructed an individually feasible and balanced mechanism Nash-implementing the *constrained Walrasian solution* in pure exchange economies. The constrained Walrasian solution is the Walrasian one where each member's consumption of some commodity does not exceed the aggregate initial endowment of it. Since as long as we give attention to only interior feasible allocations as the range of the solution, the Walrasian solution is always constrained, in the following, we will not particularly pay attention to the difference between the Walrasian solution and the constrained one. In the following, we will assume that any solution always assigns some interior feasible allocations.

While the mechanism in Hurwicz, Maskin and Postlewaite (1984) is not continuous, Postlewaite and Wettstein (1989) proposed a continuous and individually feasible mechanism Nash-implementing the Walrasian solution. Their mechanism is, however, only weakly balanced — that is, the total demand does not exceed the total supply, but may be not equal. It was Tian (1989, 1992) that constructed a continuous, individually feasible and balanced mechanism Nash-implementing the Walrasian solution in pure exchange economies. Tian (1989) also constructed a continuous, individually feasible and balanced mechanism Nash-implementing the Lindahl solution in public goods economies. Hong (1995) constructed a continuous, individually feasible and balanced mechanism Nash-implementing the Walrasian solution in private ownership production economies. These mechanisms require every member to announce a price and a net trade as his strategy.

Notice that though those mechanisms introduced in this subsection are reasonable ones, they only apply to a specific solution (the Walrasian or the Lindahl). So, to check implementability of the different solution such as PR and/or EB, we must reconsider whether or not there exists a reasonable mechanism implementing that solution.

4.4. Characterizations of Nash Implementation by Reasonable Mechanisms in Pure Exchange Economies

Recently, there is a new approach in Nash implementation theory that is to explore the ground between the above two approaches. This approach is to impose several conditions that a reasonable mechanism should satisfy, and then characterize the class of solutions implementable by such a mechanism in pure exchange economies. This approach is promoted by Sjöström (1991), Dutta, Sen and Vohra (1995), and Saijo, Tatamitani and Yamato (1995).

Sjöström (1991) proposed the use of a *quantity* mechanism: each member announces just his consumption bundle as his strategy. He showed that neither the no envy and Pareto efficient solutions nor the Pareto efficient solutions is Nash-implementable.

Dutta, Sen and Vohra (1995) proposed as reasonable mechanisms elementary mechanisms, and characterized the class of solutions Nash-implemented by those mechanisms in differentiable pure exchange economies. Elementariness of mechanisms requires that the attainable set of each member in an equilibrium be contained in a closed half space. That closed half space is given by the marginal rate of substitution at the equilibrium allocation. This implies that the dimension of the strategy space in elementary mechanisms is quitely lower than the canonical Maskin-type mechanisms, because in an elementary mechanism, the committee is constrained to the attainable sets which are tangential approximations to the lower contour set, while in the Maskin-type mechanism, the committee must be able to use the entire reported, lower contour set as the attainable set. As a specially interested class of elementary mechanisms, Dutta, Sen and Vohra (1995) proposed elementary price-quantity mechanisms where members announce a price and a consumption quantity vector. Moreover, they imposed to elementary price-quantity mechanisms a requirement of *truthful implementation*. This implies that the strategy profile composed of truthful announcements must constitute an equilibrium where the truthful announcements are the strategy profile consistent with some allocation and its associating efficiency price in the solution of the current economy. As a characterization result, they showed that the Walrasian solution is implementable by an elementary price-quantity mechanism while neither the no envy and Pareto efficient solution nor the Pareto efficient solution is implementable by that mechanism.

Succeeding to Sjöström (1991) and Dutta, Sen and Vohra (1995), Saijo, Tatamitani and Yamato (1995) identified some fundamental conditions that reasonable mechanisms should satisfy in differentiable pure exchange economies. The first condition is that the dimension of the strategy space should be finite and low enough. As such mechanisms, they consider six types of mechanisms: quantity, quantity^2, allocation, price-quantity, price-quantity^2, and price-allocation mechanisms. Quantity^2 implies that a strategy of one person consists of announcing a pair of her and her neighbor's consumption bundles. The second condition is *forthrightness* that is the same as the truthful implementation in Dutta, Sen and Vohra (1995). They imposed this as a condition of easiness in computing the outcome of an equilibrium strategy profile. The third condition is that the mechanisms be individually feasible and balanced. The fourth demands that the mechanisms should satisfy the *best response property* due to Jackson, Palfrey and Srivastava (1994): for every strategy combination of the other members, each member has a best response. It is required to justify the use of Nash equilibrium as an equilibrium concept. Saijo, Tatamitani and Yamato (1995) called the mechanisms satisfying the above four conditions *natural* *mechanisms*. They provided necessary and sufficient conditions for solutions to be Nash-implemented by the above six types of natural mechanisms. Moreover, as several main results, first, they showed that the Walrasian solution is not implemented by any natural quantity mechanism. Second, both the Walrasian and the no envy and Pareto solutions are implementable by natural price-quantity mechanisms. The reason why the no envy and Pareto solution is implementable by a natural price-quantity mechanism, that different from the result in Dutta, Sen and Vohra (1995), is that natural mechanisms do not require the elementariness.

These works restricted the class of economies to that of pure exchange ones. It is interesting to research in the case of production economies the implementability of solutions by reasonable mechanisms. In the case of production economies, it is difficult to construct individually feasible and balanced mechanisms, because the total supply will not be known to the committee *ex ante*, even if the distribution of initial endowment and production technology are known.

4.5. Natural and Double Implementation in Production Economies

It is Chapter IV of this thesis that studies the class of solutions in production economies implementable by reasonable mechanisms. In Chapter IV of this thesis, I also impose on reasonable mechanisms the four conditions Saijo, Tatamitani and Yamato (1995) did. Moreover, I also impose the following three conditions: First, the *informational decentralization* (Schmeidler (1980)) property that each member announces information only about himself. The Maskin-type mechanism requires each member to announce the preferences of all the members. This implies that each member's strategy space includes the space of other member's possible preferences. Thus, in this mechanism, the committee has the authority to compel each member to announce the traits of others, which is usually objectionable in actual democratic societies. Among the six types of natural mechanisms Saijo, Tatamitani and Yamato (1995) offered, we can regard quantity and price-quantity mechanisms as satisfying the informational decentralization property.

Second, we also impose that the natural mechanism should implement solutions not only in Nash, but also in strong Nash equilibria. The Maskin-type mechanism can be used only in the environment where the social planner is convinced that members will never take any cooperative strategies. However, it seems to be usual in actual economic contexts that the planner cannot know whether members will cooperate or not. So, it is more desirable to construct a mechanism doubly implementing solutions in Nash and in strong Nash equilibria. Double implementation in Nash and strong Nash equilibria is originally discussed by Maskin (1979). The Schmeidler (1980) mechanism is an example of doubly implementing mechanisms. There are also several works on double implementation in Nash and undominated Nash equilibria — Jackson (1992), Yamato (1993), Tatamitani (1993), and Jackson, Palfrey and Srivastava (1994).

Moreover, we require easiness of constructing the attainable set of each member. Many mechanisms with preference announcements possess the property that in equilibrium the attainable set of each member is precisely the reported lower contour set. However, since in our models the committee does not collect reports about lower contour sets, the problem is how to construct attainable sets of members to successfully implement solutions. In the case of production economies where the production technology is fixed, one method to resolve that problem is to construct the mechanism such that the attainable set of each member in equilibrium be contained in the closed half space defined by announcing quantities and some production-supporting price. The production-supporting price is determined by the production possibility frontier and some efficient production point inferred by quantity announcements.

In Chapter IV of this thesis, I first characterize the class of solutions satisfying PO doubly implementable in Nash and strong Nash equilibria by a natural quantity mechanism. Sjöström (1991), and Saijo, Tatamitani and Yamato (1995) have already clarified that no natural quantity mechanism can implement Pareto efficient solutions in pure exchange economies. It is generally true in the case of production economies, too.

However, as long as the production set has the smooth boundary (or the technology is representable by a differentiable production function), several solutions satisfying PO are doubly implementable by a natural quantity mechanism. The Walrasian solution in private production economies is such an example. In one-input and one-output differentiable production economies, PR and EB are also doubly implementable. In Chapter III of my thesis, assuming one-input and one-output differentiable production economies, I concretely construct two natural quantity mechanisms each of which doubly implements PR or EB respectively. With respect to EB, I ascertained that it is doubly implementable in a more general multi-input and multi-output differentiable production economies. Notice that all these referred solutions satisfy the axiom of SPI, which is shown in Chapter II of my thesis. In this chapter, I show that SPI and some axiom, Condition QP, are necessary and sufficient for solutions satisfying PO in differentiable convex production economies to be doubly implementable by natural quantity mechanisms. Condition QP gives a feasible punishment condition in the case that all members be potential deviators. As a corollary, all solutions satisfying PO and SPI in differentiable convex production economies are doubly implementable by individually feasible and weak balanced quantity mechanisms.

Second, in general convex production economies, we characterize the class of solutions satisfying PO doubly implementable in Nash and strong Nash equilibria by a natural price-quantity mechanism. The above three solutions are doubly implementable by natural price-quantity mechanisms in multi-input and multi-output convex production economies. It is also shown that SPI and some axiom, Condition PQP, are necessary and sufficient for solutions satisfying PO in convex production economies to be doubly implementable by natural price-quantity mechanisms. Condition PQP also gives a feasible punishment condition in the case that all members be potential deviators. As a corollary, all solutions satisfying PO and SPI in convex production economies are doubly implementable by individually feasible and weak balanced price-quantity mechanisms.

Now, there are still several problems remained. As mentioned in subsection 4.3, many authors have imposed continuity of the outcome function as a condition of reasonable mechanisms. However, as well as Dutta et al. (1995) and Saijo et al. (1995), all of my constructed mechanisms in Chapter III and IV do not satisfy this requirement. In contrast, the Hong (1995) mechanism which implements the Walrasian solution in production economies is individually feasible, balanced and continuous, and satisfies forthrightness and the best response property, although the strategy spaces of her mechanism are rather larger than ours, and her mechanism cannot doubly implement. It is interesting to explore the possibility of continuous natural mechanisms the strategy spaces of which are less than the Hong (1995). Second, as well as Dutta et al. (1995) and Saijo et al. (1995), all of my constructed mechanisms in Chapter III and IV also contain "modulo game", though they do not contain "integer game". Thus, although no pure Nash equilibrium exists in a modulo game, if members have von Neumann-Morgenstern utility functions over lotteries on the sets of feasible allocations, then there may exist mixed Nash equilibria which lead to allocations out of the solution with positive probability. Saijo, Tatamitani and Yamato (1995) conjectured some degree of trade-off between none of "modulo game" and both of the best response property and balancedness. Third, I only consider the case of more than three members economies. It remains to consider Nash implementability of solutions in production economies with two persons by natural mechanisms. Fourth, with respect to CRE, we have not yet obtained convincing answers. This problem also relates to the problem of subgame perfect implementation by reasonable mechanisms.

5. A Concluding Remark

By these results, as long as PR or EB is concerned, we can see that the performance of public ownership economic system from the point of "Natural implementation" is as well as the one of the private ownership system in which the allocation rule is the Walrasian solution. However, notice the existence of another type of incentive problem. That is a shirking problem in firm production — one of moral hazard problems, which I mentioned a little in subsection 2.1 as the problem of contested exchange in the realistic capitalist economy. In the case of public ownership systems also, such a moral hazard problem shall have developed because of cooperative production and costly monitoring. The implementation theory is not concerned with the problem of costly monitoring, so that it cannot give an adequate answer to this another problem. It seems to be interesting to evaluate the performance of various economic systems from the point of the moral hazard problem.

REFERENCES

Abreu, D., Sen, A. (1990): "Subgame Perfect Implementation: A Necessary and Almost Sufficient Condition," *Journal of Economic Theory* **50**, 285-299.

Arrow, K. J., Hahn, F. H. (1971): *General Competitive Analysis*, San Francisco: Holden Day.

Bowles, S., Gintis, H., (1981): "Structure and practice in the labor theory of value," *Review of Radical Political Economics* **12**, pp. 1-26.

Bowles, S., Gintis, H., (1988): "Contested Exchange: Political Economy and Modern Economic Theory," *American Economic Review* **78**(2), pp. 145-150.

Bowles, S., Gintis, H., (1990): "Contested Exchange: New Microfoundation for the Political Economy of Capitalism," *Politics and Society* **18**, pp. 165-222.

Cohen, G. A. (1985): "Nozick on Appropriation," *New Left Review* **150**, 89-107.

Cohen, G. A. (1986): "Self-Ownership, World Ownership, and Equality," *Social Philosophy and Policy* **3**, 76-96.

Cohen, G. A. (1989): "On the Currency of Egalitarian Justice," *Ethics* **99**, 906-944.

Cohen, G. A. (1993): "Equality of What ? On Welfare, Goods, and Capatibilities," in Nussbaum, M. C. and Sen, A. K. eds., *The Quality of Life*, 9-29, Clarendon Press, Oxford.

Dasgupta, P., Hammond, P. and Maskin, E. (1979): "The Implementation of Social Choice Rules," *Review of Economic Studies* **46**, 185-216.

Debreu, G. (1959): *Theory of Value: An Axiomatic Analysis of Economic Equilibrium*, Yale University Press, New Haven / London.

Dworkin, R. (1981a): "What is Equality ? Part 1: Equality of Welfare," *Philosophy and Public Affairs* **10**, 185-246.

Dworkin, R. (1981b): "What is Equality ? Part 2: Equality of Resources," *Philosophy and Public Affairs* **10**, 283-345.

Dutta, B. (1993): "Theory of Implementation," mimeo.

Dutta, B., Sen, A. (1991a): "Necessary and Sufficient Condition for Two-Person Nash Implementation," *Review of Economic Studies* **58**, 121-128.

Dutta, B., Sen, A. (1991b): "Implementation under strong equilibrium: complete characterization," *Journal of Mathematical Economics* **20**, 49-67.

Dutta, B., Sen, A. and Vohra, R. (1995): "Nash Implementation Through Elementary Mechanism in Economic Environments", *Economic Design* **1**, 173-204.

Foley, D. (1967): "Resource Allocation and the Public Sector", *Yale Economic Essays*, **7**, 45-98.

Fujimori, Y., (1982): *Modern Analysis of Value Theory*, Springer-Verlag, Berlin Heidelberg New York.

Gaertner, W., Pattanaik, P. K. and Suzumura, K. (1992): "Individual Rights Revisited," *Economica* **59**, 161-177.

Gevers, L. (1986): "Walrasian social choice: Some simple axiomatic approaches", in Heller *et al.*, eds., *Social Choice and Public Decision Making: Essays in Honor of K. J. Arrow*, **1**, 97-114. Cambridge Univ. Press, London / New York.

Gibbard, A. (1973): "Manipulation of Voting Schemes: A General Result," *Econometrica* **41**, 587-602.

Hong, L. (1995): "Nash Implementation in Production Economies," *Economic Theory* **5**, 401-417.

Hurwicz,L. (1972): "On Informationally Decentralized Systems," in R. Radner and C. B. McGuire, eds., *Decesion and Organization: A Volume in Honor of J. Marschak*, 297-336. North-Holland Publishing Company, Amsterdam.

Hurwicz,L. (1979): "Outcome Functions Yielding Walrasian and Lindahl Allocations at Nash Equilibrium Points," *Review of Economic Studies* **46**, 217-225.

Hurwicz,L., Maskin, E. and Postlewaite, A. (1984): "Feasible Implementation of Social Choice Correspondences by Nash Equilibria," mimeo.

Jackson,M.O. (1992): "Implementation in Undominated Strategies: A Look at Bounded Mechanism," *Review of Economic Studies* **59**, 757-775.

Jackson, M. O., Palfrey, T., Srivastava, S. (1994): "Undominated Nash Implementation in Bounded Mechanisms," *Games and Economic Behavior* **6**, 474-501.

Krause, U., (1982): *Money and Abstract Labor*, New Left Books, London.

Locke, J. (1690): *Of Civil Government*, Dent & Sons, London.

Marx, K., (1867): *Das Kapital, Volume I*, Dietz Verlag, Berlin.

Maskin, E. (1977): "Nash Equilibrium and Welfare Optimality," (mimeo).

Maskin, E. (1979): "Implementation and Strong Nash-Equilibrium," in Lafont, J. (ed.), *Aggregation and Revelation of Preferences* **8**, 432-439. North-Holland Publishing Company, Amsterdam.

Mill, J. S. (1859): *On Liberty*, Penguin, Harmondsworth.

Moore, J. (1992): "Implementation, Contracts and Renegotiation in Environments with Complete Information," in Lafont, J. (ed.), *Advances in Economic Theory: Invited Papers for the Sixth World Congress of the Econometric Society*, Vol. I, 182-282. Cambridge University Press, Cambridge.

Moore, J., Repullo, R. (1988): "Subgame Perfect Implementation," *Econometrica* **56**, 1191-1220.

Moore, J., Repullo, R. (1990): "Nash Implementation: A Full Characterization," *Econometrica* **58**, 1083-1099.

Morishima, M., (1973): *Marx's Economics*, Cambridge University Press, Cambridge.

Morishima, M., (1978): *Value, Exploitation and Growth*, McGraw Hill, London.

Moulin,H. (1987): "A Core Selection for Pricing a Single Output Monopoly", *The Rand Journal of Economics* **18**, 397-407.

Moulin,H. (1990a): "Joint Ownership of a Convex Technology: Comparison of Three Solutions", *Review of Economic Studies* **57**, 439-452.

Moulin,H. (1990b): "Fair Division under Joint Ownership: Recent Results and Open Problems", *Social Choice and Welfare* **7**, 149-170.

Moulin,H., Roemer,J. (1989): "Public Ownership of the External World and Private Ownership of Self", *Journal of Political Economy* **97**, 347-367.

Nagahisa, R. (1991): "A Local Independence Condition for Characterization of Walrasian Allocations Rule", *Journal of Economic Theory* **54**, 106-123.

Nagahisa, R. (1994): "A Necessary and Sufficient Condition for Walrasian Social Choice", *Journal of Economic Theory* **62**, 186-208.

Nash, J. (1950): "The Bargaining Problem," *Econometrica* **28**, 155-162.

Nozick, R. (1974): *Anarchy, State and Utopia*, Basic Books, New York.

Okishio, N., (1963): "A mathematical note on Marxian theorems," *Weltwirtschaftsliches Archiv.*

Palfrey, T., Srivastava, S. (1991): "Nash Implementation Using Undominated Strategies," *Econometrica* **59**, 479-501.

Pazner, E., Schmeidler, D. (1978): "Egalitarian Equivalent Allocations: A New Concept of Economic Equity," *Quarterly Journal of Economics* **92**, 671-687.

Postlewaite, A., Wettstein, D. (1989): "Feasible and Continuous Implementation," *Review of Economic Studies* **56**, 603-611.

Rawls, J. (1971): *A Theory of Justice*, Harvard University Press, Cambridge, Massachusetts.

Roemer, J.E., (1980): "A general equilibrium approach to Marxian economics," *Economerica* **48**, pp. 505-530.

Roemer, J.E., (1982): *A General Theory of Exploitation and Class*, Harvard University Press, Cambridge.

Roemer, J.E., (1986a): *Value, Exploitation and Class*, Harwood Academic Publishers, London.

Roemer, J. (1986b): "The Mismarriage of Bargaining Theory and Distributive Justice," *Ethics* **97**, 88-110.

Roemer, J. (1988): "A Challenge to Neo-Lockeanism," *Canadian Journal of Philosophy* **18**, 697-710.

Roemer, J. (1989): "A Public Ownership Resolution of the Tragedy of the Commons," *Social Philosophy and Policy* **6**, 74-92.

Roemer,J., Silvestre,J. (1989), "Public Ownership: Three proposals for Resource Allocation", mimeo.

Roemer,J., Silvestre,J. (1993), "The Proportional Solution for Economies with Both Private and Public Ownership", *Journal of Economic Theory* **59**, 426-444.

Saijo,T., Tatamitani,Y., and Yamato,T. (1995): "Toward Natural Implementation", forthcoming in *International Economic Review.*

Samuelson, P., (1982): "The normative and positive inferiority of Marx's values paradigm," *Southern Economic Journal* **49** , 1, pp. 11-18.

Satterthwaite, M. A. (1975): "Strategy-proofness and Arrow's Condition: Existence and Correspondence Theorem for Voting Procedures and Social Welfare Functions," *Journal of Economic Theory* **10**, 187-217.

Schmeidler, D. (1980): "Walrasian Analysis via Strategic Outcome Functions," *Econometrica* **40**, 1585-1594.

Sen, A. K. (1970): "The Impossibility of a Paretian Liberal," *Journal of Political Economy* **78**, 152-157.

Sen, A. K. (1979): "Utilitarianism and Welfarism," *Journal of Philosophy* **76**, 463-489.

Sen, A. K. (1980): "Equality of What ?," in McMurrin, S. (ed.), *Tanner Lectures on Human Values* I, Cambridge University Press, Cambridge.

Sen, A. K. (1985): *Commodities and Capatibilities*, North-Holland Publishing Company, Amsterdam.

Sen, A. K. (1992): "Minimal Liberty," *Economica* **59**, 139-159.

Sjöström, T. (1991): "Implementation by Demand Mechanisms," (mimeo).

Steedman, I., (1977): *Marx after Sraffa*, New Left Books, London.

Tatamitani, Y.(1993): "Double Implementation in Nash and Undominated Nash Equilibria in Social Choice Environments," *Economic Theory* **3**, 109-117.

Tian, G. (1989): "Implementation of Lindahl Correspondence by a Single Valued,Feasible and Continuous Mechanism," *Review of Economic Studies* **56**, 613-621.

Tian, G. (1992): "Completely Feasible and Continuous Implementation of the Walrasian Correspondence without Continuous, Convex and Ordered Preferences," *Social Choice and Welfare* **9**, 117-130.

Walker, M. (1981): "A Simple Incentive Compatible Scheme for Attaining Lindahl Allocations," *Econometrica* **49**, 65-73.

Weitzman, M. (1974): "Free Access vs. Private Ownership as Alternative Systems for Managing Common Property," *Journal of Economic Theory* **8**, 225-234.

Yamato, T. (1993): "Double Implementation in Nash and Undominated Nash Equilibria," *Journal of Economic Theory* **59**, 311-323.

Yoshihara, N. (1995): "Wealth, Exploitation and Labor Discipline in the Contemporary Capitalist Economy," forthcoming in *Metroeconomica* (Chapter I of this thesis).

Yoshihara, N. (1995a): "Natural and Double Implementation of Public Ownership Solutions in Differentiable Production Economies," mimeo, (Chapter III of this thesis).

Yoshihara, N. (1995b): "Full Characterizations of Public Ownership Solutions," mimeo, (Chapter II of this thesis).

Yoshihara, N. (1995c): "A Characterization of Natural and Double Implementation in Production Economies," mimeo, (Chapter IV of this thesis).

CHAPTER I

Wealth, Exploitation and Labor Discipline in the Contemporary Capitalist Economy

Abstract: Synthesizes, in one Leontief economic model, the arguments of exploitation and unequal distribution of wealth by Roemer (1982, 1986) and of the power relationship between employers and employees concerning the labor extraction by Bowles and Gintis (1988, 1990). The author introduces the level of the agent's labor-discipline as measured by the ratio of labor effort per unit of labor time to the real wage rate. The connection between this kind of power index and both exploitation status and wealth distribution is then examined. The result obtained is that, under some reasonable assumptions, the exploitation status and the level of labor-discipline accurately reflect the unequal distribution of wealth.

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1. Introduction

During the 1970's, there were remarkable developments in the discussion about exploitation in Marxian economic theory. The "Fundamental Marxian Theorem (FMT)" was originally proved by Okishio (1963) and later named by Morishima (1973). The FMT showed the equivalence between the existence of positive profit and the existence of exploitation. It purports to prove the classical Marxian argument that the exploitation of labor is the source of positive profits in the capitalist economy. After the seminal work by Morishima (1973), there were many generalizations and discussions of the FMT. While the original FMT is discussed in the simple Leontief economy with homogeneous labor, the generalization of the FMT to the Leontief economy with heterogeneous labor is discussed by Fujimori (1982), Krause (1982), e.t.c. The problem in generalizing the FMT to the von Neumann economy is discussed by Steedman (1977) and one resolution is proposed by Morishima (1978). Furthermore, Roemer (1980) generalized the theorem to a convex cone economy. These arguments may reflect the robustness of the FMT.

However, this theorem has a crucial problem: it does not follow from the FMT that the exploitation of labor is the source of positive profits. The reason is that every commodity can be shown as exploited in a system with positive profits whenever the exploitation of labor exists. This observation was pointed out by Brody (1970), Bowles and Gintis (1981), Samuelson (1982), and was named the "Generalized Commodity Exploitation Theorem (GCET)" by Roemer (1982).

After the GCET was proven, there were two remarkable developments in works on Marxian economic theory. The first one was the "General Theory of Exploitation and Class" by Roemer (1982, 1986). This argued that in a capitalist economy if *the labor supplied by agents is inelastic with respect to their wealth* (that is, the value of their own capital), then the theories of Class-Exploitation Correspondence, Class-Wealth Correspondence and Wealth-Exploitation Correspondence can be proven. The second was the "Contested Exchange in the Capitalist Economy" by Bowles and Gintis (1988, 1990)[1]. This argued that in the capitalist production process, by creating employment rents and utilizing the contingent renewal, the employer has the power over employees to extract a desired level of labor efforts.

Some remarks can be made about both arguments. First, Roemer (1982, 1986) stated that the emergence of exploitation and class can be explained, without referring to the enforceability of inducing labor effort in the production process, as a consequence of unequal wealth distribution. However, replacing the neoclassical labor market in Roemer (1982, 1986) by the contested exchange labor market, the robustness of the Wealth-Exploitation Correspondence is not necessarily guaranteed. The reason is that under a given economic environment the inelasticity condition of labor supply is not necessarily ensured in the contested exchange labor market even if it is ensured in the neoclassical labor market.

In contrast, Bowles and Gintis (1988, 1990) argued, without referring to unequal distribution of wealth, *the existence of a power relationship* between employer and employee in which a better labor performance is extracted in the capitalist production process. However, their argument does not explicitly analyze whether or not the strength of an employer's power over its employees can guarantee profitability in the capitalist economy. As far as this problem is concerned, it implies that there is *an effect of distributive inequality of wealth in tightening the power of the employer over the employee.* Hence, the problem to be solved is how to relate the unequal distribution of wealth to the problem of labor discipline.

In this paper, we synthesize, in one Leontief economic model, the analysis of wealth and exploitation by Roemer (1982) and the analysis of the labor discipline by Bowles and Gintis (1988, 1990). We propose *a method for measuring the level of the agent's labor-discipline* in this paper. Moreover, we show that the status of exploitation and the level of labor-discipline correspond to the status of personal wealth in the capitalist economy given several reasonable assumptions. Our objective for this research is not to construct a general theory of wealth and exploitation just as Roemer (1982, 1986) did, but to analyze explicitly the relation of wealth distribution, exploitation and labor discipline in the contemporary capitalist economy[2)] with more reasonable restrictions.

This paper proceeds as follows. Section 2 presents the basic economic model. Section 3 examines the existence of equilibrium in the modeled economy. In section 4, the correspondence between the level of labor-discipline and the status of personal wealth is examined. Finally, section 5 discusses the relation of exploitation, labor discipline and the unequal distribution of wealth.

2. The Basic Model

We consider the economy to be in the following: There are $|\overline{N}|$ agents, and identical firms[3)]. Each agent has some or no capital and one unit of labor endowment. The agent can invest his capital in firms and sell his labor power to firms. All firms have free access to common Leontief technology through production.

Notation:

A is an $n \times n$ productive, indecomposable input matrix

L is a strictly positive $1 \times n$ vector of direct labor inputs

b is a strictly positive $n \times 1$ vector of the real wage basket

p is a $1 \times n$ price vector normalized to $pb = 1$

$\Omega^{\nu} \in R_{+}$ is a scalar of an agent ν's real wage rate

$\omega \in R^{n}_{++}$ is an $n \times 1$ vector of aggregate endowments of capital

$\omega^{\nu} \in R^{n}_{+}$ is an $n \times 1$ vector of an agent ν's capital endowment.

An economy is specified by the list $\{A,\ L,\ b,\ \omega^{1}, \cdots, \omega^{|\overline{N}|}\}$.

Assumption 1: In the economy $\{A,\ L,\ b, \omega^{1}, \cdots, \omega^{|\overline{N}|}\}$, all agents have the same preference and the same level of labor skills. Furthermore, each firm knows the preferences and capital endowments of its agents[4)].

In this economy, agents and firms are engaging in two stages of decisions in one production period. In the first stage, each firm offers labor contracts to agents employed as workers. Suppose that in this period, there are $|N|$ ($\leq|\overline{N}|$) agents employed by firms[5)]. One labor contract with one agent, v, consists of two variables: one, a real wage rate, Ω^v, which the firm will pay to him; and, two, a monitoring project, f^v. Through the monitoring projects, the firm announces to its employees that if some agent is detected as not being productive enough in his work for his offered wage level, he will be fired at the end of the production period. Let $\{\Omega^v, f^v\}_{v\in V}$ be a list of labor contracts offered by firms. Each firm also offers a list of expected profit rates to agents who will invest their finance capital. Then each agent decides in which firm or industry he will invest his capital.

The second stage corresponds to a production process where each employed agent decides his labor effort level. In this stage, there exists a fundamental informational asymmetry between agents and the firm with respect to the effort level of each agent. Namely, while each agent's effort can be perfectly known to the other agents in the workplace, it is only known to the firm by costly monitoring. Hence, the firm's knowledge of its employees only increases with the level of monitoring intensity. At the end of the production period, each firm will fire the agent who is detected as not performing satisfactorily in comparison with his fellow agents employed at the same wage. If it is advantageous for every agent to renew his employment in the next period, such a system serves to induce labor efforts.

In this economy, the role of any firm is to implement profit maximization for capital-owning persons. That object is accomplished through the labor extraction process in production and competition against other firms in the market. There exist two types of competition among firms. First, firms compete in terms of a list of labor contracts. Since all firms are identical, in equilibrium they will adopt the same list of labor contracts in which the same type agents are offered the same wage rate. Second, firms compete in quantities, and in equilibrium a uniform profit rate prevails among all industries. Thus, in our setting, the equilibrium state of competition among firms inter and intra industries is characterized by the same list of labor contracts and an equal profit rate (EPR)[6]. Since describing such competition explicitly is beyond the scope of this paper, it is assumed for simplicity that the number of firms is equal to unity. Thus, the notation N represents the set of employed agents in the representative firm.

In such a situation, the program of the firm is described below.

Notation:

x^{ν} is an $n \times 1$ vector of activity levels operated by agent ν

$x \equiv \sum_{\nu \in N} x^{\nu}$ is an $n \times 1$ vector of aggregate activity levels

e^{ν} is the labor effort of agent ν per unit of labor time[7]

l is the labor time decided by the firm $(0 \leq l \leq 1)$

y^{ν} is an $n \times 1$ vector of activity levels operable through ν s' investing capital.

Facing a set of unemployed agents, $\overline{N} - N$, and a price vector p,

$$\max_{(\{x^{\nu}\}_{\nu \in N}, \{\Omega^{\nu}, f^{\nu}\}_{\nu \in N})} \sum_{\nu \in N} [(p - pA)x^{\nu} - (\Omega^{\nu} + s(f^{\nu}))l] \qquad \text{(p-1)}$$

$$\text{subject to } e^{\nu} l = Lx^{\nu},\ x^{\nu} \geq 0\ (\forall \nu \in N),$$

$$\sum_{\nu \in N} pAx^{\nu} \leq \sum_{\eta \in \overline{N}} pAy^{\eta},\ 0 \leq l \leq 1,$$

where $s(f)$ denotes the monitoring cost per employee corresponding to the monitoring intensity, $f \in R_{+}$. Assume that $s(f)$ is a continuously differentiable function such that $s(f) > 0$ if $f > 0$, $s(0) = 0$, $s'(\cdot) > 0$ and $s''(\cdot) \geq 0$. Note that $\sum_{\nu \in N} [(p - pA)x^{\nu} - (\Omega^{\nu} + s(f^{\nu}))l] = \sum_{\nu \in N} [p - pA - \frac{(\Omega^{\nu} + s(f^{\nu}))}{e^{\nu}} L]x^{\nu} =$ $[p - pA - \left(\frac{\sum_{\nu \in N} (\Omega^{\nu} + s(f^{\nu}))}{\sum_{\nu \in N} e^{\nu}} \right) L]x$. This equation implies that in (p-1), each agent's labor cost (wage and monitoring cost) per unit of labor effort should be minimized.

The firm offers a list of labor contracts $\{\Omega^v, f^v\}_{v\in N}$ and a list of capital contracts $[\pi_i] = [\pi_1,\ldots,\pi_n]$ where $\pi_i = \dfrac{[p_i - pA_i - \left(\dfrac{\sum_{v\in N}(\Omega^v + s(f^v))}{\sum_{v\in N} e^v}\right)L_i]x_i}{p_i\omega_i}$ so as to implement the program (p-1). Then each agent v in N or $\overline{N} - N$ faces his optimization program in supplying his labor effort and or his capital.

Notation:

$(e|_{\Omega^v, f^v})^{-v}$ is a list of the labor efforts of other employed agents offered the same labor contract as v's

α is the probability of being hired in the next period for any unemployed agent

$\delta \geq 0$ is the growth factor of population

r is the rate of time preference

$\Pi(p\omega^v, \Omega^v l)$ is the revenue of agent v when price is p, the real wage rate of agent v is Ω^v, and the labor time is l

V_E^v is agent v's present value of being employed

V_U^v is agent v's present value of being unemployed

d^v is the probability of dismissal for agent v

$\beta(f)$ is the rate of people who are detected shirking under f monitoring where $f = [f^1, \cdots, f^{|M|}]$.

Any agent v s' preference is represented by a utility function $u(\Pi(p\omega^v, \Omega^v l), e^v l)$ of his revenue, $\Pi(p\omega^v, \Omega^v l)$, and his labor supply, $e^v l$. Assume that u is a twice continuously differentiable function as follows:

$$u_e \equiv u_{el}\cdot l < 0 \text{ if } l > 0,\ u_\Pi > 0,\ u_{ee} \leq 0,\ u_{\Pi\Pi} \leq 0,\ u_{\Pi e} = 0 \text{ and}$$

$$\mu(\Pi^v) \text{ is non-increasing in } \Pi^v \text{ where } \mu(\Pi^v) = -u_{\Pi\Pi}/u_\Pi.$$

The assumption of $u_{\Pi e}$ is reasonable in this context, since every agent can receive his revenue of profit and/or wage irrespective of his real labor performance. The assumption of $u_{\Pi\Pi}$ and $\mu(\Pi^v)$ relates to the risk attitude of agents. The non-positive

$u_{\Pi\Pi}$ implies that all agents are not risk-seeking, and the non-increasing $\mu(\Pi^v)$ implies that all agents are either risk-neutral or non-increasing risk averse. These settings are natural assumptions under uncertainty (Kreps (1990)).

If v is unemployed in this period, then his program is only to maximize $u(\Pi(p\omega^v, \hat{w}), 0)$ subject to $\Pi(p\omega^v, \hat{w}) = [\pi_i pA_i] y^v + \hat{w}$ and $pAy^v \leq p\omega^v$, where $\hat{w}$ represents a reservation wage exogenously supplied by the non-capitalist sector. In the following, for simplicity, we assume that $\hat{w} = 0$.

If v is employed in this period, then his program can be written as follows[8]: For a given $(\Omega^v,\ l,\ f^v,\ \alpha,\ (e|_{\Omega^v, f^v})^{-v},\ [\pi_i],\ p)$ and given v's expected utility of being unemployed V_U^v,

$$\max_{e^v, y^v} V_E^v = \frac{u(\Pi(p\omega^v, \Omega^v l), e^v l) + d(f^v, e^v, (e|_{\Omega^v, f^v})^{-v}) V_U^v}{r + d(f^v, e^v, (e|_{\Omega^v, f^v})^{-v})} \qquad \text{(p-2)}$$

$$\text{subject to } \Pi(p\omega^v, \Omega^v l) = [\pi_i pA_i] y^v + \Omega^v l,$$

$$pAy^v \leq p\omega^v,$$

where $V_U^v = \dfrac{1-\alpha}{r+\alpha} u(\Pi(p\omega^v, 0), 0) + \dfrac{\alpha(1+r)}{r+\alpha} V_E^v$ and $\alpha = \dfrac{\beta|N|}{\delta|\overline{N}| - (1-\beta)|N|}$. Note that all agents have a common subjective probability function of dismissal, $d(\cdot,\cdot,\cdot)$, since they are assumed identical. Assume that $d(\cdot,\cdot,\cdot)$ is a continuously differentiable function as follows:

$$\text{for any given } e^{-v},\ d_e < 0,\ d_{ee} > 0,\ d_f > 0,\ d_{fe} < 0 \text{ and } d(0,\cdot,\cdot) = 0.$$

In the following, the probability function is specified as

$$d(f^v, e^v, (e|_{\Omega^v, f^v})^{-v}) = \min\left[\frac{f^v}{\{e^v\}^2}\phi(v)\ ,\ 1\right],$$

where $\phi(v) = \dfrac{e^v + \psi(v)}{2}$, $\psi(v) = \dfrac{\sum_{\eta \in N(v) - \{v\}} (e|_{\Omega^v, f^v})^\eta}{|N(v)|}$ and

$N(v) = \{\eta \in N \mid (\Omega^\eta, f^\eta) = (\Omega^v, f^v)\}$. It is easy to check that such a formulation satisfies the above assumptions of $d(\cdot,\cdot,\cdot)$[9].

We can now define equilibrium. Let us denote $A(p)$, the set of solution triples of (p-1). Also, denote $B(p, \Omega^{\nu}, f^{\nu}, [\pi_i], V_U^{\nu})$, the set of solution pairs of agent ν's optimization program. Then the equilibrium concept is[10)]:

Definition 1. A tuple $(p, \{x^{\nu}\}_{\nu\in N}, \{\Omega^{\nu}, f^{\nu}\}_{\nu\in N}, \{e^{\nu}, y^{\nu}\}_{\nu\in N}, \{V_U^{\nu}\}_{\nu\in N})$ is a reproducible solution for the economy $\{A, L, b, \omega^1, \cdots, \omega^{|\overline{N}|}\}$ if:

(a) $\exists(\{x^{\nu}\}_{\nu\in N}, \{\Omega^{\nu}, f^{\nu}\}_{\nu\in N}) \in A(p)$, and (profit maximization)

(b) $\forall \nu \in \overline{N}$, $\exists(e^{\nu}, y^{\nu}) \in B(p, \Omega^{\nu}, f^{\nu}, [\pi_i], V_U^{\nu})$ such that (individual optimization)

(c) $x = \sum_{\nu\in N} x^{\nu}$ and $x \geq Ax$ (reproducibility)

(d) $Ax \leq \sum_{\nu\in\overline{N}} Ay^{\nu} \leq \omega \equiv \sum_{\nu\in\overline{N}} \omega^{\nu}$ (feasibility of production)

Notice that this equilibrium concept permits the existence of unemployment.

3. The existence of reproducible solutions

In this section, we characterize solutions for the optimization programs of the firm and agents, and show the existence of reproducible solutions in the economy. First, it is shown that under the stationary expectation, there exists an equilibrium labor contract implemented in subgame perfection. In this state, the optimal capital contract is also characterized. Second, it is shown that there exists a unique stationary expectation under some reasonable assumptions. Finally, the existence of reproducible solutions with unemployment is shown under some assumptions.

3.1. The decision processes of labor and capital contracts

Consider the second stage of decision in the production period. Let $\{\Omega^{\nu}, f^{\nu}\}_{\nu\in N}$ and $[\pi_i]$ be labor and capital contracts offered by the firm. If agent ν has some capital endowment, then he invests his finance capital such that $pAy^{\nu} = W^{\nu}$ ($\equiv p\omega^{\nu}$) whenever $[\pi_i]$ has some positive components. If agent ν is employed, he solves the problem (p-2). In the following, suppose that for any $\nu \in N$, $V_E^{\nu} > V_U^{\nu}$ — a reasonable assumption. Solving the problem (p-2), we obtain the following condition:

$$\frac{d_e^{\nu}\{u(\Pi^{\nu}, e^{\nu} l) - rV_U^{\nu}\}}{u_e^{\nu}} = r + d(f^{\nu}, e^{\nu}, (e|_{\Omega^{\nu}, f^{\nu}})^{-\nu}) \tag{1}$$

From (1), we define an implicit function, F, as follows:

$$F(e^{\nu} l, \Pi(p\omega^{\nu}, \Omega^{\nu} l), f^{\nu}, rV_U^{\nu}, (e|_{\Omega^{\nu}, f^{\nu}})^{-\nu}) \tag{1'}$$

$$\equiv d_e^{\nu}\{u(\Pi^{\nu}, e^{\nu} l) - rV_U^{\nu}\} - u_e^{\nu}\{r + d(f^{\nu}, e^{\nu}, (e|_{\Omega^{\nu}, f^{\nu}})^{-\nu})\} = 0$$

In the neighborhood of optimal labor effort e^{ν}, the function $F(\cdot) = 0$ is continuous, and also F_e, F_l, F_{Π}, F_{rV_U}, F_f and $F_{e^{-\nu}}$ are continuous. Moreover, $F_e \neq 0$, since F_e = $d_{ee}(u - rV_U) - u_{ee}(r + d) > 0$ by the above assumptions $V_E^{\nu} > V_U^{\nu}$, $d_{ee} > 0$ and $u_{ee} \leq 0$. Thus, by applying the implicit function theorem, we can obtain a labor extraction function as follows:

$$e^{\nu} = e(\Pi(p\omega^{\nu}, \Omega^{\nu} l), l, f^{\nu}, rV_U^{\nu}, (e|_{\Omega^{\nu}, f^{\nu}})^{-\nu}) \tag{2}$$

Lemma 1: Under the stationary expectation, for a given p and $[\pi_i]$, any two agent ν, $\eta \in N$, who have the same value of capital, $W^{\nu} = W^{\eta}$, have the same optimal effort level of labor for any given (Ω, f).

Proof : Since the only difference of initial conditions among agents in N is the value of capital endowment, the difference of labor efforts for any given (Ω, f) is only generated by the difference of capital endowments. Q.E.D.

Lemma 1 implies that, to minimize labor costs per unit of labor effort, the firm should differentiate labor contracts among agents according to their capital endowments.

Lemma 2: Under the above assumptions, for any given agent in N, the optimal labor effort increases with the real wage.

Proof: From (1)', for any $v \in N$,

$$\frac{\partial e^v}{\partial \Omega} = \frac{d_e \cdot u_\Pi \cdot l - (r+d) u_{e\Pi} \cdot l}{u_{ee}(r+d) - d_{ee}(u - rV_U)} \tag{3}$$

Since $u_\Pi > 0$, $u_{ee} \leq 0$, $u_{\Pi e} = 0$, $d_e < 0$, $d_{ee} > 0$ and $V_E^v > V_U^v$, clearly $\frac{\partial e^v}{\partial \Omega} > 0$. Q.E.D.

Lemma 2 is a necessary condition for the efficiency wage labor market.

Next, consider the first stage of decision in the production period. Notice that the firm can calculate any agent v's labor effort for any given (Ω^v, f^v), since the firm knows the agent's preference and capital endowment. Hence, the problem (p-1) can be separated into the following two steps:

(First) For given $(\bar{N} - N,\ p)$ and $l \in [0, 1]$,

$$\min_{\Omega^v, f^v} \frac{\Omega^v + s(f^v)}{e(\Pi(p\omega^v, \Omega^v l),\ l,\ f^v, rV_U^v, (e|_{\Omega^v, f^v})^{-v})} \quad (\forall v \in N) \qquad \text{(p-1-1)}$$

where $e(\Pi(p\omega^v, \Omega^v l),\ l,\ f^v, rV_U^v, (e|_{\Omega^v, f^v})^{-v})$ is induced from (1)'.

(Second) For given $(\bar{N} - N,\ p)$ and n-tuple solution pairs of (p-1-1), $\{\Omega^{*v}(l), f^{*v}(l)\}_{v \in N}$ $(\forall\ l \in [0, 1])$,

$$\max_{l \in [0,\ 1]} \sum_{v \in N} [p - pA - \left(\frac{\Omega^{*v}(l) + s(f^{*v}(l))}{e(\Pi(W^v, \Omega^{*v}(l) l),\ l,\ f^{*v}(l),\ rV_U^v)} \right) L] x^v(l) \qquad \text{(p-1-2)}$$

subject to $e(\Pi(W^v, \Omega^{*v}(l) \cdot l),\ f^{*v}(l),\ rV_U^v) l = Lx^v(l) \quad (\forall v \in N)$

$pAx(l) \leq p\omega$, $x(l) \geq 0$ and $0 \leq l \leq 1$.

Let denote a solution triple of (p-1-1) and (p-1-2) by $(\{\Omega^{*v}, f^{*v}\}_{v \in N},\ l^*)$. Then, the first order condition of (p-1-1) is as follows[11]:

$$(\Omega^{*v}, f^{*v}) = \arg\min_{\Omega^v, f} \frac{(\Omega^v + s(f^v))}{e^v} \Rightarrow e^v_{\Omega^{*v}} = \frac{e^{*v}}{\Omega^{*v} + s(f^{*v})},\ s'(f^*) = \frac{e^v_{f^*}}{e^v_{\Omega^{*v}}} \tag{4}$$

[Insert Figure 1]

Lemma 3: Under the above assumptions, for any agent v in N, if (Ω^{*v}, f^{*v}) satisfies the condition (4), then (Ω^{*v}, f^{*v}) satisfies the second order condition.

Proof: See Appendix. Q.E.D.

Proposition 1: Under the stationary expectation, a list of labor contracts $\{\Omega^{*v}, f^{*v}\}_{v \in N}$ and a list of labor efforts $\{e^{*v}\}_{v \in N}$ are implemented in subgame-perfect equilibrium if they satisfy the following conditions:

$\{\Omega^{*v}, f^{*v}\}_{v \in N}$ satisfies the condition (4) for each agent v in N, and

$$e^{*v} = e(\Pi(W^v, \Omega^{*v} l^*), l^*, f^{*v}, rV_U^v) \quad (\forall v \in N).$$

Proof: Consider the following strategy combination:

The firm's strategy: Offer $\{\Omega^{*v}, f^{*v}\}_{v \in N}$ satisfying (4), and renew the employment in the next period for agent v if $e^v \geq e(\Pi(W^v, \Omega^{*v} l^*), l^*, f^{*v}, rV_U^v)$, and dismiss him at the end of this period if $e^v < e(\Pi(W^v, \Omega^{*v} l^*), l^*, f^{*v}, rV_U^v)$.

Any agent v's strategy: Supply $e^v = e(\Pi(W^v, \Omega^v l^*), l^*, f^v, rV_U^v)$ when (Ω^v, f^v) is offered.

Clearly, this strategy combination constitutes a subgame-perfect equilibrium. Q.E.D.

The firm's threatening strategy would be credible even in the case of full employment if the rate of population growth exogenously given exceeds the maximal rate of capital accumulation. In such a case, any agent is threatened by the existence of potential reserve armies that will appear in the next period of production[12)].

3.2. The existence of a stationary expectation of reservation utilities

The above analysis assumes that the economy is under the stationary expectation. That is, the ex-ante value of reservation utility of each agent coincides with the ex-post value of reservation utility 13). Now, we will show that there exists a unique value which sustains the stationary expectation.

Let's denote the ex-ante value of reservation utility of each agent v by $z_0^v \equiv rV_U^v$. Because of the dependence of the solution satisfying condition (4) on the value of z_0^v for each v, we can define continuous functions, $\Omega^{*v}(z_0^v)$, $f^{*v}(z_0^v)$, $l^*(z_0)$ and $e^{*v}(z_0)$ where $z_0 = (z_0^1, \cdots, z_0^v, \cdots, z_0^{|N|})$.

By definition, $\alpha = \min\{\frac{\beta|N|}{\delta|\overline{N}|-(1-\beta)|N|}, 1\}$. Consider β which represents the rate of agent dismissals. Let's denote $\theta \equiv \sum_{v \in N} \max(\theta_1^v, \theta_2^v) f^v$ where

$$\theta_1^v = \begin{cases} 1 & if\, e(\Omega^{*v}, f^{*v}, l^*) - e^v > 0 \\ 0 & if\, e(\Omega^{*v}, f^{*v}, l^*) - e^v \leq 0 \end{cases}, \quad \theta_2^v = \begin{cases} 1 & if\, \psi(v) - e^v > 0 \\ 0 & if\, \psi(v) - e^v \leq 0 \end{cases}$$

and $e(\Omega^{*v}, f^{*v}, l^*)$ is the critical labor effort derived from (4). Then we define a continuous function $\beta(\theta)$ such that $\beta(0) = 0$, $\beta'(\cdot) \geq 0$ and $\beta(\theta) \in [0,1]$ ($\forall \theta$). Since $f^{*v}(z_0^v)$ and $e^{*v}(z_0)$, clearly $\beta(\theta(z_0))$. Thus, we obtain $\alpha = \alpha(z_0)$.

By the above arguments, we can obtain the ex-post value of reservation utility continuously corresponding to the ex-ante value of reservation utility as follows:

$$rV_U^v(z_0) = \frac{r(1-\alpha(z_0))}{r+\alpha(z_0)} u(\Pi(W^v, 0), 0) + \frac{(1+r)\alpha(z_0)}{r+\alpha(z_0)} rV_E^{*v}(z_0)$$

where $rV_E^{*v}(z_0) =$

$$\frac{r}{r + d(f^{*v}(z_0^v), e^{*v}(z_0))} u(\Pi(W^v, \Omega^{*v}(z_0^v) l^*(z_0)), e^{*v}(z_0) l^*(z_0)) + \frac{d(f^{*v}(z_0^v), e^{*v}(z_0))}{r + d(f^{*v}(z_0^v), e^{*v}(z_0))} z_0^v$$

By condition (1), we can rewrite the above equation as follows:

$$rV_U^v(z_0) = \frac{r(1-\alpha(z_0))}{r+\alpha(z_0)} [u(\Pi(W^v, 0), 0) + g^v(z_0)] + \frac{(1+r)\alpha(z_0)}{r+\alpha(z_0)} z_0^v$$

where $g^v(z_0) = A(z_0) r \frac{u_{e^{*v}}^v(z_0)}{d_{e^{*v}}^v(z_0)}$ and $A(z_0) = \frac{(1+r)\alpha(z_0)}{r(1-\alpha(z_0))}$. Note that $rV_U^v(z_0)$ is a convex combination of $[u(\Pi(W^v, 0), 0) + g^v(z_0)]$ and z_0^v.

We show that there exist $\{z_0^v\}_{v \in N}$, such that for each v, $rV_U^v(z_0) = z_0^v$ or $[u(\Pi(W^v, 0), 0) + g^v(z_0)] = z_0^v$. Note that in equilibrium of labor contracts, according to Lemma 1 and Proposition 1, any agents who have the same value of capital provide the same level of labor effort which supports the firm's cost minimization. Hence, $\theta^v = 0$ for all v in N. This implies $\beta = 0$ in the equilibrium[14]. Hence, $[u(\Pi(W^v, 0), 0) + g^v(z_0)]$ is reduced to $u(\Pi(W^v, 0), 0)$ in equilibrium. Clearly, z_0^v = $u(\Pi(W^v, 0), 0)$ is a unique value of v's reservation utility consistent with the labor contract equilibrium under the stationary expectation.

[Insert Figure 2]

Proposition 2 : Set β as the above formula. Then there exists a unique stationary expectation consistent with the labor contract equilibrium.

3.3. The existence of reproducible solutions (RS)

Assume that the economy is under the stationary expectation. Then the value of reservation utility is $rV_U^v = u(\Pi(W^v, 0), 0)$ for each v. By (p-1-1), this implies that the value of Ω^{*v}, f^{*v} and the level of labor effort, e^{*v}, are dependent on the level of v's profit revenue, πW^v, and labor time l. Thus, we obtain continuous functions, $\Omega^{*v}(\pi W^v, l)$, $f^{*v}(\pi W^v, l)$ and $e^{*}(\pi W^v, l)$ as the solutions of (p-1-1) and (p-2).

Let's denote $\mathbf{C} \equiv \{\omega \in R_+^n \mid \exists\, x \geq 0 \; s.t. \; Ax = \omega \text{ and } x \geq Ax\}$[15] and, for any given $\omega \in \mathbf{C}$, $\tilde{C}(\omega) \equiv \{(\omega^1, \cdots, \omega^{|N|}) \in R_+^{n|N|} \mid \sum_{v \in N} \omega^v = \omega\}$. Define a continuous function $\gamma(p) = \max_i \left\{ \dfrac{p_i - pA_i}{pA_i} \right\}$ on $\Delta \equiv \{p \in R_+^n \mid pb = 1\}$, and denote that $\gamma^m = \max_{p \in \Delta} \gamma(p)$. Define for each $l \in [0, 1]$, ε_N^l: $\tilde{C}(\omega) \times \Delta \times [0, \gamma^m] \rightarrow R_+$ such that $\varepsilon_N^l(\tilde{\omega}, p, \pi) = \sum_{v \in N} e(\pi p \omega^v, l)$ where $\tilde{\omega} \equiv (\omega^1, \cdots, \omega^{|N|})$. Clearly, $\varepsilon_N^l(\tilde{\omega}, p)$ is continuous at each $(\tilde{\omega}, p, \pi)$. Given $(p, \pi) \in \Delta \times [0, \gamma^m]$, there exists $\tilde{\omega}^* \in \tilde{C}(\omega)$ such that $\tilde{\omega}^* = \arg\max\, \varepsilon_N^l(\tilde{\omega}, p, \pi)$. It is well-defined because $\tilde{C}(\omega)$ is compact. $\tilde{\omega}^*$ of this kind is determined for each $(p, \pi) \in \Delta \times [0, \gamma^m]$. Thus, we can obtain an upper-hemi continuous correspondences $\tilde{\omega}^*(p, \pi)$ at every $(p, \pi) \in \Delta \times [0, \gamma^m]$. Also, $\varepsilon^l_{\bar{N}}(\tilde{\omega}^*(p, \pi), p, \pi)$ is continuous at every $(p, \pi) \in \Delta \times [0, \gamma^m]$ according to the maximum theorem of Berge. Since Δ is compact, there exists for each given $\pi \in [0, \gamma^m]$, $p^*(\pi) = \arg\max_{p \in \Delta} \varepsilon^l_{\bar{N}}(\tilde{\omega}^*(p^*(\pi), \pi), p^*(\pi), \pi)$. Since $\varepsilon^l_{\bar{N}}(\tilde{\omega}^*(p^*(\pi), \pi), p^*(\pi), \pi)$ is continuous on $[0, \gamma^m]$, there exists a $\max_{\pi \in [0, \gamma^m]} \varepsilon^l_{\bar{N}}(\tilde{\omega}^*(p^*(\pi), \pi), p^*(\pi), \pi)$. Denote this by $\max e(\overline{N}, l)$. Then, there exists $\max e(\overline{N}, l^*)l^* = \max_{l \in [0, 1]} \{\max e(\overline{N}, l)l \mid l \in [0, 1]\}$. Assuming A^{-1} exists, define the following set: $\mathfrak{W}_+ \equiv \{\omega \in \mathbf{C} \mid LA^{-1}\omega \leq \max e(\overline{N}, l^*)l^*\}$.

Assumption 2 : In this economy, $\omega \in \mathfrak{W}_+$.

Note that there exists a unique vector $x^* > 0$ such that $x^* = (1 + \lambda)Ax^*$, according to the Perron-Frobenius Theorem, where $0 < (1 + \lambda)^{-1} < 1$ is the unique associating eigen value. Since $Ax^* \in \mathbf{C}$, so that $\mathbf{C}$ is non-empty. Moreover, there exists $\rho \geq 0$ such that $\rho Lx^* \leq \max e(\overline{N}, l^*)l^*$. Hence, $\mathfrak{W}_+$ is also non-empty.

Theorem 1 : Let the economy be at a non-trivial RS. Then the associating price vector p is the EPR price with $\pi \geq 0$ as follows:

$$p = (1 + \mu)pA + CL \text{ where } \mu = \frac{\pi}{\sigma} \geq 0 \ (\exists \sigma \in (0, 1]), \text{ and } C = \frac{\sum_{v \in V}(\Omega^v + s(f^v))}{\sum_{v \in V} e^v}.$$

Assumption 3 : Either $e^v_l = 0$ $(v \in \overline{N})$, or $e^v_l < 0$ and $e^v_{ll} < 0$ $(v \in \overline{N})$.

Theorem 2 : Let Assumption 2 and 3 hold. Then, under the stationary expectation, for every $\tilde{\omega} \in \tilde{C}(\omega)$, there exists a RS.

Theorem 1 and 2 are proved in Appendix. The above theorems reveal the knife-edge property of RS. The reason is that some change in the initial distribution of endowments may drastically change the number of employed agents in equilibrium.

4. The relation between the level of an agent's labor-discipline and the level of his wealth

In this section, we analyze the relationship between the level of agent's labor-discipline and the level of his wealth. How should we measure *the level of each agent's labor-discipline* ? In this paper, I will define *the level of an agent's discipline* as *the supply of labor effort per unit of the agent's received real wage.* So, the more labor effort some agent supplies per unit of his received real wage rate, the higher the level of his labor-discipline is.

Definition 2: One agent ν is *more labor-disciplined than* the other agent η if the following is satisfied: $\frac{e^{*\nu}}{\Omega^{*\nu}} > \frac{e^{*\eta}}{\Omega^{*\eta}}$

The implication of this definition is clear. Every employed agent must work under the firm's control. However, the final factor in deciding how much labor effort is supplied is ultimately up to the individual agent even if he is threatened by the firm's dismissal policy (the monitoring project). In spite of this fact, if agent ν provides more labor effort per unit of the real wage rate than agent η , agent ν seems to demonstrate that he is more vulnerable to the firm's control than agent η . In other words, agent ν is more obedient to the firm's control than agent η [16).

In the following, without loss of generality, assume a RS with $l = 1$. It is shown first that the wealthier agent provides a higher level of labor effort than the less wealthy agent in a non-trivial RS with $\pi > 0$. Second, despite this fact, the wealthier agent's optimal labor cost per unit of his labor effort is higher than the less wealthy agent's. Thus, the less wealthy agent has a higher level of labor-discipline than the wealthier agent. Finally, as a corollary, the wealthier agent's optimal real wage rate is higher than the less wealthy agent's, thereby establishing in our economy a poverty law in capital accumulation (Marx (1986)). It is also shown that the labor effort supplied by any agent is inelastic with respect to his wealth. That is a sufficient condition for the

Wealth-Exploitation Correspondence Principle (Roemer (1982)), discussed in the next section.

Take the labor extraction function $e(\Omega^*(W^v), f^*(W^v), rV_U(W^v), W^v)$ in a non-trivial RS with $\pi > 0$, which is derived from (p-1-1) and (p-2). Note that the level of the supplied labor effort continuously corresponds to the endowed wealth of the agent, because there is no difference among agents except in capital endowments.

Proposition 3: Let the economy be in a non-trivial RS with $\pi > 0$. Then the wealthier agent supplies a level of labor effort greater than or equal to the less wealthy agent.

Proof: Consider the following calculation:

$$\frac{d\,e(W^v)}{d\,W^v} = \frac{\partial e}{\partial \Omega^*(W^v)}\frac{\partial \Omega^*(W^v)}{\partial W^v} + \frac{\partial e}{\partial f^*(W^v)}\frac{\partial f^*(W^v)}{\partial W^v} + \frac{\partial e}{\partial W^v} + \frac{\partial e}{\partial rV_U(W^v)}\frac{\partial rV_U(W^v)}{\partial W^v}$$

$$= \frac{\partial e}{\partial \Omega^*}\left\{\frac{\partial \Omega^*}{\partial W^v} + \frac{\partial \Omega^*}{\partial rV_U}\frac{\partial rV_U}{\partial W^v}\right\} + \frac{\partial e}{\partial f^*}\left\{\frac{\partial f^*}{\partial W^v} + \frac{\partial f^*}{\partial rV_U}\frac{\partial rV_U}{\partial W^v}\right\} + \frac{\partial e}{\partial rV_U}\frac{\partial rV_U}{\partial W^v} + \frac{\partial e}{\partial W^v}$$

$$= \frac{d_{e^*}\{u_{\Pi(W^v,\Omega^*)} - u_{\Pi(W^v,0)}\}\pi}{d_{e^*e^*}\{u(\Pi(W^v,\Omega^*),e^*) - rV_U\} - u_{e^*e^*}(r+d)} \quad \text{for any } W^v \geq 0.$$

Since $u(\Pi(W^v,\Omega^*),e^*) > rV_U$, $d_{e^*e^*} > 0$ and $u_{e^*e^*} \leq 0$ by the aforementioned assumptions, the denominator of the last right-hand side of the above equation is positive. On the other hand, the numerator is a non-negative because $d_{e^*} < 0$ and $u_{\Pi(W^v,\Omega^*)} \leq u_{\Pi(W^v,0)}$ which is followed by $u_{\Pi\Pi} \leq 0$ and $u_{\Pi e} = 0$. Thus, $\frac{d\,e(W^v)}{d\,W^v} \geq 0$ for any $W^v \geq 0$. This proves the statement. Q.E.D.

Proposition 4: Let the economy be in a non-trivial RS with $\pi > 0$. Then the wealthier agent's optimal labor cost per unit of his labor effort is not lower than the less wealthy agent's.

Proof: Denote the optimal labor cost per unit of the labor effort as follows:

$$C(e^*(W^v),\Omega^*(W^v),f^*(W^v)) \equiv \frac{\Omega^*(W^v)+s(f^*(W^v))}{e(\Omega^*(W^v),f^*(W^v),rV_U(W^v),W^v)}$$

By applying the envelop theorem, we can obtain the following:

$$\frac{\partial\, C(e^*(W^v),\Omega^*(W^v),f^*(W^v))}{\partial\, W^v} =$$

$$\frac{-(\Omega^*+s(f^*))}{\{e^*\}^2}\;\frac{d_{e^*}\{u_{\Pi(W^v,0)}-u_{\Pi(W^v,\Omega^*)}\}\pi}{d_{e^*e^*}\{u(\Pi(W^v,\Omega^*),e^*)-rV_U\}-u_{e^*e^*}(r+d)} \quad \text{for any } W^v \geq 0.$$

Since $\dfrac{d_{e^*}\{u_{\Pi(W^v,0)}-u_{\Pi(W^v,\Omega^*)}\}\pi}{d_{e^*e^*}\{u(\Pi(W^v,\Omega^*),e^*)-rV_U\}-u_{e^*e^*}(r+d)} \leq 0$, we can obtain:

$$\frac{\partial\, C(e^*(W^v),\Omega^*(W^v),f^*(W^v))}{\partial\, W^v} \geq 0 \quad \text{for any } W^v \geq 0.$$

This proves the statement. Q.E.D.

Lemma 4: Let the economy be in a non-trivial RS with $\pi > 0$. Then the labor effort supplied by any agent is inelastic with respect to wealth.

Proof: We show that

$$\frac{d\log e^*(W^v)}{d\log(s(f^*(W^v))+\Omega^*(W^v))}\;\frac{d\log(s(f^*(W^v))+\Omega^*(W^v))}{d\log W^v} \leq 1.$$

By Proposition 4, it is clear that $\dfrac{d\log e^*(W^v)}{d\log(s(f^*(W^v))+\Omega^*(W^v))} \leq 1$. Check that $\dfrac{d\log(s(f^*(W^v))+\Omega^*(W^v))}{d\log W^v} \leq 1$. Since, by condition (4), $s'(f^*)=$ $e_{f^*}/e_{\Omega^*}=F_{f^*}/F_{\Omega^*}$, we can obtain: $\dfrac{\partial(s(f^*(W^v))+\Omega^*(W^v))}{\partial\, W^v} = \dfrac{2\pi}{l}\left\{\dfrac{u_{\Pi(W^v,0)}}{u_{\Pi(W^v,\Omega^*)}}-1\right\}$

Since $u_{\Pi(W^v,\Omega^*)} \leq u_{\Pi(W^v,0)}$ by the concavity of u, $\dfrac{\partial(s(f^*(W^v))+\Omega^*(W^v))}{\partial\, W^v} \geq 0$.

However, since $u_{\Pi(W^v,0)}/u_{\Pi(W^v,\Omega^*)}$ is non-increasing in W^v by the assumption of

$\mu(\Pi^{v})$, the slope of $\dfrac{\partial\,(s(f^{*}(W^{v}))+\Omega^{*}(W^{v}))}{\partial\,W^{v}}$ is non-increasing in W^{v}. This implies that $\dfrac{d\log\,(s(f^{*}(W^{v}))+\Omega^{*}(W^{v}))}{d\,\log W^{v}}\leq 1$. Q.E.D.

Theorem 3: (The Correspondence of Wealth and Labor-Discipline) Let the economy be in a non-trivial RS with $\pi>0$. Then the less wealthy agent has a higher level of labor-discipline than wealthier agents if agents are risk averse.

[Insert Figure 3]

Proof: Consider a two-dimensional non-negative Euclid space where the ordinate's axis represents a level of labor effort and the abscissa's axis represents a level of the real wage rate. We call this the (Ω,e)-space in the following. Note that for any given (Ω,f),

$$\frac{\partial\,e}{\partial\,W^{v}}=\frac{d_{e}\{u_{\Pi(W^{v},0)}-u_{\Pi(W^{v},\Omega)}\}\pi}{d_{ee}\{u(\Pi(W^{v},\Omega),e\,)-rV_{U}\}-u_{ee}(r+d)}\leq 0\quad(\forall v\in N).$$

This fact implies that if there exist two agents v and η such that $W^{v}<W^{\eta}$, then η's labor extraction curve depicted in the (Ω,e)-space should shift down from v's labor extraction curve.

Let (Ω^{*v},e^{*v}) be v's labor contract point in the (Ω,e)-space. Depict a line from the origin through this point with a slope e^{*v}/Ω^{*v}. We shall call this line the $\left(e^{*v}/\Omega^{*v}\right)$-line. In the following, we show that η's labor contract point $(\Omega^{*\eta},e^{*\eta})$ cannot lie either on or above the $\left(e^{*v}/\Omega^{*v}\right)$-line except at the point of (Ω^{*v},e^{*v}).

First, in the interval $[0,\Omega^{*v})$, the 2nd component of any point in the $\left(e^{*v}/\Omega^{*v}\right)$-line cannot exceed the value $e^{*v}(W^{v})$. However, by Proposition 3, $e^{*\eta}(W^{\eta})\geq e^{*v}(W^{v})$ should be established. This implies that, in the interval $[0,\Omega^{*v})$, $(\Omega^{*\eta},e^{*\eta})$ cannot lie on the $\left(e^{*v}/\Omega^{*v}\right)$-line. Suppose that, in the interval $[0,\Omega^{*v})$, $(\Omega^{*\eta},e^{*\eta})$ lies above the $\left(e^{*v}/\Omega^{*v}\right)$-line. Then, since η's labor extraction curve is depicted below v's labor extraction curve in the (Ω, e)-space, $e^{*\eta}(W^{\eta}) < e^{*\nu}(W^{\nu})$ would be established. This is a contradiction. Of course, in the interval $[0, \Omega^{*\nu})$, $(\Omega^{*\eta}, e^{*\eta})$ cannot lie below the $\left(e^{*\nu}/\Omega^{*\nu}\right)$-line because $e^{*\eta}(W^{\eta}) \geq e^{*\nu}(W^{\nu})$ should be established. Thus, $(\Omega^{*\eta}, e^{*\eta})$ cannot lie in the sub-(Ω, e)-space, $[0, \Omega^{*\nu}) \times R_{+}$, whenever $W^{\nu} < W^{\eta}$.

Second, in the interval $(\Omega^{*\nu}, +\infty)$, any point on the $\left(e^{*\nu}/\Omega^{*\nu}\right)$-line is above ν's labor extraction curve, because the slope of ν's labor extraction curve at $(\Omega^{*\nu}, e^{*\nu})$ is equal to the slope of the line $\frac{e^{*\nu}}{s(f^{*\nu}) + \Omega^{*\nu}}$, and clearly $\frac{e^{*\nu}}{s(f^{*\nu}) + \Omega^{*\nu}} < \frac{e^{*\nu}}{\Omega^{*\nu}}$. Hence, $(\Omega^{*\eta}, e^{*\eta})$ cannot lie on the $\left(e^{*\nu}/\Omega^{*\nu}\right)$-line. Thus, $(\Omega^{*\eta}, e^{*\eta})$ should lie below both the $\left(e^{*\nu}/\Omega^{*\nu}\right)$-line and ν's labor extraction curve such that $e^{*\eta}(W^{\eta}) \geq e^{*\nu}(W^{\nu})$. This implies that

$$\frac{e^{*\eta}}{\Omega^{*\eta}} \leq \frac{e^{*\nu}}{\Omega^{*\nu}} \Leftrightarrow W^{\eta} > W^{\nu}.$$

The equation $\frac{e^{*\eta}}{\Omega^{*\eta}} = \frac{e^{*\nu}}{\Omega^{*\nu}}$ occurs only if $u_{\Pi\Pi} = 0$. Q.E.D.

Corollary 1: (The poverty law in capital accumulation) Let the economy be in a non-trivial RS with $\pi > 0$. Then the less wealthy agent is paid a lower real wage rate than the wealthier agent if agents are risk averse.

Proof: By Proposition 3 and Theorem 3, we can get the following:

$$[e^{*\eta}(W^{\eta}) \geq e^{*\nu}(W^{\nu}) \text{ and } \frac{e^{*\eta}}{\Omega^{*\eta}} \leq \frac{e^{*\nu}}{\Omega^{*\nu}}] \Leftrightarrow W^{\eta} > W^{\nu}.$$

The left-side of the above equation meets only if $\Omega^{*\eta} \geq \Omega^{*\nu}$. Q.E.D.

The results of Theorem 3 and Corollary 1 show that it is very costly for the firm to employ the wealthier agents. In spite of this, there could be cases that the wealthiest agent is employed. Such a case would occur when full employment is feasible and profitable for the firm in the current scale of aggregate capital endowments. Even in this case, the firm's dismissal strategy can be effective against every employed agent, since every employed agent is threatened by the existence of potential reserve armies who will appear in the next period of production.

The implication of Proposition 4 and Theorem 3 is that *the mass existence of a proletariat* in the capitalist economy plays an important role *for the capitalist economy to be sufficiently profitable*, because *proletarians* who own no produced assets *have the highest level of labor-discipline*, and therefore *are the most profitable agents for capitalism.*

5. The relations of wealth, exploitation and labor-discipline

In this section, we first define exploitation. Second, we classify the agents according to their level of wealth. Third, we analyze the relations of wealth, exploitation and labor discipline.

In the following, without loss of generality, we assume that the economy is at a non-trivial RS with $\pi = \mu\sigma > 0$ $(0 < \sigma \leq 1)$.

Definition 3: Let the economy be at a non-trivial RS, and $(p, \{\Omega^{\nu}\}_{\nu \in N}, x)$ support the RS. A feasible assignment for this RS is any distribution of the net output $\{D^{\nu}\}_{\nu \in \overline{N}}$, such that:

$$(1) \quad \sum_{\nu \in \overline{N}} D^{\nu} = (I - A)x, \quad (2) \quad pD^{\nu} = \Pi(p\omega^{\nu}, \Omega^{\nu}) \ (\forall \nu \in \overline{N}).$$

The class of all feasible assignments is called Γ. Let Γ^{ν} be the set of bundles D^{ν} which ν receives under various feasible assignments.

The vector of labor value is Λ, a $1 \times n$ vector, where $\Lambda = \Lambda A + L$. Since A is indecomposable and productive, it can be written that $\Lambda = L(I - A)^{-1} > 0$.

Definition 4: Let the economy be at a non-trivial RS with $\pi > 0$. Then an agent ν is exploited if and only if [17]

$$\max_{D^v \in \Gamma^v} \Lambda D^v < Lx^v .$$

Agent v is an exploiter if and only if

$$\min_{D^v \in \Gamma^v} \Lambda D^v > Lx^v .$$

The following assumption is used by Roemer (1982).

Assumption 4: (ALE) Every agent can spend all his revenue on the purchase of any one good.

Proposition 5: Let the economy be at a non-trivial RS with $\pi > 0$. Also, let ALE hold. Then : if v is in N,

$$v \text{ is exploited} \Leftrightarrow \frac{W^v}{e^v l} < \frac{1 - \rho_{\max}\left(\frac{\Omega^v}{e^v}\right)}{\pi \rho_{\max}}$$

$$v \text{ is an exploiter} \Leftrightarrow \frac{W^v}{e^v l} > \frac{1 - \rho_{\min}\left(\frac{\Omega^v}{e^v}\right)}{\pi \rho_{\min}}$$

where $\rho_{\max} = \max\left(\frac{\Lambda_i}{p_i}\right)$ and $\rho_{\min} = \min\left(\frac{\Lambda_i}{p_i}\right)$.

Proof: Notice that, in a non-trivial RS with $\pi > 0$, $\Pi(p\omega^v, \Omega^v) = \pi W^v + \Omega^v l$ and $e^v l = Lx^v$, where $W^v = p\omega^v$.

Let $\frac{W^v}{e^v l} < \frac{1 - \rho_{\max}\left(\frac{\Omega^v}{e^v}\right)}{\pi \rho_{\max}}$. Then $\max\left(\frac{\Lambda_i}{p_i}\right)(\pi W^v + \Omega^v l) < e^v l$. Suppose D^v is any assignment satisfying $pD^v = \pi W^v + \Omega^v l$. Then $\max\left(\frac{\Lambda_i}{p_i}\right) pD^v < e^v l$. This implies $\Lambda D^v < e^v l$ for all D^v such that $pD^v = \pi W^v + \Omega^v l$. Hence, $\max_{D^v \in \Gamma^v} \Lambda D^v < e^v l$. Similarly, the converse direction can be shown. For an exploiter, it can also be proven in the same way. Q.E.D.

Proposition 6: Let the economy be at a non-trivial RS with $\pi > 0$. Then, society is exhaustively partitioned into the following five pairwise disjoint sets:

$$C^{PH} = \{v \in \overline{N} \mid e^{v} l [1 + \frac{\left(\sum_{\eta \in N} s(f^{\eta}) \Big/ \sum_{\eta \in N} e^{\eta} \right)}{\pi \max \left(\frac{pA}{\sigma L} \right)_i}] < \sigma L y^{v} \ (\forall y^{v} s.t. \ pAy^{v} = W^{v}) \ if v \in N\}$$

$$C^{H} = \{v \in \overline{N} \mid e^{v} l < \sigma L y^{v} \leq e^{v} l [1 + \frac{\left(\sum_{\eta \in N} s(f^{\eta}) \Big/ \sum_{\eta \in N} e^{\eta} \right)}{\pi \max \left(\frac{pA}{\sigma L} \right)_i}] \ (\forall y^{v} s.t. \ pAy^{v} = W^{v}) \ if v \in N\}$$

$$C^{PB} = \{v \in \overline{N} \mid \sigma L y^{v} = e^{v} l \ (\forall y^{v} s.t. \ pAy^{v} = W^{v}) \ \ if v \in N\}$$

$$C^{S} = \{v \in \overline{N} \mid \sigma L y^{v} < e^{v} l, W^{v} \neq 0 \ (\forall y^{v} s.t. \ pAy^{v} = W^{v}) \ \ if v \in N\}$$

$$C^{P} = \{v \in \overline{N} \mid W^{v} = 0\}.$$

The above five sets do not have the concept of "class" as discussed by Roemer (1982), because in our setting every agent has access to the economy only through becoming an employee or a capital holder. That is, there exists "the separation of ownership and management " while in Roemer (1982) the capital owner is also the manager. This is one characteristic of a contemporary capitalist economy. If it so, why do we define the above partition in society ? The following proposition gives us our answer.

Proposition 7: Let the economy be at a non-trivial RS with $\pi > 0$. Then: $if v \in N$,

$$v \in C^{PH} \Leftrightarrow \frac{W^{v}}{e^{v} l} > \max \left(\frac{pA}{\sigma L} \right)_i + \left(\frac{1}{\pi} \right) \frac{\sum_{\eta \in N} s(f^{\eta})}{\sum_{\eta \in N} e^{\eta}}$$

$$v \in C^{H} \Leftrightarrow \max \left(\frac{pA}{\sigma L} \right)_i + \left(\frac{1}{\pi} \right) \frac{\sum_{\eta \in N} s(f^{\eta})}{\sum_{\eta \in N} e^{\eta}} \geq \frac{W^{v}}{e^{v} l} > \max \left(\frac{pA}{\sigma L} \right)_i$$

$$\nu \in C^{PB} \Leftrightarrow \max\left(\frac{pA}{\sigma L}\right)_i \geq \frac{W^\nu}{e^\nu l} \geq \min\left(\frac{pA}{\sigma L}\right)_i$$

$$\nu \in C^S \Leftrightarrow \min\left(\frac{pA}{\sigma L}\right)_i > \frac{W^\nu}{e^\nu l} > 0$$

$$\nu \in C^P \Leftrightarrow W^\nu = 0.$$

Proof: It is sufficient to show that: $\nu \in C^H \Leftrightarrow \max\left(\frac{pA}{\sigma L}\right)_i + \left(\frac{1}{\pi}\right)\frac{\sum_{\eta \in N} s(f^\eta)}{\sum_{\eta \in N} e^\eta} \geq \frac{W^\nu}{e^\nu l}$.

Another relation is followed by Roemer (1982, 1986). Let $\nu \in C^H$. Then, $\sigma L y^\nu \leq e^\nu l + \frac{e^\nu l \left(\sum_{\eta \in N} s(f^\eta) / \sum_{\eta \in N} e^\eta \right) \sigma L y^\nu}{\pi\ pAy^\nu}$ $(\forall y^\nu \, s.t.\ pAy^\nu = W^\nu)$. Then we can obtain the following:

$$\frac{W^\nu}{e^\nu l} \leq \frac{W^\nu}{\sigma L y^\nu} + \frac{\left(\sum_{\eta \in N} s(f^\eta) / \sum_{\eta \in N} e^\eta \right) L y^\nu}{\pi\ L y^\nu} \Leftrightarrow \frac{W^\nu}{e^\nu l} \leq \frac{pAy^\nu + \left(\frac{1}{\pi}\right)\left(\sum_{\eta \in N} s(f^\eta) / \sum_{\eta \in N} e^\eta \right) \sigma L y^\nu}{\sigma L y^\nu}$$

$$\Leftrightarrow \frac{W^\nu}{e^\nu l} \leq \frac{pAy^\nu}{\sigma L y^\nu} + \left(\frac{1}{\pi}\right)\left(\sum_{\eta \in N} s(f^\eta) / \sum_{\eta \in N} e^\eta \right) \Rightarrow \max\left(\frac{pA}{\sigma L}\right)_i + \left(\frac{1}{\pi}\right)\frac{\sum_{\eta \in N} s(f^\eta)}{\sum_{\eta \in N} e^\eta} \geq \frac{W^\nu}{e^\nu l}.$$

Conversely, let $\max\left(\frac{pA}{\sigma L}\right)_i + \left(\frac{1}{\pi}\right)\frac{\sum_{\eta \in N} s(f^\eta)}{\sum_{\eta \in N} e^\eta} \geq \frac{W^\nu}{e^\nu l}$. Then

$$\frac{W^\nu}{\max\left(\frac{pA}{\sigma L}\right)_i} \leq e^\nu l [1 + \frac{\left(\sum_{\eta \in N} s(f^\eta) / \sum_{\eta \in N} e^\eta \right)}{\pi \max\left(\frac{pA}{\sigma L}\right)_i}].$$ Thus, $\sigma L y^\nu \leq e^\nu l [1 + \frac{\left(\sum_{\eta \in N} s(f^\eta) / \sum_{\eta \in N} e^\eta \right)}{\pi \max\left(\frac{pA}{\sigma L}\right)_i}]$ $(\forall y^\nu \, s.t.\ pAy^\nu = W^\nu)$. Q.E.D.

Proposition 7 implies, by being combined with Lemma 4, that the order of the above five sets such that C^{PH}, C^H, C^{PE}, C^S, C^P corresponds to the level of wealth.

The following definition divides the society into three classes: "*the high labor-disciplined*"; "*the low labor-disciplined*"; and "*the middle labor-disciplined*".

Definition 4: Let the economy be at a non-trivial RS. Then define the three sets as follows:

$$C^{HD} = \{\nu \in \overline{N} \,|\, if\, \nu \in N,\ \frac{\sum_{\eta \in N} e^{\eta}}{\sum_{\eta \in N} \Omega^{\eta}} < \frac{e^{\nu}}{\Omega^{\nu}},\ otherwise\, \nu \in \overline{N} - N\},$$

$$C^{LD} = \{\nu \in \overline{N} \,|\, if\, \nu \in N,\ \frac{\sum_{\eta \in N} e^{\eta}}{\sum_{\eta \in N} \Omega^{\eta}} > \frac{e^{\nu}}{\Omega^{\nu}},\ otherwise\, \nu \in \overline{N} - N\},$$

$$C^{MD} = \{\nu \in \overline{N} \,|\, if\, \nu \in N,\ \frac{\sum_{\eta \in N} e^{\eta}}{\sum_{\eta \in N} \Omega^{\eta}} = \frac{e^{\nu}}{\Omega^{\nu}},\ otherwise\, \nu \in \overline{N} - N\}.$$

Theorem 4: Let the economy be at a non-trivial RS with $\pi > 0$. Let ALE hold. Then every agent ν in $C^{PH} \cap (C^{LD} \cup C^{MD})$ is an exploiter. On the other hand, every agent ν in $(C^{S} \cup C^{P}) \cap (C^{HD} \cup C^{MD})$ is exploited.

Proof: It is sufficient to show that:

$$(1)\quad \frac{1 - \rho_{\min}\left(\frac{\Omega^{\nu}}{e^{\nu}}\right)}{\pi \rho_{\min}} \le \max\left(\frac{pA}{\sigma L}\right)_i + \left(\frac{1}{\pi}\right)\frac{\sum_{\eta \in N} s(f^{\eta})}{\sum_{\eta \in N} e^{\eta}} \quad \text{if } \nu \in C^{PH} \cap (C^{LD} \cup C^{MD}),$$

$$(2)\quad \frac{1 - \rho_{\max}\left(\frac{\Omega^{\nu}}{e^{\nu}}\right)}{\pi \rho_{\max}} \ge \min\left(\frac{pA}{\sigma L}\right)_i \quad \text{if } \nu \in (C^{S} \cup C^{P}) \cap (C^{HD} \cup C^{MD}).$$

First, we show (1). Suppose for some $\nu \in C^{PH} \cap (C^{LD} \cup C^{MD})$,

$$\frac{1 - \rho_{\min}\left(\frac{\Omega^{\nu}}{e^{\nu}}\right)}{\pi \rho_{\min}} > \max\left(\frac{pA}{\sigma L}\right)_i + \left(\frac{1}{\pi}\right)\frac{\sum_{\eta \in N} s(f^{\eta})}{\sum_{\eta \in N} e^{\eta}}. \text{ Then:}$$

$$p(I - A) - \left(\frac{\sum_{\eta \in N} \Omega^{\eta}}{\sum_{\eta \in N} e^{\eta}} - \frac{\Omega^{\nu}}{e^{\nu}}\right) L < \frac{1}{\rho_{\min}} L \text{ because } \pi = \frac{p(I - A) - \left(\frac{\sum_{\eta \in N} \Omega^{\eta} + \sum_{\eta \in N} s(f^{\eta})}{\sum_{\eta \in N} e^{\eta}}\right) L}{pA} \sigma.$$

Since $v \in (C^{LD} \cup C^{MD})$, we can reduce to $\min\left(\frac{\Lambda_i}{p_i}\right)p(I-A) < L$. This implies $\min\left(\frac{\Lambda_i}{p_i}\right)p < \Lambda$. This is a contradiction.

Next, we show (2). Suppose for some $v \in (C^S \cup C^P) \cap (C^{HD} \cup C^{MD})$, $\frac{1-\rho_{\max}\left(\frac{\Omega^v}{e^v}\right)}{\pi\rho_{\max}} < \min\left(\frac{pA}{\sigma L}\right)_i$. Then $p(I-A) - \left(\frac{\sum_{\eta\in N}\Omega^\eta + \sum_{\eta\in N} s(f^\eta)}{\sum_{\eta\in N} e^\eta} - \frac{\Omega^v}{e^v}\right)L > \frac{1}{\rho_{\max}}L$.

Since $v \in (C^{HD} \cup C^{MD})$, $\frac{\sum_{\eta\in N}\Omega^\eta + \sum_{\eta\in N} s(f^\eta)}{\sum_{\eta\in N} e^\eta} - \frac{\Omega^v}{e^v} > 0$, so that $\max\left(\frac{\Lambda_i}{p_i}\right)p(I-A) > L$.

Hence, $\max\left(\frac{\Lambda_i}{p_i}\right)p > \Lambda$. This is also a contradiction. Q.E.D.

Notice that if $C^{PH} \cap (C^{LD} \cup C^{MD})$ is a proper subset of C^{PH}, then every agent in $C^{PH} \cap (C^{LD} \cup C^{MD})$ is wealthier than every agent in $C^{PH} - (C^{PH} \cap (C^{LD} \cup C^{MD}))$ by Theorem 3. Also, if $(C^S \cup C^P) \cap (C^{HD} \cup C^{MD})$ is a proper subset of $C^S \cup C^P$, then every agent in $(C^S \cup C^P) - (C^{HD} \cup C^{MD})$ is wealthier than every agent in $(C^S \cup C^P) \cap (C^{HD} \cup C^{MD})$. Note that $C^P \subseteq (C^{HD} \cup C^{MD})$, for all agents in C^P belong to C^{MD} whenever some agent in C^P belongs to C^{MD}. Such a case occurs only if $N \subseteq C^P$. By these facts, it can be said that Theorem 4 does not necessarily lose the implication that there is a correlation between wealth and exploitation.

The following statement is important:

Corollary 2: *Let the economy be at a non-trivial RS with $\pi > 0$. Let ALE hold. Then, if agents are non-increasing risk averters, there exists a group of wealthier agents who are exploiters and a group of less wealthy agents who are exploited. Moreover, the less wealthy exploited agents are more labor-disciplined than the wealthier exploiters.*

Proof: The first statement is followed by Lemma 4, Theorem 3 and 4, because both $C^{PH} \cap (C^{LD} \cup C^{MD})$ and $(C^{S} \cup C^{P}) \cap (C^{HD} \cup C^{MD})$ are surely non-empty. The second statement is also followed by Lemma 4, Theorem 3 and Proposition 7. Q.E.D.

There are four interesting cases of Corollary 2 :

Case 1: $C^{PH} \subset (C^{LD} \cup C^{MD})$ and $(C^{S} \cup C^{P}) \subset (C^{HD} \cup C^{MD})$. Then every agent in C^{PH} is the wealthier exploiter and every agent in $(C^{S} \cup C^{P})$ is the less wealthy exploited. Moreover, every agent in $(C^{S} \cup C^{P})$ is more labor-disciplined than every agent in C^{PH}.

Case 2: $C^{PH} \supseteq (C^{LD} \cup C^{MD})$ and $(C^{S} \cup C^{P}) \subset (C^{HD} \cup C^{MD})$. Then every agent in $(C^{LD} \cup C^{MD})$ is the wealthier exploiter and every agent in $(C^{S} \cup C^{P})$ is the less wealthy exploited. Moreover, every agent in $(C^{S} \cup C^{P})$ is more labor-disciplined than every agent in $(C^{LD} \cup C^{MD})$.

Case 3: $C^{PH} \subset (C^{LD} \cup C^{MD})$ and $(C^{S} \cup C^{P}) \supseteq (C^{HD} \cup C^{MD})$. Then every agent in C^{PH} is the wealthier exploiter and every agent in $(C^{HD} \cup C^{MD})$ is the less wealthy exploited. Moreover, every agent in $(C^{HD} \cup C^{MD})$ is more labor-disciplined than every agent in C^{PH}.

Case 4: $N \subseteq C^{P}$. Then $N = C^{MD}$ and every agent in $\overline{N} - C^{P}$ is the wealthier exploiter and every agent in C^{P} is the least wealthy exploited.

[Insert Figure 4]

Lemma 4, Theorem 4 and Corollary 2 indicate that, in the capitalist economy with the contested exchange labor market as well as the neoclassical one, Wealth-Exploitation Correspondence is established if every agent is either non-increasing risk averse or risk neutral. Both the conditions of non-increasing risk aversion and risk neutrality of agents are sufficiently plausible in the economy under uncertainty. Hence, the above result indicates that the argument of Roemer (1990) that *the unequal distribution of wealth implies the exploitation of labor* is true at least in economic environments with plausible restrictions *even if the labor market is contested.*

Theorem 3 and Corollary 2 also indicate the importance of the unequal distribution of wealth in explaining *the modest contestedness in labor markets for the capitalist economy to be sufficiently profitable*, as well as in explaining the exploitation of labor. Notice that whether the degree of contestedness in labor exchange is moderate or not depends on the levels of employees' labor-discipline. By Theorem 3, we can see that the labor market entered by the less wealthy suppliers alone is more moderately contested than the one by the wealthier suppliers alone. In real capitalism, it is usual that most of the employed workers are the agents with no or only a few productive assets. Thus, the contestedness in labor markets would be moderated to maintain enough profitability in the capitalist economy, which is inferred from Theorem 3 and Corollary 2.

6. Concluding Remarks

In the above arguments, we discussed the corresponding relationships between the status of wealth, exploitation and labor-discipline. First, we introduced the level of the agent's labor-discipline measured by the ratio of the labor effort per unit of labor time to the real wage rate. The connection of this kind of power index is then examined with respect to wealth distribution in Proposition 4, Theorem 3 and Corollary 1. The obtained results are that the less wealthy agent has a higher level of labor-discipline than the wealthier agent if agents are risk averse, and that as a consequence of capitalist production, the income gap between the wealthy and the poor widens more and more. Theorem 4 and Corollary 2 show the robustness of the Wealth-Exploitation Correspondence in the Capitalist economy with the contested exchange labor market. Moreover, we prove that the less wealthy exploited agents are more labor-disciplined than the wealthier exploiters.

The results obtained indicate the essential importance of the unequal distribution of wealth in understanding the contemporary capitalist economy. On the other hand, Theorem 4 also demonstrates, on determining the exploitation status, an influence of the power relationship in the production process. These arguments do not contradict the work of Roemer (1982, 1986) or Bowles and Gintis (1988, 1990), but reinforce both their arguments.

It is easy to extend our arguments in our Leontief economic model into a more general model where the production set is a convex cone including the case of the von Neumann technology (Roemer (1980, 1982)) if only the labor value is refined, following Roemer (1982, chapter 5). Our results would not depend on the simplicity of the Leontief model.

Of course our analysis is based on both several assumptions and a specific characteristic of labor contracts. When we remove some of our assumptions and/or take alternative types of labor contracts, our conclusion may be different. However, since our problem is to capture one feature of contemporary capitalism under some reasonable economic assumptions, our restrictions in this paper may be permissible whenever our suppositions are not so far removed from real capitalism.

Footnote

1). Related papers are by Bowles (1985), Gintis and Ishikawa (1987) and Bowles and Boyer (1988, 1990). A similar argument is the efficiency wage theory, which gives a micro foundation of Keynsian involuntary unemployment theory, surveyed by Akerlof and Yellen (1988).

2). In this paper, "contemporary" implies "the existence of the separation between ownership and management".

3). Note that in this paper the term "the firm" has the same meaning as the firm as "the employer". Hence, in this paper, "the firm" does not imply "an internal organization" composed of a particular group of producers.

4). It is commonly assumed in the literature of efficiency wage and/or labor contested exchange models that the firm knows its employees' preferences, reservation income and discount rate, since otherwise the firm could not calculate the "labor extraction function" of its employees. See, for example, Solow (1979), Gintis and Ishikawa (1987) and Bowles and Gintis (1994).

5). The member of employed agents N is determined at the end of the previous period of production. In this economy, the labor market we are considering is organized as a sequential contingent renewal market: New employment occurs either to replace a dismissed agent or to meet new demand by extending reproduction.

6). Notice that when dynamic process of such a profit rate equalization is characterized by capital mobility across sectors, it does not ensure in general convergence to any (EPR)-equilibrium price vector (Nikaido (1983)).

7). In this paper, "labor effort" is synonymous with "labor intensity", that is to say, "labor input per unit of labor time". Moreover, following the usual efficiency wage literature, we assume that there exists a common cardinal measure of all e^{v}. However, this treatment has been criticized by Currie and Steedman (1993) who argue that labor effort may be measured only ordinally. The problem whether 'effort' is only measurable cardinally or ordinally is a recent and controversial issue. See Gintis (1995) and Currie and Steedman (1993, 1995).

8). The following formulation is based on Gintis and Ishikawa (1987). However, there are several different points. First, in our setting, a dismissal occurs only when the firm detects non-best labor performance of its agents. Second, the inexactness of monitoring does not exist, while, in Gintis and Ishikawa (1987), some proportions of agents are necessarily fired in consequence of error estimations.

9). This probability function reflects the content of labor contracts that one agent who is detected with an unsatisfactory labor performance compared to the other agents employed at the same wage as his may not be renewed his employment contract in the next period.

10). The following definition of equilibrium is based on Roemer (1981, chapter 1).

11). In the following, for any function $h(x)$, when a variable maximizes or minimizes h on X, we denote such a variable by $\arg\max_{x \in X} h(x)$ or $\arg\min_{x \in X} h(x)$.

12). The potential reserve armies are from not only the natural growth of population in the capitalist sector but also a flowing from the non-capitalist sector.

13). Since our approach is a temporary equilibrium approach, there exists another type of expectation, which is about the price in the next period. However, in this paper, according to Roemer (1981, 1982), we assume that price expectations are also stationary.

14). That is, in the equilibrium of labor contracts, there is no dismissal of agents. It is because an error in monitoring has not occurred. Of course, this does not imply that the firm's monitoring is perfect. Since perfect monitoring is extremely expensive for firms, in general, the firm's monitoring may be imperfect.

15). This set is proposed by Roemer (1981).

16). Bowles and Boyer (1990) suggested that the increase in unemployment insurance makes the bargaining power of labor stronger when capital and labor are in conflict. Also, they showed that the increase in unemployment insurance brings about higher real wages per unit of labor effort. These things imply that the level of real wages per unit of labor effort is closely connected to the bargaining power of labor. This viewpoint may seem to confirm Definition 2.

17). This definition is based on Roemer (1982, 1986).

Appendix

Lemma 3: For small enough $r > 0$, for any agent v in N, if (Ω^{*v}, f^{*v}) satisfies the condition (4), then (Ω^{*v}, f^{*v}) satisfies the second order condition.

Proof: Let's denote that $C^v(\Omega^{*v}, f^{*v}) = \dfrac{(\Omega^{*v} + s(f^{*v}))}{e^v(\Omega^{*v}, f^{*v})}$ where (Ω^{*v}, f^{*v}) satisfies the condition (4). Define the following Hessian matrix:

$$H = \begin{bmatrix} \dfrac{\partial^2 C^v(\Omega^{*v}, f^{*v})}{\partial\Omega\partial\Omega} & \dfrac{\partial^2 C^v(\Omega^{*v}, f^{*v})}{\partial\Omega\partial f} \\ \dfrac{\partial^2 C^v(\Omega^{*v}, f^{*v})}{\partial f\partial\Omega} & \dfrac{\partial^2 C^v(\Omega^{*v}, f^{*v})}{\partial f\partial f} \end{bmatrix}$$

Note that $\dfrac{\partial^2 C^v(\Omega^{*v}, f^{*v})}{\partial\Omega\partial\Omega} = \dfrac{-e^v_{\Omega\Omega}(\Omega^{*v} + s(f^{*v}))}{\{e^{*v}\}^2}$ where $e^v_{\Omega\Omega} = -\dfrac{F_{\Omega\Omega}F_e - F_\Omega F_{\Omega e}}{\{F_e\}^2}$.

Since $F_{\Omega\Omega} = d_e u_{\Pi\Pi} l \geq 0$, $F_e > 0$, $F_\Omega < 0$ and $F_{\Omega e} = d_{ee} u_\Pi l + d_e u_{\Pi e} l > 0$, we get that $e^v_{\Omega\Omega} < 0$. Thus, $\dfrac{\partial^2 C^v(\Omega^{*v}, f^{*v})}{\partial\Omega\partial\Omega} > 0$. Next, note that $\dfrac{\partial^2 C^v(\Omega^{*v}, f^{*v})}{\partial f\partial f} =$ $\dfrac{s_{ff}e^v - e^v_{ff}(\Omega^{*v} + s(f^{*v}))}{\{e^{*v}\}^2}$ where $e^v_{ff} = -\dfrac{F_{ff}F_e - F_f F_{fe}}{\{F_e\}^2}$ and $F_f = d_{ef}(u - rV_U) - u_e d_f$.

Since $d^v = \min[\dfrac{f^v}{\{e^v\}^2}\phi(v), 1]$, we get that $F_{ff} = 0, F_{fe} = \dfrac{d_{ee}}{f}\{u - rV_U\} - u_{ee}\dfrac{d}{f} > 0$ and $e^v_{ff} < 0$. Thus, $\dfrac{\partial^2 C^v(\Omega^{*v}, f^{*v})}{\partial f\partial f} > 0$. Next check $\dfrac{\partial^2 C^v(\Omega^{*v}, f^{*v})}{\partial\Omega\partial f}$. Since $\dfrac{\partial^2 C^v(\Omega^{*v}, f^{*v})}{\partial\Omega\partial f} = \dfrac{e_f - e_{\Omega f}(\Omega^{*v} + s(f^{*v})) - e_\Omega s_f}{\{e^{*v}\}^2} = \dfrac{-e_{\Omega f}(\Omega^{*v} + s(f^{*v}))}{\{e^{*v}\}^2}$ and $-e_{\Omega f} = \dfrac{F_{\Omega f}F_e - F_\Omega F_{fe}}{\{F_e\}^2} = \dfrac{F_{\Omega f}}{F_e} + \dfrac{e_\Omega F_{ef}}{F_e} = -\dfrac{e_\Omega}{f} + \dfrac{e_\Omega}{f} + \dfrac{u_{ef}}{F_e} = \dfrac{u_{ef}}{F_e} \leq 0$, we get that $\dfrac{\partial^2 C^v(\Omega^{*v}, f^{*v})}{\partial\Omega\partial f} = \dfrac{1}{\{e^{*v}\}^2}\dfrac{u_{ef}}{F_e}(\Omega^{*v} + s(f^{*v})) = \dfrac{1}{\{e^{*v}\}^2}\dfrac{e^{*v}u_{ef}}{(-F_\Omega)} \leq 0$. Thus, the above Hessian is reduced to the following:

$$H = \frac{1}{e_\Omega}\begin{bmatrix} \dfrac{-e_{\Omega\Omega}}{e_\Omega} & \dfrac{1}{F_e}\dfrac{u_{ef}}{e_\Omega} \\ \dfrac{1}{F_e}\dfrac{u_{ef}}{e_\Omega} & s_{ff} - \dfrac{e_{ff}}{e_\Omega} \end{bmatrix} = \begin{bmatrix} + & - \\ - & + \end{bmatrix}$$

Then, if either $|u_{ee}| \geq 0$ or $r > 0$ is small enough, $|H| > 0$ is guaranteed. Q.E.D.

Note that the proof of Lemma 3 implies that the optimal labor contract, (Ω^{*v}, f^{*v}), is uniquely determined because for any $\Omega^v > 0$, $e_{\Omega\Omega} < 0$ in this model.

Proof of Theorem 1 : Let $x \neq 0$ be the aggregate activity vector associated with a RS. Since A is a productive matrix, there exists $(I - A)^{-1}$ and $(I - A)x \geq 0$. And since $(I - A)^{-1} > 0$ by indecomposability of A, $x > 0$. That is, all activities must be operated at a nontrivial RS.

In maximizing profits, the firm facing price p will operate only those processes generating the maximal profit rate, because any capital holder invests all his finance capital in those processes. Hence, for all processes to operate, it is necessary that the price vector p generate the same non-negative profit rate in all sectors. Let $p^* \in \Delta$ be such a price at a non-trivial RS. Denote the corresponding solutions of (p-1-1) and (p-1-2) by $\{\Omega^{*\nu}, f^{*\nu}\}_{\nu \in N}$ and $l^* \in (0, 1]$ respectively. Also, denote the corresponding solution tuple of (p-2) by $\{e^{*\nu}, y^{*\nu}\}_{\nu \in N}$. Notice that for all $\nu \in \overline{N}$, $y^{*\nu} = A^{-1}\omega^\nu$, because all sectors generate the same profit rate. Let $x^* > 0$ be the aggregate activity vector associated with a RS. Then the associated profit is

$[p_i^* - p^*A_i - \left(\dfrac{\sum_{\nu \in N}(\Omega^{*\nu} + s(f^{*\nu}))}{\sum_{\nu \in N} e^{*\nu}}\right) L_i]x_i^* \geq 0$ for each sector i. Since for all i, $x_i^* > 0$ and $L_i > 0$, $p_i^* > 0$ should be satisfied for all i. By the budget constrain of (p-1-2), $p^*Ax^* = \sigma\ (\sum_{\nu \in \overline{N}} p^*Ay^{*\nu}) = \sigma\ p^*\omega$ for some $\sigma \in (0, 1]$. The associated EPR is

$\pi = [p_i^* - p^*A_i - \left(\dfrac{\sum_{\nu \in N}(\Omega^{*\nu} + s(f^{*\nu}))}{\sum_{\nu \in N} e^{*\nu}}\right) L_i]x_i^* \Big/ p_i^*\omega_i$ for each sector i. Let

$\mu_i = [p_i^* - p^*A_i - \left(\dfrac{\sum_{\nu \in N}(\Omega^{*\nu} + s(f^{*\nu}))}{\sum_{\nu \in N} e^{*\nu}}\right) L_i]x_i^* \Big/ p^*A_ix_i^*$ for each sector i. Then, for all i, $\mu_i p^*A_ix_i^* = \pi p_i^*\omega_i$. So, $\sum_{i=1}^{n} \mu_i p^*A_ix_i^* = \pi p^*\omega = \dfrac{\pi}{\sigma} p^*Ax^*$. This implies $\sum_{i=1}^{n}(\mu_i - \dfrac{\pi}{\sigma})p^*A_ix_i^* = 0 \Leftrightarrow p^*\left(\sum_{i=1}^{n}(\mu_i - \dfrac{\pi}{\sigma})x_i^*A_i\right) = 0$. Since $p^* > 0$ and $x^* > 0$, if $\mu_j \neq \dfrac{\pi}{\sigma}$ for some j, then $rank(A) < n$. However, since A^{-1} exists, it is a contradiction. Thus, $\mu_i = \dfrac{\pi}{\sigma} = \mu$ for all i. This implies that $p^* = (1 + \mu)p^*A + C^*L$ where $C^* = \dfrac{\sum_{\nu \in N}(\Omega^{*\nu} + s(f^{*\nu}))}{\sum_{\nu \in N} e^{*\nu}}$. Q.E.D.

Proof of Theorem 2 : Let $\tilde{\omega} \in \tilde{C}(\omega)$ be an endowment assignment in this production period of the economy. Let N° be the set of agents who are renewed their employment contracts at the end of the previous period of production, and let $\overline{N}$ be the set of agents being in this production period. Then the set of employed agents in this production peroid becomes $N^* \in 2^{\overline{N}}$ such that $N^{\circ} \subseteq N^* \subseteq \overline{N}$.

By Theorem 1, at a RS, EPR is prevailed. Hence, let us restrict our analysis to the case that a capital contract offered by the firm consists of a EPR $\pi \geq 0$. Then, all capital holders invest all their-owned finance capital in all secters. Hence, consider the following problem: for given $p \in \Delta$,

$$\max_{l \in [0,\ 1]} \sum_{v \in N^*} [p - pA - \left(\frac{\Omega^{*v}(\pi p\omega^v,\ l) + s(f^{*v}(\pi p\omega^v,\ l))}{e(\pi p\omega^v,\ \Omega^{*v}(\pi p\omega^v,\ l),\ l,\ f^{*v}(\pi p\omega^v,\ l))} \right) L] x^v(l) \tag{5}$$

$$\text{subject to} \quad e(\pi p\omega^v,\ \Omega^{*v}(\pi p\omega^v,\ l),\ l,\ f^{*v}(\pi p\omega^v,\ l))l = Lx^v(l) \ \ (\forall v \in N^*),$$

$$pAx(l) \leq p\omega .$$

Denote the solution set of the above problem (5) by $\ell_{N^*}(p,\ \pi)$. By the maximum theorem of Berge, $\ell_{N^*}(p,\ \pi)$ is upper hemi-continuous at every $(p,\ \pi) \in \Delta \times [0,\ \lambda]$. Furthermore, ℓ_{N^*} is compact-valued. By Assumption 3, the aggregate cost function, $\sum_{v \in N^*}[\Omega^{*v}(\pi p\omega^v,\ l) + s(f^{*v}(\pi p\omega^v,\ l))]l$, is convex, so it is secured that $\ell_{N^*}(p,\ \pi)$ is convex-valued.

Let define a correspondence $\mu : \Delta \times [0,\ \lambda] \to\to [0,\ \lambda]$ such that for each $l^* \in \ell_{N^*}(p,\ \pi)$, $\mu_{l^*}(p,\ \pi) = \begin{cases} \lambda & if\ \pi\varsigma(l^*) \geq \lambda \\ \pi\varsigma(l^*) & if\ 0 \leq \pi\varsigma(l^*) < \lambda \end{cases}$

where $\varsigma(l^*) \geq 1$ satisfying $\varsigma(l^*)pAx = p\omega$ such that $Lx = \sum_{v \in N^*} e(\pi p\omega^v,\ l^*)l^*$. Denote that for each $l^* \in \ell_{N^*}(p,\ \pi)$, $C(\pi p\omega^v,\ l^*) = \dfrac{\sum_{v \in N^*}(\Omega^{*v}(\pi p\omega^v,\ l^*) + s(f^{*v}(\pi p\omega^v,\ l^*)))}{\sum_{v \in N^*} e^{*v}(\pi p\omega^v,\ l^*)}$.

Since $\ell_{N^*}(p,\ \pi)$ is a closed interval, and $C(\pi p\omega^v,\ l^*)$ is continuous at every $l^* \in \ell_{N^*}(p,\ \pi)$, by Bolzano's Theorem, $C(\pi p\omega^v,\ \ell_{N^*}(p,\ \pi))$ is a closed interval in

R_+. Let define a correspondence $f: \Delta \times [0, \lambda] \to\to R_+^n$ such that for each $l^* \in \ell_{N^*}(p, \pi)$, $f^{l^*}(p, \pi) = (1 + \mu_{l^*}(p, \pi))pA + C(\pi p \omega^v, l^*)L$. Moreover, let define a correspondence $g: \Delta \times [0, 1] \to\to \Delta$ such that for each $l^* \in \ell_{N^*}(p, \pi)$,

$$g_{l^*}(p, \pi) = \begin{cases} q \in \Delta \ \ s.t. \ \ q_i = \dfrac{f_i^{l^*}(p, \pi)}{f^{l^*}(p, \pi) \cdot b} \ (\forall i = 1, \cdots, n) & if \ f^{l^*}(p, \pi) \cdot b > 0 \\ \Delta & if \ f^{l^*}(p, \pi) \cdot b = 0 \end{cases}.$$

By the definitions, g is upper hemi-continuous on $\Delta \times [0, \lambda]$, and is convex compact-valued.

Let define a correspondence $\pi^{re}: \Delta \times [0, \lambda] \to\to [0, \lambda]$ such that for each $l^* \in \ell_{N^*}(p, \pi)$, $\pi_{l^*}^{re}(p, \pi) = \dfrac{[p - pA - C(\pi p \omega^v, l^*)L]x}{p\omega}$ where $\sum_{v \in N^*} e^{*v}(\pi p \omega^v, l^*)l^* = Lx$ such that $\varsigma(l^*)pAx = p\omega$. By the definitions, π^{re} is upper hemi-continuous on $\Delta \times [0, \lambda]$, and is convex compact-valued.

Let define a correspondence $\phi: \Delta \times [0, \lambda] \to\to \Delta \times [0, \lambda]$ such that for each $l^* \in \ell_{N^*}(p, \pi)$, $\phi_{l^*}(p, \pi) = (g_{l^*}(p, \pi), \pi_{l^*}^{re}(p, \pi))$. By the definitions, ϕ is upper hemi-continuous on $\Delta \times [0, \lambda]$, and is convex compact-valued. Thus, by Kakutani's fixed point theorem, there exists a pair $(p^*, \pi^*) \in \phi\ (p^*, \pi^*)$. If p^* is not the form of $p^* = (1 + \pi^* \varsigma(l^*(p^*, \pi^*)))p^*A + C(\pi^* p^* \omega^v, l^*(p^*, \pi^*))L$, then by Theorem 1, there exists a trivial RS. If p^* is the form of $p^* = (1 + \pi^* \varsigma(l^*(p^*, \pi^*)))p^*A + C(\pi^* p^* \omega^v, l^*(p^*, \pi^*))L$, then $p^* > 0$ by the Perron-Frobenius Theorem. Thus, $\varsigma(l^*(p^*, \pi^*))Ax = \omega = Ay^*$. Since $\varsigma(l^*(p^*, \pi^*)) \geq 1$, this implies that Definition 1(d) is satisfied. Since $\omega \in \mho_+$, there exists x^* such that $\varsigma(l^*(p^*, \pi^*))Ax^* = \omega$, $\sum_{v \in N^*} e^{*v} l^*(p^*, \pi^*) = Lx^*$ and $x^* \geq Ax^*$. So, Definition 1(c) is satisfied. Thus, all conditions of RS are satisfied. Q.E.D.

Reference

Akerlof, G A., Yellen, J. (1986): *Efficiency Wage Models of the Labor Market*, Cambridge University Press. Cambridge.

Bowles, S., (1985): "The Production Process in a Competitive Economy: Walrasian, Neo-Hobbesian, and Marxian Models," *American Economic Review*, 75(1), pp. 16-36.

Bowles, S., Boyer, R., (1988): "Labor Discipline and Aggregate Demand: A Macroeconomic Model," *American Economic Review*, 75(1), pp. 395-400.

Bowles, S., Boyer, R., (1990): "A Wage-led Employment Regime: Income Distribution, Labor Discipline and Aggregate Demand in Welfare Capitalism," in Marglin, S., Schor, J., (eds.) *The Golden Age of Capitalism: Reinterpreting the Postwar Experience*, Oxford University Press. Oxford.

Bowles, S., Gintis, H., (1981): "Structure and practice in the labor theory of value," *Review of Radical Political Economics*, 12, pp. 1-26.

Bowles, S., Gintis, H., (1988): "Contested Exchange: Political Economy and Modern Economic Theory," *American Economic Review*, 78(2), pp. 145-150.

Bowles, S., Gintis, H., (1990): "Contested Exchange: New Microfoundation for the Political Economy of Capitalism," *Politics and Society*, 18(2), pp. 165-222.

Bowles, S., Gintis, H., (1994): "Credit Market Imperfections and the Incidence of Worker-Owned Firms," *Metroeconomica*, 45(3), pp. 209-223.

Brody, A., (1970): *Proportions, Prices and Planning*, New York: American Elsevier.

Currie, M., Steedman, I., (1993): "Taking Effort Seriously," *Metroeconomica*, 44(2), pp. 134-145.

Currie, M., Steedman, I., (1995): "Really Taking Effort Seriously," *Metroeconomica*, 46(2), pp. 211-212.

Fujimori, Y., (1982): *Modern Analysis of Value Theory*, Springer-Verlag. Berlin Heidelberg New York.

Gintis, H., (1995): "Taking Effort Seriously : A Reply to Currie and Steedman," *Metroeconomica*, 46(2), pp. 202-210.

Gintis, H., Ishikawa, T., (1987): "Wages, Work Intensity, and Unemployment," *Journal of The Japanese and International Economies*, 1, pp. 195-228.

Krause, U., (1982): *Money and Abstract Labor*, New Left Books. London.

Kreps, D. M., (1990): *A Course in Microeconomic Theory*, Princeton University Press. Princeton.

Marx, K., (1986): *Das Kapital, Volume I*, Dietz Verlag. Berlin.

Morishima, M., (1973): *Marx's Economics*, Cambridge University Press. Cambridge.

Morishima, M., (1978): *Value, Exploitation and Growth*, McGraw Hill. London.

Nikaido, H., (1983): "Marx on Competition," *Journal of Economics* 43(4), pp. 337-362.

Okishio, N., (1963): "A mathematical note on Marxian theorems," *Weltwirtschaftsliches Archiv*.

Roemer, J.E., (1980): "A general equilibrium approach to Marxian economics," *Economerica* 48, pp. 505-530.

Roemer, J.E., (1981): *Analytical Foundation of Marxian Economic Theory*, Cambridge University Press. Cambridge.

Roemer, J.E., (1982): *A General Theory of Exploitation and Class*, Harvard University Press. Cambridge.

Roemer, J.E., (1986): *Value, Exploitation and Class*, Harwood Academic Publishers. London.

Roemer, J.E., (1990): "A Thin Thread: Comment on Bowles' and Gintis' "Contested Exchange"," *Politics and Society*, 18(2), PP. 243-249.

Samuelson, P., (1982): "The normative and positive inferiority of Marx's values paradigm," *Southern Economic Journal* 49, 1, pp. 11-18.

Solow, Robert., (1979): "Another possible source of wage stickiness," *Journal of Macroeconomics*, 1, pp. 79-82.

Steedman, I., (1977): *Marx after Sraffa*, New Left Books. London.

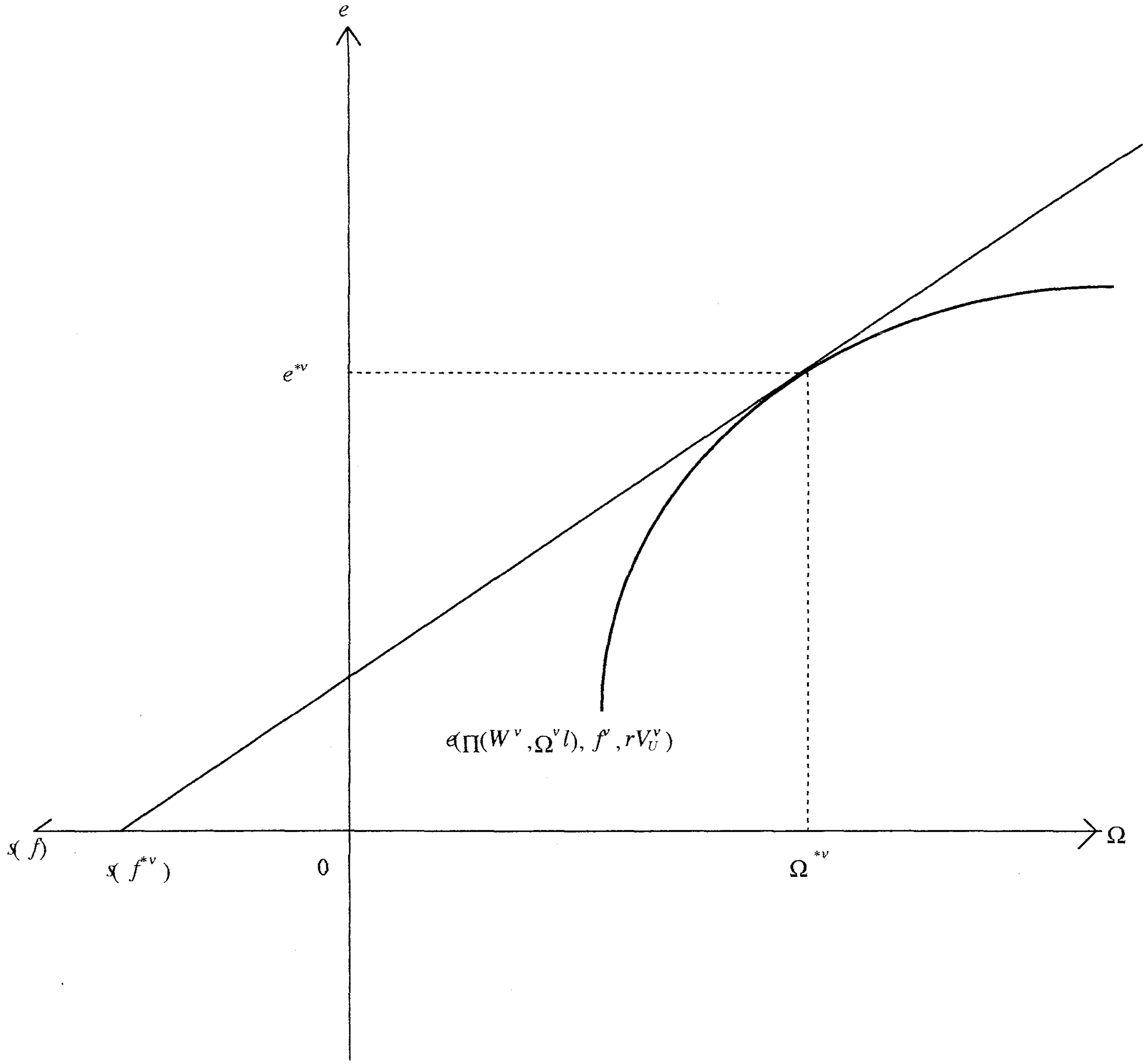


Figure 1

Determination of the labor contract equilibrium

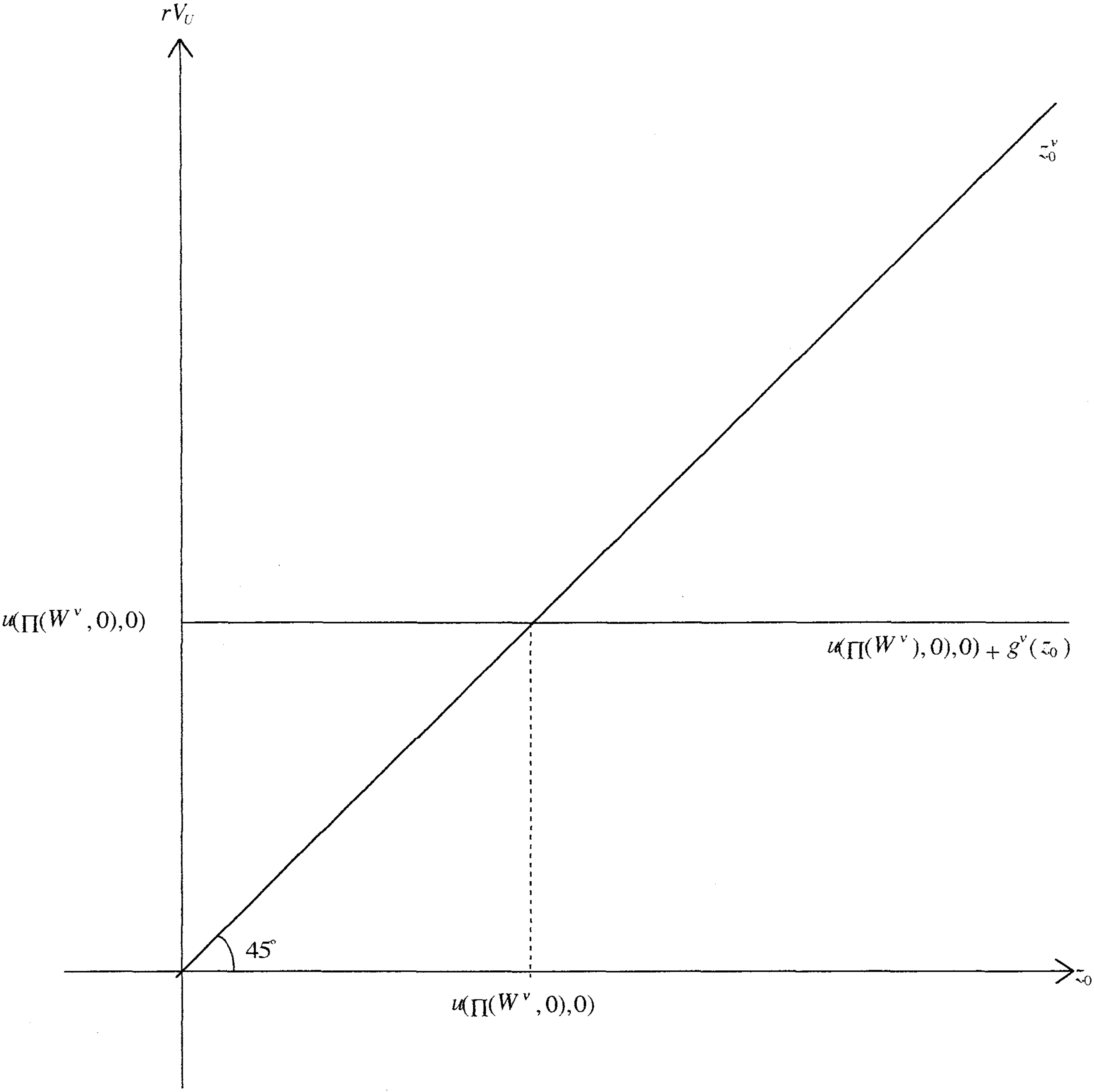


Figure 2

The stationary expectation of reservation utility

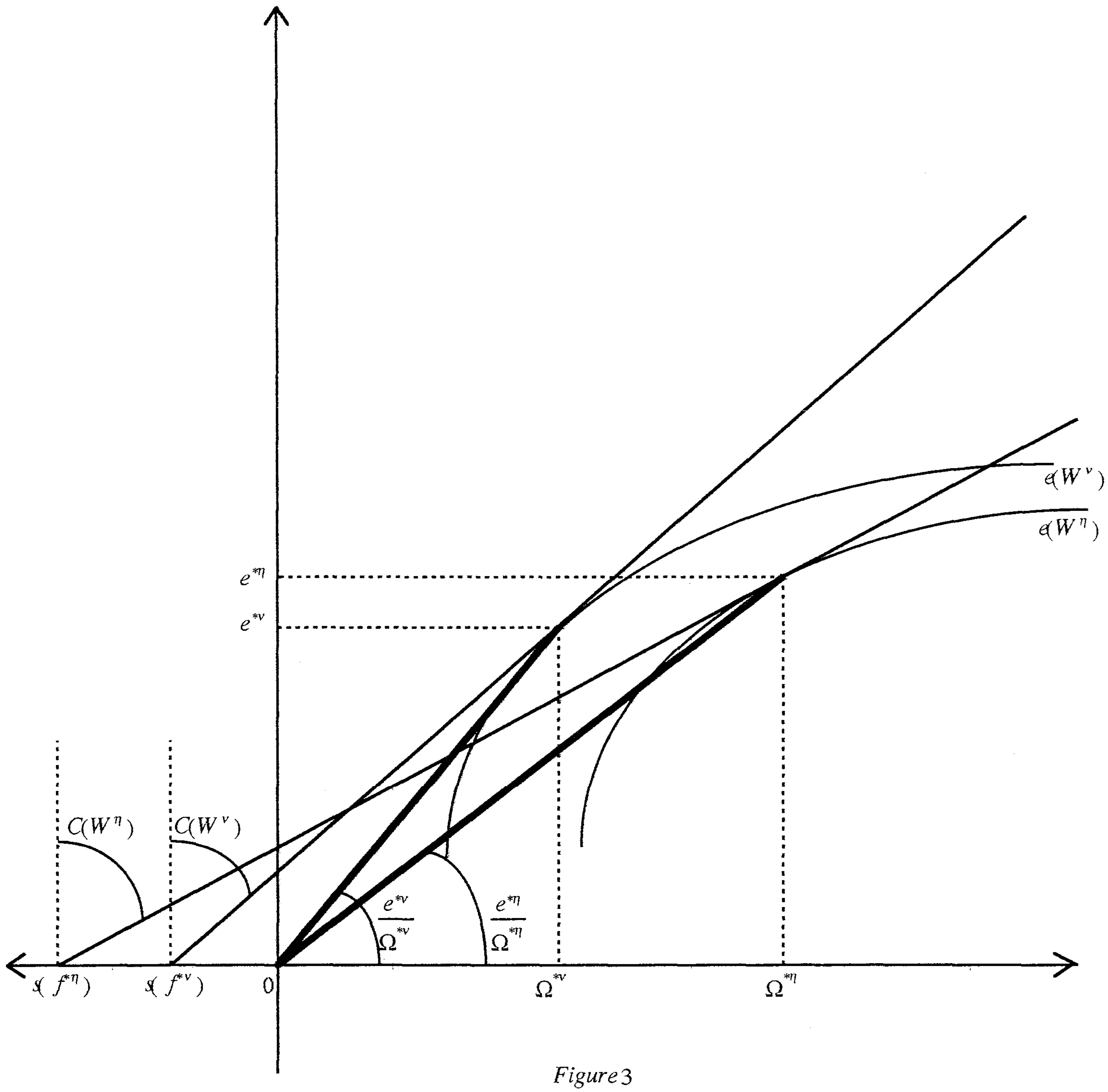


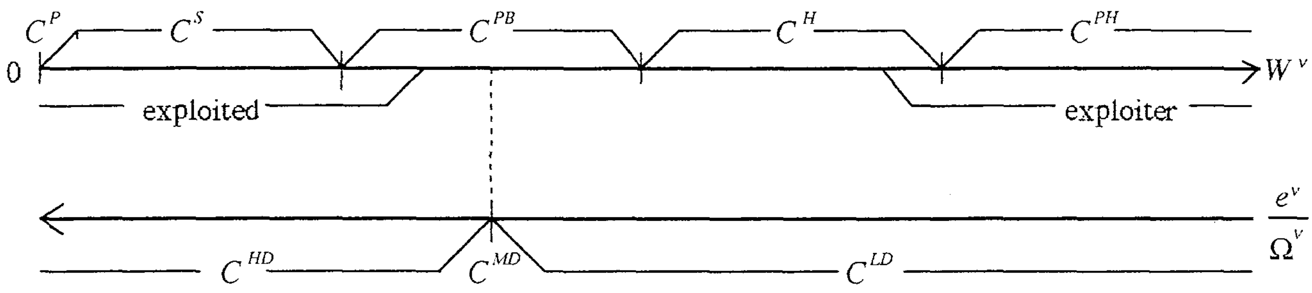
Figure 3

$$W^{\eta} > W^{\nu} \Rightarrow \begin{cases} e^{*\eta} > e^{*\nu} & \text{(Proposition 3)} \\ C(W^{\eta}) > C(W^{\nu}) & \text{(Proposition 4)} \end{cases}$$

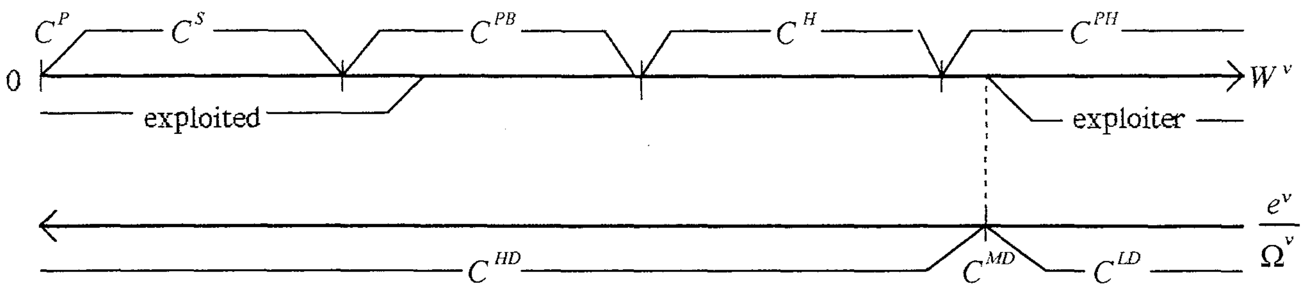
$$\Downarrow$$

$$\frac{e^{*\eta}}{\Omega^{*\eta}} < \frac{e^{*\nu}}{\Omega^{*\nu}} \quad (\textit{Theorem } 3)$$

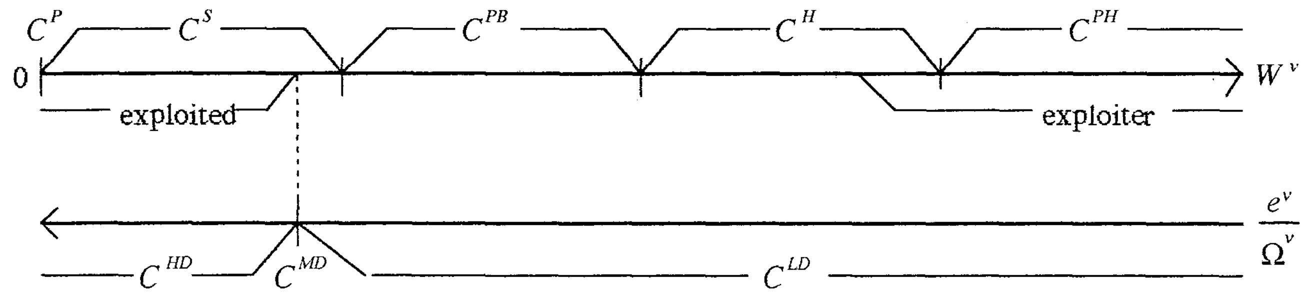
An illustration of the proof of Theorem 3



Case 1



Case 2



Case 3

Figure 4

An illustration of the statement of Theorem 4 and Corollary 2

CHAPTER II

Full Characterizations of Public Ownership Solutions

Abstract: By introducing new axioms, completely characterize two public ownership solutions in convex production economies, one of which is the Proportional Solution (PR), and the other of which is the Equal Benefit Solution (EB). The new axioms are Pareto Independence (PI) and Support Price Independence (SPI). These axioms are related to informational efficiency of allocation rules in the sense that they represent what allocation rules are attainable through as costless information transmission as possible. By adopting these axioms and the other axioms Moulin (1990a,b) adopted, it is shown that PR and EB have desirable properties from viewpoints of allocational efficiency, equity and informational efficiency. As a corollary, the Walrasian solution in private ownership production economies is also completely characterized by adopting SPI.

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1, Introduction

In production economies where some productive resource is publicly owned and the other factor is privately owned, a fundamental issue is what allocation rules are desirable to respect public ownership of the productive resource in conjunction with private ownership of the other factor. It was with regard to this issue that Roemer and Silvestre (1989) proposed three solutions. These are *the equal benefit solution* (EB), *the proportional solution* (PR) and *the constant returns equivalent solution* (CRE)[1].

In these solutions, CRE is offered some full axiomatic characterization by some literatures (Moulin (1987), Moulin (1990a,b), Moulin and Roemer (1989), and Roemer and Silvestre (1989)). On the other hand, EB and PR are also axiomatically characterized by Moulin (1990a,b), but these characterizations are not full in the following sense: the EB solution is included in any solution set satisfying Pareto Optimality (PO), Lower Bound Egalitarian (LBE) (Moulin (1990a,b)) and IIA (Moulin (1990a)), but the inverse relation is not true. Also, the PR solution is included in any solution set satisfying PO, Free Access Upper Bound (FAUB) (Moulin (1990a,b)), IIA and Maskin Monotonicity (Maskin (1977)), but the inverse relation is not true whenever preferences are weakly (not strictly) convex.

In this paper, we offer full characterization for each of EB and PR by introducing new axioms. Those are *Pareto Independence* (PI) and *Support Price Independence* (SPI). The former says that for any economy in domain, any allocation in the solution set, if the economy is changed such that the current allocation becomes Pareto efficient, then the current allocation remains in the solution set. The latter says that for any economy in domain, any allocation in the solution, some price which is supporting the current allocation as the solution, if the economy is changed such that the current price supports the current allocation as Pareto efficiency, then the current allocation remains in the solution set.

We can regard PI and SPI as a requirement of informational efficiency. Informational efficiency has an implication as in the following statements: When some economy changed its characteristic to the other one, it is necessary for attaining a new allocation as the solution in the new economy to collect information on its new characteristic (the new profile of all members' preferences and the new production set). Then, if by collecting only local information on the new economy's characteristic, the new allocation is attainable as the solution, such the solution is referred to as meeting informational efficiency. We argue in section 3 that PI and SPI represent criteria of informational efficiency in the sense that both axioms require only local information on changing economic environments. Notice that many equitable solutions do not satisfy these informationally efficiency requirements. For example, *CRE* does not satisfy these. *No-envy and efficient solution* (Foley (1967)) also does not. *Individual rational and efficient solution* and *Core solution* also do not. These solutions need to collect not local but global information of members' indifference curves at the current allocation. Hence, these solutions do not meet our requirement of informational efficiency.

The implication of PI and SPI is also related to Nash implementability of solutions by a "natural mechanism": The problem of Nash implementation by a "natural mechanism" is discussed by Dutta, Sen and Vohra (1995), and Saijo, Tatamitani and Yamato (1993). In their arguments, the mechanisms in which each participant announces his own demand quantity only or plus at most a current price are regarded as the most natural mechanisms. These literatures also introduced the axioms which completely characterize the solutions implemented by the above natural mechanisms in pure exchange economies. Those are named Condition W* and/or Condition M by Saijo, Tatamitani and Yamato (1993). These axioms can be interpreted as stronger versions of Maskin Monotonicity which is the necessary and sufficient condition for Nash Implementation by unrestricted mechanisms. In production economies where production technology is fixed, it is easily proven that *EB* and *PR* satisfy Condition M, and that they also satisfy Condition W* when the production technology is represented by a differentiable production function. Yoshihara (1994) concretely constructed, in differentiable production economies, two demand quantity-announcing mechanisms each of which implements EB or PR both in Nash and strong equilibria. As showing in section 3, the axiom PI is equivalent to the axiom SPI in differentiable production economies where production technology is fixed. Moreover, any solution satisfying Pareto Optimality and either PI or SPI satisfies Condition W* in those economies. Thus, the axioms PI and SPI imply Nash implementability of Pareto efficient solutions satisfying these axioms by a natural demand-quantity mechanism.

We show that in convex technology production economies with one input and one output, PR is a unique solution satisfying PO, FAUB and SPI even if preferences are weakly (not strictly) convex. Also, we show that EB is a unique solution satisfying PO, LBE and SPI. PO is related to allocational efficiency. Each of FAUB and LBE represents a desirability from a viewpoint of equity in the context of cooperative production with convex technology. Thus, the implication of these results is that both EB and PR not only meet some welfare criteria on desirable allocations from some viewpoints of allocational efficiency and equity in the context of cooperative production but also have a desirable property in the sense of informational efficiency and natural implementablity. Moreover, as corollaries of these results, we show that PR is a unique solution satisfying PO, PI and Individual Rationality (IR) even if preferences are weakly (not strictly) convex, and that EB is a unique solution satisfying PO, SPI and No Envy (NE) (Foley (1967)).

Next, we discuss on the case of convex technology production economies with multi-input and multi-output. In such economies, PR no longer satisfies FAUB. So we define a new axiom, "Upper Bound by Stand Alone Income (UBSAI)", that is a weaker version of FAUB in multi-input and multi-output economies. We show, in addition, that PR no longer is a unique solution satisfying PO, UBSAI and SPI when the economy has a positive commodity vector of publicly owned initial endowments.

In contrast, EB is shown to be a unique solution satisfying PO, LBE and SPI even when there exists a positive commodity vector of publicly owned initial endowments.

We also discuss on *the Walrasian solution* (W) in private ownership production economies with multi-input and multi-output. By adopting the axioms of Full Individual Rationality (FIR) (Gevers (1986)), PO and SPI, we show that W is fully characterized. Some axiomatic characterizations of W were argued by Gevers (1986) and Nagahisa (1991, 1994). Nagahisa (1991) fully characterized W in the case of differentiable pure exchange economies. In contrast, with respect to the case of W in production economies, although Gevers (1986) and Nagahisa (1994) argued, their axiomatic characterizations were not complete. A difference between our result and their results is that we succeed in fully characterizing W by adopting SPI while they did not by adopting other axioms[2].

In the following, section 2 defines the basic model and two solutions in public ownership production economies, and section 3 introduces some axioms. Section 4 gives some results. Section 5 discusses on some generalization of the results in section 4. Furthermore, section 5 discusses on axiomatic characterization of the Walrasian solution in private ownership production economies.

2. The model

There are two goods, one privately owned and utilized as an input to produce the other. Let x denote the privately owned input and y output produced. The initial endowment of x is denoted by $\Omega \in R_{--}$[3]. The production function $f: R_{-} \rightarrow R_{+}$ is such that $f(x) = y$, and is assumed to be continuous, concave and non-increasing over R_{-}. We denote by F the set of such functions. From $f \in F$, we induce one publicly owned production possibility set, $Y(f) := \{(x, y) \in R_{-} \times R_{+} \mid f(x) \geq y\}$.

There are n members ($n \geq 2$). The set of members is denoted by $I = \{1,2,\ldots,n\}$ with generic element i. Each member is endowed with a negative amount ω_i of input x and no output. The aggregation of ω_i is Ω. Each member i has $Z_i = [\omega_i, 0] \times R_+$ as his attainable consumption set. Each member i's preference is represented by a utility function $u_i : R_- \times R_+ \to R$ where u_i is continuous, quasi-concave and strictly increasing. We denote by U the set of such functions.

A *feasible allocation* for the economy $\varepsilon = (u, f) \in U^n \times F$ is an n×2-tuple $z = (z_i)_{i \in I}$ such that for all $i \in I$, $z_i \in Z_i$, and $\sum_{i \in I} z_i \in Y(f)$. Let $A(\varepsilon)$ denote the set of all feasible allocations for the economy ε.

A *public ownership solution* (POS) is a mapping S associating with every economy $\varepsilon = (u, f) \in U^n \times F$ a non-empty subset $S(\varepsilon)$ of feasible allocations $A(\varepsilon)$.

Let $\Delta := \{p \in R_+^2 | p_x + p_y = 1\}$ be the unit simplex. Let $H(p, w) := \{ w' \in R_- \times R_+ \mid pw' \leq pw\}$ and $\partial H(p,w) := \{w' \in R_- \times R_+ \mid pw' = pw \}$ for $p \in \Delta$ and $w \in R_- \times R_+$. A production point $w \in Y(f)$ is efficient at $p \in \Delta$ if $Y(f) \subseteq H(p,w)$. Agent i's demand correspondence at u, when he faces the budget constraint, $pz_i = a$, for some $p \in \Delta$ and amount $a \in R_+$ of share of surplus, is denoted by $d_i(p, a, u) := \arg\max \{ u_i(z_i) \mid z_i \in Z_i \text{ and } pz_i = a\}$. A feasible allocation z is *Pareto efficient* at $\varepsilon = (u, f) \in U^n \times F$ if there does not exist another feasible allocation z^* at ε such that $u_i(z_i^*) > u_i(z_i)$ for all $i \in I$. We denote by $P(\varepsilon)$ the set of Pareto efficient allocation at ε.

Definition (Moulin (1990a)): For any $\varepsilon = (u, f) \in U^n \times F$, $EB(\varepsilon)$ is the set of *equal benefit solutions* if for any $z \in EB(\varepsilon)$, there exists $p \in \Delta$ and $a \in R_+$ such that for all $i \in I$, $z_i \in d_i(p, a, u)$ and $\sum_{i \in I} z_i \in Y(f) \subseteq H(p, \sum_{i \in I} z_i)$.

Definition (Moulin (1990a)): For any $\varepsilon = (u, f) \in U^n \times F$, $PR(\varepsilon)$ is the set of *proportional solutions* if for any $z \in PR(\varepsilon)$, z is a Pareto efficient allocation and for any $i \in I$, $y_i = \frac{x_i}{\sum_{j \in I} x_j} f(\sum_{j \in I} x_j)$.

3. Axioms

In this section, we take several axioms each of which represents some welfare criterion on desirability of the solutions. The welfare criteria we focus on are classified into three viewpoints. The first viewpoint of the welfare criteria is related to allocational efficiency of the solutions. A representative criterion of allocational efficiency is Pareto Optimality. In the following, Axiom 1 and 2 represent welfare criteria of allocational efficiency. The second viewpoint of the welfare criteria is related to equity concepts. A representative equity concept is No Envy (Foley (1967)). Each of Axiom 3~5 represents what is the equitable allocation in the context of public ownership economies with convex technology. The third viewpoint of the welfare criteria is related to informational efficiency of the solutions. When for some economic environment, some solution is attained, one would be concerned with how much costly information transmission is involved. Informational efficiency is related to some criteria on what allocations are attainable through as costless information transmission as possible. Axiom 6~8 represent such criteria.

First, we adopt two criteria of allocational efficiency. The first criterion is total rationality. The second criterion is individual rationality.

Axiom 1: Pareto Optimality (PO): $\forall \varepsilon = (u, f) \in U^n \times F$, $S(\varepsilon) \subseteq P(\varepsilon)$.

Axiom 2: Individual Rationality (IR):
$\forall \varepsilon = (u, f) \in U^n \times F$, $[(z_i)_{i \in I} \in S(\varepsilon) \Rightarrow u_i(z_i) \geq u_i(0)\ (\forall i \in I)]$.

Second, we discuss on equity criteria which public ownership solutions should satisfy in the context of convex production economies. In commonly owned convex production economies, each member has free access to the production technology. However, in such a case, if all members behave to pursue their individually maximal welfares, it is well-known that "the tragedy of the common" is consequential, since joint utilization of convex technology brings a negative externality. This implies that to avoid such a socially inefficient state, each member should bear some share of this negative externality instead of pursuing his individually maximal welfare. In public ownership economies, it is natural that such a requirement is imposed. Thus, Axiom 3 is taken.

Axiom 3: Free Access Upper Bound (FAUB) (Moulin (1990a,b)):
$\forall \varepsilon=(u,\ f)\in U^n\times F,\ [(z_i)_{i\in I}\in S(\varepsilon)\Rightarrow u_i(z_i)\leq \max_{x_i\in[\omega_i,\ 0]} u_i(x_i,\ f(x_i))\ (\forall i\in I)].$

The next axiom is well-known as a representative equity criterion with respect to fair division problems.

Axiom 4: No Envy (NE) (Foley (1967)):
$\forall \varepsilon=(u,\ f)\in U^n\times F,\ [(z_i)_{i\in I}\in S(\varepsilon)\Rightarrow \forall i,j\in I,\ u_i(z_i)\geq u_i(z_j)].$

Axiom 5 is also related to fair division problems. In public ownership economies, identical members should be treated equally, and differences of the surplus opportunities should be caused only by the differences in preferences due to personal responsibility. In other words, all members should be guaranteed at least some minimal equality of welfare level whenever all members have identical preferences. Then, such a minimal equality of welfare imposes a lower bound on all members' welfare. Such a lower bound is constituted by the welfare that each agent is reachable by utilizing an equal share of the production set. Thus, Axiom 5 represents a requirement on equal guaranteeing of the lower bound welfare in public ownership solutions.

Axiom 5: Lower Bound Egalitarian (LBE) (Moulin (1990a,b)):
$\forall \varepsilon=(u,\ f)\in U^n\times F,\ [(z_i)_{i\in I}\in S(\varepsilon)\Rightarrow u_i(z_i)\geq \max_{x_i\in[\omega_i,\ 0]} u_i(x_i,\ \frac{1}{n}f(nx_i))\ (\forall i\in I)].$

Next, we discuss on criteria of informational efficiency. The first axiom is adopted by Moulin (1990a) to characterize public ownership solutions.

Axiom 6: Independence of Irrelevant Alternatives (IIA) (Moulin (1990a)):
$\forall \varepsilon = (u,\ f) \in U^n \times F,\ \forall g \in F\ \textit{such that}\ g(x) \leq f(x)\ (\forall x \in R_-),$
$[(z_i)_{i \in I} \in S(\varepsilon)$ and $(z_i)_{i \in I} \in A(\varepsilon') \Rightarrow (z_i)_{i \in I} \in S(\varepsilon')$ where $\varepsilon' = (u,\ g)]$.

The next two axioms are introduced first in this paper. They are Axiom 7 (Pareto Independence) and Axiom 8 (Support Price Independence). Axiom 7 says that if some economy is changed such that the current allocation assigned as the solution becomes Pareto efficient in the changed economy, then this allocation remains in the solution set of the changed economy. Such an axiom represents a criterion of informational efficiency. The reason is as follows: If some current feasible allocation is equitable solution in some economic environment, whether the current allocation is also equitable solution or not in the other economic environment is verified by only checking whether or not this allocation is Pareto efficient in the other economic environment. Checking whether or not some allocation is Pareto efficient is very easy in convex economic environments, since it is enough to collect only local information on members' indifference curves at this allocation. In other words, it is sufficient to check whether or not all members weakly prefer this allocation to the other feasible allocation in some neighborhood of this allocation whenever all possible economies have convex properties.

Axiom 7: Pareto Independence (PI): $\forall \varepsilon = (u,\ f) \in U^n \times F,\ (z_i)_{i \in I} \in S(\varepsilon)$ and
$\forall (\tilde{u},\ g) \in U^n \times F,\ [(z_i)_{i \in I} \in P(\varepsilon')$ where $\varepsilon' = (\tilde{u},\ g) \Rightarrow (z_i)_{i \in I} \in S(\varepsilon')]$.

The output distribution rule that fully divides total output independent of preferences satisfies PI. For example, the egalitarian distribution rule, which assigns equal dividend of total output to every member, and the average surplus sharing rule satisfy PI. Obviously, the Pareto solution satisfies PI. However, the Walrasian solution does not satisfy PI.

Axiom 8 (Support Price Independence) says that if some economy is changed such that some current price which supports the current allocation in the solution set becomes an efficiency price in the changed economy, then this allocation remains in the solution set of the changed economy. As well as Axiom 7, this axiom also represents a criterion of informational efficiency. The reason is that checking whether some price which supports the current allocation in the solution set becomes an efficiency price or not in some new economy is enough to collect members' preference information on some feasible allocations in some neighborhood of the current allocation.

Let $p(f, w) := \{p \in \Delta \mid \exists w \in Y(f), Y(f) \subseteq H(p, w)\}$. For $i \in I$, $u_i \in U_i$, and $z_i \in Z_i$, let $L(z_i, u_i) := \{z_i^{\circ} \in Z_i \mid u_i(z_i) \geq u_i(z_i^{\circ})\}$ be the weak lower contour set for member i with u_i at z_i.

Definition: For some $(z_i)_{i \in I} \in S(u, f)$ for $(u, f) \in U^n \times F$, a price $p \in \Delta$ *supports* $(z_i)_{i \in I}$ *at* (u, f) if $p \in \Delta$ satisfies the following conditions:
1) $p \in p(f, (\sum_{i \in I} x_i, f(\sum_{i \in I} x_i)))$, and 2) if $S \subseteq P$, then $L(z_i, u_i) \supseteq H(p, z_i)(\forall i \in I)$.

Denote for some $(z_i)_{i \in I} \in S(u, f)$ for $(u, f) \in U^n \times F$, the set of prices supporting $(z_i)_{i \in I}$ at (u, f) by $p(f, u, (z_i)_{i \in I})$.

Definition: For some $(z_i)_{i \in I} \in P(u, f)$ for $(u, f) \in U^n \times F$, a price $p \in \Delta$ *supports* $(z_i)_{i \in I}$ *at* (u, f) *as a Pareto efficient allocation* if $p \in \Delta$ satisfies the following conditions: 1) $p \in p(f, (\sum_{i \in I} x_i, f(\sum_{i \in I} x_i)))$, and 2) $L(z_i, u_i) \supseteq H(p, z_i)(\forall i \in I)$.

Denote for some $(z_i)_{i \in I} \in P(u, f)$ for $(u, f) \in U^n \times F$, the set of prices supporting $(z_i)_{i \in I}$ at (u, f) as a Pareto efficient allocation by $p^P(f, u, (z_i)_{i \in I})$. Notice that if S does not satisfy PO, then $p(f, u, (z_i)_{i \in I}) = p(f, (\sum_{i \in I} x_i, f(\sum_{i \in I} x_i)))$, while if $S \subseteq P$, then $p(f, u, (z_i)_{i \in I}) = p^P(f, u, (z_i)_{i \in I})$.

Axiom 8: Support Price Independence (SPI): $\forall \varepsilon = (u, f) \in U^n \times F$, $(z_i)_{i \in I} \in S(\varepsilon)$, $\exists p \in p(f, u, (z_i)_{i \in I})$, $\forall (\tilde{u}, g) \in U^n \times F$, $[p \in p^P(g, \tilde{u}, (z_i)_{i \in I}) \Rightarrow (z_i)_{i \in I} \in S(\tilde{u}, g)]$.

Notice that SPI requires less information than PI when attaining a new allocation as the solution in the changed economy. The reason is that checking whether some current supporting price becomes an efficient price or not requires only information of preferences on the allocations in a subset of some neighborhood of the current allocation whenever all possible economies have convex properties. Such a subset is the intersection of the neighborhood set and the half space which is defined by the current allocation and the supporting price. This fact also implies that the PI-satisfying solutions are more allocational-invariant with respect to environmental changing than SPI-satisfying solutions, since there exist many cases such that the allocation remains to be Pareto efficient while the corresponding efficiency price is changed.

As well as PI, SPI is also satisfied by any output distribution rule that fully divides total output independent of preferences. It is easy to show that the Walrasian solution satisfies SPI. On the other hand, the Pareto and Individual Rational solution and the No envy solution do not satisfy SPI.

We examine the relations of the above three axioms on informational efficiency.

Lemma 1 : PI implies SPI.

Proof: Let take $\varepsilon = (u,\ f) \in U^n \times F$, $(z_i)_{i\in I} \in S(\varepsilon)$ and any $p \in p(f,\ u, (z_i)_{i\in I})$. It is clear that for any $(\tilde{u},\ g) \in U^n \times F$ such that $p \in p^P(g,\ \tilde{u}, (z_i)_{i\in I})$, $(z_i)_{i\in I} \in P(\tilde{u},\ g)$. Since S satisfies PI, $(z_i)_{i\in I} \in S(\tilde{u},\ g)$. Q.E.D.

Let $DF := \{f \in F \mid f$ is differentiable $\}$. We denote by $U^n \times DF$ the class of differentiable production economies.

Lemma 2 : For any fixed production technology in DF, PI coincides with SPI.

Proof: Let take $\varepsilon = (u,\ f) \in U^n \times DF$ and $(z_i)_{i\in I} \in S(\varepsilon)$. Since f is differentiable, $p(f, (\sum_{i\in I} x_i,\ f(\sum_{i\in I} x_i)))$ is singleton, so that $p(f,\ u, (z_i)_{i\in I})$ is singleton. Then, if for some $\tilde{u} \in U^n$, $(z_i)_{i\in I} \in P(\tilde{u},\ f)$, $p(f,\ u, (z_i)_{i\in I}) = p^P(f,\ \tilde{u}, (z_i)_{i\in I})$ is obtained. Since S satisfies SPI, $(z_i)_{i\in I} \in S(\tilde{u},\ f)$. Q.E.D.

Lemma 3 : PO and SPI imply IIA.

Proof: Let S satisfies PO and SPI. Let take $\varepsilon = (u,\ f) \in U^n \times F$, $(z_i)_{i\in I} \in S(\varepsilon)$ and any $p \in p(f,\ u, (z_i)_{i\in I})$. Then $p \in p^P(f,\ u, (z_i)_{i\in I})$. Next, take a function $g \in F$ such that $g(x) \leq f(x)$ $(\forall x \in R_-)$ and $(z_i)_{i\in I} \in A(u,\ g)$. Then, $Y(g) \subseteq H(p,\ \sum_{i\in I} z_i)$. Thus, $p \in p^P(g,\ u, (z_i)_{i\in I})$. This implies $(z_i)_{i\in I} \in S(u,\ g)$. Q.E.D.

Lemma 4 : PO and PI imply IIA.

Proof: It is clear that by Lemma 1 and 3. Q.E.D.

We next examine the relationship between our new axioms and Nash implementability. A social choice solution S is Nash-implementable if there exists a game form such that for any possible economic environment, the set of S - solutions coincides with the set of Nash equilibrium allocations of the non-cooperative game which is defined by a pair of that game form and a profile of preferences. It is well-known that the necessary and sufficient condition for Nash implementability of S is Maskin Monotonicity (Maskin (1977)), if $n \geq 3$ and all possible preferences in the class of economic environments are strictly monotone-increasing.

Maskin Monotonicity (Maskin (1977)): $\forall\, u,\ u^* \in U^n$, $(z_i)_{i\in I} \in S(u,\ f)$,
$[L(z_i,\ u_i) \subseteq L(z_i,\ u_i^*)\ (\forall i \in I) \Rightarrow (z_i)_{i\in I} \in S(u^*,\ f)]$.

Lemma 5 : SPI and PO imply Maskin Monotonicity.

Proof: Let S satisfies PO and SPI. Let take $\varepsilon = (u,\ f) \in U^n \times F$, $(z_i)_{i\in I} \in S(\varepsilon)$ and any $p \in p(f,\ u, (z_i)_{i\in I})$. Then $p \in p^P(f,\ u, (z_i)_{i\in I})$. This implies that for all $i \in I$, $\hat{z}_i \in H(p,\ z_i) \cap Z_i \Rightarrow u_i(z_i) \geq u_i(\hat{z}_i)$. Next, take a profile $\tilde{u} \in U^n$ such that for all $i \in I$ and $z_i^* \in Z_i$, $u_i(z_i) \geq u_i(z_i^*) \Rightarrow \tilde{u}_i(z_i) \geq \tilde{u}_i(z_i^*)$. Then $p \in p^P(f,\ \tilde{u}, (z_i)_{i\in I})$. By SPI, $(z_i)_{i\in I} \in S(\tilde{u},\ f)$. Q.E.D.

Lemma 6 : PI and PO imply Maskin Monotonicity.

Proof: It is clear that by Lemma 1 and 5. Q.E.D.

By Lemma 5 and 6, the constant returns equivalent solution (*CRE*) satisfies neither PI nor SPI, because *CRE* is Pareto optimal but does not satisfy Maskin Monotonicity.

Next, we introduce Condition W* (Saijo, Tatamitani and Yamato (1993)) which characterize the solutions implemented in Nash equilibria by a natural demand-quantity mechanism. Let $f \in F$ is fixed. For $(z_i)_{i\in I} \in \times_{i\in I} Z_i$, let
$\Lambda_i^S(z_i, f) := \bigcap_{u \in S^{-1}((z_i)_{i\in I},\, f)} L(z_i, u_i)$.

Condition W* (Saijo, Tatamitani and Yamato (1993)): $\forall \varepsilon = (u, f) \in U^n \times F$, $(z_i)_{i\in I} \in S(\varepsilon)$ and $\forall u^* \in U^n$, $[\Lambda_i^S(z_i, f) \subseteq L(z_i, u_i^*)\ (\forall i \in I) \Rightarrow (z_i)_{i\in I} \in S(u^*, f)]$.

Lemma 7 : For any fixed production technology f in *DF*, SPI and PO imply Condition W*.

Proof: Let S satisfies PO and SPI. Let take $\varepsilon = (u, f) \in U^n \times DF$, $(z_i)_{i\in I} \in S(\varepsilon)$ and $p \in p(f, u, (z_i)_{i\in I})$. Then $\{p\} = p^P(f, u, (z_i)_{i\in I})$. This implies that for all $i \in I$, $\hat{z}_i \in H(p, z_i) \cap Z_i \Rightarrow u_i(z_i) \geq u_i(\hat{z}_i)$. This fact is true for all $u^\circ \in S^{-1}((z_i)_{i\in I}, f)$ because $\{p\} = p(f, (\sum_{i\in I} x_i, f(\sum_{i\in I} x_i)))$. Thus, $H(p, z_i) \cap Z_i \subseteq \Lambda_i^S(z_i, f)$ $(\forall i \in I)$.

Next, take a profile $\tilde{u} \in U^n$ such that for all $i \in I$, $\Lambda_i^S(z_i, f) \subseteq L(z_i, \tilde{u}_i)$. Then, for all $i \in I$, $H(p, z_i) \cap Z_i \subseteq L(z_i, \tilde{u}_i)$. Hence, $p \in p^P(f, \tilde{u}, (z_i)_{i\in I})$. By SPI, $(z_i)_{i\in I} \in S(\tilde{u}, f)$. Q.E.D.

Lemma 8 : For any fixed production technology f in *DF*, PI and PO imply Condition W*.

Proof: It is clear from Lemma 2 and 8. Q.E.D.

Lemma 7 and 8 implies that the solutions satisfying PI or SPI is Nash-implementable by a natural demend-quantity mechanism.

4. Some Results

This section states our main results. First, we refer to the preceding results proven by Moulin (1990a,b).

Proposition 1 (Moulin (1990a)): The proportional solution PR satisfies axioms PO, IIA, FAUB and Maskin Monotonicity. Other solutions satisfying these four axioms are welfare indistinguishable from the PR solution.

Proposition 2 (Moulin (1990a)): The equal benefit solution EB satisfies PO, UBE and IIA.

Proposition 3 (Moulin (1990b)): The equal benefit solution EB satisfies PO, UBE and NE.

While Moulin (1990a,b) showed only the necessary part of characterizing PR and EB, I completely characterize these solutions by adopting PI and SPI instead of IIA and Maskin Monotonicity.

Theorem 1: The proportional solution PR satisfies axioms PO, FAUB and SPI. Conversely, no other solutions satisfy together these three axioms.

Proof: i) Suppose a solution S satisfying the three axioms is not PR. Take an economy $\varepsilon = (u,\ f) \in U^n \times F$. Suppose that for some allocation $(z_i)_{i\in I} \in S(\varepsilon\ )$, $(z_i)_{i\in I}$ is not PR. Then, there exists a supporting price $p \in p(f,\ u, (z_i)_{i\in I})$ such that for any $(\tilde{u},\ g) \in U^n \times F$ satisfying $p \in p^P(g,\ \tilde{u}, (z_i)_{i\in I})$, $(z_i)_{i\in I} \in S(\tilde{u},\ g)$, and there exist at least two agents $j,\ k \in I$ such that $\frac{y_j}{x_j} \neq \frac{\sum_{i\in I} y_i}{\sum_{i\in I} x_i} = \frac{f(\sum_{i\in I} x_i)}{\sum_{i\in I} x_i} \neq \frac{y_k}{x_k}$. Without loss of generality, $\frac{y_j}{x_j} < \frac{f(\sum_{i\in I} x_i)}{\sum_{i\in I} x_i} < \frac{y_k}{x_k}$ is assumed. In the following, we denote by $z = (x,\ y) \equiv (\sum_{i\in I} x_i,\ f(\sum_{i\in I} x_i))$ the aggregate production plan. Consider another economy $(\bar{u}, \delta_z) \in U^n \times F$ in the following:

$$\delta_z(x') = \begin{cases} x' \cdot \left(\dfrac{y}{x}\right) & if\ x \le x' \le 0 \\ y & if\ \ x' \le x \end{cases}$$

and for all $i \in I$, $\bar{u}_i(\tilde{x}_i, \tilde{y}_i) = \begin{cases} p_x\tilde{x}_i + p_y\tilde{y}_i & if\ \tilde{x}_i \in [x_i, 0] \\ y\tilde{x}_i - x\tilde{y}_i & if\ \tilde{x}_i \in (+\infty, x_i] \end{cases}$. Then, clearly,

$p \in p^P(g, \bar{u}, (z_i)_{i\in I})$. Thus, by SPI, $(z_i)_{i\in I} \in S(\bar{u}, \delta_z)$. By the way, $\dfrac{y_j}{x_j} < \dfrac{y}{x}$ implies $\bar{u}_j(x_j, y_j) > 0$. However, $\max\limits_{x_j \in [\omega_j, 0]} \bar{u}_j(x_j, \delta_z(x_j)) = 0$, so that FAUB of S is violated.

ii) It is sufficient to show that PR satisfies PI. That is trivial because the PR allocation is the Pareto efficient proportional allocation. Thus, by Lemma 1, PR satisfies SPI. Q.E.D.

Corollary 1: The proportional solution PR satisfies axioms PO, IR and PI. Conversely, no other solutions satisfy together these three axioms.

Proof: Suppose a solution S satisfying the three axioms is not PR. Take an economy $\varepsilon = (u, f) \in U^n \times F$. Suppose that for some allocation $(z_i)_{i\in I} \in S(\varepsilon)$, there exist at least two agents $j, k \in I$ such that $\dfrac{y_j}{x_j} < \dfrac{f(\sum_{i\in I} x_i)}{\sum_{i\in I} x_i} < \dfrac{y_k}{x_k}$. We consider the same another production technology $\delta_z \in F$ as in proof of Theorem 1. Moreover, consider the following preferences: for all $i \in I$, $\forall\ (\tilde{x}_i, \tilde{y}_i) \in R_- \times R_+$, $\bar{u}_i(\tilde{x}_i, \tilde{y}_i) = y\tilde{x}_i - x\tilde{y}_i$. Clearly, $(z_i)_{i\in I} \in P(\bar{u}, \delta_z)$. Thus, by PI, $(z_i)_{i\in I} \in S(\bar{u}, \delta_z)$. However, then, $\bar{u}_k(x_k, y_k) < 0 = \bar{u}_k(0, 0)$, so that IR of S is violated. Q.E.D.

Theorem 2: The equal benefit solution EB satisfies PO, UBE and SPI. Conversely, no other solution satisfies together these three axioms.

Proof: i) Suppose a solution S satisfying the three axioms is not EB. Take an economy $\varepsilon = (u, f) \in U^n \times F$. Suppose that for some allocation $(z_i)_{i\in I} \in S(\varepsilon)$, $(z_i)_{i\in I} \notin EB(\varepsilon)$. Then, there exists a supporting price $p \in p(f, u, (z_i)_{i\in I})$ such that for any $(\tilde{u}, g) \in U^n \times F$ satisfying $p \in p^P(g, \tilde{u}, (z_i)_{i\in I})$, $(z_i)_{i\in I} \in S(\tilde{u}, g)$, and moreover, $z_i = (x_i, y_i) \in d_i(p, a_i, u_i)\ (\forall i \in I)$ where $\sum_{i\in I} a_i = \sum_{i\in I} pz_i$ and for some

$k,\ j \in I,\ a_k \neq a_j$. Without loss of generality, $a_k > \frac{\sum_{i\in I} a_i}{n} > a_j$ is assumed. Notice that for all $i \in I,\ z_i \notin \frac{1}{n}\overset{\circ}{Y}(f)$ by UBE, where $\frac{1}{n}\overset{\circ}{Y}(f) := \{(x,\ y) \in R_- \times R_+ \mid y < \frac{1}{n} f(nx)\}$.

Next consider for all $i \in I$, a utility function $\bar{u}_i \in U$ such that $\forall\ (\tilde{x}_i,\ \tilde{y}_i) \in R_- \times R_+$, $\bar{u}_i(\tilde{x}_i,\ \tilde{y}_i) = p_x\tilde{x}_i + p_y\tilde{y}_i$, and a production function $g \in F$ such that $g(x) = -\frac{p_x}{p_y}x + \sum_{i\in I} a_i$. Then, clearly, $p \in p^P(g,\ \bar{u},(z_i)_{i\in I})$. Thus, by SPI, $(z_i)_{i\in I} \in S(\bar{u},\ g)$. However, for member j, $\bar{u}_j(z_j) < \max_{x_j \in [\omega_j,\ 0]} \bar{u}_j(x_j,\ \frac{1}{n}g(nx_j))$, since $\bar{u}_j(z_j) = a_j < \frac{\sum_{i\in I} a_i}{n} = \max_{x_j \in [\omega_j,\ 0]} \bar{u}_j(x_j,\ \frac{1}{n}g(nx_j))$. It is a contradiction.

ii) It is clear that EB satisfies PO and UBE. Let us show that EB satisfies SPI. For any $\varepsilon = (u,\ f) \in U^n \times F$, $(z_i)_{i\in I} \in EB(\ \varepsilon\ )$, there exist $p(f,\ u,(z_i)_{i\in I})$ such that $\sum_{i\in I} pz_i = na = \max\ \{p\cdot(x,\ y) | y \leq f(x)\}$ and $z_i \in d_i(p,\ a,\ u_i)\,(\forall i \in I)$. Consider another economy $(\bar{u},\ g) \in U^n \times F$ such that $p \in p^P(g,\ \bar{u},(z_i)_{i\in I})$. Then, since $(z_i)_{i\in I} \in P(\bar{u},\ g)$, $z_i \in d_i(p,\ a,\ \bar{u}_i)\,(\forall i \in I)$. This implies $(z_i)_{i\in I} \in EB\,(\bar{u},\ g)$.

Q.E.D.

Corollary 2: The equal benefit solution EB satisfies PO, NE and SPI. Conversely, no other solution satisfies together these three axioms.

Proof: We follow the process of proof i) in Theorem 2, and then for member j, $\bar{u}_j(z_j) < \bar{u}_j(z_k)$, since $\bar{u}_j(z_j) = a_j < a_k = \bar{u}_j(z_k)$. Thus, NE of S is violated. Q.E.D.

5. A further discussion —Some Generalization—

In this section, some generalization of the above discussion is considered. In a general model, there are one type of labor input and m commodities partitioned into two groups: the privately owned commodities, indexed 1 to k, and the publicly owned commodities, indexed $k+1$ to m. There is a publicly owned firm with a production set

$Y \subseteq R_- \times R_-^k \times R^{m-k}$. Vector $z \in Y$ will be written as $(m+1)$ vectors, as follows: $z = (x,\ y_f,\ y_C)$ where x is the labor input, y_f is the non-positive k vector of commodity inputs supplied by members, y_C is the $(m$-$k)$ vector of commodities the negative components of which are inputs supplied by the publicly owned firm, and the positive components of which are outputs produced by the publicly owned firm. It is assumed that:

A1. $0 \in Y$.

A2. Y is closed and convex.

A3. $\forall\ z = (x,\ y_f,\ y_C) \in Y$ such that $(x,\ y_f) \in R_- \times R_-^k$,
$[\exists\ j \in \{k+1,\ \cdots,\ m\}\ s.t.\ y_{Cj} > 0 \Rightarrow x < 0]$.

A4. Y is comprehensive:
$[(x,\ y_f,\ y_C) \in Y,\ (x',\ y_f') \leq (x,\ y_f)$ and $y_C' \leq y_C\ ] \Rightarrow (x',\ y_f',\ y_C') \in Y$.

A5. Labor is productive: $[(x,\ y_f,\ y_C) \in Y$ and $x' < x\ ]$
$\Rightarrow [\exists\ (y_f',\ y_C')$ such that $(x',\ y_f',\ y_C') \in Y$ and $(y_f',\ y_C') \geq (y_f,\ y_C)]$.

We denote by Y the class of production possibilities sets satisfying A1~A5. We denote by $\partial\ Y$ the efficiency frontier of Y:
$z \in \partial\ Y \Leftrightarrow z \in Y$ and $[\{z' \in Y,\ (x',\ y_f') \geq (x,\ y_f),\ y_C' \geq y_C\} \Rightarrow z' = z]$. The aggregate initial endowment of y_f is denoted by $\Omega^f \in R_+^k$. Each member i's initial endowment of commodities is denoted by $\omega_i^f \in R_+^k$. The publicly owned initial endowment of commodity inputs is denoted by $\omega_C \in R_+^{m-k}$.

There are n members ($n \geq 2$). The set of members is denoted by $I = \{1,2,\ldots,n\}$ with generic element i. Each member i is endowed with a negative amount ω_i^0 of labor endowment. The aggregation of labor endowments is Ω^0. Each member i has $Z_i = [\omega_i^0,\ 0] \times R_+^m$ as his attainable consumption set. The generic element of i's consumption vector is denoted by $z_i = (x_i,\ y_i)$. Each member i's preference is represented by a utility function $u_i : Z_i \to R$ where u_i is continuous, quasi-concave and strictly increasing. We denote by U_i the set of such functions.

Let $Z = (Z_i)_{i\in I}$, $u = (u_i)_{i\in I}$ and $\omega = (\omega_i)_{i\in I}$ where $\omega_i = (\omega_i^0, \omega_i^f) \in R_- \times R_+^k$. An economy is specified a list $(Z, u, \omega, \omega_C, Y)$. In the following, we fix ω and ω_C, so that Z is also fixed. Then the class of possible economies is denoted by $\mathrm{E} = \underset{i\in I}{\times} U_i \times \mathrm{Y}$ with generic element $\varepsilon = (u, Y)$. An n-tuple consumption bundle $(z_i)_{i\in I}$ and an input-output combination $\tilde{z} = (x, y_f, y_C)$ constitute a feasible allocation if: 1) for each i, $z_i \in Z_i$, 2) $\tilde{z} \in Y$, and 3) $\sum_{i\in I} z_i - \sum_{i\in I} \omega_i^f - \omega_C \leq (x, y_f, y_C)$. Let $A(\varepsilon)$ denote the set of all feasible allocations for the economy ε with generic element $\zeta := ((z_i)_{i\in I}, \tilde{z})$.

A *public ownership solution* (POS) is a mapping S associating with every economy $\varepsilon = (u, Y) \in \mathrm{E}$ a non-empty subset $S(\varepsilon)$ of feasible allocations $A(\varepsilon)$. A feasible allocation $\zeta = ((z_i)_{i\in I}, \tilde{z})$ is *Pareto efficient* at $\varepsilon = (u, Y) \in \mathrm{E}$ if there does not exist another feasible allocation $\zeta^* = ((z_i^*)_{i\in I}, \tilde{z}^*)$ at ε such that $u_i(z_i^*) > u_i(z_i)$ for all $i \in I$. We denote by $P(\varepsilon)$ the set of Pareto efficient allocations at ε.

Let $\Delta := \{p \in R_+^{m+1} | p_x + p_y = 1\}$ be the unit simplex. Let $H(p, w) :=$ $\{w^* = (x^*, y_f^*, y_C^*) \in R_- \times R_-^k \times R_+^{m-k} | \; p \cdot w \geq p \cdot w^*\}$ for $p \in \Delta$ and $w \in R_- \times R_-^k \times R_+^{m-k}$. A production point $w \in Y$ is efficient at $p \in \Delta$ if $Y \subseteq H(p, w)$.

Definition: The nonzero vector $p \in \Delta$ is a vector of efficiency prices for the Pareto efficient allocation $\zeta = ((z_i)_{i\in I}, \tilde{z})$ at $\varepsilon = (u, Y) \in \mathrm{E}$ if

(a) $p \cdot z_i \leq p \cdot z_i^*$ for all $z_i^* \in Z_i$ such that $u_i(z_i) \leq u_i(z_i^*)$ $(\forall i \in I)$;

(b) $Y \subseteq H(p, \tilde{z})$, and

(c) $p \cdot (\sum_{i\in I} z_i - \sum_{i\in I} \omega_i^f - \omega_C - \tilde{z}) = 0$.

Definition (Roemer and Silvestre (1989)): An allocation $\zeta = ((z_i)_{i\in I}, \tilde{z})$ is an equal benefit solution for $\varepsilon = (u, Y) \in \mathrm{E}$ if:

(i) $\zeta \in P(\varepsilon)$;

(ii) There exists a vector of efficiency prices $p \in \Delta$ for ζ such that , for any pair of members i, h, $p \cdot z_i - p \cdot \omega_i^f = p \cdot z_h - p \cdot \omega_h^f$.

Definition (Roemer and Silvestre (1989)): An allocation $\zeta = ((z_i)_{i \in I}, \bar{z})$ is a proportional solution for $\varepsilon = (u, Y) \in \mathrm{E}$ if:

(i) $\zeta \in P(\varepsilon)$;

(ii) There exists a vector of efficiency prices $p \in \Delta$ for ζ such that

$$p \cdot z_i = p \cdot \omega_i^f + \frac{p \cdot (x_i + y_{fi})}{\sum_{h \in I} p \cdot (x_h + y_{fh})} p \cdot (\sum_{i \in I} z_i - \sum_{i \in I} \omega_i^f) \ (\forall i \in I) \text{ if } \sum_{h \in I} p \cdot (x_h + y_{fh}) < 0,$$

$$p \cdot z_i = p \cdot \omega_i^f + \frac{1}{n} p \cdot (\sum_{i \in I} z_i - \sum_{i \in I} \omega_i^f) \text{ if } \sum_{h \in I} p \cdot (x_h + y_{fh}) = 0.$$

Now we define, in the general economic environment defined above, an extended version of the axiom 3. Let $p(Y, \bar{z}) := \{p \in \Delta \mid \exists \bar{z} \in Y, Y \subseteq H(p, \bar{z})\}$. At a given economy $\varepsilon = (u, Y)$, for each $(x_i, y_{fi}) \in [\omega_i^\circ, 0] \times [-\omega_i^f, 0]$ of agent i, there exists $p \in p(Y, (x_i, y_{fi}, y_C))$ such that

$\max u_i(x_i, y_i)$ where $(x_i, y_i) \in Z_i \cap \hat{H}(p, (x_i, y_{fi} + \omega_i^f, y_C + \omega_C))$. Let denote that $\max u_i(x_i, y_i) \equiv w_i(x_i, y_{fi}, u_i, Y)$. Then, there exists $(x_i^*, y_{fi}^*) \in [\omega_i^\circ, 0] \times [-\omega_i^f, 0]$ such that $(x_i^*, y_{fi}^*) = \arg\max\ w_i(x_i, y_{fi}, u_i, Y)$. Notice that

$p \cdot (x_i^*, y_{fi}^* + \omega_i^f, y_C^* + \omega_C)$ where $(x_i^*, y_{fi}^*, y_C^*) \in \partial\ Y$ is the "agent i's Stand Alone income" realizable when he utilizes alone the technology. We then call $w_i(x_i^*, y_{fi}^*, u_i, Y)$ "agent i's welfare of Stand Alone income", and denote by $wsa(u_i, Y)$.

Axiom 3': Upper Bound by Stand Alone Income (UBSAI):

$\forall \varepsilon = (u, Y) \in \mathrm{E}, [\zeta = ((z_i)_{i \in I}, \bar{z}) \in S(\varepsilon) \Rightarrow u_i(z_i) \leq wsa(u_i, Y)\ (\forall i \in I)]$.

Notice that in the one-input and one-output economy, UBSAI is reduced to FAUB. In such a sense, UBSAI is a extended version of FAUB in the multi-input and multi-output economy.

Next, we define the axiom of (LBE) in economic environments with multi-input and multi-output. At a given economy $\varepsilon = (u, Y)$, agent i's "Lower Bound welfare"

$l\ (u_i,\ Y)$ is defined as follows:

$l\ (u_i,\ Y) = \max\{u_i(x_i,\ y_i)\ |\ (x_i,\ y_i) \in Z_i,\ y_i = (\omega_i^f + y_{fi},\ y_C + \frac{\omega_C}{n})\ s.t.\ (x_i,\ y_{fi},\ y_C) \in \frac{Y}{n},\ -y_{fi} \leq$

Then the axiom of (LBE) is represented as follows:

Axiom 5': Lower Bound Egalitarian (LBE) (Moulin (1992)):
$\forall \varepsilon = (u,\ Y) \in$ E, $[\zeta = ((z_i)_{i\in I},\ \tilde{z}) \in S(\varepsilon) \Rightarrow u_i(z_i) \geq l\ (u_i,\ Y)\ (\forall i \in I)]$.

Let $p(Y,\ \tilde{z}) := \{p \in \Delta\ |\ \exists \tilde{z} \in Y,\ Y \subseteq H(p,\ \tilde{z})\}$ and
$\hat{H}(p,\ z_i) := \{z \in R_- \times R_+^m\ |\ p \cdot z \leq p \cdot z_i\}$.

Definition: For some $\zeta = ((z_i)_{i\in I},\ \tilde{z}) \in S(u,\ Y)$ for $(u,\ Y) \in$ E, a price $p \in \Delta$ *supports* ζ *at* $(u,\ Y)$ if $p \in \Delta$ satisfies the following conditions: 1) $p \in p(Y,\ \tilde{z})$, and 2) if $S \subseteq P$, then $L(z_i,\ u_i) \supseteq \hat{H}(p,\ z_i)(\forall i \in I)$.

Let $p(Y,\ u,\ \zeta)$ be the set of prices supporting ζ at $(u,\ Y)$. Let denote the set of efficiency prices for $\zeta \in P(u,\ Y)$ by $p^P(Y,\ u,\ \zeta)$. Then the axiom of (SPI) is represented as follows:

Axiom 8': Support Price Independence (SPI):
$\forall \varepsilon = (u,\ Y) \in$ E, $\zeta = ((z_i)_{i\in I},\ \tilde{z}) \in S(\varepsilon)$, $\exists p \in p(Y,\ u,\ \zeta)$, $\forall \tilde{\varepsilon} = (\tilde{u},\ \tilde{Y}) \in$ E,
$[p \in p^P(\tilde{Y},\ \tilde{u},\ \zeta) \Rightarrow \zeta \in S(\tilde{\varepsilon})]$.

In the case of multi-input and multi-output economy, the proportional solution PR does not satisfy the axiom of Pareto Independence. However, PR satisfies SPI. Thus, Theorem 1 on PR in section 4 is changed as follows:

Theorem 3: The proportional solution PR satisfies axioms PO, UBSAI and SPI. Conversely, if ω_C=0, no other solutions satisfy together these three axioms.

Proof of Theorem 3: See Appendix.

Remark: If $\omega_C \geq (\neq) 0$, PR is not a unique solution which satisfies PO, UBSAI and SPI.

Example: Let $I=\{1,\ 2\}$, $Y(f)=\{(x,\ y)\in R_{-}\times R_{+} \mid f(x)\geq y\ \&\ f(0)=0\}$ and $\omega_C\in R_{++}$. Suppose that the production function f is differentiable and strictly concave. Each member i's initial endowment is his labor endowment $\omega_i^{\circ}(\in R_{--})$ only. Such an economy belongs to E whenever each member i's preference belongs to U_i.

Let $\tilde{z}=(x,\ y)\in\partial\ Y(f)$ and $(z_1,\ z_2)=((\frac{x}{2},\ \frac{y}{2}+\omega_C),(\frac{x}{2},\ \frac{y}{2}-\omega_C))$ where $z_i\in Z_i$ $(i=1,\ 2)$. Then $\zeta=((z_1,\ z_2),\ \tilde{z})$ is a feasible allocation. Let $\{p\}=p(Y(f),\ \tilde{z})$. Let u_i $(i=1,\ 2)$ be a utility function satisfying $U(z_i,\ u_i)\cap\hat{H}(p,\ z_i)=\{z_i\}$ where $U(z_i,\ u_i):=\{z\in R_{-}\times R_{+}^{m}\ \mid u_i(z)\geq\ u_i(z_i)\}$. Then, by this construction, ζ is a Pareto efficient allocation at $(u,\ Y(f))$. We show that ζ is an allocation having the property of FAUB at $(u,\ Y(f))$, since this example is an one-inpit and one-output economy. It is sufficient to show that $z_1\in(Y-\partial\ Y)+\{\omega_C\}$. Note that $z_1-\omega_C=(\frac{x}{2},\ \frac{y}{2})$. Since f is strictly concave, $(\frac{x}{2},\ \frac{y}{2})\in(Y-\partial\ Y)$. Next we show that ζ is an allocation having the property of SPI at $(u,\ Y(f))$. Since this example is an one-inpit and one-output economy, it is sufficient to show that for any economy $(u^*,\ Y^*)$ such that p supports ζ as a Pareto efficient allocation, ζ has the property of FAUB. Let

$$g(x^*)=\begin{cases} x^*\cdot\dfrac{y}{x} & if\ x^*\in[x,\ 0] \\ -\dfrac{p_x}{p_y}x^*+p\cdot\tilde{z} & if\ x^*\in(-\infty,\ x] \end{cases}$$ and for each $i=1,\ 2$,

$$\bar{u}_i(\tilde{x}_i,\ \tilde{y}_i)=\begin{cases} p_x\tilde{x}_i+p_y\tilde{y}_i & if\ \tilde{x}_i\in[\frac{x}{2},\ 0] \\ (y+\omega_C)\cdot\tilde{x}_i-x\tilde{y}_i & if\ \tilde{x}_i\in[\omega_i^{\circ},\ \frac{x}{2}] \end{cases}$$. Then, in $(\bar{u},\ Y(g))$, p supports ζ as a Pareto efficient allocation. ζ is also an allocation having the property of FAUB in $(\bar{u},\ Y(g))$, since for $i=1,\ 2,\ z_i\in Y(g)+\{\omega_C\}$. It is easy to show that for any other $Y^*\in$ Y such that $Y^*\subseteq H(p,\ \tilde{z})$ and $\tilde{z}\in Y^*,\ z_i\in Y^*+\{\omega_C\}$. Thus, ζ is an allocation having the property of SPI. Define a POS S satisfying PO, FAUB and SPI, which assigns the allocation ζ at $(u,\ Y(f))$. This S does not PR. Q.E.D.

In contrast, it is easy to see that the characterization of EB in section 4 is extended to the case of multi-input and multi-output economy:

Theorem 4: The equal benefit solution EB satisfies PO, LBE and SPI. Conversely, no other solution satisfies together these three axioms.

Proof of Theorem 4: See Appendix.

By adopting the axiom SPI, we can also fully characterize the Walrasian solution in private ownership production economies. Consider the following private ownership production economies: for each $i \in I$, $\underline{\omega}_i \in R_+^m$ is i's privately owned initial endowment of commodity inputs, and θ_i is i's share of Y. Following the above argument, we assume that $\{\underline{\omega}_i, \theta_i\}_{i \in I}$ is fixed, so that a couple $(u, Y) \in \mathrm{E}$ specifies one private ownership production economy. Then the Walrasian solution is defined as follows.

Definition: An allocation $\zeta = ((z_i)_{i \in I}, \tilde{z})$ is a Walrasian solution (W) for $\varepsilon = (u, Y) \in \mathrm{E}$ if $\zeta \in A(\varepsilon)$ such that there exists a price vector $p \in \Delta$ such that:

(i) $Y \subseteq H(p, \tilde{z})$;

(ii) for every $i \in I$, $z_i = \arg\max\ u_i(z_i^\circ)$ over $z_i^\circ \in Z_i$ and $p \cdot z_i^\circ \leq p \cdot \underline{\omega}_i + \theta_i p \cdot \tilde{z}$;

(iii) $p \cdot \left(\sum_{i \in I} (z_i - \underline{\omega}_i) - \tilde{z} \right) = 0$.

For characterizing the Walrasian solution, we introduce the following axiom:

Axiom 9: Full Individual Rationality (FIR) (Gevers (1986)):
$\forall \varepsilon = (u, Y) \in \mathrm{E},\ \zeta = ((z_i)_{i \in I}, \tilde{z}) \in S(\varepsilon),\ \forall i \in I,$
$u_i(z_i) \geq \max\{u_i(x_i, \underline{\omega}_i + y) \mid (x_i, \underline{\omega}_i + y) \in Z_i,\ (x_i, y) \in \theta_i \cdot Y\}.$

Theorem 5: The Walrasian solution W satisfies PO, FIR and SPI. Conversely, no other solution satisfies together these three axioms.

Proof of Theorem 5: See Appendix.

Footnote

1). In Roemer and Silvestre (1989, 1993), the existence of EB and PR is also proven in convex economic environments with multi-input and multi-output.

2). Gevers (1986) (and Nagahisa (1994)) attempted to characterize Walrasian solution by adopting the axioms of Non-discrimination (ND) and Monotonicity (M) (Generalized Monotonicity (GM)), instead of our SPI axiom. ND is a neutrality axiom on utility levels: if all members' utility levels of a feasible allocation are equal to those of the current S-solution, then this feasible allocation is also S-solution. M (GM) is a slight weaker version of Maskin Monotonicity (Maskin (1977)).

3). Notation: Throughout this paper we shall employ the symbol R to indicate the set of real numbers. The set of non-negative real numbers is denoted by R_+. The set of non-positive real numbers is denoted by R_-. The set of negative real numbers is denoted by R_{--}. Given z, $z^* \in R_- \times R_+$, vector inequalities are defined as follows: $z \geq z^*$ if $z_i \geq z_i^*$ for all $i = x, y$; $z > z^*$ if $z_i > z_i^*$ for all $i = x, y$.

APPENDIX:

Proof of Theorem 3: First, we show that PR satisfies UBSAI. Suppose $\zeta=((x_i,\ y_i)_{i\in I},\ \bar{z})\in PR(u,\ Y)$. Let y_{fi} be agent i's supply of privately owned commodity input at $\zeta\in PR(u,\ Y)$. Then for $(x_i,\ y_{fi})$, there exists $(x_i,\ y_{fi},\ y_C^{\circ})\in\partial\ Y$ such that for some $p\in p(Y,(x_i,\ y_{fi},\ y_C^{\circ}))$, $w_i(x_i,\ y_{fi},\ u_i,\ Y)$ is realized. By definition, $u_i(x_i,\ y_i)\le w_i(x_i,\ y_{fi},\ u_i,\ Y)$. Hence, $u_i(x_i,\ y_i)\le wsa(u_i,\ Y)$ for each $i\in I$. It is easy to show that PR satisfies axioms PO and SPI.

Next, prove the inverse relation when $\omega_C=0$. Suppose a solution S satisfying the three axioms is not PR. Take an economy $\varepsilon=(u,\ Y)\in\times_{i\in I}U_i\times\mathrm{Y}$. Suppose $\zeta=((z_i)_{i\in I},\ \bar{z})\in S(\varepsilon)$. Since ζ is Pareto efficient, for some $p^{\circ}\in\Delta$, $Y\subseteq H(p^{\circ},\ \bar{z})$. Hence, $p(Y,\ \bar{z})$ is non-empty. This implies that $p(Y,\ u,\ \zeta)$ is also non-empty. Suppose $p\in p^S(Y,\ u,\ \zeta)$ is a price such that for any $(\tilde{u},\ \tilde{Y})$ satisfying $p\in p^P(\tilde{Y},\ \tilde{u},\ \zeta)$, $\zeta\in S(\tilde{u},\ \tilde{Y})$. Suppose that for $\zeta=((z_i)_{i\in I},\ \bar{z})\in S(\varepsilon)$, there exist at least two agents $j,\ h\in I$ such that $\left|\dfrac{p\cdot(z_j-\omega_j^f)}{p\cdot(x_j+y_{fj})}\right|<\left|\dfrac{\sum_{i\in I}p\cdot(z_i-\omega_i^f)}{\sum_{i\in I}p\cdot(x_i+y_{fi})}\right|<\left|\dfrac{p\cdot(z_h-\omega_h^f)}{p\cdot(x_h+y_{fh})}\right|$.

Consider another economic environment. Note that $\bar{z}=(x,\ y_f,\ y_C)$. Let

$T(p,\ \bar{z},\ y_C):=\{(x^{\circ},\ y_f^{\circ})\in R_-\times R_-^k\mid p\cdot(x^{\circ},\ y_f^{\circ},\ y_C)=p\cdot\bar{z}\}$ and

$H_{y_C}(p,\ \bar{z}):=\{(x^{\circ},\ y_f^{\circ},\ y_C^{\circ})\in R_-\times R_-^k\times R^{m-k}\mid(x^{\circ},\ y_f^{\circ})\in T(p,\ \bar{z},\ y_C),\ p\cdot(x^{\circ},\ y_f^{\circ},\ y_C^{\circ})\le p\cdot\bar{z}\}$

Also, let for a given $\alpha\in[0,\ +\infty)$,

$\alpha\cdot H_{y_C}(p,\ \bar{z}):=\{\alpha\cdot(x^{\circ},\ y_f^{\circ},\ y_C^{\circ})\in R_-\times R_-^k\times R^{m-k}\mid(x^{\circ},\ y_f^{\circ},\ y_C^{\circ})\in H_{y_C}(p,\ \bar{z})\}$. Then,

$\left[\bigcup_{\alpha\in[0,\ +\infty)}\alpha\cdot H_{y_C}(p,\ \bar{z})\right]\cap H(p,\ \bar{z})\in\mathrm{Y}$. Denote

$\left[\bigcup_{\alpha\in[0,\ +\infty)}\alpha\cdot H_{y_C}(p,\ \bar{z})\right]\cap H(p,\ \bar{z})\in\mathrm{Y}$ by $Y(p,\ \bar{z})$. Note that by SPI, ζ is a S-allocation in $(u,\ Y(p,\ \bar{z}))$. For each $i\in I$, $z_i\in[\theta_i\cdot\partial\ Y(p,\ \bar{z})+\{\omega_i^f\}]\cap Z_i$ where $\theta_i=\dfrac{p\cdot(x_i+y_{fi})}{\sum_{l\in I}p\cdot(x_l+y_{fl})}$ is i's share of total profits if ζ is a PR-allocation in $(u,\ Y(p,\ \bar{z}))$.

However, since ζ is not a PR-allocation, by the above supposition, $z_h\notin[\theta_h\cdot Y(p,\ \bar{z})+\{\omega_h^f\}]\cap Z_h$. Then, by the construction of $Y(p,\ \bar{z})$, $z_h\notin[Y(p,\ \bar{z})+\{\omega_h^f\}]\cap Z_h$. Let $U(z_i^*,\ \hat{u}_i):=\{z\in R_-\times R_+^m\mid\hat{u}_i(z)\ge\hat{u}_i(z_i^*)\}$ and

$\hat{H}(p, z_i^*) := \{z \in R_- \times R_+^m \mid p \cdot z \leq p \cdot z_i^*\}$. Consider the following preference profile $\tilde{u}$:

for $h \in I$, $\{z_h\} = U(z_h, \tilde{u}_h) \cap (\hat{H}(p, z_h) \cup [Y(p, \bar{z}) + \{\omega_h^f\}] \cap Z_h)$, and

for any other $i \neq h$, $\{z_i\} = U(z_i, \tilde{u}_i) \cap \hat{H}(p, z_i)$. Clearly, $p \in p^P(Y(p, \bar{z}), \tilde{u}, \zeta)$, so

that $\zeta \in S(\tilde{u}, Y(p, \bar{z}))$ by SPI. However, since $z_h \notin [Y(p, \bar{z}) + \{\omega_h^f\}] \cap Z_h$,

$\{z_h\} = U(z_h, \tilde{u}_h) \cap (\hat{H}(p, z_h) \cup [Y(p, \bar{z}) + \{\omega_h^f\}] \cap Z_h)$ implies that ζ does not

satisfy UBSAI of S in $(\tilde{u}, Y(p, \bar{z}))$. Q.E.D.

Proof of Theorem 4: It is easy to show that EB satisfies axioms PO, LBE and SPI.

Prove the inverse relation. Suppose a solution S satisfying the three axioms is not EB.

Take an economy $\varepsilon = (u, Y) \in \times_{i \in I} U_i \times \mathrm{Y}$. Suppose $\zeta = ((z_i)_{i \in I}, \bar{z}) \in S(\varepsilon)$. Suppose

$p \in p(Y, u, \zeta)$ is a price such that for any $(\tilde{u}, \tilde{Y})$ satisfying $p \in p^P(\tilde{Y}, \tilde{u}, \zeta)$,

$\zeta \in S(\tilde{u}, \tilde{Y})$. Suppose that for $\zeta = ((z_i)_{i \in I}, \bar{z}) \in S(\varepsilon)$, there exist at least two agents

$j, h \in I$ such that $p \cdot (z_j - \omega_j^f) < \dfrac{p \cdot (\sum_{i \in I} z_i - \sum_{i \in I} \omega_i^f)}{n} < p \cdot (z_h - \omega_h^f)$. Consider another

economic environment. Note that $\bar{z} = (x, y_f, y_C)$. Let

$(x_h, y_{fh}) = \arg\max \{p \cdot (x_i + y_{fi})\}_{i \in I}$. Let $V(p, \bar{z}, (x_h, y_{fh})) :=$

$\{\hat{y}_C \in R^{m-k} \mid p \cdot (x_h, y_{fh}, \hat{y}_C) = p \cdot \bar{z}\}$, $TV(p, \bar{z}, (x_h, y_{fh})) :=$

$\{(\hat{x}, \hat{y}_f) \in R_- \times R_-^k \mid \hat{y}_C \in V(p, \bar{z}, (x_h, y_{fh})), p \cdot (\hat{x}, \hat{y}_f, \hat{y}_C) = p \cdot \bar{z}\}$ and

$HTV(p, \bar{z}, (x_h, y_{fh})) :=$

$\{(x^\circ, y_f^\circ, y_C^\circ) \in R_- \times R_-^k \times R^{m-k} \mid (x^\circ, y_f^\circ) \in TV(p, \bar{z}, (x_h, y_{fh})), p \cdot (x^\circ, y_f^\circ, y_C^\circ) \leq p \cdot \bar{z}\}$.

Also, let for a given $\alpha \in [0, +\infty)$,

$\alpha \cdot HTV(p, \bar{z}, (x_h, y_{fh})) := \{\alpha \cdot (x^\circ, y_f^\circ, y_C^\circ) \in R_- \times R_-^k \times R^{m-k} \mid (x^\circ, y_f^\circ, y_C^\circ) \in HTV(p, \bar{z}, (x_h, y_{fh}$

Then, $\left[\bigcup_{\alpha \in [0, +\infty)} \alpha \cdot HTV(p, \bar{z}, (x_h, y_{fh}))\right] \cap H(p, \bar{z}) \in \mathrm{Y}$. Denote

$\left[\bigcup_{\alpha \in [0, +\infty)} \alpha \cdot HTV(p, \bar{z}, (x_h, y_{fh}))\right] \cap H(p, \bar{z}) \in \mathrm{Y}$ by $Y(p, \bar{z}, (x_h, y_{fh}))$. Note that

by SPI, ζ is a S-allocation in $(u, Y(p, \bar{z}, (x_h, y_{fh})))$. For each $i \in I$,

$z_i \in \left[\frac{1}{n} \cdot (\partial\ Y(p, \bar{z}, (x_h, y_{fh})) + \{\omega_C\}) + \{\omega_i^f\}\right] \cap Z_i$ if ζ is a EB-allocation in

$(u, Y(p, \bar{z}, (x_h, y_{fh})))$. However, since ζ is not a EB-allocation, by the above

supposition, $z_j \notin \left[\frac{1}{n} \cdot (\partial\ Y(p, \bar{z}, (x_h, y_{fh})) + \{\omega_C\}) + \{\omega_j^f\}\right] \cap Z_j$. Since

$\dfrac{\sum_{i\in I} z_i - \sum_{i\in I}\omega_i^f}{n} \in \dfrac{1}{n}\cdot(\partial\ Y(p,\ \bar{z},(x_h,\ y_{fh}))+\{\omega_C\})$, by the above supposition,

$z_j \in \left[\dfrac{1}{n}\cdot(\text{int } Y(p,\ \bar{z},(x_h,\ y_{fh}))+\{\omega_C\})+\{\omega_j^f\}\right]\cap Z_j$ where int $Y(p,\ \bar{z},(x_h,\ y_{fh}))$ =

$Y(p,\ \bar{z},(x_h,\ y_{fh}))-\partial\ Y(p,\ \bar{z},(x_h,\ y_{fh}))$. Consider the following preference profile $\tilde{u}$: for every $i\in I$, $\tilde{u}_i(x_i^*,\ y_i^*):=p_x\cdot x_i^*+p_y\cdot y_i^*$. Clearly,

$p\in p^P(Y(p,\ \bar{z},(x_h,\ y_{fh})),\ \tilde{u},\ \zeta)$, so that $\zeta\in S(\tilde{u},\ Y(p,\ \bar{z},(x_h,\ y_{fh})))$ by SPI.

However, $z_j \in \left[\dfrac{1}{n}\cdot(\text{int } Y(p,\ \bar{z},(x_h,\ y_{fh}))+\{\omega_C\})+\{\omega_j^f\}\right]\cap Z_j$ implies

$\tilde{u}_j(z_j)< l\ (\tilde{u}_j,\ Y(p,\ \bar{z},(x_h,\ y_{fh})))$. Thus, ζ does not satisfy LBE of S in

$(\tilde{u},\ Y(p,\ \bar{z},(x_h,\ y_{fh})))$. Q.E.D.

Proof of Theorem 5: It is easy to show that W satisfies PO, FIR and SPI. Prove the inverse relation. Suppose a solution S satisfying the three axioms is not W. Take an economy $\varepsilon=(u,\ Y)\in \times_{i\in I} U_i\times \mathrm{Y}$. Suppose $\zeta=((z_i)_{i\in I},\ \bar{z})\in S(\varepsilon)$. Suppose $p\in p(Y,\ u,\ \zeta)$ is a price such that for any $(\tilde{u},\ \tilde{Y})$ satisfying $p\in p^P(\tilde{Y},\ \tilde{u},\ \zeta)$, $\zeta\in S(\tilde{u},\ \tilde{Y})$. Notice that if $S(\varepsilon)=W(\varepsilon)$, then for every $i\in I$,

$z_i\in\partial\ H(p,\ \underline{\omega}_i+\theta_i\bar{z})\cap Z_i$ because utility functions of all members are strictly increasing. However, since S is not W, there exist $j,\ l\in I$ such that

$z_j\notin H(p,\ \underline{\omega}_j+\theta_j\bar{z})\cap Z_j$ and $z_l\in \text{int}\,H(p,\ \underline{\omega}_l+\theta_l\bar{z})\cap Z_l$. We take another economy $(\tilde{u},\ Y(p,\ \bar{z},(x_h,\ y_{fh})))$ which is defined in proof of Theorem 4. Then,

$z_l\in\left[\theta_l\cdot(\text{int } Y(p,\ \bar{z},(x_h,\ y_{fh})))+\{\underline{\omega}_l\}\right]\cap Z_l$. This implies ζ does not satisfy FIR of S in $(\tilde{u},\ Y(p,\ \bar{z},(x_h,\ y_{fh})))$. Q.E.D.

REFERENCES

Dutta, B., Sen, A. and Vohra, R. (1995), "Nash Implementation Through Elementary Mechanism in Economic Environments", *Economic Design*, **1**, 173-204.

Foley, D. (1967), "Resource Allocation and the Public Sector", *Yale Economic Essays*, **7**, 45-98.

Gevers, L. (1986), "Walrasian social choice: Some simple axiomatic approaches", in Heller *et al.*, eds., *Social Choice and Public Decision Making: Essays in Honor of K. J. Arrow*, **1**, 97-114. Cambridge Univ. Press, London / New York.

Maskin,E. (1977), "Nash Equilibrium and Welfare Optimality", (Mimeo).

Moulin,H. (1987), "A Core Selection for Pricing a Single Output Monopoly", *The Rand Journal of Economics*, **18**, 397-407.

Moulin,H. (1990a), "Joint Ownership of a Convex Technology: Comparison of Three Solutions", *Review of Economic Studies*, **57**, 439-452.

Moulin,H. (1990b), "Fair Division under Joint Ownership: Recent Results and Open Problems", *Social Choice and Welfare*, **7**, 149-170.

Moulin,H. (1992), "Welfare Bounds in the Cooperative Production Problem", *Games and Economic Behavior*, **4**, 373-401.

Moulin,H., and Roemer,J. (1989), "Public Ownership of the External World and Private Ownership of Self", *Journal of Political Economy*, **97**, 347-367.

Nagahisa, R. (1991), "A Local Independence Condition for Characterization of Walrasian Allocations Rule", *Journal of Economic Theory*, **54**, 106-123.

Nagahisa, R. (1994), "A Necessary and Sufficient Condition for Walrasian Social Choice", *Journal of Economic Theory*, **62**, 186-208.

Roemer,J., and Silvestre,J. (1989), "Public Ownership: Three proposals for Resource Allocation", (Mimeo).

Roemer,J., and Silvestre,J. (1993), "The Proportional Solution for Economies with Both Private and Public Ownership", *Journal of Economic Theory*, **59**, 426-444.

Saijo,T., Tatamitani,Y., and Yamato,T. (1993), "Toward Natural Implementation", forthcoming in *International Economic Review*.

Yoshihara, N., (1994), "Natural and Double Implementation of Public Ownership Solutions in Differentiable Production Economies", (Mimeo).

CHAPTER III

Natural and Double Implementation of Public Ownership Solutions in Differentiable Production Economies

Abstract: This paper examines the implementation of various solutions in differentiable concave production economies with one privately owned input, one output, and publicly owned production technology. The public ownership solutions we focus on are the Proportional Solution (PR) and the Equal Benefit Solution (EB). Two "natural" mechanisms which doubly implement PR and EB respectively in Nash and strong Nash equilibria are proposed without assuming free disposal of the production set or the boundary condition on preferences of members.

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1. Introduction

In production economies with commonly owned technology such as "commons", it is well-known that market equilibrium allocations are Pareto inefficient. A method to resolve this problem of Pareto inefficiency is to institute *public ownership of the technology* (Roemer [12]) in those economies. Then, there are two fundamental questions. The first one is what allocation rules are desirable in those economies. It was with regard to this problem that Roemer and Silvestre [13] proposed several solutions as desirable rules. The second question is that when a desirable rule is decided, whether there exists a way of implementing this rule *in a decentralized manner*. This problem is important because the public ownership of the means of production has been viewed by many as inseparable from central planning, and so the failures of central planning in the socialist countries seem to be viewed as proof of the ineffectiveness of the public ownership. In this paper, we shall be concerned with this second problem.

This problem can be studied using the theory of Nash implementation, due to Maskin [6]. Among the public ownership solutions Roemer and Silvestre [13] have proposed, the *Proportional Solution* (PR) and the *Equal Benefit Solution* (EB) are Nash-implementable by the Maskin-type mechanism (Maskin [6])[1)] (see Roemer [12] and Moulin [10]). However, these results are not satisfactory for the second problem to be resolved in practice, because the Maskin-type mechanism seems to be an *unnatural* mechanism for implementing PR or EB *in a decentralized manner* in the following sense.

First, the Maskin-type mechanism requires each member to announce the preferences of all the members. This implies that each member usually has an infinite dimensional strategy space and which includes the space of other member's possible preferences. Thus, in this mechanism, information transmission is extremely hard, and a social planner has the authority to compel each member to announce the traits of others, which is usually objectionable in actual democratic societies. *Natural decentralized mechanisms* should not only have the finite dimensional strategy space but also satisfy the *informational decentralization* (Schmeidler [17]) property that each member announces information only about himself. Second, in the Maskin-type mechanism, the strategy profile composed of truthful announcements does not necessarily constitute an equilibrium[2)], and for every strategy combination of the other members, each member does not necessarily have the best response[3)]. These facts reveal that calculating a member's optimal strategy is quite complex. Such a mechanism is not practical in the context of an actual economy. Third, the Maskin-type mechanism can be used only in the environment where the social planner is convinced that members will never take any cooperative strategies. However, it seems to be usual in actual economic contexts that the planner cannot know whether members will cooperate or not.

Thus, to check whether the public ownership solution is implementable in a decentralized way, we must examine whether there exists a mechanism which at least overcomes the above unnatural features the Maskin-type mechanism has. By constructing such mechanisms, this paper presents a more plausible answer to this problem. In the following, we propose two feasible[4)] (i.e., individually feasible and balanced) mechanisms each of which doubly implements PR or EB in both Nash and strong Nash equilibria[5)]. Our mechanisms are of quantity-announcing type (Q-mechanism)[6)] which was originally developed by Sjöström [18], and Saijo, Tatamitani and Yamato [16].

The paper is organized as follows. Section 2 sets out the basic model. In sections 3 and 4, two Q-mechanisms, one of which doubly implements PR and the other EB, are proposed. Section 5 offers some concluding remarks.

2. The basic model

There are two goods, one privately owned and utilized as an input to produce the other. Let x denote the privately owned input and y output produced. The initial endowment of x is denoted by $\Omega \in R_{--}$[7]. There is one publicly owned firm with production set, Y, which is defined as follows: $Y := \{(x, y) \in R_{-} \times R_{+} \mid f(x) = y\}$. Notice that we do not assume free disposal. The production function $f : R_{-} \rightarrow R_{+}$ is such that $f(x) = y$ and $f(0) = 0$. We assume f is continuous, concave, decreasing over R_{-} and differentiable.

There are n members ($n \geq 3$). The set of members is denoted by $I = \{1, 2, \ldots, n\}$ with generic element i. Each member is endowed with a negative amount ω_i of input x and no output. The aggregation of ω_i is Ω. Each member i has $Z_i = [\omega_i, 0] \times [0, f(\Omega)]$ as his attainable consumption set. Define $\overset{\circ}{Z}_i := [\omega_i, 0] \times (0, f(\Omega))$. Each member i's preference is represented by a utility function $u_i : Z_i \rightarrow R$ where u_i is continuous, quasi-concave and strictly increasing.

An *economy* is specified by a list $\{I, Y, (\omega_i)_{i \in I}, (u_i)_{i \in I}\}$. However, since in the following we assume that I, Y and $(\omega_i)_{i \in I}$, are fixed and known to everyone, we denote one economy by $u := (u_i)_{i \in I} \in U := U_1 \times \cdots \times U_n$ where U_i denotes the class of utility functions for i. A *feasible allocation* is a n×2-tuple $z = (z_i)_{i \in I}$ such that for all $i \in I$, $z_i \in Z_i$, and $\sum_{i \in I} z_i \in Y$. Let A denote the set of all feasible allocations. Let $\overset{\circ}{A} := \{z = (z_i)_{i \in I} \in A \mid \text{for all } i \in I,\ z_i \in \overset{\circ}{Z}_i\}$. A feasible allocation z is *Pareto efficient* at u if there does not exist another feasible allocation z^* such that $u_i(z_i^*) > u_i(z_i)$ for all $i \in I$. We denote by $P(u)$ the set of Pareto efficient allocation at u. As Dutta, Sen and Vohra [3] did, we also assume that the class of admissible economies is restricted to the one satisfying the condition that for any $u \in U$, $\overset{\circ}{A} \cap P(u) \neq \emptyset$.

A *public ownership solution* (POS) is a *correspondence* S associating with every economy $u \in U$ a non-empty subset $S(u)$ of feasible allocations satisfying for any $u \in U$, $S(u) \subseteq \mathring{A} \cap P(u)$.

Let $\Delta := \{p \in R^2_+ | p_x + p_y = 1\}$ be the unit simplex. Let $H(p, w) := \{ w' \in R_- \times R_+ \mid pw' \leq pw\}$ and $\partial H(p,w) := \{w' \in R_- \times R_+ \mid pw' = pw \}$ for $p \in \Delta$ and $w \in R_- \times R_+$. A production point $w \in Y$ is efficient at $p \in \Delta$ if $Y \subseteq H(p,w)$. Agent i's demand correspondence at u, when he faces the budget constraint, $pz_i = a$, for some $p \in \Delta$ and amount $a \in R_+$ of share of surplus, is denoted by $d_i(p, a, u) := \arg\max \{ u_i(z_i) \mid z_i \in Z_i \text{ and } pz_i = a\}$.

Definition (Moulin [10]): For any $u \in U$, $EB(u)$ is the set of *equal benefit solutions* if for any $z \in EB(u)$, there exists $p \in \Delta$ and $a \in R_+$ such that for all $i \in I$, $z_i \in d_i(p, a, u)$ and $\sum_{i \in I} z_i \in Y \subseteq H(p, \sum_{i \in I} z_i)$.

Definition (Roemer and Silvestre [14]): For any $u \in U$, $PR(u)$ is the set of *proportional solutions* if for any $z \in PR(u)$, z is a Pareto efficient allocation and there exists a supporting price $p \in \Delta$ for z such that for any $i \in I$,

$$p z_i = \frac{p_x x_i}{\sum_{i \in I} p_x x_i} \left[\sum_{i \in I} p z_i \right].$$

A *feasible mechanism* (or game form) is a pair $\Gamma = (M, g)$ where $M = M_1 \times \cdots \times M_n$, M_i is the *strategy space of agent i*, and $g : M \to A$ is the *individual feasible and balanced outcome function* assigning to every $m \in M$ a unique element of A. Denote the i-th component of $g(m)$ by $g_i(m)$.

Given $m \in M$ and $m_i' \in M_i$, (m_i', m_{-i}) is obtained by the replacement of m_i by m_i', and $g(M_i, m_{-i})$ is the set of feasible allocations that agent i can induce when the other members select m_{-i}. Let $M_{-i} := \times_{j \neq i} M_j$.

Denote for any coalition $T \subseteq I$, $m_T = (m_i)_{i \in T}$ and $m_{-T} = (m_i)_{i \in I-T}$. Hence (m'_T, m_{-T}) is the list obtained by the replacement of m_T by m'_T, and $g(M_T, m_{-T})$ is the set of feasible allocations that the coalition T can induce when $I - T$ select m_{-T}.

For $i \in I$, $u_i \in U_i$, and $z_i \in Z_i$, let $L(z_i, u_i) := \{z'_i \in Z_i \mid u_i(z_i) \geq u_i(z'_i)\}$ be the *lower contour set for* u_i *at* z_i. Given a feasible mechanism $\Gamma = (M, g)$ and a profile of utility functions $u \in U$, the strategy profile $m \in M$ is a *Nash equilibrium of* Γ *at* u if for all $i \in I$, $g_i(M_i, m_{-i}) \subseteq L(g_i(m), u_i)$. Let $NE(\Gamma, u)$ be the set of Nash equilibria of Γ at u. Let $g(NE(\Gamma, u))$ be the set of Nash equilibrium allocations of Γ at u. Given $\Gamma = (M, g)$ and $u \in U$, the strategy profile $m \in M$ is a *strong Nash equilibrium of* Γ *at* u if for all $T \subseteq I$, for all $m'_T \in M_T$, there exists $i \in T$ such that $g_i(m'_T, m_{-T}) \in L(g_i(m), u_i)$. Let $SNE(\Gamma, u)$ be the set of strong Nash equilibria of Γ at u. Let $g(SNE(\Gamma, u))$ be the set of strong Nash equilibrium allocations of Γ at u.

The feasible mechanism $\Gamma = (M, g)$ *implements the public ownership solution (POS) S in Nash equilibria* if for all $u \in U$, $S(u) = g(NE(\Gamma, u))$. The POS S is Nash-implementable by a feasible mechanism if there exists a feasible mechanism Nash-implementing S. The feasible mechanism $\Gamma = (M, g)$ *implements the POS S in strong Nash equilibria if for all* $u \in U$, $S(u) = g(SNE(\Gamma, u))$. The POS S is strongly Nash-implementable by a feasible mechanism if there exists a feasible mechanism strong Nash-implementing S. The feasible mechanism $\Gamma = (M, g)$ *doubly implements the POS S in Nash and strong Nash equilibria* if for all $u \in U$, $S(u) = g(SNE(\Gamma, u))$ $= g(SNE(\Gamma, u))$. The POS S is doubly implementable by a feasible mechanism if there exists a feasible mechanism doubly implementing S.

3. A feasible quantity mechanism doubly implementing PR

Let $B(\omega_i) := \{ z_i = (x_i, y_i) \in R_- \times R_+ \mid y_i \leq e_i f(x_i / e_i)$ where $e_i = \omega_i / \Omega$ and $x_i \in [\omega_i, 0]\}$, and $\partial B(\omega_i) := \{ z_i = (x_i, y_i) \in R_- \times R_+ \mid y_i = e_i f(x_i / e_i)$ where $e_i = \omega_i / \Omega$ and $x_i \in [\omega_i, 0]\}$. Let $Y(f, \Omega) := \{(x, y) \in R_- \times R_+ \mid f(x) \geq y$ where $x \in [\Omega, 0]\}$ and $\partial Y(f, \Omega) := \{(x, y) \in R_- \times R_+ \mid f(x) = y$ where $x \in [\Omega, 0]\}$. Let $\{p(w, Y)\} := \{p \in \Delta \mid$ for $w \in \partial Y(f, \Omega),\ Y \subseteq H(p, w)\}$.

Let for $x^*, x^\circ \in R_-$ such that $x^* \geq x^\circ$, $Y(f, x^*, x^\circ) :=$ $\{(x, y) \in [x^\circ, x^*] \times R_+ | \ f(x) \geq y\}$. For each $i \in I$, $z_i \in Z_i$ and $w \in \partial Y(f, \Omega)$, let $Z_i(z_i, f, w)$ be the subset of Z_i defined as follows: 1) if there exist $x^+, x^- \in [\omega_i, 0]$ such that $Y(f, 0, x^+) \subseteq H(p(w, Y), z_i) \cap Z_i$,
$Y(f, x^+, x^-) \supseteq H(p(w, Y), z_i) \cap \{[x^-, x^+] \times R_+\}$ and
$Y(f, x^-, \omega_i) \subseteq H(p(w, Y), z_i) \cap Z_i$, then
$Z_i(z_i, f, w) := \{[x^-, 0] \times [0, f(\Omega)]\} \cup Y(f, x^-, \omega_i)$,
2) otherwise, $Z_i(z_i, f, w) := Z_i$.

We consider a PR-implementing mechanism $\Gamma^{PR} = (M, g^{PR})$ such that for all $i \in I$, $M_i = Z_i$ and $g^{PR} : M \to A$ defined as follows:

Rule 1: If for all $i \in I$, $z_i = (x_i, y_i) \in \overset{\circ}{Z}_i$ such that $y_i / x_i = \sum_{i \in I} y_i / \sum_{i \in I} x_i$ and $\sum_{i \in I} z_i \in \partial Y(f, \Omega)$, then $g^{PR}(m) = z$ where $m = (z_i)_{i \in I}$.

Rule 2: If for all $i \in I$, $z_i = (x_i, y_i) \in \overset{\circ}{Z}_i$ such that $y_i / x_i = \sum_{i \in I} y_i / \sum_{i \in I} x_i$ but $\sum_{i \in I} z_i \notin \partial Y(f, \Omega)$, and moreover for each $i \in I$, there exists $z_i' = (x_i', y_i') \in \overset{\circ}{Z}_i$ such that $y_i' / x_i' = y_i / x_i$ and $z_i' + \sum_{j \neq i} z_j \in \partial Y(f, \Omega)$, then $g^{PR}(m) = \bar{z}$[8] where $m = (z_i)_{i \in I}$, such that

2-1) if for each $i \in I$, $\sum_{i \in I} z_i > z_i' + \sum_{j \neq i} z_j$, then for each $i \in I$,

$$\bar{z}_i \in H(p(z_i' + \sum_{j \neq i} z_j, Y), z_i') \cap \partial B(\omega_i) \cap \overset{\circ}{Z}_i,$$

and for any $j,\ k \in I$, $\bar{y}_j/\bar{x}_j = \bar{y}_k/\bar{x}_k$, where $m = (z_i)_{i\in I}$,

2-2) if for each $i \in I$, $\sum_{i\in I} z_i < z_i' + \sum_{j\neq i} z_j$, then $\bar{z} = (z_1', z_{-1})$.

Rule 3: If for all $i \in I$, $z_i = (x_i, y_i) \in \overset{\circ}{Z}_i$ such that $y_i/x_i = \sum_{i\in I} y_i / \sum_{i\in I} x_i$ but $\sum_{i\in I} z_i \notin \partial Y(f, \Omega)$, and $1 \le \#\{ j \in I \mid \text{for all } z_j' \in \overset{\circ}{Z}_j \text{ such that } y_j'/x_j' = y_j/x_j,\ z_j' + \sum_{i\neq j} z_i \notin \partial Y(f, \Omega)\} \le n - 1$, then for $j = \min\{ j \in I \mid \text{for all } z_j' \in \overset{\circ}{Z}_j \text{ such that } y_j'/x_j' = y_j/x_j,\ z_j' + \sum_{i\neq j} z_i \notin \partial Y(f, \Omega)\}$, $g_j^{PR}(m) = (x_j, f(x_j + \sum_{i\neq j}\omega_i))$, and for all $i \neq j$, $g_i^{PR}(m) = (\omega_i, 0)$.

Rule 4: if there exists some member $i \in I$ such that for all $j,\ k \neq i$, $z_j \in \overset{\circ}{Z}_j$, $z_k \in \overset{\circ}{Z}_k$ and $y_j/x_j = y_k/x_k$, and for some $z_i' \in \overset{\circ}{Z}_i$, $y_i'/x_i' = y_j/x_j = y_k/x_k$ and $z_i' + \sum_{j\neq i} z_j \in \partial Y(f, \Omega)$, then

4-1) if for all $z_{-i}' \in \overset{\circ}{Z}_{-i}$, $PR^{-1}((z_i, z_{-i}')) = \varnothing$, then

4-1-1) if $z_i \in H(p(z_i' + \sum_{j\neq i} z_j, Y),\ z_i') \cap Z_i$,

$g_i^{PR}(m) = z_i^* = (x_i, y_i + \gamma) \in \partial\, (H(p(z_i' + \sum_{j\neq i} z_j, Y),\ z_i') \cap Z_i(z_i', f, z_i' + \sum_{j\neq i} z_j)$ (for some $\gamma \ge 0$), and for all $j \neq i$, $g_j^{PR}(m) = (\omega_j,\ x_j \cdot \dfrac{f(x_i + \sum_{j\neq i}\omega_j) - (\gamma + y_i)}{\sum_{j\neq i} x_j})$,

4-1-2) otherwise, $g^{PR}(m) = (z_i', z_{-i})$,

4-2) if there exists $z_{-i}' \in \overset{\circ}{Z}_{-i}$, $PR^{-1}((z_i, z_{-i}')) \neq \varnothing$, then

$g_j^{PR}(m) = (x_j,\ f(x_j + \sum_{k\neq j}\omega_k))$, where $j = \min\{k \in I - \{i\}\}$, and $g_k^{PR}(m) = (\omega_k, 0)$ for all $k \neq j$.

Rule 5: For any other case, the following modulo game is played and some member i^* will win the game: Let $\sum_{i\in I}(x_i^i/\omega_i) = k$. Since $(x_i^i/\omega_i) \in [0,1]$, clearly $0 \le k \le n$. Let r

$+ t = k$ where r is the largest integer less than or equal to k. Then $t \in [0, 1)$ and there is a unique $i^* \in I$ such that $t \in [(i^*-1)/n,\ i^*/n)$. Then i^* is able to receive $g_i^{PR}(m) = (f^{-1}(y^*) - \sum_{j \neq i} \omega_j,\ y^*)$ where $y^* = \max\{ f(\sum_{j \neq i} \omega_j),\ y_{i^*} \}$ and for all $j \neq i^*$, $g_j^{PR}(m) = (\omega_j, 0)$[9].

In the above mechanism, Rule 2 implies that if a strategy combination is such that all members are potential deviators, then the outcome is the feasible allocation that all members are punished. Rule 3 implies that if a strategy combination is such that at most $n - 1$ members are potential deviators, then the outcome is the feasible allocation that all members except only one of non-deviators are punished. Rule 4 corresponds to cases that either there may be a unique deviator or a coalition composed of $n - 1$ deviators.

Next, we show that the above feasible mechanism is well-defined. It is sufficient to check that the outcome attained in Rule 2 is well-defined.

Let $C(x/\Omega) := \{(z_i)_{i \in I} \in Z \mid$ for each $i \in I,\ z_i \in \partial B(\omega_i)$ such that $x_i = (x/\Omega)\omega_i \}$, and $\bigcup_{x \in [\Omega, 0]} C(x/\Omega) := C$.

Lemma 1: $C \subset A$.

Proof of Lemma 1: We show that for each $x \in [\Omega, 0]$, $C(x/\Omega) \subset A$. Suppose $x^* \in [\Omega, 0]$. Then, by definition of $C(x/\Omega)$, for each $i \in I$, $x_i^* = (x^*/\Omega)\omega_i$ and $y_i^* = e_i f(x_i^* / e_i)$. Then, it is easy to check that $(x_i^*,\ y_i^*)_{i \in I} \in A$. Q.E.D.

Lemma 2: The outcome attained in Rule 2 of $\Gamma^{PR} = (M,\ g^{PR})$ is well-defined.

Proof of Lemma 2: Suppose for all $i \in I$, $m_i = z_i = (x_i, y_i) \in \overset{\circ}{Z_i}$ such that $y_i / x_i = \sum_{i \in I} y_i / \sum_{i \in I} x_i$ but $\sum_{i \in I} z_i \notin \partial Y(f, \Omega)$, and, moreover, for each $i \in I$, there exists $z_i' = (x_i', y_i') \in \overset{\circ}{Z_i}$ such that $y_i'/x_i' = y_i / x_i$ and $z_i' + \sum_{j \neq i} z_j \in \partial Y(f, \Omega)$. Notice that such a case is not generated if f has constant returns. We show that in Rule 2-1), there exists a feasible proportional allocation $\bar{z} \in A$ such that for each $i \in I$,

$$\bar{z}_i \in H(p(z_i' + \sum_{j \neq i} z_j, Y), z_i') \cap \partial B(\omega_i) \cap \overset{\circ}{Z}_i.$$

By decreasing returns of f, for all $i \in I$, $z_i' + \sum_{j \neq i} z_j$ is uniquely determined and has the same value, $z^* = (x^*, f(x^*))$. By differentiability of f, $p(z^*, Y)$ is uniquely determined: $p(z^*, Y) = p^*$. Since, by definition, $\partial B(\omega_i) = e_i \, \partial Y(f, \Omega)$ for each $i \in I$, there exists $z_i^* \in \partial B(\omega_i)$ such that $p^* z_i^* \geq p^* z_i$ for all $z_i \in \partial B(\omega_i)$. Notice that $\sum_{i \in I} z_i^* = z^*$, so $(z_i^*)_{i \in I} \in C(x^*/\Omega) \subset A$. Consider the following two cases:

Case 1: $f(\sum_{i \in I} x_i) < \sum_{i \in I} y_i$. In this case, $\sum_{i \in I} y_i^* < \sum_{i \in I} y_i$. Since for each $i \in I$, $\sum_{i \in I} z_i^* = z_i' + \sum_{j \neq i} z_j$, it follows that for at least one member k, $y_k' - y_k^* < 0$. Define $\{k \in I \mid y_k' - y_k^* < 0\}$. Among all $j \in \{k \in I \mid y_k' - y_k^* < 0\}$, we select one member l such that $\min \frac{y_j'}{y_j^*} = \frac{y_l'}{y_l^*}$. Since $B(\omega_l)$ is convex, $z_l' \in B(\omega_l)$. This implies that there exists $\bar{z}_l \in \partial B(\omega_l) \cap \partial H(p(z^*, Y), z_l') \cap \overset{\circ}{Z}_l$ such that $\bar{x}_l \geq x_l'$. Notice that $\partial B(\omega_l) \cap \partial H(p(z^*, Y), z_l') \; (\neq \emptyset)$ has non-zero values.10) Define a function $h^l(x) = y$ such that $h^l(x)/x = \bar{y}_l / \bar{x}_l$ for any $x \in R_-$. Since for each $i \in I$, $\partial B(\omega_i) = \frac{e_i}{e_l} \partial B(\omega_l)$, there exists $\bar{z}_i = (\bar{x}_i, \bar{y}_i) \in \partial B(\omega_i) \cap \overset{\circ}{Z}_i$ such that $h^l(\bar{x}_i) = \bar{y}_i$. We show that for all $i \in I$, $\bar{z}_i \in H(p(z^*, Y), z_i')$. Notice that by definition of $\partial B(\omega_i)$ and $\partial B(\omega_l)$, $\bar{y}_i / \bar{y}_l = y_i^* / y_l^* = \omega_i / \omega_l$. Since $p^* \bar{z}_i = p^* z_l^* \frac{y_l'}{y_l^*} \frac{\bar{y}_i}{\bar{y}_l}$, $p^* z_i' = p^* z_l^* \frac{y_i^*}{y_l^*} \frac{y_i'}{y_i^*}$ and $\frac{y_i'}{y_i^*} \geq \frac{y_l'}{y_l^*}$ for all $i \in I$, it follows that $p^* \bar{z}_i \leq p^* z_i'$ for all $i \in I$. This implies $\bar{z}_i \in H(p(z^*, Y), z_i')$. By Lemma 1, $(\bar{z}_i)_{i \in I} \in \overset{\circ}{A}$.

Case 2: $f(\sum_{i \in I} x_i) > \sum_{i \in I} y_i$. This case corresponds to Rule 2-2). Clearly Rule 2-2) is well-defined. Q.E.D.

Theorem 1: The proportional solution $PR(u)$ is doubly implementable by the feasible quantity mechanism $\Gamma^{PR} = (M, g^{PR})$.

Proof of Theorem 1: See Appendix.

4. A feasible quantity mechanism doubly implementing EB

Let for any $(z_i)_{i\in I} \in \times_{i\in I} Z_i$, $p^i(EB, (z_i)_{i\in I}) := \{ p(z^i, Y) \in \Delta \mid$ for some $z^i \in \partial Y(f, \Omega)$, $EB^{-1}((z^i - \sum_{j\neq i} z_j,\ z_{-i})) \neq \varnothing\}$ and $I^{EB}((z_i)_{i\in I}) := \{i \in I \mid p^i(EB, (z_i)_{i\in I}) \neq \varnothing\}$. We can interpret $I^{EB}((z_i)_{i\in I})$ as the set of potential deviators when a strategy combination $m = (z_i)_{i\in I}$ is not consistent with any EB-optimal allocation.

We consider a EB-implementing mechanism $\Gamma^{EB} = (M,\ g^{EB})$ such that for all $i \in I$, $M_i = Z_i$ and $g^{EB}: M \to A$ defined as follows:

Rule 1: If for all $i \in I$, $z_i = (x_i, y_i) \in \mathring{Z}_i$ such that $\sum_{i\in I} z_i \in \partial Y(f, \Omega)$ and for any j, $k \in I$, $p(\sum_{i\in I} z_i, Y) z_j = p(\sum_{i\in I} z_i, Y) z_k$, then $g^{EB}(m) = z$ where $m = (z_i)_{i\in I}$.

Rule 2: If for all $i \in I$, $z_i = (x_i, y_i) \in \mathring{Z}_i$ such that $I^{EB}((z_i)_{i\in I}) = I$, but $\sum_{i\in I} z_i \notin \partial Y(f, \Omega)$, then for some $\omega^* = \max_{i\in I}\{\omega_i\}$ and $\varepsilon > 0$, $g_i^{EB}(m) = (\omega^* + \varepsilon,\ \dfrac{f(n(\omega^* + \varepsilon))}{n})$ for all $i \in I$.

Rule 3: If for all $i \in I$, $z_i = (x_i, y_i) \in \mathring{Z}_i$ but $\sum_{i\in I} z_i \notin \partial Y(f, \Omega)$, and $2 \leq \# I^{EB}((z_i)_{i\in I}) \leq n-1$, then $g_j^{EB}(m) = (x_j, f(x_j + \sum_{i\neq j} \omega_i))$ for $j = \min\{j \in I - I^{EB}((z_i)_{i\in I})\}$, and $g_i^{EB}(m) = (\omega_i, 0)$ for all $i \neq j$.

Rule 4: If $I^{EB}((z_i)_{i\in I}) = \{i\}$, then

4-1) if for all $z'_{-i} \in \mathring{Z}_{-i}$, $EB^{-1}((z_i, z'_{-i})) = \varnothing$, then

4-1-1) if $z_i \in \bigcap_{p(z^i, Y) \in p^i(EB(z_i)_{i\in I})} H(p(z^i, Y),\ z^i - \sum_{j\neq i} z_j) \cap Z_i$,

$g_i^{EB}(m) = z_i^* = (x_i, y_i + \gamma) \in \bigcap_{p(z^i, Y) \in p^i(EB(z_i)_{i\in I})} \partial\,(H(p(z^i, Y),\ z^i - \sum_{j\neq i} z_j) \cap Z_i(z^i - \sum_{j\neq i} z_j,\ f,\ z^i))$ (for some $\gamma \geq 0$), and for all $j \neq i$,

$$g_j^{EB}(m) = (\max(\omega_{-i}, \frac{\Omega}{n}),\ \frac{f(x_i + (n-1)\max(\omega_{-i}, \frac{\Omega}{n})) - (y_i + \gamma)}{n-1})$$

4-1-2) otherwise, $g_{EB}(m) = (z_*^i - \sum_{j\neq i} z_j ,\ z_{-i})$ where z_*^i= arg min { y^i },

4-2) if there exists $z'_{-i} \in \overset{\circ}{Z}_{-i}$, $EB^{-1}((z_i, z'_{-i})) \neq \varnothing$, then

$g_j^{EB}(m) = (x_j,\ f(x_j + \sum_{k\neq j}\omega_k))$, where j = min $\{k \in I - \{i\}\}$, and $g_k^{EB}(m) = (\omega_k, 0)$

for all $k \neq j$.

Rule 5: For any other case, the following modulo game is played and some member i^* will win the game: Let $\sum_{i\in I}(x_i^i/\omega_i) = k$. Since $(x_i^i/\omega_i) \in [0,1]$, clearly $0 \le k \le n$. Let $r + t = k$ where r is the largest integer less than or equal to k. Then $t \in [0, 1)$ and there is a unique $i^* \in I$ such that $t \in [(i^*-1)/n,\ i^*/n)$. Then i^* is able to receive $g_i^{EB}(m) = (f^{-1}(y^*) - \sum_{j\neq i}\omega_j,\ y^*)$ where y^* = max { $f(\sum_{j\neq i}\omega_j)$, y_{i^*} } and for all $j \neq i^*$, $g_j^{EB}(m) = (\omega_j, 0)$[9].

In the above mechanism, each Rule has the same implication as in PR-mechanism.

Let $\partial B(\Omega/n) := \{z \in R_- \times R_+ \mid y \le \frac{1}{n}f(nx)$ where $x \in [\Omega/n, 0]\}$.

Lemma 3: For any $u \in U$, for any $z \in EB(u)$, $(\sum_{i\in I} z_i/n) \in \partial B(\Omega/n)$ and $H(p(\sum_{i\in I} z_i, Y), \sum_{i\in I} z_i/n) \supseteq \partial B(\Omega/n)$. Moreover, for all $i \in I$,

$$z_i \in \partial H(p(\sum_{i\in I} z_i, Y), \sum_{i\in I} z_i/n) \cap \overset{\circ}{Z}_i.$$

Proof: It is easy to prove this lemma by the facts that $\frac{1}{n}\partial Y(f, \Omega) = \partial B(\Omega/n)$ and $\sum_{i\in I} z_i \in \partial Y(f, \Omega)$ for any $z \in EB(u)$. Q.E.D.

Theorem 2: The equal benefit solution $EB(u)$ is doubly implementable by the feasible quantity mechanism $\Gamma^{EB} = (M, g^{EB})$.

Proof of Theorem 2: See Appendix.

Saijo, Tatamitani and Yamato [16] showed, in pure exchange economies, that Walrasian solutions from equal division (WED) are not Nash-implementable by any feasible quantity mechanism even if there are only two commodities. Such a result does not apply to differentiable production economies, according to Theorem 2.

As long as the smoothness of the production possibility set is assumed, our result on EB can be generalized to economies, such as those discussed in Roemer and Silvestre [14], with multiple commodities and publicly owned endowments. Saijo, Tatamitani and Yamato [16] showed that Pareto efficient allocations are not Nash-implementable by feasible quantity mechanisms in exchange economies with at least three commodities. Such a result cannot be applied to the case of differentiable production economies because the marginal rate of transformation on the efficient production point gives us information on the price supporting some Pareto efficient allocation.

5. Concluding Remarks.

We have examined the implementability of PR and EB in decentralized procedures by constructing natural mechanisms implementing PR or EB. We constructed two feasible quantity mechanisms, each of which doubly implements PR or EB in Nash and strong Nash equilibria. Our mechanisms are more natural than the previous ones in the following ways. First, in our mechanisms, every member announces only his own demand. Hence, information transmission is as easy in our mechanisms as in usual market-like procedures. Moreover, our mechanisms satisfy informational decentralization. Second, in our mechanisms, calculating a member's optimal strategy is simpler than in the previous ones because our mechanisms satisfy the forthrightness and best response properties. Third, since our mechanisms doubly implement PR or EB, they are useful in cases in which the social planner cannot know whether members will cooperate or not.

One of our main results, that PR and EB are implementable by quantity-type mechanisms, depends on the differentiability of production function. In the case of non-differentiable production economies, even if there are only two goods, neither PR nor EB can be doubly implemented by any feasible quantity mechanism. In these cases, it can be shown that PR and EB are implementable by feasible price-quantity mechanisms as in Dutta, Sen and Vohra [3] or Saijo, Tatamitani and Yamato [16].

One undesirable feature of our "natural" mechanisms is that it lacks "continuity" of mechanisms. Tian [20, 21] and Hong [4] constructed continuous mechanisms which Nash-implement Walrasian (or Lindahl) solutions, although their strategy spaces are larger than ours. It is an open question, in our setting, whether or not there exist feasible and continuous quantity mechanisms doubly implementing PR or EB.

Footnotes

1). For complete proofs of Nash implementation, for example, see Repullo [11], McKelvery [8], Saijo [15], Moore and Repullo [9] or Dutta and Sen [1].

2). The property that the strategy profile composed of truthful announcements constitutes an equilibrium is called *forthrightness* by Saijo, Tatamitani and Yamato [16].

3). The property that every member always has a best response is called *the best response property*, proposed by Jackson, Palfrey and Srivastava [5].

4). A mechanism having either an individually infeasible or totally infeasible outcome function is unnatural. The balanced outcome function is desirable because a mechanism which has it is applicable in production economies with non-free disposal.

5). PR and EB in this paper are strongly Nash-implementable. See Dutta and Sen [2]. For arguments on double implementation, see Maskin [7] and Schmeidler [17].

6). Recently, Suh [19] constructed a mechanism doubly implementing PR in both Nash and strong Nash equilibria. This mechanism makes each member announce not only his demand but also the total output, which implies that this mechanism does not satisfy the informational decentralization property. Moreover, the Suh-mechanism does not overcome the complexity of the mechanisms we mentioned above. Also, Hong [4] constructed a mechanism which Nash-implements Walrasian solutions in private ownership production economies. It is easy to show that, in our setting, EB is Nash-implementable by the Hong [4] -type mechanism. However, the strategy spaces of the Hong-type mechanism are rather larger than ours, and these also include the space of total output.

7). Notation: Throughout this paper we shall employ the symbol R to indicate the set of real numbers. The set of non-negative real numbers is denoted by R_+. The set of non-positive real numbers is denoted by R_-. The set of negative real numbers is denoted by R_{--}. Given $z,\ z^* \in R_- \times R_+$, vector inequalities are defined as follows:

$z \geq z^*$ if $z_i \geq z_i^*$ for all $i = x, y$; $z > z^*$ if $z_i > z_i^*$ for all $i = x, y$.

8). In Rule2, we do not attain the zero allocation, since if we allow the zero allocation in Rule 2, there exist some cases that the zero allocation is a Nash equilibrium outcome, which means this mechanism does not implement PR (or EB). For example, consider an economic environment as follows:

$$\text{for all } i \in I,\ u_i = y + \frac{1}{2}x,\ \omega_i = -\frac{2}{n}.$$

$$f(x) = \begin{cases} -\frac{1}{2}x & \textit{where } x \in [-1,0] \\ -\sqrt{|x|} - \frac{1}{2} & \textit{where } x \leq -1 \end{cases}$$

Then, $(-\frac{1}{n}, \frac{1}{2n})_{i \in I} \in PR(u)$ (or $EB(u)$). Now, if every member announces $(-\frac{3}{2n}, \frac{3}{4n})$ and then Rule 2 assigns $(0,0)_{i \in I}$, such a strategy combination constitutes a Nash equilibrium.

9). The modulo game in Rule 5 is due to Dutta, Sen and Vohra [3].

10). If f is strictly concave, clearly, $\bar{z}_i$ is non-zero. If f has constant returns at least over $[x_i' + \sum_{j \neq i} x_j, 0]$, then $\bar{z}_i = z_i'$ satisfies the condition.

Appendix

Proof of theorem 1.

(1) For any $u \in U$, $PR(u) \subseteq g(SNE(\Gamma, u))$.

Pick any $z = z_i = (x_i, y_i) \in PR(u)$ for any given $u \in U$.

Case 1: No individual can be made better off by deviation.

Consider a deviation of member i from $m_i = z_i$ to $m_i^* = z_i^*$ in the following. Then by Rule 2, 3 or 4, $g_i^{PR}(m_i^*, m_{-i}) \in L(z_i, u_i)$.

Case 2: No coalition can be made better off by deviation.

Consider a coalition $T \subseteq I$, $\#T \geq 2$, deviating from $m_T = (z_i)_{i \in T}$ to $m_T^* = (z_i^*)_{i \in T}$.

Case 2-1: $\sum_{i \in T} z_i^* = \sum_{i \in T} z_i$ and for all $i \in T$, $y_i^*/x_i^* = y_i/x_i$.

2-1-1) If for all $i \in T$, $z_i^* \in \overset{\circ}{Z}_i$, Rule 1 is applied and then there exists $l \in T$ such that

$u_l(z_l^*) \leq u_l(z_l)$.

2-1-2) If for some $i \in T$, $z_i^* \in Z_i - \overset{\circ}{Z}_i$, Rule 4 or 5 is applied because this case means for all $\tilde{z}_l^* \in \overset{\circ}{Z}_l$, $\tilde{z}_l^* + \sum_{h \in T - \{l\}} z_h^* + \sum_{k \in I - T} z_k \notin \partial Y(f, \Omega)$. When for all $i \in T$, $z_i^* \in Z_i - \overset{\circ}{Z}_i$, Rule 5 is applied. When there exists only one $i \in T$ such that, $z_i^* \in Z_i - \overset{\circ}{Z}_i$, then if f is constant returns over $[\sum_{i \in I} x_i + \varepsilon, 0]$ $(\varepsilon < 0)$, Rule 4-1-1 is applied. Any other case corresponds to Rule 5. Thus, there exists at least one member $l \in T$, $g_l^{PR}(m_T^*, m_{-T}) \in L(z_l, u_l)$.

Case 2-2: $\sum_{i \in T} z_i^* = \sum_{i \in T} z_i$ and there exists at least two $i, j \in T$, such that $y_i^*/x_i^* \neq y_i/x_i$ and $y_j^*/x_j^* \neq y_j/x_j$. Then notice that $y_i^*/x_i^* \neq y_j^*/x_j^*$. Thus, Rule 5 is applied and there exists at least one member $l \in T$ such that $g_l^{PR}(m_T^*, m_{-T}) = (\omega_l, 0)$.

Case 2-3 : $\sum_{i\in T} z_i^* \neq \sum_{i\in T} z_i$ and for all $i \in T$, $z_i^* \in \overset{\circ}{Z}_i$ such that $y_i^*/x_i^* = y_i/x_i$.

2-3-1) If for all $i \in T$, there exists $\tilde{z}_i^* \in \overset{\circ}{Z}_i$ such that $\tilde{y}_i^*/\tilde{x}_i^* = y_i/x_i$ and

$\tilde{z}_i^* + \sum_{h\in T-\{i\}} z_h^* + \sum_{k\in I-T} z_k \in \partial Y(f, \Omega)$, and for all $j \in I-T$, there exists $z'_j \in \overset{\circ}{Z}_j$ such that

$y'_j/x'_j = y_j/x_j$ and $z'_j + \sum_{h\in T} z_h^* + \sum_{k\in I-T-\{j\}} z_k \in \partial Y(f, \Omega)$. Then Rule 2 is applied and for all $i \in T$, $g_i^{PR}(m_T^*, m_{-T}) \in H(p(\tilde{z}_i^* + \sum_{h\in T-\{i\}} z_h^* + \sum_{k\in I-T} z_k, Y), \tilde{z}_i^*)$. Notice that

$p(\tilde{z}_i^* + \sum_{h\in T-\{i\}} z_h^* + \sum_{k\in I-T} z_k, Y) = p(\sum_{i\in I} z_i, Y)$. Since $z \in PR(u)$ is Pareto efficient and supported by $p(\sum_{i\in I} z_i, Y)$, if there exists $i \in T$, $u_i(z_i^*) > u_i(z_i)$, then there exists $l \in T$, $u_l(z_l^*) \leq u_l(z_l)$. When for all $i \in T$, $u_i(z_i^*) \leq u_i(z_i)$, there exists at least one $h \in T$, $|z_h^*| < |z_h|$. Then, by Rule 2-2), there exists at least one member $l \in T$, $g_l^{PR}(m_T^*, m_{-T}) = z_l^*$. This means $g_l^{PR}(m_T^*, m_{-T}) \in L(z_l, u_l)$.

2-3-2) If for some $i \in T$, for all $\tilde{z}_i^* \in \overset{\circ}{Z}_i$ such that $\tilde{y}_i^*/\tilde{x}_i^* = y_i/x_i$,

$\tilde{z}_i^* + \sum_{h\in T-\{i\}} z_h^* + \sum_{k\in I-T} z_k \notin \partial Y(f, \Omega)$, or, for some $j \in I-T$, for all $z'_j \in \overset{\circ}{Z}_j$ such that

$y'_j/x'_j = y_j/x_j$, $z'_j + \sum_{h\in T} z_h^* + \sum_{k\in I-T-\{j\}} z_k \notin \partial Y(f, \Omega)$, then Rule 3 is applied if the

above statement is not true for every member in I, and otherwise, Rule 5 is applied.

Case 2-4: $\sum_{i\in T} z_i^* \neq \sum_{i\in T} z_i$ and $T = I - \{j\}$ such that for all $i, h \in T$, $y_i^*/x_i^* = y_h^*/x_h^* \neq y_j/x_j$. Then Rule 4-2) or 5 is applied.

Case 2-5: $\sum_{i\in T} z_i^* \neq \sum_{i\in T} z_i$ and $T=I-\{j\}$ such that for all $i, h \in T$, $y_i^*/x_i^* \neq y_i/x_i$, $y_h^*/x_h^* \neq y_h/x_h$ and $y_i^*/x_i^* \neq y_h^*/x_h^*$. Then Rule 5 is applied.

Case 2-6: $\sum_{i\in T} z_i^* \neq \sum_{i\in T} z_i$ and $n \geq 4$, $2 \leq \#T \leq n-2$, and there exists at least two members $i, h \in T$, $y_i^*/x_i^* \neq y_i/x_i$ and $y_h^*/x_h^* \neq y_h/x_h$. Then Rule 5 is applied.

Case 2-7: $\sum_{i\in T} z_i^* \neq \sum_{i\in T} z_i$ and there exists only $i \in T$ such that for all $h\ (\neq\ i) \in T$,

$y_h^*/x_h^* = y_h/x_h = y_i/x_i \neq y_i^*/x_i^*$, and for any $z'_{-i} \in \overset{\circ}{Z}_{-i}$, $PR^{-1}((z_i^*, z'_{-i})) = \varnothing$.

2-7-1) Then if there exists $\tilde{z}_i^* \in \overset{\circ}{Z}_i$ such that $\tilde{y}_i^*/\tilde{x}_i^* = y_i/x_i$,

$\tilde{z}_i^* + \sum_{h\in T-\{i\}} z_h^* + \sum_{k\in I-T} z_k \in \partial Y(f, \Omega)$, Rule 4-1 is applied and

$g_i^{PR}(m_T^*, m_{-T}) \in \partial\ (H(p(\tilde{z}_i^* + \sum_{h\in T-\{i\}} z_h^* + \sum_{k\in I-T} z_k, Y), \tilde{z}_i^*) \cap Y(f, \Omega)) \cap Z_i$. Notice that $p(\tilde{z}_i^* + \sum_{h\in T-\{i\}} z_h^* + \sum_{k\in I-T} z_k, Y) = p(\sum_{i\in I} z_i, Y)$. If Rule 4-1-2 is applied, then for all $h\ (\neq i) \in T$, $g_h^{PR}(m_T^*, m_{-T}) \in H(p(\sum_{i\in I} z_i, Y), z_h^*) \cap Z_h$. Since $z \in PR(u)$ is Pareto efficient and $(\tilde{z}_i^*, z_{T-\{i\}}^*, z_{-T})$ is a proportional allocation, if for all $h\ (\neq i) \in T$, $u_h(z_h^*) > u_h(z_h)$, then $u_i(\tilde{z}_i^*) \leq u_i(z_i)$. Otherwise, for some $h\ (\neq\ i) \in T$, $u_h(z_h^*) \leq u_h(z_h)$. If Rule 4-1-1 is applied and $g_i^{PR}(m_T^*, m_{-T}) \notin L(z_i, u_i)$, then $|\tilde{z}_i^*| > |z_i|$ and $|z_h^*| < |z_h|$ for some $h \in T$. Define $T^\circ := \{h \in T - \{i\} \mid |z_h^*| < |z_h|\}$. Let denote $g_i^{PR}(m_T^*, m_{-T}) = (x_i^*, y_i^* + \gamma)$, $\sum_{i\in I} z_i = (x, f(x))$ and $(p_x, p_y) = p(\sum_{i\in I} z_i, Y)$. Notice that $(x_i^*, y_i^* + \gamma) \notin L(z_i, u_i)$ implies $(y_i^* + \gamma) = -\frac{p_x}{p_y} x_i^* + y_i + \frac{p_x}{p_y} x_i + \Delta$ for some $\Delta > 0$.

By concavity of f, $f(x_i^* + \sum_{j\neq i} \omega_j) - (y_i^* + \gamma) + \frac{p_x}{p_y}(\sum_{j\neq i} \omega_j) < f(x) - y_i + \frac{p_x}{p_y}(x - x_i)$.

Notice that $\dfrac{\sum_{h\in T^\circ} x_h^*}{\sum_{l\in T-\{i\}} x_l^* + \sum_{k\in I-T} x_k} < \dfrac{\sum_{h\in T^\circ} x_h}{\sum_{l\in T-\{i\}} x_l + \sum_{k\in I-T} x_k}$. This implies that for at least one

member $h \in T^\circ$, $\dfrac{x_h^*}{\sum_{l\in T-\{i\}} x_l^* + \sum_{k\in I-T} x_k} < \dfrac{x_h}{\sum_{l\in T-\{i\}} x_l + \sum_{k\in I-T} x_k}$. Hence,

$$\frac{x_h^*}{\sum_{l\in T-\{i\}} x_l^* + \sum_{k\in I-T} x_k}\{ f(x_i^* + \sum_{j\neq i} \omega_j) - (y_i^* + \gamma) + \frac{p_x}{p_y}(\sum_{j\neq i} \omega_j)\} <$$

$$\frac{x_h}{\sum_{l\in T-\{i\}} x_l + \sum_{k\in I-T} x_k}\{ f(x) - y_i + \frac{p_x}{p_y}(x - x_i)\}.$$ This implies $g_h^{PR}(m_T^*, m_{-T}) =$

$$(\omega_h, \frac{x_h^*}{\sum_{l\in T-\{i\}} x_l^* + \sum_{k\in I-T} x_k}\{f(x_i^* + \sum_{j\neq i} \omega_j) - (y_i^* + \gamma)\}) \in H(p(\sum_{i\in I} z_i, Y), z_h).$$

2-7-2) If there is no $\tilde{z}_i^* \in \overset{\circ}{Z}_i$ such that $\tilde{y}_i^*/\tilde{x}_i^* = y_i/x_i$,

$\tilde{z}_i^* + \sum_{h\in T-\{i\}} z_h^* + \sum_{k\in I-T} z_k \in \partial Y(f,\Omega)$, then Rule 5 is applied.

Case 2-8: $\sum_{i\in T} z_i^* \neq \sum_{i\in T} z_i$ and there exists only $i\in T$ such that for all $h\ (\neq i) \in T$,

$y_h^*/x_h^* = y_h/x_h = y_i/x_i \neq y_i^*/x_i^*$, and there exists $z'_{-i} \in \overset{\circ}{Z}_{-i}$, $PR^{-1}((z_i, z'_{-i})) \neq \varnothing$.

2-8-1) Then if there exists $\tilde{z}_i^* \in \overset{\circ}{Z}_i$ such that $\tilde{y}_i^*/\tilde{x}_i^* = y_i/x_i$, and

$\tilde{z}_i^* + \sum_{h\in T-\{i\}} z_h^* + \sum_{k\in I-T} z_k \in \partial Y(f,\Omega)$, Rule 4-2 is applied.

2-8-2) If there is no $\tilde{z}_i^* \in \overset{\circ}{Z}_i$ such that $\tilde{y}_i^*/\tilde{x}_i^* = y_i/x_i$, and

$\tilde{z}_i^* + \sum_{h\in T-\{i\}} z_h^* + \sum_{k\in I-T} z_k \in \partial Y(f,\Omega)$, Rule 5 is applied.

Case 2-9: $T = I$. It is clear because $z \in PR(u)$ is Pareto efficient.

Thus, $z \in g^{PR}(SNE(\Gamma^{PR}, u))$.

(2) For any $u \in U$, $g^{PR}(NE(\Gamma^{PR}, u)) \subseteq PR(u)$.

Let $m \in NE(\Gamma^{PR}, u)$. Clearly, m cannot correspond to Rule 3, Rule 4 nor Rule 5. Suppose m corresponds to Rule 2. Then $g^{PR}(m) = \bar{z} \in \overset{\circ}{Z}$. We will show that for all $i \in I$, $H(p(\sum_{j\in I}\bar{z}_j, Y), \bar{z}_i) \cap Z_i \subseteq L(\bar{z}_i, u_i)$. Suppose not so for some $j \in I$. Then j can induce Rule 4-1 by announcing $m'_j = (x'_j, 0)$ and get

$z'_j \in \partial H(p(\sum_{i\in I}\bar{z}_i, Y), \bar{z}_j) \cap Z_j$ such that $z'_j \in \arg\max\ u_j(z_j^*)$ over

$z_j^* \in \partial H(p(\sum_{i\in I}\bar{z}_i, Y), \bar{z}_j) \cap Z_j$. This is a contradiction. Suppose m corresponds to Rule 1. Then $g^{PR}(m) = z \in \overset{\circ}{Z}$. We can show that for all $i \in I$, $H(p(\sum_{j\in I} z_j, Y), z_i) \cap Z_i \subseteq L(z_i, u_i)$ by applying the same argument for Rule 2. Thus

$g^{PR}(NE(\Gamma^{PR}, u)) \subseteq PR(u)$. Q.E.D.

Proof of theorem 2.

(1) For any $u \in U$, $EB(u) \subseteq g(SNE(\Gamma, u))$.

Pick any $z = z_i = (x_i, y_i) \in EB(u)$ for any given $u \in U$. For each $i \in I$, let $m_i = z_i$. Then, by Rule 1, $g^{EB}(m) = z$.

Case 1: No individual can be made better off by deviation.

Consider a deviation of member i from $m_i = z_i$ to $m_i^* = z_i^*$. Then i can induce Rule 2, 3 or 4. In all these cases, $g_i^{EB}(m_i^*, m_{-i}) \in L(z_i, u_i)$.

Case 2: No coalition can be made better off by deviation.

Consider a coalition $T \subseteq I$, $\#T \geq 2$, deviating from $m_T = (z_i)_{i \in T}$ to $m_T^* = (z_i^*)_{i \in T}$. T can induce Rule 1, 2, 3, 4 or 5. If T induces Rule 3, 4-2 or 5, there exists at least one member $l \in T$, $g_l^{EB}(m_T^*, m_{-T}) \in L(z_l, u_l)$. If $i \in T$ such that $I^{EB}((z_i)_{i \in I}) = \{i\}$, T can induce Rule 4-1. If T induces Rule 4-1-1 and $g_i^{EB}(m_T^*, m_{-T}) \notin L(z_i, u_i)$, then for any $j \in T - \{i\}$, $g_j^{EB}(m_T^*, m_{-T}) \in H(p(\sum_{k \in I} z_k, Y), z_j) \cap Z_j \subseteq L(z_j, u_j)$. If T induces Rule 4-1-2, then $g^{EB}(m_T^*, m_{-T}) = (z_*^i - \sum_{h \in T - \{i\}} z_h^* - \sum_{k \in I - T} z_k,\ z_{T-\{i\}}^*,\ z_{-T})$. Since any member $k \in I - T$ receives z_k and $z \in EB(u)$ is Pareto efficient, there exists $l \in T$, $u_l(z_l^*) \leq u_l(z_l)$. If T induces Rule 1, $z \in EB(u)$ is Pareto efficient, there exists $l \in T$, $u_l(z_l^*) \leq u_l(z_l)$. If T induces Rule 2, then by Lemma 3, for any $j \in T$, $g_j^{EB}(m_T^*, m_{-T}) \subseteq L(z_j, u_j)$.

Consider $T = I$. It is clear because $z \in EB(u)$ is Pareto efficient.

Thus, $z \in g^{EB}(SNE(\Gamma^{EB}, u))$.

(2) For any $u \in U$, $g^{EB}(NE(\Gamma^{EB}, u)) \subseteq EB(u)$.

Let $m = (z_i)_{i \in I} \in NE(\Gamma^{EB}, u)$. Clearly, m cannot correspond to Rule 3, Rule 4 nor Rule 5. Suppose m corresponds to Rule 1 or 2. Then there exists $p \in \Delta$ and $a \in R_+$ such that for all $i \in I$, $p\, g_i^{EB} = a$. By the same argument in the proof of Theorem 1, we can show that $g^{EB}(m)$ is a Pareto efficient allocation. Thus, $g^{EB}(m) \in EB(u)$. Q.E.D.

REFERENCES

1. B. Dutta and A. Sen, Necessary and Sufficient Condition for Two-Person Nash Implementation, *Review of Economic Studies* 58 (1991), 121-128.

2. B. Dutta and A. Sen, Implementation under strong equilibrium: complete characterization, *Journal of Mathematical Economics* 20 (1991), 49-67.

3. B. Dutta, A. Sen, and R. Vohra, Nash Implementation Through Elementary Mechanism in Economic Environments, *Economic Design* 1 (1995), 173-204.

4. L. Hong, Nash Implementation in Production Economies, forthcoming in *Economic Theory*, 1990.

5. M. O. Jackson, T. Palfrey, and S. Srivastava, Undominated Nash Implementation in Bounded Mechanisms, *Games and Economic Behavior* 6 (1994), 474-501.

6. E. Maskin, Nash Equilibrium and Welfare Optimality, mimeo, 1977.

7. E. Maskin, Implementation and Strong Nash-Equilibrium," in Lafont, J. (ed.), *Aggregation and Revelation of Preferences* 8, 432-439. North-Holland Publishing Company, Amsterdam, (1979).

8. R. D. McKelvey, Game Forms for Nash Implementation of General Social Choice Correspondences, *Social Choice and Welfare* 6 (1989).

9. J. Moore, and R. Repullo, Nash Implementation: A Full Characterization, *Econometrica* 58 (1990), 1083-1099.

10. H. Moulin, Joint Ownership of a Convex Technology: Comparison of Three Solutions, *Review of Economic Studies* 57 (1990), 439-452.

11. R. Repullo, A Simple Proof of Maskin's Theorem on Nash Implementation, *Social Choice and Welfare* 4 (1987), 39-42.

12. J. Roemer, A Public Ownership Resolution of the Tragedy of the Commons, *Social Philosophy and Policy* 6 (1989), 74-92.

13. J. Roemer, and J. Silvestre, Public Ownership: Three proposals for Resource Allocation, mimeo, 1988.

14. J. Roemer, and J. Silvestre, The Proportional Solution for Economies with Both Private and Public Ownership, *Journal of Economic Theory* 59 (1993), 426-444.

15. T. Saijo, Strategy Space Reduction in Maskin's Theorem: Sufficient Condition for Nash Implementation, *Econometrica* 56 (1988), 693-700.

16. T. Saijo, Y. Tatamitani, and T. Yamato, Toward Natural Implementation, forthcoming in *International Economic Review*, 1993.

17. D. Schmeidler, Walrasian Analysis via Strategic Outcome Functions, *Econometrica* 40 (1980), 1585-1594.

18. T. Sjöström, Implementation by Demand Mechanisms, mimeo, 1991.

19. S-C. Suh, Doubly Implementing the Proportional Solution, mimeo, 1993.

20. G. Tian, Implementation of Lindahl Correspondence by a Single Valued,Feasible and Continuous Mechanism, *Review of Economic Studies* 56 (1989), 613-621.

21. G. Tian, Completely Feasible and Continuous Implementation of the Walrasian Correspondence without Continuous, Convex and Ordered Preferences, *Social Choice and Welfare* 9 (1992), 117-130.

CHAPTER IV

A Characterization of Natural and Double Implementation in Production Economies

Abstract: Succeeding to Dutta, Sen and Vohra (1995) and Saijo, Tatamitani and Yamato (1995), define several conditions of natural mechanisms in production economies, and proposed two types of natural mechanisms, that is, the quantity and price-quantity types. First, in differentiable convex production economies, characterize the class of Pareto efficient-social choice solutions doubly implementable in Nash and strong Nash equilibria by a natural quantity mechanism. Second, in convex production economies, characterize the class of Pareto efficient-social choice solutions doubly implementable in Nash and strong Nash equilibria by a natural price-quantity mechanism. Third, examine the double implementability of the Walrasian solution in private ownership economies, and the Equal Benefit and Proportional Solutions in public ownership economies by the two natural mechanisms.

1. Introduction

In this paper, we consider production economies with convex technology. Some resource is privately owned, while the other resource or production technology is either publicly owned or privately owned. For example, the labor input is privately owned, while other resources are publicly owned in public ownership economies, or those are privately owned in private ownership economies. For each given economy, a desirable social goal on resource allocations is specified according to whether the economy is under public ownership or private ownership. The social goal on resource allocations is expressed by a social choice solution (SCS) which is a correspondence assigning to every economy some feasible allocations. In this paper, we consider the class of social choice solutions assigning at least Pareto efficient allocations. When a social choice solution is given, the succeeding problem is how to assign allocations consistent with true characteristics of the current economy. It is a difficult problem when true characteristics of the current economy are under members' private information, because each member does not necessarily reveal his true private information if he can gain by misrepresenting his information. When such a misrepresentation is beneficial to some member, the current social choice solution is said to be "manipulable". That is an important problem because many social choice solutions which have some desirable properties are manipulable (Hurwicz (1972)).

The implementation theory tries to resolve this manipulation problem by designing mechanisms which can achieve the social goal consistent with members' incentives. Under a noncooperative game defined by a mechanism, each member takes some strategic behavior, and so an outcome is assigned by the rule of the mechanism and the combination of members' strategic behavior. If the equilibrium outcomes of a mechanism coincide with the outcome of the SCS under a given equilibrium concept, then the mechanism implements the SCS. After the seminal work of Maskin (1977) on Nash implementation, that identified the class of Nash-implementable solutions in social choice environments, and that constructed mechanisms implementing them, many mechanisms were designed in different equilibrium concepts; for example, subgame perfect implementation in Moore and Repullo (1988) and Abreu and Sen (1990), undominated Nash implementation in Palfrey and Srivastava (1988), and strong Nash implementation in Dutta and Sen (1991). Since these works have sought to characterize the class of SCS's implementable in various equilibrium concepts, they have not paid attention to "desirability" of constructed mechanisms. Thus, these constructed mechanisms have many *unnatural* properties: the typical unnatural property of the mechanisms is their strategy space— each member's strategy contains at least the announcement of a preference profile.

In contrast to the above works, there was the other works in implementation theory, which tried to construct a desirable mechanism implementing a *specific* SCS in economic environments. For example, there are several desirable mechanisms which implement the Walrasian solution or the Lindahl solution in Nash equilibria (Schmeidler (1980), Hurwicz (1979), Walker (1981), Postlewaite and Wettstein (1989), Tian (1989) (1992), and Hong (1995)). Since these mechanisms only apply to a specific SCS, it must be checked for different SCS's whether there exists a desirable mechanism implementing them.

Recently, there is a new approach in Nash implementation theory that is to explore the ground between the above two approaches. This approach is to impose several conditions that a desirable mechanism should satisfy, and then characterize the class of SCS's implementable by such a mechanism in pure exchange economies. This approach is promoted by Sjöström (1991), Dutta, Sen and Vohra (1995), and Saijo, Tatamitani and Yamato (1995).

This paper also belongs to the above new approach of implementation theory. Here, our interest is in production economies. We impose some conditions on *natural* mechanisms, and characterize the class of SCS's Nash-implementable by the natural mechanism in production economies.

The first condition is that the dimension of the strategy space should be finite and low enough. The strategy space of the canonical Maskin-type mechanism are of infinite dimension. This implies that information transmission in playing the game defined by the mechanism is costly and very complicated.

The *informational decentralization* (Schmeidler (1980)) property is the second requirement we impose. This property implies that each member announces information only about himself. On the contrary, since the Maskin-type mechanism requires each member to announce the preferences of all the members, in that mechanism, each member's strategy space includes the space of other member's possible preferences. Thus, in the Maskin-type mechanism, the planner has the authority to compel each member to announce the traits of others, which is usually objectionable in actual democratic societies.

We can propose two types of mechanisms as satisfing the above two requirements: the first one is a *quantity mechanism* where each member is required to announce his own demand quantity, while the second one is a *price-quantity* where each member is required to announce his own demand quantity and a price.

Third, we also impose that the natural mechanism should implement solutions not only in Nash, but also in strong Nash equilibria. The Maskin-type mechanism can be used only in the environment where the social planner is convinced that members will never take any cooperative strategies. However, it seems to be usual in actual economic contexts that the planner cannot know whether members will cooperate or not. So, it is more desirable to construct a mechanism doubly implementing solutions in Nash and in strong Nash equilibria. Double implementation in Nash and strong Nash equilibria is originally discussed by Maskin (1979). The Schmeidler (1980) mechanism is an example of doubly implementing mechanisms. There are also several works on double implementation in Nash and undominated Nash equilibria — Jackson (1992), Yamato (1993), Tatamitani (1993), and Jackson, Palfrey and Srivastava (1994).

As well as Dutta, Sen and Vohra (1995), and Saijo, Tatamitani and Yamato (1995), the mechanisms we consider are required to be *individually feasible and balanced* (Hurwicz, Maskin and Postlewaite (1984)). The mechanisms are also required to satisfy the *best response property* (Jackson, Palfrey, and Srivastava (1994)). To avoid quite a complicated computation to obtain the equilibrium outcome, the mechanisms are also required to satisfy the *forthrightness* (Saijo, Tatamitani and Yamato (1995)).

Moreover, we require easiness of constructing the attainable set of each member. The attainable set for one member at a given strategy combination of other members is the set of allocation vectors she can obtain by her strategic behavior. Let some SCS be implementable by some mechanism. When some allocation is a SCS-optimal allocation for some true preference profile, there must be an equilibrium strategy whose outcome under the mechanism becomes that allocation. This implies that the attainable set of each member in equilibrium must be contained by the lower contour set of each member with her preference at that allocation. If the social planner knew the true preference profile, it would be possible to construct a mechanism satisfying such a property about attainable sets. In fact, many mechanisms with preference announcements possess the property that in equilibrium the attainable set of each member is precisely the reported lower contour set. However, since in our models the planner does not collect reports about lower contour sets, the problem is how to construct attainable sets of members to successfully implement the SCS. In the case of production economies where the production technology is fixed, one method to resolve that problem is to construct the mechanism such that the attainable set of each member in equilibrium be contained in the closed half space defined by announcing quantities and some production-supporting price. The production-supporting price is determined by the production possibility frontier and some efficient production point inferred by quantity announcements. Since, by the property of the SCS, the equilibrium outcomes should be Pareto efficient, it would be able to infer that the lower contour set of each member contains such a closed half space.

In this paper, we first characterize the class of Pareto efficient-SCS's doubly implementable in Nash and strong Nash equilibria by the natural quantity mechanism. Sjöström (1991), and Saijo, Tatamitani and Yamato (1995) have already clarified that no natural quantity mechanism can implement Pareto efficient-SCS's in pure exchange economies. It is generally true in the case of production economies, too. However, as long as the production set has the smooth boundary (or the technology is representable by a differentiable production function), several Pareto efficient social choice solutions are doubly implementable by the natural quantity mechanism. The Walrasian solution in private production economies is such an example. In public ownership economies, the equal benefit solution (Roemer and Silvestre (1989)) is doubly implementable by a natural quantity mechanism. In one input and one output differentiable production economies, the proportional solution (Roemer and Silvestre (1989)) is also doubly implementable (Yoshihara (1995a)). Notice that all these referred solutions satisfy the axiom of *Support Price Independence* (SPI) (Yoshihara (1995a)). In this paper, I show that SPI and some axiom, *Condition QP*, which gives a feasible punishment condition in the case that all members be potential deviators, are necessary and sufficient for Pareto efficient-SCS's in differentiable convex production economies to be doubly implementable by a natural quantity mechanism.

Second, in general convex production economies, we characterize the class of Pareto efficient-SCS's doubly implementable in Nash and strong Nash equilibria by a natural price-quantity mechanism. The above three solutions are doubly implementable by a price-quantity mechanism. It is also shown that SPI and some axiom, Condition PQP, are necessary and sufficient for a Pareto efficient-SCS in convex production economies to be doubly implementable by a natural price-quantity mechanism. Condition PQP also gives a feasible punishment condition in the case that all members be potential deviators.

In the following, section 2 presents our basic models. In section 3, we state some conditions about natural mechanisms, and define the natural quantity mechanism and the price-quantity mechanism, each of which satisfies those conditions. Moreover, we discuss about necessary and sufficient conditions for SCS's to be doubly implementable by each of the two type mechanisms. Finally, in section 4, we check that whether each of the Walrasian solution, the equal benefit solution and the proportional solution satisfies the necessary and sufficient conditions or not.

2. The basic model

We consider production economies in the following: There are one type of labor input and m commodities. There is a firm with a production set $Y \subseteq R_{-} \times R^{m}$. Vector $z \in Y$ will be written as $(m+1)$ vectors, as follows: $z = (x, y)$ where x is the labor input, y is the m vector of commodities. It is assumed that:

A1. $0 \in Y$.

A2. Y is closed and convex.

A3. $\forall\ z = (x, y) \in Y$, $[\exists j \in \{1, \cdots, m\}$ *s.t.* $y^{j} > 0 \Rightarrow x < 0]$.

A4. Labor is productive: $[(x, y) \in Y$ and $x^{\circ} < x\ ] \Rightarrow [\exists y^{\circ} \geq y$ such that $(x^{\circ}, y^{\circ}) \in Y]$.

We denote by Y the class of production possibilities sets satisfying A1~A4. We denote by $\partial\ Y$ the efficiency frontier of Y:

$z \in \partial\ Y \Leftrightarrow z \in Y$ and $[\{ z^{\circ} \in Y, (x^{\circ}, y^{\circ}) \geq (x, y)\} \Rightarrow z^{\circ} = z]$. The aggregate initial endowment of y is denoted by $\underline{\Omega} \in R^{m}_{+}$. The distribution of commodity endowments is supposed to be known and fixed.

There are n members ($n \geq 3$). The set of members is denoted by $I = \{1,2,\ldots,n\}$ with generic element i. Each member i is endowed with a negative amount ω^{0}_{i} of labor endowment. The aggregation of labor endowments is Ω^{0}. Each member i has $Z_{i} = [\omega^{0}_{i}, 0] \times R^{m}_{+}$ as his attainable consumption set. The generic element of i's consumption vector is denoted by $z_{i} = (x_{i}, y_{i})$. Let denote that $\overset{\circ}{Z}_{i} \equiv (\omega^{0}_{i}, 0) \times R^{m}_{++}$. In the following, we also fix $(\omega_i^0)_{i\in I}$, so that $Z = (Z_i)_{i\in I}$ is also fixed. Each member i's preference is represented by a utility function $u_i : Z_i \to R$ where u_i is continuous, quasi-concave and strictly increasing. We denote by U_i the set of such functions.

Suppose that Y is known and fixed. Then an economy is specified by a list $u = (u_i)_{i\in I} \in U \equiv \underset{i\in I}{\times} U_i$. An n-tuple consumption bundle $z = (z_i)_{i\in I}$ constitutes a feasible allocation if: 1) for each i, $z_i \in Z_i$, and 2) $\sum_{i\in I} z_i - \underline{\Omega} \in Y$. Let A denote the set of all feasible allocations with generic element $z = (z_i)_{i\in I}$. Let denote that $\overset{\circ}{A} \equiv \{z = (z_i)_{i\in I} \in A \mid \forall i \in I,\ z_i \in \overset{\circ}{Z}_i\}$. A feasible allocation $z = (z_i)_{i\in I}$ is *Pareto efficient* at $u \in U$ if there does not exist another feasible allocation $z^* = (z_i^*)_{i\in I}$ at u such that $u_i(z_i^*) > u_i(z_i)$ for all $i \in I$. We denote by $P(u)$ the set of Pareto efficient allocations at u. As Dutta, Sen and Vohra (1995) did, we also assume that the class of admissible economies is restricted to the one satisfying the condition that for any $u \in U,\ P(u) \cap \overset{\circ}{A} \neq \varnothing$.

A *social choice solution* (SCS) is a mapping S associating with every economy $u \in U$ a non-empty subset $S(u)$ of feasible allocations satisfying for any $u \in U$, $S(u) \subseteq P(u) \cap \overset{\circ}{A}$. Note that by the definition, for any $u \in U$ and $z = (z_i)_{i\in I} \in S(u)$, for each i, $x_i < 0$ and there exists $j \in \{1, \cdots, m\}$ such that $y_i^j > 0$.

A *mechanism* (or game form) is a pair $\Gamma = (M, g)$ where $M = M_1 \times \cdots \times M_n$, M_i is the *strategy space of agent i*, and the outcome function, $g : M \to R^{n(m+1)}$, assigns to every $m \in M$ a unique element of $R^{n(m+1)}$. Denote the i-th component of $g(m)$ by $g_i(m)$. The mechanism $\Gamma = (M, g)$ is *individually feasible* if $g(m) \in Z$ for all $m \in M$. The mechanism $\Gamma = (M, g)$ is *weakly balanced* if for all $m \in M$, for some $\tilde{z} \in \partial\ Y$, $\sum_{i\in I} g_i(m) - \underline{\Omega} \leq \tilde{z}$. The mechanism $\Gamma = (M, g)$ is *balanced* if for all $m \in M$, for some $\tilde{z} \in \partial\ Y$, $\sum_{i\in I} g_i(m) - \underline{\Omega} = \tilde{z}$. Thus, an individually feasible and balanced mechanism is a pair $\Gamma = (M, g)$ where g assigns to every $m \in M$ a unique element of A. In this paper, we focus on individually feasible and balanced mechanisms.

The list $m \in M$ will be written as (m_i, m_{-i}), where $m_{-i}=(m_1, \cdots, m_{i-1}, m_{i+1}, \cdots, m_n) \in M_{-i} \equiv \times_{j \neq i} M_j$. Given $m \in M$ and $m_i' \in M_i$, (m_i', m_{-i}) is obtained by the replacement of m_i by m_i'. Let $g(M_i, m_{-i})$ is the attainable set of member i at m_{-i}, i.e., the set of consumption bundles that member i can induce when the other members select m_{-i}.

Denote for any coalition $T \subseteq I$, $m_T = (m_i)_{i \in T}$ and $m_{-T} = (m_i)_{i \in I-T}$. Hence (m_T', m_{-T}) is the list obtained by the replacement of m_T by m_T', and $g(M_T, m_{-T})$ is the attainable set of the coalition T at m_{-T}, i.e., the set of consumption bundles that the coalition T can induce when $I - T$ select m_{-T}.

For $i \in I$, $u_i \in U_i$, and $z_i \in Z_i$, let $L(z_i, u_i) := \{z_i' \in Z_i \mid u_i(z_i) \geq u_i(z_i')\}$ be the *lower contour set for* u_i *at* z_i. Given a feasible mechanism $\Gamma = (M, g)$ and a profile of utility functions $u \in U$, the strategy profile $m \in M$ is a *Nash equilibrium of* Γ *at* u if for all $i \in I$, $g_i(M_i, m_{-i}) \subseteq L(g_i(m), u_i)$. Let $NE(\Gamma, u)$ be the set of Nash equilibria of Γ at u. Let $g(NE(\Gamma, u))$ be the set of Nash equilibrium allocations of Γ at u. Given $\Gamma = (M, g)$ and $u \in U$, the strategy profile $m \in M$ is a *strong Nash equilibrium of* Γ *at* u if for all $T \subseteq I$, for all $m_T' \in M_T$, there exists $i \in T$ such that $g_i(m_T', m_{-T}) \in L(g_i(m), u_i)$. Let $SNE(\Gamma, u)$ be the set of strong Nash equilibria of Γ at u. Let $g(SNE(\Gamma, u))$ be the set of strong Nash equilibrium allocations of Γ at u.

The mechanism $\Gamma = (M, g)$ *implements the SCS S in Nash equilibria* if for all $u \in U$, $S(u) = g(NE(\Gamma, u))$. The SCS S is Nash-implementable if there exists a mechanism which implements S in Nash equilibria. The mechanism $\Gamma = (M, g)$ *implements the SCS S in strong Nash equilibria if for all* $u \in U$, $S(u) = g(SNE(\Gamma, u))$. The SCS S is strongly Nash-implementable if there exists a mechanism which implements S in strong Nash equilibria. The mechanism $\Gamma = (M, g)$ *doubly implements the SCS S in Nash and strong Nash equilibria* if for all $u \in U$, $S(u) =$

$g(SNE(\Gamma, u)) = g(SNE(\Gamma, u))$. The SCS S is doubly implementable if there exists a mechanism which doubly implements S in Nash and strong Nash equilibria.

3. Natural mechanisms

First, we discuss on the conditions that mechanisms should satisfy if they are natural. The first and second conditions are related to characteristics of the strategy spaces.

Condition 1: The strategy spaces of natural mechanisms must be of enough low or at least finite dimensional.

This condition is a plausible requirement, since the mechanism with low dimensional strategy spaces simplifies information transimission between the planner and members. As such mechanisms, Saijo, Tatamitani and Yamato (1995 consider six types of mechanisms: quantity, quantity^2, allocation, price-quantity, price-quantity^2, and price-allocation mechanisms. In the case of general social choice environment, Saijo (1988) and McKelvey (1989) address this issue.

While the first conditon is related to quantitative characteristics of strategy spaces, the next condition is related to their qualitative characteristics.

Condition 2: All members' strategy spaces are composed of their own admissible characteristics (*Informationally decentralization* property (Schmeidler (1980)).

It is unnatural to give the planner the authority to compel each member to announce the traits of others, because the traits of others be private information of others. Among the six type mechanisms Saijo, Tatamitani and Yamato (1995) proposed, only two types, — quantity, and price-quantity, seem to be regarded as passing the Condition 2.

The next condition we require is the best response property of the mechanism:

Definition 1 (Jackson, Palfrey, and Srivastava (1994)) : The mechanism $\Gamma = (M, g)$ satisfies the *best response property* if for all $i \in I$, all $u_i \in U_i$, and all $m_{-i} \in M_{-i}$, there exists $m_i \in M_i$ such that $u_i(g_i(m_i, m_{-i})) \geq u_i(g_i(m_i^\circ, m_{-i}))$ for all $m_i^\circ \in M_i$.

Condition 3: Natural mechanisms should satisfy the best response property.

This condition is necessary to justify the use of Nash equilibrium as an equilibrium concept.

The next condition is related to easiness of constructing the attainable set of each member.

Condition 4: It is no information about preferences to construct each agent's attainable strategy space in equilibrium.

This condition represents a requirement of simple punishments. The canonical Maskin-type mechanism must collect reports about lower contour sets to punish the member deviating from a desirable solution. It implies that the planner must check whether or not be the announced preferences of the deviator consistent with that solution. Such calculation is costly for the planner. Thus, if there is a mechanism meeting the Condition 4, it is simplified the process of assigning punishing outcomes. Notice that quantity and price-quantity mechanisms in both Sjöström (1991) and Saijo, Tatamitani and Yamato (1995) do not satisfy this condition, because in their mechanisms, each member's attainable set in equilibrium is composed of the intersection of his possible lower contour sets.

The next condition represents a requirement of easiness for each member to calculate allocations induced from equilibrium strategies.

Condition 5: Natural mechanisms should be simple in the sense that it is easy to compute the outcome of an equilibrium strategy.

Saijo, Tatamitani and Yamato (1995) formalized this condition concretely as *forthrightness*. The forthrightness requires that in equilibrium, each member receives what he has announced as his own consumption bundle. This seems to meet the requirement of Condition 5.

The last condition is the feasibility of mechanisms:

Condition 6: Natural mechanisms should be individually feasible and balanced.

In the case of production economies, it is difficult to construct individually feasible and balanced mechanisms, because the total supply will not be known to the committee *ex ante*, even if the distribution of initial endowment and production technology are known.

In the following, we consider two types of natural mechanisms satisfying the above six conditions. First, we consider a natural quantity mechanism where each member announce his own consumption only, and implementation by one. Second, consider a natural price-quantity mechanism where each member announce his own consumption and a price, and implementation by one.

3.1. Double Implementation by a Natural Quantity Mechanism

Now, we define implementation by a natural quantity mechanism:

Definition 2 : The SCS S is doubly implementable by a natural quantity mechanism if there exists a mechanism $\Gamma = (M, g)$ such that

(i) Γ doubly implements S ;

(ii) for all $i \in I$, $M_i = Z_i$;

(iii) for all u $\in$ U and all $z = (z_i)_{i \in I} \in S(u)$, if $m_i = z_i$ for all $i \in I$, then $g(m) = z \in g(NE(\Gamma, u)) = g(SNE(\Gamma, u))$;

(iv) Γ is individually feasible and balanced; and

(v) Γ has the following property: if $m \in NE(\Gamma, u)$ for all $u \in U$, then for some $p \in \Delta$, for all $i \in I$, $g_i(M_i, m_{-i}) \subseteq \hat{H}(p, z_i) \cap Z_i$ where $(z_i)_{i \in I} = g(m)$ and $p \in p(Y, \sum_{i \in I} z_i - \Omega)$; and

(vi) Γ satisfies the best response property.

The above Definition 2 (iii), which is introduced and named "*forthrightness*" by Saijo, Tatamitani and Yamato (1995), represents a characteristic of mechanisms satisfying Condition 5. Notice that Definition 2 (ii) implies that the mechanism Γ satisfies Condition 1 and 2. Also, notice that Definition 2 (v) implies that the mechanism Γ satisfies Condition 4, because the half space can be constructed without information about preferences.

Next step of ours is to find necessary and sufficient conditions for a SCS to be doubly implementable by a natural quantity mechanism.

Let $\Delta := \{p \in R_+^{m+1} | p_x + p_y = 1\}$ be the unit simplex. Let $H(p, w) :=$ $\{w^* = (x^*, y^*) \in R_- \times R^m | \ p \cdot w \geq p \cdot w^*\}$ for $p \in \Delta$ and $w \in R_- \times R^m$. A production point $w \in Y$ is efficient at $p \in \Delta$ if $Y \subseteq H(p, w)$. Let $p(Y, \tilde{z}) :=$ $\{p \in \Delta \mid \exists \tilde{z} \in Y, Y \subseteq H(p, \tilde{z})\}$ and $\hat{H}(p, z_i) := \{z \in R_- \times R_+^m \mid p \cdot z \leq p \cdot z_i\}$ for some $z_i \in Z_i$. The nonzero vector $p \in \Delta$ is a vector of efficiency prices for the Pareto efficient allocation $z = (z_i)_{i \in I}$ at $u \in U$ if $p \in p(Y, \tilde{z})$ where $\tilde{z} = \sum_{i \in I} z_i - \Omega \in \partial\ Y$, and $L(z_i, u_i) \supseteq \hat{H}(p, z_i)(\forall i \in I)$.

Definition 3 : For some $z = (z_i)_{i \in I} \in S(u)$ for $u \in U$, a price $p \in \Delta$ *supports z at u as an allocation of the S solution* if $p \in \Delta$ satisfies the following conditions:
1) $p \in p(Y, \tilde{z})$ where $\tilde{z} = \sum_{i \in I} z_i - \Omega \in \partial\ Y$, and

2) if $S \subseteq P$, then $L(z_i, u_i) \supseteq \hat{H}(p, z_i)(\forall i \in I)$.

Let $p(Y, u, z)$ be the set of prices supporting z at (u, Y). Let denote the set of efficiency prices for $z \in P(u)$ by $p^P(Y, u, z)$. By definition, if S does not require

Pareto optimality, $p^P(Y, u, z) \subseteq p(Y, u, z)$. But, in this paper, since $S \subseteq P$, $p^P(Y, u, z) = p(Y, u, z)$.

Axiom of Support Price Independence (SPI) (Yoshihara (1995b)):
For all u, $z = (z_i)_{i \in I} \in S(u)$, there exists $p \in p(Y, u, z)$ such that for all $u^* \in U$, $[p \in p^P(Y, u^*, z) \Rightarrow z \in S(u^*)]$.

This condition was introduced and studied by Yoshihara (1995b) in the context of characterizing public ownership solutions. Yoshihara (1995b) showed that any Pareto efficient-SCS satisfying SPI satisfies Maskin Monotonicity (Maskin (1977)).

Let for any $(z_i)_{i \in I} \in \times_{i \in I} Z_i$, $p^i(S, (z_i)_{i \in I}) := \{ p(\tilde{z}^i, Y) \in \Delta \mid$ for some $\tilde{z}^i \in \partial Y$, $S^{-1}((\tilde{z}^i + \Omega - \sum_{j \neq i} z_j, z_{-i})) \neq \varnothing \}$ and $I^S((z_i)_{i \in I}) := \{ i \in I \mid p^i(S, (z_i)_{i \in I}) \neq \varnothing \}$. We can interpret $I^S((z_i)_{i \in I})$ as the set of potential deviators when a strategy combination $m = (z_i)_{i \in I}$ is not consistent with any S-optimal allocation.

Condition QP (QP): For every $(z_i)_{i \in I} \in \times_{i \in I} Z_i$ such that $I^S((z_i)_{i \in I}) = I$, there exists $v((z_i)_{i \in I}) \in A$ such that (i) $\sum_{i \in I} v_i((z_i)_{i \in I}) - \Omega \in \partial Y$;
(ii) $v_i((z_i)_{i \in I}) \in \bigcap_{p(\tilde{z}^i, Y) \in p^i(S, (z_i)_{i \in I})} H(p(\tilde{z}^i, Y), \tilde{z}^i + \Omega - \sum_{j \neq i} z_j) \cap Z_i$ for all $i \in I$; and

(iii) if there exists $u^* \in U$ such that for all $i \in I$,
$\bigcap_{p(\tilde{z}^i, Y) \in p^i(S, (z_i)_{i \in I})} H(p(\tilde{z}^i, Y), \tilde{z}^i + \Omega - \sum_{j \neq i} z_j) \cap Z_i \subseteq L(v_i((z_i)_{i \in I}), u_i^*)$, then $v((z_i)_{i \in I}) \in S(u^*)$.

Condition QP gives a feasible punishment condition in the case that all members be potential deviators. This condition is a generalization of Condition Q (Saijo, Tatamitani and Yamato (1995)) in the context of production economies.

We show that Axiom SPI and Condition QP are necessary and sufficient for natural quantity implementation of a Pareto efficient-SCS in differentiable production economies.

Theorem 1: Suppose that the production possibility set Y has the smooth boundary. Then, a SCS S is doubly implementable by a natural quantity mechanism if and only if it satisfies SPI and QP.

Proof of Theorem 1: See Appendix.

As a corollary of Theorem 1, it is easily shown that if free disposal of the production set is assumed, any solution satisfying Pareto efficiency and SPI in differentiable convex production economies are doubly implementable by a individually feasible and weakly balanced quantity mechanism.

Notice that if the production set has nonsmooth boundary, solutions satisfying Pareto efficiency and SPI are no longer implementable by any natural quantity mechanism. The reason is that when the production point induced by aggregating all members' announcing quantities is on a kink boundary, it is no longer determined uniquely the supporting price of that production, so that the planner does not induce information about the common marginal rate of substitution. In such a case, the planner cannot construct members' attainable sets.

3.2. Double Implementation by a Natural Price-Quantity Mechanism

Next, we define implementation by a natural price-quantity mechanism:

Definition 4: The SCS S is doubly implementable by a natural price-quantity mechanism if there exists a mechanism $\Gamma = (M, g)$ such that

(i) Γ doubly implements S;

(ii) for all $i \in I$, $M_i = \Delta \times Z_i$;

(iii) for all $u \in U$ and all $z = (z_i)_{i \in I} \in S(u)$, there exists $p \in p(Y, u, z)$ such that [$m_i = (p, z_i)$ $(\forall i \in I)$ $\Rightarrow$ $g(m) = z \in g(NE(\Gamma, u)) = g(SNE(\Gamma, u))$];

(iv) Γ is individually feasible and balanced; and

(v) Γ has the following property: if for all $u \in U$, $m = ((m_i^1, m_i^2)) \in NE(\Gamma, u)$ such that $p = m_i^1$ for all $i \in I$, then for all $i \in I$, $g_i(M_i, m_{-i}) \subseteq \hat{H}(p, z_i) \cap Z_i$ where $(z_i)_{i \in I} = g(m)$ and $p \in p(Y, \sum_{i \in I} z_i - \Omega)$; and

(vi) Γ satisfies the best response property.

The above Definition 4 (iii), which is "*forthrightness*" for a price-quantity mechanism, represents a characteristic of mechanisms satisfying Condition 5. Notice that Definition 4 (ii) implies that the mechanism Γ satisfies Condition 1 and 2.

In the following, we find necessary and sufficient conditions for a SCS to be doubly implementable by a natural price-quantity mechanism.

For each $(z_i)_{i \in I} \in \times_{i \in I} Z_i$ and $p \in \Delta$, let $p^i(S, (z_i)_{i \in I}) := \{ p \in p(\bar{z}^i, Y) \mid$ for some $\bar{z}^i \in \partial Y$, $S^{-1}((\bar{z}^i + \Omega - \sum_{j \neq i} z_j, z_{-i})) \neq \varnothing \}$ and $I^S(p, (z_i)_{i \in I}) := \{ i \in I \mid p \in p^i(S, (z_i)_{i \in I}) \}$. We can interpret $I^S(p, (z_i)_{i \in I})$ as the set of potential deviators when a strategy combination $m = (p, z_i)_{i \in I}$ is not consistent with any S-optimal allocation.

Condition PQP (PQP): For every $(p, (z_i)_{i \in I}) \in \Delta \times Z$ such that $I^S(p, (z_i)_{i \in I}) = I$, there exists $v(p, (z_i)_{i \in I}) \in A$ such that (i) $\sum_{i \in I} v_i(p, (z_i)_{i \in I}) - \Omega \in \partial Y$;
(ii) $v_i(p, (z_i)_{i \in I}) \in \bigcap_{p \in p(\bar{z}^i, Y)} H(p, \bar{z}^i + \Omega - \sum_{j \neq i} z_j) \cap Z_i$ for all $i \in I$; and

(iii) if there exists $u^* \in U$ such that for all $i \in I$,
$\bigcap_{p \in p(\bar{z}^i, Y)} H(p, \bar{z}^i + \Omega - \sum_{j \neq i} z_j) \cap Z_i \subseteq L(v_i(p, (z_i)_{i \in I}), u_i^*)$, then $v((z_i)_{i \in I}) \in S(u^*)$.

Condition PQP gives a feasible punishment condition in the case that all members be potential deviators. This condition is a generalization of Condition PQ (Saijo, Tatamitani and Yamato (1995)) or Condition B (Dutta, Sen and Vohra (1995)) in the context of production economies.

Theorem 2 : A SCS S is doubly implementable by a natural quantity mechanism if and only if it satisfies SPI and PQP.

Proof of Theorem 2: See Appendix.

As a corollary of Theorem 2, it is easily shown that if free disposal of the production set is assumed, any solution satisfying Pareto efficiency and SPI in convex production economies are doubly implementable by a individually feasible and weakly balanced price-quantity mechanism.

4. Characterization Results

In this section, we define three solutions, and investigate whether or not each of three solutions is implementable respectively by the two types of natural mechanism discussed in the above section.

Consider the following private ownership production economies: for each $i \in I$, $\underline{\omega}_i \in R^m_+$ is i's privately owned initial endowment of commodity inputs such that $\sum_{i \in I} \underline{\omega}_i = \underline{\Omega}$, and θ^W_i is i's share of Y. Then the Walrasian solution is defined as follows.

Definition 5: An allocation $z = (z_i)_{i \in I}$ is a *Walrasian solution* (*W*) for $u \in U$ if $z \in A$ such that there exists a price vector $p \in \Delta$ such that:

(i) $Y \subseteq H(p, \tilde{z})$ where $\tilde{z} = \sum_{i \in I}(z_i - \underline{\omega}_i)$;

(ii) for every $i \in I$, $z_i = \arg\max u_i(z^\circ_i)$ over $z^\circ_i \in Z_i$ and $p \cdot z^\circ_i \leq p \cdot \underline{\omega}_i + \theta^W_i p \cdot \tilde{z}$.

Consider the following public ownership solutions: the initial endowment of commodity inputs $\underline{\Omega}$ is publicly owned. The production technology Y is also publicly owned. Then:

Definition 6 (Roemer and Silvestre (1989)): An allocation $z = (z_i)_{i \in I}$ is a *Proportional solution* (*PR*) for $u \in U$ if $z \in A$ such that:

(i) $z \in P(u)$;

(ii) There exists a vector of efficiency prices $p \in \Delta$ for z such that

$$p \cdot z_i = \frac{p \cdot x_i}{\sum_{h \in I} p \cdot x_h}\left(\sum_{i \in I} p \cdot z_i\right) \; (\forall i \in I) \text{ if } \sum_{h \in I} p \cdot x_h < 0,$$

$$p \cdot z_i = \frac{1}{n}\left(\sum_{i \in I} p \cdot z_i\right) \; (\forall i \in I) \text{ if } \sum_{h \in I} p \cdot x_h = 0.$$

Definition 7 (Roemer and Silvestre (1989)) : An allocation $z = (z_i)_{i \in I}$ is a *Equal Benefit solution* (EB) for $u \in U$ if $z \in A$ such that:

(i) $z \in P(u)$;

(ii) There exists a vector of efficiency prices $p \in \Delta$ for z such that

$$p \cdot z_i = \frac{1}{n}\left(\sum_{i \in I} p \cdot z_i\right) \; (\forall i \in I).$$

Note that by the above definitions, if $S = W$, $\theta_i^S = \theta_i^W$ $(\forall i \in I)$. If $S = PR$, $\theta_i^S = \frac{x_i}{\sum_{h \in I} x_h}$, and if $S = EB$, $\theta_i^S = \frac{1}{n}$.

First, we investigate the Walrasian solution. In the case of differentiable pure exchange economies, both Dutta, Sen and Vohra (1995) and Saijo, Tatamitani and Yamato (1995) showed that the Walrasian solution is Nash-implementable by elementary and natural price-quantity mechanisms defined respectively by them. Saijo, Tatamitani and Yamato (1995) also showed that the Walrasian solution in pure exchange economies is not Nash-implementable by any natural quantity mechanism. In the case of production economies, however, it is shown that the Walrasian solution is double implementable by a natural quantity mechanism, as long as the production technology is differentiable.

Lemma 1: The Walrasian solution satisfies SPI.

Proof of Lemma 1: See Yoshihara (1995b).

Lemma 2: The Walrasian solution satisfies QP when the production set Y has the smooth boundary.

Proof of Lemma 2: Let $(z_i)_{i\in I} \in Z$ such that $I^W((z_i)_{i\in I}) = I$ be given. Then for all $i \in I$, there exists $\bar{z}^i \in \partial Y$ such that $\{p\} = p(\bar{z}^i, Y)$ and $W^{-1}((\bar{z}^i + \underline{\Omega} - \sum_{j\neq i} z_j, z_{-i})) \neq \varnothing$. Let $\omega_k^0 = \max\{\omega_i^0 \mid i \in I\}$ and $\underline{\omega}_j^{\min} = \min\{\underline{\omega}_{ij} \mid i \in I\}$ for each $j = 1, \ldots, m$. Then, let denote $\underline{\omega}^{\min} = (\underline{\omega}_j^{\min})_{j\in\{1,\cdots,m\}}$.

Let take a vector $\tilde{y} \in \{\partial Y(n(\omega_k^0 - \varepsilon)) + \underline{\omega}^{\min}\} \cap R_{++}^m$ for some $\varepsilon > 0$. Then, each $i \in I$, $(\omega_k^0 - \varepsilon, \theta_i^W \cdot (\tilde{y} - \underline{\omega}^{\min}) + \underline{\omega}_i) \in \mathring{Z}_i$. It is clear that $(\omega_k^0 - \varepsilon, \theta_i^W \cdot (\tilde{y} - \underline{\omega}^{\min}) + \underline{\omega}_i)_{i\in I} \in \mathring{A}$ and $\sum_{i\in I}(\omega_k^0 - \varepsilon, \theta_i^W \cdot (\tilde{y} - \underline{\omega}^{\min}) + \underline{\omega}_i) \in \partial Y$.

Notice that for all $i \in I$, for all $\bar{z}^i \in \partial Y$ such that $\{p\} = p(\bar{z}^i, Y)$ and $W^{-1}((\bar{z}^i + \underline{\Omega} - \sum_{j\neq i} z_j, z_{-i})) \neq \varnothing$, $(\theta_i^W \cdot \partial Y + \{\underline{\omega}_i\}) \cap Z_i \subseteq H(p, \bar{z}^i + \underline{\Omega} - \sum_{j\neq i} z_j) \cap Z_i$.

Since $(\omega_k^0 - \varepsilon, \theta_i^W \cdot (\tilde{y} - \underline{\omega}^{\min}) + \underline{\omega}_i) \in (\theta_i^W \cdot \partial Y + \{\underline{\omega}_i\}) \cap Z_i$, $(\omega_k^0 - \varepsilon, \theta_i^W \cdot (\tilde{y} - \underline{\omega}^{\min}) + \underline{\omega}_i) \in H(p, \bar{z}^i + \underline{\Omega} - \sum_{j\neq i} z_j) \cap Z_i$ for all

$p(\bar{z}_i, Y) \in p^i(W, (z_i)_{i\in I})$. Now let define $v((z_i)_{i\in I}) = (\omega_k^0 - \varepsilon, \theta_i^W \cdot (\tilde{y} - \underline{\omega}^{\min}) + \underline{\omega}_i)_{i\in I}$.

Then $v((z_i)_{i\in I})$ satisfies QP (i) and (ii). Suppose that there exists $u^* \in U$ such that for all $i \in I$, $\bigcap_{p(\bar{z}^i, Y)\in p^i(W, (z_i)_{i\in I})} H(p(\bar{z}^i, Y), \bar{z}^i + \underline{\Omega} - \sum_{j\neq i} z_j) \cap Z_i \subseteq L(v_i((z_i)_{i\in I}), u_i^*)$.

Then, for $\bar{z} = (n(\omega_k^0 - \varepsilon), \tilde{y} - \underline{\omega}^{\min})$, $p(\bar{z}, Y) \in p^i(W, v((z_i)_{i\in I}))$ for all $i \in I$. Thus, for all $i \in I$, $H(p(\bar{z}, Y), v_i((z_i)_{i\in I})) \cap Z_i \subseteq L(v_i((z_i)_{i\in I}), u_i^*)$. This implies $v((z_i)_{i\in I}) \in W(u^*)$. Q.E.D.

Corollary 1: The Walrasian solution is doubly implementable by a natural quantity mechanism when the the production set has the smooth boundary.

Proof of Corollary 1: It is followed by Theorem 1, Lemma 1 and 2. Q.E.D.

The following lemma indicates that the Walrasian solution is doubly implementable by a natural price-quantity mechanism even if the production technology is not differentiable.

Lemma 3: The Walrasian solution satisfies PQP.

Proof of Lemma 3: Let $(p, (z_i)_{i\in I}) \in \Delta \times Z$ such that $I^W(p, (z_i)_{i\in I}) = I$ be given. Then for all $i \in I$, there exists $\tilde{z}^i \in \partial Y$ such that $p \in p(\tilde{z}^i, Y)$ and $W^{-1}((\tilde{z}^i + \underline{\Omega} - \sum_{j\neq i} z_j, z_{-i})) \neq \varnothing$. Construct a feasible allocation

$v(p, (z_i)_{i\in I}) = (\omega_k^0 - \varepsilon, \theta_i^W \cdot (\tilde{y} - \underline{\omega}^{\min}) + \underline{\omega}_i)_{i\in I}$ as the same manner as in the proof of Lemma 2. Then it is clear that $v(p, (z_i)_{i\in I})$ satisfies PQP (i) and (ii). Suppose that there exists $u^* \in U$ such that for all $i \in I$,
$\bigcap_{p\in p(\tilde{z}^i, Y)} H(p, \tilde{z}^i + \underline{\Omega} - \sum_{j\neq i} z_j) \cap Z_i \subseteq L(v_i(p, (z_i)_{i\in I}), u_i^*)$ where $\tilde{z}^i \in \partial Y$ such that $W^{-1}((\tilde{z}^i + \underline{\Omega} - \sum_{j\neq i} z_j, z_{-i})) \neq \varnothing$. Then, for $\tilde{z} = (n(\omega_k^0 - \varepsilon), \tilde{y} - \underline{\omega}^{\min})$, $p \in p(\tilde{z}, Y)$.

Thus, for all $i \in I$, $H(p, v_i((z_i)_{i\in I}) \cap Z_i \subseteq L(v_i((z_i)_{i\in I}), u_i^*)$. This implies $v(p, (z_i)_{i\in I}) \in W(u^*)$. Q.E.D.

Corollary 2: The Walrasian solution is doubly implementable by a natural price-quantity mechanism.

Proof of Corollary 2: It is followed by Theorem 2, Lemma 1 and 3. Q.E.D.

We next check the implementability of the equal benefit solution. Yoshihara (1995a) has already shown that in one input and one output differentiable production economies, there is a natural quantity mechanism doubly implementing the equal benefit solution. This result is robust in general convex differentiable production economies.

Lemma 4: The equal benefit solution satisfies SPI.

Proof of Lemma 4: See Yoshihara (1995b).

Lemma 5: The equal benefit solution satisfies QP when the production set has the smooth boundary.

Proof of Lemma 5: See Appendix.

Corollary 3: The equal benefit solution is doubly implementable by a natural quantity mechanism whenever the production set has the smooth boundary.

Proof of Corollary 3: It is followed by Theorem 1, Lemma 4 and 5. Q.E.D.

The following lemma indicates that the equal benefit solution is doubly implementable by a natural price-quantity mechanism even if the production technology is not differentiable.

Lemma 6: The equal benefit solution satisfies PQP.

Proof of Lemma 6: See Appendix.

Corollary 4: The equal benefit solution is doubly implementable by a natural price-quantity mechanism.

Proof of Corollary 4: It is followed by Theorem 2, Lemma 4 and 6. Q.E.D.

With respect to the proportional solution, Yoshihara (1995a) has already shown that in one input and one output differentiable production economies, there is a natural quantity mechanism doubly implementing this solution. In general convex production economies, the following lemma indicates that the proportional solution is doubly implementable by a natural price-quantity mechanism even if the production technology is not differentiable.

Lemma 7: The proportional solution satisfies SPI.

Proof of Lemma 7: See Yoshihara (1995b).

Lemma 8: The proportional solution satisfies PQP.

Proof of Lemma 8: See Appendix.

Corollary 5: The proportional solution is doubly implementable by a natural price-quantity mechanism.

Proof of Corollary 5: It is followed by Theorem 2, Lemma 7 and 8. Q.E.D.

Lemma 9: The proportional solution satisfies QP in one input and one output economies when the production function is differentiable.

Proof of Lemma 9: See Yoshihara (1995a). Q.E.D.

Corollary 6: The proportional solution is doubly implementable by a natural quantity mechanism in one input and one output economies when the production function is differentiable.

Proof of Corollary 6: See Yoshihara (1995a). Q.E.D.

5. Concluding Remarks

In this paper, we defined several conditions that natural mechanisms should satisfy, and proposed two types of natural mechanisms — that is, quantity, and price-quantity. Moreover, first, we show that in differentiable convex production economies, any Pareto efficient-SCS is doubly implementable by a natural quantity mechanism if and only if it satisfies the axiom SPI and the Condition QP. Second, in convex production economies, any Pareto efficient-SCS is doubly implementable by a natural price-quantity mechanism if and only if it satisfies the axiom SPI and the Condition PQP. Third, both the Walrasian and the equal benefit solutions satisfy SPI, QP and PQP. The proportional solution satisfies SPI and PQP in multi input and multi output economies, and moreover, QP in one input and one output economies.

Now, there are still several problems remained. As well as Dutta et al. (1995) and Saijo et al. (1995), all of my constructed mechanisms in this paper do not have the property of continuity of the outcome function. In contrast, the Hong (1995) mechanism which implements the Walrasian solution in production economies is individually feasible, balanced and continuous, and satisfies forthrightness and the best response property, although the strategy spaces of her mechanism are rather larger than ours, and her mechanism cannot doubly implement. It is interesting to explore the possibility of continuous natural mechanisms the strategy spaces of which are less than the Hong (1995). Second, as well as Dutta et al. (1995) and Saijo et al. (1995), all of my constructed mechanisms in this paper also contain "modulo game", though they do not contain "integer game". Thus, although no pure Nash equilibrium exists in a modulo game, if members have von Neumann-Morgenstern utility functions over lotteries on the sets of feasible allocations, then there may exist mixed Nash equilibria which lead to allocations out of the solution with positive probability. Saijo, Tatamitani and Yamato (1995) conjectured some degree of trade-off between none of "modulo game" and both of the best response property and balancedness. Third, we only consider the case of more than three members economies. It remains to consider Nash implementability of solutions in production economies with two persons by natural mechanisms.

APPENDIX

Proof of Theorem 1: First, show the necessity of SPI for double implementation by a natural quantity mechanism. Suppose that S is doubly implementable by a natural quantity mechanism $\Gamma = (M, g)$. Take a profile $u \in U$ and an allocation $z = (z_i)_{i \in I} \in S(u)$. The corresponding S-support price of z at u is $p \in p(Y, u, z)$. Notice that by smoothness of Y, $p(Y, u, z)$ is singleton so that $\{p\} = p(Y, u, z)$. Consider a strategy profile m such that $m_i = z_i$ for all $i \in I$. By forthrightness, $g(m) = z \in g(NE(\Gamma, u^\circ)) = g(SNE(\Gamma, u^\circ))$ for all $u^\circ \in S^{-1}(z)$. Then, by Definition 2(v), $g_i(M_i, m_{-i}) \subseteq \hat{H}(p, z_i) \cap Z_i$ for all $i \in I$. Notice that for all $u^\circ \in S^{-1}(z)$, $\hat{H}(p, z_i) \cap Z_i \subseteq L(z_i, u_i^\circ)$ for all $i \in I$, because $S \subseteq P$. Consider u^* such that $\{p\} = p^P(Y, u^*, z)$. Then, also, $\hat{H}(p, z_i) \cap Z_i \subseteq L(z_i, u_i^*)$ for all $i \in I$. This implies that $g(m) = z \in g(NE(\Gamma, u^*))$, since $g_i(M_i, m_{-i}) \subseteq L(z_i, u_i^*)$ for all $i \in I$. By double implementability, $z = (z_i)_{i \in I} \in S(u^*)$.

Second, show the necessity of QP for double implementation by a natural quantity mechanism. Suppose that S is doubly implementable by a natural quantity mechanism $\Gamma = (M, g)$. Let $(z_i)_{i \in I} \in \times_{i \in I} Z_i$ such that $I^S((z_i)_{i \in I}) = I$ be given. Then for all $i \in I$ and all $u^\circ \in S^{-1}((\bar{z}^i + \Omega - \sum_{j \neq i} z_j, z_{-i}))$ where $\bar{z}^i \in \partial Y$ such that $p(\bar{z}^i, Y) \in p^i(S, (z_i)_{i \in I})$, $g(m^i) = m^i \in g(NE(\Gamma, u^\circ)) = g(SNE(\Gamma, u^\circ))$ is established by forthrightness where $m_i^i = \bar{z}^i + \Omega - \sum_{j \neq i} z_j$ and $m_j^i = z_j$ for all $j \neq i$. Then, by Definition 2(v), $g_i(M_i, m_{-i}^i) \subseteq \bigcap_{p(\bar{z}^i, Y) \in p^i(S, (z_i)_{i \in I})} \hat{H}(p(\bar{z}^i, Y), \bar{z}^i + \Omega - \sum_{j \neq i} z_j) \cap Z_i$ for all $i \in I$.

Let $v((z_i)_{i \in I}) = g((z_i)_{i \in I})$. Then for all $i \in I$, $v_i((z_i)_{i \in I}) \in \bigcap_{p(\bar{z}^i, Y) \in p^i(S, (z_i)_{i \in I})} \hat{H}(p(\bar{z}^i, Y), \bar{z}^i + \Omega - \sum_{j \neq i} z_j) \cap Z_i$. Since Γ is a balanced mechanism, QP(i) is also satisfied by $v((z_i)_{i \in I})$. Moreover, suppose that there exists $u^* \in U$ such that for all $i \in I$, $\bigcap_{p(\bar{z}^i, Y) \in p^i(S, (z_i)_{i \in I})} H(p(\bar{z}^i, Y), \bar{z}^i + \Omega - \sum_{j \neq i} z_j) \cap Z_i \subseteq L(v_i((z_i)_{i \in I}), u_i^*)$. Then, $v((z_i)_{i \in I}) = g((z_i)_{i \in I}) \in g(NE(\Gamma, u^*)) = S(u^*)$.

Next, we prove the sufficiency. Let S be an SCS satisfying SPI and QP. Consider the following natural quantity mechanism. For each $i \in I$, define the strategy space by $M_i = Z_i$. A generic element of M_i is denoted by $m_i = z_i$. The outcome function is defined as follows:

Rule 1: If for all $i \in I$, $m_i = z_i \in \overset{\circ}{Z}_i$ such that $\sum_{i \in I} z_i - \underline{\Omega} \in \partial\ Y$ and $S^{-1}((z_i)_{i \in I}) \neq \varnothing$, then $g(m) = (z_i)_{i \in I}$.

Rule 2: If for all $i \in I$, $m_i = z_i \in \overset{\circ}{Z}_i$ such that $I^S((z_i)_{i \in I}) = I$, and either $\sum_{i \in I} z_i - \underline{\Omega} \notin \partial\ Y$ or $S^{-1}((z_i)_{i \in I}) = \varnothing$, then $g(m) = v((z_i)_{i \in I})$.

Rule 3: If for all $i \in I$, $m_i = z_i$ such that $2 \leq \# I^S((z_i)_{i \in I}) \leq n\text{-}1$, then for $j = \min\{h \in I - I^S((z_i)_{i \in I})\}$, $g_j(m) = \alpha y_j \in \partial\ \overline{Y}(x_j + \sum_{i \neq j} \omega_i^0) + \underline{\Omega}$ for some $\alpha \geq 1$, and for all $i \neq j$, $g_i(m) = (\omega_i^0,\ 0)$, where $\partial\ \overline{Y}(x) \equiv \{y \in R^m \mid (x,\ y) \in \partial\ Y\}$.

Rule 4: If for all $i \in I$, $m_i = z_i$ and $I^S((z_i)_{i \in I}) = \{i\}$, then

4-1) if for all $z_{-i}^{\circ} \in \overset{\circ}{Z}_i$, $S^{-1}((z_i,\ z_{-i}^{\circ})) = \varnothing$, then

4-1-1) if $z_i \in \bigcap_{p(\tilde{z}^i,\ Y) \in p^i(S,\ (z_i)_{i \in I})} \hat{H}(p(\tilde{z}^i,\ Y),\ \tilde{z}^i + \underline{\Omega} - \sum_{j \neq i} z_j) \cap Z_i$,

$g_i(m) = (x_i + \gamma,\ y_i) \in \bigcap_{p(\tilde{z}^i,\ Y) \in p^i(S,\ (z_i)_{i \in I})} \partial\ [\hat{H}(p(\tilde{z}^i,\ Y),\ \tilde{z}^i + \underline{\Omega} - \sum_{j \neq i} z_j) \cap ([\omega_i^0,\ 0] \times \{\overline{Y}(x_{\min}^i) + \underline{\Omega}\}$

(for some $\gamma \in R$), and for all $j \neq i$, $g_j(m) = (\frac{\theta_j^S}{\theta_l^S}\omega_k^0,\ \frac{\theta_j^S}{\sum_{h \neq i}\theta_h^S}(\alpha - 1)y_i)$ where

$x_{\min}^i = \min\{x^i \in R_- \mid (x^i,\ y^i) = \tilde{z}_i,\ p(\tilde{z}^i,\ Y) \in p^i(S,(z_i)_{i \in I})\}$, $\omega_k^0 = \max\{\omega_{-i}^0\}$ and $\theta_l^S = \max\{\theta_{-i}^S\}$, and for some $\alpha \geq 1$, $\alpha y_i \in \partial\ \overline{Y}(x_i + \sum_{h \neq i} \frac{\theta_h^S}{\theta_l^S}\omega_k^0) + \underline{\Omega}$, and θ_h^S is S-specific surplus sharing rate of member h,

4-1-2) otherwise, $g(m) = (\tilde{z}_*^i - \sum_{j \neq i} z_j,\ z_{-i})$ where

$\tilde{z}_*^i = \arg\max\{x^i \in R_- \mid \tilde{z}^i = (x^i,\ y^i) \in \partial\ Y\ \ s.t.\ p(\tilde{z}^i,\ Y) \in p^i(S,(z_i)_{i \in I})\}$,

4-2) if there exists $z_{-i}^{\circ} \in \overset{\circ}{Z}_i$, $S^{-1}((z_i,\ z_{-i}^{\circ})) \neq \varnothing$, then for $j = \min\{h \in I - \{i\}\}$, $g_j(m) = \alpha y_j \in \partial\ \overline{Y}(x_j + \sum_{i \neq j} \omega_i^0) + \underline{\Omega}$ for some $\alpha \geq 1$, and for all $k \neq j$, $g_k(m) = (\omega_k^0,\ 0)$.

Rule 5: For any other case, the following modulo game is played and some member i^* will win the game: Let $\sum_{i\in I}(x_i/\omega_i) = k$. Since $(x_i/\omega_i)\in[0,1]$, clearly $0 \le k \le n$. Let $r + t = k$ where r is the largest integer less than or equal to k. Then $t \in [0, 1)$ and there is a unique $i^*\in I$ such that $t \in [(i^*-1)/n,\ i^*/n)$. Then i^* is able to receive $g_i(m) = (\partial\ \underline{Y}(y^* - \underline{\Omega}) - \sum_{h\neq i}\omega_h^0,\ y^*)$, and for all $j \neq i^*$, $g_j(m) = (\omega_j^0,\ 0)$9), where $y^* = \max\{\hat{y}_{i^*},\ y_{i^*}\}$ such that $\hat{y}_{i^*} = \beta y_{i^*} \in \partial\ \overline{Y}(\sum_{h\neq i}\omega_h^0) + \underline{\Omega}$ for some $\beta \ge 0$, and $\partial\ \underline{Y}(y) \equiv \{x \in R_-|\ (x,\ y) \in \partial\ Y\}$.

First, we prove that for all $u \in U$, $S(u) \subseteq g(SNE(\Gamma,\ u))$. Pick any $z = (z_i)_{i\in I} \in S(u)$ for any given $u \in U$. For each $i \in I$, let $m_i = z_i$. Then, by Rule 1, $g(m) = (z_i)_{i\in I}$.

Case 1: No individual can be made better off by deviation.

Consider a deviation of member i from $m_i = z_i$ to $m_i^* = z_i^*$. Then i can induce Rule 2, 3 or 4. In all cases, $g_i(m_i^*,\ m_{-i}) \in L(z_i,\ u_i)$.

Case 2: No coalition can be made better off by deviation.

Consider a coalition $T \subseteq I$, $\#T \ge 2$, deviating from $m_T = (z_i)_{i\in T}$ to $m_T^* = (z_i^*)_{i\in T}$. T can induce Rule 1, 2, 3, 4 or 5. If T induces Rule 3, 4-2 or 5, there exists at least one member $l \in T$, $g_l(m_T^*,\ m_{-T}) \in L(z_l,\ u_l)$. When $i \in T$ such that $I^S((z_i)_{i\in I}) = \{i\}$, T can induce Rule 4-1. If T induces Rule 4-1-1 and $g_i(m_T^*,\ m_{-T}) \notin L(z_i,\ u_i)$, then for any $j \in T - \{i\}$, $g_j(m_T^*,\ m_{-T}) \in \theta_j^S \cdot (\mathring{Y} + \underline{\Omega}) \cap Z_j$ where $\mathring{Y} = \partial\ Y - Y$, while $z_j \in \partial\ \hat{H}((p(\theta_j^S \cdot (\sum_{k\in I} z_k - \underline{\Omega}),\ \theta_j^S \cdot Y),\ \theta_j^S(\sum_{k\in I} z_k)) \cap Z_j$. This implies $g_j(m_T^*,\ m_{-T}) \in L(z_j,\ u_j)$. If T induces Rule 4-1-2, then $g(m_T^*,\ m_{-T}) = (\tilde{z}_*^i - \sum_{h\in T-\{i\}} z_h^* - \sum_{k\in I-T} z_k,\ z_{T-\{i\}}^*,\ z_{-T})$. Since $z = (z_i)_{i\in I} \in S(u)$ is Pareto efficient allocation and $g_k(m_T^*,\ m_{-T}) = z_k$ for $k \in I - T$, there exists $l \in T$ such that $g_l(m_T^*,\ m_{-T}) \in L(z_l,\ u_l)$. If T induces Rule 1, then there exists $l \in T$ such that $g_l(m_T^*,\ m_{-T}) \in L(z_l,\ u_l)$ because $z = (z_i)_{i\in I} \in S(u)$ is Pareto efficient allocation and $g_k(m_T^*,\ m_{-T}) = z_k$ for $k \in I - T$. If T induces Rule 2, then there exists $l \in T$ such that

$g_t(m_T^*, m_{-T}) \in L(z_t, u_t)$ by QP. Consider $T = I$. It is clear because $z = (z_i)_{i\in I} \in S(u)$ is Pareto efficient allocation. Thus, $z \in g(SNE(\Gamma, u))$.

Second, we prove that for all $u \in U$, $g(NE(\Gamma, u)) \subseteq S(u)$. Let $m \in NE(\Gamma, u)$ be given. It is easy to see that m cannot correspond to Rule 3, 4 nor 5. Suppose that m corresponds to Rule 1. Let for each $i \in I$, $m_i = z_i$. Then $g(m) = z = (z_i)_{i\in I} \in S(u^*)$ for some $u^* \in U$. Note that each member i can deviate to Rule 4-1 by announcing some $m_i^* = (\omega_i^0, y_i^*)$ and attain $z_i^* \in \partial\ \hat{H}(p(\sum_{i\in I} z_i, Y), z_i) \cap Z_i$. Hence, by quasi-concavity of $u \in U$, $\hat{H}(p(\sum_{i\in I} z_i, Y), z_i) \cap Z_i \subseteq L(z_i, u_i)$ for all $i \in I$, since if for some $j \in I$, $\hat{H}(p(\sum_{i\in I} z_i, Y), z_j) \cap Z_j \subseteq L(z_j, u_j)$ is not tree, $m \notin NE(\Gamma, u)$. Thus, $p(\sum_{i\in I} z_i, Y) \in p^P(Y, u, z)$ so that $g(m) = z = (z_i)_{i\in I} \in S(u)$ by SPI.

Next, suppose that m corresponds to Rule 2. Let for each $i \in I$, $m_i = z_i$. Then $g(m) = v((z_i)_{i\in I})$. By Rule 4-1, for each $i \in I$,

$$\bigcap_{p(\tilde{z}^i, Y) \in p^i(S, (z_i)_{i\in I})} \hat{H}(p(\tilde{z}^i, Y), \tilde{z}^i + \Omega - \sum_{j\neq i} z_j) \cap Z_i \subseteq g_i(M_i, m_{-i}) \subseteq L(v_i((z_i)_{i\in I}), u_i).$$

Thus, by QP, $g(m) = v((z_i)_{i\in I}) \in S(u)$. Q.E.D.

Proof of Theorem 2: First, show the necessity of SPI for double implementation by a natural price-quantity mechanism. Suppose that S is doubly implementable by a natural price-quantity mechanism $\Gamma = (M, g)$. Take a profile $u \in U$ and an allocation $z = (z_i)_{i\in I} \in S(u)$. Since $S \subseteq P$, $p(Y, u, z)$ is non-empty. Then, there exists $p \in p(Y, u, z)$ such that if $m_i = (p, z_i)$ for all $i \in I$, then $g(m) = z \in g(NE(\Gamma, u)) = g(SNE(\Gamma, u))$. Then, by Definition 5(v), $g_i(M_i, m_{-i}) \subseteq \hat{H}(p, z_i) \cap Z_i$ for all $i \in I$. Consider u^* such that $p \in p^P(Y, u^*, z)$. Then, $\hat{H}(p, z_i) \cap Z_i \subseteq L(z_i, u_i^*)$ for all $i \in I$. This implies that $g(m) = z \in g(NE(\Gamma, u^*))$, since $g_i(M_i, m_{-i}) \subseteq L(z_i, u_i^*)$ for all $i \in I$. By double implementability, $z = (z_i)_{i\in I} \in S(u^*)$.

Second, show the necessity of PQP for double implementation by a natural price-quantity mechanism. Suppose that S is doubly implementable by a natural price-quantity mechanism $\Gamma = (M, g)$. Let $(p, (z_i)_{i\in I}) \in \Delta \times Z$ such that $I^S(p, (z_i)_{i\in I}) = I$ be given. Then for all $i \in I$, there exists $\tilde{z}^i \in \partial\ Y$ such that $p \in p(\tilde{z}^i, Y)$ and

$S^{-1}((\bar{z}^i + \underline{\Omega} - \sum_{j \neq i} z_j,\ z_{-i})) \neq \emptyset$, and for all $u^\circ \in S^{-1}((\bar{z}^i + \underline{\Omega} - \sum_{j \neq i} z_j,\ z_{-i}))$,

$g(m^i) = m^i \in g(NE(\Gamma,\ u^\circ)) = g(SNE(\Gamma,\ u^\circ))$ is established by forthrightness where $m_i^i = (p,\ \bar{z}^i + \underline{\Omega} - \sum_{j \neq i} z_j)$ and $m_j^i = (p,\ z_j)$ for all $j \neq i$. Then, by Definition 5(v), $g_i(M_i,\ m_{-i}^i) \subseteq \bigcap_{p \in p(\bar{z}^i,\ Y)} H(p,\ \bar{z}^i + \underline{\Omega} - \sum_{j \neq i} z_j) \cap Z_i$ for all $i \in I$. Let

$v(p, (z_i)_{i \in I}) = g((p,\ z_i)_{i \in I})$. Then for all $i \in I$, $v_i(p, (z_i)_{i \in I}) \in \bigcap_{p \in p(\bar{z}^i,\ Y)} H(p,\ \bar{z}^i + \underline{\Omega} - \sum_{j \neq i} z_j) \cap Z_i$. Since Γ is a balanced mechanism,

PQP(i) is also satisfied by $v(p, (z_i)_{i \in I})$. Moreover, suppose that there exists $u^* \in U$ such that for all $i \in I$, $\bigcap_{p \in p(\bar{z}^i,\ Y)} H(p,\ \bar{z}^i + \underline{\Omega} - \sum_{j \neq i} z_j) \cap Z_i \subseteq L(v_i(p, (z_i)_{i \in I}),\ u_i^*)$.

Then, $v(p, (z_i)_{i \in I}) = g((p,\ z_i)_{i \in I}) \in g(NE(\Gamma,\ u^*)) = S(u^*)$.

Next, we prove the sufficiency. Let S be an SCS satisfying SPI and PQP. For all u, $z = (z_i)_{i \in I} \in S(u)$, let $p^S(Y,\ u, (z_i)_{i \in I})$ be the set of price vectors satisfying the following property : $p \in p^S(Y,\ u, (z_i)_{i \in I})$ implies $p \in p(Y,\ u, (z_i)_{i \in I})$ and for all $u^\circ \in U$ such that $p \in p^P(Y,\ u^\circ, (z_i)_{i \in I})$, $(z_i)_{i \in I} \in S(u^\circ)$. Since S is a SCS satisfying SPI, for all u, $z = (z_i)_{i \in I} \in S(u)$, $p^S(Y,\ u, (z_i)_{i \in I}) \neq \emptyset$. Let for $(p, (z_i)_{i \in I}) \in \Delta \times Z$, $S^{-1}((z_i)_{i \in I},\ p) \equiv \{u^\circ \in U \mid p \in p^S(Y,\ u^\circ, (z_i)_{i \in I})\ \&\ (z_i)_{i \in I} \in S(u^\circ)\}$. Consider the following natural price-quantity mechanism. For each $i \in I$, define the strategy space by $M_i = \Delta \times Z_i$. A generic element of M_i is denoted by $m_i = (p^i,\ z_i)$. The outcome function is defined as follows:

Rule 1: If for all $i \in I$, $m_i = (p,\ z_i)$ such that $z_i \in \overset{\circ}{Z}_i$, $\sum_{i \in I} z_i - \underline{\Omega} \in \partial\ Y$ and

$S^{-1}((z_i)_{i \in I},\ p) \neq \emptyset$, then $g(m) = (z_i)_{i \in I}$.

Rule 2: If for all $i \in I$, $m_i = (p,\ z_i)$ such that $I^S(p,\ (z_i)_{i \in I}) = I$, and either $\sum_{i \in I} z_i - \underline{\Omega} \notin \partial\ Y$ or $S^{-1}((z_i)_{i \in I},\ p) = \emptyset$, then $g(m) = v(p, (z_i)_{i \in I})$.

Rule 3: If for all $i \in I$, $m_i = (p,\ z_i)$ such that $1 \leq \# I^S(p,\ (z_i)_{i \in I}) \leq n\text{-}1$, then for $j = \min\{h \in I - I^S(p,\ (z_i)_{i \in I})\}$, $g_j(m) = \alpha y_j \in \partial\ \overline{Y}(x_j + \sum_{i \neq j} \omega_i^0) + \underline{\Omega}$ for some $\alpha \geq 1$,

and for all $i \neq j$, $g_i(m) = (\omega_i^0,\ 0)$, where $\partial\ \overline{Y}(x) \equiv \{y \in R^m \mid (x,\ y) \in \partial\ Y\}$.

Rule 4: If for some $i \in I$ and some $p \in \Delta$, $p^j = p$ for all $j \neq i$, $p^i \neq p$, and $i \in I^S(p, (z_i)_{i\in I})$, then

4-1) if for all $z_{-i}^{\circ} \in \overset{\circ}{Z}_i$, $S^{-1}((z_i, z_{-i}^{\circ}), p^i) = \varnothing$, then

4-1-1) if $z_i \in \bigcap_{p\in p(\tilde{z}^i, Y)} H(p, \tilde{z}^i + \underline{\Omega} - \sum_{j\neq i} z_j) \cap Z_i$ where $\tilde{z}^i \in \partial\ Y$ such that $S^{-1}((\tilde{z}^i + \underline{\Omega} - \sum_{j\neq i} z_j,\ z_{-i}),\ p) \neq \varnothing$,
$g_i(m) = (x_i + \gamma,\ y_i) \in \bigcap_{p\in p(\tilde{z}^i, Y)} \partial\ [\hat{H}(p,\ \tilde{z}^i + \underline{\Omega} - \sum_{j\neq i} z_j) \cap ([\omega_i^0,\ 0] \times \{\overline{Y}(x_{\min}^i) + \underline{\Omega}\})]$ (for some $\gamma \in R$), and for all $j \neq i$, $g_j(m) = (\frac{\theta_j^S}{\theta_l^S}\omega_k^0,\ \frac{\theta_j^S}{\sum_{h\neq i}\theta_h^S}(\alpha - 1)y_i)$ where

$x_{\min}^i = \min\{x^i \in R_- | (x^i, y^i) = \tilde{z}_i,\ p(\tilde{z}^i, Y) \in p^i(S, (z_i)_{i\in I})\}$, $\omega_k^0 = \max\{\omega_{-i}^0\}$ and $\theta_l^S = \max\{\theta_{-i}^S\}$, and for some $\alpha \geq 1$, $\alpha y_i \in \partial\ \overline{Y}(x_i + \sum_{h\neq i}\frac{\theta_h^S}{\theta_l^S}\omega_k^0) + \underline{\Omega}$, and θ_h^S is S-specific surplus sharing rate of member h,

4-1-2) otherwise, $g(m) = (\tilde{z}_*^i - \sum_{j\neq i} z_j,\ z_{-i})$ where
$\tilde{z}_*^i = \arg\max\ \{x^i \in R_- |\ \tilde{z}^i = (x^i,\ y^i) \in \partial\ Y\ \ s.t.\ \ p \in p(\tilde{z}^i, Y)\ \&$
$S^{-1}((\tilde{z}^i + \underline{\Omega} - \sum_{j\neq i} z_j,\ z_{-i})) \neq \varnothing\}$,

4-2) if there exists $z_{-i}^{\circ} \in \overset{\circ}{Z}_i$, $S^{-1}((z_i, z_{-i}^{\circ}), p^i) \neq \varnothing$, then for $j = \min\{h \in I - \{i\}\}$, $g_j(m) = \alpha y_j \in \partial\ \overline{Y}(x_j + \sum_{i\neq j}\omega_i^0) + \underline{\Omega}$ for some $\alpha \geq 1$, and for all $k \neq j$, $g_k(m) = (\omega_k^0,\ 0)$.

Rule 5: For any other case, the following modulo game is played and some member i^* will win the game: Let $\sum_{i\in I}(x_i/\omega_i) = k$. Since $(x_i/\omega_i) \in [0,1]$, clearly $0 \leq k \leq n$. Let $r + t = k$ where r is the largest integer less than or equal to k. Then $t \in [0, 1)$ and there is a unique $i^* \in I$ such that $t \in [(i^*-1)/n,\ i^*/n)$. Then i^* is able to receive $g_i(m) = (\partial\ \underline{Y}(y^* - \underline{\Omega}) - \sum_{h\neq i}\omega_h^0,\ y^*)$, and for all $j \neq i^*$, $g_j(m) = (\omega_j^0,\ 0)$[9], where $y^* = \max\{\hat{y}_{i^*},\ y_{i^*}\}$ such that $\hat{y}_{i^*} = \beta y_{i^*} \in \partial\ \overline{Y}(\sum_{h\neq i}\omega_h^0) + \underline{\Omega}$ for some $\beta \geq 0$, and

$\partial\ \underline{Y}(y) \equiv \{x \in R_- |\ (x,\ y) \in \partial\ Y\}$.

First, we prove that for all $u \in U$, $S(u) \subseteq g(SNE(\Gamma, u))$. Pick any $z = (z_i)_{i \in I} \in S(u)$ for any given $u \in U$. For each $i \in I$, let $m_i = (p, z_i)$ where $p \in p^S(Y, u, z)$. Then, by Rule 1, $g(m) = (z_i)_{i \in I}$.

Case 1: No individual can be made better off by deviation.

Consider a deviation of member i from m_i to m_i^*. Then i can induce Rule 2, 3 or 4. In all cases, $g_i(m_i^*, m_{-i}) \in L(z_i, u_i)$.

Case 2: No coalition can be made better off by deviation.

Consider a coalition $T \subseteq I$, $\#T \geq 2$, deviating from $m_T = (p, z_i)_{i \in T}$ to m_T^*. T can induce Rule 1, 2, 3, 4 or 5. If T induces Rule 3, 4-2 or 5, there exists at least one member $l \in T$, $g_l(m_T^*, m_{-T}) \in L(z_l, u_l)$. When $i \in T$ such that $p^i \neq p$ and $i \in I^S(p, (z_i)_{i \in I})$, T can induce Rule 4-1. If T induces Rule 4-1-1 and $g_i(m_T^*, m_{-T}) \notin L(z_i, u_i)$, then for any $j \in T - \{i\}$, $g_j(m_T^*, m_{-T}) \in \theta_j^S \cdot (\mathring{Y} + \Omega) \cap Z_j$ where $\mathring{Y} = \partial\ Y - Y$, while $z_j \in \partial\ \hat{H}(p, \theta_j^S(\sum_{k \in I} z_k)) \cap Z_j$. This implies $g_j(m_T^*, m_{-T}) \in L(z_j, u_j)$, since $p \in p(\theta_j^S \cdot (\sum_{k \in I} z_k - \Omega), \theta_j^S \cdot Y)$. If T induces Rule 4-1-2, then $g(m_T^*, m_{-T}) = (\bar{z}_*^i - \sum_{h \in T - \{i\}} z_h^* - \sum_{k \in I - T} z_k, z_{T-\{i\}}^*, z_{-T})$. Since $z = (z_i)_{i \in I} \in S(u)$ is Pareto efficient allocation and $g_k(m_T^*, m_{-T}) = z_k$ for $k \in I - T$, there exists $l \in T$ such that $g_l(m_T^*, m_{-T}) \in L(z_l, u_l)$. If T induces Rule 1, then there exists $l \in T$ such that $g_l(m_T^*, m_{-T}) \in L(z_l, u_l)$ because $z = (z_i)_{i \in I} \in S(u)$ is Pareto efficient allocation and $g_k(m_T^*, m_{-T}) = z_k$ for $k \in I - T$. If T induces Rule 2, then there exists $l \in T$ such that $g_l(m_T^*, m_{-T}) \in L(z_l, u_l)$ by PQP. Consider $T = I$. It is clear because $z = (z_i)_{i \in I} \in S(u)$ is Pareto efficient allocation. Thus, $z \in g(SNE(\Gamma, u))$.

Second, we prove that for all $u \in U$, $g(NE(\Gamma, u)) \subseteq S(u)$. Let $m \in NE(\Gamma, u)$ be given. It is easy to see that m cannot correspond to Rule 3, 4 nor 5. Suppose that m corresponds to Rule 1. Let for each $i \in I$, $m_i = (p, z_i)$ such that $z_i \in \mathring{Z}_i$, $\sum_{i \in I} z_i - \Omega \in \partial\ Y$ and $S^{-1}((z_i)_{i \in I}, p) \neq \emptyset$. Then $g(m) = z = (z_i)_{i \in I} \in S(u^*)$ for some $u^* \in U$. Note that each member i can deviate to Rule 4-1 by announcing some

$m_i^* = (p^*, (\omega_i^0, y_i^*))$ and attain $z_i^* \in \partial\ \hat{H}(p, z_i) \cap Z_i$. Hence, by quasi-concavity of $u \in U$, $\hat{H}(p, z_i) \cap Z_i \subseteq L(z_i, u_i)$ for all $i \in I$, since if for some $j \in I$, $\hat{H}(p, z_j) \cap Z_j \subseteq L(z_j, u_j)$ is not tree, $m \notin NE(\Gamma, u)$. Thus, $p \in p^P(Y, u, z)$ so that $g(m) = z = (z_i)_{i\in I} \in S(u)$ by SPI.

Next, suppose that m corresponds to Rule 2. Let for each $i \in I$, $m_i = (p, z_i)$ such that $I^S(p, (z_i)_{i\in I}) = I$, and either $\sum_{i\in I} z_i - \underline{\Omega} \notin \partial\ Y$ or $S^{-1}((z_i)_{i\in I}, p) = \emptyset$.

Then $g(m) = v(p, (z_i)_{i\in I})$. By Rule 4-1, for each $i \in I$, $\bigcap_{p\in p(\tilde{z}^i, Y)} H(p, \tilde{z}^i + \underline{\Omega} - \sum_{j\neq i} z_j) \cap Z_i \subseteq g_i(M_i, m_{-i}) \subseteq L(v_i(p, (z_i)_{i\in I}), u_i)$. Thus, by PQP, $g(m) = v((z_i)_{i\in I}) \in S(u)$. Q.E.D.

Proof of Lemma 5: Let $(z_i)_{i\in I} \in Z$ such that $I^{EB}((z_i)_{i\in I}) = I$ be given. Then for all $i \in I$, there exists $\tilde{z}^i \in \partial\ Y$ such that $\{p\} = p(\tilde{z}^i, Y)$ and $EB^{-1}((\tilde{z}^i + \underline{\Omega} - \sum_{j\neq i} z_j, z_{-i})) \neq \emptyset$. Let $\omega_k^0 = \max\{\omega_i^0 \mid i \in I\}$.

Let take a vector $\tilde{y} \in \{\partial\ Y(n(\omega_k^0 - \varepsilon)) + \underline{\Omega}\} \cap R^m_{++}$ for some $\varepsilon > 0$. Then, each $i \in I$, $(\omega_k^0 - \varepsilon, \frac{1}{n}\tilde{y}) \in \mathring{Z}_i$. It is clear that $(\omega_k^0 - \varepsilon, \frac{1}{n}\tilde{y})_{i\in I} \in \mathring{A}$ and $\sum_{i\in I}(\omega_k^0 - \varepsilon, \frac{1}{n}\tilde{y}) \in \partial\ Y + \underline{\Omega}$.

Notice that for all $i \in I$, for all $\tilde{z}^i \in \partial\ Y$ such that $\{p\} = p(\tilde{z}^i, Y)$ and $EB^{-1}((\tilde{z}^i + \underline{\Omega} - \sum_{j\neq i} z_j, z_{-i})) \neq \emptyset$, $(\frac{1}{n} \cdot \{\partial\ Y + \underline{\Omega}\}) \cap Z_i \subseteq H(p, \tilde{z}^i + \underline{\Omega} - \sum_{j\neq i} z_j) \cap Z_i$. Since $(\omega_k^0 - \varepsilon, \frac{1}{n}\tilde{y}) \in (\frac{1}{n} \cdot \{\partial\ Y + \underline{\Omega}\}) \cap Z_i$, $(\omega_k^0 - \varepsilon, \frac{1}{n}\tilde{y}) \in H(p, \tilde{z}^i + \underline{\Omega} - \sum_{j\neq i} z_j) \cap Z_i$ for all $p(\tilde{z}_i, Y) \in p^i(EB, (z_i)_{i\in I})$. Now let define $v((z_i)_{i\in I}) = (\omega_k^0 - \varepsilon, \frac{1}{n}\tilde{y})_{i\in I}$. Then $v((z_i)_{i\in I})$ satisfies QP (i) and (ii). Suppose that there exists $u^* \in U$ such that for all $i \in I$, $\bigcap_{p(\tilde{z}^i, Y)\in p^i(EB, (z_i)_{i\in I})} H(p(\tilde{z}^i, Y), \tilde{z}^i + \underline{\Omega} - \sum_{j\neq i} z_j) \cap Z_i \subseteq L(v_i((z_i)_{i\in I}), u_i^*)$. Then, for $\tilde{z} = (n(\omega_k^0 - \varepsilon), \tilde{y} - \underline{\Omega})$, $p(\tilde{z}, Y) \in p^i(EB, v((z_i)_{i\in I}))$ for all $i \in I$. Thus, for all $i \in I$, $H(p(\tilde{z}, Y), v_i((z_i)_{i\in I})) \cap Z_i \subseteq L(v_i((z_i)_{i\in I}), u_i^*)$. This implies $v((z_i)_{i\in I}) \in EB(u^*)$. Q.E.D.

Proof of Lemma 6: Let $(p, (z_i)_{i\in I}) \in \Delta \times Z$ such that $I^{EB}(p, (z_i)_{i\in I}) = I$ be given. Then for all $i \in I$, there exists $\tilde{z}^i \in \partial Y$ such that $p \in p(\tilde{z}^i, Y)$ and $EB^{-1}((\tilde{z}^i + \underline{\Omega} - \sum_{j\neq i} z_j, z_{-i})) \neq \varnothing$. Construct a feasible allocation $v(p, (z_i)_{i\in I}) = (\omega_k^0 - \varepsilon, \frac{1}{n}\tilde{y})_{i\in I}$ as the same manner as in the proof of Lemma 5. Then it is clear that $v(p, (z_i)_{i\in I})$ satisfies PQP (i) and (ii). Suppose that there exists $u^* \in U$ such that for all $i \in I$, $\bigcap_{p\in p(\tilde{z}^i, Y)} H(p, \tilde{z}^i + \underline{\Omega} - \sum_{j\neq i} z_j) \cap Z_i \subseteq L(v_i(p, (z_i)_{i\in I}), u_i^*)$ where $\tilde{z}^i \in \partial Y$ such that $EB^{-1}((\tilde{z}^i + \underline{\Omega} - \sum_{j\neq i} z_j, z_{-i})) \neq \varnothing$. Then, for $\tilde{z} = (n(\omega_k^0 - \varepsilon), \tilde{y} - \underline{\Omega})$, $p \in p(\tilde{z}, Y)$. Thus, for all $i \in I$, $H(p, v_i((z_i)_{i\in I}) \cap Z_i \subseteq L(v_i((z_i)_{i\in I}), u_i^*)$. This implies $v(p, (z_i)_{i\in I}) \in EB(u^*)$.

Q.E.D.

Proof of Lemma 8: Let $(p, (z_i)_{i\in I}) \in \Delta \times Z$ such that $I^{PR}(p, (z_i)_{i\in I}) = I$ be given. When $PR^{-1}((z_i)_{i\in I}) \neq \varnothing$, it is trivial that PR satisfies PQP. Consider the case, $PR^{-1}((z_i)_{i\in I}) = \varnothing$. When the production set is convex-cone, it does not occurred such a case. Hence, we only consider the case that the production set is not convex-cone.

By $I^{PR}(p, (z_i)_{i\in I}) = I$, $\frac{py_i}{x_i} = \frac{py_j}{x_j}$ for all i, j ($i \neq j$). Then, for each i, there exists $\tilde{z}^i = (\tilde{x}^i, \tilde{y}^i) \in \partial Y$ such that $p \in p(\tilde{z}^i, Y)$ and $\frac{p(\tilde{y}^i + \underline{\Omega})}{x_i} = \frac{py_h}{x_h}$ for all $h \neq i$. Suppose that there exist i, j ($i \neq j$) such that $\tilde{z}^i \neq \tilde{z}^j$. Since $\#I \geq 3$, $\frac{p(\tilde{y}^i + \underline{\Omega})}{\tilde{x}^i} = \frac{p(\tilde{y}^j + \underline{\Omega})}{\tilde{x}^j}$. Then $\tilde{x}^i = \tilde{x}^j = \tilde{x}$, because the production set is not convex-cone. Hence, $\tilde{y}^i = \tilde{y}^j = \tilde{y}$. Thus, there exists a unique $\tilde{z} = (\tilde{x}, \tilde{y}) \in \partial Y$ such that $p \in p(\tilde{z}, Y)$ and $PR^{-1}(\tilde{z} + \underline{\Omega} - \sum_{j\neq i} z_j, z_{-i}) \neq \varnothing$ for all $i \in I$.

Since $\sum_{j\neq i} y_j \in R^m_{++}$, $\tilde{y} + \underline{\Omega} \in R^m_{++}$ for all $i \in I$. Then, if $x_i^\bullet = \tilde{x} - \sum_{j\neq i} x_j$ and $y_i^\bullet = \frac{x_i^\bullet}{\tilde{x}}(\tilde{y} + \underline{\Omega})$, $(x_i^\bullet, y_i^\bullet) \in \mathring{Z}_i$ for all $i \in I$. By definition of PR, for all $i \in I$, $p \cdot (x_i^\bullet, y_i^\bullet) = p \cdot (\tilde{z} + \underline{\Omega} - \sum_{j\neq i} z_j)$.

Suppose $\tilde{x} \leq \sum_{h \in I} x_h$. Then, $x_i^{\bullet} \leq x_i$ for all $i \in I$. Let $v_1((z_i)_{i \in I}) = (x_1^{\bullet},\ \tilde{y} + \underline{\Omega} - \sum_{j \neq 1} y_j)$ and $v_j((z_i)_{i \in I}) = (x_j,\ y_j)$ for all $j \neq 1$. Then $v((z_i)_{i \in I})$ satisfies PQP (i), (ii) and (iii).

Suppose $\tilde{x} > \sum_{h \in I} x_h$. Then, $x_i^{\bullet} > x_i$ for all $i \in I$. Let $y_i^{\bullet\bullet} = y_i^{\bullet} - \frac{\omega_i^0}{\Omega^0}\underline{\Omega} = \frac{x_i^{\bullet}}{\tilde{x}}\tilde{y} + \left(\frac{x_i^{\bullet}}{\tilde{x}} - \frac{\omega_i^0}{\Omega^0}\right)\underline{\Omega}$. Since $\tilde{x} < \sum_{h \in I} x_h^{\bullet}$, $\sum_{h \in I} \frac{x_h^{\bullet}}{\tilde{x}}\tilde{y} < \tilde{y}$. Then either $\tilde{y} \leq \sum_{h \in I} y_h^{\bullet\bullet}$ or $\tilde{y} > \sum_{h \in I} y_h^{\bullet\bullet}$. $\tilde{y} \leq \sum_{h \in I} y_h^{\bullet\bullet}$ implies $\sum_{h \in I} y_h^{\bullet} \geq \tilde{y} + \underline{\Omega}$. This also implies $\sum_{h \in I} \frac{x_h^{\bullet}}{\tilde{x}}(\tilde{y} + \underline{\Omega}) \geq \tilde{y} + \underline{\Omega}$, so that $\sum_{h \in I} x_h^{\bullet} \leq \tilde{x}$. It is a contradiction. Thus, $\tilde{y} > \sum_{h \in I} y_h^{\bullet\bullet}$. Then, there exists at least one member $k \in I$ such that $(x_k^{\bullet},\ y_k^{\bullet\bullet}) \in \frac{\omega_k^0}{\Omega^0}(Y - \partial\ Y)$. Select a member $l \in I$ such that $\min \frac{y_k^{\bullet\bullet}}{y_k^{\circ}} = \frac{y_l^{\bullet\bullet}}{y_l^{\circ}}$ where $(x_k^{\circ},\ y_k^{\circ}) = \frac{\omega_k^0}{\Omega^0}(\tilde{x},\ \tilde{y})$ and $(x_k^{\bullet},\ y_k^{\bullet\bullet}) \in \frac{\omega_k^0}{\Omega^0}(Y - \partial\ Y)$. Then there exists $\bar{z}_l = (\bar{x}_l,\ \bar{y}_l) \in \frac{\omega_k^0}{\Omega^0}\partial\ Y \cap \partial\ H(p, (x_l^{\bullet},\ y_l^{\bullet\bullet}))$ such that $\bar{y}_l = \alpha\tilde{y}$ ($\exists \alpha \in (0, 1)$). Define a function $h^l(x) = y$ such that $h^l(x)/x = \bar{y}_l/\bar{x}_l$ for any $x \in R_-$. There exists $\bar{z}_i = (\bar{x}_i,\ \bar{y}_i) \in \frac{\omega_i^0}{\Omega^0}\partial\ Y$ such that $h^l(\bar{x}_i) = \bar{y}_i$ for all $i \in I$. Notice that for all $i \in I$, $\bar{y}_i/\bar{y}_l = y_i^{\circ}/y_l^{\circ} = \omega_i/\omega_l$. Since $p\bar{z}_i = pz_i^{\circ}\frac{y_l^{\bullet\bullet}}{y_l^{\circ}}\frac{\bar{y}_i}{\bar{y}_l}$, $pz_i^{\bullet\bullet} = pz_i^{\circ}\frac{y_i^{\circ}}{y_i^{\circ}}\frac{y_i^{\bullet\bullet}}{y_i^{\circ}}$ and $\frac{y_i^{\bullet\bullet}}{y_i^{\circ}} \geq \frac{y_l^{\bullet\bullet}}{y_l^{\circ}}$ for all $i \in I$, it follows that $p\bar{z}_i \leq pz_i^{\bullet\bullet}$ for all $i \in I$. Thus, $\bar{z}_i \in H(p, (x_i^{\bullet},\ y_i^{\bullet\bullet}))$ for all $i \in I$. This implies $\bar{z}_i + \frac{\omega_i^0}{\Omega^0}\underline{\Omega} \in H(p, (x_i^{\bullet},\ y_i^{\bullet})) \cap \overset{\circ}{Z}_i$ for all $i \in I$. Let $v((z_i)_{i \in I}) = (\bar{z}_i)_{i \in I}$. Then $v((z_i)_{i \in I})$ satisfies PQP (i) and (ii). Since $v((z_i)_{i \in I})$ is a proportional allocation, PQP (iii) is also satisfied. Q.E.D.

REFERENCES

Abreu, D., Sen, A. (1990): "Subgame Perfect Implementation: A Necessary and Almost Sufficient Condition," *Journal of Economic Theory* **50**, 285-299.

Dasgupta, P., Hammond, P. and Maskin, E. (1979): "The Implementation of Social Choice Rules," *Review of Economic Studies* **46**, 185-216.

Dutta, B., Sen, A. (1991a): "Necessary and Sufficient Condition for Two-Person Nash Implementation," *Review of Economic Studies* **58**, 121-128.

Dutta, B., Sen, A. (1991b): "Implementation under strong equilibrium: complete characterization," *Journal of Mathematical Economics* **20**, 49-67.

Dutta, B., Sen, A. and Vohra, R. (1995): "Nash Implementation Through Elementary Mechanism in Economic Environments", *Economic Design*, **1**, 173-204.

Foley, D. (1967): "Resource Allocation and the Public Sector", *Yale Economic Essays*, **7**, 45-98.

Gevers, L. (1986): "Walrasian social choice: Some simple axiomatic approaches", in Heller *et al.*, eds., *Social Choice and Public Decision Making: Essays in Honor of K. J. Arrow*, **1**, 97-114. Cambridge Univ. Press, London / New York.

Hong, L. (1995): "Nash Implementation in Production Economies," *Economic Theory* **5**, 401-417.

Hurwicz,L. (1972): "On Informationally Decentralized Systems," in R. Radner and C. B. McGuire, eds., *Decesion and Organization: A Volume in Honor of J. Marschak*, 297-336. North-Holland Publishing Company, Amsterdam.

Hurwicz,L. (1979): "Outcome Functions Yielding Walrasian and Lindahl Allocations at Nash Equilibrium Points," *Review of Economic Studies* **46**, 217-225.

Hurwicz,L., Maskin, E. and Postlewaite, A. (1984): "Feasible Implementation of Social Choice Correspondences by Nash Equilibria," mimeo.

Jackson,M.O. (1992): "Implementation in Undominated Strategies: A Look at Bounded Mechanism," *Review of Economic Studies* **59**, 757-775.

Jackson, M. O., Palfrey, T., Srivastava, S. (1994): "Undominated Nash Implementation in Bounded Mechanisms," *Games and Economic Behavior* **6**, 474-501.

Maskin, E. (1977): "Nash Equilibrium and Welfare Optimality," (mimeo).

Maskin, E. (1979): "Implementation and Strong Nash-Equilibrium," in Lafont, J. (ed.), *Aggregation and Revelation of Preferences* **8**, 432-439. North-Holland Publishing Company, Amsterdam.

McKelvey, R.D., (1989): "Game Forms for Nash Implementation of General Social Choice Correspondences," *Social Choice and Welfare*, **6**, 139-156.

Moore, J. (1992): "Implementation, Contracts and Renegotiation in Environments with Complete Information," in Lafont, J. (ed.), *Advances in Economic Theory: Invited Papers for the Sixth World Congress of the Econometric Society*, Vol. I, 182-282. Cambridge University Press, Cambridge.

Moore, J., Repullo, R. (1988): "Subgame Perfect Implementation," *Econometrica* **56**, 1191-1220.

Moore, J., Repullo, R. (1990): "Nash Implementation: A Full Characterization," *Econometrica* **58**, 1083-1099.

Moulin,H. (1987): "A Core Selection for Pricing a Single Output Monopoly", *The Rand Journal of Economics*, **18**, 397-407.

Moulin,H. (1990a): "Joint Ownership of a Convex Technology: Comparison of Three Solutions", *Review of Economic Studies*, **57**, 439-452.

Moulin,H. (1990b): "Fair Division under Joint Ownership: Recent Results and Open Problems", *Social Choice and Welfare*, **7**, 149-170.

Moulin,H., and Roemer,J. (1989): "Public Ownership of the External World and Private Ownership of Self", *Journal of Political Economy*, **97**, 347-367.

Nagahisa, R. (1991): "A Local Independence Condition for Characterization of Walrasian Allocations Rule", *Journal of Economic Theory* **54**, 106-123.

Nagahisa, R. (1994): "A Necessary and Sufficient Condition for Walrasian Social Choice", *Journal of Economic Theory* **62**, 186-208.

Palfrey, T., Srivastava, S. (1991): "Nash Implementation Using Undominated Strategies," *Econometrica* **59**, 479-501.

Postlewaite, A., Wettstein, D. (1989): "Feasible and Continuous Implementation," *Review of Economic Studies* **56**, 603-611.

Repullo, R. (1987): "A Simple Proof of Maskin's Theorem on Nash Implementation," *Social Choice and Welfare* **4**, 39-42.

Roemer, J. (1986): "The Mismarriage of Bargaining Theory and Distributive Justice," *Ethics* **97**, 88-110.

Roemer, J. (1989): "A Public Ownership Resolution of the Tragedy of the Commons," *Social Philosophy and Policy* **6**, 74-92.

Roemer,J., and Silvestre,J. (1989), "Public Ownership: Three proposals for Resource Allocation", (Mimeo).

Roemer,J., and Silvestre,J. (1993), "The Proportional Solution for Economies with Both Private and Public Ownership", *Journal of Economic Theory*, **59**, 426-444.

Saijo, T. (1988): "Strategy Space Reduction in Maskin's Theorem: Sufficient Condition for Nash Implementation," *Econometrica* **56**, 693-700.

Saijo,T., Tatamitani,Y., and Yamato,T. (1995), "Toward Natural Implementation", forthcoming in *International Economic Review.*

Schmeidler, D. (1980): "Walrasian Analysis via Strategic Outcome Functions," *Econometrica* **40**, 1585-1594.

Sjöström, T. (1991): "Implementation by Demand Mechanisms," (mimeo).

Suh, S-C. (1993): "Doubly Implementing the Proportional Solution," (mimeo).

Tatamitani, Y.(1993): "Double Implementation in Nash and Undominated Nash Equilibria in Social Choice Environments," *Economic Theory* **3**, 109-117.

Tian, G. (1989): "Implementation of Lindahl Correspondence by a Single Valued,Feasible and Continuous Mechanism," *Review of Economic Studies* **56**, 613-621.

Tian, G. (1992): "Completely Feasible and Continuous Implementation of the Walrasian Correspondence without Continuous, Convex and Ordered Preferences," *Social Choice and Welfare* **9**, 117-130.

Walker, M. (1981): "A Simple Incentive Compatible Scheme for Attaining Lindahl Allocations," *Econometrica* **49**, 65-73.

Yamato, T. (1993): "Double Implementation in Nash and Undominated Nash Equilibria," *Journal of Economic Theory* **59**, 311-323.

Yoshihara, N. (1995a): "Natural and Double Implementation of Public Ownership Solutions in Differentiable Production Economies," mimeo.

Yoshihara, N. (1995b): "Full Characterizations of Public Ownership Solutions," mimeo.