Multi-Profile Intergenerational Social Choice*

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Abstract. Ferejohn and Page transplanted a stationarity axiom from Koopmans' theory of impatience into Arrow's social choice theory with an infinite horizon and showed that the Arrow axioms and stationarity lead to a dictatorship by the first generation. We prove that the negative implications of their stationarity axiom are more far-reaching: there is no Arrow social welfare function satisfying their stationarity axiom. We propose a more suitable stationarity axiom, and show that an Arrow social welfare function satisfies this modified version if and only if it is a lexicographic dictatorship where the generations are taken into consideration in chronological order. *Journal of Economic Literature* Classification No.: D71.

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1 Introduction

As is well-known, the validity of Arrow's celebrated general impossibility theorem (Arrow, 1951; 1963) hinges squarely on the finiteness of population. Fishburn (1970), Sen (1979) and Suzumura (2000) presented their respective method of proving Arrow's theorem and highlighted the crucial role played by the assumption of finiteness of population. Kirman and Sondermann (1972) and Hansson (1976) cast a new light on the structure of an Arrovian social welfare function with an infinite population, revealing the structure of decisive coalitions for an Arrow social welfare function as *ultrafilters.* In their analysis, however, there was no explicit consideration of a sequential relationship among the members of an infinite population. It was a pioneering analysis due to Ferejohn and Page (1978) that introduced time explicitly into the analysis. Time flows only unidirectionally, and two members t and t' of the society, to be called generation t and generation t', respectively, are such that generation t' appears in the society only after generation t appears in the society if and only if $t < t'$ holds. As a result of introducing this time structure of infinite population, Ferejohn and Page (1978) also opened a new gate towards marrying Arrovian social choice theory and the theory of evaluating infinite intergenerational utility streams, which was initiated by Koopmans (1960) and Diamond (1965). This paper is an attempt to reexamine the Ferejohn and Page analysis of intergenerational social choice theory in a multi-profile setting.

Starting out with Hansson's (1976) result that any social welfare function satisfying Arrow's (1951; 1963) axioms must be such that the set of decisive coalitions is an ultrafilter, we strengthen one of Ferejohn and Page's (1978) results in an intergenerational context. Ferejohn and Page propose a stationarity condition in an infinite-horizon multi-profile social choice model and show that if a social welfare function satisfying Arrow's conditions and stationarity exists, generation one must be a dictator. We prove that the implications of their stationarity condition are even more far-reaching: there exist no Arrow social welfare functions satisfying stationarity, not even dictatorial ones. The same conclusion holds if individual preferences are restricted to those that are history-independent. We propose an alternative framework where the preferences of any generation are restricted to depend on the outcome for this generation only. In that case, there do exist Arrow social welfare functions satisfying stationarity but they are all such that generation one has even more dictatorial power than established in the Ferejohn-Page result. Adding Pareto indifference as a requirement leads again to an impossibility. Finally, we propose what we suggest is a more suitable multi-profile version of stationarity and characterize lexicographic dictatorships where the generations are taken into consideration in chronological order. The main conclusion is that, although the infinite-population version of Arrow's social choice problem permits, in principle, non-dictatorial rules, these additional possibilities all but vanish even if an alternative stationarity axiom is imposed. The relationship between the Ferejohn and Page analysis and our extensions thereof, on the one hand, and the Koopmans-Diamond analysis of the evaluation of infinite intergenerational utility streams and their subsequent extensions, on the other, is discussed in the Concluding Remarks.

2 Infinite-Horizon Social Choice

Suppose there is a set of per-period alternatives X containing at least three elements, that is, $|X| \geq 3$ where $|X|$ denotes the cardinality of X. These per-period alternatives could be consumption bundles, for example, but we do not restrict attention to one particular interpretation. We identify the population with a sequence of generations indexed by the positive integers $\mathbb{N}$. Let X^∞ be the set of all streams of per-period alternatives $\mathbf{x} = (x_1, x_2, \ldots, x_t, \ldots)$ where, for each $t \in \mathbb{N}$, $x_t \in X$ is the period-t alternative experienced by generation t. We also refer to x_t as the factor of $\mathbf{x}$ relevant for generation t.

That the set of feasible per-period alternatives is the same for all generations appears to be a rather restrictive assumption; for example, technological progress is likely to generate dramatically different feasible sets of consumption bundles several decades into the future. The above more restrictive formulation is chosen because it is needed in order to define Ferejohn and Page's (1978) version of stationarity and their relevant results using this property; this is the case because, to apply their axiom, some generations must be able to assess not only their own per-period alternatives but those of other generations as well. However, the new approach we develop in our analysis can easily accommodate a framework where the per-period feasible sets may be period-dependent. The reason is that our proposed model is based on axioms that do not require a generation t to be capable of comparing per-period alternatives other than those relevant for t.

The set of all orderings on X^∞ is denoted by $\mathcal{R}$. A social ordering is an element R of $\mathcal{R}$. Each generation $t \in \mathbb{N}$ has an ordering $R_t \in \mathcal{R}$. A (preference) profile is a stream $\mathbf{R} = (R_1, R_2, \ldots, R_t, \ldots)$ of orderings on X^∞. The set of all such profiles is denoted by $\mathcal{R}^\infty$. Let $t \in \mathbb{N}$. For $\mathbf{x} \in X^\infty$, we define

$$\mathbf{x}_{\geq t} = (x_t, x_{t+1}, \ldots) \in X^\infty$$

and, analogously, for $\mathbf{R} \in \mathcal{R}^\infty$,

$$\mathbf{R}_{\geq t} = (R_t, R_{t+1}, \ldots) \in \mathcal{R}^\infty.$$

Two subsets of the *unlimited domain* $\mathcal{R}^\infty$ will be of importance in this paper. We define the *forward-looking domain* $\mathcal{R}^\infty_F \subseteq \mathcal{R}^\infty$ by letting, for all $\mathbf{R} \in \mathcal{R}^\infty$, $\mathbf{R} \in \mathcal{R}^\infty_F$ if, for each $t \in \mathbb{N}$, there exists an ordering Q_t on X^∞ such that, for all $\mathbf{x}, \mathbf{y} \in X^\infty$,

$$\mathbf{x} R_t \mathbf{y} \;\Leftrightarrow\; \mathbf{x}_{\geq t} Q_t \mathbf{y}_{\geq t}.$$

Analogously, the *selfish domain* $\mathcal{R}^\infty_S \subseteq \mathcal{R}^\infty_F \subseteq \mathcal{R}^\infty$ is obtained by letting, for all $\mathbf{R} \in \mathcal{R}^\infty$, $\mathbf{R} \in \mathcal{R}^\infty_S$ if, for each $t \in \mathbb{N}$, there exists an ordering $\succeq_t$ on X such that, for all $\mathbf{x}, \mathbf{y} \in X^\infty$,

$$\mathbf{x} R_t \mathbf{y} \;\Leftrightarrow\; x_t \succeq_t y_t.$$

For a relation $R \in \mathcal{R}$, the *asymmetric part* $P(R)$ of R is defined by

$$\mathbf{x} P(R) \mathbf{y} \;\Leftrightarrow\; [\mathbf{x} R \mathbf{y} \text{ and } \neg\, \mathbf{y} R \mathbf{x}]$$

for all $\mathbf{x}, \mathbf{y} \in X^\infty$. The *symmetric part* $I(R)$ of R is defined by

$$\mathbf{x} I(R) \mathbf{y} \;\Leftrightarrow\; [\mathbf{x} R \mathbf{y} \text{ and } \mathbf{y} R \mathbf{x}]$$

for all $\mathbf{x}, \mathbf{y} \in X^\infty$. Furthermore, for all $\mathbf{x}, \mathbf{y} \in X^\infty$ and for all $R \in \mathcal{R}$, $R|_{\{\mathbf{x},\mathbf{y}\}}$ is the restriction of R to the set $\{\mathbf{x}, \mathbf{y}\}$.

In the infinite-horizon context studied in this paper, a *social welfare function* is a mapping $f\colon \mathcal{D} \to \mathcal{R}$, where $\mathcal{D} \subseteq \mathcal{R}^\infty$ with $\mathcal{D} \neq \emptyset$ is the domain of f. The interpretation is that, for a profile $\mathbf{R} \in \mathcal{D}$, $f(\mathbf{R})$ is the social ranking of streams in X^∞.

Arrow (1951; 1963) imposed the axioms of *unlimited domain, weak Pareto* and *independence of irrelevant alternatives* on a social welfare function and showed that, in the case of a finite population, the resulting social welfare functions are *dictatorial:* there exists an individual such that, whenever this individual strictly prefers one alternative over another, this strict preference is reproduced in the social ranking, irrespective of the preferences of other members of society. In our context, these axioms are defined as follows.

Unlimited domain. $\mathcal{D} = \mathcal{R}^\infty$.

Weak Pareto. For all $\mathbf{x}, \mathbf{y} \in X^\infty$ and for all $\mathbf{R} \in \mathcal{D}$,

$$\mathbf{x} P(R_t) \mathbf{y} \; \forall t \in \mathbb{N} \;\Rightarrow\; \mathbf{x} P(f(\mathbf{R})) \mathbf{y}.$$

Independence of irrelevant alternatives. For all $\mathbf{x}, \mathbf{y} \in X^\infty$ and for all $\mathbf{R}, \mathbf{R}' \in \mathcal{D}$,

$$R_t|_{\{\mathbf{x},\mathbf{y}\}} = R'_t|_{\{\mathbf{x},\mathbf{y}\}} \;\forall t \in \mathbb{N} \;\Rightarrow\; f(\mathbf{R})|_{\{\mathbf{x},\mathbf{y}\}} = f(\mathbf{R}')|_{\{\mathbf{x},\mathbf{y}\}}.$$

Let $f\colon \mathcal{D} \to \mathcal{R}$ be a social welfare function and let $\mathbf{x}, \mathbf{y} \in X^\infty$. A set $T \subseteq \mathbb{N}$ is *almost decisive for* $\mathbf{x}$ *over* $\mathbf{y}$ *for* f (in short, T is $ad_f(\mathbf{x}, \mathbf{y})$) if, for all $\mathbf{R} \in \mathcal{D}$,

$$\mathbf{x}P(R_t)\mathbf{y} \;\forall t \in T \;\text{ and }\; \mathbf{y}P(R_t)\mathbf{x} \;\forall t \in \mathbb{N} \setminus T \;\Rightarrow\; \mathbf{x}P(f(\mathbf{R}))\mathbf{y}.$$

Analogously, T is *decisive for* $\mathbf{x}$ *over* $\mathbf{y}$ *for* f (in short, T is $d_f(\mathbf{x}, \mathbf{y})$) if, for all $\mathbf{R} \in \mathcal{D}$,

$$\mathbf{x}P(R_t)\mathbf{y} \;\forall t \in T \;\Rightarrow\; \mathbf{x}P(f(\mathbf{R}))\mathbf{y}.$$

Finally, a set $T \subseteq \mathbb{N}$ is *decisive for* f if, for all $\mathbf{x}, \mathbf{y} \in X^\infty$ and for all $\mathbf{R} \in \mathcal{D}$,

$$\mathbf{x}P(R_t)\mathbf{y} \;\forall t \in T \;\Rightarrow\; \mathbf{x}P(f(\mathbf{R}))\mathbf{y}.$$

Clearly, $\mathbb{N}$ is decisive for any social welfare function f satisfying weak Pareto. If there is a generation $t \in \mathbb{N}$ such that $\{t\}$ is decisive for f, generation t is a *dictator for* f. Let $\mathcal{T}(f)$ denote the set of all decisive sets for a social welfare function f.

Hansson (1976) has shown that if a social welfare function f satisfies unlimited domain, weak Pareto and independence of irrelevant alternatives, then $\mathcal{T}(f)$ must be an *ultrafilter.* An ultrafilter on $\mathbb{N}$ is a collection $\mathcal{U}$ of subsets of $\mathbb{N}$ such that

1. $\emptyset \notin \mathcal{U}$;
2. $\forall T, T' \subseteq \mathbb{N}$, $[[T \in \mathcal{U}$ and $T \subseteq T'] \;\Rightarrow\; T' \in \mathcal{U}]$;
3. $\forall T, T' \in \mathcal{U}$, $T \cap T' \in \mathcal{U}$;
4. $\forall T \subseteq \mathbb{N}$, $[T \in \mathcal{U}$ or $\mathbb{N} \setminus T \in \mathcal{U}]$.

The conjunction of properties 1 and 4 implies that $\mathbb{N} \in \mathcal{U}$ and, furthermore, the conjunction of properties 1 and 3 implies that the disjunction in property 4 is exclusive—that is, T and $\mathbb{N} \setminus T$ cannot both be in $\mathcal{U}$.

We conclude this section with a statement and discussion of Hansson's (1976) observation regarding decisive sets in our framework. We do not provide a proof at this point because it is analogous to our proof of a new variant of the theorem that applies to a different domain; see Section 4.

Theorem 1 (Hansson, 1976) *If a social welfare function f satisfies unlimited domain, weak Pareto and independence of irrelevant alternatives, then $\mathcal{T}(f)$ is an ultrafilter.*

An ultrafilter $\mathcal{U}$ is *principal* if there exists a $t \in \mathbb{N}$ such that, for all $T \subseteq \mathbb{N}$, $T \in \mathcal{U}$ if and only if $t \in T$. Otherwise, $\mathcal{U}$ is a *free* ultrafilter. It can be verified easily that if $\mathbb{N}$ is replaced with a finite set, then the only ultrafilters are principal and, therefore, Hansson's theorem reformulated for finite populations reduces to Arrow's (1951; 1963) theorem—that is, there exists an individual (or a generation) t which is a dictator. In the infinite-population case, a set of decisive coalitions that is a principal ultrafilter corresponds to a dictatorship just as in the finite case. Unlike in the finite case, however, there also exist free ultrafilters but they cannot be defined explicitly; the proof of their existence relies on non-constructive methods such as the axiom of choice. These free ultrafilters are non-dictatorial.

3 Stationarity and the Forward-Looking Domain

None of the above-defined axioms invoke the intertemporal structure imposed by our intergenerational interpretation. In contrast, the following *stationarity* property proposed by Ferejohn and Page (1978) is based on the unidirectional nature of time. The intuition underlying stationarity is that if two streams of per-period alternatives agree in the first period, their relative social ranking is unchanged if this common first-period alternative is eliminated. To formulate a property of this nature in a multi-profile setting, the profile under consideration for each of the two comparisons must be specified. In Ferejohn and Page's (1978) contribution, the same profile is employed before and after the first-period alternative is eliminated. It seems to us that this leads to a rather demanding requirement because the preferences of the first generation continue to be present even though the alternative relevant for this generation has been eliminated. We will show that, as a consequence, an impossibility result is obtained, thus strengthening Ferejohn and Page's (1978) theorem establishing that generation one is a dictator. Ferejohn and Page's (1978) stationarity axiom, which is due originally to Koopmans (1960) in a related but distinct context, is defined as follows.

Stationarity. For all $\mathbf{x}, \mathbf{y} \in X^\infty$ and for all $\mathbf{R} \in \mathcal{D}$, if $x_1 = y_1$, then

$$\mathbf{x} f(\mathbf{R}) \mathbf{y} \;\Leftrightarrow\; \mathbf{x}_{\geq 2} f(\mathbf{R}) \mathbf{y}_{\geq 2}.$$

Ferejohn and Page's (1978) result is the following. We prove a variant of the theorem on a different domain in Section 4 of this paper. As is the case for Hansson's (1976) theorem, it will be obvious how the proof can be modified so as to accommodate the unlimited-domain assumption.

Theorem 2 (Ferejohn and Page, 1978) *If a social welfare function f satisfies unlimited domain, weak Pareto, independence of irrelevant alternatives and stationarity, then generation one is a dictator for f.*

Our first new result strengthens Theorem 2. In particular, the following theorem shows that not even the generation-one dictatorships identified in the previous result satisfy stationarity in addition to Arrow's axioms. We obtain the following theorem, which settles an open question posed by Ferejohn and Page (1978, p.273) in the negative.

Theorem 3 *There exists no social welfare function that satisfies unlimited domain, weak Pareto, independence of irrelevant alternatives and stationarity.*

Proof. Suppose, by way of contradiction, that f is a social welfare function that satisfies the axioms of the theorem statement. Let $x, y \in X$ and define the streams

$$\begin{aligned} \mathbf{x} &= (x, x, x, \ldots) = (x, \mathbf{x}), \\ \mathbf{y} &= (y, y, y, \ldots) = (y, \mathbf{y}), \\ \mathbf{z} &= (y, x, x, \ldots) = (y, \mathbf{x}). \end{aligned}$$

Let $\mathbf{R} \in \mathcal{R}_F^\infty \subseteq \mathcal{R}^\infty$ be a profile such that

$$\mathbf{z} P(R_1) \mathbf{y}$$

and

$$\mathbf{y} = \mathbf{y}_{\geq t} P(R_t) \mathbf{x}_{\geq t} = \mathbf{x}$$

for all $t \in \mathbb{N}$. By Theorem 2, generation one is a dictator and, thus,

$$\mathbf{z} P(f(\mathbf{R})) \mathbf{y}. \tag{1}$$

By stationarity,

$$\mathbf{x} f(\mathbf{R}) \mathbf{y} \;\Leftrightarrow\; \mathbf{z}_{\geq 2} f(\mathbf{R}) \mathbf{y}_{\geq 2} \;\Leftrightarrow\; \mathbf{z} f(\mathbf{R}) \mathbf{y}$$

and, thus, $\mathbf{x} P(f(\mathbf{R})) \mathbf{y}$ by (1). But $\mathbf{y} P(f(\mathbf{R})) \mathbf{x}$ by weak Pareto, a contradiction. ■

Theorem 3 shows that Ferejohn and Page's (1978) stationarity axiom is too strong in the presence of the Arrow axioms—not even dictatorial rules survive if this property is added. A possible way of weakening Arrow's axioms is to dispense with the unlimited-domain assumption. This seems to be a natural way to proceed because it is not clear why generation t's preferences should depend on the situation of their distant ancestors. Thus, a plausible domain restriction is obtained if the preferences of generation t are restricted to depend on the per-period alternatives involving $\{t, t+1, \ldots\}$ only.

Forward-looking domain. $\mathcal{D} = \mathcal{R}_F^\infty$.

Replacing unlimited domain with forward-looking domain, however, does not resolve the impossibility result of Theorem 3. This is the case because Hansson's (1976) theorem and Ferejohn and Page's (1978) dictatorship result remain true if unlimited domain is replaced with forward-looking domain (see Section 4 for details) and the profiles used in the proof of Theorem 3 are all forward-looking profiles. Thus, the conclusions of Theorems 1, 2 and 3 remain true if the unlimited-domain assumption is replaced with forward-looking domain.

Theorem 4 *If a social welfare function f satisfies forward-looking domain, weak Pareto and independence of irrelevant alternatives, then $\mathcal{T}(f)$ is an ultrafilter.*

Theorem 5 *If a social welfare function f satisfies forward-looking domain, weak Pareto, independence of irrelevant alternatives and stationarity, then generation one is a dictator for f.*

Theorem 6 *There exists no social welfare function that satisfies forward-looking domain, weak Pareto, independence of irrelevant alternatives and stationarity.*

In view of Theorem 6, we require more stringent domain restrictions than that implied by the forward-looking domain assumption in order to accommodate properties such as stationarity in an intergenerational infinite-horizon setting. Alternatively, we may consider modifying the stationarity axiom. Both of these routes are explored in the following section.

4 The Selfish Domain and Multi-Profile Stationarity

The selfish-domain assumption requires preference profiles to be members of $\mathcal{R}_S^\infty$, that is, each generation t cares exclusively about the factor x_t of an alternative $\mathbf{x}$ that is relevant for this generation.

Selfish domain. $\mathcal{D} = \mathcal{R}_S^\infty$.

Recall that, for any profile $\mathbf{R} \in \mathcal{R}_S^\infty$ and for any $t \in \mathbb{N}$, there exists an ordering $\succeq_t$ on X such that $\mathbf{x} R_t \mathbf{y}$ if and only if $x_t \succeq_t y_t$.

This domain restriction allows us to resolve the impossibility results of Theorems 3 and 6. However, the resulting social welfare functions are dictatorial in a strong sense. We show that even with a selfish domain, Ferejohn and Page's (1978) version of stationarity requires that generation one determines the social ranking of two alternatives $\mathbf{x}$ and $\mathbf{y}$ if it has a strict preference between x_1 and y_1. If $x_1 = y_1$, the preferences *of generation one* are consulted regarding period-two outcomes, and so on. In the context of social welfare functions satisfying selfish domain, we say that generation one is a *strong dictator for f* if, for all $\mathbf{x}, \mathbf{y} \in X^\infty$ and for all $\mathbf{R} \in \mathcal{R}_S^\infty$,

$$[\exists t \in \mathbb{N} \ \text{ such that } \ [x_\tau = y_\tau \ \forall \tau < t \ \text{ and } \ x_t P(\succeq_1) y_t]] \ \Rightarrow \ \mathbf{x} P(f(\mathbf{R}))\mathbf{y}.$$

We begin by proving a version of Hansson's (1976) theorem that applies to the selfish domain. It is straightforward to see that the proof can be adapted easily to apply to the unlimited domain and the forward-looking domain as well; replacing all occurrences of $\mathcal{R}_S^\infty$ with $\mathcal{R}^\infty$ or with $\mathcal{R}_F^\infty$ leaves all arguments intact.

The two following preliminary results will be of convenience in establishing Hansson's theorem on our modified domain. Again, it is straightforward to see that the proofs go through without difficulties under either of the two alternative domain assumptions considered in this paper.

Lemma 1 *Let f be a social welfare function that satisfies selfish domain, weak Pareto and independence of irrelevant alternatives. Let $\mathbf{x}, \mathbf{y} \in X^\infty$ and let $T \subseteq \mathbb{N}$. If T is $ad_f(\mathbf{x}, \mathbf{y})$, then T is $d_f(\mathbf{x}, \mathbf{y})$.*

Proof. Let f be a social welfare function that satisfies the three requisite axioms, let $\mathbf{x}, \mathbf{y}, \mathbf{z} \in X^\infty$ be pairwise distinct and let $T \subseteq \mathbb{N}$ be $ad_f(\mathbf{x}, \mathbf{y})$. Consider a profile $\mathbf{R} \in \mathcal{R}_S^\infty \subseteq \mathcal{R}_F^\infty \subseteq \mathcal{R}^\infty$ such that

$$\begin{aligned} &\mathbf{z} P(R_t) \mathbf{x} P(R_t) \mathbf{y} \qquad \forall t \in T, \\ &\mathbf{y} P(R_t) \mathbf{z} P(R_t) \mathbf{x} \qquad \forall t \in \mathbb{N} \setminus T. \end{aligned}$$

By weak Pareto, $\mathbf{z} P(f(\mathbf{R}))\mathbf{x}$. Because T is $ad_f(\mathbf{x}, \mathbf{y})$, we have $\mathbf{x} P(f(\mathbf{R}))\mathbf{y}$. By transitivity, $\mathbf{z} P(f(\mathbf{R}))\mathbf{y}$. This implies that T is $ad_f(\mathbf{z}, \mathbf{y})$ by independence of irrelevant alternatives.

Now consider a profile $\mathbf{R}' \in \mathcal{R}_S^\infty \subseteq \mathcal{R}_F^\infty \subseteq \mathcal{R}^\infty$ such that

$$\begin{aligned} \mathbf{x}P(R'_t)\mathbf{z}P(R'_t)\mathbf{y} \qquad & \forall t \in T, \\ \mathbf{x}P(R'_t)\mathbf{z} \text{ and } \mathbf{y}P(R'_t)\mathbf{z} \qquad & \forall t \in \mathbb{N} \setminus T. \end{aligned}$$

By weak Pareto, $\mathbf{x}P(f(\mathbf{R}'))\mathbf{z}$. Because T is $ad_f(\mathbf{z}, \mathbf{y})$ as just established, it follows that $\mathbf{z}P(f(\mathbf{R}'))\mathbf{y}$. By transitivity, $\mathbf{x}P(f(\mathbf{R}'))\mathbf{y}$. Because the relative ranking of $\mathbf{x}$ and $\mathbf{y}$ is not specified in the preferences of the individuals in $\mathbb{N} \setminus T$, independence of irrelevant alternatives implies that T is $d_f(\mathbf{x}, \mathbf{y})$. ■

Lemma 2 *Let f be a social welfare function that satisfies selfish domain, weak Pareto and independence of irrelevant alternatives. Let $\mathbf{x}, \mathbf{y} \in X^\infty$ and let $T \subseteq \mathbb{N}$. If T is $d_f(\mathbf{x}, \mathbf{y})$, then $T \in \mathcal{T}(f)$.*

Proof. The proof proceeds as the proof of Lemma 1 by establishing that $d_f(\mathbf{x}, \mathbf{y})$ implies $d_f(\mathbf{z}, \mathbf{w})$ for all constellations of $\mathbf{z}$ and $\mathbf{w}$. ■

Our version of Hansson's (1976) theorem is formulated for the selfish domain but it is easy to see that every occurrence of $\mathcal{R}_S^\infty$ can be replaced with $\mathcal{R}^\infty$ or with $\mathcal{R}_F^\infty$ without affecting the validity of the arguments employed.

Theorem 7 *If a social welfare function f satisfies selfish domain, weak Pareto and independence of irrelevant alternatives, then $\mathcal{T}(f)$ is an ultrafilter.*

Proof. Suppose f satisfies selfish domain, weak Pareto and independence of irrelevant alternatives. We need to show that $\mathcal{T}(f)$ has the four properties of an ultrafilter.

1. If $\emptyset \in \mathcal{T}(f)$, we obtain $\mathbf{x}P(f(\mathbf{R}))\mathbf{y}$ and $\mathbf{y}P(f(\mathbf{R}))\mathbf{x}$ for any two alternatives $\mathbf{x}, \mathbf{y} \in X^\infty$ and for any profile $\mathbf{R} \in \mathcal{R}_S^\infty$, which is impossible. Thus, $\emptyset \notin \mathcal{T}(f)$.

2. This property follows immediately from the definition of decisiveness.

3. Suppose $T, T' \in \mathcal{T}(f)$. Let $\mathbf{x}, \mathbf{y}, \mathbf{z} \in X^\infty$ be pairwise distinct and let $\mathbf{R} \in \mathcal{R}_S^\infty \subseteq \mathcal{R}_F^\infty \subseteq \mathcal{R}^\infty$ be such that

$$\begin{aligned} \mathbf{x}P(R_t)\mathbf{y}P(R_t)\mathbf{z} \qquad & \forall t \in T \setminus T', \\ \mathbf{z}P(R_t)\mathbf{x}P(R_t)\mathbf{y} \qquad & \forall t \in T \cap T', \\ \mathbf{y}P(R_t)\mathbf{z}P(R_t)\mathbf{x} \qquad & \forall t \in T' \setminus T. \end{aligned}$$

Because T is decisive, we have $\mathbf{x}P(f(\mathbf{R}))\mathbf{y}$. Because T' is decisive, we have $\mathbf{z}P(f(\mathbf{R}))\mathbf{x}$. By transitivity, $\mathbf{z}P(f(\mathbf{R}))\mathbf{y}$. This implies that $T \cap T'$ is $ad_f(\mathbf{z}, \mathbf{y})$ by independence of irrelevant alternatives. Lemma 1 implies that $T \cap T'$ is $d_f(\mathbf{z}, \mathbf{y})$ and, by Lemma 2, $T \cap T' \in \mathcal{T}(f)$.

4. Let $T \subseteq \mathbb{N}$. Let $\mathbf{x}, \mathbf{y}, \mathbf{z} \in X^\infty$ and $\mathbf{R} \in \mathcal{R}_S^\infty \subseteq \mathcal{R}_F^\infty \subseteq \mathcal{R}^\infty$ be such that

$$\begin{aligned} &\mathbf{x}P(R_t)\mathbf{z}P(R_t)\mathbf{y} \qquad \forall t \in T, \\ &\mathbf{y}P(R_t)\mathbf{x}P(R_t)\mathbf{z} \qquad \forall t \in \mathbb{N} \setminus T. \end{aligned}$$

If $\mathbf{x}P(f(\mathbf{R}))\mathbf{y}$, T is $ad_f(\mathbf{x}, \mathbf{y})$ by independence of irrelevant alternatives and, by Lemma 1, T is $d_f(\mathbf{x}, \mathbf{y})$. Now Lemma 2 implies that $T \in \mathcal{T}(f)$.

If $\neg(\mathbf{x}P(f(\mathbf{R}))\mathbf{y})$, we have $\mathbf{y}f(\mathbf{R})\mathbf{x}$ by completeness. Furthermore, $\mathbf{x}P(f(\mathbf{R}))\mathbf{z}$ by weak Pareto. Transitivity implies $\mathbf{y}P(f(\mathbf{R}))\mathbf{z}$ and, by independence of irrelevant alternatives, $\mathbb{N} \setminus T$ is $ad_f(\mathbf{y}, \mathbf{z})$. Lemma 1 implies that $\mathbb{N} \setminus T$ is $d_f(\mathbf{y}, \mathbf{z})$ and Lemma 2 yields $\mathbb{N} \setminus T \in \mathcal{T}(f)$. ■

Ferejohn and Page's (1978) conclusion regarding the dictatorial role played by generation one also remains valid on the selfish domain. Thus, we obtain

Theorem 8 *If a social welfare function f satisfies selfish domain, weak Pareto, independence of irrelevant alternatives and stationarity, then generation one is a dictator for f.*

Proof. Suppose f satisfies selfish domain, weak Pareto, independence of irrelevant alternatives and stationarity. By Theorem 7, $\mathcal{T}(f)$ is an ultrafilter. Let x and y be two distinct elements of X and let $\succeq$ be an ordering on X such that $xP(\succeq)y$. Define a profile $\mathbf{R} \in \mathcal{R}_S^\infty \subseteq \mathcal{R}_F^\infty \subseteq \mathcal{R}^\infty$ by letting, for all $t \in \mathbb{N}$ and for all $\mathbf{x}, \mathbf{y} \in X^\infty$,

$$\mathbf{x}R_t\mathbf{y} \;\Leftrightarrow\; x_t \succeq y_t.$$

Now consider the streams

$$\begin{aligned} \mathbf{x} &= (x, y, x, y, x, y, \ldots) = (x, \mathbf{y}), \\ \mathbf{y} &= (y, x, y, x, y, x, \ldots) = (y, \mathbf{x}), \\ \mathbf{z} &= (x, x, y, x, y, x, \ldots) = (x, \mathbf{x}), \\ \mathbf{w} &= (y, y, x, y, x, y, \ldots) = (y, \mathbf{y}). \end{aligned}$$

We have $\mathbf{x}P(R_t)\mathbf{y}$ for all odd t and $\mathbf{y}P(R_t)\mathbf{x}$ for all even t. Because of property 4 of an ultrafilter, one of the two sets $\{2,4,6,\ldots\}$ and $\{1,3,5,\ldots\}$ must be decisive. If $\{2,4,6,\ldots\}$ is decisive, we have

$$\mathbf{y}P(f(\mathbf{R}))\mathbf{x}. \tag{2}$$

By stationarity,

$$\mathbf{x}f(\mathbf{R})\mathbf{z} \;\Leftrightarrow\; \mathbf{x}_{\geq 2}f(\mathbf{R})\mathbf{z}_{\geq 2} \;\Leftrightarrow\; \mathbf{y}f(\mathbf{R})\mathbf{x}$$

and, thus, $\mathbf{x}P(f(\mathbf{R}))\mathbf{z}$ by (2). Analogously, stationarity implies

$$\mathbf{w}f(\mathbf{R})\mathbf{y} \;\Leftrightarrow\; \mathbf{w}_{\geq 2}f(\mathbf{R})\mathbf{y}_{\geq 2} \;\Leftrightarrow\; \mathbf{y}f(\mathbf{R})\mathbf{x}$$

and (2) implies $\mathbf{w}P(f(\mathbf{R}))\mathbf{y}$. By transitivity, $\mathbf{w}P(f(\mathbf{R}))\mathbf{z}$. We have $\mathbf{z}P(R_t)\mathbf{w}$ for all $t \in \{1,2,4,6,\ldots\}$ and, because $\{2,4,6,\ldots\}$ is decisive and $\{2,4,6,\ldots\} \subseteq \{1,2,4,6,\ldots\}$, property 2 of an ultrafilter implies that $\{1,2,4,6,\ldots\}$ is decisive. Thus, $\mathbf{z}P(f(\mathbf{R}))\mathbf{w}$, a contradiction. Therefore, $\{1,3,5,\ldots\}$ must be decisive and, thus,

$$\mathbf{x}P(f(\mathbf{R}))\mathbf{y}. \tag{3}$$

By stationarity,

$$\mathbf{z}f(\mathbf{R})\mathbf{x} \;\Leftrightarrow\; \mathbf{z}_{\geq 2}f(\mathbf{R})\mathbf{x}_{\geq 2} \;\Leftrightarrow\; \mathbf{x}f(\mathbf{R})\mathbf{y}$$

and, thus, $\mathbf{z}P(f(\mathbf{R}))\mathbf{x}$ by (3). Analogously, stationarity implies

$$\mathbf{y}f(\mathbf{R})\mathbf{w} \;\Leftrightarrow\; \mathbf{y}_{\geq 2}f(\mathbf{R})\mathbf{w}_{\geq 2} \;\Leftrightarrow\; \mathbf{x}f(\mathbf{R})\mathbf{y}$$

and (3) implies $\mathbf{y}P(f(\mathbf{R}))\mathbf{w}$. By transitivity, $\mathbf{z}P(f(\mathbf{R}))\mathbf{w}$. Because $\mathbf{z}P(R_t)\mathbf{w}$ for all $t \in \{1,2,4,6,\ldots\}$ and $\mathbf{w}P(R_t)\mathbf{z}$ for all $t \in \{3,5,7,\ldots\}$, $\{3,5,7,\ldots\}$ cannot be decisive. By property 4 of an ultrafilter, it follows that $\{1,2,4,6,\ldots\}$ is decisive.

We have thus established that $\{1,3,5,\ldots\}$ and $\{1,2,4,6,\ldots\}$ are decisive and, by property 3 of an ultrafilter, $\{1\} = \{1,3,5,\ldots\} \cap \{1,2,4,6,\ldots\}$ is decisive, which means generation one is a dictator. ■

With selfish domain, we do not obtain an impossibility but all that is left as a possibility if stationarity is added to the list of axioms is a strong dictatorship of generation one.

Theorem 9 *If a social welfare function f satisfies selfish domain, weak Pareto, independence of irrelevant alternatives and stationarity, then generation one is a strong dictator for f.*

Proof. Suppose f satisfies the axioms of the theorem statement. Let $t \in \mathbb{N}$ and suppose $\mathbf{x}, \mathbf{y} \in X^\infty$ and $\mathbf{R} \in \mathcal{R}_S^\infty$ are such that

$$x_\tau = y_\tau \ \forall \tau < t \ \text{ and } \ x_t P(\succeq_1) y_t.$$

Apply stationarity $t-1$ times to obtain

$$\mathbf{x} f(\mathbf{R}) \mathbf{y} \ \Leftrightarrow \ \mathbf{x}_{\geq t} f(\mathbf{R}) \mathbf{y}_{\geq t}. \tag{4}$$

By Theorem 8, generation one is a dictator so that $x_t P(\succeq_1) y_t$ implies $\mathbf{x}_{\geq t} P(f(\mathbf{R})) \mathbf{y}_{\geq t}$ and, thus, $\mathbf{x} P(f(\mathbf{R})) \mathbf{y}$ by (4). ■

The set of social welfare functions satisfying the axioms of Theorem 9 is not empty. For example, the social welfare function f defined by letting, for all $\mathbf{x}, \mathbf{y} \in X^\infty$ and for all $\mathbf{R} \in \mathcal{R}_S^\infty$, $\mathbf{x} f(\mathbf{R}) \mathbf{y}$ if and only if

$$[x_\tau I(\succeq_1) y_\tau \ \forall \ \tau \in \mathbb{N}] \ \text{ or } \ [\exists t \in \mathbb{N} \ \text{ such that } \ [x_\tau I(\succeq_1) y_\tau \ \forall \tau < t \ \text{ and } \ x_t P(\succeq_1) y_t]]$$

satisfies the required axioms. However, it does not satisfy *Pareto indifference,* another Pareto condition that requires unanimous indifference to be respected by a social aggregation procedure.

Pareto indifference. For all $\mathbf{x}, \mathbf{y} \in X^\infty$ and for all $\mathbf{R} \in \mathcal{D}$,

$$\mathbf{x} I(R_t) \mathbf{y} \ \forall t \in \mathbb{N} \ \Rightarrow \ \mathbf{x} I(f(\mathbf{R})) \mathbf{y}.$$

More generally, adding Pareto indifference to the axioms of Theorem 9 produces an impossibility.

Theorem 10 *There exists no social welfare function that satisfies selfish domain, weak Pareto, independence of irrelevant alternatives, stationarity and Pareto indifference.*

Proof. Suppose f satisfies the axioms of the theorem statement. By Theorem 9, generation one is a strong dictator for f. Let $x, y \in X$ and consider the two streams

$$\begin{aligned} \mathbf{x} &= (x, x, x, x, \ldots) = (x, x, \mathbf{x}), \\ \mathbf{y} &= (x, y, x, x, \ldots) = (x, y, \mathbf{x}). \end{aligned}$$

Let $\mathbf{R} \in \mathcal{R}_S^\infty$ be a profile such that $x P(\succeq_1) y$ and $x I(\succeq_2) y$. By Pareto indifference and reflexivity, $\mathbf{x} I(f(\mathbf{R})) \mathbf{y}$. Because generation one is a strong dictator, $\mathbf{x} P(f(\mathbf{R})) \mathbf{y}$ and we have a contradiction. ■

In Ferejohn and Page's (1978) stationarity axiom, the same profile $\mathbf{R}$ is applied in both comparisons even though the period-one alternative is no longer present. This seems to us to be rather counter-intuitive and, consequently, we propose the following version that takes this point into consideration by eliminating the first-period factor not only from the alternatives but also from the profile.

Multi-profile stationarity. For all $\mathbf{x}, \mathbf{y} \in X^\infty$ and for all $\mathbf{R} \in \mathcal{D}$, if $x_1 = y_1$, then

$$\mathbf{x} f(\mathbf{R}) \mathbf{y} \;\Leftrightarrow\; \mathbf{x}_{\geq 2} f(\mathbf{R}_{\geq 2}) \mathbf{y}_{\geq 2}.$$

Unlike stationarity, multi-profile stationarity does not require generation t to be able to compare per-period alternatives other than those relevant for period t itself. Because this is the case for all other axioms as well, our two final results remain true if the per-period sets of alternatives are period-dependent, thus providing a more realistic framework. For simplicity of presentation, we do not state these alternative versions explicitly and leave it to the reader to verify that if X is replaced with X_t for each $t \in \mathbb{N}$, all arguments continue to go through, provided that each X_t contains at least three elements.

We now examine the implications of our multi-profile stationarity axiom. If it is used to replace stationarity in the statement of Theorem 10, a *chronological dictatorship* is characterized. In contrast to the strongly dictatorial rules of Theorem 9, a chronological dictatorship consults generation one first but, in the case of indifference, then moves on to consult *generation two* regarding the ranking of two streams, and so on. Thus, there still is a very strong dictatorship component but it is not as extreme as that generated by stationarity—and it is compatible with Pareto indifference.

The chronological dictatorship f^{CD} is defined as follows. For all $\mathbf{x}, \mathbf{y} \in X^\infty$ and for all $\mathbf{R} \in \mathcal{R}_S^\infty$, $\mathbf{x} f^{CD}(\mathbf{R}) \mathbf{y}$ if and only if

$$[x_\tau I(\succeq_\tau) y_\tau \ \forall\, \tau \in \mathbb{N}] \ \text{ or } \ [\exists t \in \mathbb{N} \ \text{ such that } \ [x_\tau I(\succeq_\tau) y_\tau \ \forall \tau < t \ \text{ and } \ x_t P(\succeq_t) y_t]].$$

In a first step towards our characterization result, we show that Ferejohn and Page's (1978) dictatorship result is true even on a selfish domain *and* with multi-profile stationarity instead of stationarity. Some steps in the proof of Theorem 8 have to be amended in order to establish this new version of the result.

Theorem 11 *If a social welfare function f satisfies selfish domain, weak Pareto, independence of irrelevant alternatives and multi-profile stationarity, then generation one is a dictator for f.*

Proof. Suppose f satisfies selfish domain, weak Pareto, independence of irrelevant alternatives and multi-profile stationarity. By Theorem 7, $\mathcal{T}(f)$ is an ultrafilter. Let x and y be two distinct elements of X and let $\succeq$ be an ordering on X such that $xP(\succeq)y$. Define a profile $\mathbf{R} \in \mathcal{R}_S^\infty \subseteq \mathcal{R}_F^\infty \subseteq \mathcal{R}^\infty$ by letting, for all $t \in \mathbb{N}$ and for all $\mathbf{x}, \mathbf{y} \in X^\infty$,

$$\mathbf{x} R_t \mathbf{y} \;\Leftrightarrow\; x_t \succeq y_t.$$

Now consider the streams

$$\begin{aligned}
\mathbf{x} &= (x, y, x, y, x, y, \ldots) = (x, \mathbf{y}),\\
\mathbf{y} &= (y, x, y, x, y, x, \ldots) = (y, \mathbf{x}),\\
\mathbf{z} &= (x, x, y, x, y, x, \ldots) = (x, \mathbf{x}),\\
\mathbf{w} &= (y, y, x, y, x, y, \ldots) = (y, \mathbf{y}).
\end{aligned}$$

We have $\mathbf{x}P(R_t)\mathbf{y}$ for all odd t and $\mathbf{y}P(R_t)\mathbf{x}$ for all even t. Because of property 4 of an ultrafilter, one of the two sets $\{2, 4, 6, \ldots\}$ and $\{1, 3, 5, \ldots\}$ must be decisive. If $\{2, 4, 6, \ldots\}$ is decisive, we have

$$\mathbf{y}P(f(\mathbf{R}))\mathbf{x} \;\text{ and }\; \mathbf{y}P(f(\mathbf{R}_{\geq 2}))\mathbf{x}. \tag{5}$$

By multi-profile stationarity,

$$\mathbf{x}f(\mathbf{R})\mathbf{z} \;\Leftrightarrow\; \mathbf{x}_{\geq 2}f(\mathbf{R}_{\geq 2})\mathbf{z}_{\geq 2} \;\Leftrightarrow\; \mathbf{y}f(\mathbf{R}_{\geq 2})\mathbf{x}$$

and, thus, $\mathbf{x}P(f(\mathbf{R}))\mathbf{z}$ by (5). Analogously, multi-profile stationarity implies

$$\mathbf{w}f(\mathbf{R})\mathbf{y} \;\Leftrightarrow\; \mathbf{w}_{\geq 2}f(\mathbf{R}_{\geq 2})\mathbf{y}_{\geq 2} \;\Leftrightarrow\; \mathbf{y}f(\mathbf{R}_{\geq 2})\mathbf{x}$$

and (5) implies $\mathbf{w}P(f(\mathbf{R}))\mathbf{y}$. By transitivity, $\mathbf{w}P(f(\mathbf{R}))\mathbf{z}$. We have $\mathbf{z}P(R_t)\mathbf{w}$ for all $t \in \{1, 2, 4, 6, \ldots\}$ and, because $\{2, 4, 6, \ldots\}$ is decisive and $\{2, 4, 6, \ldots\} \subseteq \{1, 2, 4, 6, \ldots\}$, property 2 of an ultrafilter implies that $\{1, 2, 4, 6, \ldots\}$ is decisive. Thus, $\mathbf{z}P(f(\mathbf{R}))\mathbf{w}$, a contradiction. Therefore, $\{1, 3, 5, \ldots\}$ must be decisive and, thus,

$$\mathbf{x}P(f(\mathbf{R}))\mathbf{y} \;\text{ and }\; \mathbf{x}P(f(\mathbf{R}_{\geq 2}))\mathbf{y}. \tag{6}$$

By multi-profile stationarity,

$$\mathbf{z}f(\mathbf{R})\mathbf{x} \;\Leftrightarrow\; \mathbf{z}_{\geq 2}f(\mathbf{R}_{\geq 2})\mathbf{x}_{\geq 2} \;\Leftrightarrow\; \mathbf{x}f(\mathbf{R}_{\geq 2})\mathbf{y}$$

and, thus, $\mathbf{z}P(f(\mathbf{R}))\mathbf{x}$ by (6). Analogously, multi-profile stationarity implies

$$\mathbf{y}f(\mathbf{R})\mathbf{w} \;\Leftrightarrow\; \mathbf{y}_{\geq 2}f(\mathbf{R}_{\geq 2})\mathbf{w}_{\geq 2} \;\Leftrightarrow\; \mathbf{x}f(\mathbf{R}_{\geq 2})\mathbf{y}$$

and (6) implies $\mathbf{y}P(f(\mathbf{R}))\mathbf{w}$. By transitivity, $\mathbf{z}P(f(\mathbf{R}))\mathbf{w}$. Because $\mathbf{z}P(R_t)\mathbf{w}$ for all $t \in \{1, 2, 4, 6, \ldots\}$ and $\mathbf{w}P(R_t)\mathbf{z}$ for all $t \in \{3, 5, 7, \ldots\}$, $\{3, 5, 7, \ldots\}$ cannot be decisive. By property 4 of an ultrafilter, it follows that $\{1, 2, 4, 6, \ldots\}$ is decisive.

We have thus established that $\{1, 3, 5, \ldots\}$ and $\{1, 2, 4, 6, \ldots\}$ are decisive and, by property 3 of an ultrafilter, $\{1\} = \{1, 3, 5, \ldots\} \cap \{1, 2, 4, 6, \ldots\}$ is decisive, which means generation one is a dictator. ■

The final result of this paper characterizes f^{CD}.

Theorem 12 *A social welfare function f satisfies selfish domain, weak Pareto, independence of irrelevant alternatives, Pareto indifference and multi-profile stationarity if and only if $f = f^{CD}$.*

Proof. That f^{CD} satisfies the required axioms can be verified by the reader. To prove the converse implication, suppose f satisfies the required axioms. It is sufficient to show that, for all $\mathbf{x}, \mathbf{y} \in X^\infty$ and for all $\mathbf{R} \in \mathcal{R}_S^\infty$,

$$\mathbf{x}I(f^{CD}(\mathbf{R}))\mathbf{y} \;\Rightarrow\; \mathbf{x}I(f(\mathbf{R}))\mathbf{y} \tag{7}$$

and

$$\mathbf{x}P(f^{CD}(\mathbf{R}))\mathbf{y} \;\Rightarrow\; \mathbf{x}P(f(\mathbf{R}))\mathbf{y}. \tag{8}$$

(7) follows immediately from Pareto indifference. To prove (8), suppose $t \in \mathbb{N}$, $\mathbf{x}, \mathbf{y} \in X^\infty$ and $\mathbf{R} \in \mathcal{R}_S^\infty$ are such that

$$x_\tau I(\succeq_\tau) y_\tau \;\forall \tau < t \;\text{ and }\; x_t P(\succeq_t) y_t.$$

Apply multi-profile stationarity $t - 1$ times (and Pareto indifference where required) to obtain

$$\mathbf{x}f(\mathbf{R})\mathbf{y} \;\Leftrightarrow\; \mathbf{x}_{\geq t} f(\mathbf{R}_{\geq t})\mathbf{y}_{\geq t}. \tag{9}$$

Applying the result of Theorem 11, the relative ranking of $\mathbf{x}_{\geq t}$ and $\mathbf{y}_{\geq t}$ according to $\mathbf{R}_{\geq t}$ is determined by the strict preference for $\mathbf{x}$ over $\mathbf{y}$ according to the first generation in the profile $\mathbf{R}_{\geq t}$ (which is generation t in $\mathbf{R}$), so that $\mathbf{x}_{\geq t}P(f(\mathbf{R}_{\geq t}))\mathbf{y}_{\geq t}$ and, by (9), $\mathbf{x}P(f(\mathbf{R}))\mathbf{y}$. ■

5 Concluding Remarks

In concluding this paper, it may be worthwhile to clarify the relationship between the multi-profile intergenerational social choice theory developed in this paper, on the one hand, and the theory of evaluating infinite intergenerational utility streams, which capitalizes on the Koopmans (1960) analysis of impatience and the Diamond (1965) impossibility theorem on the existence of continuous evaluation orderings over the set of infinite utility streams satisfying the Sidgwick (1907) anonymity principle and the Pareto efficiency principle, on the other. Among many contributions that appeared after Diamond (1965), those which are most relevant in the present context include Asheim, Mitra and Tungodden (2007), Basu and Mitra (2003; 2007), Bossert, Sprumont and Suzumura (2007), Hara, Shinotsuka, Suzumura and Xu (2007) and Svensson (1980). Although these two lines of inquiry are related in the sense that both are concerned with aggregating generational evaluations of their well-beings into the overall social evaluation, they contrast sharply in at least two respects. In the first place, the latter investigation is *welfaristic* in the sense of basing the overall social evaluation on the infinite-generational utility streams, whereas the former exercise is free from such an early commitment to this informational basis. In the second place, while the latter approach hinges squarely on the continuity assumption even in a vestigial form, the former has nothing to do with any continuity assumption on social welfare orderings. More substantially, the Sidgwick anonymity principle, which plays a crucial role in establishing the Diamond impossibility theorem and related work, has nothing to do with our impossibility theorems. The same observation also applies to the Hammond (1976) equity axiom, which plays an important role in some recent developments in the theory of evaluating infinite-generational utility streams. Since continuity is a requirement which is rather technical in nature, to get rid of the dependence on this assumption may be counted as a virtue rather than a vice. Although the Sidgwick anonymity principle and the Hammond equity principle have an obvious intuitive appeal, it is fortunate that we need not go against these appealing axioms in defending our approach. It can surely be added to the list of axioms but all that is thereby obtained is another set of Arrow-type impossibility results, some of which will even contain redundancies.

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