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THE EFFECT OF IT INNOVATION ON INDUSTRIAL OUTPUT ELASTICITIES

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Abstract

Over the past decade, IT investment has been regarded as a key factor in enhancing productivity and economic development in Korea. This paper will assess whether the IT industry can positively affect structural change using an Input-Output model. Changes in Korean industries are traced using assumptions of IT innovation based on data from 1995 through 2000. Analysis reveals that the response of the economy falls short of our expectation that the development of the IT industry would generate growth in the productivity of the Korean economy. Government policy has been oriented toward cultivating IT industry through heavy investment, while neglecting efforts to make the overall industrial structure compatible with IT. We conclude that IT policy should be market-oriented to make the overall economy IT friendly so that industrial structures will respond more positively to IT development.

Keywords: IT Industry, Output Elasticity, Productivity, Industrial Structure

JEL Classification: L16, L63

I. Introduction

With a view toward transitioning to a knowledge based economy, the Korean government has promoted a policy of emphasizing Information and Communication Technology (hereafter IT or Information Technology) industry as a means of progress toward this goal. Particular emphasis has been placed on investment in the IT sector as a means of achieving a breakthrough toward the goal of $20,000 per capita income in the next few years. Investment in IT is believed to facilitate gains in efficiency by reducing production cost and increasing national competitiveness. Between 1995 and 2000, the IT industry has grown from 7.6% to 11.4% of the overall Korean economy. This high growth rate was made possible through strategic support and investment measures intended to promote economic development pursuant to the government-led “informatization policy.”

We can find many positive results from studies that have examined the question of the

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“Productivity Paradox.” Analytical results ranging from productivity studies (Jorgenson, 2001; Jorgenson, Ho, and Stiroh, 2003; Oliner and Sichel, 1994; Morrison and Berndt, 1990), labor related productivity studies (Oliner and Sichel, 2003; Autor et al., 1998), and firm-level analyses (Hitt and Brynjolfsson, 1996) all demonstrated that investment in the IT sector generated positive effects on the economy.

Many studies by Korean researchers concerning the impact of IT on the Korean economy have also shown positive results. These studies have contributed to the Korean Government’s overconfidence in the IT industry. However, Kim (2002) is one of a few researchers who have disputed the conventional wisdom concerning the impact of IT on the Korean economy. His study proved that the Korean economy has experienced growth in GDP primarily through massive commitments of capital and labor since the 1990s rather than through TFP (Total Factor Productivity) growth. Inefficient investment by the government in IT related firms for almost a decade created a bubble effect in the IT industry, which was a factor in the overall growth in GDP. Accordingly, the purpose of this paper is to analyze whether the IT industry has generated a positive impact on the Korean economy according to the level of industry.

Using Input-Output data, Klein (2003) found increasing returns to scale and average productivity gains in an industry-level analysis of the automotive sector. Klein focused on the automotive industry rather than studying a broader cross-section of all industries because the automotive industry is a major user of IT. An analysis done by Leontief and Duchin (1986) also used an Input-Output model to examine the impact of IT innovation. This analysis revealed an increasing trend in output and employment, but bigger aggregate gross output and lower employment growth rates occurred in the scenario where technical progress was occurred more rapidly.

Many reports and studies by Korean researchers using Input-Output technique also show a positive relationship between the IT industry and the level of production in the Korean economy. Those studies, however, ignore the reasons for the change in production and focus merely on explaining the induced coefficients of the production relating to the final demand. On the contrary, this paper explains how output elasticities are affected by cost variations in the prices of commodities. The comparison of output elasticities of industries for the years 1995, 1998 and 2000 will be tested for structural changes in production. The study of IT’s ability to facilitate production (Kim and Oh, 2004) employed the same scenario of capital cost reduction of 1% resulting from technological innovation in the IT industry. That study compared the results from 1995 and 1998 to analyze the rate of output change for three years.

In this paper, the same method is used for output elasticity rather than the rate of output change. The empirical framework is based on the Input-Output tables for 1995, 1998 and 2000. These tables were originally composed of more than 400 industries, but were further consolidated to 31 industries. The IT industry was adjusted according to the IT industry classification system developed by the Ministry of Information and Communication of

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1 Analysis revealed that the Government’s informatization policy was ineffective in promoting industrial productivity and led only to an increase in the share of the overall economy of the IT industry itself. In contrast, primary, manufacturing, and service industries atrophied in terms of their share of the overall economy. The investment in IT turned out to be an inefficient resource allocation for enhancing the production process of the overall economy.


The same classification method as was used in Kim and Oh (2004) is applied to adjust for 31 industries overall.

The result demonstrates that the effects of IT innovation on the overall Korean economy fell short of our expectation that IT would enhance productivity. This result is disappointing to those who have implicitly trusted in the ability of IT to spread productivity increase to other industries through technological innovation. One possible interpretation of this result is that Korea’s economic structure has not yet reached an IT friendly stage and that the development of the IT industry does not achieve the economies of scale in other industries. The result of this analysis leads to the conclusion that the Korean government’s investment program for the development of the IT industry has been largely ineffective.

This paper consists of four sections: Section 2 describes the basic idea of Leontief’s Input-Output model and its compensating idea of derivation of output elasticities of industries. The empirical findings of industrial output elasticities for 1995, 1998 and 2000 are contained in Section 3. Concluding remarks along with policy implications are in Section 4.

II. Model

The model used in this paper was introduced under the name of the Variable Input-Output (hereafter referred to as the VIO) model. The characteristics of the VIO model allow it to trace the change in output of industries resulting from a change in production cost in a specific industry. The details of this model can be found in Appendix I. The VIO model can be differentiated from the general idea of the traditional Leontief Input-Output model.

Leontief’s model is represented by two equations explaining the input and output sides. The equation of input side shows the relation of price change with the change in value added items, such as primary input use of labor or capital.

1) The price equation of the input side: \[ p = (I - A')^{-1}v \]
When there is a change in the use of labor or capital \( v \) in a certain industry, the equation of the input side will reflect the change of price \( p \) of all commodities of the economy.

2) The output equation of the output side: \[ x = (I - A)^{-1}f \]
When there is a change in the final demand \( f \) in household’s consumption, firms’ investment, and government expenditure, the equation of the output side will reflect the change in output \( x \) of all industries of the economy.

What we can notice from Leontief’s model is that the technical coefficient, \( a_{ij} \) in the matrix A is assumed to be fixed. This coefficient matrix A represents the use and the flow of commodities of entire industries as intermediate inputs. If there is a change in the cost of primary input for the production of a commodity in a specific industry, this will influence the

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3 The Ministry of Information and Communication defined a basic communication service, broadcasting service, communication device, and software industries as IT industries. In order to comply with such systems, it is to carry out analysis on electric appliances, video/audio & communication devices, computer & office devices, and communication, broadcasting, and computer service industries regarded as an IT industry. As a result, this paper has a relatively wider classification range of IT industries than that found in previous IT industry studies. For example, most of international papers by such as Morrison and Berndt(1991), Lichtenberg(1993), and Jorgenson (2001) treated electronic computer, communication device, scientific equipment, and copy machine industries as IT capital.
whole economy by changing the production process through substitution of primary and intermediate inputs. However, Leontief’s model misses an essential real world point, in that there is no factor that connects the input and the output side.

The VIO model derives the profit maximizing optimal level of primary($L_k$) and intermediate inputs($x_{ij}$) as can be seen in equation (3) in Appendix I. The intermediate inputs($x_{ij}$) involves the relative price($p_j/p_i$) that affects the technical coefficient($((p_j/p_i)\alpha_{ij})$ of intermediate input use matrix in the VIO model. The technical coefficient($((p_j/p_i)\alpha_{ij})$ becomes larger or smaller according to the change in prices of commodity i and j. If the changed price of i is smaller and the changed price of j is less small than before, the value of the relative price becomes larger and it also makes the technical coefficient larger. This means the substitution of commodity i for commodity j as intermediate inputs in the production process. Through the substitution of intermediate inputs, we can trace the change in output of all industries.

The price equation derived from the profit maximization function is denoted in equation 4 in the Appendix I.

3) The price equation(equation (4) in the Appendix I) of the input side:
\[ \ln p = (I - A')^{-1}[\sum_k \beta_k \ln w_k]. \]

The price ($p$) in the above equation shows the relation with the price of primary input $k(w_k)$.

The price affected by the change in primary input cost will affect the output by influencing the technical coefficient in the matrix A as cannot be seen in Leontief’s output equation discussed above. As a result, this compensates for the problem of the missing connection between the input and the output side by involving the factor of price change.

4) The output equation (equation (7) in the Appendix I) of the output side:
\[ \frac{d\ln x}{d\ln w_k} = (\hat{x})^{-1}[(\hat{\beta}^{-1}(I-A)^{-1}\hat{p} - \hat{x})\frac{d\ln p}{d\ln w_k}]. \]

With this output equation of $d\ln x$, we can find the output elasticities of all industries with respect to a cost variation of a specific industry. We will compare the industrial output elasticities ($d\ln x/d\ln w_k$) of the respective year and test whether there has been a change in productivity of the economy.

III. Empirical Results

This paper conducts a comparison of output elasticities for the years 1995, 1998, and 2000 to ascertain the change in industrial effects resulting from IT innovation. The scenario for IT innovation is assumed to be a 1% decrease in capital cost in IT industry for each year. The change in output elasticities of all industries can be interpreted as the susceptibility of the industrial structure to IT innovation. The effect from heavy governmental support of the IT

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4 In this paper, we can set $k=1$ to be a labor input and $k=2$ to be a capital input. Only capital input in the IT industry is addressed in this analysis.
industry through a strategic investment policy will be tested by comparing the elasticity of the respective years.

Table 1 shows the industrial output elasticities for 1995, 1998 and 2000 derived from applying the scenario of a capital cost reduction of 1%.

The negative signs indicate that the capital cost decrease in the IT industry increases output. The cost and price decreases in IT product, which occur through the improvement of production processes in the IT industry, will lead to output increases in the industries denoted with negative signs. Industries using IT products to promote commodity production at cheaper costs find their business will flourish in contrast with industries placing lesser emphasis on the use of IT products. On the contrary, the positive signs annotated on Table 1 indicate that the capital cost decrease in the IT industry decreases the output of the relevant industries. One can interpret this data as revealing that the development of the IT industry affects negatively the production process in those particular industries.

The changes in output of all industries are derived from the transactions of commodities produced in all sectors in the economy. As we assume no change in the final demand (no
income effect), the value of output change in the output elasticity reflects only the change in the intermediate input for the production of the good or service produced by each particular industry (substitution effect). The change in use of intermediate goods and services by a single industry is the aggregation of the intermediate input flow of its inter-industry and intra-industry transactions. This change implies substitution of intermediate input to produce goods or services by changing the input combinations.

The industries that could be interpreted as IT-friendly industries are those industries showing negative figures in Table 1. Of these three industries, #15, Precision instruments industry, changes into a non-IT friendly industry over the course of five years. Only one industry, #14, Household electrical appliances, remains IT friendly as the IT industry itself develops. Three out of thirty one industries show increases in output in 1995 and 1998 while two industries show increases in 2000. These fractional numbers such as 3/31 (1/10) or 2/31 (1/15) are much less than the number, 10/58 (1/6) that was drawn in the study by Gill et al. (1997) for the US economy.

Contrary to our expectation that there would be an increase in the negative figures annotated in Table 1 with the passage of time, we see the opposite effect instead: the industrial structures reveal a trend in the direction of more positive figures in the output elasticities. In fact, most industries show increasing trends in positive elasticities, which implies that the cost decrease associated with the use of IT leads to less production. Despite having been targeted by the Korean government as a strategic industry with the intent of alleviating the economic crisis of 1997, even the IT industry itself shows decreasing effect in its elasticity with the passage of time. The IT industry's output elasticity is gradually decreasing (in absolute value) from -0.059724 in 1995, -0.052387 in 1998, to -0.042919 in 2000. A 1% decrease in production cost resulting from technological innovation in the IT industry lessens its effect on production even after the decision by the government to target it as a strategic investment industry. These trends can be interpreted as revealing that the policy implementation by the Korean government has been inefficient.

To see how the industrial elasticities change over five years, we can compare the elasticities of 1995 and 2000. Figure 1 shows the changes in the elasticity of the individual industry of manufacture and service over the course of five years. From this bar chart, we can see that most of the industries show increased output elasticities, which means decreases in output with respect to the cost reduction resulting from the use of IT products. In other words, the growth of output increase has decreased as a result of the development of the IT industry during this period. The industrial structure for production became less sensitive to the growth of IT industry over the course of the five year period.

The single exception to this general trend and the one industry that actually demonstrated a production increase with respect to cost reduction in the IT industry is #14, Household electrical appliances, even if its decrease seems to be minute in the graph. The decrease in the output elasticity (the output increase with respect to the cost reduction) from -0.00068 in 1995 to -0.00068 in 1995

5 This transaction of industry-level is the extension of B2B of firm-level for using the products of other firms for production process.

6 Primary industry (#1 Agriculture, forestry and fishery and #2 Mining and quarrying) and #31 Dummy sector are left out and not considered. Industry 3 through 17 is categorized as manufacturing industries and Industry 18 through 30 except 24 are categorized as service industries. Industry 24 is Information Technology industry involving Information + Communication Technology related goods and services.
to -0.000736 in 2000 in Table 1 occurs only in this industry out of all the 31 industries under review. This industry shows a positive response of increased production in response to technological innovations in the IT industry. On the contrary, the IT industry itself shows the highest bar in growing by 0.017 (from -0.059724 in 1995 to -0.042919 in 2000) which indicates it became less productive over the five year period. This might be interpreted as demonstrating inefficient investment in the IT industry. In summary, most of the positive values of change in output elasticities in Figure 1 provide a clue to the conclusion that the growth rate of production in the overall economy actually decreased in this period.

**Table 2. Changes in Industrial Sector by Year**

<table>
<thead>
<tr>
<th>Year</th>
<th>Sector</th>
<th>Output data (million won)</th>
<th>%</th>
<th>Changed effect (million won)</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>1995</td>
<td>Primary</td>
<td>35,197,190</td>
<td>4.1826</td>
<td>35,192,826</td>
<td>4.1822</td>
</tr>
<tr>
<td></td>
<td>Manufacturing</td>
<td>352,079,319</td>
<td>41.8386</td>
<td>352,041,965</td>
<td>41.8356</td>
</tr>
<tr>
<td></td>
<td>Service</td>
<td>390,297,696</td>
<td>46.3802</td>
<td>390,271,678</td>
<td>46.3787</td>
</tr>
<tr>
<td></td>
<td>IT</td>
<td>63,944,358</td>
<td>7.5987</td>
<td>63,982,788</td>
<td>7.6035</td>
</tr>
<tr>
<td></td>
<td>Total</td>
<td>841,518,563</td>
<td>100</td>
<td>841,489,257</td>
<td>100</td>
</tr>
<tr>
<td>1998</td>
<td>Primary</td>
<td>36,082,172</td>
<td>3.3945</td>
<td>36,075,342</td>
<td>3.3940</td>
</tr>
<tr>
<td></td>
<td>Manufacturing</td>
<td>426,833,063</td>
<td>40.1552</td>
<td>426,781,400</td>
<td>40.1520</td>
</tr>
<tr>
<td></td>
<td>Service</td>
<td>500,580,000</td>
<td>47.0931</td>
<td>500,543,207</td>
<td>47.0915</td>
</tr>
<tr>
<td></td>
<td>Total</td>
<td>1,062,958,081</td>
<td>100</td>
<td>1,062,915,225</td>
<td>100</td>
</tr>
<tr>
<td>2000</td>
<td>Primary</td>
<td>40,934,810</td>
<td>2.9388</td>
<td>40,924,544</td>
<td>2.9382</td>
</tr>
<tr>
<td></td>
<td>Manufacturing</td>
<td>534,341,825</td>
<td>38.3611</td>
<td>534,259,842</td>
<td>38.3578</td>
</tr>
<tr>
<td></td>
<td>Service</td>
<td>658,699,898</td>
<td>47.2889</td>
<td>658,627,187</td>
<td>47.2869</td>
</tr>
<tr>
<td></td>
<td>IT</td>
<td>158,951,238</td>
<td>11.4113</td>
<td>159,019,880</td>
<td>11.4170</td>
</tr>
<tr>
<td></td>
<td>Total</td>
<td>1,392,927,771</td>
<td>100</td>
<td>1,392,831,453</td>
<td>100</td>
</tr>
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</table>
When we trace the structural change from 1995 to 2000, IT industry has grown itself as we can see in Table 2. Due to the government's strategic support on IT industry, we can see its rapid growth from 7.5987% to 11.4113% over the past 5 years. When we compare the actual output data with the changed effect converted from the output elasticities, we can see the IT's growth without making other industrial sector bigger. This implies that the IT development does not accompany development of other sectors because of the lack of IT friendly input substitution.

By comparing the elasticities, we can see the decreasing trend in production activities. However, a statistical test is needed to make a decisive conclusion that the development of IT is unable to affect change in the production of the Korean economy. To make an overall assessment as to whether IT has affected the production of the Korean economy, we used Repeated Measures Design. The standard errors of difference between 1995 and 1998 ($\sigma_1$), between 1995 and 2000($\sigma_2$), and between 1998 and 2000($\sigma_3$) were used for testing the hypothesis.\(^7\) The F value for this test turns out to be 3.00 with the P value(Pr > F = 0.0573) with which we cannot reject the null hypothesis at 5% level of significance. Or as the P value is closer to 5%, we can barely reject the null hypothesis for concluding that IT has a capability to affect the production. Because the F value for rejecting the null hypothesis is near and around the cutoff value, we could interpret it in both ways.

What is clear from the changing trend of elasticities is that the development of the IT industry ineffectively affects the production activities of the Korean economy. The strategically chosen IT industry did not lead to the change in the productivity of the economy if we fail to reject the null hypothesis. Alternatively, it had a negative effect on productivity if we reject the null hypothesis. Either way, we cannot say that positive contributions from IT do not exist in the industrial structure for production.\(^8\) We can conclude that price reductions in IT products cannot provide most industries with the chance to use more IT product and other intermediate inputs that become relatively cheaper. Korean industrial production structure has not yet matured sufficiently to internalize the impact of technological innovation from the IT industry.

### IV. Concluding Remarks

Over the span of five years, the response to the development of the IT sector falls short of our expectations. Instead, the results of our analysis were contrary to our expectations in that they revealed an ineffective trend in output change. The industries that were favorable to the development of IT declined from three to two out of thirty one industries over the course of five years. Not only were the number of IT friendly industries in the whole industrial structure few and far between, but overall industrial output decreased, showing a lack of receptivity to the IT industry.

In 2000, as Korea began emerging from the economic impasse associated with IMF

\(^7\) Ho: $\sigma_1 = \sigma_2 = \sigma_3$

\(^8\) Paired observations test(Ho: $\mu_2 \geq 0$, Ha: $\mu_2 < 0$) dealing with two year data, 1995 and 2000, reveals the t value (-1.73) with the P value(0.047). This means that there has been a decrease in industrial output with respect to the development of IT industry in five years at 5% level of significance. This supports the negative effect of IT on the economy even if the F value represents a positive value(3.00).
controls, we can see that the Korean economic structure is not yet on the economies of scale through these unfavorable responses to IT innovation. The development of IT did not influence the other industries, except for a few industries producing consumer durables. This is a result similar to that found by Gordon (2000). The assertion that there is a positive correlation between the overall share of the economy enjoyed by the IT industry and the rate of growth in productivity can no longer be accepted as valid based on the results of this analysis. The result is in contrast to the study produced by Jorgenson (2001) that productivity growth in IT-producing industries has risen in importance and that a productivity revival is under way in the rest of the U.S. economy.

The government has emphasized the cultivation of the IT industry as a potential breakthrough for easing the economic crisis in 1997 and entering into a knowledge based economy. The Government-led “informatization policy” has mainly concentrated on support of capital investment in hardware infrastructure. Korea has established an IT infrastructure that is on a par with the level of advanced countries. Certainly, Korea could not have achieved this level of infrastructure and IT development without the positive role of government. In this environment, however, inter-industry transactions were not positively stimulated to yield more efficient production processes among most of the industries. This beneficial infrastructure has not been utilized as a production means among the economic actors to lower the production costs through inter-industry transactions. From this we can infer that firm-level transactions, such as B2B, were not initiated for the substitution of input due to the policy of supporting development of the IT industry. There had been not much B2B where on-line transactions between firms can create value and wealth on the basis of competition.

In an effort to compensate for the IT industry’s inability to facilitate inter-industry transactions, incentives should be provided for software development that will encourage economic actors to use the existing IT infrastructure. There should be a shift from a short term focus on developing a world class IT infrastructure to a focus on developing strategies that will encourage economic actors in the Korean economy to actually use and obtain optimum benefits from the infrastructure. To date, governmental intervention, not the private sector, has been the primary factor responsible for Korea’s progress as an information based economy. This governmental strategy finds its roots in a fundamental belief in the infinite potential of IT rather than rational applications of market principles. The government should implement market-oriented IT policies, so as to encourage economic actors to adopt the new technology, with incentives to drive down costs. This will enhance industrial productivity.

REFERENCES


The Variable Input-Output (VIO) model (Liew, 1979) is established by transforming a Cobb-Douglas' production function into log-linear type:

\[
\ln x_j = \alpha_{0j} + \sum_i \alpha_{ij} \ln x_i + \sum_k \beta_{kj} \ln L_{kj}
\]  

(1)

The production equation (1) is assumed to be homogeneous degree of one, which can be expressed as \(\sum \alpha_{ij} + \sum \beta_{kj} = 1\). The equation (1) is used as a technical constraint for the profit maximization of firms or industries. The profit equation is based upon the assumption of zero profit.

Profit equation: 

\[
p_j x_j - \sum_i p_i x_i - \sum_k w_{kj} L_{kj} = 0
\]  

(2)
Where

- $x_j$: output of j industry,
- $x_{ij}$: commodity (intermediate medium) of i industry purchased by j industry,
- $L_{kj}$: amount of labor or capital employed in j industry as value added,
- $p_j$: commodity price of j industry,
- $p_i$: commodity price of i industry,
- $w_j$: unit price of primary production factors (labor or capital) purchased by j industry.

Using eq(1) and eq(2), the Lagrangian Equation for profit maximization is set up by enlarging the difference between revenue and cost subject to technical constraint.

$$\text{Max } \Pi = \sum_j (p_j x_j - \sum_i p_i x_{ij} - \sum_k w_k L_{kj})$$

$$+ \sum_{l} \lambda \left( \ln x_j - \alpha_{oj} - \sum_i \alpha_{ij} \ln x_{ij} - \sum_k b_{kj} \ln L_{kj} \right)$$

From the first order condition, the optimal levels of employment of intermediate input ($x_{ij}$) and primary input ($L_{kj}$) are derived.

$$x_{ij} = p_j \alpha_{ij} x_j / p_i,$$
$$L_{kj} = p_k \beta_{kj} x_j / w_k$$

These optimum values of intermediate medium and primary input in (3) into eq.(1) in order to obtain a following price function.

$$\ln p = (I - A')^{-1} [\sum_k \beta_k \ln w_k]$$

This price equation is the price frontier derived from duality condition of production frontier of technical constraint.

In the above equation(4), primary input (labor or capital) price ($w_k$) is the only variable to affect the commodity prices ($p_i$). The cost variation in the primary input of a certain industry affects prices of all industries’ commodities. These changed prices are reflected in a basic equation of IO table as follows:

$$x = (I - \hat{p}^{-1} A \hat{p})^{-1} f = \hat{p}^{-1} (I - A)^{-1} \hat{p} f$$

where the “$\wedge$” sign indicates a diagonal matrix of price.

As the VIO model is based on the Cobb-Douglas production function that is homogeneous of degree one, the system allows input substitution.\(^9\) Thus the output equation with the changes of relative prices is for the production activities change through the varied technological coefficients, $A$. If we premultiply the output equation (5) by $(I - A)\hat{p}$ on both sides, we get $\hat{p} x - A \hat{p} x = \hat{p} f$.

When we totally differentiate $\hat{p} x - A \hat{p} x = \hat{p} f$, we obtain $d\hat{p} x + \hat{p} d x = (A (d \hat{p} x + \hat{p} d x)) = d \hat{p} f + \hat{p} df$.

\(^9\) For a detailed description about Leontief’s works, see P. A. Samuelson, Activity Analysis of Production and Allocation (Wiley, 1951), p.142
So we derive $dx$ consisting of two terms in the following equation (6).

$$dx = \hat{p}^{-1}(I-A)^{-1}(d\hat{p}f + \hat{p}df) - \hat{p}^{-1}d\hat{p}x$$

$$= \hat{p}^{-1}(I-A)^{-1}(\hat{p} \hat{p}^{-1}d\hat{p}f + df) - \hat{p}^{-1}d\hat{p}x$$

$$= \hat{p}^{-1}(I-A)^{-1}(\hat{p} \hat{p}^{-1}(\hat{p} \hat{p}^{-1}d\hat{p}) + df) - \hat{p}^{-1}d\hat{p}x$$

$$= \hat{p}^{-1}(I-A)^{-1}(\hat{p} \hat{p}^{-1}(\hat{p} \hat{p}^{-1}d\hat{p}) + df) - \hat{p}^{-1}d\hat{p}x$$

$$= [\hat{p}^{-1}(I-A)^{-1}\hat{p} \hat{p}^{-1}d\hat{p}] + \hat{p}^{-1}(I-A)^{-1}df$$

$$= [\hat{p}^{-1}(I-A)^{-1}\hat{p} \hat{p}^{-1}d\hat{p}] + \hat{p}^{-1}(I-A)^{-1}df$$

$$= \hat{p}^{-1}(I-A)^{-1}\hat{p} \hat{p}^{-1}d\hat{p} + [\hat{p}^{-1}(I-A)^{-1}\hat{p} \hat{p}^{-1}d\hat{p}] + \hat{p}^{-1}(I-A)^{-1}df$$

$$= \hat{p}^{-1}(I-A)^{-1}\hat{p} \hat{p}^{-1}d\hat{p} + [\hat{p}^{-1}(I-A)^{-1}\hat{p} \hat{p}^{-1}d\hat{p}] + \hat{p}^{-1}(I-A)^{-1}df$$

We call the first term of the equation above “substitution effect” and the second term “income effect.” To compute the output elasticities with respect to cost, we take the substitution portion of the differential output equation and multiply an inverse of a diagonal matrix of output$(\hat{x})$.

$$(\hat{x})^{-1}dx = (\hat{x})^{-1}[\hat{p}^{-1}(I-A)^{-1}\hat{p} \hat{p}^{-1}d\hat{p} + [\hat{p}^{-1}(I-A)^{-1}\hat{p} \hat{p}^{-1}d\hat{p}] + \hat{p}^{-1}(I-A)^{-1}df]$$

$$dlnx = (\hat{x})^{-1}[\hat{p}^{-1}(I-A)^{-1}\hat{p} \hat{p}^{-1}d\hat{p} + [\hat{p}^{-1}(I-A)^{-1}\hat{p} \hat{p}^{-1}d\hat{p}] + \hat{p}^{-1}(I-A)^{-1}df]$$

$$dlnx/dlnw_k = (\hat{x})^{-1}[\hat{p}^{-1}(I-A)^{-1}\hat{p} \hat{p}^{-1}d\hat{p} + [\hat{p}^{-1}(I-A)^{-1}\hat{p} \hat{p}^{-1}d\hat{p}] + \hat{p}^{-1}(I-A)^{-1}df]$$

As we deal with only the intermediate demand change, the second term $df(=0)$ in as an income effect in equation (6) will be not considered and the first term having $dlnw_k$ as an input substitution effect will be left in equation (7) above. In other words, through this equation we derive the changes of output of all industries resulting from the decrease in capital cost($w_k$). The output elasticities with respect to the input price change, $(dlnx/dlnw_k)$, of industries are represented in the relevant column $(24^{th}$ column) of the matrix in equation (7).

$$(dlnx/dlnw_k) = (\partial x/x)/(\partial w_k/w_k) = (\partial x/\partial w_k)(w_k/x);$$

In the input-output table, the capital cost($w_k$) includes royalties paid to the foreign firms. We assume that the cost on royalties in IT industry is decreased by 1% through the technological innovation. The percentage decrease in royalties($dlnw_k$) will lead to percentage changes in output($dlnx$) of all industries through the affected prices($\hat{p}$) of commodities of all 31 industries on the right side of equation (7). The changes in ($\hat{p}$) are derived by taking anti-log on $lnp$ in equation (4) and used in equation (7) for obtaining the output elasticities.