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<td>Ogawa, Eiji; Kudo, Takeshi</td>
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How much depreciation of the US dollar for sustainability of the current accounts?*

Eiji Ogawa† and Takeshi Kudo‡

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† Graduate School of Commerce and Management of Hitotsubashi University, e-mail: ogawa.eiji@srv.cc.hit-u.ac.jp
‡ Graduate School of Economics of Hitotsubashi University, e-mail: ged2403@srv.cc.hit-u.ac.jp
1. Introduction

The United States have been faced with the increasing current account deficits in the recent years. Its current account deficits were recorded over 5 percent of GDP in 2002. We remember that the current account deficits were over 3 percent of GDP in the mid of 1980s when the US dollar made a large depreciation after the Plaza Accord in September 1985. It is regarded that the recent current account deficits are going beyond a dangerous level by comparing the recent situation with that in the mid of 1980s.

Though the US dollar began to depreciate several months before the Plaza Accord, the depreciation of the US dollar gained momentum by the Plaza Accord. The real effective exchange rate of the US dollar depreciated nearly 40% from the peak in the early 1985 to the early 1988. Following the depreciation, the current account deficit was reduced from 3.4% in the last quarter of 1986 to 1.4% in the second quarter of 1990 (see Figure 1).

Some researchers doubt that such the current account deficits of the United States are sustainable in the current level of the exchange rates because the current account deficits began to increase again and have reached to 5% of GDP. This paper investigates how much the US dollar should be depreciated for reducing the current account deficits in the United States.

This paper quotes our empirical analytical results from Kudo and Ogawa (2003) to explain unsustainable current account deficits in the United States in the next section. In Section 3, we conduct a simulation analysis to investigate how much depreciation of the US dollar is needed to reduce the current account deficits in the near future. We use some VAR models to estimate relationships between the exchange rate of the US dollar and the current accounts in the United States. Then we use the estimated VAR models to conduct the simulation analysis about impacts of hypothetical exchange rate movements on the current account deficits. We suppose five scenarios of exchange rate movements; 10%, 30%, and 50% of depreciation of the US dollar in the second quarter in 2004 and depreciations of the US dollar in the same ways as the post Plaza Accord and the Indonesian currency crisis from 1997 to 1998.
2. Unsustainable Current Account Deficits in the United States

In this section, our empirical analytical results in Kudo and Ogawa (2003) are explained as for sustainability of the current account deficits in the United States. We used the method of Bohn (1995) and Ahmed and Rogers (1995) in order to derive the necessary and sufficient conditions. The sustainability of the current account deficits was empirically analyzed from a perspective based on international capital flows in addition to perspectives of domestic investment-saving relationship and international trade flows according to Mann (2002). We investigated whether the current account is sustainable in the sense of the external debt solvency.

2.1. Methodology and Data

Kudo and Ogawa (2003) conducted empirical analyses on the sustainability of the current account deficits from perspectives of the domestic investment-saving relationship, the international trade flows, and the international capital flows according to Mann (2002). Their theoretical backgrounds are explained in the Appendix.

In our empirical analysis based on investment-saving balance, we represent the repayment for the external debts \( r_tD_{t-1} \) as RD, the private savings \( S_t \) as PS, the private investments \( I_t \) as PI. We use data on the private gross savings and investments as PS and PI, respectively. We replace the government expenditure \( G_t \) by the government gross investment \( GE_t \) and the tax revenue \( T_t \) by the government gross saving \( GS_t \). In addition, we make data series of the national gross saving \( NS_t \) and the national gross investment \( NI_t \). We also make data series of the investment-saving balances of the private sector \( PIS_t \) and the public sector \( GIS_t \) as well as the national investment-saving balance \( NIS_t \).

In our empirical analysis based on international trade flows, we represent the exports of goods and services \( X_t \) as EX and the imports of goods and services \( M_t \) as IM. In addition, we make data series of a sum the repayment for external debt \( r_tD_{t-1} \) and
the imports $M_t$, which is represented as MM. We also use the trade balance TB. We also test directly whether the current account deficit CAD is stationary.

In our empirical analysis based on international capital flows, we represent the change in foreign reserve $\Delta R_t$ as RES, the capital inflows $Fin_t$ as FIN, and the capital outflows $Fout_t$ as FOUT. In the analysis on the items in financial account, we use the direct investment inflow DIIN, the portfolio investment inflow PIIN, and the other investment inflow OIIN, and the direct investment outflow DIOUT, the portfolio investment outflow PIOUT, and the other investment outflow OIOUT. In addition, we make data series on the direct investment balance DIB, the portfolio investment balance PIB, and the other investment balance OIB.

We used the Johansen's method to investigate whether the relevant variables are cointegrated. We used the unit-root tests on the relevant variables in the systems to investigate whether all the variables are the elements of the cointegration in advance. If the variables are relevant to the cointegration system, they are expected to follow the same order integration processes. As the result, we can find that the system is cointegrated.

We test whether the conditions of the cointegration vector are satisfied, for the systems in which all variables are cointegrated. If the system passes all of the tests, we can conclude that the condition of the current account sustainability is satisfied. Based on the analysis in the preceding section, we analyze the sustainability of the current account.

The original variables and the standardized variables by GDP are prepared for all of the data. Most of the data in the analysis based on the domestic investment-saving balance are taken from the “National Income and Production Account Tables” by the Bureau of Economic Analysis. The balance of payments data are taken from the “International Transactions Accounts”. All of the data were seasonally adjusted.

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1 We use the table 1 in Osterwald-Lenum (1992) as the critical value here.
2 Noticing that the linear restriction which is described in previous section is imposed on the cointegration vector, Miyao (2001) tests the cointegration by using the framework of the Engle-Granger test. Though he carries out unit-root test on the series of RD+IM-EX, this is similar to carry out the Engle-Granger test on the system of RD, IM, EX by imposing the restriction (1,1,-1) on the cointegration vector.
sample period of the data covers from the first quarter of 1960 to the fourth quarter of 2002. The number of observations is 172.

2.2. Empirical Results from the Perspective on the Domestic Investment-Saving Balance

In this subsection, we investigate the current account sustainability from the perspective based on the domestic investment-saving balance. We consider the following pattern as

(1) \( RD+PI+GE-PS-GS \),
(2) \( RD+NI-NS \),
(3) \( RD+PIS+GIS \),
(4) \( RD+NIS \).

Equation (1) is the same as the system in equation (A7) in Appendix. In equation (2), we define the national investments NI as a sum of private investments PI plus government investments GE and the national savings NS as a sum of private savings PS plus government savings GS. This means we analyze the whole economy’s investment-saving relationship. In equation (3), we use investment-saving balance of both the private and public sectors. We analyze the national investment-saving balance in equation (4).

In the case of using the non-standardized data, the ADF test rejected a unit-root for the government savings GS in equation (1) (Table 1.1). In the case of using the data standardized by GDP, a unit-root is rejected in the private investments PI and the government savings GS in equation (1).

In the case of using the non-standardized data, the ADF test did not reject any unit-root for all variables in equation (2) (Tables 1.1 and 1.2). The cointegration test showed that this system has full rank in the cointegration relationship but that this is contradiction to the assumption of this test (Table 1.3). In the case of using the standardized data, a unit-root is rejected for the national savings NS in equation (2).

In the case of using the non-standardized data, the ADF test rejected a unit-root for the private and public sectors’ investment-saving balances, PIS and GIS in
equation (3) (Table 1.1). In the case of using the data standardized by GDP, a unit-root is rejected for the private and public sectors’ investment-saving balances, PIS and GIS in equation (3) (Table 1.1).

In the case of using the non-standardized data, every variable follows a first-order integrated process in equation (4) (Tables 1.1 and 1.2). We conducted the cointegration test for the system of equation (4). The cointegration test cannot reject that the system has no cointegration vector in terms of both the non-standardized data. In the case of using the standardized data, a unit-root is rejected for the national investment-saving balance NIS.

Therefore, each of the systems of equation (1), (2), and (3) is not cointegrated in terms of both the non-standardized and standardized data. On one hand, the system of equation (4) is not cointegrated in terms of the standardized data.

2.3 Empirical Results from the Perspective on the International Trade Flows

We investigate the current account sustainability from the perspective based on the international trade flows. For the cointegration relationship in equation (A10), we consider the following pattern as

\[\begin{align*}
(5) & \quad RD+IM-EX, \\
(6) & \quad MM-EX, \\
(7) & \quad RD-TB, \\
(8) & \quad CAD.
\end{align*}\]

Equation (5) follows directly the definition in equation (A10) in Appendix. Next, we use MM rather than RD and IM in equation (6). In equation (7), we use the trade balance TB rather than the imports and the exports. In addition, we conduct a unit-root test for the current account deficit CAD itself in equation (8).

In the case of using the non-standardized data, the ADF tests show that a unit-root is rejected for the imports IM in equation (5) (Table 2.1). Therefore, this system has no cointegration relationship in terms of the non-standardized data. In the case of using the standardized data, we cannot reject the repayment for the external
debt RD and imports IM following an I(2) process while the exports EX follows a first-order integrated process (Tables 2.1 and 2.2). We regard that the power of the ADF test is very weak and conduct the cointegration test for this system.

In the case of using the non-standardized data, we can find that the sum of the imports and repayment for the external debts MM follows a first-order integrated process and that the exports EX follows a second-order integrated process in equation (6). Since the power of the ADF test is weak, we conduct the cointegration test for the system of equation (6). We obtain a result that the system has a cointegration vector. We also test whether a linear restriction on the cointegration vector is satisfied. As a result, the test rejected the null hypothesis of a linear restriction on the cointegration vector. One hand, in the case of using the standardized data, a unit-root is rejected for the exports EX.

In the case of using the non-standardized data, all variables in this system follow first-order integrated processes in equation (7) (Tables 2.1 and 2.2). The cointegration test found that this system has no cointegration vector (Table 2.3). In the case of using the standardized data, a unit-root is rejected for the trade balance TB.

In equation (8), the stationarity of the current account deficit CAD is the condition of the current account sustainability. We investigate whether this condition is satisfied. Table 3.1 shows that we cannot reject any unit-root for the current account deficit.

Therefore, each of the systems of equations (5), (6), and (7) are not cointegrated. On one hand, the system of equation (8) has a unit root for the current account deficit. Thus, these results show that the U.S. current account deficit is unsustainable from the perspective based on the international trade flows.

2.4. An Analysis on the Finance for Current Account Deficits

We investigated the U.S. current account sustainability from the perspectives based on the domestic investment-saving relationships and on the international trade flows. These analytical results show that the U.S. current account deficit is not
sustainable. Next, we investigate which items in the international capital inflows finance the current account deficit in the long run.

First, we analyze the cointegration relationship among the current account deficit, the international capital flows, and the change in the foreign reserves. We conduct unit-root tests for relevant variables in advance. The results are shown in Table 3.1. The results is that the unit-root is rejected for the change in the foreign reserves $\Delta R_t$. The empirical results in the previous section showed that the current account deficit $CAD_t$ is non-stationary. Therefore, the current account deficit $CAD_t$ and the international capital flows $FB_t$ should be cointegrated in equation (A11) in Appendix in order to be consistent with the fact that the change in the foreign reserves $\Delta R_t$ is stationary.

The results of unit-root and cointegration tests on the current account deficit and the international capital flows are shown in Tables 3.1, 3.2, and 3.3. The results of unit-root tests in the case of using the non-standardized data is that a second-order integration is not rejected for the financial balance $FB$ while the current account deficit $CAD$ follows a first-order integration process. In the case of using the standardized data, the financial balance $FB$ and the current account deficit $CAD$ follow a first-order integration process.

We also conduct cointegration tests between the current account deficit and the financial balance. The results are shown in Table 3.3. In the case of using the non-standardized data, the rank of cointegration is full-rank and it contradicts with the assumptions. In the case of using the standardized data, we can find a cointegration vector in the system that includes the current account deficit $CAD$ and the financial balance $FB$.

Next, we conduct the analysis by decomposing the financial balance $FB$ into the direct investment balance $DIB$, the portfolio investment balance $PIB$ and the other investment balance $OIB$. Because the change in foreign reserves $\Delta R$ is stationary, some of the other variables ($DIB$, $PIB$, and $OIB$) in equation (A12) in Appendix should

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3 Though it is not rejected for $FB$ to follow the second-order integrated process, we carried out the cointegration test on the system since it is said that the power of ADF test is weak.
be cointegrated. The unit-root tests show that the current account deficit and the portfolio investment balance follow first-order integrated processes.

Table 3.3 shows that the cointegration rank is 2 among the variables in the case of using the non-standardized data. The cointegration rank is 1 among the variables in the case of using the standardized data. Thus, the cointegration has full-rank and it contradicts with the assumptions of the analysis in the case of using the non-standardized data. On one hand, there is a cointegration vector in the system which includes the current account deficit and the portfolio investment balance in the case of using the standardized data. Accordingly, we can conclude that the huge current account deficit in the United States has been financed by the portfolio investment from other countries in the long run in terms of the stationary relationship.

3. Simulation Analysis on Depreciation of the US Dollar for Sustainable Current Account Deficits

In this section, we investigate how impact depreciation of the US dollar would give on the current account deficits in the United States and how much depreciation of the US dollar is needed to make the current account deficits sustainable.

3.1. Methodology and Data

We simulate how much depreciation the US dollar is needed for its current account sustainability by using the estimated parameters of vector autoregression (VAR) models. Three VAR models are estimated in our analysis. The first model (Model 1) is a 2 variables VAR model which contains the exchange rate and the current account. The second model (Model 2) is a 3 variables VAR model which contains the exchange rate, trade balance and factor income receipt from abroad from a viewpoint of international trade flows. The last model (Model 3) is a 3 variables VAR model which contains the exchange rate, saving-investment balances for the private and the public sectors from a viewpoint of domestic investment saving balance.
We suppose some cases of exchange rate movements in order to simulate their effects on the current account deficits. The supposed cases are that the US dollar will sharply depreciate in the second quarter of 2004. We suppose three cases where the US dollar will depreciate against its trading partners’ currencies in terms of the real effective exchange rates by 10%, 30%, and 50% in the second quarter of 2004. In addition, we suppose two hypothetical movements of the exchange rate. One is that the US dollar is supposed to make similar movements as the actual movements after the Plaza Accord during the three years after the Plaza Accord. The other is that the US dollar is supposed to make similar movements as the actual movements during the Indonesian currency crisis period from the third quarter of 1997 to the second quarter of 1998.

In addition to the data used in the previous sections, we use the real effective exchange rate of the US dollar as one of the vector in the three VAR models. The real effective exchange rate data is taken from the IMF’s *International Financial Statistics*.

Before we estimate the three VAR models, we test the stationarity of relevant variables by using the Augmented Dickey-Fuller (ADF) tests. The results are shown in Table 4. The null hypothesis of non-stationarity is not rejected for all of the variables at 5% significance level. Next, we test cointegration for the three VAR models. The results are shown in Table 5. The first and third VAR models are not cointegrated while the second VAR model is cointegrated. The estimated cointegration vector of the second VAR model is shown in Table 6. Considering the results as stated above, we estimate the differenced variables VARs in addition to the original data VARs, and the vector error correction model (VECM) for the second VAR model.

### 3.2. VAR Models

We estimate the three VAR models in this analysis. The first VAR model (Model 1) is the two-variable VAR contains the exchange rate and the current account. In the second model (Model 2), we decompose the current account into the trade balance and the income receipt. On the other hand, from a viewpoint of the domestic investment
saving balance, the third VAR model (Model 3) contains the exchange rate and the saving-investment balances for the private and the public sectors.

The results of estimating Model 1 estimation are shown in Table 7. Almost all of the estimates in terms of levels are significant at 5% level while all of the estimates in terms of log difference are not significant. The estimated parameters of Model 2 are shown in Table 8. Most of the parameters are significantly estimated at 5% significance level in the original variables estimation, while the estimates in difference variables estimation are not significant. In the error correction model estimation, all variables except for the income receipt equation are not significant. The results of Model 3 are shown in Table 9. In the VAR estimation using original level variables, almost all of the estimates are significantly estimated though all of the estimates are not significant in the VARs using the difference variables.

3.3. Results of Simulation Analysis
3.3.1. Impacts of Depreciation of the US Dollar on the Current Account Deficits

In this subsection, we show results of the simulation analysis based on the three estimated VAR models for some scenarios of the US dollar depreciation. At first, we suppose three cases where the US dollar will depreciate against its trading partners’ currencies in terms of the real effective exchange rates by 10%, 30%, and 50% in the second quarter of 2004. In addition, we suppose two hypothetical movements of the exchange rate. One is that the US dollar is supposed to make similar movements as the actual movements after the Plaza Accord during the three years after the Plaza Accord. The other is that the US dollar is supposed to make similar movements as the actual movements during the Indonesian currency crisis period from the third quarter of 1997 to the second quarter of 1998.

At first, we simulate the current account behavior if the US dollar were sharply depreciated by 10% in the second quarter of 2004. Figure 2 shows a current account behavior that is obtained by the simulation analysis based on Model 1. Figure 3 shows a
current account behavior based on that is obtained by the simulation analysis based on Model 2. Figure 4 shows a current account behavior based on that is obtained by the simulation analysis based on Model 3. The 10% depreciation would gradually reduce the current account deficits to 2% of GDP by 2018 in the cases of Models 1 and 2. On one hand, it would reduce the current account deficits to 2% of GDP by 2008.

Next, we simulate the current account behavior if the US dollar were sharply depreciated by 30% in the second quarter of 2004. Figure 5 shows a current account behavior that is obtained by the simulation analysis based on Model 1. Figure 6 shows a current account behavior based on that is obtained by the simulation analysis based on Model 2. Figure 7 shows a current account behavior based on that is obtained by the simulation analysis based on Model 3. The 30% depreciation would reduce the current account deficits to 2% of GDP by 2011 and then to 1.6% of GDP in 2018 in the cases of Models 1 and 2. On one hand, it would reduce the current account deficits to 1.3% of GDP in 2008 and then increase it to 2.5% in 2020 in the case of Model 3.

Moreover, we simulate the current account behavior if the US dollar were sharply depreciated by 50% in the second quarter of 2004. Figure 8 shows a current account behavior that is obtained by the simulation analysis based on Model 1. Figure 9 shows a current account behavior based on that is obtained by the simulation analysis based on Model 2. Figure 10 shows a current account behavior based on that is obtained by the simulation analysis based on Model 3. The 50% depreciation would reduce the current account deficits to 0.8% of GDP by 2013 in the case of Model 1 and to 1% of GDP by 2015 in the case of Model 2. On one hand, it would reduce the current account deficits to 0.5% of GDP in 2008 and then increase it to 2.8% in 2020 in the case of Model 3.

We suppose two more scenarios of the US dollar depreciation. The first case is that the exchange rate of the US dollar from the last quarter of 2003 to the third quarter of 2006 move in the same way as the exchange rate of the US dollar actually moved after the Plaza Accord. Figure 11 shows a current account behavior that is obtained by the simulation analysis based on Model 1. Figure 12 shows a current account behavior based on that is obtained by the simulation analysis based on Model 2. Figure 13 shows a current account behavior based on that is obtained by the simulation analysis based on Model 3.
analysis based on Model 3. The exchange rate movements would reduce the current account deficits to 2% of GDP by 2010 and then to about 1% in 2016 in the cases of Models 1 and 2. On one hand, it would reduce the current account deficits to about 1% of GDP in 2009 and then increase it to 2.8% of GDP in 2020.

The second case is that the exchange rate of the US dollar depreciates from the last quarter of 2003 in the same way as the Indonesia rupiah depreciation in the Asian currency crisis from the second quarter of 1997 to the first quarter of 1998. Figure 14 shows a current account behavior that is obtained by the simulation analysis based on Model 1. Figure 15 shows a current account behavior based on that is obtained by the simulation analysis based on Model 2. Figure 16 shows a current account behavior based on that is obtained by the simulation analysis based on Model 3. The exchange rate movements would sharply reduce the current account deficits to 2% of GDP by 2006 in the cases of Models 1 and 2. The current accounts would be surplus in the case of Model 1 and equilibrium in the case of Model 2 in 2013. After then, the current account deficits would be 1% of GDP in 2020. On one hand, the current account deficits would reduce to 2% of GDP in 2005 and then turn to surplus by 2007. However, the current accounts would turn to deficit and then increase to about 3% in 2017.

3.3.2. Depreciation of the US Dollar and Sustainability of the Current Account

We investigate whether each series of the simulated current account deficits is sustainable. The Augmented Dickey-Fuller (ADF) test is used to investigate the sustainability of the current account deficits. The analytical results can conclude that the simulated current account deficits would be sustainable if the null hypothesis of unit-root is rejected by the ADF test. We conduct the unit-root test not only for during the full sample period (from the first quarter of 1976 to the fourth quarter of 2020) but for the forecasted for the sub-sample period (from the fourth quarter of 2003 to the fourth quarter of 2020). While Table 10 shows results of the unit-root tests for the estimated values during a estimation period from the first quarter of 1976 to the third
quarter of 2003, results for the estimated and simulated values in each of the VAR models during the full sample period and the sub-sample period are shown in Table 11.

We find the same tendency from the results of Model 1 and 2. In these models, the null hypothesis of unit-root of the simulated current account cannot be rejected for the full-sample period while the null hypothesis of unit-root can be rejected for the forecasted sub-sample period except for the case of exchange rate movements in the same way as the post Plaza Accord (Case 4).

From the results of the unit-root tests for the simulated current account data based on the third VAR model (Model 3) contains the exchange rate and the saving-investment balances for the private and the public sectors, we find that the null hypothesis of unit-root for the series can be rejected not only for the sub-sample period but also for the full-sample period. Accordingly, we can regard that the simulated current account deficits based on Model 3 are sustainable for all of the cases of supposed exchange rate movements.

4. Conclusion

This paper investigated how much the US dollar should be depreciated for reducing the current account deficits in the United States. We conclude that some scenarios of the US dollar depreciation would reduce the current account deficits to a level under 2% of GDP in the next several years. The results are regarded as robust for each of the scenarios thought they depend on our supposed VAR models. The results were derived from the 2 variables VAR model and the 3 variables VAR models by taking into account relationships between the current accounts and the exchange rates without exogenously reducing fiscal deficits. It is expected that smaller depreciation of the US dollar should reduce the current account deficits if the US government reduced the fiscal deficits at the same time. In other words, the US government should reduce the fiscal deficits in order that it should prevent a large depreciation of the US dollar for reducing the current account deficits and make them sustainable in the near future.

We can regard that the simulated current account deficits based on the third
VAR model (Model 3) contains the exchange rate and the saving-investment balances for the private and the public sectors are sustainable for all of the cases of supposed exchange rate movements. It is not so robust to conclude sustainability of the simulated current account deficits because the result is obtained in only Model 3. However, it is possible to obtain sustainable current account series by taking into account relationships among the exchange rate, the private sector’s saving-investment balance, and fiscal deficits according to Model 3. The result enables us to speculate that the fiscal deficits are the most important factors that would make the current account deficits in the United States sustainable in the near future.

Appendix
A.1. A Perspective Based on the Domestic Investment-Saving Balance

In this appendix, we explain the econometric methods that we use in our analysis and summarize the three perspectives that Mann (2002) pointed out.

As the first perspective, we investigate the relationship among the domestic investment-saving balance, the current account deficit, and the external debts. As we described above, we investigate the investment-saving balance for each of the sectors (private and public sectors). First, the relationship between the change in the external debts in the end of the period \( D_t \) and the current account deficit \( CAD_t \) is represented by

\[
(A1) \quad D_t - D_{t-1} = CAD_t.
\]

The current account deficit increases the external debts as the current account deficit is financed the international capital inflows. This can be interpreted as a “budget constraint” of the whole economy in period \( t \).

Next, we consider both the domestic investment and saving behavior of each of the sectors.\(^4\) The budget constraint of the private sector in period \( t \) is represented by

\[
(A2) \quad A_t - A_{t-1} = r_t A_{t-1} + S_t - I_t,
\]

\(^4\) Matsubayashi (2002) analyzes that each sector’s budget constraint is satisfied from the view of the necessary condition and sufficient condition. But, we will not consider each sector’s budget constraint for focusing on the current account sustainability.
where \( r_t \) is the interest rate, \( A_t \) is the asset holdings by the private sector, which include the claims on the public sectors and foreigners, \( S_t \) is the savings of the private sector, and \( I_t \) is the investments of the private sector.

The budget constraint of the public sector (government) is represented by
\[
(A3) \quad B_t - B_{t-1} = r_t B_{t-1} + G_t - T_t,
\]
where \( B_t \) is the government debts, \( G_t \) is the government expenditures, and \( T_t \) is the tax revenues. The government bonds are held by the private sector and foreigners.

We obtain \( B_t - A_t = D_t \) since the government bond holdings by the private sector equal to the liabilities of the public sector to the private sector. From equations (A2) and (A3), we derive the relationship between the current account deficit and the domestic investment-saving balance as
\[
(A4) \quad CAD_t = r_t D_{t-1} + I_t + G_t - S_t - T_t.
\]

We define the stochastic discount factor of the private sector as
\[
Q_{i,t+1} = \left[ \beta^t u'(C_{i,t+1}) / u'(C_i) \right], \quad \text{where} \quad C_t \text{ is consumption, } u() \text{ is utility function and } u'(t) > 0, u''(t) < 0 \text{ are satisfied, and } Q_{i,0} = 1. \quad \text{The Euler equation of intertemporal consumption is}
\]
\[
(A5) \quad E_t \left[ Q_{i,t+k} \left( \prod_{j=0}^{k} (1+r_{t+j}) \right) \right] = 1.
\]

Substituting equation (A4) into equation (A1), we obtain a difference equation of \( D_t \). We solve forward the equation and use equation (A5) to derive the whole economy's intertemporal budget constraint based on the domestic investment-saving balance:
\[
(A6) \quad E_t \sum_{k=0}^{\infty} (Q_{i,t+k} I_{t+k}) + E_t \sum_{k=0}^{\infty} (Q_{i,t+k} G_{t+k}) - E_t \sum_{k=0}^{\infty} (Q_{i,t+k} S_{t+k})
\]
\[
- E_t \sum_{k=0}^{\infty} (Q_{i,t+k} T_{t+k}) + (1 + r_t) D_{t-1} = \lim_{K \to \infty} E_t (Q_{i,t+k} D_{t+k}).
\]

Now, we consider solvency of the external debts based on the equation (A6). We suppose that the transversality condition \( \lim_{K \to \infty} E_t (Q_{i,t+k} D_{t+k}) = 0 \) to obtain
\[
(1 + r_t) D_{t-1} = E_t \sum_{k=0}^{\infty} Q_{i,t+k} (S_{t+k} + T_{t+k} - I_{t+k} - G_{t+k}).
\]
This means that the external debts at the present time should be equal to the present
value of the net savings in the present and the future because the present value of the external debts in the terminal period to converge to zero in order to satisfy the transversality condition. Thus, the current account sustainability condition of the economy is that the external debts at the present time have to be repaid by the net savings in the present and the future.

Ahmed and Rogers (1995) derived the necessary and sufficient conditions of the current account sustainability by transforming the equation (A6) to an applicable econometric method. According to them, we difference the both sides of equation (A6) to obtain:

\[
\Delta E_t \sum_{k=0}^{\infty} (Q_{t+k} I_{t+k}) + \Delta E_t \sum_{k=0}^{\infty} (Q_{t+k} G_{t+k}) - \Delta E_t \sum_{k=0}^{\infty} (Q_{t+k} S_{t+k}) = \lim_{k \to \infty} E_t (Q_{t+k} T_{t+k}) + (r_i D_{t-i} + I_t + G_t - S_t - T_t)
\]

(A7)

where \(\Delta\) is the difference operator.

From this equation, Ahmed and Rogers (1995) show that the necessary and sufficient conditions of the current account sustainability or the transversality condition is that \(r_i D_{t-i}, I_t, G_t, S_t, T_t\) are cointegrated and have the cointegration vector \((1, 1, 1, -1, -1)\) under some assumptions. We analyze the cointegration among these variables to investigate whether the current account sustainability condition is satisfied.

A.2. A Perspective on the International Trade Flows

Next, we consider the solvency of the external debts from the international trade flows as the second perspective of the current account sustainability. By

---

5 The following conditions should be satisfied. (i) \(I_t, G_t, S_t, T_t\) follow I(1) processes, (ii) the utility function is separable for time, the marginal utility of consumption \(u'(C_t)\) follows a random-walk process, and the subjective discount factor satisfies \(\beta \in (0, 1)\), (iii) all risks are invariant for any time period i.e. the covariance between the stochastic discount factor and each variable is constant, (iv) the series of the external debt follows I(1) process, and (v) the expectation operator \(E_t\) represents the rational expectation. Under these assumptions, Ahmed and Rogers (1995) show that the stationarity of the right hand side of equation (7) is identical to cointegrate the relevant variables.
abstracting the net receipts of labor income and the current transfers in the balance of payments, we can represent the current account deficit as

\[ \text{CAD}_t = r_tD_{t-1} - X_t + M_t, \]

where \( X_t \) is exports of goods and services and \( M_t \) is imports of goods and services.

We substitute equation (A8) into equation (A1) to obtain a difference equation of \( D_t \). We solve forward the difference equations and use equation (A5) to derive the economy's intertemporal budget constraint based on the international trade flows:

\[ \text{E}_t \sum_{k=0}^{\infty} (Q_{t,k+1}M_{t,k}) - \text{E}_t \sum_{k=0}^{\infty} (Q_{t,k+1}X_{t,k}) + (1 + r_t)D_{t-1} \]

\[ = \lim_{K \to \infty} E_t (Q_{t,k+K}D_{t+K}). \]

The transversality condition in equation (A9) means that the initial external debts are repaid by the net exports in the present and the future. We difference the both sides of equation (A9) to obtain:

\[ \Delta \text{E}_t \sum_{k=0}^{\infty} (Q_{t,k+1}M_{t,k}) - \Delta \text{E}_t \sum_{k=0}^{\infty} (Q_{t,k+1}X_{t,k}) + (r_tD_{t-1} - X_t + M_t) \]

\[ = \lim_{K \to \infty} E_t (Q_{t,k+K}D_{t+K}) - \lim_{K \to \infty} E_t (Q_{t,k+K-1}D_{t+K-1}). \]

According to equation (A10), the necessary and sufficient conditions of the current account deficit sustainability should be that \( r_tD_{t-1}, X_t, M_t \) are cointegrated and have the cointegration vector \((1, -1, 1)\). Thus, from the perspective on the international trade flows, we analyze this cointegration relationship to investigate the current account sustainability.

A.3. A Perspective on the International Capital Flows

Finally, we consider the condition of the current account sustainability from the perspective on the international capital flows. The definition of the balance of payments tells us that the relationship between the current account deficit and the international capital flows should be represented by the following equation:

\[ \text{CAD}_t = \text{Fin}_t - \text{Fout}_t - \Delta R_t, \]

where \( \text{Fin}_t \) is the capital inflows, \( \text{Fout}_t \) is the capital outflows, and \( R_t \) is the foreign
reserves.

The definition of the balance of payments tells us that equation (A11) always holds. Accordingly, we should analyze whether the private capital flows finance the current account deficit. We analyze the cointegration relationship by omitting the change in foreign reserves in equation (A11).

If we find the cointegration between the current account deficit and the capital flows in equation (A11), then we will consider which items in the financial account finance the current account deficit. Focusing on each of the international capital flows in equation (A11), we can rewrite equation (A11) as

\[ \text{(A12)} \quad CAD_t = DIB_t + PIB_t + OIB_t - \Delta R_t, \]

where \( DIB_t \) is direct investment in the financial account, \( PIB_t \) is portfolio investment in the financial account, and \( OIB_t \) is other investment in the financial account. If variables in the sub-system including the current account deficit and some of the times in equation (A12) are cointegrated, then the items would support the current account deficit in the long run. Thus, we also test the cointegration relationship in the sub-system of the equation (A12).

References


Kudo, T. and E. Ogawa, (2003) “The U.S. Current Account Deficit is supported by the
International Capital Inflows?" Hitotsubashi University, Faculty of Commerce and Management, Working Papers, No. 92.


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<td>Jane</td>
<td>Accountant</td>
<td>Finance</td>
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<td>1234</td>
<td><a href="mailto:jane@email.com">jane@email.com</a></td>
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<tr>
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<td>Bob</td>
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<tr>
<td>3/1/20</td>
<td>Alice</td>
<td>Designer</td>
<td>Marketing</td>
<td>Los Angeles</td>
<td>9012</td>
<td><a href="mailto:alice@email.com">alice@email.com</a></td>
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**Notes:**
- Jane has been with the company for 5 years.
- Bob has a degree in Mechanical Engineering.
- Alice is currently working on a new design project.
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備考

この表は、計画と実績の比較を示しています。計画は灰色で、実績はより詳細に記載されています。この表は、計画と実績の差異を理解し、必要な対策を講じることを目的としています。
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<tr>
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<tr>
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27
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なお、上記収入の内訳は、年金収入が全体収入の70%を占め、役員報酬はその30%を占めています。
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注：以上数据仅供参考，具体项目金额和完成情况以实际为准。
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图像描述：
- 图表显示了不同时间段的数据波动情况。
- 图表的X轴表示日期，Y轴表示数值范围。
- 图表中有实线和虚线两条线，表示不同的数据系列。