The Optimality of Delegation under Imperfect Commitment

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#### Abstract

Should a boss (a principal) delegate authority (a decision right) to his or her subordinate (agent) if the subordinate has private information? This paper answers this question under "imperfect commitment" assumption that compensation schemes are contractable but decisions are not verifiable. Our conclusions are that (i) the principal strictly prefers delegation to centralization if the decision is sufficiently important to the principal; (ii) the principal should adopt performance-based compensation scheme under both delegation and centralization, but the optimal compensation schemes are quite different; (iii) the principal more prefers delegation to centralization in comparison with no contract case or complete contract case.

#### 1 Introduction

The purpose of this paper is to reexamines whether a boss delegate authority (decision right) to his or her subordinate or not if the subordinate has private information. The existing literature has studied this question under two different assumptions. One approach adopts "no commitment assumption," in which the boss cannot write any contract and points out a trade-off as follows (Alonso and Matouschek 2005, Dessein 2002, and Harris and Raviv 2002, Holmstrom 1984, Jensen and Meckling 1992). On one hand, if a boss holds authority (centralization), an ignorant boss about subordinates' information cannot avoid to make an inappropriate decision. On the other

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hand, if the boss delegates authority to his or her subordinate (decentralization), the subordinate, who is not under control of the boss, may abuse his authority. The answer in this approach, therefore, is that if a conflict of purpose between a boss and his or her subordinate is not serious, authority should be delegated to the subordinate. Another approach of the existing studies are based on "complete contract assumption" in which a contract can be written completely. Under the assumption, the revelation principle says that it is optimal that a boss always keeps his authority (Melumad and Shibano 1991and Harris and Raviv, 1998).

Both approaches, however, has a trouble in analyzing the relationship between the allocation of authority and compensation scheme. The studies under no commitment assumption explain the reason why a boss should delegate authority to his or her subordinate, but cannot analyze the optimal compensation scheme from the definition of no commitment assumption. By adopting complete commitment assumption instead, we can find the optimal compensation scheme. The optimal compensation scheme under a decentralized process, however, cannot be analyzed as it is optimal that a boss always keeps his authority. This analytical difficulty is serious, for we cannot answer the important questions for practical organization designers as follows: should firms adopt different incentive system according to different allocations of authority?

In order to answer this question, this paper analyzes an intermediate circumstance between no commitment assumption and complete contract assumption. Our model is as follows. There is a principal (a boss), and an agent (her subordinate) who are involved in decision-making such as choosing a project from available projects.<sup>1</sup> Only the agent has private information concerning with the decision, i.e. his ability to implement the discussing project (i.e. whether marginal cost is high or low). Firstly the agent sends a message about her private information to the principal and the holder of authority makes a decision. A decision process is centralized if the principal makes a decision. A decision process is decentralized or authority is delegated to the subordinate if the agent makes a decision. After the execution of the chosen project, the principal receives verifiable performance measure about a project (a verifiable signal about the decision).

Our important assumption, which distinguishes our paper from the others, is that the decision is not verifiable but the agent's message and performance about the unverifiable decision are verifiable (imperfect commitment assumption). In other words, the principal cannot design a decision rule but can design compensation scheme contingent on the agent's message and the performance. This is an intermediate assumption between the two assumptions in the existing literature, in that the principal can write a contract but

<sup>&</sup>lt;sup>1</sup>According to the custom in contract theory literature, we use "she" ("he") as a pronoun of a principal or a boss (an agent or a subordinate).

the ability for contracting about decisions is limited.

By analyzing our model, we find a new trade-off between delegation and centralization. The existing literature considers the trade-off between information loss by centralization and a loss of control by delegation (Dessein 2002, Holmstrom 1984, and Jensen and Meckling 1992). Our new trade-off is between a self-commitment cost by centralization and an incentive cost by delegation. When the decision process is centralized, the principal bears a self-commitment cost to make the decision which is desiable before receiving the agent's private information, because after receiving the agent's private information, she tends to choose the excessive difficult project. Under a decentralized decision process, the principal bears an incentive cost to lead the agent to make a desirable decision, because the agent prefers to choose an easy project

The analysis of the trade-off gives us three results. Firstly, we show that although the principal can write contract, the principal strictly prefers delegation to centralization if the self-commitment cost outweights the incentive cost. Second, the principal should design a compensation scheme contingent on performance under both delegation and centralization, but the optimal schemes are quite different. When authority is delegated to the agent, the principal should offer to high ability agent a compensation more dependent on his performance than to low ability agent, as the principal wants the high ability agent to implement more difficult project. Under centralization, the principal uses a performance-based compensation scheme to low ability agent as the principal should refrain to choose excessive difficult project. The scheme under centralization corresponds with overtime pay and substitute holiday in a real world, while the scheme under decentralization corresponds with the performance-based payment. Finally, we find a new benefit of delegation in that the principal sometimes bears no incentive cost when authority is delegated to the agent, because (i) the high ability agent more tolerates a risk (the movement in compensation) than the low ability agent, as the high ability agent has information rent like a standard adverse selection model; (ii) under delegation, performance-contingent compensation is paid to the high ability agent. When the decision process is centralized, the principal pays performance-based compensation to the low ability agent who has less information rent. These provide a benefit of delegation.

While most studies, except for the papers cited, above deals with noinformation side of delegation (Aghion and Tirole, 1997, Baliga and Sjostrom, 1998, Baker et al., 1999, Bolton and Farrell (1990), Athey and Roberts 2001), our model is closely related to Krishna and Morgan (2005) and Ottaviani (2000). They also examine the optimality of delegation under assumption that the decision is not verifiable and compensation is feasible. Their papers, however, is different from ours in that (i) the ability of the principal to contract is more limited: the contract is contingent only on the agent's message; (ii) they implicitly assume incentive systems in organizations (the agent's utility function is single-peaked). As the result, they analyze a different trade-off from this paper's.

The remainder of the paper is organized as follows. Section 2 develops our model. In section 3, we compute complete contract case and no contract case as a benchmark. Section 4 analyzes delegation and centralization is analyzed in section 5. In section 6, both cases are compared. Section 7 provides some concluding remarks.

#### 2 Framework

We consider a principal (a boss) and an agent (a subordinate) who have to make a decision x within available decisions  $X = \{x_H, x_L\}$  ( $\Delta x = x_H - x_L > 0$ ) such as setting the subordinate's goal or choosing a project from available projects. When a decision x is executed, the principal obtains  $v_P(x)$  ( $\Delta v_P = v_P(x_H) - v_P(x_L) > 0$ ) and the agent bears cost  $v_A(x, \theta) = -\theta x$  such as some cost to acheive the goal or implement the project.  $\theta \in \Theta = \{\theta_0, \theta_1\}$  is the random variable which represents the nature of the decisions such as the marginal cost of the goal or the difficulty of the projects ( $\Delta \theta = \theta_1 - \theta_0 > 0$ ) and whose density function is  $f(\theta)$ . For convenience, we denote  $f = f(\theta_H)$ and  $1 - f = f(\theta_L)$ . Only the agent is informed about the nature  $\theta$ .

The execution of the decision also generates a noisy signal  $(y \in Y = \{G, B\})$  about the decision. The probability on y when a decision x is executed, is denoted by g(y; x). Let  $\Delta g = g(G; x_H) - g(G; x_L) > 0$ . We assume that y is independent of  $\theta$ . Examples of y involve performance in choosing a project x, work time under management by objectives, and so on. To avoid unessential classification, we assume that  $g(G; x_H) > g(G; x_L) \geq 1/2$  and  $\theta_1 \geq (f+1)\theta_0$ .

The decision-making process has two stages: (i) the agent sends a message,  $m \in M$ , about private information to the principal; (ii) after the message is sent, either the principal or the agent who has control right chooses x. Authority is delegated to the agent (a decision process is decentralized) if the subordinate chooses x. A decision process is centralized if the principal chooses x.

The principal can design a contract but the principal's ability to write the contract is limited. We use the following terminology of the principal's ability to commit in this paper. The ability is called "complete" if message m, signals y and decision x are verifiable, while "no ability to commit" means that neither message m, signals y nor decision x is verifiable. We call the ability imperfect if message m and y are verifiable while  $\theta$  and x are not verifiable. The imperfect commitment assumption reflects the situations where firms can design compensation scheme but cannot design the decision rule. We therefore adopt the imperfect assumption in most of this paper,

	message $(m)$	performance $(y)$	decision $(x)$
Complete Commitment	V	V	V
Imperfect Commitment	V	V	Not V
No Commitment	Not V	Not V	Not V

Table 1: Definition of complete, imperfect, no commitment assumptions ("V" stands for "Verifiable" )

while the complete commitment assumption and no commitment assumption are treated as benchmarks.<sup>2</sup>Under the imperfect commitment assumption, the principal designs a compensation scheme contingent on m and a signal y, i.e., w(y, m).

When  $\theta$  and y are generated, the utilities of the principal and the agent are given by

$$U_P(x, w; \theta, y) = v_P(x) - w(m; y), \tag{1}$$

$$U_A(x,w;\theta,y) = v_A(x;\theta) + w(m;y).$$
(2)

The *interim* payoffs are denoted by

$$E_y[U_P(x,w;\theta,y)] = \sum_{y \in Y} U_P(x,w;\theta,y)g(y;x),$$
(3)

$$E_y[U_A(x,w;\theta,y)] = \sum_{y \in Y} U_A(x,w;\theta,y)g(y;x).$$
(4)

The *ex ante* payoffs are denoted by

$$E_{y,\theta}[U_P(x,w;\theta,y)] = \sum_{\theta \in \Theta, y \in Y} U_P(x,w;\theta,y) f(\theta) g(y;x),$$
(5)

$$E_{y,\theta}[U_A(x,w;\theta,y)] = \sum_{\theta \in \Theta, y \in Y} U_A(x,w;\theta,y) f(\theta) g(y;x).$$
(6)

The timing of the game is given as follows.

- 1. The principal allocates the control right.
- 2. The agent privately observes  $\theta$ .

<sup>&</sup>lt;sup>2</sup>Together with the imperfect commitment assumption, we implicitly assume that the decision right X is transferred contractably. One might have the question why the decision set X is contractable while elements in the decision set are not verifiable, in paticularwhy the principal does not turn over the authority when delegating it to the agent. The answer is that if the delegation is *ex ante* beneficial for the principal, the principal can honor the delegation of authority by utilizing the following procedure: (i) the principal does not observing the decision (e. g., increasing the physical distance between the principal and the agent, or not introducing the monitor institution); (ii) at the same time the agent sends message and implements the decision. Under the procedure, the principal cares her *ex ante* payoff whenever the turn-over of authority is feasible and thus keeps the promise.

- 3. The principal offers a contract  $\{w(m, y)\}$ .
- 4. The agent accepts or rejects the contract.
- 5. The agent sends a message  $m \in M$ .
- 6. The principal or the agent makes a decision.
- 7. y is observed and the contract is executed.

### 3 Benchmarks

In this section, we establish two benchmark results under the complete commitment assumption and the no commitment assumption.

A Benchmark under the Complete Commitment Assumption We begin with the analysis under the complete commitment assumption that x, m, and y is verifiable. Let h(m, y) = (x(m), w(m, y)) be an allocation when m is sent and y is observed. The principal's problem is to choose a mechanism  $H = \{h(m, y) \mid m, y\}$  to maximizes her expected utility. In this case, the problem is substantially equivalent to a standard adverse selection problem.<sup>3</sup> Although the model differs from the standard adverse selection model in that a signal y is verifiable, the use of y does not improve the principal's utility as y is only a noisy signal of x and x is verifiable. As the standard model, we make the following assumptions without loss of generality; (i)  $M = \Theta$ ; (ii)  $w(\theta, G) = w(\theta, B)$ , and  $x(\theta, G) = x(\theta, B)$  for any  $\theta$ .

Furthermore, it is (weakly) optimal that the decision right belongs to the principal because revelation principle implies that the principal's payoff when the authority is delegated to the agent is always realized when the principal keeps authority. We therefore consider the following problem,

$$\begin{array}{ll} [\text{P-1}] \\ \max_{h(\theta_i,y)} & E_{y,\theta_i}[U_P(h(\theta_i,y);\theta_i,y)] \\ \text{s.t.} & E_y[U_A(h(\theta_i,y);\theta_i,y)] \ge 0 & \text{for any } \theta_i, \ (\text{PC}i) \\ & E_y[U_A(h(\theta_i,y);\theta_i,y)] \ge E_y[U_A(h(\theta_j,y);\theta_i,y)] & \text{for any } \theta_i,\theta_j, \\ & (\text{IC}i) \\ & U_A(h(\theta_i,y);\theta_i,y) \ge 0 & \text{for any } \theta_i,y. \\ & (\text{LLC}iy) \end{array}$$

 $<sup>^{3}</sup>$ See contract theory textbooks such as Bolton and Dewatripont (2005), Itoh (2003) or Laffont and Martimort (2002).

The participation constraint (PCi) implies that *i*-type agent must obtain at least his reservation utility, which we normalize to zero. The limited liability constraint (LLCiy) comes from the fact that *i*-type agent can leave the contract after observing y. The incentive compatibility constraint (ICi) is imposed on the problem in order to guarantee that *i*-type agent reports the truth. By the standard procedure, the constraints in the problem are reduced to the binding (PC1), the binding (IC0), and the so-called monotonic condition, i.e.,

$$w(\theta_1, y) - \theta_1 x(\theta_1) = 0, \qquad (PC1')$$

$$w(\theta_0, y) - w(\theta_1, y) = \theta_0(x(\theta_0) - x(\theta_1)),$$
 (IC0')

$$x(\theta_0, y) \ge x(\theta_1, y). \tag{M}$$

Before driving an optimal mechanism, we consider an optimal compensation scheme,  $w(\theta, y)$ , given the decisions  $(x(\theta_0), x(\theta_1)) = (x_H, x_L)$ . In this case, the principal utilizes the agent's private information to make a decision but cannot avoid to bear so-called information rent. The reason is represented in Figure 1. If  $\theta$  is verifiable, the optimal compensation scheme is  $w(\theta_0, G) = w(\theta_0, B) = \theta_0 x_H$  and  $w(\theta_1, G) = w(\theta_1, B) = \theta_1 x_L$ . The principal's expected payoff is  $f(v_p(x_H) - \theta_0 x_H) + (1 - f)(v_p(x_L) - \theta_1 x_L)$ . When  $\theta$  is not verifiable, the scheme is not incentive compatible, as the  $\theta_0$ -type agent obtains a rent  $\Delta \theta x_L$  by reporting the false type  $\theta_1$  (Region B), i.e., the scheme violates (IC0'). To keep the agent's truth-telling, the principal must increases by  $\Delta \theta x_L$  the compensation to the  $\theta_0$ -type agent (Region A) and thus bears information rent  $(f \Delta \theta x_L)$ 

**Lemma 1.** Suppose that the ability to commit is complete and the principal's decision is  $(x(\theta_0), x(\theta_1)) = (x_H, x_L)$ . An optimal compensation scheme is  $(w(\theta_0, y), w(\theta_1, y)) = (\theta_0 x_H + \Delta \theta x_L, \theta_1 x_L)$  for any y. The principal's payoff are given by

$$E_{y,\theta_i}[U_P(h^*(\theta_i, y); \theta_i, y)] = f(v_p(x_H) - \theta_0 x_H - \Delta \theta x_L) + (1 - f)(v_p(x_L) - \theta_1 x_L) \equiv \pi_{HL}^*.$$

When the principal freely chooses a decision pair  $(x(\theta_0), x(\theta_1)), (x_H, x_L)$ is not always optimal, for differently from the standard model, we restrict the available decision set X to a binary set  $\{x_H, x_L\}$ . The principal's optimal payoff and an optimal mechanism,  $H^*$ , is shown in the following proposition.

**Proposition 1.** Suppose that complete contract is feasible. We denote two threshold values by

$$k_1 = \left(\theta_1 + \frac{f}{1 - f}\Delta\theta\right)\Delta x,$$
  
$$k_2 = \theta_0\Delta x.$$

The optimal mechanism and the principal's expected payoff are as follows.



1.  $\Delta v_p > k_1$ 

 $\begin{aligned} (x(\theta_0), x(\theta_1)) &= (x_H, x_H), w(\theta, y) = \theta_1 x_H \text{ for any } \theta, y \\ E_{y,\theta_i}[U_P(h^*(\theta_i, y); \theta_i, y)] &= v_p(x_H) - \theta_1 x_H \equiv \pi^*_{HH} \end{aligned}$ 

2.  $k_1 \ge \Delta v_p \ge k_2$ 

$$\begin{aligned} (x(\theta_0), x(\theta_1)) &= (x_H, x_L), \\ (w(\theta_0, y), w(\theta_1, y)) &= (\theta_0 x_H + \Delta \theta x_L, \theta_1 x_L) \text{ for any } y \\ E_{y,\theta_i}[U_P(h^*(\theta_i, y); \theta_i, y)] \\ &= f(v_p(x_H) - \theta_0 x_H - \Delta \theta x_L) + (1 - f)(v_p(x_L) - \theta_1 x_L) \equiv \pi^*_{HL} \end{aligned}$$

3.  $k_2 > \Delta v_p$ 

$$(x(\theta_0), x(\theta_1)) = (x_L, x_L), w(\theta, y) = \theta_1 x_L \text{ for any } \theta, y$$
$$E_{y,\theta_i}[U_P(h^*(\theta_i, y); \theta_i, y)] = v_p(x_L) - \theta_1 x_L \equiv \pi^*_{HH}$$

*Proof.* See contract theory textbook, e.g. Bolton and Dewatripont (2005), Laffont and Martimort (2002) and Itoh (2003).  $\Box$ 

A Benchmark under the No Commitment Assumption In this case, the holder of authority makes a decision to maximize his or her *interim* payoff, as x is not verifiable. When the principal keeps authority, she maximizes  $v_P(x)$  subject to (PC1) and (PC2). As the result, the principal always chooses  $x_H$  and pays  $\theta_1 x_H$  to the agent. The principal's payoff is  $v_p(x_H) - \theta_1 x_H$ .<sup>4</sup>

If authority is delegated to the agent, the agent always chooses  $x_L$  and the principal pays a compensation  $\theta_1 x_L$ . The principal's payoff is  $v(x_L) - \theta_1 x_L$ . Therefore we obtain the following proposition.

**Proposition 2** (Alonso and Matouschek 2005, Dessein 2002, Holmstrom 1984, Jensen and Meckling 1992). Suppose that the principal cannot write any contract. If  $\Delta v_p < \theta_1 \Delta x$ , delegation is strictly preferred to centralization. Otherwise, the principal prefers centralization.

Proof. Straightforward.

#### 4 Imperfect Commitment Case: Centralization

We next assume that x is unverifiable and consider a centralized case in which the principal chooses x. In that case, the principal chooses x to maximize her payoff after receiving the agent's report (*interim* payoff), i.e.,

$$E_y[U_p(x,w;\theta,y)] = v_p(x;\theta) - \sum_{y \in Y} g(y;x)w(\theta,y), \tag{7}$$

where we assume that  $M = \Theta$  without loss of generality.<sup>5</sup> This implies that the principal faces a self-commitment problem. If y is not utilized  $(w(\theta, G) = w(\theta, B))$ , the principal chooses  $x_H$  regardless of the agent's report, as  $v_p(x_H) > v_p(x_L)$  for any  $\theta$ . In the other words, the principal tends to choose excessively difficult a goal or project. The choice, however, is not always optimal from a perspective of *ex ante* payoff. To implement ex ante optimal choice  $x(\theta)$ , the principal should design the compensation scheme satisfy the following conditions (namely, Self-Commitment Constraints), i.e., for each  $\theta_i$ 

$$x(\theta_i) = \arg\max E_y[U_p(x, w; \theta_i, y)], \qquad (SCCi)$$

equivalently,

$$[w(\theta_i, G) - w(\theta_i, B)] \ge \frac{v_p(x(\theta_i)) - v_p(x')}{g(G; x(\theta_i)) - g(G; x'))} \text{ for any } x' \neq x(\theta_i).$$

<sup>&</sup>lt;sup>4</sup>In this setting, communication as cheap-talk game is infeasible, as the conflict of both parties is sufficiently large. In fact, the agent's optimal report strategy is to say  $\theta_L$ , as telling  $\theta_H$  implies that the principal chooses  $x_H$ . So, any separating equilibrium does not exist.

<sup>&</sup>lt;sup>5</sup>Bester and Strausz (2001) points out that revelation principle does not hold if some control variable can not be committed (i.e. x in our model). However, it is shown that if X is bianary, the revelation principle holds.

Let S be the right-hand side in the above equality when  $x(\theta_i) = x_H$  and  $x' = x_L$ , i.e.,  $S = \frac{v_p(x_H) - v_p(x_L)}{g(G; x_H) - g(G; x_L))} = \frac{\Delta v_p}{\Delta g}$ . The problem therefore is modified as [P-2],

$$\begin{array}{l} [\text{P-2}] \\ \max_{h(\theta_i, y)} \\ \text{s.t.} \end{array} \qquad E_{y, \theta_i}[U_P(h(\theta_i, y); \theta_i, y)] \\ \text{s.t.} \qquad (\text{PC}i), \ (\text{IC}i), \ (\text{LLC}iy), \ \text{and} \ (\text{SCC}i) \end{array}$$

We next show that [P-2] can be replaced by [P-2'].

**Lemma 2.** Suppose that x is not verifiable. [P-2] is equivalent to the following problem.

$$\begin{array}{ll} \left[P-2'\right] \\ \max_{h(\theta_i,y)} & E_{y,\theta_i}[U_P(h(\theta_i,y);\theta_i,y)] \\ s.t. & E_y[U_A(x(\theta_0),w(\theta_0,y);\theta_0,y)] = E_y[U_A(x(\theta_1),w(\theta_1,y);\theta_0,y)], \\ & (\text{ICO'}) \\ & x(\theta_0) \ge x(\theta_1), \end{array}$$

$$(M)$$

$$(LLCiy)$$
 and  $(SCCi)$ .

*Proof.* It is easily shown that (i) (IC0) and (IC1)  $\rightarrow$  (M); (ii) (M) and  $(IC0') \rightarrow (IC0)$  and (IC1); (iii) (IC0) and  $(PC1) \rightarrow (PC0)$ ; (iv) (LLC1G)and  $(LLC1B) \rightarrow (PC1)$ . By showing an optimal H in [P-2] satisfies (IC0'), we obtain the lemma.

Suppose that  $E_y[U_A(x(\theta_0), w(\theta_0, y); \theta_0, y)] > E_y[U_A(x(\theta_1), w(\theta_1, y); \theta_0, y)].$ Then we obtain

$$E_y[U_A(x(\theta_0), w(\theta_0, y); \theta_0, y)] > E_y[U_A(x(\theta_1), w(\theta_1, y); \theta_0, y)]$$
  
$$\geq E_y[U_A(x(\theta_1), w(\theta_1, y); \theta_1, y)] \ge 0,$$

where the last inequality is obtained from (PC1). Decreasing  $E_{y \in Y}[w(\theta_0, y)]$ slightly reduces the principal's expected payment, while relaxing (IC1) without violating (PC1), (SCCi) and (LLCiy). This contradicts the assumption. 

Before driving an optimal mechanism, we consider an optimal compensation scheme,  $w(\theta, y)$ , given the decisions  $(x(\theta_0), x(\theta_1)) = (x_H, x_L)$ 

Lemma 3. Suppose that the ability of commitment is imperfect and the principal's decision is  $(x(\theta_0), x(\theta_1)) = (x_H, x_L)$ . An optimal compensation scheme is

$$w(\theta_0, G) = w(\theta_0, B) = \theta_0 x_H + \Delta \theta x_L + g(G; x_L)S,$$
(9)

$$w(\theta_1, G) = \theta_1 x_H + S, w(\theta_1, B)) = \theta_1 x_L.$$

$$(10)$$

The principal's payoff are given by

$$\mathbb{E}_{y,\theta_i}[U_P(h^C(\theta_i, y); \theta_i, y)] = \pi^*_{HL} - g(G; x_L)S.$$

Proof. By some manipulation, (SCC1) becomes

$$w(\theta_1, G) - w(\theta_1, B) \ge S$$

where  $S = \frac{v_p(x_H) - v_p(x_L)}{g(G;x_H) - g(G;x_L)} > 0$ . Since  $U_A(x_L, w(\theta_1, G)) = U_A(x_L, w(\theta_1, B)) + g(G; x_L)(w(\theta_1, G) - w(\theta_1, B))$ , (LLC1B) and (SCC1) implies (LLC1G). We can ignore (LLC1G). On the other hand, by substituting (IC0') into the objective function, it yields

$$E_{\theta_i}[v_p(x(\theta_i))] + f\theta_0 \Delta x - w(\theta_1, B) - [w(\theta_1, G) - w(\theta_1, B)].$$

This shows (i)  $w(\theta_1, B) = \theta_1 x_L$  is optimal; (ii) (SCC1) is binding. Therefore,  $w(\theta_1, G) = \theta_1 x_H + S$  and  $w(\theta_1, B) = \theta_1 x_L$ . Although there are many  $w(\theta_0, G)$  and  $w(\theta_0, B)$  satisfying (LLC0G), (LLC0B) and (IC0'), the principal's expected payoffs given by the different scheme are same. A simple example of the schemes is  $w(\theta_0, G) = w(\theta_0, B) = \theta_0 x_H + \Delta \theta x_L + g(G; x_L)S$ .

This lemma shows that the principal should offer a compensation contingent on y to  $\theta_1$ -agent when the principal adopts a centralized decision process and wants to choose  $(x(\theta_0), x(\theta_1)) = (x_H, x_L)$  from perspective of ex ante payoff (see Figure 2). If the principal offers an optimal compensation scheme under the complete commitment assumption, i.e.,  $(w(\theta_0, y), w(\theta_1, y)) = (\theta_0 x_H + \Delta \theta x_L, \theta_1 x_L)$  for any y, the principal chooses  $x_H$  if the agent's message is  $\theta_1$ , as she maximize her *interim* payoff. To commit to choose  $x_L$ , the principal offers an optimal scheme satisfying

$$w(\theta_1, y) - w(\theta_1, y) \ge S.$$

This constraint requires that the performance-based compensation scheme is offered to  $\theta_1$ -agent.

This self-commitment problem causes costs to the principal in two ways. The first one is "direct effect." To commit  $x_L$ , the principal must increases by S compensation to  $\theta_1$ -type agent if performance y is G, as the agent is protected by the limited-liability constraint. The increase of  $w(\theta_1, G)$ decreases the principal's payoff by  $(1 - f)g(G; x_L)S$ . At the same time, the change in the compensation to  $\theta_1$ -type agent causes another cost to the principal. The increase of  $w(\theta_1, G)$  increases  $\theta_1$ -type agent's expected payoff. This means that  $\theta_0$ -type agent obtains more rent (Region B in Figure 2) by sending a false message  $\theta_1$ . To maintain truth-telling, the payment to  $\theta_1$ -type is also increased by Region A in Figure 2. The principal's payoff decreases by  $fg(G; x_L)S$ . This is "indirect effect." As the result, the principal bears a self-commitment cost by  $g(G; x_L)S$ .

The following proposition shows an optimal mechanism and the principal's payoff when the principal keeps authority.



**Proposition 3.** Suppose that the ability of commitment is imperfect. The optimal mechanism and the principal's payoff are as follows.

1.  $\Delta v_p > k_1 - \frac{g(G;x_L)S}{1-f}$   $(x(\theta_0), x(\theta_1)) = (x_H, x_H), w(\theta, y) = \theta_1 x_H \text{ for any } \theta, y$   $E_{y,\theta_i}[U_P(h^C(\theta_i, y); \theta_i, y)] = \pi^*_{HH}$ 2.  $k_1 - \frac{g(G;x_L)S}{1-f} \ge \Delta v_p \ge k_2$   $(x(\theta_0), x(\theta_1)) = (x_H, x_L),$   $w(\theta_0, G) = w(\theta_0, B) = \theta_0 x_H + \Delta \theta x_L + g(G; x_L)S$  $w(\theta_1, G) = \theta_1 x_H + S, w(\theta_1, B)) = \theta_1 x_L$ 

$$E_{y,\theta_i}[U_P(h^C(\theta_i, y); \theta_i, y)] = \pi_{HL}^* - g(G; x_L)S$$

3.  $k_2 > \Delta v_p$ 

$$(x(\theta_0), x(\theta_1)) = (x_L, x_L),$$
  

$$w(\theta, G) = \theta_1 x_L + S, w(\theta, y) = \theta_1 x_L \text{ for any } \theta,$$
  

$$E_{y,\theta_i}[U_P(h^C(\theta_i, y); \theta_i, y)] = \pi_{LL}^* - g(G; x_L)S$$

*Proof.* We first compute a optimal compensation scheme given  $(x(\theta_0), x(\theta_1))$ . By a comparison of the principal's payoffs in these cases, we obtain the proposition. 1.  $(x(\theta_0), x(\theta_1)) = (x_H, x_H)$ In this case, (IC0'), (SCC*i*) and (LLC*iy*) are equivalent to (SCC*i*), (LLC*iy*) and

$$E_y[w(\theta_0, y)] = E_y[w(\theta_1, y)] = \theta_1 x_H.$$

$$(11)$$

The proof is trivial. Although there are many compensation schemes satisfying (11), and (SCC*i*) and (LLC*iy*), the different schemes gives the principal the same payoff, i.e.,  $v_p(x_H) - \theta_1 x_H$ . A simple example of the schemes is  $w(\theta_i, y) = \theta_1 x_H$  for any i, y.

2.  $(x(\theta_0), x(\theta_1)) = (x_L, x_L)$ In this case, (IC0') implies  $E_y[w(\theta_0, y)] = E_y[w(\theta_1, y)]$ . The objective function therefore becomes

$$v_p(x_L) - E_y[w(\theta_0, y)].$$

As it is shown that (i) (SCC1) and (SCC2) are binding; (ii) (LLC0*B*) and (LLC1*B*) are binding. Therefore, a optimal scheme is  $w(\theta, G) = \theta_1 x_L + S$  and  $w(\theta, B) = \theta_1 x_L$  for any  $\theta$ .

We remark the principal's payoff when  $(x(\theta_0), x(\theta_1)) = (x_H, x_H)$  and  $(x(\theta_0), x(\theta_1)) = (x_L, x_L)$  in comparison with the result of the benchmark under perfect commitment assumption. When the principal implements  $(x(\theta_0), x(\theta_1)) = (x_L, x_L)$ , the payoff of the principal decreases by the self-commitment cost  $g(G; x_L)S$ , as the principal utilizes the performance-based compensation to choose  $x_L$ . Similarly, the principal's payoff does not change when the principal implements  $(x(\theta_0), x(\theta_1)) = (x_H, x_H)$ , as the principal prefers  $x_H$  even without compensation.

#### 5 Imperfect Commitment Case: Delegation

We next consider delegation. Delegation of authority changes [P-1] in two ways. Firstly, the principal uses the compensation scheme so that the agent takes the decision desirable for principal. To induce the desirable decision, the compensation scheme satisfies, for each  $\theta_i$ ,

$$x(\theta_i) = \arg \max_x \sum_{y \in Y} g(y; x) w(\theta_i, y) - \theta_i x \text{ for } \theta_i.$$
(DCi)

equivalnetly,

$$[w(\theta_i, G) - w(\theta_i, B)] \ge \frac{\theta_i(x(\theta_i) - x')}{g(G; x(\theta_i)) - g(G; x')} \text{ for any } x' \neq x(\theta_i).$$

Let  $I_0 = \frac{\theta_0 \Delta x}{\Delta g}$  and  $I_1 = \frac{\theta_1 \Delta x}{\Delta g}$ . In comparison with centralization, the agent always chooses  $x_L$  if  $w(\theta, G) = w(\theta, B)$ .

Secondly, (ICi) in the benchmark problem is more severe, as the agent not only tells a false but also makes a decision to maximize his payoff, i.e.,

$$E_y[U_A(h(\theta_i, y); \theta_i, y)] \ge \max_x E_y[U_A(h(\theta_j, y); \theta_i, y)] \quad \text{for any } \theta_i, \theta_j.$$
(D-ICi)

We denote a maximal x in the right hand by  $\tilde{x}(\theta_i)$ . The principal solves the following problem.

$$\begin{array}{ll} [\text{P-3}] \\ \max_{h(\theta_i,y)} & E_{y,\theta_i}[U_P(h(\theta_i,y);\theta_i,y)] \\ \text{s.t.} & (\text{PC}i), (\text{D-IC}i), (\text{LLC}iy), \text{ and } (\text{DC}i). \end{array}$$

We firstly compute an optimal compensation scheme when the principal implements  $(x(\theta_0), x_{(\theta_1)}) = (x_H, x_L).$ 

**Proposition 4.** Suppose that the ability of commitment is imperfect, authority is delegated to the agent, and the principal implements  $(x(\theta_0), x_i(\theta_1)) =$  $(x_H, x_L)$ . If  $\Delta \theta x_L \geq g(G; x_H) I_0$ , the following compensation scheme is optimal,

$$w(\theta_0, G) = \theta_0 x_H + \Delta \theta x_L + (1 - g(G; x_H)) I_0,$$
(12)

$$w(\theta_0, B) = \theta_0 x_H + \Delta \theta x_L - g(G; x_H) I_0, \qquad (13)$$

$$w(\theta_1, G) = w(\theta_1, B) = \theta_1 x_L.$$
(14)

The principal achieves the payoff in the complete contract benchmark,  $\pi_{HL}^*$ . Otherwise, the following compensation scheme is optimal.

$$w(\theta_0, G) = \theta_0 x_H + I_0, \tag{15}$$

$$w(\theta_0, B) = \theta_0 x_H, \tag{16}$$

$$w(\theta_1, G) = w(\theta_1, B) = g(G; x_L)I_0 + \theta_0 x_H.$$
(17)

Under the compensation, the principal's payoff is  $\pi^*_{HL} - [g(G; x_H)I_0 - \Delta\theta x_L]$ *Proof.* See appendix.

If the principal delegates authority to the agent and want to induce  $(x(\theta_0), x_{(\theta_1)}) = (x_H, x_L)$ , a performance-based compensation is offered to  $\theta_0$ -agent, as the principal gives  $\theta_0$ -type agent incentive to choose  $x_H$ , i.e.,

$$[w(\theta_0, G) - w(\theta_0, B)] \ge I_0.$$
(18)

One might think that this additional constraint brings the principal additional cost, as the agent is protected by limited liability. However, the



Figure 3: The Delegation Case

principal's payoff does not change if  $\Delta \theta x_L \geq g(G; x_H) I_0$ , for  $\theta_0$ -type agent already has information rent and thus he can stand the movement in compensation (See Figure 3). The principal therefore does not bear the incentive cost to lead  $\theta_0$ -type agent to choose  $x_H$ . Otherwise, delegation requires the incentive cost  $[g(G; x_H)I_0 - \Delta\theta x_L].$ 

We next compute the optimal mechanism when authority is delegated to the agent.

**Proposition 5.** Suppose that x is not verifiable and authority is delegated to the agent. The optimal mechanism and the principal's expected payoff are as follows. If  $\Delta \theta x_L \geq g(G; x_H) I_0$ ,

1. 
$$\Delta v_p > k_1 + \frac{g(G;x_H)}{1-f}I_1$$
  
 $(x(\theta_0), x(\theta_1)) = (x_H, x_H),$   
 $w(\theta_0, G) = w(\theta_1, G) = \theta_1 x_H + I_1,$   
 $w(\theta_0, B) = w(\theta_1, B) = \theta_1 x_H,$   
 $E_{y,\theta_i}[U_P(h^D(\theta_i, y); \theta_i, y)] = \pi^*_{HH} - g(G; x_H)I_1$ 

2. 
$$k_1 + \frac{g(G;x_H)}{1-f} \ge \Delta v_p \ge k_2$$
$$(x(\theta_0), x(\theta_1)) = (x_H, x_L),$$
$$w(\theta_0, G) = \theta_0 x_H + \Delta \theta x_L + (1 - g(G; x_H))I_0,$$
$$w(\theta_0, B) = \theta_0 x_H + \Delta \theta x_L - g(G; x_H)I_0,$$
$$w(\theta_1, G) = w(\theta_1, B) = \theta_1 x_L,$$
$$E_{y,\theta_i}[U_P(h^D(\theta_i, y); \theta_i, y)] = \pi_{HL}^*$$

3.  $k_2 > \Delta v_p$ 

$$w(\theta_i, y) = \theta_1 x_L, \text{ for any } i, y$$
$$E_{y,\theta_i}[U_P(h^D(\theta_i, y); \theta_i, y)] = \pi_{LL}^*$$

$$If \ \Delta \theta x_L < g(G; x_H) I_0,$$

$$1. \ \Delta v_p > k_1 + \frac{g(G; x_H) [I_1 - I_0] + \Delta \theta x_L}{1 - f}$$

$$(x(\theta_0), x(\theta_1)) = (x_H, x_H),$$

$$w(\theta_0, G) = w(\theta_1, G) = \theta_1 x_H + I_1,$$

$$w(\theta_0, B) = w(\theta_1, B) = \theta_1 x_H,$$

$$E_{y, \theta_i} [U_P(h^D(\theta_i, y); \theta_i, y)] = \pi_{HH}^* - g(G; x_H) I_1$$

2. 
$$k_{1} + \frac{g(G;x_{H})[I_{1}-I_{0}] + \Delta \theta x_{L}}{1-f} \geq \Delta v_{p} \geq k_{2} + \frac{g(G;x_{H})I_{0} - \Delta \theta x_{L}}{f}$$
$$(x(\theta_{0}), x(\theta_{1})) = (x_{H}, x_{L}),$$
$$w(\theta_{0}, G) = \theta_{0} x_{H} + I_{0}, \quad w(\theta_{0}, B) = \theta_{0} x_{H},$$
$$w(\theta_{1}, G) = w(\theta_{1}, B) = g(G; x_{L})I_{0} + \theta_{0} x_{H},$$
$$E_{y,\theta_{i}}[U_{P}(h^{D}(\theta_{i}, y); \theta_{i}, y)] = \pi_{HL}^{*} - [g(G; x_{H})I_{0} - \Delta \theta x_{L}]$$

3.  $k_2 + \frac{g(G;x_H)I_0 - \Delta\theta x_L}{f} > \Delta v_p$  $w(\theta_i, y) = \theta_1 x_L$ , for any i, y $E_{y,\theta_i}[U_P(h^D(\theta_i, y); \theta_i, y)] = \pi_{LL}^*.$ 

*Proof.* We can easily show that the following compensation scheme is optimal if the principal implements  $(x_H, x_H)$  or  $(x_L, x_L)$ .

1.  $(x_H, x_H)$ 

The following compensation scheme is optimal.

$$w(\theta_0, G) = w(\theta_1, G) = \theta_1 x_H + I_1,$$
  
 $w(\theta_0, B) = w(\theta_1, B) = \theta_1 x_H.$ 

Under the compensation, the principal's payoff is  $\pi^*_{HH} - g(G; x_H)I_1$ 

2.  $(x_L, x_L)$ 

The following compensation scheme is optimal.

 $w(\theta_i, y) = \theta_1 x_L$ , for any i, y

Under the compensation, the principal's payoff is  $\pi_{LL}^*$ 

By a comparison of the principal's payoffs in these cases, we obtain the proposition.  $\hfill \Box$ 

We remark the principal's payoff when $(x(\theta_0), x(\theta_1)) = (x_H, x_H)$  and  $(x(\theta_0), x(\theta_1)) = (x_L, x_L)$  in comparison with the result of the benchmark under perfect commitment assumption. When the principal implements  $(x(\theta_0), x(\theta_1)) = (x_H, x_H)$ , the incentive constraints reduces the principal's payoff, as the principal offers the performance-based compensation to induce  $x_H$ . Similarly the principal's payoff does not change when the principal implements  $(x(\theta_0), x(\theta_1)) = (x_H, x_H)$ , as the agent prefers  $x_L$  without performance-based compensation.

#### 6 Delegation v.s. Centralization

In this section, we will compare delegation with centralization if the ability of commitment is imperfect. Firstly we consider it when the principal implements  $(x(\theta_0), x_{(\theta_1)}) = (x_H, x_L)$ . If  $\Delta \theta x_L \geq g(G; x_H)I_0$ , delegation dominates centralization, because delegation does not cause the incentive cost while centralization always causes the self-commitment cost  $g(G; x_L)S$ (See Proposition 4 and Lemma 3). If  $\Delta \theta x_L < g(G; x_H)I_0$ , the principal faces the trade-off between the incentive cost by delegating the authority and the self-commitment cost by keeping the authority. The difference of principal's payoffs between both decision processes is given by

$$(\text{Delegation}) - (\text{Centralization})$$

$$= [\pi_{HL}^* - (g(G; x_H)I_0 - \Delta\theta x_L)] - [\pi_{HL}^* - g(G; x_L)S],$$

$$= -[g(G; x_H)I_0 - \Delta\theta x_L] + g(G; x_L)S.$$
(19)

The first term is the incentive cost, while the second term is the selfcommitment cost. If the self-commitment cost outweighs the incentive cost, it is optimal that the principal delegates authority to the agent. Otherwise, the centralized decision process is optimal.

**Proposition 6.** Suppose that x is not verifiable, and the principal implements  $(x(\theta_0), x(\theta_1)) = (x_H, x_L)$ . If  $\Delta \theta x_L \ge g(G; x_H)I_0$ , delegation dominates centralization. If  $\Delta \theta x_L < g(G; x_H)I_0$  and  $g(G; x_L)S \ge (g(G; x_H)I_0 - \Delta \theta x_L)$ , delegation dominates centralization, the principal prefers to delegating authority rather than to keeping it. Otherwise, the principal prefers to keeping authority rather than to delegating it. Inequality (19) shows that the principal more prefer delegation as the information is more important ( $\Delta \theta$  increases). The similar result is obtained in Dessein (2002), but the logics are different. The reason in Dessein (2002), is that the increases of  $\Delta \theta$  increases the demerit of centralization, as the principal cannot utilize the agent's information when a decision process is centralized. On the other hands, the reason in our paper is that the increase of  $\Delta \theta$  enhances the merit of delegation, because when the decision process is decentralized it enlarges the agent's information rent.

**Corollary 1.** Suppose that x is not verifiable, and the principal implements  $(x(\theta_0), x(\theta_1)) = (x_H, x_L)$ . As  $\Delta \theta$  increases, the principal more prefers a decentralized decision process to a centralized decision process.

**Proposition 7.** Suppose that x is not verifiable. If  $\Delta\theta x_L \ge g(G; x_H)I_0$  and  $k_1 \ge \Delta v_p$ , then delegation dominates centralization. If  $\Delta\theta x_L < g(G; x_H)I_0$  and  $\Delta v_p \le \max\{k_2 + \frac{g(g; x_H)I_0 - \Delta\theta x_L}{f}, k_2 + \frac{\Delta g(\theta_0 x_H - \theta_1 x_L)}{g(G, x_L)}\}$ , then delegation dominates centralization. Otherwise, centralization dominates delegation.

*Proof.* See appendix.

Even if  $\Delta \theta x_L \geq g(G; x_H)I_0$ , the principal does not always prefers delegation, as  $(x_H, x_H)$  can be optimal. To induce  $(x_H, x_H)$ , centralization is always optimal, as the principal always bears incentive cost when the decision process is decentralized while under the centralized decision process it causes no self-commitment cost. Therefore, there is the condition that delegation is optimal, i.e.  $k_1 < \Delta v_p$ .

If  $\Delta\theta x_L < g(G; x_H)I_0$ , there is two important points to determine the threshold that the principal prefers delegation to centralization. First one is the condition that the self-commitment cost outweighs the incentive cost, which is given by

$$g(G; x_L)S \ge [g(G; x_H)I_0 - \Delta\theta x_L] \leftrightarrow \Delta v_p \ge k_2 + \frac{\Delta g}{g(g; x_L)}(\theta_0 x_H - \theta_1 x_L).$$

This determines the threshold when the principal wants to implement  $(x_H, x_L)$ . Second, delegation is always optimal if the principal want to induce  $(x_L, x_L)$  decision, because if the decision process is decentralized, the principal bears no incentive cost, while if the decision process is centralized, it causes the self-commitment cost to induce  $(x_L, x_L)$ . The condition that the principal wants to prefer  $(x_L, x_L)$  is  $\Delta v_p \leq k_2 + \frac{g(g; x_H)I_0 - \Delta \theta x_L}{f}$ . These two point determine the threshold value in which delegation is optimal.

**Corollary 2.** • The principal is more likely to prefers delegation to centralization when the ability to commit is imcomplete than when the ability to commit is complete.

- If  $\Delta \theta x_L \geq g(G; x_H) I_0$ , the principal is more likely to prefers delegation to centralization when the ability to commit is imcomplete than when there is no ability to commit.
- Otherwise, the principal is more likely to prefers centralization to delegation when the ability to commit is imcomplete than when there is no ability to commit.

The principal more prefers delegation to centralization if x is not verifiable than if it is verifiable. Certainly, both delegation and centralization require that the principal offers performance-based compensation scheme. However, the effect of the scheme is different: while the necessity of the principal's self-commitment always reduces her payoff under centralization, the principal sometimes avoid additional cost if authority is delegated to the agent. The reason is that  $\theta_0$ -type agent can more tolerate performance-based compensation than  $\theta_1$ -type agent, since  $\theta_0$ -type agent obtains information rent as standard adverse selection model. This is why delegation is more preferable.

Furthermore, this result shows that delegation is more preferable in comparison to no contract case if  $\Delta \theta x_L \geq g(G; x_H)I_0$ . Recall that if the principal cannot write any contract,  $\Delta v_p \leq \theta_1 \Delta x$  is condition for optimality of delegation. Within  $\Delta v_p \in (\theta_1 \Delta x, k_1)$ , delegation is optimal only under imperfect commitment.

#### 7 Concluding Remarks

We examine the optimality of delegation under the imperfect commitment assumption. Our conclusion is that (i) the principal strictly prefers delegation to centralization if the decision is sufficiently important to the principal; (ii) the principal should adopt performance-based compensation scheme under both delegation and centralization, but the structure of the optimal compensation schemes are quite different; (iii) the principal more prefers delegation to centralization in comparison with no contract case or complete contract case.

We close this paper with a discussion of some possible extensions. In our model, we focus on the interaction between the allocation of authority and compensation schemes. There, however, are the other incentive systems in organization such as promotion, career concerns, and so on. How these incentive systems affect the optimal allocation of authority is an interesting future direction for our research.

	$\Delta w_1 < I_0$	$I_0 \le \Delta w_1 \le I_1$
$I_0 \le \Delta w_0 < I_1$	$(x_L, x_L)$	$(x_H, x_L)$
$I_1 \le \Delta w_0$	$(x_L, x_H)$	$(x_H, x_H)$

Table 2: 4 cases of  $(\tilde{x}(\theta_0), \tilde{x}(\theta_1))$ 

## Appendix

#### A Proof of Proposition 4

*Proof.* When the principal implements  $(x(\theta_0), x_{(\theta_1)}) = (x_H, x_L)$ , (DC0) and (DC1) become

$$\Delta w_0 \ge I_0,\tag{20}$$

$$\Delta w_1 \le I_1,\tag{21}$$

where  $\Delta w_1 = w(\theta_1, G) - w(\theta_1, B)$  and  $\Delta w_0 = w(\theta_0, G) - w(\theta_0, B)$ . As setting  $\Delta w_1$  and  $\Delta w_0$  changes the optimal  $\tilde{x}$  in left-hand side of (D-IC). This problem is divided into 4 cases according to  $\tilde{x}(\theta)$ . Table 2 represents  $(\tilde{x}(\theta_0), \tilde{x}(\theta_1))$ . If  $\Delta w_1 < I_0$  and  $I_0 \leq \Delta w_1 < I_1$ ,  $(\tilde{x}(\theta_0), \tilde{x}(\theta_1)) = (x_L, x_L)$ . We will compute the principal's optimal payoff in each case and obtain the proposition by a comparison of the principal's payoffs in these cases.

1. 
$$\Delta w_1 < I_0 \leq \Delta w_0 < I_1$$

Step 1: (D-IC0) and (D-IC1)  $\rightarrow$  (DC0) (D-IC0) and (D-IC1) become

$$\sum_{m} g(y; x_L) w(\theta_1, y) + \theta_0 \Delta x \le \sum_{m} g(y; x_H) w(\theta_0, y)$$
(22)

$$\sum g(y; x_H) w(\theta_0, y) \le \sum g(y; x_L) w(\theta_1, y) - \Delta g \Delta w_0, \qquad (23)$$

equivalently,

$$w(\theta_0, B) \ge \sum g(y; x_L) w(\theta_1, y) + \theta_0 \Delta x - g(G; x_H) \Delta w_0$$
(24)

$$w(\theta_0, B) \le \sum g(y; x_L) w(\theta_1, y) - g(G; x_L) \Delta w_0.$$
<sup>(25)</sup>

By subtracting (24) from (25), we obtain

$$\theta_0 \Delta x \le \Delta g \Delta w_0. \tag{26}$$

This is (DC0).

**Step 2:** If  $\Delta \theta x_L \geq g(G; x_H)I_0$ , the principal achieves the payoff in benchmark by using the following compensation scheme.

$$w(\theta_0, G) = \theta_0 x_H + \Delta \theta x_L + (1 - g(G; x_H)) I_0,$$
(27)

$$w(\theta_0, B) = \theta_0 x_H + \Delta \theta x_L - g(G; x_H) I_0, \qquad (28)$$

$$w(\theta_1, G) = w(\theta_1, B) = \theta_1 x_L \tag{29}$$

We can easily check that this scheme satisfies (I-IC*i*) and (LLC*iy*) and the principal's payoff is  $\pi_{HL}^*$ . Therefore, the scheme is optimal compensation to implement  $(x_H, x_L)$  as the scheme gives the principal the same payoff as that in [P-1].  $\Delta \theta x_L \geq g(G; x_H)I_0$  is a condition to satisfy (LLC0*B*).

**Step 3:** If  $\Delta \theta x_L < g(G; x_H)I_0$ , both (D-IC0) and (D-IC1) are binding.

By some manipulation, binding (D-IC0) and binding (D-IC1) is

$$\Delta w_0 = I_0,\tag{30}$$

$$w(\theta_0, B) = \sum g(y; x_L) w(\theta_1, y) + \theta_0 \Delta x - g(G; x_L) I_0.$$
(31)

Suppose that  $\Delta w_0 = I_0 + \epsilon_1$  and  $w(\theta_0, B) = \sum g(y; x_L)w(\theta_1, y) - g(G; x_L)\Delta w_0 - \epsilon_2$  is solution, where  $\epsilon_1, \epsilon_2 \ge 0$  are chosen to satisfy (D-IC0) and (D-IC1).<sup>6</sup> By substituting them into the objective function, [P-3] becomes

$$\min \sum g(y; x_L) w(\theta_1, y) - f \Delta g I_0 + f(g(G; x_H)\epsilon_1 - \epsilon_2), \qquad (32)$$

s.t. 
$$w(\theta_1, y) - \theta_1 x_L \ge 0$$
 for any  $y$ , (33)

$$\sum g(y; x_L) w(\theta_1, y) \ge g(G; x_L) I_0 + \theta_0 x_H + \epsilon_2, \tag{34}$$

$$\Delta w_1 \le I_1. \tag{35}$$

where (34) is (LLC0B), and  $\Delta w_0 = I_0$  implies (LLC0G). As we can easily show that (34) is binding, the principal's payoff is

$$g(G;x_L)I_0 + \theta_0 x_H + (1-f)\epsilon_2 - f\Delta gI_0 + fg(G;x_H)\epsilon_1$$
(36)

The payoff is increasing with respect to  $\epsilon_1$  and  $\epsilon_2$ . This contradicts the optimality of  $\Delta w_0$  and  $w(\theta_0, B)$ .

<sup>&</sup>lt;sup>6</sup>The other  $\epsilon_1 < 0$  or  $\epsilon_2 < 0$  cannot satisfy (D-IC0) and (D-IC1).

**Step 4:** If  $\Delta \theta x_L < g(G; x_H)I_0$ , the following compensation scheme is optimal.

$$w(\theta_0, G) = \theta_0 x_H + I_0, \tag{37}$$

$$w(\theta_0, B) = \theta_0 x_H,\tag{38}$$

$$w(\theta_1, G) = w(\theta_1, B) = g(G; x_L)I_0 + \theta_0 x_H.$$
(39)

From step 2 and step 3, we easily show that the scheme is optimal. Under the compensation, the principal's payoff is

$$\pi_{HL}^* - [g(G; x_H)I_0 - \Delta\theta x_L]. \tag{40}$$

2.  $\Delta w_1 < I_0 < I_1 \le \Delta w_0$ (D-IC0) and (D-IC1) becomes

$$\sum g(y; x_H)w(\theta_0, y) - \theta_0 x_H \ge \sum g(y; x_L)w(\theta_1, y) - \theta_0 x_L, \quad (41)$$

$$\sum g(y; x_L) w(\theta_1, y) - \theta_1 x_L \ge \sum g(y; x_H) w(\theta_0, y) - \theta_1 x_H.$$
(42)

This is the same as (IC0) and (IC1). We can use the standard procedure to obtain the solution. We can easily show that (i) (D-IC0) and (D-IC1) are equivalent to binding (D-IC0) (ii)  $\Delta w_0 = I_1$ .

min 
$$f[w(\theta_1, B) + g(G; x_L)\Delta w_1 + \theta_0\Delta x]$$
 (43)

$$+ (1-f)[w(\theta_1, B) + g(G; x_L)\Delta w_1]$$
(44)

s.t. 
$$\Delta w_0 = I_1$$

$$\Delta w_1 < I_0 \tag{46}$$

$$w(\theta_1, B) + g(G; x_L)\Delta w_1 - g(G; x_H)\Delta w_0 + \theta_0\Delta x \ge \theta_0 x_H,$$

(45)

$$w(\theta_1, B) \ge \theta_1 x_L,\tag{48}$$

$$w(\theta_1, B) + g(G; x_L) \Delta w_1 \ge \theta_1 x_L, \tag{49}$$

Therefore, the optimal compensation scheme is given as follows. If  $\Delta \theta x_L \geq g(G; x_H) I_1$ , the optimal compensation scheme is

$$w(\theta_0, G) = \theta_0 x_H + \Delta \theta x_L + (1 - g(G; x_H)) I_1,$$
(50)

$$w(\theta_0, B) = \theta_0 x_H + \Delta \theta x_L - g(G; x_H) I_1,$$
(51)

$$w(\theta_1, G) = w(\theta_1, B) = \theta_1 x_L.$$
(52)

Under the compensation, the principal's payoff is

$$\pi_{HL}^*.\tag{53}$$

If  $\Delta \theta x_L < g(G; x_H) I_1$ , the optimal compensation scheme is

$$w(\theta_0, G) = \theta_0 x_H + I_1, \tag{54}$$

$$w(\theta_0, B) = \theta_0 x_H,\tag{55}$$

$$w(\theta_1, G) = w(\theta_1, B) = g(G; x_H)I_1 + \theta_0 x_L.$$
 (56)

Under the compensation, the principal's payoff is

$$\pi_{HL}^* - [g(G; x_H)I_1 - \Delta\theta x_L].$$
(57)

3.  $I_0 \leq \Delta w_1 \leq I_1, I_0 \leq \Delta w_0 < I_1$ (D-IC0) and (D-IC1) become

$$\sum g(y; x_L) w(\theta_1, y) \ge \sum g(y; x_L) w(\theta_0, y), \tag{58}$$

$$\sum g(y; x_H) w(\theta_0, y) \ge \sum g(y; x_H) w(\theta_1, y).$$
(59)

It is shown these equalities are equivalent to  $w(\theta_0, B) = w(\theta_1, B)$ . The problem becomes

$$\min \quad w(\theta_1, B) + fg(G; x_L)\Delta w_0 + (1 - f)g(G; x_L)\Delta w_1$$
s.t. 
$$I_0 \leq \Delta w_0 < I_1$$

$$I_0 \leq \Delta w_1 \leq I_1$$

$$w(\theta_1, B) \geq \theta_1 x_L,$$

$$w(\theta_1, B) \geq \theta_0 x_H,$$

We can easily show that (i)  $\Delta w_0 = \Delta w_1 = I_0$ ; (ii)  $w(\theta_0, B) = w(\theta_1, B) = \theta_0 x_H$ . The principal's payoff is

$$\pi_{HL}^* - \left[g(G; x_H)I_0 - f\Delta\theta x_L\right] \tag{60}$$

4.  $I_0 \leq \Delta w_1 \leq I_1, I_1 \leq \Delta w_0$ (D-IC0) and (D-IC1) become

$$\sum_{m} g(y; x_L) w(\theta_1, y) - \theta_1 \Delta x \ge \sum_{m} g(y; x_H) w(\theta_0, y), \qquad (61)$$

$$\sum g(y; x_H) w(\theta_0, y) \ge \sum g(y; x_H) w(\theta_1, y).$$
(62)

To satisfy both conditions, it requires

$$\sum g(y; x_L) w(\theta_1, y) - \theta_1 \Delta x - \sum g(y; x_H) w(\theta_1, y) \ge 0, \tag{63}$$

equivalently,  $\Delta w_1 \leq -I_1$ . This contradicts  $I_0 \leq \Delta w_1$  which is the condition of this case. Therefore there is no direct mechanism in this case.

#### **B** Proof of Proposition 7

If  $\Delta \theta x_L \geq g(G; x_H)I_0$ , we can easily show the former part in the proposition. In this appendix, we consider the situation satisfying  $\Delta \theta x_L < g(G; x_H)I_0$ .

We easily observe that delegation is optimal if  $\Delta v_p \in (0, k_2]$  because  $(x_L, x_L)$  is optimal under both decentralized and centralized decision process and the payoff of the principal in decentralized decision process is equal to the one in the benchmark under perfect commitment. Similarly, centralization is optimal if  $\Delta v_p \in (k_1, \infty)$ .

1.  $\Delta v_p \in (k_2, k_2 + \frac{g(g;x_H)I_0 - \Delta\theta x_L}{f}]$ (Centralization) - (Delegation) =  $\pi^*_{HL} - g(G; x_L)S - \pi^*_{HH}$ =  $f[\frac{f\Delta g - g(G; x_L)}{f\Delta g}\Delta v_p - k_2]$ 

By  $g(G; x_H) > g(G; x_L) \ge 1/2$  (which implies  $\Delta g \le 1/2$ ),  $f\Delta g - g(G; x_L)$  is negative. Therefore, delegation is optimal.

2.  $\Delta v_p \in (k_2 + \frac{g(g;x_H)I_0 - \Delta\theta x_L}{f}, \frac{(1-f)\Delta g}{(1-f)q(g;x_H) + fg(g;x_L)}]$ 

$$(\text{Centralization}) - (\text{Delegation})$$
  
=  $\pi_{HL}^* - g(G; x_L)S - [\pi_{HL}^* - (g(G; x_H)I_0 - \Delta\theta x_L)]$   
=  $\frac{q_L}{\Delta g} [\Delta v_p - k_2 - \frac{\Delta g(\theta_0 x_H - \theta_1 x_L)}{q_L}]$ 

Therefore, delegation optimal if  $\Delta v_p \leq \max\{k_2 + \frac{g(g;x_H)I_0 - \Delta\theta x_L}{f}, k_2 + \frac{\Delta g(\theta_0 x_H - \theta_1 x_L)}{q_L}\}$ .

3.  $\Delta v_p \in \left(\frac{(1-f)\Delta g}{(1-f)q(g;x_H) + fg(g;x_L)}, k_1\right]$ 

$$(\text{Centralization}) - (\text{Delegation})$$
$$= \pi_{HL}^* - (g(G; x_H)I_0 - \Delta\theta x_L) - \pi_{HH}^*$$
$$= (1 - f)[\Delta v_p - (k_1 - \frac{g(G; x_H)I_0 - \Delta\theta x_L}{1 - f})]$$

From  $\theta_1 \ge (f+1)\theta_0$ , we can show  $\frac{(1-f)\Delta g}{(1-f)q(g;x_H)+fg(g;x_L)} \ge k_1 - \frac{g(G;x_H)I_0 - \Delta\theta x_L}{1-f}$ . Therefore, centralization is optimal.

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