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On Interaction between the Monetary Environment and Incentive Compatibility:  
A Case of Simple Dynamic Insurance Contracts

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On Interaction between the Monetary Environment and Incentive Compatibility: A Case of Simple Dynamic Insurance Contracts

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Abstract.  This paper examines interaction between the monetary environment and dynamic insurance contracts. The incentive compatibility condition of dynamic contracts is primarily determined by returns on alternative investment opportunities. Therefore, in an economic environment where money is circulated as a precautionary savings device, inflation rates as alternative returns may have significant effects on the extent to which incentive compatibility conditions are binding. In particular, the paper demonstrates that incentive compatibility conditions could be relaxed by higher rates of inflation, so that more efficient insurance contracts could be implemented in an inflationary environment. This dependence of incentive compatibility conditions on inflation rates mitigates the welfare costs of holding money, and thereby raises optimal steady-state rates of inflation. Given greater degrees of relative risk aversion, stronger constraint-relaxing effects augment optimal rates of inflation.

JEL classification: E51, E52.

Keywords: optimal monetary policy, incentive compatibility constraint, irreversible investment, Tobin effect.

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1. Introduction This paper examines interaction between the monetary environment and the incentive compatibility conditions of dynamic insurance contracts. As explored in previous papers, including Townsend (1982), Green (1987) and Thomas and Worrall (1990), dynamic insurance contracts combine intertemporal resource allocation with intratemporal risk sharing in a sophisticated manner. However, with respect to the former, dynamic insurance contracts potentially compete with alternative financial instruments such as intertemporal allocation devices.

In Townsend (1982), who assumed no alternative investment opportunity, the extent to which incentive compatibility conditions are binding depends crucially on how low the risk-free returns implied by an autarkic resource allocation are. More concretely, the lower is the implied return, the less binding the incentive compatibility condition, and consequently, the better the potential performance of dynamic insurance contracts.

Conversely, given alternative investment opportunities with good returns, the provider of a dynamic insurance contract (the principal) is forced to compete with such alternatives by offering high returns to the insured (the agent) to the detriment of insurance performance. For example, Allen (1985) showed that when the agent is able to secretly borrow and lend at the same rate as the principal, no incentive compatible contract can improve on self-insurance by lending and borrowing at that rate.

In addition, Cole and Kocherlakota (2001) demonstrated that when the agent is able to take a long position (but no short position) on alternative investment opportunities with sufficiently high returns, the trading of risk-free bonds between agents mimics the efficient resource allocation induced by dynamic incentive compatible contracts. In other words, given alternative investment opportunities with relatively low returns, a dynamic principal-agent contract improves on self-insurance by lending and borrowing among agents.

This paper considers a simple dynamic insurance contract within the monetary environment in which money is circulated as a precautionary savings measure, and analyzes the interaction between this dynamic insurance contract and money-holdings, particularly in terms of welfare. Borrowing from Dutta and Kapur’s (1998) overlapping generations
model comprising three cohorts, our framework endogenizes money demand as a consequence of the lack of insurance for unobservable income shocks and the irreversibility of productive capital as well as the presence of liquidity constraints. As Dutta and Kapur (1998) showed, the optimal rate of inflation is positive (but relatively low) regardless of costly money-holdings, because steady-state welfare is improved by a lump-sum transfer of seigniorage to the youngest generation that makes irreversible decisions.

We introduce into their model a two-period insurance contract for a proportion of the idiosyncratic income fluctuations of middle-aged agents à la Townsend (1982). Our dynamic insurance contract incorporates an incentive compatibility condition specifying that consumers with higher incomes may prefer insurance payoffs with ‘back-loaded’ features to payoffs from alternative investment opportunities.

In this framework, our model allows an alternative welfare-improving route for moderately inflationary policies. Since holding money is an alternative to an insurance contract in our model, the value of dishonest reporting to high-income agents largely depends on the monetary environment. More concretely, higher rates of inflation lead to lower returns on money as a savings device, and thereby relax the incentive compatibility constraint specified by a dynamic insurance contract. We demonstrate that such a constraint-relaxing effect, in addition to the seigniorage-transfer effect emphasized by Dutta and Kapur (1998), can offset the welfare costs of holding money when steady-state rates of inflation are relatively low.

This welfare effect is surprising. On the one hand, in the context of incomplete markets, it is well-known that seigniorage redistribution among either heterogeneous agents or different generations can explain the positive welfare effects of monetary expansion (see, for example, the survey by Woodford, 1990). On the other, the introduction of sophisticated financial contracts into Bewley (1983)-type models with incomplete markets helps to reduce market incompleteness, and thereby lower money demand. Hence, it is likely to undermine the positive role of redistribution policies by inducing more costly money-financing due to reduced money demand. A novel feature of our model is that moderately inflationary
policy improves welfare through interaction between the monetary environment and dynamic insurance contracts, even though the enhanced insurance effect in general weakens the effect of the seigniorage redistribution.

Of the numerous papers that analyze the coexistence of money and either credit or insurance, Aiyagari and Williamson (2000) is the one most closely related to ours. An infinite-horizon insurance contract is incorporated into their model, while money demand is motivated by limited participation. They analyze a situation in which higher inflation contributes to relaxing a constraint on a commitment to a long-run insurance contract (that is not an incentive compatibility constraint). In their case, the optimal rate of inflation is also higher with relaxed commitment constraints.

This paper is organized as follows. In section 2, we present a monetary model with a two-period insurance contract. In section 3, we explore several numerical examples, incorporating logarithmic and non-logarithmic preferences. In section 4, we discuss our findings and present conclusions.

2. A monetary model with dynamic insurance contracts

2.1. A modified version of Dutta and Kapur (1998)  Our monetary model borrows from Dutta and Kapur’s (1998) overlapping generations model comprising three cohorts, although our model substitutes idiosyncratic income shocks for preference shocks. We are interested in their model in three respects. First, demand for fiat money is endogenized elegantly by taking into consideration the absence of insurance and the irreversibility of investment in addition to the presence of liquidity constraints. In their model, fiat money plays an essential role as insurance for unobservable preference shocks. Second, intergenerational allocation can be explicitly represented by government-issued fiat money. Third, a framework in which agents live for three periods enables the introduction of a two-period insurance contract as an intragenerational device for mitigating the moral hazard problem, which could not be resolved by a static (one-period) contract.

The economy has three overlapping generations, the young, middle-aged and old. The
size of each cohort is constant over time and standardized at unity. A young consumer
is endowed with $y_0$ units of goods. A middle-aged consumer earns an independently and
identically distributed (i.i.d.) income of $y_h$ with probability $\frac{1}{2}$ and one of $y_l$ with probability
$\frac{1}{2}$, where $y_h > y_l$. The average middle-aged income is defined as $y_a (= \frac{y_h + y_l}{2})$. As already
mentioned, middle-aged income is not publicly observable. There is no endowment for old
consumers.

A young consumer can make a physical investment of $I$ units, and earns $1 + r$ per unit
of investment when old. As in Dutta and Kapur (1998), investors can neither reverse nor
collateralize their investment in middle age. These assumptions are reasonable when in-
vestment by young consumers is not observable. Given that investment is irreversible and
cannot be collateralized, middle-aged consumers cannot insure themselves against idiosyn-
cratic shocks once they are known.

Both young and middle-aged consumers may hold fiat money, the value of which de-
preciates according to the per-period rate of inflation $\pi$. In the absence of an insurance
contract, only fiat money can play an essential role as self-insurance against the income
fluctuations of middle-aged consumers in this environment. As in Dutta and Kapur (1998),
any seigniorage revenues $s$ are redistributed to young consumers in the form of a lump sum.

Henceforth, we restrict our attention to the steady-state equilibrium, and omit time sub-
scripts. A young consumer maximizes an expected logarithmic utility function by choosing
middle-aged consumption and old consumption as follows:

$$
\max_{\{c^m_h, c^m_l, c^o_h, c^o_l\}} V = \frac{1}{2} (\ln c^m_h + \ln c^o_h) + \frac{1}{2} (\ln c^m_l + \ln c^o_l)
$$

(1)

where $c^m_h$ ($c^m_l$) is middle-aged consumption, given a realization of middle-aged income of
$y_h$ ($y_l$), while $c^o_h$ ($c^o_l$) is old-age consumption, given a realization of middle-aged income of
$y_h$ ($y_l$). Logarithmic preferences are used mainly for analytical convenience in this section.
Non-logarithmic preferences are solved numerically in Section 3.
Given the assumption,
\[ y_a = y_0(1 + r) = 1, \]
it is fairly easy to demonstrate that the first-best steady-state allocation, which is attainable from complete markets, is the achievement of complete consumption smoothing, in which the consumption distribution is given by \[ c^m_h = c^m_l = c^o_h = c^o_l = y_a(= 1). \] The corresponding expected utility \( V \) is equal to zero.

As in Dutta and Kapur (1998), neither insurance contracts nor self-insurance by borrowing is available, and only fiat money can provide self-insurance against income fluctuations during middle age. In the current framework, utility maximization is subject to the following budget constraints. With a realization of \( y_h \), we have

\[ I + m = y_0 + s, \]
\[ c^m_h \leq (1 - \pi)m + y_h, \quad (2) \]
and
\[ c^o_h = (1 + r)I + (1 - \pi)[(1 - \pi)m + y_h - c^m_h]. \]

With a realization of \( y_l \), we have

\[ I + m = y_0 + s, \]
\[ c^m_l \leq (1 - \pi)m + y_l, \quad (3) \]
and
\[ c^o_l = (1 + r)I + (1 - \pi)[(1 - \pi)m + y_l - c^m_l], \]

where \( I \) is irreversible physical investment, and \( m \) is the real value of fiat money held by a young consumer. Notice that \( m \) is non-negative, or \( m \geq 0 \).

If \( 1 + r > (1 - \pi)^2 \), then one of the liquidity constraints (2) or (3) should be binding. In this paper, we focus on the most plausible case, in which the budget constraint is binding only for middle-aged consumers with a low realization of income \( y_l \), or assume that only
equation (3) is binding. That is,

\[ c^m_l = (1 - \pi)m + y_l, \]  

(4)

and

\[ c^o_l = (1 + r)I \]  

(5)

for low-income earners as a result of liquidity constraints, while

\[(1 - \pi)c^m_h = c^o_h = \frac{1}{2} \left[ (1 + r)(y_0 + s - m) + (1 - \pi)y_h + (1 - \pi)^2 m \right] \]  

(6)

for high-income earners as a result of consumption smoothing. In fact, with respect to idiosyncratic shocks to time preferences, Dutta and Kapur (1998) consider essentially the same case; that is, the case in which the budget constraint is binding only for impatient consumers.

Since there are only two markets (goods and fiat money), the following market-clearing condition for the goods market implies equilibrium in the money market thanks to Walras’ law.

\[ I + \frac{c^m_l + c^m_l}{2} + \frac{c^o_h + c^o_h}{2} = y_0 + y_a + (1 + r)I \]  

(7)

With respect to monetary policy, the money supply \( M \) should increase by \( \Delta M = s(1 + \pi) + \pi M \) in each period to maintain this steady-state equilibrium. It is straightforward to show that zero inflation \( (\pi = 0) \) implies zero seigniorage \( (s = 0) \).

Given money demand \( m \), indirect utility \( V \) is derived by substituting equations (4), (5), and (6) into equation (1). For money demand \( m \) to be positive, the first derivative of \( V \) with respect to \( m \) at \( m = 0 \) should be positive, or

\[ \left. \frac{\partial V}{\partial m} \right|_{m=0} = \frac{1}{2} \frac{1}{2} \left[ (1 + r)(y_0 + s) + (1 - \pi)y_h \right] \left[ -(1 + r) + (1 - \pi)^2 \right] + \frac{1}{2} \left[ \frac{1 - \pi}{y_t} - \frac{1}{y_0 + s} \right] > 0. \]  

(8)

In the above equation, the first term on the right-hand side (marginal utility in the
high-income state) is negative, while the second term (marginal utility in the low-income state) may be positive. The condition expressed by equation (8) implies that with greater middle-aged income fluctuations \((y_h - y_l)\), lower rates of inflation \((\pi)\), or poorer returns on irreversible investment \((r)\), money demand \(m\) is more likely to be positive.

As the following proposition demonstrates, the above framework shares with Dutta and Kapur (1998) the local property that welfare improves when a small-scale monetary expansion is implemented at zero inflation \((s = \pi = 0)\).

**Proposition:** If only low-income earners are subject to liquidity constraints, and \(m\) is positive, then steady-state welfare \(V\) improves with a marginal increase in \(s\) from \(s = 0\) \((\pi = 0)\).

Proof: see the appendix.

As demonstrated in numerical examples in the next section, the current model exhibits a global property with respect to monetary expansion, which is confirmed numerically by Dutta and Kapur (1998). That is, when money demand \(m\) is positive, an expansionary monetary policy with \(s > 0\) is welfare-improving if the costs of holding money are not excessive at moderate rates of inflation. Hence, there is an optimal rate of inflation. These policy consequences, local and global, are due mainly to a seigniorage transfer to young consumers who make commitments to irreversible, but productive investment.

**2.2. Introduction of dynamic insurance contracts** As Dutta and Kapur (1998) explain, the unobservability of middle-aged income fluctuations may allow for the absence of insurance contracts, while the unobservability of investment behavior by young consumers may justify the irreversibility of, and the inability to collateralize physical investment. However, consideration of periods following the one in which middle-aged incomes are realized, as well as minor changes in either of these assumptions, may enable two-period contracts to be introduced for middle-aged incomes in this framework. In practice, the introduction
of dynamic contracts requires that agents on contracts can be observed throughout their lives.

**The replacement of fiat money by dynamic contracts**  A simple way to introduce a dynamic contract is to allow low-income middle-aged earners to borrow against future fruits from irreversible investment by allowing collateralizable physical investment. Replacing completely government-issued fiat money with privately-issued risk-free bonds \((m = 0)\), low-income earners maximize \(\ln(y_l + \rho_l) + \ln[(1 + r)y_0 - d\rho_l] \) or \(\ln(y_l + \rho_l) + \ln(1 - d\rho_l)\), while high-income earners maximize \(\ln(y_h - \rho_h) + \ln(1 + d\rho_h)\) with respect to \(\rho_l\) or \(\rho_h\) (the amount of privately-issued risk-free bonds) given \(d\) (gross risk-free returns). Note that \((1 + r)y_0 = 1\) by assumption.

Given the market-clearing condition \(\rho_l = \rho_h\), the equilibrium risk-free rate is equal to unity, while equilibrium lending from high-income to low-income earners is

\[
\rho^* = \frac{y_h - y_l}{4}\tag{9}
\]

Note that even low-income earners are not subject to liquidity constraints by construction, and hence, there is no demand for money as a precautionary savings device in this case.

As shown by Townsend (1982), if two-period contracts are made not \textit{ex post}, but prior to the realization of middle-aged incomes, dynamic insurance contracts not only mitigate the moral hazard problem relating to incentive compatibility conditions, but they also yield higher expected utility for young investors. The most important feature of such optimal dynamic insurance is that two-period insurance payoffs are contingent on voluntary reports of realized incomes from each middle-aged consumer.

Following Townsend (1982), we adopt the following parameterization for insurance payoffs for simplicity and convenience. On the one hand, insurance payoffs for high-income earners are ‘back-loaded’ with contributions in middle age and benefits in old age. That is, we assume that \((-\rho, +d\rho)\), where \(\rho > 0\) and \(d > 0\). On the other hand, payoffs for low-income earners are ‘front-loaded’ or \((+\rho, -d\rho)\).
In the context of the above payoff structure, a larger $\rho$ and a smaller $d$ imply greater insurance cover for fluctuations in middle-aged incomes. On the one hand, the transfer of $\rho$ from high-income earners to low-income earners during middle age function as standard insurance payoffs. On the other hand, as shown subsequently, the transfer of $d\rho$ from low-income earners to high-income earners during old age serves as an incentive device for forcing high-income earners to be honest. Hence, $d$ can be interpreted as the agency cost of mitigating asymmetric information problems. A dynamic contract with both $\rho = \frac{y_h - y_l}{2}$ and $d = 0$ (zero agency costs), if enforceable, would provide complete insurance cover.

Suppose that fiat money is replaced completely by dynamic insurance ($m = 0$). To keep high-income earners from dishonestly reporting their incomes, the following incentive compatibility condition must be satisfied:

$$\ln(y_h - \rho) + \ln(1 + d\rho) \geq \ln(y_h + \rho) + \ln(1 - d\rho).$$

Note that $(1 + r)y_0 = 1$ by assumption.

Approximating both sides of the above equation to the second order yields $\frac{1}{y_h} \leq d$. The left-hand side of this equation is equal to the risk-free rate implied by the autarkic allocation for high-income earners ($y_h$ for middle-aged consumers and unity for old-age consumers). It implies that the internal rate of dynamic insurance $d$ should equal or exceed the autarkic risk-free return. Notice that the autarkic rate is below the equilibrium one-period risk-free rate ($\frac{1}{y_h} < 1$), and hence, dynamic insurance incurs lower agency costs than does trading risk-free bonds.

With this binding incentive compatibility constraint $\frac{1}{y_h} = d$, the optimality condition requires that

$$\frac{1}{2} [\ln(y_h - \rho) + \ln(1 + d\rho)] + \frac{1}{2} [\ln(y_l + \rho) + \ln(1 - d\rho)]$$

must be maximized with respect to $\rho$. Then, the optimal level of transfer $\rho^{**}$ is equal to
\[ \rho^{**} = \frac{1}{4} (v - 2 + \sqrt{9v^2 + 4v + 4}) > \rho^* = \frac{v}{2}, \]

where \( v = \frac{y_h - y_l}{2} \).

It follows from the above optimality condition that the two-period incentive compatibility contract provides greater insurance cover than does self-insurance from issuing risk-free bonds (that is, there are larger transfers during middle age or \( \rho^{**} > \rho^* \)), and lower agency costs are incurred during old age or \( d = \frac{1}{y_n} < 1 \). Clearly, given this optimal dynamic insurance contract, there is no demand for money as a precautionary savings device.

The coexistence of dynamic contracts with fiat money We have so far examined cases in which dynamic contracts, whether financial or insurance, completely replace fiat money. However, one potentially serious problem with dynamic contracts involving large-scale transfers between contracting parties, particularly between aged consumers at contract maturity, is that such contracts may not be fully enforceable. There may be limited enforceability because irreversible investment may not be collateralizable or verifiable, or because some of the returns from physical investment may be in the form of private benefits, or because legal institutions may be unable to enforce dynamic contracts to a full extent.

Considering these issues, we assume that transfers within the same cohort are smaller than those for self-insurance that use risk free bonds \( \rho^* = \frac{y_h - y_l}{4} \). An important implication of this is that low-income earners may be subject to liquidity constraints, and in addition, young investors may hold fiat money as a precautionary savings device. In other words, dynamic insurance contracts for small transfers may coexist with fiat money.

Defining the size of the transfer \( \rho \) as

\[ \rho = \alpha \rho^* = \alpha \frac{(y_h - y_l)}{4}, \]

we choose \( \alpha \) (\( 0 < \alpha < 1 \)) so that the money demand of young investors can be positive.
Although there is no explicit mechanism for determining optimal transfers by construction, the incentive compatibility condition is still able to endogenously fix the internal return on dynamic contracts $d$. Because fiat money now competes with dynamic insurance contracts as an alternative investment opportunity, high-income earners (savers) compare the performance of dynamic insurance with that of holding money. Therefore, provided that the internal return on back-loaded payoffs ($d$) is greater than or equal to the return on fiat money as an alternative ($1 - \pi$), the nature of the back-loaded payoff prevents high-income earners from reporting dishonestly. In this case, the incentive compatibility condition is given by

$$d \geq 1 - \pi.$$  \hfill (11)

As before, $d$ represents the degree of insurance efficiency; the lower $d$, the smaller are agency costs.

If $\alpha$ is substantially less than unity, the following budget constraints are still obtainable. With a realization of low income, we have

$$c^m_l = (1 - \pi)m + y_l + \rho,$$

and

$$c^o_l = (1 + r)I - d\rho,$$

while with a realization of high income, we have

$$(1 - \pi)c^m_h = c^o_h = \frac{1}{2} \left[ (1 + r)(y_0 + s - m) + d\rho + (1 - \pi)(y_h - \rho) + (1 - \pi)^2m \right]$$ \hfill (12)

with the binding incentive compatibility condition $d = 1 - \pi$.

3. **Numerical examples** The most interesting feature of the coexistence of dynamic insurance contracts and fiat money is that there is interaction between the monetary environment and incentive compatibility constraints. Because of this feature, two competing effects on the optimal steady-state rates of inflation may arise. In asset pricing models with
limited insurance, greater insurance cover generally leads to higher real returns on risk-free assets (that is, closer to time preference rates) in the real economy, and leads to lower rates of inflation (higher rates of deflation) as well as reduced money demand in the monetary economy. In this sense, the introduction of dynamic insurance contracts generates overall downward pressure on equilibrium rates of inflation $\pi$.

However, as equation (11) implies, incentive compatibility conditions would be relaxed by higher rates of inflation, so that more efficient insurance contracts could be implemented in an inflationary environment. This dependence of incentive compatibility constraints on inflation rates would mitigate the welfare costs of holding money due to inflation, and thereby raise equilibrium rates of inflation with welfare improvement.

These competing effects on the optimal steady-state rates of inflation are illuminated by the following numerical examples. As suggested in the previous section, if middle-aged income fluctuations ($y_h - y_l$) increase, returns on irreversible investment ($r$) fall, or if insurance coverage ($\alpha$) is narrower, money demand $m$ is more likely to be positive. For example, Figure 1 depicts ranges for insurance coverage ($\alpha$) and income volatility ($v = \frac{y_h - y_l}{2}$) for positive values of money demand ($m$), when $r = 0.2$, $d = 1$, $\pi = 0$, and $s = 0$.1

3.1. Steady-state equilibrium Considering these properties of our model, we choose a set of parameters as follows. To obtain conditions under which money demand is positive, we choose low returns on irreversible investment ($r = 0.05$), volatile income fluctuations ($v = 0.4$), and medium insurance coverage ($\alpha = 0.3$). We set a seigniorage level ($s$) that is consistent with a given steady-state rate of inflation ($\pi$) so that equation (7) is satisfied, and calculate the welfare gain relative to autarky corresponding to each steady-state rate of inflation. Our choice of structural parameters is illustrative, not necessarily realistic. More realistic sets of parameters are used subsequently.

We construct each figure for the following cases: (1) there exists no insurance contract;

\footnote{Note that the incentive compatibility condition (11) and the market-clearing condition (7) are satisfied when $d = 1$, $\pi = 0$ and $s = 0$.}
(2) insurance contracts promise payoffs that are independent of inflation rates; and (3) insurance contracts have binding incentive compatibility. More concretely, the case in which there is no insurance contract, represented by the thin solid line in each figure, corresponds to the one analyzed in Dutta and Kapur (1998) where there is no alternative to holding money as a precaution against idiosyncratic income shocks. In case (2), represented by the dotted line, the degree of insurance efficiency $d$ is constant, and is set to be unity for all positive rates of inflation $\pi$. In the case in which there are binding incentive compatible contracts, represented by the thick solid line, the back-loaded payoffs $(-\rho, +d\rho)$ of the contracts are determined so that the incentive compatibility condition is binding, or so that $d = 1 - \pi$.

**Positive aspects** Although our primary interest is in the welfare impact of dynamic insurance contracts, particularly the effect on optimal rates of inflation, comments on positive aspects of the steady-state equilibrium of this model are worth making. Figures 2 and 3 indicate steady-state levels of money demand and seigniorage in the above three cases. As shown by Figure 2, higher inflation raises the cost of holding money and thereby lowers money demand in all cases. In addition, the absence of insurance contracts raises consumers’ money demand, and thereby lowers physical investment. Figure 3 shows that increased money demand due to the lack of insurance also augments seigniorage levels when there are no insurance contracts, while seigniorage levels are represented by hump-shaped Laffer curves due to the combined effects of higher inflation and lower money demand.

Unlike the inflation-independent contracts, the binding incentive compatible contracts provide consumers with opportunities for more efficient insurance, in which lower agency costs are associated with a given level of transfer payments $\rho$. Due to this insurance effect, the binding incentive compatible contracts tend to generate less demand for money as a precautionary savings measure.

However, the inflation-independent contracts are imperfect substitutes for money as a precautionary savings measure, and higher rates of inflation generate an excess premium.

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1 In this case, equation (11) is not binding, but it is satisfied.
for the inflation-independent contracts relative to holding money. In this regard, when there are inflation-independent contracts, money demand is lower.

With logarithmic preferences, given a steady-state level of inflation, the latter effect (lower money demand in inflation-independent contracts) just dominates the former (lower money demand in binding incentive compatible contracts). That is, when there are inflation-independent contracts, consumers prefer to hold slightly less money, and there are greater commitments to physical investment among young consumers.

**Normative aspects** Figure 4 depicts the welfare gain relative to autarky corresponding to each steady-state rate of inflation, and thereby indicates the optimal rates of inflation for the three cases. When there are no insurance contracts, as in Dutta and Kapur (1998), the optimal rate of inflation (denoted by a square) is positive because of the effect of the seigniorage transfer to young consumers who make irreversible decisions. More precisely, such a seigniorage transfer promotes irreversible, but productive investment among young consumers, so that it may mitigate the welfare costs of holding money due to inflation. Since the dominant welfare costs of money-holdings are associated with rising inflation, the optimal rate of inflation is moderate. It should be emphasized that redistribution between generations is largely responsible for the positive welfare effects of expansionary policy in this context.\(^3\)

When insurance payoffs are independent of inflation rates, overall welfare is higher due to the insurance effects, while steady-state rates of inflation are lower. In that case, the curve represented by the dotted line shifts to the northwest in Figure 4. Consequently, the optimal rate of inflation (denoted by a triangle) in Figure 4 is lower than in the previous case, in which there are no insurance contracts.

When incentive compatible contracts are binding, as a result of the constraint-relaxing effects, the optimal rate of inflation (denoted by a circle) in Figure 4 is higher than when insurance has inflation-independent payoffs. In other words, the constraint-relaxing effect

\(^3\) A survey on the optimum quantity of money by Woodford (1990) suggested that the positive welfare effects of expansionary monetary policy may be due to the redistribution of seigniorage among either heterogeneous agents or different generations.
cancels out the downward effect of insurance on steady-state rates of inflation. As equation (12) indicates, the optimal rate of inflation is the same as when there are no insurance contracts because the budget constraint of high-income earners reduces to the one that applies in the absence of dynamic insurance contracts due to indifference between insurance and money. Consequently, the first-order conditions of the optimization problems are the same as each other.

**Non-logarithmic preferences** The above welfare (normative) aspects of our numerical results do not depend on the logarithmic form of utility function; in fact, the welfare effects are enhanced by higher degrees of risk aversion. Assuming the same set of parameters other than with respect to preferences, we compute a list of optimal steady-state values for instances of power function preferences \( c^{1-\gamma} \) with varying degrees of relative risk aversion \( \gamma \) from 0.4 to 10, for the three cases: (1) no insurance contracts \( (\alpha = 0) \); (2) binding incentive compatible contracts \( (d = 1 - \pi) \) and (3) the inflation-independent payoff contracts \( (d = 1) \). Table 1 reports these numerical results. In addition, optimal steady-state values of inflation \( \pi \) and money demand \( m \) are summarized for different degrees of relative risk aversion in Figures 5 and 6 respectively.

Welfare gains are greater when there are binding incentive compatible contracts than when there are inflation-independent-payoff contracts or no insurance contracts. As Figure 5 shows, optimal rates of inflation increase substantially with higher degrees of relative risk aversion in each case. However, the ranking of optimal inflation between the three cases is the same as that for the logarithmic utility function \( \gamma = 1 \). Optimal rates of inflation are higher when there are binding incentive compatible contracts than when there are inflation-independent contracts. Optimal rates of inflation are the same (with rounding computational tolerances) whether there are binding incentive compatible contracts or no insurance contracts.

Figure 6 indicates that money demand at each optimal rate of inflation is, regardless of risk aversion, much larger without insurance than with insurance contracts. One interesting feature is that the difference in money-holdings when there are binding incentive
compatible contracts and inflation-independent payoff contracts depends on the degree of relative risk aversion. As already discussed, because of greater insurance coverage due to the constraint-relaxing effect, binding incentive compatible contracts generate less money demand, while because of an excess premium over money-holdings with an increase in optimal rates of inflation, the inflation-independent contracts result in less money demand. As Figure 6 demonstrates, the former effect dominates at each optimal rate of inflation when $\gamma$ is relatively small, but the latter effect dominates when $\gamma$ is greater than around three. Similarly, as $\gamma$ increases, the inflation-independent-payoff contracts generate less seigniorage levels than do the binding incentive compatible contracts.

As discussed above, the parameters chosen in the numerical examples are not necessarily realistic, but are illustrative. We now have a realistic set of parameters, in which two-period returns on irreversible investment are set equal to 0.5 instead of 0.05. Supposing that the length of a period in our overlapping generations model is 25 years, this implies that the return on physical investment, subtracting aggregate risk premiums, is around one percent per year. The numerical results suggest that money demand is non-negative only when the degree of relative risk aversion exceeds two. The annual optimal rate of inflation, with $\gamma = 8$ in the binding incentive compatible constraint case, is 3.1% per annum, which implies about four percent differences between physical investment and money-holdings.

**Distributional aspects** From a distributional point of view, we report the relative performance of three monetary environments in terms of inequality in consumption or welfare between high-income and low-income states. As reported in Table 1, our measures are the variances of middle-aged consumption $c_m$ and old-age consumption $c_o$ in high-income and low-income states, and welfare differences in utility between income states. These measures indicate inequality at the steady-state equilibrium, evaluated \textit{ex ante}, that is, before agents experience idiosyncratic income shocks.

With respect to the variance of middle-aged consumption, the inflation-independent contracts outperform any other contract when $\gamma$ is less than four, because the lower optimal rate of inflation under the inflation-independent-payoff contracts preserves the higher value
of money from youth to middle age. However, as the degree of risk aversion increases beyond four, money demand does not grow when there are inflation-independent contracts by as much as it does in any other monetary environment, and consequently, holding money as a precautionary savings measure declines substantially.

With regard to the variance of old-age consumption, both the environments with incentive compatible contracts and without insurance outperform inflation-independent-payoff contracts, since the latter with $d$ fixed at unity have higher agency costs, which are ultimately borne by low-income earners in their old age. Note that the optimal rates of inflation are the same when there are binding incentive compatible contracts or when there is no insurance.

Figure 7 illustrates a welfare criterion measured using differences in utility between income states. Of the three monetary environments, the most efficient insurance contracts with binding incentive compatibility constraints result in the largest improvement in welfare equality, while the inflation-independent contracts with $d = 1$ generate the lowest improvement because low-income earners bear the highest agency costs in their old age.

4. Discussion A dynamic insurance contract combines intertemporal resource allocation with intratemporal risk sharing in a sophisticated manner, while it may compete potentially with alternative investment opportunities. This paper has examined interaction between monetary policies and dynamic insurance contracts when fiat money is an intertemporal alternative to dynamic contracts. In particular, it demonstrates that incentive compatibility constraints could be relaxed with higher rates of inflation, so that more efficient insurance contracts can be implemented in an inflationary environment. This dependence of incentive compatibility constraints on inflation rates mitigates the welfare costs of holding money due to inflation, and thereby raises optimal steady-state inflation rates. Given greater relative risk aversion, stronger constraint-relaxing effects augment optimal rates of inflation.

Viewed from a rather different perspective, our model introduces a dynamic insurance contract as an intragenerational transfer into Dutta and Kapur (1998), who consider fiat
money as an intergenerational transfer. In our model, the first-best allocation cannot be achieved either intergenerationally or intragenerationally. Therefore, by considering the coexistence of insurance and fiat money, we have analyzed the connection between intragenerational and intergenerational devices through incentive compatibility constraints in the context of incomplete markets.

In the broader context of macroeconomics, the effect explored in this paper is related to the so-called Tobin effect (Tobin, 1955, 1965). Although there is no direct substitution of physical investment for money through inflation in our model, we showed that more efficient insurance contracts assume the role of money as insurance in a moderately inflationary environment, and thereby promote irreversible, but productive, physical investment. In addition, expansionary monetary policy improves welfare when inflation rates are relatively low.

In Dutta and Kapur (1998), a seigniorage redistribution between generations is largely responsible for the Tobin effect in that an inflationary policy generates not only investment-enhancing effects but also generates welfare-improving effects. This Tobin effect driven by redistribution is typically reduced as the role of money diminishes due to the introduction of credit or insurance. A novel feature of our model is that the incentive compatible constraint that is relaxed by moderate inflation helps to sustain the Tobin effect.

Appendix: a proof of the proposition in section 2

When only low-income earners are subject to liquidity constraints, given money demand $m$, an indirect utility $V$ is derived by substituting equations (4), (5), and (6) into equation (1). The derivative of this indirect utility $V$ with respect to $s$ evaluated at $s = \pi = 0$ reduces to

$$\frac{\partial V}{\partial s} = \frac{0.5}{y_0 - m} + \frac{1 + r}{(1 + r)(y_0 - m) + m + y_h} + \left[ \frac{(1 + r)(y_h - m) - m}{(1 + r)(y_0 - m) + m + y_h} - \frac{0.5m}{m + y_l} - 0.5 \right] \frac{\partial \pi}{\partial s}. \quad (13)$$

From total differentiation of the market-clearing condition (equation (7)),
\[ \frac{\partial \pi}{\partial s} = \frac{1}{0.5(c_h^o - c_l^o) + m} > 0, \]  

(14)
is obtained when \( s = \pi = 0 \).

Substituting equation (14) into equation (13), and rearranging yields

\[ \frac{\partial V}{\partial s} = \frac{0.25}{0.5(c_h^o - c_l^o) + m} (c_h^o - c_l^o)(\frac{1}{c_l^m} - \frac{1}{c_h^m}) > 0, \]

because \( c_h^o > c_l^o \) and \( c_h^m > c_l^m \) when only low-income earners are subject to liquidity constraints.

REFERENCES


Table 1: Results in Cases of Power Functions

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Figure 1: Conditions for Positive Money Demand
Figure 2: Steady-State Level of Money Demand

The figure illustrates the steady-state level of money demand in a model, with three scenarios depicted:

- **w/o insurance**: This line represents the scenario without insurance, showing a steady decline in money demand as the steady-state inflation rate increases.
- **w/i insurance (IC binding)**: This line indicates the scenario with insurance where the central commitment (IC) is binding, also showing a declining trend, albeit at a different rate compared to the no-insurance scenario.
- **w/i insurance (independent payoff)**: This line represents the scenario with insurance where the central commitment is independent, again showing a decline in money demand with increasing inflation, but at a still different rate.

The x-axis represents the steady-state inflation rate, ranging from -0.020 to 0.100, while the y-axis measures the level of money demand, ranging from 0.000 to 0.200. The chart highlights the impact of insurance on the steady-state level of money demand under different conditions.
Figure 3: Steady-State Level of Seigniorage
Figure 4: Welfare Gain Relative to Autarky

steady-state inflation rate

- w/o insurance
- w/i insurance (IC binding)
- w/i insurance (independent payoff)
Figure 5: Optimal Inflation and Relative Risk Aversion

- w/o insurance
- w/i insurance (IC binding)
- w/i insurance (independent payoff)
Figure 6: Money Demand at Optimal Inflation
Figure 7: Differences in Welfare between High Income and Low Income Earners at Optimal Inflation