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<th>Entrepreneurship and Asymmetric Information in Input Markets</th>
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Entrepreneurship and Asymmetric Information in Input Markets

by

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Abstract

Entrepreneurs starting new firms face two sorts of asymmetric information problems. Information about the quality of new investments may be private, leading to adverse selection in credit markets. And, entrepreneurs may not observe the quality of workers applying for jobs, resulting in adverse selection in labor markets. We construct a simple model to illustrate some consequences of new firms facing both sorts of asymmetric information. Multiple equilibria can occur. Stable equilibria can be in the interior, or at a corner in which no entrepreneurs enter. Stable interior equilibria can involve involuntary unemployment, as well as credit rationing. Equilibrium outcomes mismatch workers to firms, and will generally result in an inefficient number of new firms. With involuntary unemployment, there will be too few new firms, but with full employment, there may be too many or too few. Taxes or subsidies on new firms and employment can be used to achieve a second-best optimum. Alternative information assumptions are explored.

Key Words: entrepreneurship, asymmetric information, adverse selection

JEL Classification: D82, G14, H25

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1. Introduction

New firms and the entrepreneurs that initiate them are beset by problems of asymmetric information with respect to their prospects for success, as well as with respect to the quality of labor they are able to hire and their ability to obtain credit on good terms. The fact that new entrepreneurs are to a large extent indistinguishable from one another means that creditors are unable to tailor financial terms to entrepreneurs’ project qualities. As well, since they are hiring workers for the first time, they do not have the experience to discern the quality of potential workers and whether they will be a good match for the particular projects being initiated. This puts new firms at a significant disadvantage with respect to existing firms whose track records have been proven, who may have internal finance, and who have had a chance to sort out good or suitable workers from bad or unsuitable ones. These problems naturally lead to the question of whether public policies should actively encourage the entry of new entrepreneurial firms. That is the focus of this paper.

The literature has recognized in a piecemeal way some of the problems that new firms face due to asymmetric information, and has in some cases come to surprising results. The seminal paper of Stiglitz and Weiss (1981) studied the adverse selection problems that arise when banks are unable to distinguish high- from low-quality projects and must offer the same financial terms to all. In their case, the expected return could be observed, but not the riskiness of individual projects of a given expected return. In this setting, too few projects would be financed—those with the highest risk—suggesting a subsidy on the financing of new firms. Moreover, the possibility of credit rationing exists which exacerbates the underinvestment. Subsequently, de Meza and Webb (1987) considered the case in which banks could observe ex post project returns but not the probability of success. In this setting, the findings of Stiglitz and Weiss were reversed: there would be overinvestment in low-probability projects and no possibility of credit rationing, leading to a presumption of taxing new firms. These results have been generalized by Boadway and Keen (2005) to allow for more general patterns of project characteristics in the pool, and to allow for alternative forms of finance. What emerges is a general presumption of overinvestment as low-risk investments opt for debt finance and high-risk ones for equity
finance.\(^1\)

Asymmetric information has also been the focus of the venture capital (VC) literature where the financing of new entrepreneurs is combined with managerial advice. Here, the emphasis has been more on moral hazard problems associated with the effort of both the VC and the entrepreneur. Keuschnigg and Neilsen (2003) have argued that these moral hazard problems can be addressed by a tax on new firms combined with a reduced capital gains tax. Dietz (2002) has added adverse selection to the VC problem and allowed entrepreneurs the choice between VC financing (with managerial advice) and bank financing. He finds that high-risk projects choose the former and low-risk projects the later, but that too many low-risk projects end up being financed by VCs.

There has been a limited amount of attention paid to the consequences for new firms of imperfect information on other markets. Weiss (1980) considers the case of adverse selection in labor markets whereby firms cannot observe the quality of workers they hire. He shows that workers will tend to be drawn from the bottom of the skill distribution—since a common wage is paid regardless of quality—and too few will be hired. Moreover, there is a possibility of excess supply for labor, or involuntary unemployment, in equilibrium. A subsidy on employment would be welfare-improving in this context. The Weiss model focuses entirely on adverse selection in labor markets: there is no heterogeneity of entrepreneurs and there is no uncertainty of project success. Presumably other forms of asymmetric information in labor markets would particularly affect new firms as well, such as unobservable effort (Shapiro and Stiglitz, 1984) or search problems (Diamond, 1982).

There are potentially many other ways in which the entry of new firms is rendered inefficient because of asymmetric information or externalities. Some of these include signaling problems, knowledge externalities, and strategic barriers to entry. The potential consequences of various sources of inefficiency for tax policy toward new firms are surveyed in broad terms in Boadway and Tremblay (2005). Rosen (2005) surveys the empirical effects of existing policies on entrepreneurship in the United States.

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\(^1\) This overinvestment result is in sharp contrast to the consequences of asymmetric information for already existing firms. Myers and Majluf (1984) argue that managers of existing firms will pass up good projects when new equity finance is used if insiders have more information about the value of a firm’s projects that outside investors.
Our purpose is to study how adverse selection in both credit markets and labor markets affects equilibrium and efficiency in the formation of new firms, and to consider the consequences for policy. To do so, we develop a simple but rather specific model designed to capture the main features of information asymmetry facing new firms, while at the same time avoiding needless complications. This asymmetry is two-sided: potential entrepreneurs do not know quality of individual workers, and workers do not know quality, or ability, of new entrepreneurs. And, banks and governments know neither.

The model we use builds on the one used by de Meza and Webb (1987) to study adverse selection in credit markets by adding an employment dimension. Entrepreneurs of varying ability each hire a fixed number of workers who are of varying quality. If successful, an entrepreneur’s firm produced a fixed output, where the possibility of success depends jointly on the ability of entrepreneurs and the quality of workers. Entrepreneurs have no initial wealth, so must rely on credit to finance their operations. Equilibrium will involve the best entrepreneurs hiring workers randomly from the set of lowest-quality workers. There will be several sorts of inefficiencies in this context. Workers of different qualities will be mismatched with entrepreneurs of different ability. Neither labor markets nor credit markets may clear: there may be involuntary unemployment or credit rationing. And, there will be an inefficient number of entrepreneurs, either too many or too few depending on the nature of the equilibrium outcome. This will lead to the possibility of efficiency-enhancing policy intervention.

The basic model is outlined in the following section, and the full-information equilibria in Section 3. This is followed by the analysis of equilibrium when the quality of workers and the ability of entrepreneurs are both private. In Section 5, we investigate the efficiency of markets outcomes under asymmetric information and the implications for policy. The following section briefly discusses some extensions to the basic model. These include the consequences of asymmetric information applying only to one of credit or labor markets, and of allowing for the possibility of more complicated wage contracts to separate workers by quality. A final section concludes.
2. Elements of the Model

The model we use has several specific features, many of which are chosen to facilitate the analysis. It is constructed to highlight the kinds of issues that can arise when there is asymmetric information in labor and credit markets, while at the same time avoiding complications that can obscure the phenomena we are trying to illustrate and can also lead to excessively complex analysis. Many of our simplifications parallel those found in the literature on adverse selection in credit markets, such as the simple structure of project returns and the limitation in the number of dimensions of decision-making. Naturally, this leads to results that are model-specific, but hopefully are still suggestive.

The model is partial equilibrium in the sense that it focuses on the entrepreneurial sector of the economy, that is, the sector consisting of new entrepreneurial firms. There is a continuum of potential entrepreneurs, as well as a (separate) continuum of potential workers. Entrepreneurs differ in a single dimension called ‘ability’, denoted \( a \), while workers differ by quality, denoted \( q \). For simplicity, assume that the distributions of both \( a \) and \( q \) are uniform over \([0, 1]\), and that their total populations are the same and are normalized to be unity. The assumption that the distributions of entrepreneurs and workers are the same is important because, as we shall see, it leads to perfect matching of workers and entrepreneurs in the full-information outcome, thereby avoiding the complications that arise when matching is imperfect. In fact, the supports of the two distributions need not be the same, and are assumed to be so only for simplicity. Only a portion of both potential entrepreneurs and workers end up choosing the entrepreneurial sector, and those who do not have a fallback option as discussed below.

Every potential entrepreneur has a project that may either be successful or unsuccessful. Success occurs with probability \( p \) and yields a return \( R \), where \( R \) is fixed exogenously and is the same for all projects. If the project fails, zero revenue is obtained \((R = 0)\).\(^3\) To

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\(^2\) An alternative approach would be to assume that entrepreneurs and workers come from the same population, as in Kanbur (1981) for example, and discussed in de Meza (2002). In our approach, differences in attributes of entrepreneurs and workers play a key role, and it simplifies matters to abstract from the possibility that individuals may possess various amounts of each attribute. This would add an occupational choice dimension to our analysis that would complicate things considerably.

\(^3\) These assumptions are equivalent to the de Meza and Webb (1987) case, as opposed to the
undertake a project, an entrepreneur hires a fixed number of workers, taken to be one for simplicity. The probability of success of a project \( p \) depends upon both the ability of the entrepreneur \( a \) and the quality of the worker hired \( q \) according to:

\[
p(a, q) = \beta a q^\alpha, \quad 0 < \beta, \alpha < 1
\]

Both \( a \) and \( q \) may be private information to the entrepreneur or the worker respectively. Moreover, neither can be inferred ex post since all that might be observed is whether the project has succeeded or failed and not the probability of success.

When a project is undertaken, the worker is paid a wage \( w \) up front, and for simplicity this is the only cost to the entrepreneur. Entrepreneurs are assumed to have no wealth so the amount \( w \) must be obtained from credit markets.\(^5\) We assume credit takes the form of a loan extended by a bank at a gross interest rate of \( r \) (i.e., one plus the market interest rate). If the project is successful, the entrepreneur repays \( rw \) to the bank; otherwise, the firm is bankrupt and the bank receives no payment. We assume that banks can costlessly observe whether the project succeeds or fails. Since it may be in the interest of entrepreneurs to declare bankruptcy even if the project is successful, it may be more realistic to require that banks monitor project returns ex post in the event of such a declaration. Adding an ex post monitoring cost would not change the results, so we leave it out for simplicity. Some consequences of ex post monitoring costs when new firms face adverse selection in credit markets are discussed in Boadway and Keen (2005).

All agents—entrepreneurs, workers, and banks—are assumed to be risk-neutral. Potential entrepreneurs choose whether or not to undertake a project. Those who do not

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\(^4\) Equation (1) could be generalized so that \( p(a, q) = \beta a^\gamma q^\alpha \), but this would not have a qualitative effect on the results. An alternative approach would be to allow the quality of workers to affect the return \( R \) rather than the probability of success, as in Weiss (1980). This also leads to qualitatively similar results.

\(^5\) Adding a fixed capital cost that must be financed by credit, as in Stiglitz and Weiss (1981) and de Meza and Webb (1987), adds nothing of substance in our context since credit is already required to finance wage costs.
undertake their project have a perfectly certain alternative income, denoted $\pi_0$. The expected profit of the project to an active entrepreneur is then:

\begin{equation}
\pi = p(a, q)(R - rw) \geq \pi_0
\end{equation}

where $p(a, q)$ is determined by (1) and $\pi_0$ is assumed to be the same for all entrepreneurs.

Workers can seek employment either in the entrepreneurial sector or elsewhere in a ‘traditional’ sector, where there is full information and no uncertainty. To allow for the possibility of unemployment, we assume that there is ex post immobility between the two sectors: a worker who chooses to seek employment in the entrepreneurial sector cannot move to the traditional sector in the same period if a job is not obtained. Those who opt for the traditional sector produce an output equal to their quality $q$ for certain and earn a wage equal to $q$. There is no need for loan intermediation in the traditional sector since wage payments are perfectly certain and there is no bankruptcy. However, in the entrepreneurial sector, there may be asymmetric information in the sense that entrepreneurs cannot observe the quality of workers they hire. In this case, following Weiss (1980), all those who are employed in the entrepreneurial sector earn the same wage $w$ despite their quality. Later, we consider the possibility that contracts can be constructed that will separate workers by type. These will consist of a certain upfront wage payment as well as a bonus payment in the event the project is successful. As we shall see, there may be involuntary unemployment, in which case jobs are filled randomly from those who have opted for that sector.

Let $e$ be the proportion of workers in the entrepreneurial sector who become employed. Then, given risk neutrality, workers will seek work in the entrepreneurial sector only if the following reservation constraint is satisfied:

\begin{equation}
e w \geq q
\end{equation}

where $e \leq 1$. Let $\tilde{q}$ be the quality of workers who are just indifferent between the traditional and the entrepreneurial sectors, so $\tilde{q} = ew$. All workers with $q \leq \tilde{q}$ enter the entrepreneurial sector, so the number of workers in the entrepreneurial sector, given the uniform distribution assumed, is $\tilde{q}$. This result that the lowest quality workers enter the
entrepreneurial sector only applies when worker quality cannot be observed. As we shall see, higher-quality workers will be attracted to the entrepreneurial sector when \( q \) is observable by entrepreneurs. This constitutes an important source of inefficiency induced by asymmetric information.

In the credit market, banks are perfectly competitive. Let \( \rho \) be the risk-free gross rate of return that banks must pay to their depositors. If banks know the probability of success \( p \) of a given project—that is, they know the ability of the entrepreneur and the quality of the worker as in the full-information case—they will in equilibrium charge a gross interest rate \( r \) that, by their zero-expected profit condition, satisfies:\(^6\)

\[
(4) \quad r(p) = \frac{\rho}{p}
\]

However, if \( p \) for individual entrepreneurs is not known, a common gross interest rate \( \bar{r} \) must be charged that satisfies:

\[
(5) \quad \bar{r} = \frac{\rho}{\bar{p}}
\]

where \( \bar{p} \) is the expected probability of success of entrepreneurs who obtain bank finance.

In what follows, our analysis focuses on two cases, the benchmark full-information case in which both \( a \) and \( q \) are public information, and the asymmetric-information case in which both are private information. As we shall see, while the former is fully efficient, the latter is generally not even constrained efficient, thereby motivating policy intervention. In a later section, we briefly discuss the intermediate cases where either \( a \) or \( q \) are public information.

3. Equilibrium and Optimality with Full Information

With full information, both the ability of each entrepreneur and the quality of each worker is known to all agents, including the government. Equilibrium is characterized first by the set of potential entrepreneurs and workers who choose to enter the entrepreneurial

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\(^6\) This assumes that there are no operating costs for banks and that banks need not monitor projects ex post to verify that bankruptcy has in fact occurred. As mentioned, monitoring costs would have no qualitative effect on the results.
sector, and second by the assignment of workers to entrepreneurs. The nature of the equilibrium outcome depends on the parameters of the problem as well as the return $R$. For concreteness, we shall focus on a particular case: that is which the highest-quality workers opt for the entrepreneurial sector, although it is possible that the set of workers choosing the entrepreneurial sector may be in the interior of the quality distribution. The lowest-quality workers will go to the traditional sector since the probability of success will be too low in the entrepreneurial sector: for example, no entrepreneur would hire a worker with $q = 0$ since $p = 0$ in that case. (The highest-ability entrepreneurs will always enter the sector.) In the case where the highest-quality workers enter, the nature of the equilibrium outcome is intuitive.

Consider first the matching of workers by quality with entrepreneurs by ability. The probability of success for a type-$a$ entrepreneur employing a type-$q$ worker is $p(a, q) = \beta a q^\alpha$, which is known to the worker and the banks. For a given distribution of worker qualities and entrepreneurial abilities in the entrepreneurial sector, aggregate expected output will be highest if higher-quality workers are matched with higher-ability entrepreneurs. To see this, consider two entrepreneurs of ability $a_2 > a_1$ and two workers of quality $q_2 > q_1$. Expected output will be highest if $q_2$ is matched with $a_2$, and $q_1$ with $a_1$, since:

$$\beta a_1 q_1^\alpha R + \beta a_2 q_2^\alpha R > \beta a_1 q_2^\alpha R + \beta a_2 q_1^\alpha R, \quad \text{or} \quad a_2(q_2^\alpha - q_1^\alpha) > a_1(q_2^\alpha - q_1^\alpha)$$

With full information, wage rates will be specific to workers’ abilities, so there will be no adverse selection and no involuntary unemployment (unlike in the asymmetric-information case as we shall see in the next section). Therefore, there will be an equal number of workers and entrepreneurs in the entrepreneurial sector. Given that the densities of $q$ and $a$ are identical by assumption, we might expect that the full-information equilibrium will entail perfect matching of workers to entrepreneurs with $q$ increasing in $a$. Moreover, if the highest-ability entrepreneurs and the highest-quality workers are the ones that opt for the entrepreneurial sector, which is the case we shall assume, the matching outcome will imply $q = a$ since the upper support of both distributions is the same. We proceed by showing that $q = a$ is in fact an equilibrium in the full-information case when both entrepreneurs and workers are drawn from the top of their respective distributions.
The cutoff ability of active entrepreneurs will then be determined by a zero-net-expected-profit condition, and since each entrepreneur hires one worker, that will determine the cutoff quality of workers in the entrepreneurial sector.

To see that perfect matching will be an equilibrium, suppose the wage function is \( w(q) \), where \( w(q) \geq q \) to ensure participation. Given that \( a \) and \( q \) are public information, the banks charge an entrepreneur-specific gross interest rate of \( r(p) = \rho/p \). An entrepreneur with ability \( a \) chooses \( q \) to maximize expected profits, given the wage function \( w(q) \):

\[
\max_{q} \pi = p(a,q)(R-r(p)w(q)) = \beta a q^\alpha R - \rho w(q)
\]

The first-order condition is:

\[
w'(q) = \frac{\alpha \beta R}{\rho} a q^{\alpha - 1}
\]

This determines \( q(a) \), the quality of worker most preferred by an entrepreneur of type \( a \), given the market wage function \( w(q) \).

Our approach is to suppose that \( q = a \), and show that the wage function that supports that outcome can be an equilibrium. If \( q = a \), (6) may be written as the following differential equation:

\[
w'(q) = \frac{\alpha \beta R}{\rho} q^{\alpha}
\]

Integrating this yields:

\[
w(q) = \frac{\alpha \beta R}{\rho(1 + \alpha)} q^{\alpha + 1} + F
\]

where \( F \) is a constant of integration, whose value will be determined below. We assume that \( w(q) \) is defined over \([0, 1]\) and is a continuous function. Moreover, we assume that \( R \) is sufficiently large that in \((6')\), \( w'(q) > 1 \) for all workers in the entrepreneurial sector. In fact, this will be the case if \( w'(q) > 1 \) for the marginal worker, since \( w'(q) \) is increasing in \( q \) along the wage profile defined by \((6')\). This is sufficient to ensure that the highest-quality workers are the ones that enter the entrepreneurial sector.
Let $\tilde{q}$ be the ability of the marginal worker. Then, competition for workers will ensure that $w(\tilde{q}) = \tilde{q}$, where $\tilde{q}$ is the wage rate that can be obtained in the traditional sector.\footnote{To see this, note that if $w(\tilde{q}) > \tilde{q}$, a worker of slightly lower quality, say $\tilde{q} - \varepsilon$, can offer to work for a slightly lower wage. The marginal entrepreneur would then prefer to employ this worker than one with quality $\tilde{q}$.
} Since $w'(q) > 1$, $w(q) > q$ for all workers with $q > \tilde{q}$, implying that the highest-quality workers enter the entrepreneurial sector. (Of course, had it been the case that $w'(q) < 1$ for the marginal worker, a segment of workers from the interior of the wage distribution would enter.) Since, as we shall confirm below, the highest-ability entrepreneurs are the active ones, the marginal entrepreneur with lowest ability $\tilde{a}$ will hire the marginal worker $\tilde{q}$. And, since the densities of the two distributions are the same, this implies that $q(a) = a$ so there is perfect matching. Therefore, from $w(\tilde{q}) = \tilde{q} = \tilde{a}$, we can infer by applying (7) for the marginal worker that $F$ satisfies:

\begin{equation}
F = \tilde{a} - \frac{\alpha \beta R}{\rho (1 + \alpha)} \tilde{a}^{\alpha + 1}
\end{equation}

The wage profile will adjust so that (8) is satisfied. All workers with $q > \tilde{a}$ will therefore earn $w(q) > q$ by (7) and the fact that $w'(q) > 1$, implying that they will earn a surplus.

Consider now the entrepreneurs. The expected profit of an entrepreneur of ability $a$ is given by: $\pi = \beta a q^\alpha R - \rho w(q)$, where $w(q)$ satisfies (7) and (8). The solution to the entrepreneur’s first-order condition will be $q = a$, and it will be unique since the second-order condition for the entrepreneur’s problem is satisfied. Therefore, expected profits may be written:

\begin{equation}
\pi(a) = \beta a^{1+\alpha} R - \rho \left( \frac{\alpha \beta R}{\rho (1 + \alpha)} a^{\alpha + 1} + F \right) = \frac{\beta R}{(1 + \alpha)} a^{\alpha + 1} - \rho F
\end{equation}

where $F$ is given by (8). Since $\pi(a)$ is increasing in $a$, that implies that the marginal entrepreneur will have the lowest ability among active entrepreneurs. The ability of the marginal entrepreneur $\tilde{a}$ is determined, using (8), by:

\begin{equation}
\pi_0 = \beta R \tilde{a}^{\alpha + 1} - \rho \tilde{a}
\end{equation}

All entrepreneurs with $a \geq \tilde{a}$ will enter, and the number of active entrepreneurs, denoted $m$, will be given by $m = 1 - \tilde{a}$.
It is apparent that this full-information equilibrium is efficient. To see that, we only have to show that $\tilde{a}$ is optimal. This determines the number of active entrepreneurs and workers, and we already know that output is maximized when matching is perfect, which will be the case when $q = a$. Social surplus is given by the following, where $q = a$ in equilibrium:

$$S = \int_{\tilde{a}}^{1} [p(a, q)R - \rho w(q) - \pi_0] da + \rho \int_{\tilde{q}}^{1} [w(q) - q] dq$$

(10)  

where the first term is the surplus obtained by entrepreneurs and the second term the surplus of workers, both measured in terms of second-period income.\(^8\) Differentiating (10) by $\tilde{a}$, we obtain:

$$\frac{dS}{d\tilde{a}} = - [\beta \tilde{a}^{\alpha+1} R - \rho w(a) - \pi_0] - \rho [w(\tilde{a}) - \tilde{a}] = - [\beta \tilde{a}^{\alpha+1} R - \rho \tilde{a} - \pi_0] = 0$$

where the last equality follows from (9). Therefore, the number of entrepreneurs is optimal, and the full-information equilibrium is efficient.

The equilibrium outcome we have described in this section is only one of many that can occur. We have chosen it partly for simplicity, but partly because of the stark differences that will exist between it and equilibria under asymmetric information discussed below.\(^\ast\) As mentioned, if we had assumed that $w'(q) < 1$ in (6'), the set workers who opt for the entrepreneurial sector would be along an interval of dimension $1 - \tilde{a}$ in the interior of the quality distribution. Of course, higher-quality workers would be matched with higher-entrepreneurs, and the equilibrium outcome under full information would still be socially optimal.

\(^8\) Alternatively, dividing through by $\rho$ would yield the surplus in present value terms. Since workers get paid in the first period, their surplus occurs then, while entrepreneurs’ surplus occurs in the second period.
4. Equilibrium with Asymmetric Information

In this case, both $a$ and $q$ are private information. We suppose for now that wages are paid up front and are therefore independent of the success of the firm. Then, all workers who are employed in the entrepreneurial sector obtain a common wage rate $w$, while all active entrepreneurs face a common interest rate $\bar{r}$. We begin with a general overview of the relationships that must hold in equilibrium before turning to the qualitative features of equilibria.

Equilibrium Relationships

Consider first the decision of workers to seek employment in the entrepreneurial versus the traditional sector. Suppose workers believe—correctly in equilibrium—that the probability of being employed in the entrepreneurial sector is $e$. All entrepreneurs will offer the same wage rate $w$ in equilibrium since all workers have the same expected quality from the point of view of entrepreneurs. Given $w$, the expected income of workers in the entrepreneurial sector is $ew$. Since a worker of quality $q$ can receive a wage of $q$ in the traditional sector, the cutoff quality of workers by (3) is $\tilde{q} = ew$. All workers with $q < \tilde{q}$—those with the lowest quality—choose the entrepreneurial sector regardless of the parameters of the problem. This is in contrast with the social optimum achieved with full information where a segment of higher-quality workers will be attracted to the entrepreneurial sector.

Each entrepreneur also takes $e$ as given. Recall from (1) that the probability of success for any entrepreneur depends on $q^\alpha$, which cannot be observed here. When an entrepreneur hires a worker, that worker is selected randomly from the pool of available workers with $q \in [0, \tilde{q}]$. Given $e$ and the uniform distribution of workers, the expected value of $q^\alpha$ is:

\[ E[q^\alpha] = \frac{1}{\tilde{q}} \int_0^{\tilde{q}} q^\alpha dq = \frac{\tilde{q}^{\alpha+1}}{1+\alpha} = \frac{(ew)^\alpha}{1+\alpha} \]

Then, for an entrepreneur of ability $a$, the expected probability of success will be given by

\[ E[p] = \beta a E[q^\alpha] = \frac{\beta a}{1+\alpha}(ew)^\alpha \]

Expected profits for this entrepreneur can therefore be written, using (2), as:

\[ \pi = E[p](R - \bar{r}w) = \frac{\beta a}{1+\alpha}(ew)^\alpha(R - \bar{r}w) \]
This assumes that all entrepreneurs who choose to become active can receive a loan at the rate $\pi$. We return later to the issue of whether there can be credit rationing in this context, which would complicate matters considerably.

Each active entrepreneur can be thought of as choosing a wage rate to maximize profits, given $e$. As we shall see, all entrepreneurs who become active offer the same wage rate $w$ in equilibrium. Moreover, workers will be indifferent among active entrepreneurs since they are paid the wage $w$ in advance, so are not affected by bankruptcy. Let $m$ be the number of active entrepreneurs. Since there are $\tilde{q}$ workers in the entrepreneurial sector and since each entrepreneur hires one worker, the employment rate $e$ is given by:

\begin{equation}
\tag{14}
e = \min \left\{ \frac{m}{\tilde{q}}, 1 \right\}
\end{equation}

For the case in which $m < \tilde{q}$, there is unemployment ($e < 1$) and we have by (3) and (14), $\tilde{q} = ew = mw/\tilde{q}$. Therefore, the labor force in the entrepreneurial sector and the rate of employment can be expressed respectively as $\tilde{q} = \sqrt{mw}$ and $e = \sqrt{m/w}$. This implies that the quality of the marginal worker with full employment and unemployment may be written:

\begin{equation}
\tag{15} \tilde{q} = ew = \begin{cases} w & \text{if} \quad m = w \\ \sqrt{mw} & \text{if} \quad m < w \end{cases}
\end{equation}

Banks can observe neither $a$ nor $q$. Assuming that projects are allocated randomly among banks, the interest rate they offer will be $\pi = \rho/p$ by (5), where $p$ is the expected probability of success of any given project. Using (11) and (15), $p$ is given by:

\begin{equation}
\tag{16} p = \frac{\beta a}{1+\alpha} w^\alpha \quad \text{if} \quad m = w \\
\frac{\beta a}{1+\alpha} (mw)^\alpha \quad \text{if} \quad m < w
\end{equation}

where $\pi$ is the average quality of active entrepreneurs, discussed below. Expected profits of an entrepreneur of ability $a$ in equilibrium can then be written, using (13), (14) and (15), as:

\begin{equation}
\tag{17} \pi = \begin{cases} \frac{\beta a}{1+\alpha} w^\alpha (R - \pi w) & \text{if} \quad m = w \\ \frac{\beta a}{1+\alpha} (mw)^\alpha \pi (R - \pi w) & \text{if} \quad m < w \end{cases}
\end{equation}
Finally, we can use these expressions for expected profits to determine the surplus accruing to society from a given allocation of resources. Note that inactive entrepreneurs (who obtain reservation profits $\pi_0$) and workers who remain in the traditional sector earn no surplus. Moreover, workers who enter the entrepreneurial sector but are unemployed produce nothing. (If there were credit rationing, active entrepreneurs who are unable to obtain a loan would earn no surplus if there is ex post immobility between the two sectors.) Let $A$ and $Q$ be the respective sets of entrepreneurs and workers in the entrepreneurial sector, where $Q = \{q \mid q \leq \tilde{q}\}$ and $A$ is discussed below. Social surplus $S$ is given by:

$$S = \int_A [p \cdot (R - \bar{\tau}w) - \pi_0] \, da + \rho \int_Q [ew - q] \, dq = \bar{\tau}m[R - \bar{\tau}w] - m\pi_0 + \rho \left[ ew\tilde{q} - \frac{\tilde{q}^2}{2} \right]$$

where, recall, $m$ is the number of entrepreneurs and $\bar{\tau}$ is the expected probability of success of all active entrepreneurs. Since, $e\tilde{q} = m$ and $\bar{\tau} = \rho/\bar{p}$, social surplus can be written, using (15) and (16), as:

$$\text{(18)} \quad S = m(\bar{\tau}R - \pi_0) - \frac{\rho\tilde{q}^2}{2} = \begin{cases} m \left[ \frac{\beta_m}{1+\alpha} m^\alpha - \pi_0 - \frac{\rho m}{2} \right] & \text{if } m = w \\ m \left[ \frac{\beta_m}{1+\alpha} (mw)^\alpha - \pi_0 - \frac{\rho m}{2} \right] & \text{if } m < w \end{cases}$$

We turn now to the determination of the two key endogenous variables in the model, the wage rate (which determines the number of workers who opt for the entrepreneurial sector) and the number of entrepreneurs.

**Determination of the Wage Rate**

Consider first the wage rate preferred by any given entrepreneur. Using (13), the value of $w$ that maximizes the profits of an entrepreneur of any ability, given the employment rate $e$, is the solution to the following problem:

$$\max_{\{w\}} \ w^\alpha \cdot (R - \bar{\tau}w)$$

Using the first-order conditions and (5), the solution, denoted $\hat{w}$, is given by:

$$\hat{w} = \frac{\alpha R}{(1 + \alpha)\bar{\tau}} = \frac{\alpha R\bar{p}}{(1 + \alpha)\rho}$$

which is independent of the ability level of the entrepreneur. The second-order conditions are satisfied given our assumption that $\alpha < 1$. Starting at $w = 0$, profits will initially rise
with \( w \) and eventually reach a peak at \( \hat{w} \), assumed to be at \( \hat{w} < 1 \) so that we have an interior solution. The intuition here is that the rise in \( w \) attracts better quality workers, which increases the probability of success, but also increases labor costs. Given that \( p \) is concave in \( q \), the latter eventually offsets the former.

Denote by \( w^e \) the market clearing, or equilibrium, wage rate, that is, the wage rate such that \( e = 1 \). Given \( m \), the market clearing wage will be \( w^e = m \), since then the number of workers just equals the number of entrepreneurs, \( \tilde{q} = m \). Whether or not involuntary unemployment exists depends upon the relative size of \( w^e \) and \( \hat{w} \). If \( \hat{w} > w^e = m \), entrepreneurs will bid up the wage rate above the market clearing level, attracting excess workers into the entrepreneurial sector and generating involuntary unemployment. On the other hand, if \( \hat{w} \leq w^e = m \), the wage rate will be bid up only to \( w^e \) by competition for workers, so there will be full employment. Consider the consequences for entrepreneurial profits of each outcome in turn.

**Unemployment Case:** \( \hat{w} > w^e = m \)

In this case, the market wage is \( \hat{w} \) given by (19) and \( \bar{p} \) is given by the second row of (16). These consist of two equations in \( \hat{w} \) and \( \bar{p} \) whose solutions are:

\[
(20) \quad \bar{p}(\bar{a}, m) = \left( \frac{\beta \bar{a}}{1 + \alpha} \right)^{\frac{2}{\alpha}} \left( \frac{\alpha}{1 + \alpha} \frac{mR}{\rho} \right)^{\frac{2}{\alpha}}
\]

\[
(21) \quad \hat{w}(\bar{a}, m) = \left[ \frac{\beta \bar{a}}{1 + \alpha} \frac{\alpha}{1 + \alpha} \frac{R}{\rho} \right]^{\frac{2}{\alpha}} m^{\frac{\alpha}{2 - \alpha}}
\]

where both \( \bar{p}(\bar{a}, m) \) and \( \hat{w}(\bar{a}, m) \) are increasing in \( \bar{a} \) and \( m \).

The expected profits of an entrepreneur of ability \( a \) can be written as follows, using the second row of (17) with \( w = \hat{w} \):

\[
\pi = \frac{\beta a (m \hat{w})^{\frac{2}{\alpha}}}{1 + \alpha} (R - \bar{w} \hat{w})
\]

Using (21), this yields:

\[
(22) \quad \hat{\pi}(a, \bar{a}, m) = a \left( \frac{\beta}{1 + \alpha} \frac{R}{1 + \alpha} \right)^{\frac{2}{\alpha}} \left( \frac{\alpha \bar{a} m}{\rho} \right)^{\frac{\alpha}{2 - \alpha}}
\]
where \( \hat{\pi}(\cdot) \) refers to expected profits when the wage rate is \( \hat{w} \). This function for expected profits is increasing in all three arguments, \( a, \bar{a} \) and \( m \).

**Full Employment Case:** \( \hat{w} \leq w^e = m \)

In this case, by the first row of (16), we have

\[
\bar{p} = \frac{\beta \bar{a} m^\alpha}{1 + \alpha}
\]

Expected profits of an entrepreneur of ability \( a \) can immediately be written, using \( \bar{p} = \rho/\bar{p} \) and the above expression for \( \bar{p} \), as:

\[
\pi^e(a, \bar{a}, m) = \frac{\beta a m^\alpha}{1 + \alpha} (R - \bar{a} m) = a \left( \frac{\beta R m^\alpha}{1 + \alpha} - \frac{\rho m}{\bar{a}} \right)
\]

In this case, \( \pi^e(\cdot) \), expected profits when \( w = w^e \), is increasing in \( a \) and \( \bar{a} \), but the effect of \( m \) is ambiguous.

**The Number of Active Entrepreneurs**

Given that expected profits are increasing in ability \( a \) with or without unemployment, there is a cutoff ability level \( \tilde{a} \) such that all entrepreneurs with \( a \geq \tilde{a} \) become active and the remainder obtain their reservation profits \( \pi_0 \). Then, the number of active entrepreneurs is \( m = 1 - \tilde{a} \), and given the uniform distribution that we have assumed, their average (expected) ability is \( \bar{a} = (1 + \tilde{a})/2 \), so

\[
m = 2 - 2\bar{a}
\]

Recall from (21) that the profit-maximizing wage rate \( \hat{w} \) depends on \( \bar{a} \) and \( m \). Therefore, whether \( \hat{w} \gtrless w^e = m \) depends on \( \bar{a} \). In particular, using (21) and (24), we obtain that:

\[
\hat{w} > w^e \iff \frac{\beta \bar{a}}{1 + \alpha} \frac{\alpha}{1 + \alpha} \frac{R}{\rho} > (2 - 2\bar{a})^{1-\alpha}
\]

The left-hand side of (25) is increasing in \( \bar{a} \), while the right-hand side is decreasing. Moreover, at \( \bar{a} = 0 \), the right-hand side exceeds the left-hand side. Therefore, there will be a value of \( \bar{a} \), denoted \( \bar{a}' \) such that

\[
\frac{\beta \bar{a}'}{1 + \alpha} \frac{\alpha}{1 + \alpha} \frac{R}{\rho} = (2 - 2\bar{a}')^{1-\alpha}
\]

16
For $\overline{\alpha} \leq \overline{\alpha}'$, $\hat{w} \leq w^e$ (there is full employment), and vice versa. It must be the case that $0 < \overline{\alpha}' < 1$ (since the right-hand side is less than the left-hand side at $\overline{\alpha} = 1$). Note that $\overline{\alpha} \geq 1/2$ since if $m = 0, \overline{a} = 1/2$. In what follows, we shall assume that $\overline{\alpha}' > 1/2$ to allow for the possibility that there is full employment. If $\overline{\alpha}' < 1/2$, $\overline{\alpha}$ would always exceed $\overline{\alpha}'$ so there would always be involuntary unemployment.

Whether the equilibrium involves full employment or unemployment depends on the relationship between $\overline{\alpha}$ and $\overline{\alpha}'$. In turn, the market wage $w$ as well as the expected probability of success $\overline{p}$ and expected profits $\pi$ depend on this relationship. We can summarize these results for future reference as follows:

\begin{equation}
    w(\overline{\alpha}) = \begin{cases}
        m & \text{if } \overline{\alpha} \leq \overline{\alpha}' \\
        \hat{w}(\overline{\alpha}, m) & \text{if } \overline{\alpha} > \overline{\alpha}'
    \end{cases}
\end{equation}

\begin{equation}
    \overline{p}(\overline{\alpha}) = \begin{cases}
        \frac{\beta m^\alpha}{(1+\alpha)} & \text{if } \overline{\alpha} \leq \overline{\alpha}' \\
        \left( \frac{\beta m^\alpha}{1+\alpha} \right)^{\frac{2-\alpha}{2-\frac{\alpha}{\rho}}} \left( \frac{\alpha m R}{\rho} \right)^{\frac{2-\alpha}{2-\frac{\alpha}{\rho}}} & \text{if } \overline{\alpha} > \overline{\alpha}'
    \end{cases}
\end{equation}

\begin{equation}
    \pi(a, \overline{\alpha}) = \begin{cases}
        a \left( \frac{\beta m^\alpha}{1+\alpha} - \frac{\rho m}{\overline{\alpha}} \right) & \text{if } \overline{\alpha} \leq \overline{\alpha}' \\
        a \left( \frac{\beta m^\alpha}{1+\alpha} \right)^{\frac{2-\alpha}{2-\frac{\alpha}{\rho}}} \left( \frac{\alpha m R}{\rho} \right)^{\frac{2-\alpha}{2-\frac{\alpha}{\rho}}} & \text{if } \overline{\alpha} > \overline{\alpha}'
    \end{cases}
\end{equation}

where $\hat{w}(\overline{\alpha}, m)$ is given by (21), $\overline{\alpha}'$ is given by (26) and $m = 2 - 2\overline{\alpha}$ by (24).

It remains to determine $\overline{\alpha}$, the average quality of entrepreneurs, which depends upon how many entrepreneurs become active. For the marginal entrepreneur, $\pi(\hat{a}, \overline{\alpha}) = \pi_0$. By (27.3), for $a > \hat{a}$, we have $\pi(a, \overline{\alpha}) > \pi_0$ since $\pi(a, \overline{\alpha})$ is increasing in $a$, confirming our presumption that active entrepreneurs are those such that $a \geq \hat{a}$. In characterizing the number of active entrepreneurs, we can focus on the expected profit function for the marginal entrepreneur, defined as $\tilde{\pi}(\overline{\alpha}) \equiv \pi(\hat{a}, \overline{\alpha})$.

Using (27.3), we obtain

\begin{equation}
    \tilde{\pi}(\overline{\alpha}) = \pi(\hat{a}, \overline{\alpha}) = \begin{cases}
        \hat{a} \left[ \frac{\beta R (2-2\overline{\alpha})}{1+\alpha} - \frac{\rho (2-2\overline{\alpha})}{\overline{\alpha}} \right] & \text{if } \overline{\alpha} \leq \overline{\alpha}' \\
        \hat{a} \left[ \frac{\beta}{1+\alpha} \right]^{\frac{2-\alpha}{2-\frac{\alpha}{\rho}}} \left[ \frac{\alpha m R (2-2\overline{\alpha})}{\rho} \right]^{\frac{2-\alpha}{2-\frac{\alpha}{\rho}}} & \text{if } \overline{\alpha} > \overline{\alpha}'
    \end{cases}
\end{equation}
where \( \dot{a} = 2a - 1 \). Differentiating \( \pi(\pi) \) with respect to \( \pi \) gives:

\[
\frac{d\pi(\pi)}{d\pi} = \frac{\partial\pi(\dot{a}, \pi)}{\partial \dot{a}} \frac{d\dot{a}}{d\pi} + \frac{\partial\pi(\dot{a}, \pi)}{\partial \pi} = 2 \frac{\partial\pi(\dot{a}, \pi)}{\partial \dot{a}} + \frac{\partial\pi(\dot{a}, \pi)}{\partial \pi}
\]

The first term on the right-hand side of (29) is positive since \( \partial\pi / \partial \dot{a} > 0 \). The second term, \( \partial\pi / \partial \pi \), is initially positive and then becomes negative, as differentiation will confirm. Moreover, at \( \pi = \frac{1}{2} \) and \( \pi = 1 \), \( \pi(\pi) = 0 \). A typical shape for the \( \pi(\pi) \) function might be single-peaked as shown in Figure 1.\(^9\)

Interior equilibrium values for \( \pi \) will be those such that \( \pi(\pi) = \pi_0 \). Figure 1 depicts possible equilibria. For given values of \( \pi_0 \), there are generally two interior equilibria, one stable and the other unstable. The stable one is denoted \( \pi^* \), and is to the left of the peak of the \( \pi(\pi) \) curve. The other equilibrium \( \pi^u \) is unstable: for \( \pi > \pi^u \), entrepreneurs will exit causing \( \pi \) to rise, and vice versa. That implies that the two stable equilibria will be the interior one at \( \pi = \pi^* \) and the corner equilibrium \( \pi = 1 \) (where there are no active entrepreneurs).\(^10\) Depending on the size of \( \pi \) relative to \( \pi' \), the stable equilibrium may involve unemployment. The higher the value of \( \pi_0 \), the the higher will be \( \pi^* \), and the more likely will there be unemployment in equilibrium.

**The Possibility of Credit Rationing**

Stiglitz and Weiss (1981) found that credit rationing could arise when projects were pooled by their expected return \( (pR) \), which was exogenously given. In the de Meza and Webb (1987) case where projects were pooled by their return \( R \) and the distribution of the probability of success \( p \) across entrepreneurs was given, credit rationing could not arise. Our model is an extension of the de Meza-Webb model to allow for \( p \) for a given entrepreneur to be endogenously determined by the quality of worker hired. It turns out that in this case, credit rationing might arise. We simply show that possibility here without exploiting its consequences for the form of equilibrium achieved and its efficiency properties.

---

\(^9\) Twice differentiating (28) with respect to \( \pi \), we obtain that for \( \pi > \pi' \), \( d^2 \pi / d\pi^2 < 0 \). However, for \( \pi < \pi' \), the sign of \( d^2 \pi / d\pi^2 \) is ambiguous. Differentiating (28) with respect to \( \pi \) also reveals that \( \pi(\pi) \) can be increasing at \( \pi = \pi' \) as shown in Figure 1. In fact, the slope of \( \pi(\pi) \) will generally be discontinuous at the point \( \pi = \pi' \), but it can either rise of fall discontinuously.

\(^10\) Of course, if \( \pi_0 \) is very high, the only equilibrium will be one in which there are no entrepreneurs. In Figure 2, the \( \pi_0 \) curve lies above the peak of the \( \pi(\pi) \) curve. We are ruling this out as being not interesting for our purposes.
Consider first the marginal entrepreneur. Using \( \rho = \pi \bar{\pi} \) by (5) and the expressions for \( \bar{\pi} \) in (27.2), the expected profits for the marginal entrepreneur in (28) can be rewritten as follows:

\[
\tilde{\pi}(\pi, \bar{\pi}) = \begin{cases} 
\frac{(\pi - 1)\beta}{1+\alpha}(2 - 2\pi)^{\alpha}(R - (2 - 2\pi)\bar{\pi}) & \text{if } \pi \leq \pi' \\
\frac{(\pi - 1)\beta R}{(1+\alpha)^2} \left[ \frac{\alpha R (2 - \pi)}{(1+\alpha)^2} \right]^{\frac{1}{2}} & \text{if } \bar{\pi} > \bar{\pi}'
\end{cases}
\]

where, in equilibrium, \( \tilde{\pi}(\pi, \bar{\pi}) = \pi_0 \). Suppose we focus on a stable interior equilibrium, which requires that \( \partial \tilde{\pi}(\pi, \bar{\pi})/\partial \bar{\pi} > 0 \). Differentiating condition \( \tilde{\pi}(\pi, \bar{\pi}) = \pi_0 \), we obtain:

\[
\left. \frac{d\pi}{d\bar{\pi}} \right|_{\tilde{\pi} = \pi_0} = - \left. \frac{\partial \tilde{\pi}/\partial \pi}{\partial \tilde{\pi}/\partial \bar{\pi}} \right|_{\tilde{\pi} = \pi_0} > 0
\]

where the sign follows from the stability condition and the fact that \( \tilde{\pi}(\bar{\pi}, \bar{\pi}) \) is decreasing in \( \bar{\pi} \). Intuitively, an increase in the interest rate pushes the lowest-ability entrepreneurs out of the sector and increases the average quality of those remaining, \( \bar{\pi} \).

Next, turn to the banks. The expected profit per unit of lending is \( \Pi_B = \bar{\pi} \bar{\pi} - \rho \). Using (27.2) and the fact that \( m = 2 - 2\pi \) by (24), expected profits per unit can be written:

\[
\Pi_B = \begin{cases} 
\frac{\rho}{1+\alpha}(1 - \bar{\pi})^{\bar{\pi}} - \rho & \text{if } \bar{\pi} \leq \bar{\pi}' \\
\frac{\beta}{1+\alpha} \left( \frac{2\alpha R}{1+\alpha} \right)^{\frac{1}{2}} \bar{\pi} \bar{\pi}^{1 - \frac{1}{2} - \frac{1}{2}} - \rho & \text{if } \bar{\pi} > \bar{\pi}'
\end{cases}
\]

Credit rationing can only occur if an increase in the interest rate \( \bar{\pi} \) causes bank expected profits to fall. Given that \( \bar{\pi} \) is increasing in \( \bar{\pi} \) as shown above, a necessary condition for this is that \( \partial \Pi_B/\partial \bar{\pi} < 0 \). From (31), we find by differentiation that for the full employment case where \( \bar{\pi} \leq \bar{\pi}' \), \( \partial \Pi_B/\partial \bar{\pi} < 0 \) if \( \bar{\pi} > 1/(1 + \alpha) \), which is clearly possible. Similarly, in the unemployment case, we obtain that \( \partial \Pi_B/\partial \bar{\pi} < 0 \) if \( \bar{\pi} > 2/(2 + \alpha) \), which is again possible. Thus, unlike in the de Meza-Webb case, credit rationing could occur in our model.

Intuitively, \( \bar{\pi} \) can fall in \( \bar{\pi} \) since lower-quality workers are left in the entrepreneurial sector when the number of entrepreneurs decreases (\( \bar{\pi} \) increases). Exploring the consequences of that would be rather complicated and would take us too far afield from our present purpose.

To summarize the results of this section, equilibrium in the asymmetric-information case will have the following features. The highest-ability entrepreneurs and the lowest-quality workers will enter the entrepreneurial sector, in contrast with the full-information
case. As well, workers will be randomly assigned to entrepreneurs contrary to the efficient matching of the full-information case. Assuming wages are paid upfront, there will be a single wage rate and a single interest rate facing all firms. There will generally be multiple equilibria, unless \( \pi_0 \) is high enough to rule out an entrepreneurial sector entirely. Two equilibria will be stable and one unstable. The stable equilibria will include one interior one and one corner solution in which there are no entrepreneurs. The interior stable equilibrium may or may not involve involuntary unemployment.

5. Efficiency and Policy with Asymmetric Information

In this section, we study the optimality properties of equilibrium outcomes with asymmetric information in credit and labor markets. Our main interest will be in stable interior equilibria with and without unemployment. We begin by investigating the efficiency of market equilibria, and then look at the consequences for government policy.

Local Efficiency Properties of Equilibria

Recall the expressions for social surplus \( S \) in (18). Rewriting these using the fact that \( m = 2 - 2\pi \), we obtain:

\[
S = \begin{cases} 
(2 - 2\pi) \left[ \frac{\alpha R}{1 + \alpha} (2 - 2\pi)^\alpha - \pi_0 - (1 - \pi) \rho \right] & \text{if } \pi \leq \pi' \\
(2 - 2\overline{\pi}) \left[ \frac{\alpha R}{1 + \alpha} (2 - 2\overline{\pi})^{\frac{\alpha}{2}} \hat{w}^{\frac{\alpha}{2}} - \pi_0 - \frac{\rho \hat{w}^2}{2} \right] & \text{if } \overline{\pi} > \overline{\pi}'
\end{cases}
\]

where \( \hat{w} \) in the case of unemployment is given by (21), with \( m = 2 - 2\pi \). The efficiency of the equilibrium outcomes can be investigated by considering the effects on social surplus of incremental changes in \( \pi \) and, in the case of an unemployment equilibrium, in \( \hat{w} \). Consider the unemployment case first, concentrating on the interior equilibrium \( (\overline{\pi} < 1) \).

Unemployment Equilibrium: \( 1 > \overline{\pi} > \overline{\pi}' \)

In this case, \( S \) depends on \( \overline{\pi} \) directly and also indirectly via \( \hat{w}(\overline{\pi}) \). Consider the two effect in turn. Differentiating the second row in (32) partially with respect to \( \overline{\pi} \), we obtain, after
straightforward manipulation and using the expressions for $\overline{p}$ and $\hat{w}$ in (27.2) and (19),\(^{11}\)

\[(33)\quad \frac{\partial S}{\partial \overline{a}} \bigg|_{\hat{w}} = \left[ \frac{2}{\overline{a}} - 4 - \alpha \right] \rho \hat{w} < 0\]

where the sign follows from the fact that $\overline{a} > 1/2$. Then, differentiating $S$ with respect to $\hat{w}$ and using (27.2), (24) and (19), we obtain:

\[(34)\quad \frac{\partial S}{\partial \hat{w}} \bigg|_{\overline{a}} = \frac{m \rho \alpha}{2} > 0\]

Thus, (33) and (34) indicate that an unemployment equilibrium is inefficient. If $\overline{a}$ and $\hat{w}$ could be manipulated separately, $\overline{a}$ should be reduced (the number of entrepreneurs increased) and $\hat{w}$ should be increased (more high-ability workers should be attracted into the entrepreneurial sector).

However, $\hat{w}$ depends on $\overline{a}$ through (21). Substituting $m = 2 - 2\overline{a}$ in (21) and differentiating with respect to $\overline{a}$, we obtain:

$$\frac{\partial \hat{w}}{\partial \overline{a}} \bigg|_{\hat{w}} \lesssim 0 \quad \text{as} \quad \frac{2}{2 + \alpha} \gtrsim \overline{a}$$

This relationship between $w$ and $\overline{a}$ applies only for $\overline{a} > \overline{a}'$. For $\overline{a} \leq \overline{a}'$, full employment exists, so $w = m = 2 - 2\overline{a}$ implying that $w$ is declining in $\overline{a}$. The two panels of Figure 2 depict possible cases for the relationship between the $w$ and $\overline{a}$, depending on whether $\overline{a}' \gtrsim 2/(2 + \alpha)$. If $\overline{a}' \geq 2/(2 + \alpha)$, the market wage is monotonically decreasing in $\overline{a}$, while if $\overline{a}' < 2/(2 + \alpha)$, the wage rate is hump-shaped in the range where there is unemployment. These figures will be useful again below.

\(^{11}\) Specifically, differentiating (32) and using (27.2), we obtain:

$$\frac{1}{2} \frac{\partial S}{\partial \overline{a}} \bigg|_{\hat{w}} = \overline{p} R \left( \frac{1 - 2\overline{a}}{\overline{a}} \right) + \pi_0 + \frac{\rho \hat{w}}{2} - \frac{\alpha \overline{p} R}{2}$$

Since $\tilde{a} = 2\overline{a} - 1$, $\overline{p} \tilde{a}/\overline{a} = \tilde{p}$ (by (1)), and $\tilde{p}(R - \rho \hat{w}/\overline{p}) = \pi_0$ for the marginal entrepreneur, this becomes:

$$\frac{1}{2} \frac{\partial S}{\partial \overline{a}} \bigg|_{\hat{w}} = \rho \hat{w} \left( \frac{1}{\overline{a}} - \frac{3}{2} \right) - \frac{\alpha \overline{p} R}{2}$$

which reduces to the expression in the text using (19).
The total effect of a change in \( \bar{\alpha} \) on surplus can be expressed as follows:

\[
\frac{dS}{d\bar{\alpha}} = \frac{\partial S}{\partial \bar{\alpha}} + \frac{\partial S}{\partial \hat{w}} \frac{\partial \hat{w}}{\partial \bar{\alpha}}
\]

Given (33) and (34), this will be unambiguously negative if \( \partial \hat{w} / \partial \bar{\alpha} < 0 \), which will be the case if \( \bar{\alpha} > 2/(2 + \alpha) \). That is, an increase in the number of entrepreneurs would increase efficiency. Otherwise, it will be ambiguous. We return below to how policy might be used to enhance efficiency.\(^{12}\)

**Full Employment Equilibrium:** \( 1/2 < \bar{\alpha} \leq \bar{\alpha}' \)

With full employment, \( \bar{\alpha} \) has only a direct effect on \( S \) in the first row of (32). Differentiating with respect to \( \bar{\alpha} \), we obtain after similar manipulation:

\[
\frac{1}{2} \frac{dS}{d\bar{\alpha}} = \frac{2\rho(1-\bar{\alpha})^2}{4\bar{\alpha}} - \alpha \bar{\rho} R
\]

To interpret this, use \( w^e = m = 2 - 2\bar{\alpha} \) and \( \alpha \bar{\rho} R = \rho(1+\alpha)\hat{w} \) by (19) to find:

\[
\frac{1}{2} \frac{dS}{d\bar{\alpha}} = \rho \left( \frac{1-\bar{\alpha}}{\bar{\alpha}} w^e - (1+\alpha)\hat{w} \right) \geq 0
\]

since \( w^e > \hat{w} \). Thus, there may be too few or too many entrepreneurs in the full-employment equilibrium.

These efficiency results contrast with those of de Meza and Webb (1987) who show that in the absence of heterogeneous worker quality, there are unambiguously too many entrepreneurs in equilibrium (\( \bar{\alpha} \) is too low in our notation). In the de Meza-Webb model, there is an adverse selection effect that allows low-quality entrepreneurs to take advantage of a common interest rate, and too many do so. That effect is present in our model as well, but in addition the quality of workers hired in the entrepreneurial sector tends to

\(^{12}\) Note that an increase in the number of entrepreneurs will increase employment, even if it also increases \( \hat{w} \). To see this, use \( m = 2 - 2\bar{\alpha} \) and (21) to give:

\[
e = \sqrt{m/\hat{w}} = \left[ (2 - 2\bar{\alpha})^{1-\frac{\alpha}{1+\alpha}} \left( \frac{\beta\bar{\alpha}}{1+\alpha} \frac{\alpha}{1+\alpha} \frac{R}{\rho} \right)^{\frac{\alpha}{2-\alpha}} \right]^{\frac{1}{2}}
\]

Differentiating this with respect to \( \bar{\alpha} \), we obtain \( de/d\bar{\alpha} < 0. \)
be too low. Increasing the number of entrepreneurs is a way of attracting higher-quality workers, but at the expense of taking lower-quality entrepreneurs. Either of those effects can dominate.

The above welfare effects are local ones. Unfortunately in our model, social surplus in (32) is not globally concave. Differentiation with respect to $\bar{a}$ indicates that $d^2S/d\bar{a}^2$ is generally of ambiguous sign. Therefore, there are various possibilities for global optima, as our discussion next illustrates.

**Policy Implications**

In the above analysis, we considered a hypothetical perturbation of $\bar{a}$, and thus $\hat{w}$ in the unemployment case, around the equilibrium. The government cannot control $\bar{a}$ directly since that is determined by the decision of entrepreneurs to become active. Instead, in a decentralized market economy, the government can influence equilibrium outcomes by intervening with taxes or subsidies. Two kinds of policy instruments might be used to influence $\bar{a}$ and $\hat{w}$: a tax or subsidy on entrepreneurs who become active and a tax or subsidy on wages. We begin by analyzing how these policy instruments affect equilibrium values of $\bar{a}$ and $\hat{w}$. Then, we turn to the effect of policies on the social surplus, $S$. In evaluating the potential for policy intervention, it is useful to note that since the government can observe neither the quality of workers nor the ability of entrepreneurs, the full-information optimum cannot be achieved. In particular, workers cannot be optimally matched to firms, and nothing can be done to avoid the fact that it will be the lowest quality of workers that will be attracted to the entrepreneurial sector. A more far-reaching analysis might consider ways in which information about $q$ and $a$ could be elicited, such as by signaling or ex ante monitoring.

**Effect of Policies on Equilibrium Outcomes**

Let $\tau$ be a subsidy on entrepreneurs, and $\sigma$ a wage subsidy. Then, (2) can be revised to give the after-subsidy expected profit of an active entrepreneur:

$$\pi = E[p](R - (1 - \sigma)w\bar{p}) + \tau \geq \pi_0$$

where $E[p] = \beta a(ew)^\alpha/(1 + \alpha)$ by (12) and $\bar{p} = \rho/\bar{p}$ by (5). The equilibrium value of $\bar{p}$
is determined by the zero-net-profit condition of the marginal entrepreneur. Revising (28) to incorporate the subsidies and using \( \tilde{a} = 2\pi - 1 \), we obtain:

\[
\tilde{\pi}(\pi, \sigma, \tau) = \begin{cases} 
(2\pi - 1) \left[ \frac{\beta R (2 - 2\pi)^{\alpha}}{1 + \alpha} - \frac{(1 - \sigma) \rho (2 - 2\pi)}{\pi} \right] + \tau & \text{if } \pi \leq \tilde{\pi} \\
(2\pi - 1) \left[ \frac{\beta R}{1 + \alpha} \right]^{\frac{2}{\alpha}} \left[ \frac{\alpha \pi (2 - 2\pi)}{(1 - \sigma) \rho} \right]^{\frac{1}{\alpha}} + \tau & \text{if } \pi > \tilde{\pi}
\end{cases}
\]

where \( \tilde{\pi}(\pi, \sigma, \tau) = \pi_0 \) in equilibrium. Equation (37) determines how \( \bar{a} \), and therefore \( \tilde{a} \) responds to changes in \( \sigma \) and \( \tau \). Let us focus on the stable interior solution in Figure 1. Stability requires that \( \partial \bar{\pi} / \partial \sigma > 0 \) for both the full employment and unemployment cases. Since \( \partial \bar{\pi} / \partial \sigma > 0 \) and \( \partial \bar{\pi} / \partial \tau > 0 \) in (37), we have that:

\[
\frac{\partial a}{\partial \sigma} = -\frac{\partial \bar{\pi}}{\partial \sigma} < 0, \quad \frac{\partial a}{\partial \tau} = -\frac{\partial \bar{\pi}}{\partial \tau} < 0
\]

which imply that \( \partial \bar{a} / \partial \sigma < 0 \) and \( \partial \bar{a} / \partial \tau < 0 \) as well. An increase in either subsidy increases the number of active entrepreneurs by attracting more low-ability ones into the entrepreneurial sector.

Next, consider the wage rate. In the full employment case, the market-clearing wage is \( w = m = 2 - 2\pi \) as before. An increase in the number of entrepreneurs reduces \( \pi \) and therefore \( w \). Lower-ability entrepreneurs and lower-quality workers enter the entrepreneurial sector.

With unemployment, the preferred wage rate in (19) becomes:

\[
(19') \quad \hat{w} = \frac{\alpha R \bar{\pi}}{(1 - \sigma)(1 + \alpha) \rho}
\]

which, combined the expression for \( \bar{\pi} \) in (16), then leads to a revised version of (21):

\[
(21') \quad \hat{w}(\pi) = \left[ \frac{\beta}{1 + \alpha} \frac{\alpha}{1 + \alpha} \frac{R}{(1 - \sigma) \rho} \right]^{\frac{2}{\alpha}} \left[ \frac{\alpha \pi (2 - 2\pi)}{(1 - \sigma) \rho} \right]^{\frac{1}{\alpha}}
\]

In equilibrium, \( \bar{a} \) will be determined by \( \tilde{\pi}(\pi, \sigma, \tau) = \pi_0 \) in (37). Combining the second row in (37) with (21') yields \( \hat{w} \) as a function of the equilibrium value of \( \bar{a} \) and the subsidies:

\[
(39) \quad \hat{w}(\bar{a}, \tau, \sigma) = \frac{(\pi_0 - \tau) \alpha}{(1 - \sigma) \rho} \frac{\bar{\pi}}{2\bar{a} - 1} \quad \text{if } \bar{a} > \tilde{a}'
\]

24
where

\[ \frac{\partial \hat{w}}{\partial a} < 0, \quad \frac{\partial \hat{w}}{\partial \tau} < 0, \quad \frac{\partial \hat{w}}{\partial \sigma} > 0 \]

We are now in a position to investigate the effect of subsidy policy on social surplus. We begin with local welfare analysis, evaluating the effect of introducing small subsidies starting at laissez-faire equilibria. Then, optimal policies are considered.

**The Efficiency Effect of Incremental Policies**

Social surplus is again given by (32), where now \( \bar{a} \) and \( \hat{w} \) depend on \( \tau \) and \( \sigma \). We consider the effect of changes in \( \tau \) and \( \sigma \) on \( S \) in the unemployment and the full-employment equilibria in sequence.

**Unemployment Case**

Differentiating the second row of (32) with respect to \( \bar{a} \) and \( \hat{w} \) and using (27.2) and (19'), we obtain the analogs of (33) and (34):\(^{13}\)

\[ \frac{\partial S}{\partial a} = \left[ \frac{2}{a} - 4 - \alpha + \frac{\sigma}{1 - \sigma} \right] (1 - \sigma) \rho \hat{w} + 2\tau \]

\[ \left. \frac{\partial S}{\partial \hat{w}} \right|_{a} = [\alpha - (1 + \alpha)\sigma] \frac{mp}{2} \]

At the no-subsidy equilibrium with \( \tau = \sigma = 0 \), these reduce to (33) and (34) with \( \partial S/\partial \bar{a} < 0 \) and \( \partial S/\partial \hat{w} > 0 \).

Consider now the effect of small changes in subsidies on social surplus. Differentiating the second row in (32) with respect to \( \tau \) and \( \sigma \) yields:

\[ \frac{\partial S}{\partial \tau} = \frac{\partial S}{\partial \bar{a}} \frac{\partial \bar{a}}{\partial \tau} + \frac{\partial S}{\partial \hat{w}} \frac{d \hat{w}}{d \tau}, \quad \frac{\partial S}{\partial \sigma} = \frac{\partial S}{\partial \bar{a}} \frac{\partial \bar{a}}{\partial \sigma} + \frac{\partial S}{\partial \hat{w}} \frac{d \hat{w}}{d \sigma} \]

where \( d \hat{w}/d \tau \) and \( d \hat{w}/d \sigma \) include both the direct effects of these policies on \( \hat{w} \) by (40) and the indirect effect through changes in \( \bar{a} \) using (38). These imply \( d \hat{w}/d \tau > 0 \) and \( d \hat{w}/d \sigma < 0 \).

\(^{13}\) We are assuming an interior solution with \( \hat{w} < 1 \), although technically a corner solution with all workers choosing the entrepreneurial sector is possible.
\(d\hat{w}/d\sigma > 0\). Suppose we evaluate this starting at the no-intervention equilibrium. Then, using the signs obtained from (41) and (42) when \(\tau = \sigma = 0\), we obtain:

\[
\left. \frac{\partial S}{\partial \tau} \right|_{\tau=\sigma=0} \geq 0, \quad \left. \frac{\partial S}{\partial \sigma} \right|_{\tau=\sigma=0} > 0
\]

Thus, starting from the laissez-faire unemployment equilibrium, welfare will be unambiguously increased if we impose a small subsidy on wages.

### Full Employment Case

In this case, \(S\) depends only on \(\overline{a}\). Differentiating the first row of (32) with respect to \(\overline{a}\) and using (27.2) yields the analog of (36) with subsidies incorporated:

\[
(44) \quad \frac{1}{2} \frac{dS}{d\overline{a}} = \frac{2\rho(1 - \overline{a})^2}{\overline{a}} - \alpha p R + \sigma \rho \frac{(2\overline{a} - 1)(2 - 2\overline{a})}{\overline{a}} + \tau
\]

This again reduces to (36) and has an ambiguous sign in the no-intervention case.

The effects of small changes in \(\tau\) and \(\sigma\) on social surplus are now:

\[
(45) \quad \frac{\partial S}{\partial \tau} = \frac{dS}{d\overline{a}} \frac{\partial}{\partial \tau}, \quad \frac{\partial S}{\partial \sigma} = \frac{dS}{d\overline{a}} \frac{\partial}{\partial \sigma}
\]

Both of these have ambiguous signs at the no-intervention full-employment equilibrium.

### Optimal Policies

Suppose now that the government can choose subsidies \(\sigma\) and \(\tau\) to maximize social surplus. It turns out that the social optimum may involve either full employment or unemployment depending on the parameters of the problem.

If the optimum involves unemployment, the government will choose \(\tau\) and \(\alpha\) such that in (43), \(\partial S/\partial \tau = \partial S/\partial \sigma = 0\). This will be the case if \(\partial S/\partial \overline{a} = 0\) and \(\partial S/\partial \hat{w} = 0\), where these are given by (41) and (42). This leads to a straightforward characterization of optimal policies. Setting (42) to zero, we immediately obtain the optimal wage subsidy:

\[
\sigma = \frac{\alpha}{1 + \alpha} > 0
\]

Then, setting (41) to zero and using this expression for \(\sigma\) (which implies that \(\sigma/(1-\sigma) = \alpha\)), the optimal subsidy on entrepreneurs satisfies:

\[
\left. \frac{\partial S}{\partial \overline{a}} \right|_{\tau=\sigma=0} = \left[ \frac{2}{\overline{a}} - 4 \right] \rho(1 - \sigma)\hat{w} + 2\tau = 0 \quad \implies \quad \tau > 0
\]
where the sign of $\tau$ follows from the fact that $\overline{\pi} > 1/2$. Thus, if unemployment exists in the optimum, both the wage subsidy and the subsidy on entrepreneurs should be positive.

On the other hand, if the optimum involves full employment, $S$ depends only on $\overline{\pi}$. Optimal subsidy policies require setting (44) to zero, and this requires only one policy instrument. It is apparent that either $\tau$ or $\sigma$ can be used. Here, the sign of the optimal subsidy is ambiguous. Suppose, for example, that $\tau$ is used. Then, its sign depends on the sign of the first two terms on the right-hand side of (44), or equivalently the right-hand side of (36), which is ambiguous. This parallels the result found above for incremental policy changes.

Whether there is full employment or unemployment in the optimum depends upon the parameters of the problem. To see this, consider how $S$ varies with $\overline{\pi}$. In the case of unemployment, differentiating the second row of (32) with respect to $\overline{\pi}$ and using (27.2), we obtain:

\begin{equation}
\frac{\partial S}{\partial \overline{\pi}} \bigg|_{\overline{\pi} > \overline{\pi}^*} = \overline{\pi} R \left[ \frac{2}{\overline{\pi}} - \alpha - 4 \right] + \rho \hat{w} + 2\pi_0
\end{equation}

For the full employment case, we found earlier that differentiating the first row of (32) gives:

\begin{equation}
\frac{dS}{\overline{\pi}} \bigg|_{\overline{\pi} < \overline{\pi}^*} = \overline{\pi} R \left[ \frac{2}{\overline{\pi}} - 2\alpha - 4 \right] + (2 - 2\pi)\rho + 2\pi_0
\end{equation}

As $\overline{\pi}$ approaches $\overline{\pi}'$, we move from one case to the other. Let $\overline{\pi} \rightarrow \overline{\pi}'^+$ denote $\overline{\pi}$ approaching $\overline{\pi}'$ from above (in the unemployment region), and vice versa for $\overline{\pi} \rightarrow \overline{\pi}'^-$. Then we obtain from (46) and (47), and using the fact that $\hat{w} = 2 - 2\overline{\pi}$ at $\overline{\pi} = \overline{\pi}'$:

\begin{equation*}
\lim_{\overline{\pi} \rightarrow \overline{\pi}'^+} \frac{dS}{\overline{\pi}} - \lim_{\overline{\pi} \rightarrow \overline{\pi}'^-} \frac{dS}{\overline{\pi}} = \alpha \rho \hat{w} > 0
\end{equation*}

Therefore, the slope of $S(\overline{\pi})$ increases discontinuously at $\overline{\pi} = \overline{\pi}'$, implying that $S(\overline{\pi})$ cannot be concave. Moreover, whether the slope of $S(\overline{\pi})$ at $\overline{\pi} = \overline{\pi}'$ is positive or negative depends upon the parameters of the problem, as inspection of (46) and (47) indicates.

---

14 This is just the first equation in footnote 11. Note that we are assuming that $\sigma$ is chosen optimally so that $\partial S/\partial \hat{w} = 0$. Therefore, we need not take account of changes in $\hat{w}$ as $\overline{\pi}$ changes.
Suppose first that \( S(\pi) \) is single-peaked in \( \pi \). Then, if \( \partial S/\partial \pi < 0 \) at \( \pi = \pi' \), it will be optimal to induce a reduction in \( \pi \) thereby moving into the range of full employment. By the same token, if \( \partial S/\partial \pi > 0 \), there will be unemployment in the optimum. From (46) and (47), we can see that \( \partial S/\partial \pi \) will be positive at \( \pi = \pi' \) if \( \pi_0 \) is large enough. A large value of \( \pi_0 \) will make it more difficult to attract entrepreneurs into the sector, thereby increasing the chances of an unemployment equilibrium.

However, it is quite possible that \( S(\pi) \) is not single-peaked. That is, \( \lim_{\pi \to \pi^+} dS/d\pi \) may be positive, while \( \lim_{\pi \to \pi^-} dS/d\pi \) is negative. This can occur if the wage function is as depicted in Panel B of Figure 3. Then, a reduction in \( \pi \) will cause \( S \) to rise. However, starting at \( \pi = \pi' \), increases in \( \pi \) will also cause \( \hat{w} \) to rise in this case. This increase in \( \hat{w} \) together with the increase in \( \pi \) could cause the right-hand side of (46) to rise. There would be local optima in both the full-employment and unemployment ranges of \( \pi \), and either one could be the global optimum.

Thus, policy prescriptions depend on the parameters of the problem: with unemployment both \( \sigma \) and \( \tau \) should be positive, while with full employment only one of \( \sigma \) or \( \tau \) is needed and it could be positive or negative. Indeed, optimal policies are even more ambiguous when we recall that the laissez-faire equilibrium could be a corner solution in which no entrepreneurs are active. In this case, it will be necessary to impose sufficiently large subsidies to move the initial equilibrium to a stable interior one. To study this case properly would involve an explicitly dynamic analysis.

6. Extensions

In the previous analysis, we assumed that both worker qualities and entrepreneur abilities were private information. This results in adverse selection in two markets, which leads to various sorts of ambiguity: ambiguity about the possibility of unemployment, ambiguity about policy prescriptions, and multiple equilibria. Moreover, equilibrium outcomes vary considerably from the full-information case in terms of the quality of workers that opt for the entrepreneurial sector, the number of entrepreneurs, and the mismatch between worker quality and entrepreneurial ability. In this section, we consider two sorts of extensions that might serve to reduce the divergence between asymmetric-information and full-information
outcomes. First, we relax the information assumptions by either $a$ or $q$ to be public information. Then, we consider the possibility that firms can elicit information about worker quality by offering contracts that allow separating equilibria. This involves introducing an ex post wage payment to workers in successful firms in addition to the ex ante wage that we have already considered. In each case, we shall simply sketch the outlines of the analysis and summarize the results rather than providing a full-fledged treatment, which would be too space-consuming. The intuition will apparent given what we have learned in the case already considered.

**Entrepreneurial Ability Known**

Suppose first that $a$ is public information, but $q$ remains private. Since workers cannot be distinguished, a common wage $w$ will be offered, and the expected quality of workers employed by all entrepreneurs will be the same. Banks can offer ability-specific gross interest rates $r(a) = \rho/p(a)$, where $p(a)$ is given by the expression for $E[p]$ in (12). The expected profit of a type-$a$ entrepreneur then becomes:

$$\pi = p(R - r(a)w) = \frac{\beta a R (ew)^\alpha}{1 + \alpha} - \rho w$$

where $e \leq 1$ is the employment rate for workers who opt for the entrepreneurial sector.

As before, entrepreneurs take $e$ as given and choose a wage to offer. As we shall see, each entrepreneur will have a different preferred wage rate, but competition among entrepreneurs will cause a common wage rate to emerge (which may or may not clear the labor market). To see this, consider the population of active entrepreneurs. Suppose that all entrepreneurs with $a \geq \tilde{a}$ will become active, and let $\hat{w}(a)$ be the wage offered by a type-$a$ entrepreneur. Workers who seek a job in the entrepreneurial sector will apply to the entrepreneurs offering the highest $\hat{w}(a)$. We assume, critically, that there is ex post immobility not only between sectors as above, but also from one entrepreneur to another once a job application is made. This implies that competition for workers will equalize $\hat{w}(a)$ among entrepreneurs, so we can simply write $\hat{w}$ in what follows. Given the probability of employment $e$, workers with $q \leq e \hat{w}$ are attracted to the entrepreneurial sector, and all are indifferent among the active entrepreneurs.
Suppose as before that there are \( m \) active entrepreneurs. In equilibrium, all entrepreneurs will perceive the same \( e \) (although out of equilibrium they may well perceive different ones), and that will be given by (14) as before. Moreover, given that \( \hat{w} \) is the same for all entrepreneurs—the highest one preferred by any active entrepreneur—the equilibrium wage rate will again be given by \( \hat{w} \) with \( \hat{q} = \sqrt{m\hat{w}} \) if there is unemployment by (15), and \( w^e = m \) if there is full employment. Unemployment will occur if \( \hat{w} > w^e \) as before.

Consider now what determines the common value of \( \hat{w} \) that is offered by all entrepreneurs in the unemployment case. Given \( e \), an entrepreneur of ability \( a \) prefers the wage rate \( w \) that maximizes expected profits \( \pi \) given by (48). The solution to this problem yields:

\[
\hat{w} = \left( \frac{\beta a R \alpha}{(1 + \alpha) \rho} \right)^{1/(1-\alpha)} e^{\alpha/(1-\alpha)}
\]

which is increasing in \( a \). The entrepreneur with \( a = 1 \) prefers to offer the highest wage rate, and all other entrepreneurs will be obliged to follow. Therefore, \( \hat{w} = \hat{w}(1) \). As long as \( \hat{w} > m \), that will be the prevailing wage rate. In effect, perfect information in capital markets intensifies competition for workers thereby pushing up \( \hat{w} \). Using \( e = \sqrt{m/\hat{w}} \), the above equation for entrepreneur \( a = 1 \) becomes:

\[
(49) \quad \hat{w} = \left( \frac{\beta R \alpha}{(1 + \alpha) \rho} \right)^{2/(2-\alpha)} \left( m^{\alpha/(2-\alpha)} \right)
\]

The characterization of equilibrium is similar to that for the asymmetric-information case considered earlier. The market clearing wage rate will be \( w^e = m \), where \( m = 2 - 2\bar{a} \). There will be a value of \( \bar{a} \), say \( \bar{a}' \), such that \( \hat{w} = w^e \). The equilibrium wage rate will be \( w = w^e \) if \( \bar{a} \leq \bar{a}' \), and \( w = \hat{w} \) if \( \bar{a} > \bar{a}' \), where \( \hat{w} \) is given by (49). We can proceed as above to derive expected profits for the marginal entrepreneur, \( \hat{\pi}(\bar{a}) \), the analog of (28). The analog of Figure 1 applies here as well. There will generally be two interior equilibria, only one of which is stable. The other stable equilibrium is that at which \( \bar{a} = 1 \) where there are no active entrepreneurs. In the stable equilibrium, there may be either full employment or unemployment.

It is apparent that this case has similar qualitative features to the asymmetric-information case analyzed earlier. In both cases, there may be full employment or involuntary unemployment. Workers are paid a common wage and are drawn from the bottom of
the quality distribution. There is, however, no adverse selection problem in credit markets in the sense that the interest rate can be conditioned on the ability of entrepreneurs. This implies some differences in the equilibrium possibilities under the two regimes. It will be the case that $\bar{a}'$ will be lower and $\hat{w}$ higher in this case than in the case where both $a$ and $q$ are private information. This is a consequence of the fact that the wage rate is bid up to that preferred by the highest-ability entrepreneur. If there is full employment, wage rates will be the same for a given number of entrepreneurs. Unfortunately, although one might expect there to be fewer entrepreneurs financed in this case given the absence of adverse selection, that turns out not to be unambiguously the case. Nonetheless, from a social point of view, the number of entrepreneurs is now unambiguously too low ($\bar{a}$ is too high) if there is full employment: the tendency for over-entry due to adverse selection in credit markets (the De Meza-Webb effect) no longer applies.

**Worker Quality Known**

In this case, worker quality can be observed by all agents, but each entrepreneur’s ability is private knowledge. This information structure leads to a somewhat more complicated outcome. Since the quality of the worker hired by a given entrepreneur is known to the bank, the interest rate charged can reflect that quality. For any entrepreneur that hires a worker of quality $q$, the gross interest rate is $r(q) = \rho / \bar{p}(q)$ with $\bar{p} = \beta \bar{a} q^\alpha$. Since worker quality is known, a wage function $w(q)$ can be offered. The expected profit of a type-$a$ entrepreneur is then given by:

$$\pi = \beta a q^\alpha (R - r(q) w(q)) = a \left[ \beta q^\alpha R - \frac{\rho}{a} w(q) \right] \equiv a \pi_a(q) \geq \pi_0$$

where $\pi_a(q)$ represents profits per unit of ability when worker quality is $q$.

With worker quality observable to entrepreneurs, there will be full employment. But, wage setting is more complicated than before since wages can be quality-specific. First note that offering wages equal to each worker’s quality cannot be an equilibrium in the entrepreneurial sector. That is because for all entrepreneurs, the same quality of worker would maximize profits $a \pi_a$ when $w = q$, so wages of this worker would be bid up. To determine the equilibrium pattern of wages, consider the expression for profits in (50).
Competition among entrepreneurs implies that \( w(q) \) must be such that \( \pi_a(q) \) will be equalized for workers of all qualities \( q \), where \( w(q) \geq q \) (since workers always have the option of obtaining \( w(q) = q \) in the traditional sector). All entrepreneurs will then be indifferent about the quality of workers they hire. Given that \( \pi_a(q) = \pi_a \) for all \( q \), we have from the definition of \( \pi_a(q) \) in (50):

\[
(51) \quad w(q) = \frac{\alpha(\beta q^\alpha R - \pi_a)}{\rho}
\]

Since \( \alpha < 1 \), \( w(q) \) is increasing and strictly concave: \( w'(q) > 0 > w''(q) \). Given that \( w(0) < 0 \) in (50), there will be two values of \( q \), denoted \( \underline{q} \) and \( \overline{q} \) with \( \overline{q} > \underline{q} \), such that \( w(\underline{q}) = q > 0 \), and \( w(\overline{q}) = \overline{q} > q \). Since the upper bound of \( q \) is unity, we assume \( \overline{q} = 1 \) if \( w(1) \geq 1 \). Figure 3 illustrates.

For worker qualities \( q \) such that \( \underline{q} < q < \overline{q} \), the wage payment \( w(q) > q \), so these workers are attracted into the entrepreneurial sector. Workers with \( q < \underline{q} \) or \( q > \overline{q} \) (if there are any of the latter) will choose the traditional sector. Then the supply of labor in the entrepreneurial sector, denoted \( s \), will be given by \( s(\overline{\pi}, \pi_a) = \overline{q} - q \), where \( \underline{q} \) and \( \overline{q} \leq 1 \) are defined as above. Note that by (51), \( s(\overline{\pi}, \pi_a) \) is increasing in \( \overline{\pi} \) and decreasing in \( \pi_a \). In terms of Figure 3, an increase in \( \overline{\pi} \) shifts the curve \( w(q) \) up, while an increase in \( \pi_a \) shifts it down.

Since the number of entrepreneurs is \( m \) and each entrepreneur hires one worker, equilibrium in the labor market requires \( s(\overline{\pi}, \pi_a) = m \). Given \( m \), \( \pi_a \) adjusts to satisfy this equilibrium condition. Since, \( s(\cdot) \) is monotonic in \( m \), we can solve the equilibrium condition for the value of \( \pi_a \) that ensures labor market clearing, \( \pi_a(\overline{\pi}, m) \), where \( \pi_a(\cdot) \) is increasing in \( \overline{\pi} \) and decreasing in \( m \).

Since entrepreneurs that are not active obtain \( \pi_0 \), the profit of the marginal entrepreneur will satisfy \( \pi_0 = \tilde{\alpha} \pi_a(\overline{\pi}, m) \). Entrepreneurial expected profits are increasing in \( a \), so it will be the case as before that entrepreneurs with ability \( a \geq \tilde{a} \) will become active, while the remainder will choose the alternative option. Therefore, \( m = 1 - \tilde{a} \) and \( \overline{\pi} = (1 + \tilde{a})/2 \). Using these relationships, the condition determining the quality of the marginal entrepreneur may be written:

\[
\tilde{a} \pi_a \left( \frac{1 + \tilde{a}}{2}, 1 - \tilde{a} \right) = \pi_0
\]
The value of $\tilde{a}$, or equivalently $\bar{a} = (1 + \tilde{a})/2$, that satisfies this equation will be uniquely determined.

Given that entrepreneurs are indifferent about the quality of workers they hire, and workers are indifferent to whom they work for, there will not be perfect matching of $a$ and $q$. Indeed, we might expect that matching is random. That being the case, there will be three sources of inefficiency. There will be a tendency for too many entrepreneurs to enter due to the standard adverse selection effect on credit markets. There will be a mismatch of workers with entrepreneurs. And, the set of workers by quality may not be correct. However, unlike in the case where $q$ is private, here higher-quality workers will generally be attracted into the entrepreneurial sector. However, it is not clear whether the average quality of workers is too high or too low.

**Separating Equilibria**

So far, we have assumed that wages are paid up front, so that in the case where worker quality is unobservable, all workers receive the same wage. In this extension, we allow for the possibility that firms can offer workers a two-part contract consisting of an ex ante wage rate $w$ and an ex post bonus of $b$ that is paid in the successful state. Given that workers of different quality will have induce a different probability of project success, it will be possible for firms to offer contracts that separate workers by type. To keep matters tractable, consider the case of two ability-types of entrepreneurs $a_1 > a_0$ and two qualities of workers $q_1 > q_0$. This allows us to focus on the matching properties of equilibrium at the expense of the cutoff values for $a$ and $w$. Assume the number of type-1 workers and entrepreneurs is the identical, and the same for type-0s. Consider first the full-information allocation followed by the asymmetric-information case.

**Full Information**

In this case, higher-quality workers are employed by higher-ability entrepreneurs, and lower-quality workers are employed by lower-ability entrepreneurs. Entrepreneur and worker types are public knowledge, so fair interest rates are offered and there are no incentive constraints for workers. In equilibrium, wage rates $w_0, w_1$ must be such that worker participation constraints are satisfied: $w_0 \geq q_0, w_1 \geq q_1$. In addition, high-ability
entrepreneurs must not prefer low-quality workers and vice versa, so the following ‘hiring constraints’ must be satisfied:

\[(52.1) \quad \pi_0 = \beta a_0 q_0^\alpha R - \rho w_0 \geq \beta a_0 q_1^\alpha R - \rho w_1 \quad \implies \quad \rho(w_1 - w_0) \geq \beta a_0 R(q_1^\alpha - q_0^\alpha)
\]

\[(52.2) \quad \pi_1 = \beta a_1 q_1^\alpha R - \rho w_1 \geq \beta a_1 q_0^\alpha R - \rho w_0 \quad \implies \quad \rho(w_1 - w_0) \leq \beta a_1 R(q_1^\alpha - q_0^\alpha)
\]

We assume that competition among workers for jobs enables entrepreneurs to obtain all the surplus.\(^{15}\)

Three cases are possible. Suppose first that all workers receive their reservation wage \((w_0 = q_0, \ w_1 = q_1)\). If neither hiring constraint is binding, high-ability entrepreneurs will prefer high-quality workers and low-ability entrepreneurs will prefer low-quality workers. The equilibrium wage rates will then be the reservation wages.

Second, if the hiring constraint for low-ability entrepreneurs is violated at the reservation wages, \(w_1\) will be bid up to prevent low-ability entrepreneurs from preferring to hire high-quality workers. In this case, the equilibrium wage rates are \(w_0 = q_0\) and \(w_1 > q_1\), where \(w_1\) satisfies \(a_0\)’s hiring constraint:

\[\rho(w_1 - w_0) = \beta a_0 R(q_1^\alpha - q_0^\alpha) \quad \implies \quad w_1 = w_0 + \frac{\beta a_0 R}{\rho} (q_1^\alpha - q_0^\alpha) > q_1\]

Finally, if the hiring constraint for high-ability entrepreneurs is violated at the reservation wages, \(w_0\) will be bid up to prevent them from preferring to hire low-quality workers. In this case, the equilibrium wage rates are \(w_1 = q_1\) and \(w_0 > q_0\), where \(w_0\) satisfies \(a_1\)’s hiring constraint:

\[\rho(w_1 - w_0) = \beta a_1 R(q_1^\alpha - q_0^\alpha) \quad \implies \quad w_0 = w_1 - \frac{\beta a_1 R}{\rho} (q_1^\alpha - q_0^\alpha) > q_0\]

These cases are shown in Figure 4. Which case occurs depends upon the parameters of the economy.

\(^{15}\) For example, suppose \(w_1 > q_1\). A type-1 entrepreneur can reduce the wage rate. The worker employed by this entrepreneur has an incentive to seek employment elsewhere, but since all positions are already filled, this worker will bid down the wage rate elsewhere. This process will continue until wages are bid down as low as is feasible, given the participation and incentive constraints.
Asymmetric Information

If neither \( a_i \) nor \( q_i \) are observable, two problems can arise that prevent the full-information outcome from occurring. First, type-0 workers will prefer to mimic type-1 workers since the latter obtain a higher wage. Second, type-0 entrepreneurs may prefer to mimic type-1’s by hiring type-1 workers since then they will obtain a lower interest rate from the banks. To address the former problem, wage packages \( \{w_0, b_0\} \) and \( \{w_1, b_1\} \) can be used to separate the two types of workers. To address the latter, the wage package offered to the type-1 workers would have to be bid up to dissuade type-0 entrepreneurs from preferring to hire type-1 workers. In addition, it may still be the case that type-1 entrepreneurs prefer type-0 workers, although now they will be taken for type-0’s and will have to pay the higher interest rate.

We begin with some preliminaries. In a separating equilibrium, the probabilities of success of the two types of entrepreneurs will be given by \( p_i = \beta a_i q_i^\alpha, \ i = 0, 1 \). Assuming that wage package offers are observable, the banks will be able to distinguish entrepreneurs by type and will know the quality of their workers. Thus, ability-specific interest rates will be \( r_i = \rho / p_i = \rho / (\beta a_i q_i^\alpha), \ i = 0, 1 \). Expected profits of the entrepreneurs then become \( \pi_i = \beta a_i q_i^\alpha (R - r_i w_i - b_i) = \beta a_i q_i^\alpha (R - b_i) - \rho w_i, \ i = 0, 1 \). We can then construct iso-profit curves for each type of entrepreneur in \((b, w)\)-space. Their slopes will be as follows:

\[
\frac{dw}{db}\bigg|_{\pi_0} = \frac{-\beta a_0 q_0^\alpha}{\rho} > \frac{-\beta a_1 q_1^\alpha}{\rho} = \frac{dw}{db}\bigg|_{\pi_1}
\]

Therefore, the single-crossing property holds: iso-profit curves are downward-sloping straight lines with those for type-1 entrepreneurs being steeper than those for type-0’s.

The expected payments to workers in present value terms, denoted \( u_0 \) and \( u_1 \), are \( u_i = w_i + b_i / r_i = w_i + \beta a_i q_i^\alpha b_i / \rho, \ i = 0, 1 \). Note that workers discount the bonus at a risk-adjusted rate since they only receive it with some probability. Indifference curves for the two types of workers then have the following slopes:

\[
\frac{dw}{db}\bigg|_{u_0} = \frac{-\beta a_0 q_0^\alpha}{\rho} > \frac{-\beta a_1 q_1^\alpha}{\rho} = \frac{dw}{db}\bigg|_{u_1}
\]

Thus, workers indifference curves are parallel to those of entrepreneurs, and the single-crossing property again holds. Note the important point that type-0 workers place a
relatively higher value on up-front wages than do type-1 workers, implying that they can be separated using the bonus payment \( b \). Since the bonus need only be paid to type-1 workers, it will turn out to be the case that in equilibrium, wage packages will be of the form \( \{w_0, 0\}, \{w_1, b_1\} \).

Consider now the hiring constraints for the entrepreneurs. The analogs of (52.1) and (52.2) are:

\[
\begin{align*}
\pi_0 &= \beta a_0 q_0^\alpha R - \rho w_0 \geq \beta a_0 q_1^\alpha (R - r_1 w_1 - b_1) = \beta a_0 q_1^\alpha R - \frac{a_0}{a_1} (\rho w_1 + \beta a_1 q_1^\alpha b_1) \\
\pi_1 &= \beta a_1 q_1^\alpha (R - b_1) - \rho w_1 \geq \beta a_1 q_0^\alpha (R - r_0 w_0) = \beta a_1 q_0^\alpha R - \frac{a_1}{a_0} \rho w_0
\end{align*}
\]

Suppose, hypothetically, that both workers earned their reservation wage so that \( w_0 = q_0 \) and \( w_1 + b_1/r_1 = q_1 \). We can show that this contradicts one or the other of these hiring constraints. Rewrite the constraints using the reservation wages as follows:

\[
\begin{align*}
\pi_0 &= \beta a_0 q_0^\alpha R - \rho q_0 \geq \beta a_0 q_1^\alpha R - \frac{a_0}{a_1} \rho q_1 \quad \Rightarrow \quad \beta a_0 R (q_0^\alpha - q_1^\alpha) \geq \rho \left( q_0 - \frac{a_0}{a_1} q_1 \right) \\
\pi_1 &= \beta a_1 q_1^\alpha R - \rho q_1 \geq \beta a_1 q_0^\alpha R - \frac{a_1}{a_0} \rho q_0 \quad \Rightarrow \quad \beta a_0 R (q_1^\alpha - q_0^\alpha) \geq \rho \left( \frac{a_0}{a_1} q_1 - q_0 \right)
\end{align*}
\]

Clearly one of these will be violated when workers are paid their reservation wages. Therefore, the interior range of Figure 4 in the full-information case vanishes since at least one type of entrepreneur must always be constrained from mimicking the other’s hiring. There are two cases to consider, one in which the low-ability entrepreneurs must be prevented from hiring high-quality workers, and the other in which high-ability entrepreneurs must be constrained from hiring low-quality workers. Consider each in turn.

**Case 1: Hiring Constraint on Type-0 Entrepreneurs Binding**

We proceed by stating the problem facing entrepreneurs. The two types simultaneously offer their respective wage packages \( \{w_0, b_0\} \), and \( \{w_1, b_1\} \), and must satisfy all incentive constraints. Let IC\(_i\) be the incentive constraint facing type-\( i \) workers and IR\(_i\) be the participation (individual-rationality) constraint. And, let IC\(_{a0}\) be the incentive constraint facing type-0 entrepreneurs, which is the binding one in this case.
The problem of the type-1 entrepreneurs, given the wage offers of the other, is to choose \( \{w_1, b_1\} \) to maximize \( \pi_1 = \beta a_1 q_1^\alpha (R - b_1) - \rho w_1 \) subject to:

\[
\text{IR}_1 \quad w_1 + \frac{\beta a_1 q_1^\alpha}{\rho} b_1 \geq q_1 \\
\text{IC}_0 \quad w_1 + \frac{\beta a_1 q_1^\alpha}{\rho} b_1 \leq w_0 + \frac{\beta a_0 q_0^\alpha}{\rho} b_0 \\
\text{IC}_a \quad \pi_0 = \beta a_0 q_0^\alpha (R - b_0) - \rho w_0 \geq \beta a_0 q_1^\alpha (R - r_1 w_1 - b_1) = \beta a_0 q_1^\alpha (R - b_1) - \frac{a_0}{a_1} \rho w_1
\]

Similarly, type-0 entrepreneurs maximize \( \pi_0 = \beta a_0 q_0^\alpha (R - b_0) - \rho w_0 \) subject to:

\[
\text{IR}_0 \quad w_0 + \frac{\beta a_0 q_0^\alpha}{\rho} b_0 \geq q_0 \\
\text{IC}_1 \quad w_1 + \frac{\beta a_1 q_1^\alpha}{\rho} b_1 \geq w_0 + \frac{\beta a_0 q_1^\alpha}{\rho} b_0
\]

Information constraints IC\(_0\) and IR\(_0\) will be binding in equilibrium: type-0 workers must be precluded from mimicking type-1’s, and will receive no rents. Constraints IC\(_1\) and IR\(_1\) will be slack: there will be no incentive for type-1 workers to mimic type-0’s, and type-1 workers will receive enough rent to preclude type-0 entrepreneurs from wanting to hire them. And, of course, IC\(_a\) will be binding in this case by assumption.

Figure 5 depicts this case. The steep lines represent indifference curves for the type-1 workers. The flat curve is an indifference curve for the type-0 workers. In the full-information case, \( w_0 = q_0 \) and \( w_1 = q_1 \). However, with asymmetric information, this is not incentive-compatible. Instead, the two equilibrium contracts are depicted as \( E_0 \) and \( E_1 \). Type-1 workers obtain an information rent equal to \( u_1 - q_1 \), while type-0 workers receive their full-information wage and reservation utility. The utility level \( u_1 \) is determined by the wage package that just precludes type-0 entrepreneurs from mimicking type-1’s, that is, that satisfies IC\(_a\).
Case 2: Hiring Constraint on Type-1 Entrepreneurs Binding

In this case, constraint IC\(_{a0}\) is not binding, but an analogous constraint IC\(_{a1}\) must be added to the problem of type-0 entrepreneurs to preclude type-1’s from mimicking them. This constraint is:

\[
\text{IC}_{a1} \quad \pi_1 = \beta a_1 q_1^a (R - b_1) - \rho w_1 \geq \beta a_1 q_0^a (R - r_0 w_0 - b_0) = \beta a_1 q_0^a (R - b_0) - \frac{a_1}{a_0} \rho w_0
\]

Figure 6 depicts this case, and compares it with the previous case. The points \(E'_0\) and \(E'_1\) depict the equilibrium wage contracts. Now, type-0 workers obtain an informational rent, while type-1’s receive their reservation utility levels. Once again, the incentive constraint on type-\(q_0\)s is binding.

Naturally, efficiency will be improved under separation compared with the pooling equilibrium case. Indeed, in this simple two-by-two case the separating equilibria will have perfect matching and will replicate the outcome of the full-information case, although the rents will be shared in a different way. Moreover, if we had allowed for more types of entrepreneurs, the efficient number of them would have entered.

7. Concluding Comments

The results in this paper are obviously model-specific. Nonetheless, they are suggestive and do indicate that once one combines adverse selection in labor markets with those in credit markets, matters become much more complicated and policy prescriptions less clearcut. Multiple stable equilibria exist, one of which can be a corner solution in which no entrepreneurs are active (and therefore no surplus is generated). Even if the market equilibrium is interior, it may involve involuntary unemployment, or even credit rationing. Depending on the equilibrium, efficiency consequences, and therefore policy prescriptions, may differ. If there is involuntary unemployment, a presumption exists that there will be too few entrepreneurs and therefore too much unemployment, although even that depends on parameter values. In a full-employment equilibrium, no such presumption exists. Unlike the case with adverse selection applying only in credit markets, there may be too few or too many entrepreneurs. Adverse selection in credit markets tends to induce too many low-ability entrepreneurs to enter since the interest rate they face is too generous given
their ability. At the same time, the entry of more entrepreneurs mitigates the adverse selection problem in labor markets which results in too-few high-quality workers.

There are a number of ways in which the model could be fruitfully enriched, albeit at the further expense of simplicity. A straightforward extension would be to allow firms to vary the number of workers they employ, as in Weiss (1980). As he shows, firms will tend to hire too few workers because of adverse selection, leading to an argument for subsidizing employment in the entrepreneurial sector. A more ambitious extension would be to have both old firms and new firms competing with one another both for workers and in output markets. Assuming that established firms have informational advantages over new ones, one would expect to obtain a case for differential tax treatment of the two sorts of firms, although to which type’s advantage may not be obvious. Finally, we have assumed in our basic model, following the literature, pooling on both labor and credit markets, although we have explored some of the implications of separating wage contracts. One can imagine extending the model to allow for the possibility of separating firms either on the basis of information acquired from ex ante monitoring by banks or signalling by entrepreneurs, or on the basis of other firm characteristics or behavior, such as firm size or the ability to provide collateral.\footnote{Elsewhere, we have analyzed the efficiency and policy consequences of ex ante monitoring (Boadway and Sato, 1999). For a consideration of the consequences of banks separating firms using collateral and variable loan size, see Boadway and Keen (2005).} These extensions would complicate the analysis considerably.

References

Boadway, R. and M. Keen (2005), ‘Financing and Taxing New Firms under Asymmetric Information,’ Queen’s University, mimeo.


Figure 1
Figure 2, Panel A

\[ w^c = 2 - 2w \]
Figure 2, Panel B
Figure 3

The graph shows the function $w(q)$, which increases with $q$. The figure illustrates the relationship between $w$ and $q$, with $w$ on the vertical axis and $q$ on the horizontal axis. The graph highlights the behavior of the function as $q$ changes.
\[
\begin{align*}
\beta a_0 R (q_1^\alpha - q_0^\alpha) & \quad \beta a_1 R (q_1^\alpha - q_0^\alpha) & \rho (q_1 - q_0) \\
\omega_0 = q_0 & \quad \omega_0 = q_0 & \quad \omega_0 > q_0 \\
w_1 > q_1 & \quad w_1 = q_1 & \quad w_1 = q_1 \\
\end{align*}
\]

Figure 4
Figure 5
Figure 6