Stay or Leave? : 
Choice of Plant Location with Cost Heterogeneity

Jota Ishikawa
(Hitotsubashi University)
Yoshimasa Komoriya
(Hitotsubashi University)
Stay or Leave?:
Choice of Plant Location with Cost Heterogeneity*

Jota Ishikawa† Yoshimasa Komoriya
Hitotsubashi University Hitotsubashi University

April 26, 2006

Abstract

In a two-country model, we examine location choices by two domestic firms when they serve only domestic market and their cost structures are different. Whether the firm, that has more incentive for foreign direct investment, is more efficient or less efficient than the other depends on the difference between domestic and foreign marginal costs and the presence of fixed costs. We may have multiple equilibria. A small change in trade costs may reverse plant locations. Moreover, a decrease in trade costs may reduce domestic welfare.

Keywords: foreign direct investment; heterogeneous firms; duopoly; location choices

JEL Classification: F12, F21, F23

*We are grateful to Kenzo Abe, Satya Das, Hiroshi Mukunoki, participants of the Otago Workshop, and seminar participants at Hitotsubashi University and Ritsumeikan University for helpful comments. Jota Ishikawa acknowledges financial support from the Ministry of Education, Culture, Sports, Science and Technology of Japan under both the Grant-in-Aid for Scientific Research and the 21st Century Center of Excellence Project, the Japan Economic Research Foundation, and the Japan Securities Scholarship Foundation.

†Corresponding author: Faculty of Economics, Hitotsubashi University, Kunitachi, Tokyo 186-8601, Japan; Fax: +81-42-580-8882; E-mail: jota@econ.hit-u.ac.jp
1 Introduction

Foreign direct investment (FDI) has been growing rapidly. In particular, the world flow of FDI has dramatically increased in the last decade. Although there are a number of reasons for FDI, a typical reason is low production costs in the host country. Many firms shift their production facilities to developing countries such as China because of cheap labor. Recently, China has attracted huge amount of investment from developed countries and become the “world’s factory”. When FDI is made in developing countries due to low wages, the main purpose of FDI is usually not to serve the host market but to export products to other markets including the source country, because the host market is not very attractive due to the low income level.\(^1\) For example, a number of Japanese firms invest in China and ASEAN countries to serve the Japanese market.\(^2\)

In 2004, the share of Japanese reverse imports reached 19.1% of the total Japanese imports and about 80% of reverse imports is from Asia. Japanese plants located in Asia export 20% of their products to Japan (Nikkei Shimbun, April 25, 2006).

It is observed in many industries that some firms undertake FDI, while some others stay at home. An interesting question is why this occurs. This paper tackles this question when products are consumed only in the source country. Specifically, we pay our attention to inter-firm cost asymmetry. We examine which firm has more incentive for FDI, a more efficient one or a less efficient one. To this end, we construct a simple oligopoly model with cost heterogeneity and investigate the relationship between firms’ location choices and trade costs. In our model, there are two countries (domestic and foreign) and two domestic firms whose marginal costs (MCs) are different. The two firms choose their production locations to serve the domestic market. We find that the cost difference within a firm, i.e., the difference between domestic and foreign MCs plays a crucial role. Moreover, in the presence of fixed costs (FCs), multiple equilibria may exist and a small change in trade costs may reverse plant locations.

There are many studies that analyze location choices of multinational firms (MNFs): the choice between exports and FDI (local production) and the choice between domestic production and FDI.\(^3\) However, the cost asymmetry among firms with the same nationality has been paid little attention.\(^4\) To our best knowledge, location choices among heterogeneous firms with the same nationality have not been analyzed.\(^5\)

Qiu and Tao (2001) examine the choice between FDI and exports by two heterogeneous firms. In their paper, FCs are assumed away and heterogeneity stems from different MCs. In contrast,\(^6\)

---

\(^1\) When products are exported to the source country, it is called vertical FDI. It is also sometimes called “reverse imports” (from the viewpoint of the source country). When products are exported to countries other than the source country, it is called export-platform FDI.

\(^2\) We should mention that the Chinese market has been getting more attractive for foreign firms because of rapid economic growth and a huge population.

\(^3\) For recent studies of MNFs, see Markusen (2002) and Barba Navaretti and Venables (2004).

\(^4\) A typical model assumes a single firm in each country. See, for example, Dei (1990) and Horstmann and Markusen (1992). In their models, firms serve both domestic and foreign markets.

\(^5\) Assuming two identical domestic firms (potential MNFs), Yomogida (2004) considers the choice between FDI and domestic production. He shows the possibility of socially undesirable FDI.
FCs play an important role in our analysis. Moreover, their main focus is on the relationship between local content requirement and location choice.\(^\text{6}\) They show that the less efficient firm undertakes FDI if two firms are located in different countries. Helpman et al. (2004) show a reason why exporting firms and MNFs coexist. In their model, cost heterogeneity also plays a crucial role. However, their model is a monopolistic competition model originally developed by Melitz (2003) who considers the coexistence of exporting firms and non-exporting firms.

Ishikawa and Miyagiwa (2005) extend our static analysis to a dynamic framework. They specifically investigate the relationship between the inter-firm cost asymmetry and the timing of international outsourcing or FDI. In particular, they show that a more efficient firm does not always undertake FDI before a less efficient one.

The rest of the paper is organized as follows. Section 2 presents the basic model. We examine the effects of trade-cost reductions on firms’ profits in different regimes. We analyze the location choices without plant-specific FCs in Section 3 and that with plant-specific FCs in Section 4. Section 5 concludes the paper.

2 Basic Model

We consider a duopoly model where there are two countries (domestic and foreign) and two domestic firms (firms 1 and 2). Both firms produce a homogeneous good in either domestic country or foreign country and serve only domestic market.\(^\text{7}\) The model involves two stages of decision. In stage 1, both firms simultaneously choose their plant locations.\(^\text{8}\) Plant locations are determined by Nash equilibrium. In stage 2, the firms compete in quantities with Cournot conjectures. Only one firm serves the market under certain demand and cost conditions, but we focus on equilibrium where both firms serve the market. The game is solved by backward induction.

The inverse demand function is given by

\[
P = P(X); \quad P' < 0, \quad (1)
\]

where \(X\) and \(P\) are, respectively, the demand and consumer price. We define the elasticity of the slope of the inverse demand function for the following analysis:

\[
\epsilon(X) = \frac{XP''(X)}{P'(X)}. \quad (2)
\]

The (inverse) demand curve is concave if \(\epsilon(X) \leq 0\) and convex if \(\epsilon(X) \geq 0\). In the following analysis, we assume \(\epsilon(X) < 1\), which implies that goods produced by two firms are strategic.

\(^{\text{6}}\) In the literature of international trade theory, only a few studies focus on inter-firm cost asymmetry within the framework of oligopoly. Exceptions include Long and Soubeyran (1997).

\(^{\text{7}}\) For example, all goods produced in export processing zones must be exported. We deal with a case where both domestic and foreign markets are served as well as a case where only foreign market is served elsewhere (Ishikawa and Komoriya, 2006).

\(^{\text{8}}\) Ishikawa and Miyagiwa (2005) consider the case of preemption in a dynamic framework.
substitutes (i.e., $P_i + P'' x_i < 0$ where $x_i$ is the output of firm $i$ ($i = 1, 2$)).

The profits of firm $i$ ($i = 1, 2$) are given by

$$
\Pi_i(x_i; t) = (P(X) - t)x_i - C_i(x_i),
$$

where $t$ is a specific trade cost such as transport costs and $C_i(\cdot)$ is the cost function. The firms incur the trade cost when they produce in the foreign country. The cost function of firm $i$ ($i = 1, 2$) is given by

$$
C_i(x_i) = \begin{cases} 
    c_i x_i + f_i, \\
    c^*_i x_i + f^*_i,
\end{cases}
$$

where $c_i$ and $f_i$ are, respectively, a constant MC and a plant-specific FC. An asterisk denotes foreign variables or parameters. We assume that firm 1 is more efficient than firm 2 in the sense that $c_1 < c_2$, $c^*_1 < c^*_2$, $f_1 = f_2$, and $f^*_1 = f^*_2$; and that the MC is lower in the foreign country, i.e., $c_i > c^*_i$ for $i = 1, 2$.

The first-order conditions for profit maximization are ($i = 1, 2$)

$$
\frac{\partial \Pi_i}{\partial x_i} = P + P' x_i - (C'_i + t) = 0.
$$

The second-order sufficient conditions ($i = 1, 2$):

$$
2P' + P'' x_i = P'(2 - \epsilon \sigma_i) < 0
$$

and

$$
|\Omega| = P'(3P' + P'' X) = (P')^2(3 - \epsilon) > 0
$$

where $\sigma_i$ is the market share of firm $i$ (i.e., $\sigma_i \equiv x_i/X$) and

$$
\Omega \equiv \begin{pmatrix} 2P' + P'' x_1 & P' + P'' x_1 \\
    P' + P'' x_2 & 2P' + P'' x_2 \end{pmatrix}
$$

are satisfied with $\epsilon(X) < 1$.

We first examine the effects of a change in $t$ on equilibrium profits. For this, we need to obtain the effects of a change in $t$ on outputs. When firm $i$ produces in the domestic country but firm $j$ produces in the foreign country, we have

$$
\begin{pmatrix} dx_i \\ dx_j \end{pmatrix} = \frac{1}{|\Omega|} \begin{pmatrix} 2P' + P'' x_j & -(P' + P'' x_i) \\
    -(P' + P'' x_j) & 2P' + P'' x_i \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix}.
$$

Thus, the effects on outputs are

$$
\frac{dx_i}{dt} = -\frac{P' + P'' x_i}{|\Omega|} > 0, \quad \frac{dx_j}{dt} = \frac{2P' + P'' x_i}{|\Omega|} < 0, \quad \frac{dX}{dt} = \frac{P'}{|\Omega|} < 0. \tag{6}
$$

9 For details, see Furusawa et al. (2003).

10 Even if FCs are different between the firms, our main results are still valid.
Using the first-order condition and (6), we can obtain
\[
\frac{d\Pi_i}{dt} = \frac{P' x_i (2P' + P'' x_i)}{|\Omega|} > 0. \tag{7}
\]
\[
\frac{d\Pi_j}{dt} = -\frac{(P')^2 x_j}{|\Omega|} (4 - \epsilon - \epsilon \sigma_i) < 0. \tag{8}
\]
Thus, when \( t \) lowers, the profits of firm \( i \) decrease and those of firm \( j \) increase.

When both firms produce in the foreign country, we have
\[
\left( \begin{array}{c}
\frac{dx_1}{dt} \\
\frac{dx_2}{dt}
\end{array} \right) = \frac{1}{|\Omega|} \left( \begin{array}{cc}
2P' + P'' x_2 & -(P' + P'' x_1) \\
-(P' + P'' x_2) & 2P' + P'' x_1
\end{array} \right) \left( \begin{array}{c}
1 \\
1
\end{array} \right).
\]
Thus, the effects on outputs are
\[
\frac{dx_1}{dt} = \frac{P' + P' x_2 - x_1}{|\Omega|} = \frac{P'}{|\Omega|} \{1 - \epsilon (1 - \sigma_1) + \epsilon \sigma_1\} = \frac{P'}{|\Omega|} (1 - \epsilon + 2 \epsilon \sigma_1) < 0, \tag{9}
\]
\[
\frac{dx_2}{dt} = \frac{P' + P' x_1 - x_2}{|\Omega|} = \frac{P'}{|\Omega|} (1 + \epsilon - 2 \epsilon \sigma_1) < 0, \tag{10}
\]
\[
\frac{dX}{dt} = \frac{2P'}{|\Omega|} < 0, \tag{11}
\]
where the first two inequalities are from \( \epsilon < 1 \) and \( 0 \leq \sigma_1 \leq 1 \). Using the first-order condition and equations (9) through (11), we can obtain (\( i = 1, 2 \))
\[
\frac{d\Pi_i}{dt} = -\frac{2P' x_i}{|\Omega|} (P' + P'' x_j) < 0. \tag{12}
\]
Therefore, when both firms produce in the foreign country, they gain from a lower \( t \).

### 3 Location Choices without FCs

We examine firms’ location choices. As a benchmark, we examine a case where there exist no FCs in this section. Without FCs, the firm’s decision does not depend on the other firm’s decision. That is, there exist dominant strategies for both firms. Firm \( i \) (\( i = 1, 2 \)) produces in the domestic country if and only if
\[
\Delta c_i \equiv c_i - c^*_i \leq t.
\]
If \( t \) is high enough, both firms choose domestic production. If \( t \) is low enough, on the other hand, both firms choose foreign production. It is possible that one firm produces in the domestic country while the other produces in the foreign country. Whereas firm 1 produces in the domestic country and firm 2 produces in the foreign country if \( \Delta c_1 \leq t < \Delta c_2 \), firm 1 produces in the foreign country and firm 2 produces in the domestic country if \( \Delta c_2 \leq t < \Delta c_1 \). Intuitively, the firm which can save the real MC more by foreign production has more incentive for FDI. When \( \Delta c_1 = \Delta c_2 \), both firms simultaneously shift their production from the domestic country to the foreign country as \( t \) falls.
Proposition 1 Suppose that there exist no FCs. If $\max\{\Delta c_1, \Delta c_2\} \leq t$, both firms produce in the domestic country. If $\Delta c_i \leq t < \Delta c_j$ ($i, j = 1, 2; i \neq j$), firm $i$ produces in the domestic country while firm $j$ produces in the foreign country. If $\min\{\Delta c_1, \Delta c_2\} > t$, both firms produce in the foreign country.

To obtain some more insight, we specify the cost function of firm $i$ ($i = 1, 2$) as follows:

$$C_i(x_i) = \begin{cases} c_i x_i = a_i w x_i \\ c_i^* x_i = a_i^* w^* x_i \end{cases}$$

where $a_i$ and $w$ are, respectively, labor coefficient and the wage rate, which are exogenously given and constant. It is assumed that $a_1 < a_2$, $w > w^*$, $a_i \leq a_i^*$, and $a_i w > a_i^* w^*$. Then firm $i$ ($i = 1, 2$) produces in the domestic country if and only if

$$a_i w \leq a_i^* w^* + t.$$

Figure 1 illustrates this condition. Firm 1 produces in the domestic (foreign) country in the region below (above) line 1, while firm 2 produces in the domestic (foreign) country in the region below (above) line 2. Whereas Panel (a) shows the case where $a_1^*/a_1 > a_2^*/a_2$ holds, Panel (b) shows the case where $a_1^*/a_1 < a_2^*/a_2$. $a_1^*/a_1 > a_2^*/a_2$ ($a_1^*/a_1 < a_2^*/a_2$) could be the case if it is relatively difficult (easy) to transfer more efficient technology to the foreign country.

There are three regions in Panel (a) and four regions in Panel (b). Both firms produce in the domestic (foreign) country when $w$ is relatively low (high) and $t$ is relatively high (low), that is, $(t, w)$ is in region DD (region FF). Whereas firm 1 produces in the domestic country and firm 2 produces in the foreign country in region DF, firm 1 produces in the foreign country and firm 2 produces in the domestic country in region FD. We should note that region FD never appears in Panel (a).\(^{11}\)

The location choice depends on the relative size of the labor-coefficient ratio, $a_i^*/a_i$ ($i = 1, 2$). If $a_1^*/a_1 > a_2^*/a_2$, then firm 2 (i.e., the less efficient firm) always has more incentive for FDI than firm 1 (i.e., the more efficient firm). In the case where $a_1^*/a_1 < a_2^*/a_2$ holds, however, firm 2 (firm 1) has more incentive to undertake FDI if both $t$ and $w$ and are relatively high (low).

To obtain economic intuition, we first consider an extreme case where $a_i^* = a_i$ ($i = 1, 2$), that is, foreign and domestic production of firm $i$ shares the same technology. When they invest in the foreign country, both firms face the same trade costs, and hence the share of trade costs in the “effective” MC is larger for firm 1 (i.e., the more efficient firm) than for firm 2 (i.e., the less efficient firm).\(^{12}\) Thus, the advantage of firm 1 is relatively small when both firms produce abroad. In this case, therefore, firm 2 always has more incentive for FDI. (see Figure 1 (a)).

As long as $a_1^*/a_1 > a_2^*/a_2$, the same economic intuition goes through. If $a_1^*/a_1 < a_2^*/a_2$, on the other hand, Figure 1 (b) shows that firm 1 always has more incentive for FDI. (see Figure 1 (b)).

\(^{11}\)It can be seen in panel (b) that as $t$ falls, both firms simultaneously shift their locations from the domestic country to the foreign country at point $S$ (where two lines intersect).

\(^{12}\)The effective MC includes the trade costs. That is, the effective MC of firm $i$ is $c_i$ (i.e., the real MC) if it produces at home and $c_i^* + t$ if it produces abroad.
other hand, firm 1 faces a trade-off between relatively high trade costs and more efficient foreign technology. Thus, if trade costs are low enough, firm 1 has more incentive to locate its plant in the foreign country.

Next we examine domestic welfare, measured by the sum of consumer surplus and firms’ profits:\footnote{In the welfare analysis, we assume for simplicity that $t$ is transport costs. Since both firms are domestic, a tariff is just a transfer within the domestic country.}

$$W \equiv U(X) - P(X)X + \Pi_1 + \Pi_2$$

where $dU/dX = P$. Obviously, a change in $t$ does not affect welfare in region $DD$. In region $FF$, both firms gain from a decrease in $t$ (see (12)). Consumers also benefit from a lower $t$, because the price falls. Thus, a lower $t$ leads to higher welfare in region $FF$. In region $DF$ (region $FD$), a decrease in $t$ benefits firm 2 (firm 1) and hurts firm 1 (firm 2) (see (7) and (8)). In order to investigate the case where firm $i$ produces in the domestic country and firm $j$ produces in the foreign country ($i, j = 1, 2; i \neq j$), we differentiate (14) with respect to $t$ to obtain

$$\frac{dW}{dt} = -XP' \frac{dX}{dt} + \frac{d\Pi_1}{dt} + \frac{d\Pi_2}{dt} = \frac{(P')^2}{11}(2\sigma_i^2 - 6\sigma_i + 5 - \epsilon).$$

When $\epsilon \neq 0$, $dW/dt < 0$ (that is, a decrease in $t$ improves domestic welfare) if and only if $\sigma_i < \sigma_i^* \equiv (-\sqrt{-10\epsilon + 2\epsilon^2 + 9} + 3)/2\epsilon$. When $\epsilon = 0$, $dW/dt < 0$ if and only if $\sigma_i < 5/6$.\footnote{Assuming linear demand: $P(X) = A - aX$, $c_j^* + t < (4A + 7c_i)/11$ is equivalent to $\sigma_i < 5/6$.} Thus, if $x_i$ is not very large, then $dW/dt < 0$ holds. In particular, Appendix B shows $1/2 < \sigma_i^* < 1$. Thus, $dW/dt < 0$ holds if $\sigma_i \leq 1/2$, which always holds with $i = 2$.

Intuitively, a lower $t$ is beneficial, because the total supply rises and the domestic consumers gain. However, an increase in the output of the less efficient firm at the expense of the more efficient firm is detrimental.\footnote{See also Lahiri and Ono (1988).} When the latter effect dominates the former, domestic welfare deteriorates. In Figure 1, therefore, a decrease in $t$ reduces domestic welfare only if $(t, w)$ is in region $DF$.

We obtain the following proposition.

**Proposition 2** When both firms produce in the foreign country, a decrease in $t$ improves domestic welfare. A lower $t$ raises domestic welfare if only the more efficient firm produces in the foreign country, but may reduce it if only the less efficient firm produces in the foreign country and its market share is small.

### 4 Location Choices with FCs

In this section, we introduce plant-specific FCs into our analysis. Once FCs are present, the production-location decisions also depend on the output levels. The decision by a firm affects
that of the other firm. This leads to potentially an unlimited number of cases to examine. In the following analysis, therefore, we focus on the case with linear demand: \( P = A - aX \) (i.e., \( \epsilon = 0 \)).

The analysis further simplifies if we assume \( \Delta f_i \equiv f_i^* - f_i > 0 \) (i = 1, 2) and \( f_i = 0 \).\(^{17}\) We let \( DD \) (\( FF \)) and \( DF \) (\( FD \)) respectively denote the case where both firms are located in the domestic (foreign) country and the case where firm 1 is located in the domestic (foreign) country while firm 2 is located in the foreign (domestic) country. For example, \( \Pi_{FD} \) is the profits of firm \( i \) when firm 1 produces abroad and firm 2 produces at home.

Given that the rival firm (i.e., firm 2) produces in the domestic country, firm 1 will undertake FDI if \( \Delta \Pi_i^D \equiv \Pi_i^{FD} - \Pi_i^{DD} > 0 \). Similarly, given that firm 1 produces at home, firm 2 will produce abroad if \( \Delta \Pi_i^F \equiv \Pi_i^{FF} - \Pi_i^{DD} > 0 \). Since \( \Delta f_i = f_i^* > 0 \), firm \( i \) has now no incentive to locate its plant in the foreign country when \( \Delta c_i = t \). We let \( t_i^D \) denote the trade cost that makes \( \Delta \Pi_i^D = 0 \) hold. That is, at \( t_i^D \), firm \( i \) is indifferent between domestic and foreign production, given that the rival firm stays in the domestic country. Similarly, given that the rival firm produces in the foreign country, firm \( i \) will undertake FDI if \( \Delta \Pi_i^F > 0 \) (where \( \Delta \Pi_i^F \equiv \Pi_i^{FF} - \Pi_i^{DF} \) and \( \Delta \Pi_i^D \equiv \Pi_i^{FD} - \Pi_i^{DD} \)).\(^{18}\) We also let \( t_i^F \) denote the trade cost that leads to \( \Delta \Pi_i^F = 0 \). Obviously, \( \max \{t_i^D, t_i^F\} < \Delta c_i \) holds.

\( \Delta \Pi_i^k \) (\( i = 1, 2; k = D, F \)) is derived in Appendix A (see (A1) and (A2)). To facilitate the following analysis, we illustrate \( \Delta \Pi_i^k = 0 \) in Figures 2 and 3. Whereas Figure 2 shows the case where \( \Delta c_1 > \Delta c_2 \) holds, Figure 3 shows the case where \( \Delta c_1 < \Delta c_2 \). When \( f_i^* = 0 \), \( \Delta \Pi_i^k = 0 \) holds at \( t = \Delta c_i \). Moreover, when \( f_i^* > 0 \), \( \Delta \Pi_i^k = 0 \) holds at some \( t \) which is less than \( \Delta c_i \); and \( \Delta \Pi_i^k = 0 \) is downward-sloping.\(^{19}\) By noting

\[
\Delta \Pi_i^D - \Delta \Pi_i^F = \frac{4(\Delta c_i - t)(\Delta c_j - t)}{9a} \quad (j = 1, 2; i \neq j),
\]

\( \Delta \Pi_i^D = 0 \) and \( \Delta \Pi_i^F = 0 \) intersect with each other at \( t = \Delta c_j \) as well as at \( t = \Delta c_i \). In Figure 2 (Figure 3), \( \Delta \Pi_i^D = 0 \) is located above \( \Delta \Pi_i^F = 0 \) when \( 0 \leq t < \Delta c_2 \) (\( 0 \leq t < \Delta c_1 \)) and vice versa when \( \Delta c_2 < t < \Delta c_1 \) (\( \Delta c_1 \leq t < \Delta c_2 \)).

The following lemma is immediate.

**Lemma 1** Regardless of the rival’s location, firm \( i \) produces in the domestic country when \( t \geq \max \{t_i^D, t_i^F\} \) but in the foreign county when \( t < \min \{t_i^D, t_i^F\} \). When \( t_i^D \leq t \leq t_i^F \) (\( t_i^F \leq t < t_i^D \)), firm \( i \) is located in the domestic (foreign) country if the rival produces in the domestic country, but in the foreign (domestic) country if the rival produces in the foreign country. Moreover, \( t_i^D < \Delta c_i \) and \( t_i^F < \Delta c_i \) when \( f_i^* > 0 \), while \( t_i^D = t_i^F = \Delta c_i \) in the absence of FCs (i.e., \( f_i^* = 0 \)).

Depending on the relative sizes of \( t_i^D \) and \( t_i^F \) (\( i = 1, 2 \)), we have different location patterns. Since there are four critical values, there are 24 possible orders. However, some of them are not possible. The following Lemmas are useful to eliminate those irrelevant cases.

\(^{17}\)This type of FCs may be monitoring costs and/or communication costs.

\(^{18}\)Neary (2005) calls \( \Delta \Pi_i^D \) and \( \Delta \Pi_i^F \) the offshoring gain.

\(^{19}\)\( t \leq \Delta c_i \) is necessary for firm \( i \) to undertake FDI.
Lemma 2 If \( t_i^D < t_i^F \), then \( t_j^F < t_j^D < \Delta v_j < t_i^D < t_i^F \).

**Proof.** See Appendix B. ■

Lemma 3 If \( t_i^D \leq t_i^D \), then \( t_i^F < t_i^F \).

**Proof.** See Appendix B. ■

Using Lemma 2, we can eliminate 16 cases. And using Lemma 3, we can eliminate one more case (i.e., \( t_i^F < t_j^F < t_i^D < t_j^D \)). That is, the following seven cases are possible:  

- \( t_i^F < t_j^D < t_i^D < t_j^F \), \( t_j^F < t_i^D < t_i^F < t_j^D \), \( i, j = 1, 2; i \neq j \), and \( t_i^F < t_j^F < t_i^D < t_j^D \).20

Invoking Lemma 1, we examine the plant locations determined by Nash equilibrium. For example, suppose \( t_i^F < t_j^F < t_i^D < t_j^D \). We first consider the strategy of firm 1. Recalling the definition of \( t_i^D \) and \( t_i^F \), firm 1 produces in the domestic country if \( t \geq t_i^F \) and in the foreign country if \( t < t_i^F \) regardless of firm 2’s strategy. If \( t_i^D \leq t < t_i^F \), firm 1 chooses the same location as firm 2 does. The strategy of firm 2 is as follows. Regardless of firm 1’s strategy, firm 2 produces in the domestic country if \( t \geq t_i^D \) and in the foreign country if \( t < t_i^D \). Given firm 1’s location, firm 2 chooses the different location if \( t_i^D < t \leq t_i^D \) and firm 2 produces in the foreign country. Thus, we obtain the following Nash equilibrium. If \( t_i^D \), both firms produce in the domestic county. If \( t_i^F < t \leq t_i^D \), firm 1 produces in the foreign country while firm 2 produces in the domestic country. If \( t < t_i^F \), both firms produce in the foreign country. Thus, a lower \( t \) leads to more incentive for the more efficient firm (i.e., firm 1) to undertake FDI. In this manner, we can find Nash equilibrium.

In view of Table 1, we can summarize the location patterns as follows.

1. **Cases I and II.** \( t_j^F < t_i^D < t_i^D < t_i^F \) (i.e., \( t_i^F < t_j^D < t_i^D < t_j^F \), \( t_j^F < t_i^D < t_i^F < t_j^D \), \( i, j = 1, 2; i \neq j \)) : In these cases, if \( t \geq t_i^D \) (\( t < t_i^F \)), both firms produce in the domestic (foreign) country. If \( t_i^F \leq t < t_i^D \), firm \( i \) produces in the foreign country while firm \( j \) produces in the domestic country.\(^{21}\)

2. **Case III.** \( t_j^F < t_i^F < t_i^D < t_i^F \) (i.e., \( t_j^F < t_i^D < t_i^F < t_i^F \), \( i, j = 1, 2; i \neq j \)) : As in Cases I and II, both firms produce in the domestic (foreign) country if \( t \geq t_i^D \) (\( t < t_i^F \)). If either \( t_j^F \leq t < t_i^D \) or \( t_i^D \leq t < t_i^F \), firm \( i \) produces in the foreign country while firm \( j \) produces in the domestic country. However, if \( t_i^F \leq t < t_i^D \), there are two possible equilibria. In one equilibrium, firm \( i \) produces in the foreign country while firm \( j \) produces in the domestic country; and vice versa in the other equilibrium.

3. **Case IV.** \( t_j^F < t_i^F < t_i^D < t_i^F \) (i.e., \( t_i^D \leq t < t_i^F \)), both firms produce in the domestic (foreign) country. If \( t_i^D \leq t < t_i^F \) (\( t_i^F \leq t < t_i^F \)), firm 1 produces in the domestic (foreign)

\(^{20}\)In Appendix A, we verify that these seven cases actually exist.

\(^{21}\)In Cases I, II, and III, \((F.D) \ (D.F))\) means that firm \( i \) produces abroad (at home) and firm \( j \) produces at home (abroad) (i.e., \( i, j = 1, 2; i \neq j \)). This should be distinguished from above-defined \( F.D \ (D.F) \) which means that firm 1 produces abroad (at home) and firm 2 produces at home (abroad).
produce in the foreign country, competition becomes so intense that neither location patterns are similar to those in the case without FCs. That is, as shifts from $DD$ located in the foreign country. The presence of FCs plays a crucial role here. Relatively high FCs and complete reversal of the location patterns may occur at both $t_{ij}^F$ and $t_{ij}^D$ occurs at either $t_{ij}^F$ or $t_{ij}^D$ in Case IV. It should be noted that all orders of Cases III and IV (i.e., $t_{ij}^F < t_{ij}^D < t_{ij}^P < t_{ij}^D$, $t_{ij}^P < t_{ij}^D < t_{ij}^P$ and $t_{ij}^P < t_{ij}^D < t_{ij}^P$) are possible if $\Delta c_1 < \Delta c_2$.

<table>
<thead>
<tr>
<th>Case I. $t_{ij}^F &lt; t_{ij}^D &lt; t_{ij}^P &lt; t_{ij}^D$</th>
<th>$t &lt; t_{ij}^D$</th>
<th>$t_{ij}^F &lt; t &lt; t_{ij}^D$</th>
<th>$t_{ij}^D &lt; t &lt; t_{ij}^P$</th>
<th>$t_{ij}^P &lt; t &lt; t_{ij}^D$</th>
<th>$t_{ij}^D &lt; t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Best response of firm $i$ ($R_i(D), R_i(F)$)</td>
<td>(F,F)</td>
<td>(F,F)</td>
<td>(F,F)</td>
<td>(D,F)</td>
<td>(D,D)</td>
</tr>
<tr>
<td>Best response of firm $j$ ($R_j(D), R_j(F)$)</td>
<td>(F,F)</td>
<td>(F,D)</td>
<td>(D,D)</td>
<td>(D,D)</td>
<td>(D,D)</td>
</tr>
<tr>
<td>Nash equilibrium (firm $i$, firm $j$)</td>
<td>(F,F)</td>
<td>(F,D)</td>
<td>(F,D)</td>
<td>(D,D)</td>
<td>(D,D)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Case II. $t_{ij}^F &lt; t_{ij}^D &lt; t_{ij}^P &lt; t_{ij}^D$</th>
<th>$t &lt; t_{ij}^F$</th>
<th>$t_{ij}^F &lt; t &lt; t_{ij}^D$</th>
<th>$t_{ij}^D &lt; t &lt; t_{ij}^P$</th>
<th>$t_{ij}^P &lt; t &lt; t_{ij}^D$</th>
<th>$t_{ij}^D &lt; t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Best response of firm $i$ ($R_i(D), R_i(F)$)</td>
<td>(F,F)</td>
<td>(F,F)</td>
<td>(F,F)</td>
<td>(F,D)</td>
<td>(D,D)</td>
</tr>
<tr>
<td>Best response of firm $j$ ($R_j(D), R_j(F)$)</td>
<td>(F,F)</td>
<td>(F,D)</td>
<td>(D,D)</td>
<td>(D,D)</td>
<td>(D,D)</td>
</tr>
<tr>
<td>Nash equilibrium (firm $i$, firm $j$)</td>
<td>(F,F)</td>
<td>(F,D)</td>
<td>(F,D)</td>
<td>(D,D)</td>
<td>(D,D)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Case III. $t_{ij}^F &lt; t_{ij}^D &lt; t_{ij}^P &lt; t_{ij}^D$</th>
<th>$t &lt; t_{ij}^F$</th>
<th>$t_{ij}^F &lt; t &lt; t_{ij}^D$</th>
<th>$t_{ij}^D &lt; t &lt; t_{ij}^P$</th>
<th>$t_{ij}^P &lt; t &lt; t_{ij}^D$</th>
<th>$t_{ij}^D &lt; t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Best response of firm $i$ ($R_i(D), R_i(F)$)</td>
<td>(F,F)</td>
<td>(F,F)</td>
<td>(F,D)</td>
<td>(D,D)</td>
<td>(D,D)</td>
</tr>
<tr>
<td>Best response of firm $j$ ($R_j(D), R_j(F)$)</td>
<td>(F,F)</td>
<td>(F,D)</td>
<td>(D,D)</td>
<td>(D,D)</td>
<td>(D,D)</td>
</tr>
<tr>
<td>Nash equilibrium (firm $i$, firm $j$)</td>
<td>(F,F)</td>
<td>(F,D)</td>
<td>(F,D)</td>
<td>(D,D)</td>
<td>(D,D)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Case IV. $t_{ij}^F &lt; t_{ij}^D &lt; t_{ij}^P &lt; t_{ij}^D$</th>
<th>$t &lt; t_{ij}^F$</th>
<th>$t_{ij}^F &lt; t &lt; t_{ij}^D$</th>
<th>$t_{ij}^D &lt; t &lt; t_{ij}^P$</th>
<th>$t_{ij}^P &lt; t &lt; t_{ij}^D$</th>
<th>$t_{ij}^D &lt; t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Best response of firm 1 ($R_1(D), R_1(F)$)</td>
<td>(F,F)</td>
<td>(F,F)</td>
<td>(F,D)</td>
<td>(D,D)</td>
<td>(D,D)</td>
</tr>
<tr>
<td>Best response of firm 2 ($R_2(D), R_2(F)$)</td>
<td>(F,F)</td>
<td>(F,D)</td>
<td>(D,D)</td>
<td>(D,D)</td>
<td>(D,D)</td>
</tr>
<tr>
<td>Nash equilibrium (firm 1, firm 2)</td>
<td>(F,F)</td>
<td>(F,D)</td>
<td>(F,D)</td>
<td>(D,D)</td>
<td>(D,D)</td>
</tr>
</tbody>
</table>

Table 1: Best response of each firm and Nash equilibrium

...country while firm 2 produces in the foreign (domestic) country. If $t_{ij}^F < t < t_{ij}^D$, there are two possible equilibria.

The following should be noted. First, in all cases, both firms produce in the domestic country if $t \geq \max\{t_{ij}^P, t_{ij}^P\}$ but in the foreign country if $t < \min\{t_{ij}^F, t_{ij}^F\}$. Second, in the first four cases, the location patterns are similar to those in the case without FCs. That is, as $t$ falls, the regime shifts from $DD$ to $DF$ and then to $FF$, or from $DD$ to $FD$ and then to $FF$. This similarity arises, because the two critical values of firm $i$ are greater than those of firm $j$ in those four cases, that is, $\max\{t_{ij}^P, t_{ij}^P\} < \min\{t_{ij}^P, t_{ij}^P\}$ holds. This is likely to arise when the difference between $\Delta c_j$ and $\Delta c_i$ is large ($\Delta c_j < \Delta c_i$) and the FCs are small. A small FC of firm $i$ (j) implies that the gap between $\Delta c_i$ ($\Delta c_j$) and $t_{ij}^P$ ($t_{ij}^F$) or $t_{ij}^P$ ($t_{ij}^F$) is small. Third, multiple equilibria arise when neither of the firms has a dominant strategy. The intuition for multiple equilibria is as follows. For both firms, the effective MCs are lower if they produce in the foreign country. However, if both firms produce in the foreign country, competition becomes so intense that neither firm can cover its FC, and hence one of them would rather stay in the domestic country. Thus, only one firm is located in the foreign country. The presence of FCs plays a crucial role here. Relatively high FCs lead to a relatively large gap between $t_{ij}^P$ ($t_{ij}^F$) and $t_{ij}^P$ ($t_{ij}^F$). In addition, if the gap between $\Delta c_j$ and $\Delta c_i$ is small, multiple equilibria are likely arise. Lastly, because of multiple equilibria, the complete reversal of the location patterns may occur at both $t_{ij}^F$ and $t_{ij}^D$ in Case III, and actually occurs at either $t_{ij}^F$ or $t_{ij}^D$ in Case IV. It should be noted that all orders of Cases III and IV (i.e., $t_{ij}^F < t_{ij}^D < t_{ij}^P$, $t_{ij}^F < t_{ij}^D < t_{ij}^P$ and $t_{ij}^F < t_{ij}^D < t_{ij}^P$) are possible if $\Delta c_1 < \Delta c_2$, ...
while only $t_2^F < t_1^F < t_2^D < t_1^D$ is possible if $\Delta c_1 > \Delta c_2$. This is because firm 1 always has more incentive for FDI if both $\Delta c_1 > \Delta c_2$ and $f^*_1 = f^*_2$ hold.

We obtain the following proposition.

**Proposition 3** If $t_j^F < t_i^F < t_j^D < t_i^D$ ($i, j = 1, 2; i \neq j$), a small change in $t$ may completely reverse the location choices at both $t_i^F$ and $t_j^D$. If $t_2^F < t_1^F < t_1^D < t_2^D$, the complete reversal does occur at either $t_1^F$ or $t_2^D$.

We should note that Proposition 2 is still valid with FCs. However, in contrast to the case without FCs, at the critical levels, the profits of the firm which undertakes FDI are the same, while those of the other firm discontinuously drop. This is because the effective MC of the firm undertaking FDI becomes lower, which in turn decreases the price. In particular, Appendix B proves the following propositions.

**Proposition 4** At critical levels of $t$, domestic welfare jumps up if only the more efficient firm switches its production from the domestic country to the foreign country but goes down if only the less efficient firm switches its production from the domestic country to the foreign country.

**Proposition 5** When plant locations are completely reversed, consumer surplus jumps up if only the firm, whose FDI can save its real MC more than the rival’s, undertakes FDI, the profits of a firm are larger when it produces abroad than when the rival does, and the effects of complete reversal on domestic welfare are generally ambiguous. However, if $\Delta c_1 > \Delta c_2$, then the complete reversal, under which the more efficient firm switches its location from the domestic country to the foreign country and vice versa for the less efficient firm, improves domestic welfare.

## 5 Concluding Remarks

Using a simple, two-country, duopoly model, we have analyzed location choices by the firms. Specifically, both firms are domestic; they are heterogeneous in the sense that their MCs are different; and they serve only domestic market. When the trade costs are neither very high nor very low, one of the two firms has incentive to undertake FDI in the foreign country. In the absence of FCs, the difference between domestic and foreign MCs is crucial. It should be emphasized that what is crucial is not the cost difference between firms but the cost difference between domestic and foreign production within a firm. In the presence of FCs, we may have multiple equilibria. Moreover, the production location may not monotonically change as the trade costs change. However, the difference between domestic and foreign MCs still plays an important role here. We have also shown that a lower trade cost may lead to reduce domestic welfare.

The domestic government may be able to affect the trade cost, $t$. For instance, this is the case if a part of the trade cost is non-tariff barriers. In this case, the government may intervene to improve domestic welfare. When the less efficient firm has incentive for FDI, for example, the
government may keep the trade cost so high as to discourage the incentive. And it may lift the non-tariff barriers once the transport cost becomes so low that both firms have incentive for FDI.

In our analysis, we can reinterpret FDI as international outsourcing. In particular, it is often considered that firms have to incur FCs to undertake FDI but do not in the case of outsourcing. Thus, one may think that FDI and outsourcing, respectively, correspond to the case with and without FCs.

Moreover, we have focused on the case where firms choose to produce either at home or abroad. A firm may shift a part of its production facilities to the foreign country, or have both domestic and foreign plants. The analysis including those cases is left for the future research.

Appendix A

In this appendix, we show that the location patterns obtained with FCs actually exist. In the following equations, \( i, j = 1, 2; i \neq j \). The profits are given by

\[
\begin{align*}
\Pi_{i}^{DD} &= a \left( \frac{A - 2c_i + c_j}{3a} \right)^2, \\
\Pi_{i}^{FD} &= a \left( \frac{A - 2c_i^* + c_j - 2t}{3a} \right)^2 - f_i^*, \\
\Pi_{i}^{DF} &= a \left( \frac{A - 2c_i + c_j^* + t}{3a} \right)^2, \\
\Pi_{i}^{FF} &= a \left( \frac{A - 2c_i^* + c_j^* - t}{3a} \right)^2 - f_i^*. 
\end{align*}
\]

The firm \( i \)'s incentive to undertake FDI is determined by

\[
\begin{align*}
\Delta \Pi_{i}^{D} &= a \left( \frac{A - 2c_i^* + c_j - 2t}{3a} \right)^2 - a \left( \frac{A - 2c_i + c_j}{3a} \right)^2 - f_i^*, \\
\Delta \Pi_{i}^{F} &= a \left( \frac{A - 2c_i + c_j^* + t}{3a} \right)^2 - a \left( \frac{A - 2c_i + c_j + t}{3a} \right)^2 - f_i^*.
\end{align*}
\]

Differentiating above two equations with respect to \( t \), we obtain

\[
\begin{align*}
\frac{d\Delta \Pi_{i}^{D}}{dt} &= -\frac{4}{9a} (A - 2c_i^* + c_j - 2t), \\
\frac{d\Delta \Pi_{i}^{F}}{dt} &= -\frac{4}{9a} (A - c_i - c_i^* + c_j^*).
\end{align*}
\]

Given that firm \( j \) produces in the domestic country, firm \( i \) will undertake FDI if the following condition holds:

\[
\Delta \Pi_{i}^{D} > 0 \iff t < \frac{1}{2} \left\{ (A - 2c_i^* + c_j) - \sqrt{(A - 2c_i + c_j)^2 + 9af_i^*} \right\} = t_i^{DP}. 
\]

Similarly, given that firm \( j \) produces in the foreign country, firm \( i \) will undertake FDI if the following holds:

\[
\Delta \Pi_{i}^{F} > 0 \iff t < (c_i - c_i^*) - \frac{9af_i^*}{4(A - c_i - c_i^* + c_j^*)} = t_i^{FP}. 
\]
We can easily verify that $t_1^D < \Delta c_i$ and $t_1^F < \Delta c_i$ when $f_i^* > 0$, while $t_1^D = t_1^F = \Delta c_i$ in the absence of FCs (i.e., $f_i^* = 0$).

Suppose $A = 20$, $a = 2$, $c_1 = 4$, $c_2 = 5$, $c_2^* = 3$, and $f_1^* = f_2^* = 3$. Then,

1. $t_2^F < t_2^P < t_1^P < t_1^F$ ($t_1^P = 2.240, t_1^F = 2.250, t_2^P = 1.094, t_2^F = 0.962$) holds when $c_1^* = 1.00$;

and $t_2^F < t_2^P < t_2^P < t_2^F$ ($t_1^P = 0.290, t_1^F = 0.209, t_2^P = 1.094, t_2^F = 1.097$) when $c_1^* = 2.95$.

2. $t_2^F < t_2^P < t_2^F < t_1^F$ ($t_1^P = 1.740, t_1^F = 1.729, t_2^P = 1.094, t_2^F = 1.000$) when $c_1^* = 1.50$; and $t_2^F < t_2^P < t_2^P < t_2^F$ ($t_1^P = 0.740, t_1^F = 0.682, t_2^P = 1.094, t_2^F = 1.069$) when $c_1^* = 2.50$.

3. $t_2^F < t_2^P < t_2^P < t_1^F$ ($t_1^P = 1.120, t_1^F = 1.080, t_2^P = 1.094, t_2^F = 1.044$) when $c_1^* = 2.12$; and $t_2^F < t_2^P < t_2^P < t_2^F$ ($t_1^P = 1.060, t_1^F = 1.017, t_2^P = 1.094, t_2^F = 1.048$) when $c_1^* = 2.18$.

4. $t_2^F < t_2^P < t_2^P < t_1^F$ ($t_1^P = 1.090, t_1^F = 1.049, t_2^P = 1.094, t_2^F = 1.046$) when $c_1^* = 2.15$

**Appendix B**

**Proof of Proposition 2.** We show $1/2 < \sigma_i^* \equiv (-\sqrt{-10\epsilon + 2\epsilon^2 + 9} + 3)/2\epsilon < 1$ when $\epsilon < 1$ and $\epsilon \neq 0$. Defining $f(\sigma_i) \equiv 2\epsilon\sigma_i^2 - 6\sigma_i + 5 - \epsilon$, we have

$$f(\sigma_i) = 2\epsilon(\sigma_i - \frac{3}{2\epsilon}) + (5 - \epsilon - \frac{9}{2\epsilon}).$$

By noting $f(\sigma_i)$ is a quadratic function, $f(\sigma_i)$ reaches its minimum of $5 - \epsilon - 9/2\epsilon$ at $3/2\epsilon$ if $\epsilon$ is positive and its maximum $5 - \epsilon - 9/2\epsilon$ at $3/2\epsilon$ if $\epsilon$ is negative. Since $3/2\epsilon > 1$ when $0 < \epsilon < 1$ and $3/2\epsilon < 0$ when $\epsilon < 0$, $f(\sigma_i)$ is monotonically decreasing for $\sigma_i \in [0,1]$. Since $f(1/2) = 2 - \epsilon/2 > 0$ and $f(1) = \epsilon - 1 < 0$, we obtain $1/2 < \sigma_i^* < 1$. ■

**Proof of Lemma 2.** First, we prove $\Delta c_j < t_1^P < t_1^F$. In Figures 2 and 3, $t_1^D < t_1^F$ at some $f_i^*(>0)$ implies that $\Delta \Pi^F = 0$ is located to the right of $\Delta \Pi^D = 0$, which holds if and only if $\Delta c_j < t < \Delta c_i$. Thus, $\Delta c_j < t_1^P < t_1^F$ must be the case when $t_1^D = t_1^F$. Next, we prove $t_j^F < t_j^P < t_j^F$. Since max{$t_1^D, t_1^F$} < $\Delta c_j$, either $t_j^F < t_1^D$ or $t_j^F < t_1^P$ holds. Suppose $t_1^P < t_1^F$. Then, in view of the first part of the proof, $\Delta c_i < t_1^P < t_1^F$ is necessary. This is contradiction, because $\Delta c_j < t_1^P < t_1^F < \Delta c_i$. Thus, $t_j^F < t_j^P < \Delta c_j$. ■

**Proof of Lemma 3.** When $t_1^P < t_1^F$ holds, it is obvious that $t_2^F < t_2^F$ from Lemma 2. Thus, it is sufficient to consider only the case where $t_1^P > t_1^F$ holds. First, we suppose a combination of FCs ($f_1^*$ and $f_2^*$) under which $t_2^P = t_2^P$ holds. Because $t_1^P < \Delta c_i$ ($i = 1, 2$), $t_1^P = t_2^P < \min\{\Delta c_1, \Delta c_2\}$ holds. Using (15), we find $\Delta \Pi^P = \Delta \Pi^F = \Delta \Pi^P - \Delta \Pi^F > 0$ when $t = t_1^P = t_1^P$. Because $\Delta \Pi^P = \Delta \Pi^P = 0$ at $t_1^P (t_1^P)$, we find $\Delta \Pi^P = \Delta \Pi^P < 0$. Differentiating $\Delta \Pi^F$ (A1) and $\Delta \Pi^F$ (A2) with respect to $t$ and rearranging those, we can obtain

$$\left|\frac{d\Delta \Pi^F}{dt}\right| = \left|\frac{d\Delta \Pi^P}{dt}\right| + \frac{4}{9\alpha} \{2(c_2^* - c_1^*) + (c_2 - c_1)\}. \quad \text{(A7)}$$

13
Since $c_1 < c_2$ and $c_1^* < c_2^*$, (A7) means that the absolute value of the slope of $\Delta \Pi_i^F$ is greater than that of $\Delta \Pi_j^F$. This implies that the required additional reduction of $t$ for $\Delta \Pi_i^F = 0$ is smaller than that for $\Delta \Pi_j^F = 0$. Thus, $t_i^F < t_j^F$ holds. Next we suppose a combination of FCs under which $t_2^F < t_1^F$ holds. Because $t_2^D < t_i^F$ and $t_1^D < \Delta c_i$ ($i = 1, 2$), we have three possible orders, $t_2^D < t_1^D < \Delta c_1 < \Delta c_2$, $t_2^D < t_1^D < \Delta c_2 < \Delta c_1$ and $t_2^D < \Delta c_2 < t_1^D < \Delta c_1$. In view of Figure 2, the third order implies $t_1^D < t_2^F$ and hence $t_2^F < t_1^F$ from Lemma 2. Thus, it is sufficient to consider the first two orders. These orders imply $t_2^D < t_1^D < \min\{\Delta c_1, \Delta c_2\}$. Then, $\Delta \Pi_i^D - \Delta \Pi_j^D$ (or $-\Delta \Pi_i^F$) at $t_1^D$ is smaller than $\Delta \Pi_2^D - \Delta \Pi_2^F$ (or $-\Delta \Pi_2^F$) at $t_2^D$. Thus, we can also prove $t_1^F < t_2^F$ as in the case of $t_2^D = t_1^D$.

Proof of Proposition 4. When the effective marginal costs of two firms are $m_i$ and $m_j$ ($i, j = 1, 2$ and $i \neq j$), consumer surplus is

$$CS = \frac{a}{2} \left( \frac{2A - m_i - m_j}{3a} \right)^2.$$  

Firm $i$’s FDI makes its effective marginal cost lower. The effect of a change in $m_i$ on consumer surplus is

$$\frac{\partial CS}{\partial m_i} = -\left( \frac{2A - m_i - m_j}{9a} \right) < 0$$

whose sign is always negative (as long as Cournot interior solutions exist). Since the profits of firm $j$ are

$$\Pi_j = a \left( \frac{A + m_i - 2m_j}{3a} \right)^2,$$

the effect of a change in $m_i$ on the profits is given by

$$\frac{\partial \Pi_j}{\partial m_i} = \frac{2}{3} \left( \frac{A + m_i - 2m_j}{3a} \right) > 0.$$

By noting that firm $i$’s profits are continuous at the critical levels of $t$: $t_i^D$ and $t_i^F$, the change of domestic welfare is given by

$$\frac{\partial CS}{\partial m_i} + \frac{\partial \Pi_j}{\partial m_i} = \frac{m_i - m_j}{3a}.$$

Thus, the following condition holds:

$$m_i > m_j \Leftrightarrow \frac{\partial CS}{\partial m_i} + \frac{\partial \Pi_j}{\partial m_i} > 0.$$

This means that FDI undertaken by the firm with lower effective marginal cost always improves domestic welfare. Taking this finding into account, we investigate the following four cases where only one firm changes its location: from $DD$ to $FD$, from $DD$ to $DF$, from $FD$ to $FF$, and from $DF$ to $FF$. In the first and second cases, since the effective marginal costs at $DD$ are $c_1$ and $c_2$ and $c_1 < c_2$. Thus, domestic welfare rises in the first case and falls in the second case. In the third case, the effective marginal costs are $c_1^*$ and $c_2$ at $FD$. Suppose $c_1^* + t \geq c_2$. Then $c_1^* + t > c_1$ because $c_1 < c_2$. Obviously, firm 1 will not undertake FDI at this trade cost. Thus, $c_1^* + t < c_2$ and domestic welfare deteriorates. In the fourth case, the effective marginal costs are
Suppose \( c_1 \geq c_2 + t \). Then \( c_1 > c_2 + t \) because \( c_1 > c_2 \). Obviously, firm 1 will not produce at home at this trade cost. Thus, \( c_1 < c_2 + t \) and domestic welfare improves.

**Proof of Proposition 5.** The complete reversal arises in Cases III and IV. We have two possible cases: from \( DF \) to \( FD \), and from \( FD \) to \( DF \). The difference of consumer surplus between two regimes is,

\[
CS^{FD} - CS^{DF} = \frac{a}{2} \left( \frac{2A - c_1^* - t - c_2}{3a} \right)^2 - \frac{a}{2} \left( \frac{2A - c_1 - c_2^* - t}{3a} \right)^2
\]

\[
= \frac{1}{18a} (c_1 - c_1^* - c_2 + c_2^*) \{2A - c_1^* - t - c_2 + (2A - c_1 - c_2^* - t) \}.
\]

Therefore,

\[
c_1 - c_1^* > c_2 - c_2^* \iff c_2 - c_1^* > c_2 - c_1 \iff CS^{FD} > CS^{DF}.
\]

This implies that consumer surplus is larger when the firm which can save the real marginal cost more by FDI switches its production from the domestic country to the foreign country than when the other firm does.

The difference of firm \( i \)'s profits (\( i = 1, 2 \)) is,

\[
\Pi_i^{FD} - \Pi_i^{DF} = a \left( \frac{A - 2c_2 - 2t + c_2}{3a} \right)^2 - a \left( \frac{A - 2c_1 + c_2 + t}{3a} \right)^2 - f_i^1,
\]

\[
\Pi_2^{FD} - \Pi_2^{DF} = a \left( \frac{A - 2c_2 + c_1^* + t}{3a} \right)^2 - a \left( \frac{A - 2c_2 + 2t + c_1^*}{3a} \right)^2 + f_2^*.
\]

When \( t_2^f < t_2^f < t_2^d < f_2^d \), \( \Pi_1^{FD} > \Pi_1^{DF} \) holds at \( t \in [t_1^f, t_2^d] \). Because \( \Delta c_1 > t_1^d \), the rival's FDI lowers the firm 1's profits (\( \Pi_1^{DD} > \Pi_1^{DF} \)). Thus, \( \Pi_1^{FD} > \Pi_1^{DD} > \Pi_1^{DF} \). Similarly, using \( \Pi_2^{FD} > \Pi_2^{DD} \) and \( \Delta c_1 > t_1^D \), we have \( \Pi_2^{DF} > \Pi_2^{DD} > \Pi_2^{FD} \). Thus, \( \Pi_1^{FD} > \Pi_1^{DF} \) and \( \Pi_2^{DF} > \Pi_2^{FD} \). This finding is also applicable when \( t_2^f < t_2^f < f_1^D < t_2^D \). When \( t_2^f < t_2^f < t_2^D < t_2^D \) (i.e., in Case IV), \( \Pi_1^{FD} > \Pi_1^{DF} \) holds at \( t \in [t_1^f, t_2^D] \). Using \( \Delta c_2 > t_2^D \), \( \Pi_2^{FD} > \Pi_2^{DD} > \Pi_2^{DF} \). Similarly, using \( \Pi_2^{DF} > \Pi_2^{DD} \) and \( \Delta c_1 > t_1^D \), we have \( \Pi_2^{DF} > \Pi_2^{DD} > \Pi_2^{FD} \). Thus, again \( \Pi_1^{FD} > \Pi_1^{DF} \) and \( \Pi_2^{DF} > \Pi_2^{FD} \) hold. This means that the profits of firm \( i \) are larger when it produces abroad and the rival produces at home than vice versa.

Moreover, we have

\[
\Pi^{FD} - \Pi^{DF} = (\Pi_1^{FD} + \Pi_2^{FD}) - (\Pi_1^{DF} + \Pi_2^{DF})
\]

\[
= \{(A - 2c_1 + c_2) + (A - 2c_1^* + c_2^* - t)\} \{2(c_1 - c_1^*) + (c_2 - c_2^*) - 3t\} \}/9a
\]

\[
- \{(A + c_1 - 2c_2) + (A + c_1^* - 2c_2^* - t)\} \{c_1 - c_1^* + 2(c_2 - c_2^*) - 3t\} \}/9a.
\]

Since \( A - 2c_1 + c_2 > A + c_1 - 2c_2 \) and \( A - 2c_1^* + c_2^* - t > A + c_1^* - 2c_2^* - t \), \( \Pi^{FD} - \Pi^{DF} > 0 \) if the following holds:

\[
2(c_1 - c_1^*) + (c_2 - c_2^*) - 3t > (c_1 - c_1^*) + 2(c_2 - c_2^*) - 3t \iff c_1 - c_1^* > c_2 - c_2^*.
\]

Noting (A8), we can claim that \( W^{FD} > W^{DF} \) if \( c_1 - c_1^* > c_2 - c_2^* \).
Finally, we can verify that the sign of the following equation is generally ambiguous if $c_1 - c_1^* < c_2 - c_2^*$:

$$W^{FD} - W^{DF} = \{8Ac_1 - 11c_1^2 - 8Ac_1^* + 11c_1^2 - 8Ac_2 + 11c_2^2 + 8Ac_2^* - 11c_2^2$$

$$- 14c_1^2c_2 + 14c_1c_2^* + 14c_1t + 22c_1^*t - 14c_2t - 22c_2^*t\}/18a.$$

For example, if $A = 20$, $a = 2$, $c_1 = 4$, $c_1^* = 2.915$, $c_2 = 4.1$, $c_2^* = 3$, $f_1^* = f_2^* = 3$, then $\Pi^{FD} > \Pi^{DF}$, $CS^{FD} < CS^{DF}$, and $W^{FD} > W^{DF}$. If $A = 20$, $a = 2$, $c_1 = 4$, $c_1^* = 2.93$, $c_2 = 4.1$, $c_2^* = 3$, $f_1^* = f_2^* = 3$, then $\Pi^{FD} > \Pi^{DF}$, $CS^{FD} < CS^{DF}$, and $W^{FD} < W^{DF}$. If $A = 20$, $a = 2$, $c_1 = 4$, $c_1^* = 2.95$, $c_2 = 4.1$, $c_2^* = 3$, $f_1^* = f_2^* = 3$, then $\Pi^{FD} < \Pi^{DF}$, $CS^{FD} < CS^{DF}$, and $W^{FD} < W^{DF}$. Thus, when the complete reversal occurs with $c_1 - c_1^* < c_2 - c_2^*$, domestic welfare may deteriorate regardless of which firm makes FDI. ■
References


Figure 1: Domestic Market
Figure 2: $\Delta c_1 > \Delta c_2$
Figure 3: $\Delta c_1 < \Delta c_2$