The Second-Order Dilemma of Public Goods and Capital Accumulation

Akira Okada[1]

October, 2007

Abstract The second-order dilemma arises from each individual's incentive to free ride on a mechanism to solve the public goods provision problem (the first-order dilemma). We show by a voluntary participation game that if the depreciation rate is low, public goods can be accumulated through voluntary groups for provision, and that the accumulation is effective in solving the second-order dilemma. This analysis also shows that population growth increases the accumulation of public goods in the long run.

Keywords Public goods · Second-order dilemma · Voluntary participation · Accumulation · Group formation

JEL Classifications: C72, D7, H41

[1]Graduate School of Economics, Hitotsubashi University, 2-1 Naka, Kunitachi, Tokyo 186-8601, Japan
e-mail: aokada@econ.hit-u.ac.jp

1 Introduction

Since the seminal work of Olson (1965), it has been widely argued that public goods should be undersupplied by voluntary contribution due to free-riding incentives (see e.g. Bergstrom, Blume and Varian 1986; Andreoni 1988). To solve the free-riding problem, a large volume of literature has investigated the mechanisms that can achieve an efficient provision of public goods. The principle theoretical approaches are listed below.

First, the theory of repeated games considers ways in which long-term relationships can facilitate cooperation among selfish individuals. It considers the level of contributions attained under decentralized punishment rules. Second, the theory of mechanism design investigates the use of mechanisms to encourage or force an efficient provision of public goods. Groves and Ledyard (1977) proposed a mechanism that achieves Pareto efficient allocation in a public good economy. For a survey of mechanism design, see Groves and Ledyard (1987). More recently Moore and Repullo (1988) have shown that almost any social choice rule can be implemented by the subgame perfect equilibria of multi-stage mechanisms, in an economic environment with at least one private good. Finally, other works have used cooperative game theory (especially core theory) to study how individuals can reach an efficient solution through voluntary bargaining. For examples of this type of work, see Foley (1970) and Mas-Colell (1980).

Most previous studies of the free-riding problem implicitly assume a restrictive property. It is usually assumed either that all individuals in question have already participated (or been forced to participate) in a mechanism[2], or that individuals are only willing to

[2] We interpret the notion of a mechanism broadly so that it includes repeated game strategies and

participate if they become better off by participating than by maintaining the status quo. The latter condition is called *the participation constraint* in the literature of mechanism design. It should be remarked, however, that *any* mechanism which achieves an efficient provision of public goods is itself a kind of public good. Individuals may thus have an incentive to free ride on the mechanism, which may in turn lead to the failure of the mechanism. This problem is called *the second-order dilemma* of public goods (Oliver 1980 and Ostrom 1990). No external power can force individuals to participate in a mechanism; participation should always be voluntary.

The voluntary participation problem has recently been studied by several authors (Palfrey and Rosenthal 1984; Okada 1993; Saijo and Yamato 1999; Dixit and Olson 2000). These works use a two-stage game model to analyze whether or not individuals will voluntarily participate in a mechanism which implements an efficient provision of public goods.[3] It has been shown that individuals do not necessarily participate in such a mechanism, and that the likelihood of all individuals participating becomes lower as the population increases. These results partially support a widely held negative view in the literature, namely that efficient provision of public goods is not possible due to the second-order dilemma. To solve the second-order dilemma, recent literature has extensively explored a number of social and behavioral factors: norm, community, social sanctions, trust, fairness, reciprocity, and inequity-averse preferences. These devices can significantly change individuals' free-riding incentives.

---

cooperative bargaining games.

[3]Palfrey and Rosenthal (1984) consider a one-stage model of voluntary participation in providing binary public goods. The payoff structure of their model is similar to that obtained in the reduced form of the two-stage games studied by other authors.

In this paper, we reexamine the voluntary participation game from the viewpoint of public goods accumulation. The motivation of our analysis is twofold. First, most studies on voluntary participation have ignored the possibility of accumulation. Many public goods, however, can be accumulated for generations. Some examples include forests, parks, common lands, irrigation systems, public libraries, and museums. The relationship between accumulation and strategic behavior of group formation is subtle and worth investigating. Second, individuals' free-riding incentives will naturally change as public goods accumulate. We thus consider a role of accumulation in the second-order dilemma.

We present a dynamic model of a public good economy with Cobb-Douglas utility functions. A voluntary participation game is played by (non-overlapping) generations. In the first stage, every individual decides independently whether or not to participate in a group. In the second stage, all participants negotiate over the mechanism (for example, the Groves and Ledyard mechanism) that will implement efficient provision within the group. It is assumed that if all participants agree on a mechanism, then it can be effectively created and implemented at a cost to be borne by the participants.[4] Non-participants are allowed to free ride on the mechanism. Depending on the outcome of voluntary participation, public goods may be accumulated and inherited by the next generation. We will prove that if the depreciation rate of public goods is low, then the stock of public goods will monotonically increase owing to consistent group formation. Furthermore, it will continue to increase until group formation is no longer beneficial. In this sense, accumulation is effective in solving the second-order dilemma of public goods. The

[4]The mechanism implementation problem is an important research field in its own right. Since we wish to focus on the second-order dilemma, however, this issue is beyond the scope of the paper.

maximum level of public goods is an increasing function of the population. We will also show that population growth increases the accumulation of public goods in the long run.

To our knowledge, there is little literature on voluntary provision games with accumulation. Fershtman and Nitzan (1991) and Marx and Matthews (2000) have considered dynamic contribution games with infinitely long-lived individuals. They obtain a result similar to the folk theorem of repeated games, describing two polar cases of voluntary contribution. In a linear-quadratic differential game, Fershtman and Nitzan (1991) showed the inefficiency of voluntary contribution by a feedback Nash equilibrium in Markovian strategies. Marx and Matthews (2000) demonstrated that efficient results could be implemented by trigger-type strategies with a long horizon. Pecorino and Temimi (2007) analyze a repeated game of voluntary contributions to rival public goods without accumulation, and show that a small participation cost prevents the largest possible group from being sustained by a grim trigger strategy as group size grows sufficiently. Their result is based on the property of rival public goods that the cooperative payoff is driven towards zero in a large group. In all these studies, the second-order dilemma of public goods is not an issue.

The paper is organized as follows. Section 2 presents a dynamic model of the voluntary provision of public goods. Section 3 analyzes group formation. Section 4 examines the accumulation of public goods. Section 5 discusses implications of the model. All proofs are given in the Appendix.

2 The model

Consider a simple economy with one private good and one public good, which evolves over several generations t (=1, 2,$\cdots$). The public goods are assumed to be non-excludable and non-rival. Let n_t be the number of individuals in generation t where $n \leq n_t \leq N$. In each generation, every individual i is initially endowed with wealth ω and voluntarily contributes an amount x_i ($0 \leq x_i \leq \omega$) of the private goods to the provision of public goods. The public goods are produced using technology that provides a constant return to scale, and the production of public goods is given by $\beta \sum_{i=1}^{n_t} x_i$ where β (> 0) is the marginal productivity of private goods.

The total amount K_t of public goods in generation t is given by

$$K_t = (1 - \delta)K_{t-1} + \beta \sum_{i=1}^{n_t} x_i, \tag{1}$$

where K_{t-1} is the stock of public goods in the last generation $(t-1)$ and δ $(0 < \delta < 1)$ is the depreciation rate of public goods. For clarity of analysis, in this paper we employ a Cobb-Douglas utility function for individuals, $u_i(y_i, K_t) = y_i^{\alpha} K_t^{1-\alpha}, \ \ 0 < \alpha < 1$, where y_i is the private consumption, satisfying $y_i + x_i = \omega$, and α is the preference weight of private goods. Each generation lives for only one period, and does not care about the welfare of future generations. The population change Δn_{t-1} is defined as $n_t - n_{t-1}$.

Given K_{t-1}, the voluntary contribution game of generation t is defined as follows.

Every individual i $(= 1, \cdots, n_t)$ has an action $x_i \in [0, \omega]$ and a payoff function given by

$$u_i(x_1, \cdots, x_n, K_{t-1}) = (\omega - x_i)^\alpha((1-\delta)K_{t-1} + \beta \sum_{i=1}^{n_t} x_i)^{1-\alpha}.^5 \tag{2}$$

The first proposition characterizes both a Nash equilibrium and a collectively rational (Pareto efficient) outcome of the game.

Proposition 2.1. A Nash equilibrium $x^{NE} = (x_i^{NE})_{i=1}^{n_t}$ of the voluntary contribution game at generation t exists uniquely, and is characterized as follows.

(1) If $0 \leq K_{t-1} < \frac{1-\alpha}{\alpha}\frac{\beta\omega}{1-\delta}$, then $x_i^{NE} = \frac{(1-\alpha)\beta\omega - \alpha(1-\delta)K_{t-1}}{(1+n_t\alpha-\alpha)\beta}$.

(2) If $\frac{1-\alpha}{\alpha}\frac{\beta\omega}{1-\delta} \leq K_{t-1}$, then $x_i^{NE} = 0$.

A symmetric Pareto efficient contribution $x^{PE} = (x_i^{PE})_{i=1}^{n_t}$ of the game is given as follows.

(1) If $0 \leq K_{t-1} < \frac{1-\alpha}{\alpha}\frac{\beta\omega}{1-\delta}n_t$, then $x_i^{PE} = (1-\alpha)\omega - \frac{\alpha(1-\delta)K_{t-1}}{\beta n_t}$.

(2) If $\frac{1-\alpha}{\alpha}\frac{\beta\omega}{1-\delta}n_t \leq K_{t-1}$, then $x_i^{PE} = 0$.

When the depreciated stock $(1-\delta)K_{t-1}$ of public goods is low, all individuals voluntarily contribute some positive amount of their private goods. Otherwise, the zero contribution is the dominant action for each individual. A Nash equilibrium does not in general lead to the collectively rational outcome. The proposition reveals the following three phases in the voluntary contribution game, depending on the stock K_{t-1} of public goods; (i) $0 \leq K_{t-1} < \frac{1-\alpha}{\alpha}\frac{\beta\omega}{1-\delta}$: the Nash equilibrium with positive contributions is not

[5] To save symbols we use the notation u_i to denote several different types of payoffs, whenever the type is clear from the context.

Pareto efficient, (ii) $\frac{1-\alpha}{\alpha}\frac{\beta\omega}{1-\delta} \leq K_{t-1} < \frac{1-\alpha}{\alpha}\frac{\beta\omega}{1-\delta}n_t$: the zero contribution is the dominant action of every individual, which leads to an inefficient outcome (a multi-person prisoners' dilemma), (iii) $\frac{1-\alpha}{\alpha}\frac{\beta\omega}{1-\delta}n_t \leq K_{t-1}$: the Nash equilibrium with zero contributions is Pareto efficient. In phase (i), the accumulation of public goods is possible through voluntary contribution under the dynamical system (1). The following proposition shows to what level public goods can be accumulated through voluntary contribution.

Proposition 2.2. If the initial stock K_0 of public goods satisfies $0 \leq K_0 < \frac{1-\alpha}{\alpha}\frac{\beta\omega}{1-\delta}$, the sequence of public goods stocks K_t $(t = 1, 2, \cdots)$ converges to the stationary point $K^* = \frac{(1-\alpha)\beta n_t w}{n_t\alpha+\delta(1-\alpha)}$. The stationary point K^* is smaller than $\frac{1-\alpha}{\alpha}\frac{\beta\omega}{1-\delta}$.

The proposition shows that, although the accumulation of public goods is possible through voluntary contribution when the initial stock of public goods is very low, the accumulation is only in a small-scale. It can not go beyond the boundary $\frac{1-\alpha}{\alpha}\frac{\beta\omega}{1-\delta}$ between phases (i) and (ii). Without any suitable mechanism to induce more contributions, the public goods can not be accumulated beyond this boundary. The next section considers how the mechanism design approach can effectively solve the first-order (and the second-order) dilemma of public goods when participation is voluntary.

3 The group formation

We consider a situation where individuals voluntarily form a group to provide public goods. We assume that if such a group is formed, its members can establish a suitable mechanism to implement the collective provision of public goods. The mechanism itself certainly incurs additional costs: communication, negotiations, monitoring, punishment, staffing, and maintaining the group organization. In what follows, all these costs are simply referred to as *group costs*. The participants bear the group costs equally. The focus of our analysis is on whether or not individuals will voluntarily participate in such a costly group to enforce an efficient provision of public goods, and (if they do) on how many individuals participate. All non-participants are free from any enforcement by the group; they are allowed to free ride on the group contributions without consequence.

To consider the problem of voluntary participation in a pure form, we present a two-stage game of group formation in each generation t $(= 1, 2, \cdots)$.

(1) Participation decision stage:

Given the stock K_{t-1} of public goods, all individuals of generation t decide independently whether to participate in a group or not. Let s be the number of participants in the group. If $s = 0$ or 1, then no group forms.

(2) Group negotiation stage:

All participants decide independently whether or not they should agree to establish a mechanism to enforce the optimal provision of public goods by the group. The mechanism is established if and only if all participants agree to it (unanimous rule). A group is formed if the mechanism is established, every participant bearing an equal share of the group cost. The group cost per member is denoted by $c(s)$ (> 0), where s is the group size. All non-participants can choose their contributions freely. If a group is not formed, then the voluntary contribution game described in section 2 is played. As individuals make their choice in the group negotiation stage, it is assumed that all have perfect knowledge of the participation decision outcome.

In this two-stage process, individuals decide whether or not to participate in a group by rationally anticipating the outcome of the group negotiation stage. We will characterize a subgame perfect equilibrium of the game of group formation. In this paper we consider only pure strategy equilibria.[6]

We first analyze the group-optimal level of contributions that will maximize the sum of all members' payoffs. Suppose that a group S of individuals forms. Let x_p^S be the common contribution of every participant in S, and let x_{np} be the contribution of every non-participant. A pair (x_p^S, x_{np}) is a Nash equilibrium of the voluntary contribution game with group S if x_p^S maximizes the sum of all participants' payoffs, given all non-participants' contributions x_{np}, and if x_{np} maximizes every non-participant's payoff, given all other individuals' contributions. The next lemma characterizes a Nash equilibrium of the voluntary contribution game with group S.

Lemma 3.1. Let $s(\geq 2)$ be the number of participants in a group S. The group-optimal contribution x_p^S of every participant and the non-participant's contribution x_{np} are characterized as follows.

(1) If $0 \leq K_{t-1} < \frac{1-\alpha s}{\alpha}\frac{\beta\omega}{1-\delta}$, then $x_p^S = \frac{\{(1-\alpha)s+(s-1)(n_t-s)\alpha\}\beta\omega-\alpha(1-\delta)K_{t-1}}{(1+(n_t-s)\alpha)\beta s}$, and $x_{np} = \frac{(1-\alpha s)\beta\omega-\alpha(1-\delta)K_{t-1}}{(1+(n_t-s)\alpha)\beta}$.

(2) If $\frac{1-\alpha s}{\alpha}\frac{\beta\omega}{1-\delta} \leq K_{t-1} < \frac{1-\alpha}{\alpha}\frac{\beta\omega}{1-\delta}s$, then $x_p^S = (1-\alpha)\omega - \frac{\alpha(1-\delta)K_{t-1}}{\beta s}$, and $x_{np} = 0$.

[6] Okada (1993) characterizes a mixed strategy equilibrium in the context of an n-person prisoners' dilemma.

(3) If $\frac{1-\alpha}{\alpha}\frac{\beta\omega}{1-\delta}s \leq K_{t-1}$, then $x_p^S = x_{np} = 0$.

The lemma shows that phase (i) ($0 \leq K_{t-1} < \frac{1-\alpha}{\alpha}\frac{\beta\omega}{1-\delta}$) characterized by Proposition 2.1 is further divided into two sub-phases, depending on whether or not non-participants contribute positive amounts of their private goods to the provision of public goods. The stationary point K^* of the dynamical system (1) given by Proposition 2.2 is larger than $\frac{1-2\alpha}{\alpha}\frac{\beta\omega}{1-\delta}$ when the depreciation rate δ is sufficiently low.[7] Thus, considering the accumulation of public goods through voluntary contributions by individuals, we can ignore the first case of the lemma without loss of generality. In what follows, we will assume that all non-participants make the zero contributions when any group forms.

When the group-optimal contributions are positive (the second case in Lemma 3.1), the payoff of each participant, denoted by $u_p(s, K_{t-1})$, is given by

$$u_p(s, K_{t-1}) = \frac{\alpha^\alpha(1-\alpha)^{1-\alpha}}{(\beta s)^\alpha}[\beta s\omega + (1-\delta)K_{t-1}]. \tag{3}$$

The payoff of non-participants, denoted by $u_{np}(s, K_{t-1})$, is given by

$$u_{np}(s, K_{t-1}) = \omega^\alpha(1-\alpha)^{1-\alpha}[\beta s\omega + (1-\delta)K_{t-1}]^{1-\alpha}. \tag{4}$$

When a group fails, all individuals receive the Nash equilibrium payoffs of the voluntary contribution game. For simplicity of exposition, we will focus our analysis on the case

[7] The condition is $0 < \delta < \frac{\alpha}{1-\alpha}\frac{\alpha n_t}{\alpha n_t - 2\alpha + 1}$. We remark that this condition holds for any $\delta < 1$ when $0 < \alpha \leq 1/2$.

that $K_{t-1} \geq \frac{1-\alpha}{\alpha}\frac{\beta\omega}{1-\delta}$. From Proposition 2.1 the Nash equilibrium payoff is given as

$$u(0, K_{t-1}) = \omega^{\alpha}[(1-\delta)K_{t-1}]^{1-\alpha}. \tag{5}$$

The same analysis can be applied to the case of $K_{t-1} < \frac{1-\alpha}{\alpha}\frac{\beta\omega}{1-\delta}$.

Lemma 3.2. Let $s(\geq 2)$ be the number of participants. The payoffs u_p and u_{np}, for participants and non-participants respectively, satisfy:

(1) $u_p(s, K_{t-1}) \geq u(0, K_{t-1})$,

(2) $u_p(s, K_{t-1})$ and $u_{np}(s, K_{t-1})$ are monotonically increasing in s,

(3) $\frac{u_{np}(s-1, K_{t-1})}{u_p(s, K_{t-1})}$ is monotonically increasing in s.

All participants can guarantee their payoffs in the case of zero contribution. The payoffs for both participants and non-participants increase, however, as the group becomes larger. Furthermore, the ratio of a non-participant's payoff to the payoff he would obtain by joining a group increases as the group size increases. This last property implies that if there is any incentive to not join a particular group, this incentive will also be present for any larger group.

We next analyze the group negotiation stage. When a group S implements the group-optimal contribution, the net payoff of every participant is given by $u_p(s, K_{t-1}) - c(s)$. We assume that the group cost function $c(s)$ has the following property:

Assumption 3.1. The net payoff $u_p(s, K_{t-1}) - c(s)$ of each participant is a monotonically increasing function of the group size s. Furthermore, there exists some integer $s \leq n_t$ such that the net payoff is larger than the payoff $u(0, K_{t-1})$.

The participant's (gross) payoff $u_p(s, , K_{t-1})$ increases monotonically with the group size s (Lemma 3.2). This assumption implies that the participant's net payoff has the same property. If this last assumption does not hold, then no size of group is more beneficial than the non-cooperative outcome and the problem of group formation becomes vacuous.

Definition 3.1. Given K_{t-1}, the *group size threshold* is defined as the (unique) solution s of the equation $u_p(s, K_{t-1}) - c(s) = u(0, K_{t-1})$, and is denoted by $s(K_{t-1})$.[8]

The group size threshold will play a critical role in our analysis of group formation. Assumption 3.1 guarantees its existence.[9] The next lemma shows that the group size must be larger than the threshold in order for participants to agree to form the group.

Lemma 3.3. In the group negotiation stage, unanimous agreement to form a group is reached in a strict Nash equilibrium[10] if and only if the group size s is larger than the threshold $s(K_{t-1})$.

Since the proof of the lemma is straightforward, we omit it. We remark that there are

[8]In the following analysis, we will treat the group size variable s as a real number rather than a natural number for technical convenience. This treatment does not affect our results in any crucial way.

[9]For clarity of analysis, we ignore degenerate cases where the threshold happens to be an integer.

[10]A Nash equilibrium is called *strict* if every player has a unique best response. In a non-strict Nash equilibrium, individuals may be indifferent to the actions of agreeing and disagreeing to form a group.

also many non-strict Nash equilibria, including all situations where at least two individuals do not agree to form a group. We exclude such non-strict equilibria from our analysis. An immediate implication of the lemma is that no group will form if the population of generation t is smaller than the threshold. This fact suggests that population growth is an important consideration in the accumulation of public goods, a point which will be examined in the next section.

The following lemma shows how the group size threshold depends on the stock of public goods and the group costs.

Lemma 3.4. The threshold $s(K_{t-1})$ of group size is a monotonically increasing function of K_{t-1}. If the group cost function $c(s)$ shifts upward, then the threshold $s(K_{t-1})$ also increases for any value of K_{t-1}.

An intuition for the lemma is as follows. Accumulation may increase the opportunity costs of groups since individuals enjoy a high stock of public goods without any group. To cover the opportunity costs, the group size threshold increases along with the accumulation.[11]

Finally, we analyze the participation decision stage. Each individual is faced with two conflicting incentives. First of all, there is an incentive to free ride on the contri-

[11] Dorsey (1992) reports experimental results of real time voluntary contribution games in which the total contribution level is posted and subjects may adjust their contributions prior to making a final commitment. It is observed that the final contribution increases when adjustments are limited to increases. Since subjects' contributions are costless signaling in the adjustment process, the observed behavior seems to be understood as a signaling effect. I am grateful to the editor in chief for pointing out Dorsey's work to me.

butions of others and thus not participate in a group. But if all individuals' actions are governed by this incentive, then no group will be formed and no contributions will be made to public goods. Every individual thus has a second incentive: to participate in a group, *provided that a sufficient number of other individuals also participate.* Due to these different incentives, the individual's participation decision is not trivial.

There always exists a (non-strict) Nash equilibrium in which no individuals participate and no group is formed. We are interested only in the Nash equilibrium that forms a group, and call this situation a *group equilibrium.* The following theorem characterizes the group equilibrium of the participation decision stage.

Theorem 3.1. A group equilibrium of the participation decision stage is characterized as follows.

(1) There exists a group equilibrium if and only if $s(K_{t-1}) < n_t$. That is, the number s^* of individuals in the group must satisfy $s(K_{t-1}) < s^* \leq n_t$.

(2) The largest possible group, with n_t participants, is a group equilibrium if and only if either (a) $n_t = [s(K_{t-1})]$[12] or (b) $u_p(n_t, K_{t-1}) - c(n_t) \geq u_{np}(n_t - 1, K_{t-1})$.

(3) The number of participants in a group equilibrium is uniquely determined (except in degenerate cases) if $\frac{dc(s)}{ads} \geq \frac{\partial u_p(s,K_{t-1})}{\partial s} \frac{c(s)}{u_p(s,K_{t-1})}$.[13]

Theorem 3.1 shows that if the population n_t is larger than the threshold $s(K_{t-1})$, a group may be formed voluntarily by self-interested individuals. One may wonder why self-

[12] $[x]$ represents the smallest integer which is larger than a real number x.

[13] It is assumed here that the cost function $c(s)$ is differentiable.

interested individuals would be willing to form a group for the collective provision of public goods in any situation where they have an incentive to free ride on others' contributions. The answer to this question consists of two parts. First, individuals are better off joining a group than they would be if a group doesn't form, provided that the group size is larger than the threshold. Second, when an individual opts out of the equilibrium group the remaining participants have the opportunity to respond. This is possible because group formation in this game is a two-stage process. In practice, the remaining participants decrease their contributions and in the worst case (from the defector's viewpoint) may dissolve the group entirely if the group size drops below the threshold. This counter-response thus always damages the defector. A corollary of the theorem is that when the population is large, the largest group is not supported as a Nash equilibrium for a generic class of the Cobb-Douglas utility functions (see Sac and Amado 1999 in the case of no stock of public goods). The proof is left to readers since it is straightforward.

The first-order and second-order dilemmas of public goods describe different kinds of strategic situations. The first-order dilemma arises in the voluntary contribution game, which is typically formulated as an n-person prisoners' dilemma where zero contribution is the dominant action of every individual. The second-order dilemma, on the other hand, arises in the voluntary participation game where non-participation is not the dominant action.

From the result in this section, we conclude that the voluntary participation game does not solve the second-order dilemma of public goods, in the sense that all individuals do not contribute to the provision of public goods. On the other hand, one may argue that our model is one of possible participation games, and that a different game *could* solve the second-order dilemma. It, however, should be remarked that, if anyone could design a game which forces all individuals to participate in a group to provide public goods, then the game itself would be a public good. Every individual thus has an incentive to free ride, and we are again faced with the second-order dilemma.

In the next section, we will examine how much generations of individuals accumulate the public goods through voluntary groups, and to what extent the *accumulation* itself can serve as a device to solve the second-order dilemma.

4 The accumulation of public goods

We now consider the effect of allowing public goods to accumulate when the group formation game is played repeatedly by different generations. Although each generation lives only for one period and does not care about future generations, the public goods they provide are inherited by the next generation. In this way, the action of one generation affects the welfare of the future generations.

Theorem 3.1 shows that individuals may form a group to provide public goods if the population n_t is larger than the group size threshold $s(K_{t-1})$. When a group forms, the total contribution of individuals is given by

$$\sum_{i=1}^{n_t} a_i = (1-\alpha)\omega s^*(K_{t-1}) - \frac{\alpha(1-\delta)}{\beta} K_{t-1}, \tag{6}$$

where $s^*(K_{t-1})$ is an equilibrium group size (see Lemma 3.1). When there are multiple equilibria in the group formation game, we select a particular equilibrium. Our result is not affected by the equilibrium selection in any critical way. Substituting (6) into (1), the public good stock K_t obeys the dynamic equation

$$\begin{aligned} K_t &= (1-\alpha)((1-\delta)K_{t-1} + \beta\omega s^*(K_{t-1})) \quad \text{if} \quad s(K_{t-1}) < n_t \\ &= (1-\delta)K_{t-1} \qquad\qquad\qquad\qquad\qquad\quad \text{otherwise.} \end{aligned} \tag{7}$$

Since public goods are depreciated in this model, the formation of a group may not always increase the stock of public goods.

Lemma 4.1. When $s(K_{t-1}) < n_t$ in (7), $K_{t-1} < K_t$ if and only if the equilibrium group size $s^*(K_{t-1})$ is larger than

$$a(K_{t-1}) \equiv \frac{1-(1-\alpha)(1-\delta)}{(1-\alpha)\beta\omega} K_{t-1}. \tag{8}$$

The lemma shows that the public goods produced by generation t outweigh the depreciation of the inherited stock if the group size is larger than the critical level $a(K_{t-1})$. For this reason, we call $a(K_{t-1})$ the *critical level for accumulation.*

Figure 4.1 illustrates the transition dynamics described by (7). In Figure 4.1, the transition curve l_t for generation t describes how the stock of public goods increases when a group is formed. If the group size increases in the next generation $(t+1)$, the transition curve shifts upward from l_t to l_{t+1}. l_0 is the transition curve in which no group is formed. Suppose that generation t inherits a stock K_{t-1} of public goods from the previous generation (point K_{t-1}). If the population n_t is larger than the threshold $s(K_{t-1})$, then a group with $s^*(K_{t-1})$ participants will form. The new stock K_t of public goods is then determined by the transition curve l_t as shown by the arrows; the stock of public goods will increase from K_{t-1} to K_t if the group size is larger than the critical level for accumulation $a(K_{t-1})$. The condition for accumulation is satisfied if and only if K_{t-1} lies below the intersection of the 45-degree line and the transition curve l_t. Thus, applying the same mechanism to the next generation $(t+1)$ results in another increase of public goods. If no group forms, the stock of public goods decreases due to depreciation according to the transition curve l_0.

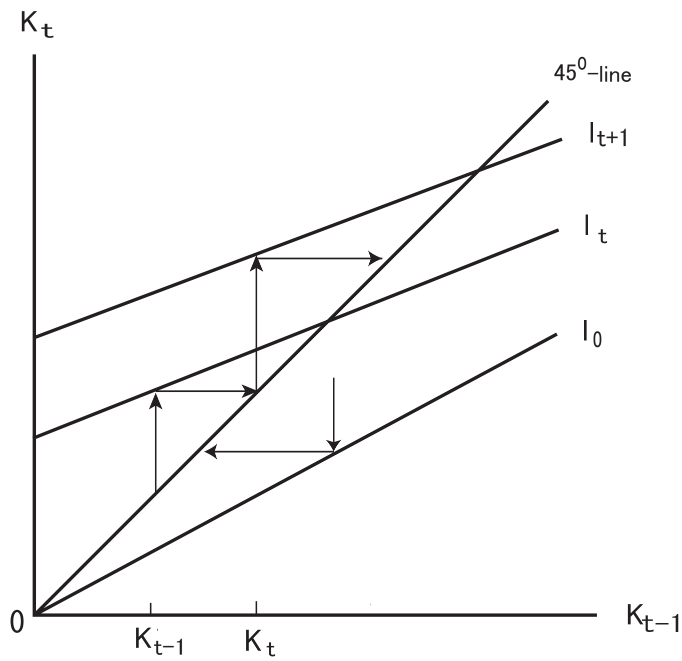

Figure 4.1 transitive dynamics of accumulation

The above arguments show that the dynamics of public goods accumulation depend on two variables: the stock K_{t-1} and the population n_t. We may thus treat the pair (K_{t-1}, n_t)

as a state variable of the economy, and investigate the dynamics of public goods in the (K_{t-1}, n_t) plane (Figure 4.2). Specifically, we can define the region of interest

$$F = \{(K_{t-1}, n_t) \in R_+ \times [n, N] \mid \frac{1-2\alpha}{\alpha}\frac{\beta\omega}{1-\delta} \leq K_{t-1} \text{ and } s(K_{t-1}) \leq n_t\}.$$

In Figure 4.2, the curve marked s illustrates the group size threshold. The region F is bounded by the quadrangle $ABCD$. The line CD indicates $K_{t-1} = \frac{1-2\alpha}{\alpha}\frac{\beta\omega}{1-\delta}$. In section 3 (after Lemma 3.1), we have shown that the stock of public goods exceeds the line CD through voluntary contributions by individuals when the depreciation rate δ is low. It follows from Lemma 3.1 and Theorem 3.1 that inside the region F groups will form. If n_t falls below the s-curve ($n_t < s(K_{t-1})$), then (K_{t-1}, n_t) is outside of region F, and no group is formed. For this reason, we refer to F as the *feasible region* of groups.

As Lemma 4.1 shows, the stock K_{t-1} increases if and only if the equilibrium group size $s^*(K_{t-1})$ is larger than the critical level $a(K_{t-1})$. Since $s^*(K_{t-1})$ is larger than the group size threshold $s(K_{t-1})$ by Theorem 3.1.(1), the public goods can increase if the threshold $s(K_{t-1})$ is larger than the critical level $a(K_{t-1})$. The following lemma shows that this is the case if the depreciation rate δ is sufficiently low.

Lemma 4.2. There exists some $\bar{\delta} > 0$ such that $s(K_{t-1}) > a(K_{t-1})$ for all $\delta < \bar{\delta}$ and all K_{t-1}.

We are now ready to investigate the accumulation of public goods derived from group formation. To examine the relationship between accumulation and group formation, we

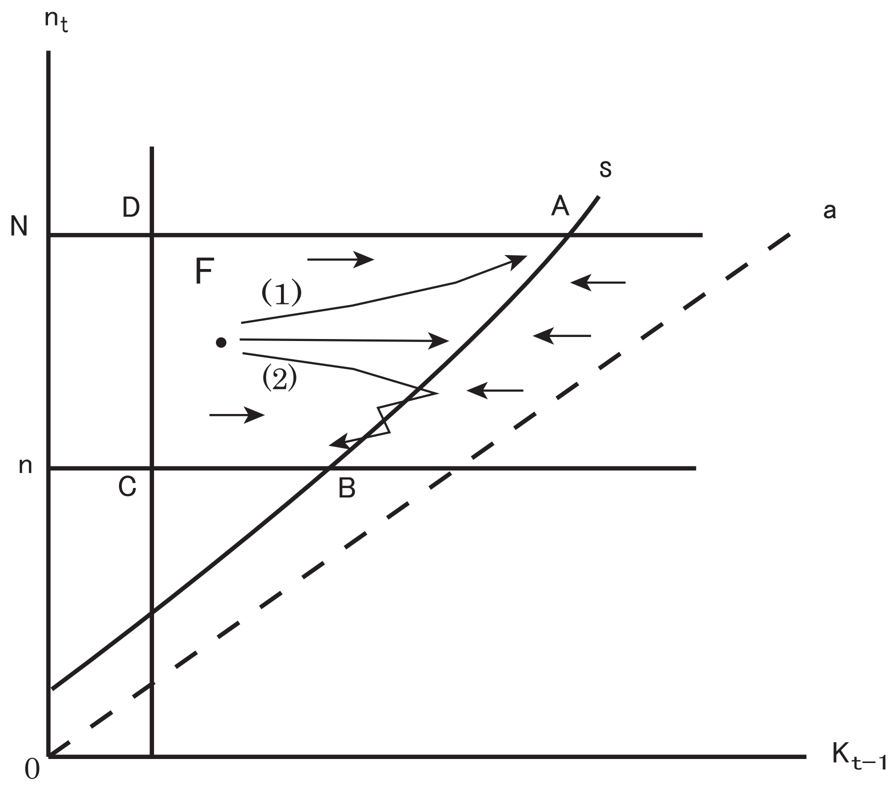

Figure 4.2 Accumulation of public goods

first consider the case of no population change ($\Delta n_t = 0$).

Theorem 4.1. Suppose that $n_t = n_1$ for all $t \geq 1$. If $\delta < \bar{\delta}$, then starting from any initial point $(K_0, n_1) \in F$, the sequence of public goods stocks K_t $(t = 1, 2, \cdots)$ generated by (7) increases monotonically. At some generation t, K_t will exceed the maximum level and exit the feasible region F.

The theorem shows that when the depreciation rate of public goods is sufficiently low, the stock of public goods increases monotonically and eventually exceeds the maximum level in the feasible set F of groups, beyond of which the second-order dilemma of public goods disappears since group formation is no longer beneficial. In Figure 4.2, this scenario of public goods accumulation is shown by the long, horizontal arrow. Short arrows illustrate ways in which the public goods stock may evolve over time.

Figure 4.3 offers an intuitive explanation for the theorem. The feasible region F can be divided into several sub-regions I_s by the smallest integers s larger than group size thresholds, as illustrated in Figure 4.3.[14] In the sub-region I_s, the equilibrium group size is greater than or equal to s (Theorem 3.1). Since the group size threshold $s(K_{t-1})$ is larger than the critical level for accumulation $a(K_{t-1})$ everywhere in this region (Lemma 4.2), the stock of public goods keeps increasing. Contrary to the theorem, suppose that state variables (K_{t-1}, n_1) remain in one sub-region I_s. Then, the stock K_{t-1} converges to the level K^* with $s = a(K^*)$ or exceeds it. Since the level K^* is greater than the boundary between I_s and I_{s+1}, this is impossible. Several possible generations move K_{t-1} from the current sub-region I_s to the next one. After a sufficiently large number of generations, K_{t-1} exceeds the maximum level M and exits the feasible region F.

Theorem 4.1 describes the accumulation of public goods only in the case of a fixed population. In the following discussion, we will sketch the accumulation patterns of public goods under various population change models with help of the (K_{t-1}, n_t) phase diagram shown in Figures 4.2 and 4.3.

Case (1): population increase

Suppose that the population n_t monotonically increases over time, eventually reaching

[14]For clarity, the lower and upper bounds on population are omitted.

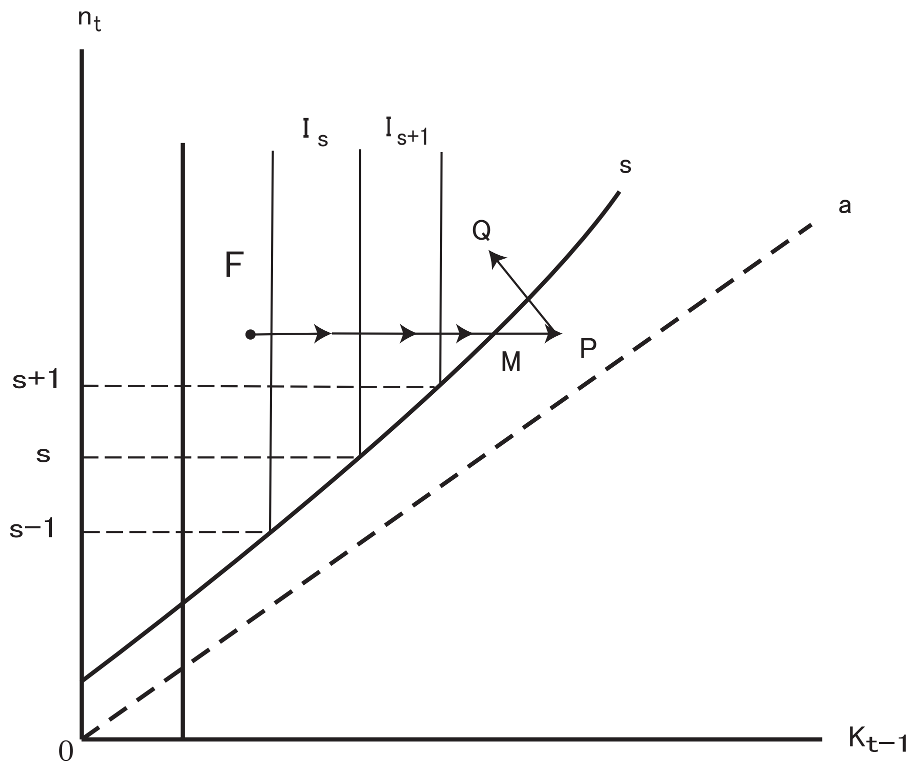

Figure 4.3 Accumulation in the feasible region of groups

some upper limit N. Two patterns of accumulation are possible. The first pattern roughly corresponds to path (1) illustrated in Figure 4.2: the stock of public goods approaches point A, but the pair (K_{t-1}, n_t) never goes outside the feasible region F. In the second pattern, however, the pair (K_{t-1}, n_t) may exit the feasible region F. If this happens then (K_{t-1}, n_t) will move towards the upper left-hand corner of the plot since K_{t-1} decreases (due to the failure of group formation) while the population n_t increases. This is pictured in the change from P to Q in Figure 4.3. Eventually (K_{t-1}, n_t) will move back into the feasible region, and the accumulation process starts again. The accumulation process thus moves the economy back and forth across the boundary of the feasible region F.

Population growth has a positive effect on the accumulation of public goods in the long run. First of all, the condition for group formation that population should exceed the group size threshold $s(K_{t-1})$ is satisfied more easily under population growth. Furthermore, since the group size threshold $s(K_{t-1})$ increases monotonically with the stock K_{t-1}, the condition is likely to be violated early in the accumulation process if there is no population growth. Second, even if at some point the condition for group formation is not met, future population growth can restart the process of accumulation.

Case (2): population decrease

Now let us suppose that population n_t monotonically decreases, eventually arriving at the lower limit n. As is the case with increasing population, the accumulation process follows one of two typical patterns. In the first pattern the public good stock monotonically decreases and approaches point B (Figure 4.2) . In the second pattern, roughly illustrated as path (2) in Figure 4.2, (K_{t-1}, n_t) may exit the feasible region F. The point (K_{t-1}, n_t) can return to the feasible region if the rate of population decrease is not too steep. By the same arguments used in case (1), the stock-population pair (K_{t-1}, n_t) moves towards lower right corner of the plot until it reaches the boundary of F. Thereafter it moves approximately along the boundary and approaches point B.

5 Discussion

The second-order dilemma of public goods arises from individuals' incentive to free-ride on a mechanism which is designed to solves the (first-order) dilemma of public goods.

Due to the second-order dilemma, the effectiveness of the institutional approach has been placed in doubt ever since the work of Parsons (1937:89-94). Parsons brought up a logical inconsistency in Hobbes' arguments on the "Leviathan": if individuals are rational egoists, then why should they act in the collective interest by establishing a coercive state? For recent criticisms of the institutional approach, see Bates (1988), Dixit and Olson (2000) among others. In his critique of the "new institutionalism", Bates (1988) contends that institutions provide a collective good and that rational individuals would seek to secure its benefits for free. The proposed institution is thus subject to "the very incentive problems it is supposed to resolve" (Bates 1988).

Although not as pessimistic as these critics, recent works on voluntary participation games (Palfrey and Rosenthal 1984; Okada 1993; Saijo and Yamato 1999; Dixit and Olson 2000) have provided partial support for this negative view. It has been shown that when individuals are given the opportunity to decide freely whether or not they should participate in a mechanism to provide public goods, then not all will participate due to the incentive of free riding.

In this paper we have shown that strategic interactions of individuals underlying the second-order dilemma can be captured well by the game of voluntary participation. Although every individual has an incentive to not participate, non-participation is not his dominant action. Individuals have the other incentive to participate in a group, provided that the number of participants exceeds the group size threshold. Even if not efficient, a positive amount of contributions can be made by self-interested individuals given the opportunity to organize themselves (Theorem 3.1). The accumulation of public goods is thus possible when the depreciation rate of public goods is low. In the long run, public goods will be accumulated until a voluntary group is no longer beneficial (Theorem 4.1).

Finally, we discuss implications of our result to real world examples of common-pool resources. Despite a negative view expressed by the " tragedy of commons", there exist a large number of empirical studies showing that people successfully self-govern common-pool resources (Ostrom, 1990). Well-known examples are mountain commons in Switzerland and Japan. In these areas, common-pool resources of mountain grazing and forest have been sustained for many centuries. Local residents established organizations formally or informally to regulate the use of commons. Although memberships of organizations and regulation rules are different, the two examples have several common properties. People lived in steep mountains, and populations have remained stable over long periods. Mutual monitoring was not very difficult. These environments made monitoring, sanctioning and other transaction costs relatively low. The two conditions for accumulation of public goods in our model, that is, the low group costs and the low depreciation rate of public goods, seem to be satisfied in these examples of mountain commons.

In conclusion, we have shown that accumulation has a positive effect in solving the second-order dilemma of public goods provision under certain conditions. It should be remarked that our result does nothing to deny the importance of social and behavioral factors such as community, norm and trust in human decision-making, which can also help solve the second-order dilemma. The accumulation of public goods has a beneficial and complementary effect. When combined with other social and behavioral factors, public goods can increase in an incremental process as multiple generations of individuals solve the second-order dilemma.

Appendix

Proof of Proposition 2.1. (the first part) Let $x = (x_i)_{i=1}^{n_t}$ be a Nash equilibrium of the voluntary contribution game in generation t. Since $y = \log x$ is monotonically increasing each x_i maximizes $\log u_i(x_1, \cdots, x_{n_t}, K_{t-1})$, given all other individuals' contributions and K_{t-1}, where u_i is defined in (2). Under $x_i \geq 0$, it can be shown that x_i satisfies

$$x_i = \max(0, M - \alpha \sum_{j \neq i} x_j) \quad \text{where} \quad M = (1-\alpha)\omega - \frac{\alpha}{\beta}(1-\delta)K_{t-1}. \tag{A.1}$$

When $\frac{1-\alpha}{\alpha}\frac{\beta\omega}{1-\delta} \leq K_{t-1}$, (A.1) implies $x_i = 0$. When $\frac{1-\alpha}{\alpha}\frac{\beta\omega}{1-\delta} > K_{t-1}$, we can show without much difficulty that (A.1) has a unique solution $x_i = \frac{(1-\alpha)\beta\omega - \alpha(1-\delta)K_{t-1}}{(1+n_t\alpha-\alpha)\beta}$. (the second part) It can be seen that the sum U of all individuals' payoffs, $U = \sum_{i=1}^{n_t} u_i(x_1, \cdots, x_{n_t}, K_{t-1})$, is maximized at a symmetric contribution profile $x = (x_i)_{i=1}^{n_t}$ where they all contribute the same amounts of private goods. Putting $x = x_1 = \cdots = x_{n_t}$ yields $U = n_t(\omega - x)^\alpha((1 - \delta)K_{t-1} + \beta n_t x)^{1-\alpha}$ where $0 \leq x \leq \omega$. Considering $x \geq 0$, the identical contributions that maximize U is given by $x = \max\,(0, (1-\alpha)\omega - \frac{\alpha(1-\delta)K_{t-1}}{\beta n_t})$. Q.E.D.

Proof of Proposition 2.2. It follows from Proposition 2.1 that every individual contributes $x_i^{NE} = \frac{(1-\alpha)\beta\omega - \alpha(1-\delta)K_{t-1}}{(1+n_t\alpha-\alpha)\beta}$. The dynamic system (1) of public goods is given by

$$K_t = \frac{1-\alpha}{1+(n_t-1)\alpha}(1-\delta)K_{t-1} + \frac{1-\alpha}{1+(n_t-1)\alpha}\beta n_t w. \tag{A.2}$$

It can be shown that (A.2) has the stationary point $K^* = \frac{(1-\alpha)\beta n_t w}{n_t\alpha+\delta(1-\alpha)}$, and that the sequence K_t converges to K^* as t goes to infinity. We can easily see that the inequality $K^* < \frac{1-\alpha}{\alpha}\frac{\beta\omega}{1-\delta}$ holds for any $0 < \alpha < 1$ and any $0 < \delta < 1$. Q.E.D.

Proof of Lemma 3.1. It can be shown that all non-participants contribute the same amounts x_{np} of private goods in a Nash equilibrium. Then, the sum U^S of all participants' payoffs is given by $U^S = s(\omega - x_p^S)^\alpha((1-\delta)K_{t-1} + \beta s x_p^S + \beta(n_t - s)x_{np})^{1-\alpha}$. The participants' contribution x_p^S which maximizes U^S satisfies $x_p^S = \max\,(0, (1-\alpha)\omega - \frac{\alpha(1-\delta)K_{t-1}}{\beta s} - \frac{(n_t-s)\alpha x_{np}}{s})$. The non-participant's contribution x_{np} satisfies (A.1). We consider the following four cases: (i) $x_p^S > 0$ and $x_{np} > 0$, (ii) $x_p^S > 0$ and $x_{np} = 0$, (iii) $x_p^S = 0$ and $x_{np} > 0$, and (iv) $x_p^S = 0$ and $x_{np} = 0$. A calculation shows that case (i) has the solution (x_p^S, x_{np}) given by (1) in the lemma, and that K_{t-1} must satisfy $0 \leq K_{t-1} < \frac{1-\alpha s}{\alpha}\frac{\beta\omega}{1-\delta}$. It is easy to show that case (ii) and case (iv) have the solutions given by (2) and (3) in the lemma, respectively, under the conditions on K_{t-1}. Finally, we can see without much difficulty that case (iii) is not possible. Q.E.D.

Proof of Lemma 3.2. (1): easy. (2): $\log\, u_p(s, K_{t-1})$ is a monotonically increasing function of s since $\frac{d}{ds}\log\, u_p = \frac{\beta\omega}{\beta s\omega+(1-\delta)K_{t-1}} - \frac{\alpha}{s} \geq 0$ from Lemma 3.1.(2). Therefore $u_p(s, K_{t-1})$ is monotonically increasing in s. It easily follows from (4) that $u_{np}(s, K_{t-1})$ is monotonically increasing in s. (3): Let $f(s) = \log\, u_{np}(s-1, K_{t-1}) - \log\, u_p(s, K_{t-1})$. It is sufficient to show that $f(s)$ is monotonically increasing in s. This can be proved by

$$\frac{df}{ds} > \frac{\alpha}{s} - \frac{\alpha\beta\omega}{\beta s\omega + (1-\delta)K_{t-1}} > 0.$$

Q.E.D.

Proof of Lemma 3.4. By definition, the threshold $s(K_{t-1})$ is the solution s of $u_p(s, K_{t-1}) - c(s) - u(0, K_{t-1}) = 0$. By applying the Implicit Function Theorem, we obtain

$$(\frac{\partial u_p}{\partial s} - \frac{dc}{ds})\frac{ds}{dK_{t-1}} = \frac{\partial u(0, K_{t-1})}{\partial K_{t-1}} - \frac{\partial u_p}{\partial K_{t-1}}. \tag{A.3}$$

By Assumption 3.1, $\frac{\partial u_p}{\partial s} - \frac{dc}{ds} > 0$. By (3) and (5), the RHS of (A.3) is rewritten as

$$\begin{aligned} \frac{\partial u(0, K_{t-1})}{\partial K_{t-1}} - \frac{\partial u_p}{\partial K_{t-1}} &= \frac{(1-\alpha)(1-\delta)^{1-\alpha}\omega^\alpha}{K_{t-1}^\alpha} - \frac{\alpha^\alpha(1-\alpha)^{1-\alpha}(1-\delta)}{(\beta s)^\alpha} \\ &= (1-\alpha)(1-\delta)((\frac{\omega}{(1-\delta)K_{t-1}})^\alpha - (\frac{\alpha}{(1-\alpha)\beta s})^\alpha). \end{aligned}$$

The RHS of the last equality is positive by Lemma 3.1.(2). Thus $\frac{ds}{dK_{t-1}} > 0$ by (A.3), which proves the first part of the lemma. The second part can be proved by the fact that the function of s, $g(s, K_{t-1}) = u_p(s, K_{t-1}) - c(s) - u(0, K_{t-1})$, shifts downward as the group cost $c(s)$ increases. The solution s of $g(s, K_{t-1}) = 0$ therefore increases. Q.E.D.

Proof of Theorem 3.1. (1): It follows from Lemma 3.3 that the group size s of a Nash equilibrium must satisfy $s(K_{t-1}) < s$. Since $s \leq n_t$, $s(K_{t-1}) < n_t$. Conversely, suppose that $s(K_{t-1}) < n_t$. Consider two distinct cases: (i) $n_t = [s(K_{t-1})]$, and (ii) $n_t > [s(K_{t-1})]$. In case (i), it follows from Lemma 3.3 that, if any participant defects from the largest group, the remaining participants do not agree to form their group since $n_t - 1 < s(K_{t-1})$. Since the defector's payoff decreases from $u_p(n_t, K_{t-1}) - c(n_t)$ to $u(0, K_{t-1})$, the largest group is a Nash equilibrium. In case (ii), even if one participant defects from the largest group, the remaining participants form the group since $s(K_{t-1}) < n_t - 1$. The defector thus receives the payoff $u_{np}(n_t - 1, K_{t-1})$. Now consider two subcases: case (iia) $u_p(n_t, K_{t-1}) - c(n_t) \geq u_{np}(n_t - 1, K_{t-1})$, and case (iib) $u_p(n_t, K_{t-1}) - c(n_t) < u_{np}(n_t - 1, K_{t-1})$. In subcase (iia), it is clear that the largest group is a Nash equilibrium.

In subcase (iib), consider further the two distinct subcases: (iib-1) $u_p(s, K_{t-1}) - c(s) < u_{np}(s-1, K_{t-1})$ for all s with $s(K_{t-1}) + 1 < s \leq n_t$, and (iib-2) $u_{np}(s-1, K_{t-1}) \leq u_p(s, K_{t-1}) - c(s)$ for some s with $s(K_{t-1}) + 1 < s \leq n_t$. In subcase (iib-1), the equilibrium group size is equal to $[s(K_{t-1})]$. In subcase (iib-2), denote by m^* the maximum integer which satisfies $u_{np}(s-1, K_{t-1}) \leq u_p(s, K_{t-1}) - c(s)$. Then one has $u_{np}(m^* - 1, K_{t-1}) \leq u_p(m^*, K_{t-1}) - c(s^*)$ and $u_p(m^* + 1, K_{t-1}) - c(m^* + 1) < u_{np}(m^*, K_{t-1})$. These two inequalities mean that a group with m^* participants is also a Nash equilibrium.

(2): (2) can be proved by the arguments in case (i) and case (iia) of (1).

(3): In the same way as Lemma 3.2.(3), it can be proved that $\frac{u_{np}(s-1, K_{t-1})}{u_p(s, K_{t-1}) - c(s)}$ is a monotonically increasing function of s, where $s \geq s(K_{t-1})$ if the assumption of the theorem holds. This fact yields the following two properties (for notational simplicity, the stock variable K_{t-1} is omitted in the payoff functions u_p and u_{np}): (a) $u_{np}(s-1) \geq u_p(s) - c(s)$ implies $u_{np}(t-1) > u_p(t) - c(t)$ for $s < t$, and (b) $u_{np}(s-1) \leq u_p(s) - c(s)$ implies $u_{np}(t-1) < u_p(t) - c(t)$ for $t < s$.

According to the proof of (1), it suffices to show that the number of participants in a group equilibrium is unique in each of three distinct cases: (iia), (iib-1), and (iib-2) (except a degenerate case). In case (iia), the largest group is a Nash equilibrium since $u_p(n_t, K_{t-1}) - c(n_t) \geq u_{np}(n_t - 1, K_{t-1})$. Property (b) yields $u_p(t, K_{t-1}) - c(t) > u_{np}(t-1, K_{t-1})$ for all t with $s(K_{t+1}) < t < n_t$. This means that no other group is a

Nash equilibrium. In subcase (iib-1), only groups with $[s(K_{t-1})]$ participants are Nash equilibria. In subcase (iib-2), as we have shown in the proof of (1), there exist some m^* which satisfy $u_{np}(m^*-1, K_{t-1}) \leq u_p(m^*, K_{t-1}) - c(m^*)$ and $u_p(m^*+1, K_{t-1}) - c(m^*+1) < u_{np}(m^*, K_{t-1})$. The first inequality and property (b) imply that

$$u_{np}(s-1) < u_p(s) - c(s) \quad \text{if} \;\; s(K_{t+1}) + 1 < s \leq m^* - 1, \tag{A.4}$$

and the second inequality and property (a) imply that

$$u_p(s+1, K_{t-1}) - c(s+1) < u_{np}(s, K_{t-1}) \quad \text{if} \;\; m^* \leq s \leq n_t. \tag{A.5}$$

By (A.4) and (A.5), any group with s $(\neq m^* - 1, m^*)$ participants is not a Nash equilibrium. It can be shown that if $u_{np}(m^* - 1, K_{t-1}) < u_p(m^*, K_{t-1}) - c(s^*)$, then only groups with m^* participants are Nash equilibria, and that if $u_{np}(m^* - 1, K_{t-1}) = u_p(m^*, K_{t-1}) - c(m^*)$, then groups with either $m^* - 1$ or m^* participants are Nash equilibria. Q.E.D.

Proof of Lemma 4.1. It follows from (7) that $K_t - K_{t-1} = \{(1-\alpha)(1-\delta) - 1\}K_{t-1} + (1-\alpha)\beta\omega s^*(K_{t-1})$. $K_t - K_{t-1} > 0$ is equivalent to $s^*(K_{t-1}) > a(K_{t-1})$. Q.E.D.

Proof of Lemma 4.2. Recall that the group size threshold $s(K_{t-1})$ is the solution of $u_p(s, K_{t-1}) - c(s) = u(0, K_{t-1})$. Since the LHS is increasing in s (Assumption 3.1), it suffices us to prove that $u_p(a, K_{t-1}) - c(a) < u(0, K_{t-1})$ at $a = a(K_{t-1})$. Substituting (3) and (5) to the inequality above, we have

$$(1-\alpha)\{\frac{\alpha}{(1-\alpha)\beta a}\}^{\alpha}[\beta a\omega + (1-\delta)K_{t-1}] - c(a) < \omega^{\alpha}[(1-\delta)K]^{1-\alpha}. \tag{A.6}$$

Substituting (8) into (A.6) and arranging it, we obtain

$$\frac{1}{1+\frac{1}{\alpha}\frac{\delta}{1-\delta}} < \{1-\delta+(\frac{1-\delta}{\omega})^{\alpha}(\frac{1}{K_{t-1}})^{1-\alpha}c(a)\}^{\frac{1}{\alpha}}. \tag{A.7}$$

(A.7) holds if

$$\delta < (\frac{1-\delta}{\omega})^{\alpha}(\frac{1}{K_{t-1}})^{1-\alpha}c(a). \tag{A.8}$$

We can prove that (A.8) holds for any sufficiently low δ. Q.E.D.

Proof of Theorem 4.1. Let $K = K(s)$ be the inverse function of the group size threshold $s = s(K_{t-1})$. By Lemma 3.2, the function $K(s)$ is well-defined and monotonically increasing. The set of all K_{t-1} with $(K_{t-1}, n_1) \in F$ is divided into distinct intervals $I_s = (K(s-1), K(s))$, for $s = 2, \cdots, n_1$. For every s, $K_{t-1} \in I_s$ implies $s(K_{t-1}) < s \leq s^*(K_{t-1})$ where $s^*(K_{t-1})$ is the equilibrium group size. By Lemma 4.2, $a(K_{t-1}) < s(K_{t-1})$. Thus, by Lemma 4.1, $K_{t-1} < K_t$. This proves the first part.

To prove the second part, suppose, on the contrary, that $K_t < K(n_1)$ for all t. Since $\{K_t\}$ is a monotonically increasing sequence, it must be true that there exists some integer $m \leq n_1$ such that $K_t \in I_m$; i.e., $K(m-1) < K_{t-1} < K(m)$ for almost all $t = 1, 2, \cdots$. By Theorem 3.1, $s^*(K_t) \geq m$ for $K_t \in I_m$. Consider two cases: (1) $s^*(K_t) = m$ for all t, and (2) otherwise. In case (1), it follows from the dynamic system (7) that $\{K_t\}$ monotonically increases and converges to the stationary point K^* where $m = a(K^*)$. By Lemma 4.2, $m < s(K^*)$, that is, $K(m) < K^*$. Therefore, $K(m) < K_t$ for any sufficiently large t. This contradicts that $K(m-1) < K_{t-1} < K(m)$ for almost all t. In case (2), $\{K_t\}$ may converge either to K^* or to some larger value, since the equilibrium group size may be larger than m. The same contradiction arises as in case (1). Q.E.D.

Acknowledgements I would like to thank an associate editor and two anonymous reviewers for their encouragement and helpful suggestions. I am also grateful to Ashutosh Sarker and seminar participants at Hitotsubashi University, University of Tokyo, Osaka University, Tinbergen Institute, Keio University, University of Nottingham and Kyoto University for their useful comments. This research was supported in part by the Ministry of Education, Science and Culture in Japan (Grant-in-aid No.10630006) and the Asahi Glass Foundation.

References

Andreoni, J. (1988). Privately provided public goods in a large economy: The limits of altruism. *Journal of Public Economics*, 35, 57-73.

Bates, R.H. (1988). Contra contractarianism: some reflections on the new institutionalism. *Politics and Society*, 16, 387-401.

Bergstrom, T., Blume, L. & Varian, H. (1986). On the private provision of public goods. *Journal of Public Economics*, 29, 25-49.

Dixit, A. & Olson, M. (2000). Does voluntary participation undermine the Coase theorem? *Journal of Public Economics*, 76, 309–335.

Dorsey, R. (1992). The voluntary contributions mechanism with real time revisions. *Public Choice*, 73, 261-282.

Groves, T. & Ledyard, J. (1977). Optimal allocation of public goods: a solution to the "free rider" problem. *Econometrica*, 45, 783-811.

Groves, T. & Ledyard, J. (1987). Incentive compatibility since 1972. In T. Groves, R. Radner & S. Reiter (Eds.), *Information, incentives, and economic mechanisms: Essays in honor of Leonid Hurwicz*, (pp. 48-111). Oxford; Blackwell.

Fershtman, C. & Nitzan, S. (1991). Dynamic voluntary provision of public goods. *European Economic Review*, 35, 1057-1067.

Foley, D.K. (1970). Lindahl's solution and the core of an economy with public goods. *Econometrica*, 38, 66–72.

Marx, L.M. & Matthews, S.A. (2000). Dynamic voluntary contribution to a public project. *Review of Economic Studies*, 67, 327-358.

Mas-Colell, A. (1980). Efficiency and decentralization in the pure theory of public goods. *Quarterly Journal of Economics*, 94, 625-641.

Moore, J. & Repullo, R. (1988). Subgame perfect implementation. *Econometrica*, 56, 1191-1220.

Okada, A. (1993). The possibility of cooperation in an n-person prisoners' dilemma with institutional arrangements. *Public Choice*, 77, 629-656.

Oliver, P. (1980). Rewards and punishments as selective incentives for collective action: Theoretical investigations. *American Journal of Sociology*, 85, 1356-1375.

Olson, M.(1965). *The logic of collective action.* Cambridge: Harvard University Press.

Ostrom, E. (1990). *Governing the commons: The evolution of institutions for collective action.* Cambridge: Cambridge University Press.

Palfrey, T.R. & Rosenthal, H. (1984). Participation and the provision of discrete public goods: A strategic analysis. *Journal of Public Economics*, 24, 171-193.

Parsons, T. (1937). *The structure of social action.* New York: MacGraw-Hill.

Pecorino, P. & A. Temimi (2007). Public good provision in a repeated game: The role of small fixed costs of participation. *Public Choice*, 130, 337-346.

Saijo, T. & Yamato, T. (1999). A voluntary participation game with a non-excludable public good. *Journal of Economic Theory*, 84, 227-242.