Hitotsubashi Journal of Economics 49 (2008), pp.75-90.

ENDOGENOUS INCOME DISTRIBUTION WITH PRODUCT OBSOLESCENCE

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Received March 2008; Accepted June 2008

Abstract

Wage inequality in U. S. and UK has increased over the past 25 years. Paradoxically, skilled labor supply has also increased in both countries. This paper develops the dynamic general equilibrium model of product innovation with product obsolescence. We develop a model to provide an explanation of inequality phenomena between skilled and unskilled labor by the channel of innovation and market structure. This paper builds on the dynamic general equilibrium model of product innovation and incorporates overhead cost of the production of intermediate goods to capture endogenous growth rate of innovation, hazard rate, product life cycle and inequality.

Keywords: wage inequality, skilled and unskilled labor, product innovation, general equilibrium
JEL Classification: O15, D2, D3

I. *Introduction*

Wage inequality in U. S. and UK has increased over the past 25 years. Paradoxically, skilled labor supply has also increased in both U.S. and UK. Starting from Kuznets (1955) who explains an inverted U-shaped curve of inequality with respect to development stage, there have been literatures explaining inequality phenomena. On the contrary to the negative effect of

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development stage on income inequality suggested by Kuznets Hypothesis, empirical studies done by Juhn, Murphy, and Pierce (1993), and Deininger and Squire (1998) find little support for Kuznets hypothesis. Juhn, Murphy, and Pierce (1993) shows that income inequality has increased in OECD countries during the past twenty years. Motivated by a rise in income inequality of OECD countries, there have been studies to explain the source of the rise in income inequality. The important sources for a rise in income inequality are explained as dualism of labor productivity by Bourguignon and Morrison (1998), technological change by Galor and Tsiddon (1997), educational cohort size by Higgins and Williamson (1999), skill-biased technical progress by Aghion and Howitt (1998, pp. 300-303) and knowledge spillover across countries by Lee (1999) and Goo, Kim, and Lee (2005). As they suggested, the skill-biased technical change increases demand for skilled workers, leading to their wage premium over unskilled workers.

Most papers, however, analyzing the source of the rise in income inequality have neglected the relations among growth, hazard rate and income inequality. Extending these relations between growth and income inequality to the product obsolescence yields more interesting results. This paper contributes to provide another explanation of inequality phenomenon in a general equilibrium model linking endogenous growth and hazard rate.

Endogenous growth models feature two directions. On one hand, there are quality ladder models that concern with the Schumpeterian process of creative destruction, which is the central to the seminal theories of Aghion and Howitt (1992), Grossman and Helpman (1991a, Ch.4; 1991b; 1991c) and Segerstrom (1991). In these models, higher quality products perfectly substitute for lower quality products, so the rents of firms producing lower quality products are completely destroyed as soon as a higher quality product is introduced to the market. On the other hand, there are R&D-driven growth models that have a characteristic of the static and dynamic scale effect by product varieties (Romer, 1990; Grossman and Helpman, 1991a, Ch.3). These models, however, not only ignore the role of market structure but also exaggerate the role of product varieties since all goods introduced into the market do not obsolete but exist forever.[1]

Without taking into account product obsolescence, the stock of knowledge resulting from innovation activity would be overstated. Introducing product obsolescence into the R&D-driven growth models will enrich the Schumpeterian growth model. This is the basis on which Lai (1998) analyzes economic growth with gradual product obsolescence in the structure of a semiendogenous rate of innovation. Since Vernon (1966) described the life cycle theory of manufactured goods, there have been several studies to analyze product life cycle. Gort and Klepper (1982), Jovanovic and MacDonald (1994a, 1994b) and Klepper (1996), among others, depict product life cycle models and its empirical evidence concerning how a market evolve from infancy to maturity. These studies, however, analyze firm survival according to given life cycle stages. That is, firms that enter market in early stages have a higher survival rate while hazard rates increase in the mature stages because of an increase in competitive intensity. Instead, in our model, we focus on steady state length of product life cycle and hence hazard

[1] The studies on innovation and market structure are found on Loury (1979), Lee and Wilde (1980), Dasgupta and Stiglitz (1980), Kamien and Schwartz (1982), Shaked and Sutton (1987), and Scherer and Ross (1990, Ch.17). Market structure, among other features, refers to the distribution of firm size and the number of firms in a market (see Scherer and Ross, 1990, Ch.1).

rate determined by market. Furthermore, we examine the endogenous income distribution.

Analyzing income distribution with product life cycle is nothing new. There have been studies to analyze the international (North-South) product cycle in dynamic models by innovation and imitation. Firms in the North (developed countries) initially develop and manufacture new products and imitation takes place in the South (less developed countries). The production moves from the North to the South because of relatively cheap labor force in the South. Krugman (1979) adopts the exogenous rate of product innovation by the North and the exogenous rate of imitation by the South and therefore an increase in labor supply in a country plays a role in lowering the relative wage of that country. Contrary to Krugman (1979), Grossman and Helpman (1991c) considers the endogenous rate of innovation and imitation and develops a model where an increase in labor supply in a country raises the relative wage to that of the other country. Imitation, however, always takes place in the South in Grossman and Helpman (1991c), as criticized by Segerstrom (1991).[2] Moreover, because their papers focus on innovation and imitation, the relation of intellectual property rights protection to economic growth becomes a main debate. Thus, what is new in this paper is that we develop a model to provide an explanation of inequality phenomena between skilled and unskilled labor within a country by the channel of innovation and market structure, but not by the channel of innovation and imitation.

This paper builds on the dynamic general equilibrium model of product innovation by Romer (1990) and Grossman and Helpman (1990) and incorporates a fixed operating cost of the production of intermediate goods to capture endogenous growth rate of innovation, hazard rate, product life cycle and inequality. We have the view that skill-biased technology development plays an important role in causing wage inequality between skilled and unskilled workers. Technology development is enhanced by skilled labor, which is the engine of growth, yielding an increase in wage premium of skilled workers. Thus, it leads to a rise in wage inequality. Growth has a positive effect on wage inequality. In the structure of general equilibrium and endogenous growth, the relative increase of skilled labor to unskilled labor has two effects on wage inequality. It enhances growth and leads to a rise in income inequality indirectly, but it directly reduces the wage inequality in that the increased factor supply leads to a decrease in the factor price. This paper shows that the direct effect dominates the indirect effect. The increase in the aggregate skilled labor contributes to play a role in reducing wage inequality while it enhances growth, suggesting the importance of education for the unskilled to become skilled workers.

An increase in productivity parameter of R&D activity plays a positive role in enhancing growth and inequality since the growth rate of technology development is directly affected by this parameter. A decrease in fixed operating cost reduces hazard rate while enhancing growth. Thus, inequality as well as product life cycle increases. Innovation of new product contributes to reducing overhead cost and speed up growth and inequality. If the decrease in fixed operating cost and the increase in the productivity of R&D activity is accelerated, the inequality will rise even though increased skilled labor supply plays a positive role in reducing inequality. Therefore, the increasing pattern of inequality phenomena with increasing pattern of skilled labor supply in U.S. and UK can be explained by overwhelming effects of fixed operating cost

[2] Segerstrom (1991) develops a model to provide an explanation of the empirical phenomenon that firms in a country imitate the products developed by other firms in the North.

and productivity of R&D activity.

The paper is organized as follows. Section II proposes a dynamic general equilibrium model in a closed economy to find the effect of exogenous hazard rate on steady state growth and inequality. Section III takes into account endogenous hazard rate and explains the effect of policy variables on the steady state growth rate, hazard rate, product life cycle and inequality. In this section, we present numerical examples to make the comparative statics concrete. Conclusion and some extensions are provided in section IV.

II. *The Model*

There are final good and intermediate goods productions as well as technology development. A consumable final good is produced by employing non-reproducible unskilled labor and intermediate inputs, whereas all intermediate inputs and new technology development by skilled labor. Each intermediate good is differentiated from the others and its producer can solely access her own new technology, maintaining her own monopoly power for the input.

On the consumer side, consumers have the identical, homothetic preferences. A time-separable utility function of consumer m, for $m=L, H$, where L and H denote unskilled and skilled labor, respectively, is given by

$$U(m,t)=\int_t^{\infty} e^{-\rho(\tau-t)} \log u(c(m,\tau))d\tau,$$

where ρ is the subjective discount rate and $c(m,\tau)$ is the consumption of final good chosen by consumer m at time τ. As in Grossman and Helpman (1990), this consumer's maximization problem can be solved in two stages. After getting an indirect utility function by solving consumer's maximization of static utility for a given level of expenditure at time t, we can solve the optimal pattern of expenditure in the second stage. Let an instantaneous sub-utility function be, $u(c(m,t))=c(m,t)$. The first stage of consumer's utility maximization problem yields her consumption of final good produced at time t and it is given by $c(m,t)=E(m,t)/P_Y(t)$, where $P_Y(t)$ denotes the price of $Y(t)$, which is denoted as final good produced in a country at time t, and $E(m,t)$ denotes a given level of expenditure of consumer m at time t. This derived demand for final good has a standard implication. It is positively related to expenditure and negatively to its price. Normalizing the price of final good as a unity yields $c(m,t)=E(m,t)$. An indirect utility function can be derived by plugging this derived demand function into the instantaneous sub-utility function. Let each consumer supply a unit of labor and let $w_m(\tau)$ denote the wage rate of consumer m at time τ and $a_H(t)$ denote the value of asset a skilled worker holds at time t. In the second stage, consumer m at time t chooses the time pattern of expenditures in order to maximize

$$V(m,t)=\int_t^{\infty} e^{-\rho(\tau-t)} \log E(m,\tau)d\tau,$$

subject to her intertemporal budget constraint given by

$$\int_t^{\infty} e^{-R(\tau)}E(m,\tau)d\tau \leq \int_t^{\infty} e^{-R(\tau)}w_m(\tau)d\tau+a_m(t),$$

where $R(\tau)=\int_t^\tau r(h)dh$ is the cumulative interest rate, $r(h)$ is the interest rate at time h and $a_L(t)=0$. Consumers face common interest rate by assuming that capital market is perfect. The necessary condition for this problem yields the optimal path for expenditure and it is given by $\dot{E}(L)/E(L)=\dot{w}_L/w_L$ and $\dot{E}(H)/E(H)=r-\rho$, for all. t If $\dot{w}_L/w_L=r-\rho$, as it will be in equilibrium, the inelastic labor supply yields the optimal path for aggregate expenditure which is the same with that for individual expenditure and hence does the optimal path for aggregate consumption and it is given by

$$\frac{\dot{E}(t)}{E(t)}=\frac{\dot{C}(t)}{C(t)}=r(t)-\rho, \tag{1}$$

where $E(t)=C(t)=c(L,t)L(t)+c(H,t)H(t)$, and $L(t)$ and $H(t)$ represent the unskilled and skilled labor force available, respectively. This optimal path for aggregate expenditure and hence optimal path for aggregate consumption are positive (negative) if interest rate of a world capital market is greater (less) than the consumer's subjective discount rate.

On the production side, a homogenous final good at time t, $Y(t)$, which is the only consumable good, is produced by an unskilled labor and varieties of intermediate inputs with constant returns to scale. Let $A(t)$ denote the level of technology development at time t and $A_0(t)$ denote the technology that becomes obsolete and exits market at time t. Hence $A(t)-A_0(t)$ are the varieties of intermediate inputs available at time t. The production function of the final good at time t, suppressed for the notational convenience, is given by

$$Y=L^\mu\int_{A_0}^{A}[x(z)]^{1-\mu}d(z),\ \text{for } 0<\mu<1, \tag{2}$$

where $x(z)$ denotes the intermediate input, indexed by $z\in[A_0, A]$. This production function for final good shows constant return to scale for the given level of technology development in a country, but exhibits dynamic scale economies due to the level of technology development, which is external to the final good producer. Assuming final good market is competitive, from this production function for final good, the necessary conditions for optimal allocations of inputs can be derived by marginal cost pricing and they are given by

$$\mu L^{\mu-1}\int_{A_0}^{A}[x(z)]^{1-\mu}d(z)=w_L,$$

$$(1-\mu)L^\mu[x(z)]^{-\mu}=p(z),$$

where w_L and $p(z)$ denote inputs prices of unskilled labor and intermediate goods demanded for the production of final good, respectively. Let each intermediate good be differentiated from the others, which means the producers of intermediate goods compete monopolistically, and a unit of skilled labor be required for production of a unit of intermediate good. The producer of each intermediate good maximizes profits $\pi(z)=[p(z)-w_H]x(z)$, where w_H denote the wage rate of skilled labor. From the derived demand for each intermediate input given by the marginal cost pricing for the production of final good, maximizing profits for each intermediate input yields the monopoly price of each intermediate good and it is given by $p(z)=w_H/[1-\mu]$, which is markup marginal cost. It shows that the prices of all the varieties of intermediate goods at a point in time are the same. Thus, the quantity of each intermediate good is the same and so is the profit of producing intermediate good, which is given by $\pi = \frac{\mu w_H}{1-\mu}\frac{H_X}{A-A_0}$, where H_X represent the aggregate skilled labor devoted to the production of intermediate goods. Let ω be defined as the index of wage inequality between skilled and unskilled labor, that is $\omega \equiv w_H/w_L$. From the above equations we can derive the index of wage inequality and it is given by

$$\omega = \frac{[1-\mu]^2}{\mu}\frac{L}{H_X}.$$

This shows that wage inequality will be higher the lower is the relative quantity of the aggregate skilled labor devoted to the activity in intermediate goods sector to the unskilled labor. This is quite intuitive in that an increase in factor supply will lead to a decrease in the price of the factor.

Following Romer (1990) in assuming that the rate of new technology development will be higher the larger are the existing stock of technologies and skilled labor devoted to new technology development, the evolution of new technology development is

$$\dot{A} = \varphi A H_A, \tag{3}$$

where φ is a productivity parameter and H_A is the skilled labor devoted to the activity in technology development. This production function of technology development implies that knowledge diffuses immediately and costlessly within a country. Since knowledge has been assumed to be a nonrival input to each individual producer of new technologies, all producers of new technologies can freely access to the entire existing stock of technologies within a country. Hence the marginal cost pricing of new technology development is $\varphi P_A A = w_H$, where P_A is the price of new technology development.

An innovator who develops new product at time t faces the possibility that the product will be out of the market from product obsolescence. When the product is out of the market, the innovator's monopoly profits are over, so the profits accrue from the time of development until the time of exit. The present value of future profits that a monopolist can get is

$$V(t_d,t_o) \equiv \int_{t_d}^{t_o} e^{-[R(\tau)-R(t_o)]}\pi(t_d,\tau)d\tau,$$

where t_d denotes the time a product is developed and t_o denotes the time the product becomes out of market. The interval over which the monopolist can get profits, that is product life cycle, is $T = t_o - t_d$. If the interest rate is constant, as will be in equilibrium, the present value of the future profits from new product development is given by

$$V(t,T) = \pi(t,t)[1-e^{-rT}]/r$$

Let $G(\tau)$ denote the cumulative probability density function for T, defined by $G(\tau) =$ Pr$ob[T \leq \tau]$. The probability density function for a product developed at time t is given by $g(t,T) = h(t)e^{-\int_t^{t+T} h(\tau)d\tau}$ where $h(\tau)$ denotes the hazard rate at time τ defined by $h(\tau) = \dot{A}_0(\tau)/[A(\tau) - A_0(\tau)]$. If the hazard rate is constant over time, as will be in equilibrium, the probability density function becomes $g(t,T) = he^{-hT}$

With competitive market for technology development, the present value of future profits that a monopolist can get should be equal to the cost of new technology development, so the price of new technology development at time t becomes

$$P_A(t)=\int_t^{\infty} e^{-R(\tau)}V(t,\tau)h[1-e^{-h[\tau-t]}]d\tau.$$

By differentiating this condition with respect to time, we get no-arbitrage condition

$$\dot{P}_A=-\pi+[r+h]P_A. \tag{4}$$

This no-arbitrage condition represents that the instantaneous sum of capital gains is just equal to the instantaneous interest rate with risk premium net of dividends of investment in new technology development.

Let γ_x be defined as the growth rate of variable x, where $x=E$, C, Y, A, w_H, w_L and P_A. The market clearing condition for final good implies that the aggregate consumption in a country is equal to the output of final good. Thus, the growth rate of aggregate consumption is the same with that of final good. Given unskilled labor, the growth relation between final good and technology development is $\gamma_Y=\mu\gamma_A$, and the market clearing condition for final good is $\gamma_Y=r-\rho$, so we get $\gamma_A=[r-\rho]/\mu$. Let $\chi(t)$ denote the fraction of goods that become obsolete at time t, defined as $\chi(t)\equiv A_0(t)/A(t)$, then the steady state fraction of goods that become obsolete is $\chi=h/[\gamma_A+h]$. The no-arbitrage condition can be derived by the growth rate of the price of technology development from the marginal cost pricing of new technology development and the growth rate of wage rate of skilled labor with a fixed aggregate unskilled labor and it is given by

$$\frac{\dot{w}_H}{w_H}-\frac{\dot{A}}{A}=-\frac{\varphi\mu H_X}{[1-\mu][1-\chi]}+r+h.$$

In the long run, skilled labor devoted to R&D activity as well as the production activity becomes constant from the assumption of a fixed aggregate quantity of skilled labor available. The growth rate of skilled labor devoted to intermediate goods production becomes zero, that is, $\gamma_{H_X}\to 0$. With the market clearing condition for skilled labor and the growth relation between final good and technology development, the no-arbitrage condition with full employment yield the growth rate of technology development

$$[1-\mu][\rho+h+\gamma_A]\gamma_A/[\gamma_A+h]=\mu[\varphi\overline{H}-\gamma_A]. \tag{5}$$

The above growth rate equation yields following lemma.

Lemma 1.
Solving this quadratic equation yields the growth rate of technology development. The positive growth rate is

$$\gamma_A=\frac{\gamma_0-h+\sqrt{[\gamma_0-h]^2+4\mu\varphi\overline{H}h}}{2}, \tag{6}$$

where $\gamma_0=\mu\varphi H-[1-\mu]\rho$ denoted as the growth rate of technology development in the economy without the hazard rate.

The growth rate is a function of the exogenous hazard rate and policy variables such as skilled labor supply and the productivity parameter of R&D activity. This growth rate implies that not only an increase in skilled labor supply but also an increase in productivity parameter of technology development enhances growth. The following proposition immediately follows Lemma 1.

Proposition 1.
The growth rate of technology development rises with exogenously given hazard rate at a decreasing rate.

The increase in hazard rate implies the increase in the risk premium, so it raises the cost of new technology development. The increase in the cost of new technology development will reduce the incentive to develop new technology. The less incentive to develop new technology leads to a fall in growth. Notwithstanding, in steady state, as can be seen from the profit function of production of intermediate goods $\pi=\frac{\mu}{1-\mu}\frac{w_H H_X[\gamma_A+h]}{A\gamma_A}$, the increase in hazard rate raises the profit for the production of intermediate goods, given the growth rate of technology development. This is so because firms with lower technologies exit market and the share in sales of each firm existing in the market becomes higher. The increased monopoly profit unambiguously enhances growth since it is the engine of growth. The result in this paper shows that the effect of the hazard rate on profit dominates its effect on the cost of new technology development. In other words, the rate of return to the producers of intermediate goods becomes higher, even though the cost of new technology development increases, which leads to a higher growth.

We also get limit value of growth rate as hazard rate approaches to zero as well as it approaches to infinite. As, $h\rightarrow 0$, $\gamma_A\rightarrow\mu\varphi\bar{H}-[1-\mu]\rho$ and as, $h\rightarrow\infty$, $\gamma_A\rightarrow\mu\varphi\bar{H}$ by L'Hopital's rule.

The endogenously determined interest rate becomes

$$r=\mu\frac{[\gamma_0-h]+\sqrt{[\gamma_0-h]^2+4\mu\varphi\bar{H}h}}{2}+\rho,$$

and the skilled labor devoted to the production of intermediate goods is

$$H_X=\bar{H}-\frac{1}{\varphi}\frac{[\gamma_0-h]+\sqrt{[\gamma_0-h]^2+4\mu\varphi\bar{H}h}}{2}.$$

The index of wage inequality can be written as a function of aggregate endowments of skilled labor and the growth rate of technology development since $H_X=\bar{H}-\gamma_A/\varphi$ from full employment condition and it is given by

$$\omega=\frac{[1-\mu]^2L}{\mu[\bar{H}-\gamma_A/\varphi]}.$$

Given the aggregate skilled and unskilled labor forces available in a country, the growth rate of technology development and hence that of final good as well as consumption is positively related to the wage inequality. This index of wage inequality shows that an increase in an aggregate supply of skilled labor force available leads to a fall in wage inequality directly while it enhances growth. Hence it leads to a rise in wage inequality indirectly, given growth rate of skilled labor devoted to the production of intermediate goods. The net effect of an increase in the aggregate skilled labor force available depends on which dominates the other. If the direct effect dominates the indirect effect, then the wage inequality rises with the aggregate skilled labor force available. Given unskilled labor as a unity, the increase in the aggregate supply of skilled labor implies that skilled labor measured as a unit of unskilled labor becomes more abundant. Since the supply of skilled labor measured as a unit of unskilled labor increases, the price of skilled labor becomes relatively cheaper than the price of unskilled labor, leading to a decrease in wage inequality.

By plugging the endogenously determined growth rate of technology development into the wage inequality equation, the wage inequality between skilled and unskilled workers is derived as

$$\omega=\frac{[1-\mu]^2 L}{\mu\left[\bar{H}-\frac{\gamma_A}{\varphi}\right]}\rightarrow\frac{[1-\mu]L}{\mu\left[\bar{H}-\frac{1}{\varphi}\frac{[\gamma_0-h]+\sqrt{[\gamma_0-h]^2+4\mu\varphi\bar{H}h}}{2}\right]}. \tag{7}$$

The wage inequality is a function of hazard rate, productivity parameters, subject discount rate and the aggregate skilled labor endowed with. This wage inequality function yields following proposition.

Proposition 2.
(i) Increasing hazard rate leads to a rise in inequality.
(ii) An increase in skilled labor force available contributes to reducing inequality.
(iii) Increasing μ leads to a fall in inequality while an increase in productivity of technology development leads to a rise in inequality.

The increase in hazard rate enhances growth that raises inequality directly. Since the increase in haze rate raises the risk premium, the rate of return to skilled labor becomes higher than that to unskilled labor, which raises inequality. A rise in the aggregate supply of skilled labor leads to a fall in wage inequality while the aggregate supply of unskilled labor has a positive effect on the wage inequality. The relative increase of skilled labor to unskilled labor will narrow the gap of the wages between skilled and unskilled workers. The long run growth rate of technology development says that an increase in the skilled labor force available enhances growth. Even though growth give a rise in wage inequality, the direct effect of a relative increase of skilled to unskilled labor force available dominates the indirect effect. Thus, the increase in the ratio of skilled labor to unskilled labor through education meets two goals: enhancing growth and reducing inequality. Increased μ means that unskilled labor becomes more productive and the producer of final good favors unskilled labor, so the wage rate of the unskilled workers becomes higher. The production parameter of φ has something to do with the flow of technology development. A rise in this parameter enhances the growth rate of technology development, giving more returns to the skilled worker and leading to a rise in the wage inequality.

So far, we analyzed growth and inequality in the economy in which the hazard rate is given exogenously. Examining growth and inequality in the economy with endogenous hazard rate will enrich the analysis and yields more interesting results. In the next section we endogenize the hazard rate and analyze growth and inequality.

III. *Endogenous Hazard Rate*

In this section, it is assumed that each intermediate good producer faces a fixed operating cost $\phi(z)$ that the firm which enters the market to produce intermediate good with technology z should incur. Let $\phi(z) \equiv \beta w_H/z$ denote the fixed operating cost of the firm with technology z when the firm entered market, where β denotes a constant. The firm's fixed operating cost increases with the age of firm's technology when the firm entered market, implying that firm's productivity increases with knowledge. Assuming that this fixed operating cost increases with the age of firm's technology, the producer of each intermediate good maximizes profits $\pi(z)=[p(z)-w_H]x(z)-\beta w_H/z$. From the derived demand for each intermediate input given by the marginal cost pricing for the production of final good, maximizing profits for each intermediate input yields the monopoly price of each intermediate good and it is given by $p(z)=w_H/[1-\mu]$, which is markup marginal cost. It shows that the prices of all the varieties of intermediate goods at a point in time are the same and hence the quantity of each intermediate good. However, the profit of producing intermediate good becomes different among firms and it is given by $\pi(z)=\dfrac{\mu w_H H_X}{[1-\mu]A[1-\chi]}-\dfrac{\beta w_H}{z}$. In equilibrium, the firm with the lowest technology A_0 should incur zero profit, so $\beta=\dfrac{\mu H_X A_0}{[1-\mu]A[1-\chi]}$, implying $\pi(z)=\dfrac{\mu w_H H_X}{[1-\mu]A}\dfrac{1-\chi(z)}{1-\chi}$, where $\chi(z)=A_0/z$. This profit function shows that it increases with technology development. In other words, at a point in time the firm entered market later the higher profit it has.

With the profit of the firm that entered market at time t with technology A given by $\pi(A)=\dfrac{\mu w_H H_X}{[1-\mu]A}$, following the same steps with the previous section yields no-arbitrage condition as follows.

$$\frac{\dot{w}_H}{w_H}-\frac{\dot{A}}{A}=-\frac{\varphi\mu H_X}{[1-\mu]}+r+h.$$

The steady state growth rate of technology development is given by

$$\gamma_A=\mu\varphi H-[1-\mu][\rho+h]. \tag{8}$$

This equation shows the negative relation between the hazard rate and the growth rate of technology development.

The hazard rate can be determined by the zero profit condition of the firm that produces intermediate good with the lowest technology. Since $\beta=\dfrac{\mu H_X\chi}{[1-\mu][1-\chi]}$ and $\chi=\dfrac{h}{\gamma_A+h}$, with skilled labor market clearing condition the hazard rate is given by

$$h=\frac{\varphi\beta[1-\mu]\gamma_A}{\mu[\varphi H-\gamma_A]}. \tag{9}$$

This equation shows the positive relation between the hazard rate and the growth rate of technology development. The relationship between the hazard rate and the growth rate of technology development driven by equations (8) and (9) shows the following lemma.

Lemma 2.
For the positive hazard rate and growth rate, there exists a unique steady state equilibrium determining endogenous hazard rate and growth rate of technology development.

We also get limit value of growth rate as hazard rate approaches to zero as well as it approaches to infinite. As $h \to 0$, $\gamma_A \to \gamma_0 = \mu\varphi\overline{H} - [1-\mu]\rho$ and as $h \to \infty$, $\gamma_A \to \varphi\overline{H}$ by L'Hopital's rule. Contrary to the result analyzed in the previous section, when we consider the endogenous hazard rate, the economy without hazard rate grows faster in the steady state than that with hazard rate. In steady state, as can be seen from the profit function of production of intermediate goods, the hazard rate does not affect the profit for the production of intermediate goods, given the growth rate of technology development. The reason is that the increase in profit with hazard rate is offset by the overhead cost in the steady state. The share in sales of each firm existing in the market remains the same with that in the economy in the absence of hazard rate. The hazard rate, however, represents the risk premium, so it raises the cost of new technology development. The increase in the cost of new technology development will reduce the incentive to develop new technologies. The less incentive to develop new technologies leads to a fall in growth.

The endogenously determined hazard rate and growth rate of technology development can be derived from equations (8) and (9). Plugging growth rate given by equation (8) into hazard rate given by equation (9) yields $\mu h^2 + Bh - \varphi\beta\gamma_0 = 0$, where $B = \gamma_0 + \varphi\beta[1-\mu] + \rho$ and $\gamma_0 = -\mu\varphi H - [1-\mu]\rho$ denoted as growth rate of technology development in the economy without hazard rate. Solving the above quadratic equation with positive hazard rate yields

$$h = \frac{-B + \sqrt{B^2 + 4\mu\varphi\beta\gamma_0}}{2\mu}. \tag{10}$$

Plugging this hazard rate into the growth rate given by equation (8) yields

$$\gamma_A = \gamma_0 + [1-\mu]\frac{B - \sqrt{B^2 + 4\mu\varphi\beta\gamma_0}}{2\mu}. \tag{11}$$

Based on the results given by equations (10) and (11), we get the following proposition.

Proposition 3.
(i) As the parameter representing fixed operating cost increases, the hazard rate increases while the growth rate of technology development decreases. That is $\frac{\partial h}{\partial \beta} > 0$ and $\frac{\partial \gamma_A}{\partial \beta} < 0$.
(ii) An increase in skilled labor supply plays a positive role in both increasing the hazard rate and enhancing growth. That is, $\frac{\partial h}{\partial H} > 0$ and $\frac{\partial \gamma_A}{\partial H} > 0$.
(iii) The productivity parameter of R&D activity leads to a rise in both the hazard rate and the growth rate of technology development. That is, $\frac{\partial h}{\partial \varphi} > 0$ and $\frac{\partial \gamma_A}{\partial \varphi} > 0$.

The effects of β on the growth rate and the hazard rates are quite intuitive. The increase in overhead cost reduces the profits of firms producing intermediate goods, which leads to a decrease in growth. It also shortens product life cycle, so the hazard rate increases. The positive effect of H on growth is nothing new since skilled labor (human capital) is the source of growth. The increase in the production parameter of φ represents the increase in the growth rate of technology development. The change in H or φ has two effects on the hazard rate. On one hand, since the increase in H or φ speeds up growth, the level of technology is accelerated. This accelerated growth has an effect of reducing overhead cost, so the increase in H or φ increases the hazard rate. On the other hand, the hazard rate increases with H or φ, because the cost of new technology development becomes higher with the increased H or φ. Hence the effect of H or φ on the hazard rate depends on which dominates the other. This paper shows the positive effect of H or φ on the hazard rate dominates the negative effect, so the net effect is positive. With the growth rate of technology development, we get long-run wage inequality in the economy with endogenous hazard rate and it is given by

$$\omega=\frac{\varphi[1-\mu]L}{\mu\left[\varphi\bar{H}+\rho+\dfrac{B-\sqrt{B^2+4\mu\varphi\beta\gamma_0}}{2\mu}\right]}. \tag{12}$$

Compared with the economy without hazard rate, the economy with endogenous hazard rate has a lower growth rate and hence a lower wage inequality. The above inequality function yields the following proposition.

Proposition 4.
(i) As the overhead cost increases, inequality decreases.
(ii) An increase in skilled labor supply plays a positive role in reducing inequality.
(iii) The productivity parameter of R&D activity leads to a rise in inequality.

The increase in overhead cost enhances growth. This favors skilled labor devoted to the R&D activity, leading to a decrease in skilled labor devoted to the production activity. The reduced skilled labor devoted to the production activity unambiguously increases inequality. The skilled labor supply plays two roles. It reduces inequality directly, but also it increases inequality indirectly through the enhanced growth. This paper shows that the direct effect dominates the indirect effect, so inequality rises with skilled labor supply, as usual labor market predicts that a rise in skilled labor supply leads to a fall in inequality while unskilled labor supply has a positive effect on inequality. The relative increase of skilled labor to unskilled labor will narrow the gap of the wages between skilled and unskilled workers. The increase in productivity parameter of R&D activity enhances growth and gives more rate of return to the skilled labor, so it increases inequality.

The steady state product life cycle T can be obtained with the endogenously determined hazard rate and the growth rate of technology development since $\chi=A_0/A=e^{-\gamma_A T}$. The product life cycle can be characterized by

$$T=-\frac{1}{\gamma_A}\log\left[\frac{h}{\gamma_A+h}\right]. \tag{13}$$

With the steady state hazard rate and growth rate, the numerical examples show that the above product life cycle function yields the following corollary.

TABLE 1. COMPARATIVE STATICS BY NUMERICAL EXAMPLES

	γ_A	h	T	ω
β	−	+	−	−
H	+	+	−	−
φ	+	+	−	+

Notes: γ_A = growth rate of technology development, h = hazard rate, T = product life cycle, ω = wage inequality, β = a parameter of overhead cost in the production of intermediate goods, H = human capital endowed in an economy and φ = a productivity parameter in R&D activity.

TABLE 2. THE EFFECTS OF β ON γ_A, h, T AND ω

β	γ_A	h	T	ω
0.5	0.0512	0.0263	21.11	0.3418
1	0.0459	0.0424	15.99	0.3079
2	0.0388	0.0635	12.29	0.2725

Notes: Numerical examples are based on equations (10) ~ (13). Unskilled labor supply in wage inequality is normalized as one. The parameters used are: $\mu = 2/3$, $\rho = 0.02$, $\varphi = 0.1$ and $H = 1$.

Corollary 1.
The steady state length of product life cycle becomes shorten with overhead cost, skilled labor supply and/or the productivity parameter of R&D activity.

The increase in overhead cost increases the hazard rate, so it will shorten the steady state length of product life cycle. Not only the enhanced growth but also the increased hazard rate will shorten the length of product life cycle. The skilled labor supply or the productivity parameter of R&D activity enhances growth and increases the hazard rate. Thus the skilled labor supply or the productivity parameter of R&D activity has the negative effect on inequality.

We present some numerical examples to make the above comparative statics more concrete. We summarize in Table 1 the effect of policy variables on endogenous variables by numerical examples. The policy variables we use are β, H and φ and the endogenous variables are γ_A, h, T and ω. The numerical examples are based on equation (10) through (13). The basic parameters we use are: $\mu=2/3$ and $\rho=0.02$. The baseline numerical example we present consists of parameters such as $\beta=1$, $\varphi=0.1$ and $H=1$. In the baseline numerical example, growth rate of technology development is about 4.5 percent, the hazard rate is about 4.2 percent, product life cycle is about 16 years and inequality shows about 31 percent.

Table 2 shows the effects of β on γ_A, h, T and ω, given $\varphi=0.1$ and $H=1$. The growth rate of technology development, product life cycle and inequality decrease with fixed operating cost, while the hazard rate increases. First, let $\beta=0.5$. Then, the growth rate of technology development increases from 4.6 percent in the baseline example to 5.1 percent. The hazard rate decreases from 4.2 percent to 2.6 percent. Product life cycle increases from 16 years to 21 years. Inequality increases from 31 percent to 34 percent. Next, raise the parameter value in fixed operating cost to $\beta=2$. Then, the growth rate of technology development decreases from 4.6 percent in the baseline example to 3.9 percent. The hazard rate increases from 4.2 percent to 6.4 percent. Product life cycle decreases from 16 years to 12 years. Inequality decreases from 31 percent to 27 percent.

The effects of H γ_A, h, T and ω, given $\varphi=0.1$ and $\beta=1$, are provided in Table 3. The

TABLE 3. THE EFFECTS OF H ON γ_A, h, T AND ω

H	γ_A	h	T	ω
0.5	0.0176	0.0272	28.37	0.5145
1	0.0459	0.0424	15.99	0.3079
2	0.1074	0.0579	9.77	0.1799

Notes: Numerical examples are based on equations (10) ~ (13). Unskilled labor supply in wage inequality is normalized as one. The parameters used are: $\mu=2/3$, $\rho=0.02$, $\varphi=0.1$ and $\beta=1$.

TABLE 4. THE EFFECTS OF φ ON γ_A, h, T AND ω

φ	γ_A	h	T	ω
0.05	0.0208	0.0177	37.33	0.2849
0.1	0.0459	0.0424	15.99	0.3079
0.2	0.0959	0.0922	7.43	0.3203

Notes: Numerical examples are based on equations (10) ~ (13). Unskilled labor supply in wage inequality is normalized as one. The parameters used are: $\mu=2/3$, $\rho=0.02$, $\varphi=0.1$ and $H=1$.

effect of H on γ_A is opposite to the effect of β on γ_A, while the effects of H on h, T and ω are similar to the effects of β on h, T and ω. The growth rate of technology development and the hazard rate increase with skilled labor supply, while product life cycle and inequality decrease. First, let $H=0.5$. Then, the growth rate of technology development decreases from 4.6 percent in the baseline example to 1.8 percent. The hazard rate decreases from 4.2 percent to 2.7 percent. Product life cycle increases from 16 years to 28 years. Inequality increases from 31 percent to 51 percent. Next, raise the parameter value in skilled labor supply to $H=2$. Then, the growth rate of technology development increases from 4.6 percent in the baseline example to 10.7 percent. The hazard rate increases from 4.2 percent to 5.8 percent. Product life cycle decreases from 16 years to 10 years. Inequality decreases from 31 percent to 18 percent.

We provide in Table 4 the effects of φ on γ_A, h, T and ω, given $H=1$ and $\beta=1$. The effect of φ on ω is opposite to the effect of H on ω, while the effects of φ on h, T and ω are similar to the effects of H on h, T and ω. The growth rate of technology development, the hazard rate and inequality increase with productivity parameter of R&D activity, while product life cycle decreases. First, let $\varphi=0.05$. Then, the growth rate of technology development decreases from 4.6 percent in the baseline example to 2.1 percent. The hazard rate decreases from 4.2 percent to 1.8 percent. Product life cycle increases from 16 years to 37 years. Inequality decreases from 31 percent to 28 percent. Next, raise the productivity parameter value of R&D activity to $\varphi=0.2$. Then, the growth rate of technology development increases from 4.6 percent in the baseline example to 9.6 percent. The hazard rate increases from 4.2 percent to 9.2 percent. Product life cycle decreases from 16 years to 7 years. Inequality increases from 31 percent to 32 percent.

Inequality phenomena in U.S. and UK, measured as Gini coefficient, are characterized as increasing pattern during past 25 years, even though skilled labor supply measured as secondary school enrollment rate increases in both countries. However, as provided in our model and numerical examples, if the fixed operating cost decreases and the productivity of R&D activity increases, growth will be enhanced and inequality will rise. Those results imply that inequality phenomena can be explained by fixed operating cost and productivity of R&D activity. In other words, if the fixed operating cost decreases and the productivity of R&D activity increases, and the positive effects of them on inequality are large enough to alleviate the negative effect of skilled labor on inequality, then inequality will rise with growth enhanced.

IV. *Conclusion and Extension*

This paper suggested another look at explaining inequality phenomena. This paper explains inequality phenomena by innovation and market structure. We presented a R&D-driven growth model that captures endogenous economic growth, hazard rate, product life cycle and inequality. The basic general equilibrium model with the endogenous hazard rate provided in this paper yields a close link between inequality and market structure. The numerical examples make the results concrete.

Interesting finding is that the decrease in overhead cost enhances growth and reduces hazard rate. Thus, inequality increases with decreasing overhead cost. We also find that the increase in skilled labor supply plays a positive role in increasing growth and hazard rate, while it contributes to a fall in product life cycle and inequality. The productivity parameter of R&D activity unambiguously enhances growth, hazard rate and thus inequality, but it reduces product life cycle. Thus, if the positive effects of the overhead cost saving and the productivity of R&D activity on inequality dominates the negative effect of skilled labor supply on inequality, then inequality increases.

This paper examined endogenous growth, hazard rate, product life cycle and inequality in a closed economy. Analyzing them in an open economy would be interesting. This paper is analyzed on the model of the varieties of product development, so incorporating quality ladder model remains in the future research.

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