Possibility and Optimality of Agreements in International Negotiations on Climate Change*

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Abstract

We build a three-stage model of international negotiations on regulation of the level of total emissions of greenhouse gases, and examine the possibility of a cooperative agreement and the Pareto optimality of the outcome. First, we derive the condition for Pareto optimal allocations, which is an extension of the celebrated Lindahl-Bowen-Samuelson condition and Chichilnisky et al.'s (2000) result. Next, we show that if the distribution rule of initial emission permits is the proportional rule to the Nash equilibrium emissions in the noncooperative game, then some cooperative agreement can be reached in the negotiations. However, for many other (equitable) distribution rules, no cooperative agreement is possible. Even if a cooperative agreement is attained, the outcome is rarely Pareto optimal.

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1 Introduction

The Kyoto Protocol to the United Nations Framework Convention on Climate Change stipulates the limitation or reduction of greenhouse gas emissions in the developed countries and others for five years from 2008 to 2012 according to the quantified commitments. The limitation of greenhouse gas emissions after this period, however, is not determined yet, and it will soon become the most important issue of international negotiations. In this paper, we build a model of international negotiations for reduction of greenhouse gas emissions with emission permits trading, and examine whether a cooperative agreement can be reached, and moreover a Pareto optimal allocation can be attained at the cooperative agreement if any.

Although every country places a positive value on reduction of greenhouse gas emissions, marginal willingness to sacrifice its own consumption for improvement of the environment may vary among countries. For such a profile of the valuation functions of the countries over consumption and greenhouse gas emissions, which we call the *welfare functions* of the countries, we derive the condition for the Pareto optimal levels of greenhouse gas emissions, production and consumption.

A new feature of our model is that it takes account of emissions of greenhouse gases *from consumption activities as well as from production activities*. It is important to incorporate emissions from consumption in the present model because under the trading system of emission permits, production and consumption may take place in different countries. For instance, if Russia sells a lot of permits, it consumes more than it produces, emitting more gases from consumption. Our condition for Pareto optimality is the extension of the celebrated Lindahl-Bowen-Samuelson condition and Chichilnisky et al.'s (2000) result to the case where consumption of commodities also generates the public bad.

Next, we introduce a three-stage model of international negotiations and emission permits trading. In the first stage, all the countries negotiate on the level of total emissions of greenhouse gases, given the distribution rule of the permits. In the second stage, each country determines its domestic rule on requirements for emission permits. In the third stage, the market of emission permits is open, and an equilibrium price of a permit is established. At the equilibrium, each country produces and consumes commodities, and emits the amount of greenhouse within the limit determined in the first stage. Hence, the total amount of emission permits determines the equilibrium level of consumption of each country, and thereby its final welfare level.

We analyze how the limitation of the total amount of greenhouse gas emissions affects the consumption of each country. In particular, we take into account the "feedback effect": an increase in the amount of emission permits for a country may raise the consumption of the country, but not as much as the same amount because the increase in consumption itself accompany greenhouse gas emissions, requiring additional permits.

Based on the analysis, we apply the Nash bargaining theory (Nash, 1950) to the negotiations in the first stage of the model. We draw the *bargaining frontier*, namely the locus of welfare vectors of the countries attained through a choice of the total amount of emission permits. We show that parts of the bargaining frontier are strictly below the *Pareto frontier*, namely the locus of welfare vectors that are technologically feasible.

Then, we define the *disagreement point* by the vector of welfare levels of the countries at a Nash equilibrium in a noncooperative emission game where no regulation is imposed on emissions. We show that if the distribution rule of initial emission permits is the proportional rule to the Nash equilibrium emission levels in the noncooperative emission game, then the disagreement point is inside the bargaining frontier. For many other distribution rules, however, the disagreement point is located outside the bargaining frontier, in which cases no cooperative agreement can be reached. Even if it is inside the bargaining frontier, the Nash bargaining solution (or any other bargaining solution) may not be Pareto optimal because the bargaining frontier is below the Pareto frontier.

There are several related works to this paper. Most closely related are Chichilnisky et al.'s (2000) and Prat (2000). Chichilnisky et al.'s (2000) considered the case where the total amount of emission permits is fixed, and obtained a striking result that almost all distributions of the initial permits are incompatible with Pareto optimality of the final allocation. Prat (2000), like the present paper, studied the case in which the distribution rule of the initial permits is given, and showed that, under some regularity conditions, only one level of total emissions is compatible with Pareto optimality of the final allocation. Prat also proved that the preferences of each country is single-peaked as to the levels of total emissions, and hence, by the median voter theorem, there exists a unique winning level of total emissions in majority voting.

Differences of the present paper from these works are as follows: First, we take account of emissions of greenhouse gases not only from production but also from consumption. Second, we apply the Nash bargaining theory to international negotiations on the levels of total emissions, instead of the theory of majority voting. It is often the case that international negotiations for reduction of greenhouse gas emissions break down without any agreement. The Nash bargaining theory is useful to explain why it is so difficult to reach an agreement in the negotiations.

Okada (2003) presented a two-stage game model of international negotiations on emission permits, which is similar to our model. However, he assumed that the total amount of emission permits is fixed, and considered negotiations on distributions of the fixed total amount among countries. He showed that the core of the voting game on distributions of emission permits is empty. In contrast, we assume that a distribution rule of permits among countries is given, and focus on negotiations about the level of global emissions.[1]

There are two issues in the abatement of global warming. One is the level of global emissions of greenhouse gases, and the other is the distribution of initial emission permits among countries. When there are multiple issues in negotiations, participants often try to make an agreement issue by issue. Hence, we may consider two procedures in negotiations such as follows:

(i) First the countries determine the total amount of emissions, and then they negotiate on distributions of the total.

(ii) First the countries agree on a distribution *rule*, and then they decide the total amount of emissions.[2]

This paper focuses on the procedure of type (ii). A possible scenario of procedure (ii) may be described as follows, using the negotiations for the Kyoto protocol as an example. The countries first agree (implicitly) on the relative burdens of reduction of greenhouse gases. For instance, the reduction rate for Japan should be one percent lower than that for the United States, which should in turn be one percent lower than that for the European Union, and so on. Then, assuming this distribution rule is (implicitly) accepted, they agree that the total emissions should be reduced by 5.2 percent below

[1]Recent contributions which shed light on other aspects of international negotiations and agreements on climate change are Carraro and Siniscalco (1993), Barrett (1994), Helm (2003), Lange and Vogt (2003), and Shiell (2003).

[2]There have been long debates on which distribution rules are fair or just, a proportional rule to past emissions, to GDPs, to populations, or to the costs of reducing emissions? In this paper, we will not study this normative issue, but consider various cases of distribution rules.

the 1990 level. This agreement then determines the absolute value of the reduction rate for each country (6 percent for Japan, 7 percent for the US, and 8 percent for EU).

The rest of the paper is organized as follows. The next section presents the basic assumptions on technology and preferences of each country. In section 3 we show the extension of the Lindahl-Bowen-Samuelson condition and Chichilnisky et al.'s (2000) result on Pareto optimal allocations. Section 4 presents and analyzes the three-stage model on international negotiations and emission permits trading. In section 5 we examine the bargaining frontier and the disagreement point in international negotiations, and discuss the possibility of a cooperative agreement and the Pareto optimality of the outcome. Section 6 presents numerical examples in which the Nash bargaining solution is not Pareto optimal, and section 7 concludes.

2 Technology and preferences

There are n countries, $N = \{1, \dots, n\}$. Let $y_i \in \mathbb{R}_+$ denote the gross domestic product (GDP) of country $i \in N$, $c_i \in \mathbb{R}_+$ the consumption of country i. Both production and consumption are accompanied by emissions of greenhouse gases. Let $x_i^p \in \mathbb{R}_+$ denote the emission of greenhouse gases from production. The relation of x_i^p and y_i is represented by the function $x_i^p = f_i(y_i)$, where $f_i' > 0$, $f_i'' > 0$. Let $x_i^c \in \mathbb{R}_+$ denote the emission of greenhouse gases from consumption. The relation of x_i^c and c_i is represented by the function $x_i^c = g_i(c)$, where $g_i' > 0$, $g_i'' \geq 0$. Let $x_i := x_i^p + x_i^c$ be the total emission of greenhouse gases of country i, and let $X := \sum_{i \in N} x_i$ be the level of global emissions of greenhouse gases.

Each country i has preferences over pairs $(c_i, X) \in \mathbb{R}_+^2$ of its own consumption and an amount of global emissions of greenhouse gases. The preferences are represented by a continuously differentiable and strictly quasi-concave function $V_i : \mathbb{R}_+^2 \to \mathbb{R}$. We call the function V_i *the welfare function of country i*.

The partial derivative of V_i with respect to the variable a is denoted by $D_a V_i$. We assume that $D_c V_i > 0$ and $D_X V_i < 0$. We also assume that for every $X \in \mathbb{R}_+$, $\lim_{c_i \to 0} \frac{D_X V_i(c_i, X)}{D_c V_i(c_i, X)} = 0$. This assumption means that the marginal willingness to sacrifice consumption for reduction of greenhouse gas emissions approaches zero as the amount of consumption goes to zero.

3 Pareto optimal allocations

A greenhouse gas is a public "bad" in the sense that an increase in greenhouse gas emissions makes every country worse off. There are several features of emissions of greenhouse gases that are distinct from other public goods or bads. First, every country produces the public bad (greenhouse gases) through *both production and consumption* of commodities. For example, suppose that country A produces automobiles and country B imports them. Then, not only country A but also country B increases its emission of greenhouse gases by driving more automobiles.

Second, the relation of emission of greenhouse gases to production or consumption varies widely among countries, depending on technology. For instance, to attain a given level of production, Japan emits a relatively smaller amount of greenhouse gases than Russia because Japan has developed technology to save oil and other resources.

The celebrated Lindahl-Bowen-Samuelson condition (Samuelson, 1954) states that at a Pareto optimal allocation, the sum of the marginal rates of substitution between the (composite) commodity and the public good over all individuals should be equal to the marginal rate of transformation between the two goods. Chichilnisky et al.(2000) extended the Lindahl-Bowen-Samuelson condition to the case where the public good (bad) is greenhouse gas emission. However, they assume that only production generates greenhouse gases. Next we derive the conditions for Pareto optimal allocations in the present model where consumption of commodities also produces greenhouse gases.

An *allocation* is a vector $(y, c, x) := (y_1, \ldots, y_n; c_1, \ldots, c_n; x_1, \ldots, x_n) \in \mathbb{R}^{3n}_+$. An allocation (y, c, x) is *technologically feasible* if and only if
(i) $\sum_{h\in N} y_h = \sum_{h\in N} c_h$ and
(ii) for every $j \in N$, $x_j = f(y_j) + g(c_j)$.
We say that an allocation (y, c, x) *Pareto dominates* an allocation $(\tilde{y}, \tilde{c}, \tilde{x})$ if and only if
(i) for every $j \in N$, $V_j(c_j, \sum_{h\in N} x_h) \geq V_j(\tilde{c}_j, \sum_{h\in N} \tilde{x}_h)$, and
(ii) for some $j \in N$, $V_j(c_j, \sum_{h\in N} x_h) > V_j(\tilde{c}_j, \sum_{h\in N} \tilde{x}_h)$.
An allocation (y^*, c^*, x^*) is *Pareto optimal* if and only if it is technologically feasible and there is no technologically feasible allocation that Pareto dominates it.

Let $(y^*, c^*, x^*) \gg 0$ be a Pareto optimal allocation. For each $j \in N$, define $\bar{V}_j := V_j(c^*_j, \sum_{h\in N} x^*_h) \in \mathbb{R}$. Let $i \in N$ be given. Then, (y^*, c^*, x^*) is a solution for the following constrained maximization problem:

$$\max_{(y,c,x)\in\mathbb{R}^{3n}_+} V_i(c_i, \sum_{i\in N} x_i)$$

subject to

$$\forall j \in N,\ x_j = f(y_j) + g(c_j) \tag{1}$$

$$\sum_{h\in N} y_h = \sum_{h\in N} c_h \tag{2}$$

$$\forall j \in N, j \neq i,\ V_j(c_j, \sum_{h\in N} x_h) = \bar{V}_j \tag{3}$$

Define the Lagrangean function as

$$\begin{aligned} & L((c_h)_{h\in N}, (x_h)_{h\in N}, (y_h)_{h\in N}, (\lambda_j)_{j\in N, j\neq i}, (\gamma_h)_{h\in N}, \delta) \\ = & \ V_i(c_i, \sum_{h\in N} x_h) - \sum_{j\neq i} \lambda_j (V_i(c_j, \sum_{h\in N} x_h) - \bar{V}_j) \\ & - \sum_{j\in N} \gamma_j (x_j - f_j(y_j) - g_j(c_j)) - \delta(\sum_{h\in N} y_h - \sum_{h\in N} c_h). \end{aligned}$$

From the first order conditions,

$$D_{c_i}L = D_{c_i}V_i(c_i^*, \sum_{h\in N} x_h^*) + \gamma_i g_i'(c_i^*) + \delta = 0 \tag{4}$$

$$D_{x_i}L = D_X V_i(c_i^*, \sum_{h\in N} x_h^*) - \sum_{j\neq i} \lambda_j D_X V_j(c_j^*, \sum_{h\in N} x_h^*) - \gamma_i = 0 \tag{5}$$

$$D_{y_i}L = \gamma_i f_i'(y_i^*) - \delta = 0 \tag{6}$$

and for each $j \neq i$,

$$D_{c_j}L = -\lambda_j D_{c_j} V_i(c_i^*, \sum_{h\in N} x_h^*) + \gamma_j g_j'(c_j^*) + \delta = 0 \tag{7}$$

$$D_{x_j}L = D_X V_i(c_i^*, \sum_{h\in N} x_h^*) - \sum_{j\neq i} \lambda_j D_X V_j(c_j^*, \sum_{h\in N} x_h^*) - \gamma_j = 0 \tag{8}$$

$$D_{y_j}L = \gamma_j f_j'(y_j^*) - \delta = 0 \tag{9}$$

From equations (5) and (7), $\gamma_i = \gamma_j$ for all $i, j \in N$. Hence, from equations (6) and (9), $f_i'(y_i^*) = f_j'(y_j^*)$ for all $i, j \in N$. Thus, we have the following result.

Proposition 1 Production Efficiency. *At a Pareto optimal allocation, the marginal emission from production is the same for all the countries.*

Solving the system of equations (4)-(9), we have

$$\sum_{j \in N} \frac{D_X V_j(c_j^*, \sum_{h \in N} x_h^*)}{D_{c_j} V_j(c_j^*, \sum_{h \in N} x_h^*)} \cdot (f_j'(y_j^*) + g_j'(c_j^*)) = -1$$

Define

$$\eta_i(c_i, X) := \left| \frac{D_X V_i(c_i, X)}{D_c V_i(c_i, X)} \right| \tag{10}$$

The value $\eta_i(c_i, X)$ is the absolute value of country i's marginal rate of substitution of its own consumption for global emission of greenhouse gases. Then,

$$\sum_{j \in N} \eta_j(c_j^*, \sum_{h \in N} x_h^*) \cdot (f_j'(y_j^*) + g_j'(c_j^*)) = 1 \tag{11}$$

Proposition 2 *At a Pareto optimal allocation, the weighted sum of the marginal rates of substitution of consumption for global emission of greenhouse gases over all the countries is equal to one where each weight is the sum of the marginal emission from production and the marginal emission from consumption in each country.*

From Proposition 1, $f_i'(y_i^*) = f_j'(y_j^*)$ for all $i, j \in N$. Hence, *if there is no external effect of consumption of commodities*, that is, $g_i' = 0$ for all $i \in N$, then equation (11) becomes

$$\sum_{j \in N} \eta_j(c_j^*, \sum_{h \in N} x_h^*) = \frac{1}{f_i'(y_i^*)} \text{ for all } i \in N. \tag{12}$$

The value $\frac{1}{f_i'(y_i^*)}$ is the marginal rate of transformation between consumption of commodities and emission of greenhouse gases, and the above equation coincides with Chichilnisky et al.'s (2000) condition. In other words, our condition is an extension of theirs to the case where consumption of commodities has also an external effect on the production of the public bad (or good).

4 Three-stage model of international negotiations and trades

In this section, we consider a three-stage model of international negotiations and emission permits trading. We assume that that a distribution rule of emission permits among countries is given. The rule may be the proportional rule to the levels of GDP at a benchmark year, the proportional rule to populations, or the proportional rule to the costs of reducing greenhouse gas gases, etc. In the first stage, all the countries negotiate on the total amount of emissions of greenhouse gases, given the distribution rule of emission permits. In the second stage, each country determines the domestic rule on the requirement for emission permits. In the third stage, firms and consumers act so as to maximize their objectives, and the levels of production, consumption and emissions of greenhouse gases are determined at a market equilibrium.

4.1 Equilibrium in the market of emission permits

To analyze the game backward, we first focus on market equilibria of emission permits in the third stage of the game. At a given price of the permit, how is the demand for emission permits determined?

Consider the following policy of the government of country $i \in N$.

Policy (A): the government requires that each producer should obtain emission permits at the international price for the amount of greenhouse gases emitted in the process of production, and that each consumer should also obtain permits for the amount emitted in the process of consumption of the commodity.

A point in this policy is that each producer should be responsible only for greenhouse gases emitted in the process of production, and should not be responsible for those emitted in the process of consumption of its product. Later we will see that the policy (A) is indeed an optimal choice of the government at the second stage.

Let $q \in \mathbb{R}_+$ be the international price of an emission permit. Then, the marginal revenue of production is one since the price of the (composite) commodity is one, while the marginal cost of production due to the requirement for emission permits is $q\, f'(y)$ under the above policy. Hence, from profit maximization of firms, the level of production in country i, denoted y_i^*, is determined by

$$1 = q\, f'(y_i^*). \tag{13}$$

Given a level of production, the level of consumption is determined by the equivalence between the national income and the national expenditure.[3] Let $\bar{x}_i \in \mathbb{R}_+$ be the initial assignment of emission permits to country $i \in N$. Let $q \in \mathbb{R}_+$ be a given international price of the emission permit. When the levels of production and consumption are y_i and c_i, respectively, the total amount of emissions of greenhouse gases in country i is $f_i(y_i) + g_i(c_i)$. Then, the revenue from (or the expenditure for, if it is negative) emission permits is equal to $q\,(\bar{x}_i - f_i(y_i) - g_i(c_i))$. Thus, the national income is given by $y_i + q\,(\bar{x}_i - f_i(y_i) - g_i(c_i))$. This should be equal to the national expenditure c_i. We therefore say that a consumption-production pair (c_i, y_i) is *feasible with emission permits trading* for country i at q and $\bar{x}_i$ if and only if

$$c_i = y_i + q\,(\bar{x}_i - f_i(y_i) - g_i(c_i)). \tag{14}$$

For each $c_i \in \mathbb{R}_+$, since $1 + q\, g_i'(c_i) > 0$, by the implicit function theorem, equation (14) can be locally solved for c_i as a function of y_i. Denote the function $c_i(y_i)$. For each $y_i \in \mathbb{R}_+$, $c_i(y_i) \in \mathbb{R}_+$ is the level of consumption when the level of production is y_i.

Let us turn to the second stage of the game in which the government of each country determines the domestic rule on emission permits trading. We assume that for each $i \in N$, the objective of the government of country i is to maximize its welfare V_i. In the second stage of the game, however, the government of country i regards the level of global emissions of greenhouse gases as fixed since it is determined in international negotiations in the first stage. Hence, in order to maximize V_i, it should choose a policy or a rule that will maximize its domestic consumption in the third stage.

To find such a rule or a policy, it is necessary to examine how the level of consumption changes as the level of production changes. By differentiating the function $c_i(\cdot)$,

$$c_i'(y_i) = \frac{1 - q\, f_i'(y_i)}{1 + q\, g_i'(c_i(y_i))} < 1 \tag{15}$$

and

$$c_i''(x_i) = \frac{A}{[1 + q\, g_i'(c_i(y_i))]^2} \tag{16}$$

[3]Notice that in the present model, there is only one (composite) commodity.

where

$$A = -q\, f_i''(y_i)[1+q\, g_i'(c_i(y_i))] - f_i'(y_i) - g_i(c_i(y_i)) - q\, g_i''(c_i(y_i))c_i'(y_i)[1-q\, f_i'(y_i)].$$

If there were no requirement to obtain emission permits, an increase in y_i raises consumption by the same amount. With the requirement for emission permits, however, a unit of increase in y_i induces more payment for emission permits, and therefore can increase consumption by less than one unit as shown in equation (15).

Let y_i^* be the level of production of country i that maximizes its own consumption. By equation (16), $c_i''(y_i) < 0$ whenever $c_i'(y_i) = 0$. Hence, the necessary and sufficient condition for a local optimum, y_i^*, is $c_i'(y_i^*) = 0$. By equation (15), we have

$$q = \frac{1}{f_i'(y_i^*)}. \tag{17}$$

In general, one unit of increase in production generates $f_i'(y_i)$ units of emission of greenhouse gases, which increases the payment for (or decreases the revenue from) emission permits by $q\, f_i'(y_i)$. If consumption also increased, then more payment for emission permits would be necessary. At the optimum, however, consumption should never increase by the change in production. Hence, the increase in production should be just offset by the increase in the payment for (or the decrease in the revenue from) emission permits from production. That is why $q\, f_i'(y_i) = 1$ and equation (17) hold at the optimal level of production y_i^*.

However, as one notice by comparing equation (17) with equation (13), the optimal amount y_i^* is exactly the amount of production attained through profit maximization of firms under the policy (A). Therefore, to choose this policy is indeed an optimal choice of the government in the second stage of the game.

Proposition 3 *Consider country $i \in N$. In the second stage of the game, the policy (A) is an optimal choice of the government of country i whose objective is to maximize the welfare V_i of country i. Under the policy, and at any given price q of the emission permit, the amount of production y_i^* of country i is determined by*

$$q = \frac{1}{f_i'(y_i^*)}.$$

Then the amount of consumption $c_i(y_i^*)$ is the maximal amount under the price q and the initial assignment $\bar{x}_i$ of emission permits.

From Proposition 3, we have $f_i'(y_i^*) = f_j'(y_j^*) = \frac{1}{q}$ for all $i, j \in N$. Hence, we obtain the following corollary.

Corollary 1 *Any allocation attained in the third stage of the game satisfies the condition of production efficiency.*

4.2 The demand function for emission permits

Having examined how the production and the consumption of country i are determined in the third stage of the model, this subsection studies the property of the demand function for emission permits of country i. That is, we investigate how the demand for emission permits changes as the price of the permit changes.

For each $q \in \mathbb{R}_+$, let $y_i(q) \in \mathbb{R}_+$ be the amount of production of country i at q in the third stage of the game. Since $f''(y_i) > 0$ for all $y_i \in \mathbb{R}_+$, the function $\frac{1}{f'(y_i)}$ is decreasing in y_i. From Proposition 3, $y_i(q)$ is decreasing in q.

Define $x_i^p(q) := f_i(y_i(q))$. $x_i^p(q)$ is the emission from production of country i at q. Define $x_i^c(q) := g_i(c_i(y_i(q)))$. $x_i^c(q)$ is the emission from consumption of country i at q. Let $x_i(q) := x_i^p(q) + x_i^c(q)$. Then, $x_i(q)$ is the gross demand for emission permits of country i at q.

We would like to examine whether the gross demand for emission permits is decreasing in q or not. As we have already seen, if q rises, then $y_i(q)$ decreases, and hence $x_i^p(q)$ also decreases. We need to check whether $x_i^c(q)$ is decreasing in q or not.

By definition, for all $q \in \mathbb{R}_{++}$,

$$c_i(y_i(q_i)) = y_i(q) + q\,[\bar{x}_i - f_i(y_i(q)) - g_i(c_i(y_i(q)))].$$

Differentiating both sides,

$$\begin{aligned} c_i'(y_i(q))y_i'(q) &= [1 - q\,f_i'(y_i(q))]y_i'(q) + \bar{x}_i - f_i(y_i(q)) - g_i(c_i(y_i(q))) \\ &\quad -q\,g_i'(c_i(y_i(q)))c_i'(y_i(q))y_i'(q). \end{aligned}$$

Since $1 - q\,f_i'(y_i(q)) = 0$ from Proposition 3, we have

$$\begin{aligned} [1 + q\,g_i'(c_i(y_i(q)))]c_i'(y_i(q))y_i'(q) &= \bar{x}_i - f_i(y_i(q)) - g_i(c_i(y_i(q))) \\ &= \bar{x}_i - x_i^p(q) - x_i^c(q). \end{aligned}$$

Hence,

$$\frac{dc}{dq} = c_i'(y_i(q))y_i'(q) = \frac{\bar{x}_i - x_i^p(q) - x_i^c(q)}{1 + q\, g_i'(c_i(y_i(q)))}. \tag{18}$$

Because $g_i'(c_i) > 0$ for all $c_i \in \mathbb{R}_+$, it follows that

$$\frac{dc}{dq} \geq 0 \Leftrightarrow \bar{x}_i - x_i^p(q) - x_i^c(q) \geq 0$$

and

$$\frac{dc}{dq} \leq 0 \Leftrightarrow \bar{x}_i - x_i^p(q) - x_i^c(q) \leq 0.$$

This means that if country i is a *net supplier* of emission permits, then an increase in the price q induces higher consumption through the income effect, which then raises the emission from consumption, x_i^c. That is, $x_i^c(q)$ is increasing in q. Therefore, it is ambiguous whether the total demand for emission permits of country i, $x_i(q) = x_i^p(q) + x_i^c(q)$, is decreasing or not. If $|x_i^{p\prime}(q)| < |x_i^{c\prime}(q)|$, then the total demand is increasing in q.

On the other hand, if country i is a *net demander* of emission permits, then both $x_i^{p\prime}(q)$ and $x_i^{c\prime}(q)$ are negative. Thus, the total demand for emission permits of country i is decreasing in q.

What about the *aggregate demand* for emission permits? Let $X^D(q) := \sum_{i \in N} x_i(q)$ be the aggregate demand for emission permits at q. Since $x_i(q)$ may be increasing or decreasing in q, it is ambiguous in general whether the aggregate demand $X^D(q)$ is decreasing in q or not. However, under an additional assumption, we can determine the sign of $X^{D\prime}(q)$ at an equilibrium price.

Assumtion L (Linearity in Emission from Consumption):
For some constant $\alpha \in \mathbb{R}_{++}$, $g_i(c_i) = \alpha c_i$ for all $i \in N$.

Assumption L means that (i) emissions from consumption are proportional to the levels of consumption, and (ii) per unit emissions from consumption are the same for all the countries. While the relations of emissions of greenhouse gases with production vary widely among countries, depending on the production technologies to save energies, emissions from consumption may be nearly proportional to the amount of consumption, and moreover, the differences among countries in per unit emissions from consumption seem small. For example, emissions of greenhouse gases from cooking, heating, driving, exercising, etc., would be proportional to the amount of consumption, and there is little difference in per unit emissions among countries.

Under Assumption L, we have

$$\begin{aligned} X^{D'}(q) &= \sum_{i\in N} x_i{}'(q) \\ &= \sum_{i\in N} x_i^{p\prime}(q) + \sum_{i\in N} x_i^{c\prime}(q) \\ &= \sum_{i\in N} x_i^{p\prime}(q) + \sum_{i\in N} g_i'(c_i(y_i(q)))\, c_i'(y_i(q))\, y_i'(q) \\ &= \sum_{i\in N} x_i^{p\prime}(q) + \alpha \sum_{i\in N} c_i'(y_i(q))\, y_i'(q). \end{aligned}$$

By equation (18),

$$X^{D'}(q) = \sum_{i\in N} x_i^{p\prime}(q) + \frac{\alpha}{1+\alpha\, q} \sum_{i\in N} [\bar{x}_i - x_i^p(q) - x_i^c(q)]. \tag{19}$$

Let $q^* \in \mathbb{R}_+$ be an equilibrium price of the emission permit when the total supply of emission permits is a given $\bar{X} \in \mathbb{R}_+$. Then,

$$\begin{aligned} \sum_{i\in N} [\bar{x}_i - x_i^p(q^*) - x_i^c(q^*)] &= \sum_{i\in N} \bar{x}_i - \sum_{i\in N} x_i(q^*) \\ &= \bar{X} - X^D(q^*) \\ &= 0 \end{aligned} \tag{20}$$

Substituting (20) into (19),

$$X'(q^*) = \sum_{i\in N} x_i^{p\prime}(q^*) < 0.$$

Thus, we have obtained the following proposition.

Proposition 4 *Let $q^* \in \mathbb{R}_+$ be an equilibrium price of the emission permit. Under Assumption L, the aggregate demand function for emission permits is decreasing in a small neighborhood of q^*.*

4.3 Maximization of the welfare of each country in international negotiations

The previous subsection showed the property of the aggregate demand function for emission permits. Based on the property, this subsection examines how the equilibrium consumption of each country changes as the total supply of emission permits changes. This reveals a trade-off between reduction of greenhouse gases and consumption of each country. Then, for each country, the condition for an optimal amount of total supply of emission permits is determined, taking the trade-off into account.

Let $q(X)$ be the equilibrium price when the total supply of emission permits is X. It follows form Proposition 4 that $q'(X) < 0$ under Assumption L.

We assume that the assignment of emission permits to each country is proportional to the total supply of emission permits. Let $(\theta_1, \ldots, \theta_n) \in]0, 1[^n$ with $\sum_{i \in N} \theta_i = 1$ be the proportional factors.

In the first stage of the game, each country claims how much the total supply of emission permits should be. Each country tries to maximize its welfare $V_i(c_i, X)$, taking account of the fact that, once the total supply X is determined in the first stage, the production and the consumption of country i are determined at the equilibrium price $q(X)$ in the final stage. Hence, the welfare of each country is a function of X.

With slight abuse of notation, let $c_i(X)$ be the consumption level of country $i \in N$ when the total supply of emission permits is X. For each $i \in N$, define

$$W_i(X) := V_i(c_i(X), X).$$

Then,

$$W_i'(X) = D_c V_i(c_i, X) c_i'(X) + D_X V_i(c_i, X). \tag{21}$$

A change in the total supply of emission permits X affects the welfare of country i through two routes: one is the direct effect represented by $D_X V_i(c_i, X)$, and the other is the indirect effect through the change in consumption, $D_c V_i(c_i, X) c_i'(X)$. To examine how the consumption changes with a change in X, notice first that by feasibility,

$$c_i(X) = y_i(q(X)) + q(X)[\theta_i X - f_i(y_i(q(X))) - g_i(c_i(y_i(q(X))))].$$

Differentiating,

$$\begin{aligned} c_i'(X) \;&=\; y_i'(q) q'(X) + q'(X)[\theta_i X - f_i(y_i) - g_i(c_i)] \\ &\quad + q(X)[\theta_i - f_i'(y_i) y_i'(q) q'(X) - g_i'(c_i) c_i'(X)]. \end{aligned} \tag{22}$$

The terms of equation (22) represent the following effects of an increase in X on c_i.

(i) $y_i'(q)q'(X) > 0$: the increase in production due to the fall in q.
(ii) $q'(X)[\theta_i X - f_i(y_i) - g_i(c_i)]$: the change in revenue from (or the payment for) emission permits due to the fall in q. If country i is a net supplier of emission permits, then $q'(X)[\theta_i X - f_i(y_i) - g_i(c_i)] < 0$, and if it is a net demander, $q'(X)[\theta_i X - f_i(y_i) - g_i(c_i)] > 0$.
(iii) $q(X)\,\theta_i > 0$: the increase in the revenue from (or the decrease in payment for) emission permits due to the increase in the initial assignment of emission permits to country i.
(iv) $-q(X)[f_i'(y_i)y_i'(q)q'(X)] < 0$: the decrease in the revenue from (or the increase in payment for) emission permits due to the increase in emission through production.
(v) $-q(X)[g_i'(c_i)c_i'(X)]$: the change in the revenue from (or the payment for) emission permits due to the change in emission through consumption.

By Proposition 3, $q(X)f'(y_i(q(X))) = 1$. Hence, the effect (i) is just offset by the effect (iv). This is because each country maximizes its consumption in the second stage of the game. Therefore,

$$c_i'(X) = \frac{\theta_i q(X) + q'(X)[\theta_i X - f_i(y_i) - g_i(c_i)]}{1 + q(X)g_i'(c_i)} \tag{23}$$

The value $c_i'(X)$ may be called the *marginal opportunity cost of reduction of greenhouse gases for country i in terms of its own consumption under the system of emission permits trading.*

Substituting (23) and (10) into (21), we have

$$\frac{W_i'(X)}{D_c V_i(c_i, X)} = \frac{\theta_i q(X) + q'(X)[\theta_i X - f_i(y_i) - g_i(c_i)]}{1 + q(X)g_i'(c_i)} - \eta_i(c_i, X). \tag{24}$$

Let X_i^* be the amount of total supply of emission permits that is optimal for country i, and let $y_i^* := y_i(q(X_i^*))$ and $c_i^* := c_i(X_i^*)$. If $X = 0$, then $c_i = 0$, which is clearly not optimal for country i. Hence, $X_i^* > 0$ and $W_i'(X_i^*) = 0$. From (24),

$$\eta_i(c_i^*, X_i^*) = \frac{\theta_i q(X_i^*) + q'(X_i^*)[\theta_i X_i^* - f_i(y_i^*) - g_i(c_i^*)]}{1 + q(X_i^*)g_i'(c_i^*)} \tag{25}$$

Let us summarize the analysis in this subsection as follows.

Proposition 5 *Suppose that the initial assignments of emission permits to the countries are proportional to the total supply X. Let X_i^* be the amount* *of the total supply of emission permits that maximizes the welfare of country i, and let c_i^* be the level of consumption at $q(X_i^*)$. At (c_i^*, X_i^*), country i's marginal rate of substitution of its own consumption for global emission of greenhouse gases is equal to country i's marginal opportunity cost of reduction of greenhouse gases in terms of its own consumption under the system of emission permits trading. The latter is equal to*

$$\frac{\theta_i q(X_i^*) + q'(X_i^*)[\theta_i X_i^* - f_i(y_i^*) - g_i(c_i^*)]}{1 + q(X_i^*) g_i'(c_i^*)}$$

where $\theta_i \in]0, 1[$ is the proportion of the initial assignment of emission permits to country i.

5 Non-optimality of the bargaining outcome

5.1 The bargaining frontier

As the analyses in the previous sections have shown, once the amount of the total supply of emission permits is determined in international negotiations in the first stage, an equilibrium price of the emission permit is established in the market of emission permits, which then determines the level of consumption of each country. Hence, the welfare level of each country depends solely on the total supply of emission permits. In other words, by tracing the vector of welfare levels attained by all the countries at each amount of the total supply of emission permits, we can draw the welfare possibility frontier of the international negotiations. Let us call the frontier the *bargaining frontier*. This gives the set of feasible welfare vectors in the Nash bargaining theory.

As shown next, however, vectors on the bargaining frontier are not necessarily Pareto optimal. That is, there exists a technologically feasible allocation for which the welfare of every country is at least as good as at the vector on the bargaining frontier, and the welfare of some country is strictly higher. We show this fact by focusing on the the level of the total supply of emission permits that maximizes the welfare of one country. Such a level of the total supply clearly supports a welfare vector on the bargaining frontier, but the associated allocation is not Pareto optimal except for a rare case. We first consider the rare case, and then the general case.

Proposition 6 *For each $i \in N$, let X_i^* be the amount of the total supply of emission permits that maximizes the welfare of country i. Under Assumption*

L, if $X_i^* = X_j^* := X^*$ for all $i, j \in N$, then the allocation attained at X^* is Pareto optimal.

Proof. By Proposition 5 and Assumption L, for all $i \in N$,

$$\eta_i(c_i^*, X^*) = \frac{\theta_i q(X^*) + q'(X^*)[\theta_i X^* - f_i(y_i^*) - g_i(c_i^*)]}{1 + \alpha q(X^*)}$$

where c_i^* and y_i^* are consumption and production of country i achieved at X^*. Summing up the n equations,

$$\begin{aligned}
\sum_{i \in N} \eta_i(c_i^*, X^*) &= \frac{1}{1 + \alpha q(X^*)} \left(q(X^*) \sum_{i \in N} \theta_i + q'(X^*)[X^* \sum_{i \in N} \theta_i - \sum_{i \in N} (f_i(y_i^*) + g_i(c_i^*))] \right) \\
&= \frac{1}{1 + \alpha q(X^*)} \left(q(X^*) + q'(X^*)[X^* - \sum_{i \in N} (f_i(y_i^*) + g_i(c_i^*))] \right) \\
&= \frac{q(X^*)}{1 + \alpha q(X^*)} \\
&= \frac{1}{\frac{1}{q(X^*)} + \alpha}
\end{aligned}$$

By Proposition 3, $\frac{1}{q(X^*)} = f_i'(y_i^*)$ for all $i \in N$. Notice also that $\alpha = g_i'(c_i^*)$ for all $i \in N$. Hence,

$$\sum_{i \in N} \eta_i(c_i^*, X^*)(f_i'(y_i^*) + g_i'(c_i^*)) = 1$$

■

Proposition 7 *Let $N = \{1, 2\}$. For each $i \in N$, let X_i^* be the amount of total supply of emission permits that maximizes the welfare of country i. Under Assumption L, if $X_1^* \neq X_2^*$, then for each $i \in N$, the allocation attained at X_i^* is not Pareto optimal.*

Proof. By Proposition 5 and Assumption L,

$$\eta_1(c_1(X_1^*), X_1^*) = \frac{\theta_1 q(X_1^*) + q'(X_1^*)[\theta_1 X_1^* - f_1(y_1^*(q(X_1^*)) - g_1(c_1^*(X_1^*)]}{1 + \alpha q(X_1^*)}$$

Since $X_1^* \neq X_2^*$,

$$\eta_2(c_2(X_1^*), X_1^*) \neq \frac{\theta_2 q(X_1^*) + q'(X_1^*)[\theta_2 X_1^* - f_2(y_2^*(q(X_1^*)) - g_1(c_2(X_1^*))]}{1 + \alpha q(X_1^*)}$$

Adding both sides,

$$\sum_{i \in N} \eta_i(c_i(X_1^*), X_1^*) \neq \frac{1}{\frac{1}{q(X_1^*)} + \alpha}$$

Hence,

$$(\frac{1}{q(X_1^*)} + \alpha) \sum_{i \in N} \eta_i(c_i(X_1^*), X_1^*) \neq 1$$

By Proposition 3,

$$\sum_{i \in N} \eta_i(c_i(X_1^*), X_1^*)(f_i'(y_i(q(X_1^*))) + g_i'(c_i(X_1^*)) \neq 1$$

■

The reason why the bargaining frontier is below the Pareto frontier is as follows. It is true that production efficiency (or equivalently cost minimization of a given amount of reduction of greenhouse gases) is ensured through emission permits trading. As Chichilnisky et al.(2000) observed, however, overall Pareto optimality requires particular combinations of a total amount of emissions and a distribution of private goods. Hence, under a given distribution rule of initial emission permits, only some levels of total emissions are compatible with Pareto optimality.

5.2 The disagreement point

We next examine the *disagreement point* in the Nash bargaining theory, namely the vector of welfare levels that the countries attain if they fail to reach an agreement in negotiations. In the present model, the disagreement point may be defined as the vector of Nash equilibrium welfare levels of the countries in an emission game with no regulation and no emission permits trading. In the emission game, a strategy of each country is an amount of emissions of greenhouse gases. Since there is no emission permits trading, for each country, there is a one-to-one relation between consumption c_i and emission x_i that is derived from the equation $x_i = f_i(c_i) + g_i(c_i)$. Let $c_i(x_i)$ be the consumption level that country i achieves when it emits x_i.

Formally, an *emission game* is a list $(S_1, \ldots, S_n; W_1, \ldots, W_n)$ where for each $i \in N$, $S_i = \mathbb{R}_+$ is the set of strategies (emissions of greenhouse gases) and $W_i : \mathbb{R}^n_+ \to \mathbb{R}$ is the payoff function defined by

$$W_i(x_1, \ldots, x_n) := V_i(c_i(x_i), \sum_{h \in N} x_h).$$

A strategy profile $(x_1^*, \ldots, x_n^*)$ is a Nash equilibrium if for every $i \in N$ and every $x_i \in S_i$, $W_i(x_1^*, \ldots, x_n^*) \geq W_i(x_1^*, \ldots, x_{i-1}^*, x_i, x_{i+1}^*, \ldots, x_n^*)$.

As we have shown above, the bargaining frontier is strictly below the Pareto frontier in the present model. In other words, the set of vectors of feasible welfare levels in the bargaining "shrinks" from that of technologically feasible welfare vectors. Hence, it is not clear whether the disagreement point is located inside the bargaining frontier or not. In fact, it depends on distribution rules of initial emission permits among the countries.

Next we show that if the distribution rule is the proportional rule to the Nash equilibrium emissions in the noncooperative emission game, then the disagreement point is within the bargaining frontier.

Proposition 8 *Let $(x_1^*, \ldots, x_n^*) \in \Pi_{i \in N} S_i$ be a Nash equilibrium in the emission game $(S_1, \ldots, S_n; W_1, \ldots, W_n)$. For each $i \in N$, let $\theta_i \in [0, 1]$ be defined by*

$$\theta_i := \frac{x_i^*}{\sum_{h \in N} x_h^*}.$$

Then, under the distribution rule $(\theta_1, \ldots, \theta_n)$, the vector of the Nash equilibrium welfare levels is inside the bargaining frontier.

Proof. Let $X^* := \sum_{h \in N} x_h^*$. Clearly, X^* is a possible choice in the first stage of the bargaining game. Suppose that X^* is chosen. Let $i \in N$ be any country. By the distribution rule, country i's initial endowment of emission permits, denoted $\bar{x}_i$, is $\bar{x}_i = \theta_i X^* = x_i^*$. If country i produces $c_i(x_i^*)$ and consumes the same amount, then $x_i^* = f_i(c_i(x_i^*)) + g_i(c_i(x_i^*))$ holds, and hence $\bar{x}_i - f_i(c_i(x_i^*)) - g_i(c_i(x_i^*)) = 0$. Therefore, for any $q \in \mathbb{R}_+$,

$$c_i(x_i^*) = c_i(x_i^*) + q[\bar{x}_i - f_i(c_i(x_i^*)) - g_i(c_i(x_i^*))].$$

This means that the consumption $c_i(x_i^*)$ is feasible for country i under emission permits trading (Equation (14)). Thus, the welfare level

$W_i(x_1^*, \ldots, x_n^*) = V_i(c_i(x_i^*), X^*)$ is feasible with the choice X^* in the first stage of the bargaining game. Since this holds for all $i \in N$, the welfare vector $(W_1^*, \ldots, W_n^*)$ is within the bargaining frontier. ■

5.3 Trade-off between equity and efficiency

Given a bargaining frontier and a disagreement point, we may apply one of various solutions in the Nash bargaining theory to predict an outcome of the international negotiations. Then, the following facts hold:

(i) If the disagreement point is outside the bargaining frontier, then the outcome of negotiations has to be the point (the Nash equilibrium welfare levels in the noncooperative emission game), and no cooperative agreement is made. Clearly, the outcome is not Pareto optimal.

(ii) If the disagreement point is within the bargaining frontier, then some cooperative agreement is established. It is certain that the outcome Pareto dominates the Nash equilibrium in the noncooperative emission game. However, because the bargaining frontier is below the Pareto frontier, the outcome may not be Pareto optimal.

Both cases result in an inefficient final allocation, but the welfare loss in case (i) is larger than in case (ii). As Proposition 8 shows, if the distribution rule is the proportional rule to the Nash equilibrium emissions in the noncooperative emission game, then case (i) never arises. However, it seems very difficult to justify this distribution rule on any grounds of equity or fairness. For many other (equitable) distribution rules, the disagreement point is outside the bargaining frontier. Our analysis thus reveals a trade-off between equity in distribution rules and efficiency in final allocations.

6 Numerical examples

This section presents numerical examples in which the outcomes of negotiations are not Pareto optimal.[4] We consider two cases. The former corresponds to case (ii) in the previous section, and the latter to case (i).

[4]We use MATHEMATICA Ver.4.2 for calculations and drawing a figure in this section.

6.1 Example

Let $N := \{1, 2\}$. The emission functions are defined as follows: for every $i \in N$,

$$f_i(y_i) = y_i^2 \tag{26}$$

$$g_i(c_i) = c_i \tag{27}$$

For each $i \in N$, the welfare function $V_i : \mathbb{R}^2_+ \to \mathbb{R}$ is defined as:

$$V_1(c_1, X) = c_1^{0.8}(10 - X)^{0.2} \tag{28}$$

$$V_2(c_2, X) = c_2^{0.2}(10 - X)^{0.8}. \tag{29}$$

6.2 The Pareto frontier

The *Pareto frontier* is the locus of the welfare vectors of the two countries that can be attained technologically. This frontier can be derived from our extension of the Samuelson condition (Equation (11)). With the above definitions of the welfare functions and the emission functions, equation (11) becomes

$$(c_1 + c_2 + 1)(\frac{c_1}{4} + 4c_2) - 10 + \frac{(c_1 + c_2)^2}{2} + c_1 + c_2 = 0$$

We solve this equation for c_1 as the function of c_2. Then, the total amount of greenhouse gas emissions X can be expressed as the function of c_2:

$$X(c_2) = \frac{(c_1 + c_2)^2}{2} + c_1(c_2) + c_2.$$

Hence, the welfare of country i, W_i, is expressed as the function of c_2:

$$W_1(c_2) = V_1(c_1(c_2), X(c_2)) \tag{30}$$

$$W_2(c_2) = V_2(c_2, X(c_2)) \tag{31}$$

Tracing the vector $(W_1(c_2), W_2(c_2))$ for the variable c_2 in an appropriate interval, we obtain the Pareto frontier.

6.3 The bargaining frontier

Next, we derive the *bargaining frontier*, namely the locus of the welfare vectors that are attained at various levels of the total emission X with a given proportional distribution rule and emission permits trading. From equation (13), for each price q of the emission permit, the production of country i is

$$y_i = \frac{1}{2q}.$$

Hence, for each $i \in N$,

$$c_i = \frac{1}{2q} + q[\theta_i X - \frac{1}{4q^2} - c_i].$$

Solving this for c_i, we express c_i as the function of q, X and θ_i:

$$c_i(q, X, \theta_i) = \frac{1}{4q(1+q)} + \frac{q\theta_i X}{1+q}. \tag{32}$$

When the total supply of emission permits is X, the equilibrium price is obtained by solving the following equation for q:

$$\frac{1}{4q^2} + \frac{1}{4q^2} + \frac{1}{q} = X.$$

Let $q(X)$ be the solution. Plugging this as well as some value of θ_i into (32), we obtain c_i as the function of X, $c_i(X)$. Then, substituting $c_i(X)$ into the welfare function of country i, we can express the welfare level of country i as the function of X. Tracing the welfare vectors of the two countries for the variable X in $[0, 10]$, we derive the bargaining frontier.

Notice that the bargaining frontier depends on (θ_1, θ_2), namely the share of initial emission permits for the two countries. We consider several cases of the values (θ_1, θ_2).

6.4 The disagreement point

The *disagreement point* of the Nash bargaining theory for this model should be the Nash equilibrium welfare levels of the two countries in the emission game with no regulation. For each $i \in N$, and for each given x_j (emission of country $j \neq i$), the welfare of country i is given by

$$W_1 = c_1^{0.8}(10 - x_2 - c_1^2 - c_1)^{0.2} \tag{33}$$
$$W_2 = c_2^{0.2}(10 - x_1 - c_2^2 - c_2)^{0.8} \tag{34}$$

Differentiating the functions with respect to c_i and setting the value at 0, we obtain the best response functions. Then, the Nash equilibrium consumptions, emissions, and welfare levels of the two countries can be derived. The emission vector at the Nash equilibrium is $(x_1, x_2) = (6.658, 0.546)$, and the vector of the welfare levels of the two countries (the disagreement point) is $(d_1, d_2) = (2.248, 1.888)$. The Nash product, $(W_1 - d_1)(W_2 - d_2)$, is then defined, and the contour sets of the Nash product can be drawn.

6.5 Results

The results are shown in Figures 1 and 2. In each figure, the Pareto frontier is the outer concave curve, and the bargaining frontier is the inner concave curve. The disagreement point is indicated by the dot. Figure 1 corresponds to the case where $(\theta_1, \theta_2) = (0.925, 0.075)$. This is the case where the distribution rule of initial emission permits is the proportional rule to the Nash equilibrium emissions in the noncooperative emission game. Figure 2 corresponds to the case where $(\theta_1, \theta_2) = (0.8, 0.2)$.

In Figure 1, the disagreement point is inside the bargaining frontier, but the Nash bargaining solution is not Pareto optimal. On the other hand, in Figure 2, the disagreement point is outside the bargaining frontier. Hence, cooperative agreement is not at all possible, and clearly the outcome (the disagreement point) is not Pareto optimal. In this example, approximately 86 to 99 percent of initial permits has to be assigned to country 1, who is less concerned about the environment, in order for some cooperative agreement to be possible.

7 Conclusion

In this paper we have analyzed international negotiations for abatement of global warming, using the framework of the Nash bargaining theory. First, we have extended the condition for Pareto optimal allocations to the case where consumption activities also generate greenhouse gases. Second, we have built a three-stage model of international negotiations on the level of total emissions, and examined whether a cooperative agreement is possible, and if any, the outcome is Pareto optimal or not. We have shown fundamental difficulties in attaining a Pareto optimal allocation.

There are some limits to the analysis in this paper. First, we assume that once negotiations break down, all the countries play a non-cooperative emission game. Hence, our model does not cover the case in which some subset of the participants reach an agreement while the other countries withdraw from the negotiations. Second, we fix the participants (the signatories of the Kyoto Protocol) in negotiations, and ignore the behavior of non-participants (non-signatories). In fact, before starting negotiations there should be another stage in which each country decides whether it participates in the mechanism or not.[5] It is left for future researches to develop a more general model which takes account of these factors.

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Figures

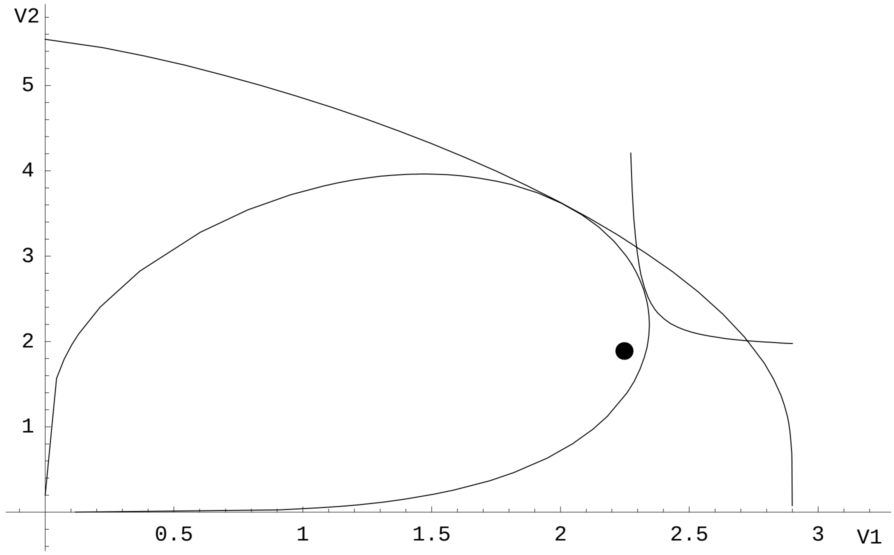

Figure 1: The Nash bargaining solution when the share of initial permits is (0.925, 0.075).
(The proportional rule to the Nash equilibrium emissions in the noncooperative game.)

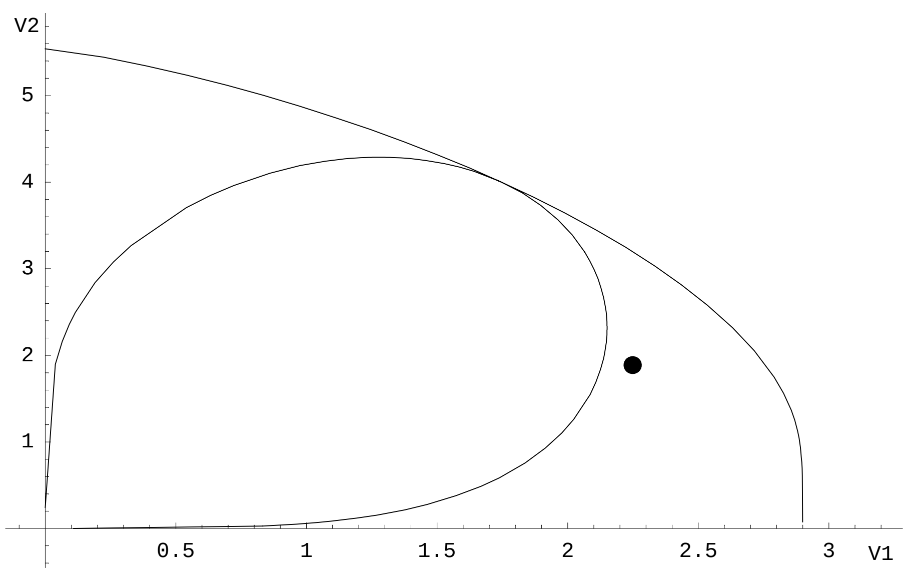

Figure 2: The Nash bargaining solution when the share of initial permits is (0.8, 0.2)