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Market research and complementary advertising under asymmetric information

by

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Abstract

We consider whether market research can always increase a seller's sales under bilateral asymmetric information. If a monopoly seller provides a high quality object, market research cannot increase sales even when the cost is sufficiently low. A low quality seller, on the other hand, can likely benefit from market research. However, this research has shown that market research alone does not improve sales and that advertising complements market research. Thus the high quality seller can increase sales by using both methods. The availability of advertising and market research to both types of seller results in disappearance of information asymmetry and efficient trade.

1 Introduction

A buyer's reservation value may depend not only on the object's quality but also on the buyer's preference level for the object. Bilateral asymmetric information prompts a seller to set an inappropriate price and may lose him potential sales. We can find a specific example of such determining effects of pricing in an annual economic report of Japan. Inappropriately set prices, in particular through a weak grasp of demand and inaccurately set high prices, have also been blamed for Japan's economic stagnation from 1998 to 1999 (Economic Planning Agency, 1998; Cabinet Office 1999).

Bilateral asymmetric information can prevent the realization of efficient trade. Myerson and Satterthwait's (1983) well-known study shows that under bilateral asymmetric information no mechanisms can enforce trade whenever a seller and a buyer can benefit from the trade. In

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their model, the seller's value is independent of the buyer's. Chatterjee and Samuelson (1983) and Lindsey, Samuelson and Zeckhauser (1996) show that the same inefficiency exists when a buyer's value is correlated with a seller's. Two marketing methods may help a seller set a price. The first method is advertising (AD), and the second is market research (MR). If a seller provides a high quality object, he may try advertising the quality in such forms as TV commercials or banner-ads on websites. AD has been widely studied. Nelson (1970, 1974) was the first to treat AD as a signal to a buyer that the object is of high quality even though AD is not directly informative. While Nelson's analysis was compelling, he did not use a formal model. Milgrom and Roberts (1986) set up a signaling model that supports Nelson's results. Nelson (1970, 1974) and Milgrom and Roberts (1986) consider the case in which the seller advertises the object. Lizzeri (1999) considers a profit maximizing intermediary who can certify the quality of a seller. In this case, the seller can ask the intermediary to certify the quality. They showed that the high quality seller can increase sales by AD or quality certification. Moreover, Shavell (1994) proposes that market efficiency can be improved if a seller discloses the quality. He considers the case in which a social welfare maximizer can force the seller to disclose the quality.

On the other hand, a seller may try seeking a buyer's reservation value or demand in such forms of MR as Interactive Marketing, Hearing, Interview or Internet research. MR is widely used in real business transactions. The expenditure in Interactive Marketing is fastly growing. According to the MET Report 2003, the trend of expenditure in Interactive Marketing for 2001-04 shows over 70 percents cumulative growth and strong growth is also seen in the US (London Business School, 2003). According to the Japan Market Research Association, research business sales in Japan doubled from 1999 to 2006 to a value of 260 billion yen (JMRA, 2007). Internet research accounted for an increasing large portion of this value has recently become popular. However, though many papers offer a variety of MR approaches, few papers analyze the effectiveness of MR. For instance, Chen, Narasimhan and Zhang (2001) offers that a seller can benefit from individual marketing in which the seller can distinguish high preference buyers from low preference buyers, but this finding is based on the assumption that MR is effective.

Two questions arise: Is MR really an effective business method? Can MR always increase sales and improve market efficiency? To address these questions, we investigate the relationship between MR and AD. The answer may seem to be affirmative since a seller can use new information to set a price through MR. This paper contributes by showing that MR is not always effective.

We adopt a pure strategy perfect Bayesian equilibrium to analyze the market with AD and MR under bilateral asymmetric information. We consider the situation where a seller has private information about his object's quality and a buyer has private information about his preference for the object. Their values for the object are correlated in the sense that a buyer's consumption value depends on both a quality and a preference. Both an advertising agent and a market researcher independently provide AD and MR respectively. A seller can use each marketing tool or both before setting a price. The quality is perfectly disclosed to a buyer if a seller uses AD, while a seller can grasp a buyer's preference if a seller uses MR. We restrict the class of a buyer's belief to "monotone belief". There exists a unique separating PBE thanks to a monotone belief. Two points are worth noting here. First, the seller provides a new object whose quality is not generally known. The seller does not of course know a buyer's preference for the object at this point. Second, the quality is exogenously given. The seller cannot choose the quality of the object.

We obtain three results: First, MR alone cannot increase a high quality seller's sales, so he does not use MR even when the cost is zero. Second, MR can improve market efficiency if the buyer's taste is high, while AD can improve market efficiency if the buyer's taste is low. Third, for a high quality seller AD complements MR. He can enjoy a synergy effect between AD and MR.

This paper is organized in the following way. Section 2 presents the model. Section 3 analyzes equilibria. Section 4 discusses the property of marketing activities and the effect to the efficiency of the market and seller's sales. Section 5 provides some extensions and section 6 concludes. All proofs are given in the Appendix.

2 Model

There are one seller (S), one buyer (B), one advertising agent (A) and one market researcher (M). The seller provides an object. The quality of the object has two possible types, either low ($\underline{\theta}_S$) or high ($\bar{\theta}_S$) ($0 < \underline{\theta}_S < \bar{\theta}_S$). We denote $\theta_S \in \Theta_S = \{\underline{\theta}_S, \bar{\theta}_S\}$ and $\Delta\theta_S = \bar{\theta}_S - \underline{\theta}_S$. The probability that the object has a high quality $\bar{\theta}_S$ is denoted by $\alpha \in (0,1)$. The seller privately knows the quality of the object.

In this paper, we assume that the reservation value v_S of the object to the seller is given by θ_S, and that the reservation value v_B of the object to the buyer are composed by the quality θ_S and her private taste θ_B, that is $v_B = \theta_S + \theta_B$. For example, a buyer's reservation value of a computer may be determined both by the intrinsic quality like a memory size and the performance of a processor, and by her private taste for an operating system and design. The buyer's taste θ_B is her private information and may have two possible types, either low ($\underline{\theta}_B$) or high ($\bar{\theta}_B$). We denote $\theta_B \in \Theta_B = \{\underline{\theta}_B, \bar{\theta}_B\}(0 < \underline{\theta}_B < \bar{\theta}_B)$ and $\Delta\theta_B = \bar{\theta}_B - \underline{\theta}_B$. The probability that the buyer's taste has a high value $\bar{\theta}_B$ is denoted by $\beta \in (0,1)$. When a trade occurs with price p, the seller receives payoff $p - \theta_S$, and the buyer receives $\theta_S + \theta_B - p$. The high (and low) type seller's payoff is denoted by $\bar{\pi}(p)$ (and $\underline{\pi}(p)$).

In the market, the seller may purchase services by an advertising agent and a market researcher. If the seller uses advertising (AD), then he can make the quality of an object public information. On the other hand, the market researcher conveys the private taste of the buyer to the seller.

The game has five stages.

Stage 1. The advertising agent and the market researcher independently choose their service fees.

Stage 2. The seller and the buyer privately know their types.

Stage 3. Knowing the quality of the object, the seller decides whether to use AD and MR.

Stage 4. The seller announces a price of the object, depending on the information about the types of the seller and the buyer which are available to him.

Stage 5. The buyer decides whether to buy the object or not. The buyer cannot observe whether the seller uses MR or not.

Strategies for the advertising agent and the market researcher are their services' fees, $a \in \mathbb{R}_+$ and $m \in \mathbb{R}_+$, respectively.

A (pure) strategy for the seller is a triplet $s = (k, l, p)$. An advertising strategy is a decision rule to decide whether to use AD or not, given his type and an AD fee, and it is formally described as a function k from $\mathbb{R}_+ \times \Theta_S$ to $\{0,1\}$ where 0 means not using AD and 1 means using AD. A market researching strategy l is a function from $\mathbb{R}_+ \times \Theta_S$ to $\{0, 1\}$ where 0 means not using MR and 1 means using MR. A pricing strategy p is a function from $\Theta_S \times \Theta_B$ to $\mathbb{R}_+$ when he uses MR, and a function from Θ_S to $\mathbb{R}_+$ when he does not.

A strategy for the buyer is a pair $b = (b_0, b_1)$ of decision rules to buy the object. The decision rule b_0 when the seller does not use AD is a function from $\Theta_B \times \mathbb{R}_+$ to $\{0, 1\}$ where 0 means not buying and 1 means buying. The decision rule b_1 when the seller uses AD is a function from $\Theta_B \times \Theta_S \times \mathbb{R}_+$ to $\{0, 1\}$.

We analyze a perfect Bayesian equilibrium (PBE) of the game. A PBE of the game is represented by a profile $(a^*, m^*, s^*, b^*, \mu^*)$ of strategies and belief where the buyer's belief μ^* is her probabilistic assessment of the seller's type, given a price set by the seller, and it is formally a function from $\mathbb{R}_+$ to the set of all probability distributions over the seller's type set Θ_S. [1] We denote by $\mu(\theta_S \mid p)$ the probability that the buyer's belief μ assigns to the seller's type θ_S, given price p.

In what follows, we make the following assumption on the buyer's belief.

Assumption 1 The buyer's belief μ is *monotone* if there exists some $\hat{p} \in \mathbb{R}_+$ such that the probability $\mu(\bar{\theta}_S \mid p)$ satisfies:

$$\mu(\bar{\theta}_S \mid p) = \begin{cases} \mu_0 & \text{if } p \geq \hat{p} \\ 0 & \text{if } p < \hat{p} \end{cases} \tag{1}$$

for some $\mu_0 \geq 0$.

It is well known that many equilibria exist in general in signaling games with incomplete information. We assume that the buyer's subjective probability for the seller to be a high type ($\bar{\theta}_S$) is non-decreasing in price p. As Fudenberg and Tirol (1983), the assumption of a monotone belief means that if the buyer is offered a high price, she considers the seller's type is high, and if the buyer is offered a low price less than a threshold, she estimates the seller's type to be low for sure. The monotone belief is intuitive and standard in the literature of sequential bargaining with asymmetric information. Without loss of generality, we adopt a

[1] Precisely, the buyer's belief includes her assessment on whether the seller uses MR or not. It is shown that a PBE of the model does not depend on this part of her belief.

step-function instead of a continuous function as a monotone belief. A PBE constructed under a step-function is preserved even if we adopt a continuous increasing function.

Definition 2 A profile $(a^*, m^*, s^*, b^*, \mu^*)$ of strategies and belief is a *perfect Bayesian equilibrium (PBE)* of the game if (i) all four agents maximize their conditional expected payoffs given all available information, [2] and (ii) the buyer's belief μ^* is monotone, and furthermore it obeys the Bayes' updating rule whenever it is possible. If not possible, the buyer's belief may be arbitrarily selected, keeping the monotonicity.

The following three types of a PBE may arise. A PBE is *separating* if different types of a seller offer different prices. In a separating PBE, the buyer can correctly infer the true quality of the object by observing a price. A PBE is *pooling* if all types of a seller offer the same price. In a pooling PBE, the buyer obtains no additional information on the quality of the object from the price. A PBE is *semi-separating* if all types of a seller offer the same price to one type of a buyer and they offer different prices to the other type of a buyer when MR is available.

3 Equilibrium

In this section, we characterize a PBE in four cases: (1) neither AD nor MR is available (benchmark), (2) only AD is available, (3) only MR is available and (4) both AD and MR are available.

The seller's types and the buyer's types satisfy:

Assumption 2 $\alpha\bar{\theta}_S + (1-\alpha)\underline{\theta}_S + \underline{\theta}_B < \bar{\theta}_S < \underline{\theta}_S + \bar{\theta}_B$.

The left inequality means that the reservation value of the high type seller ($\bar{\theta}_S$) is strictly greater than the expected reservation value of the low type buyer ($\underline{\theta}_B$) under an initial belief. If this condition holds, no trade between the high type seller and the low type buyer is possible. On the other hand, the right inequality means that the reservation value of the high type seller

[2] Since a formulation of the maximization problem of an agent's conditional expected payoff is standard, we omit it to avoid notational complexity.

is strictly lower than the expected reservation value of the high type buyer ($\bar{\theta}_B$) no matter what belief the buyer has. A trade between the high type seller and the high type buyer is possible, independent of the buyer's belief under the assumption. If the reservation value $\bar{\theta}_S$ is very small, then a trade is possible between every pair of the seller and the buyer. The purpose of our analysis examines how market research and advertising improve a probability of trading, and we exclude such a case.

Since the buyer's optimal strategy is common in all four cases, we first characterize it. Given her belief μ and price p, the buyer θ_B obtains expected payoff:

$$\begin{aligned}&\mu(\underline{\theta}_S \mid p)(\underline{\theta}_S + \theta_B - p) + \mu(\bar{\theta}_S \mid p)(\bar{\theta}_S + \theta_B - p)\\ &= z(\theta_S \mid p) + \theta_B - p\end{aligned}$$

if she buys the object. Here, $z(\theta_S \mid p) = \mu(\bar{\theta}_S \mid p)\bar{\theta}_S + \mu(\underline{\theta}_S \mid p)\underline{\theta}_S$ represents the expected quality of the object under belief μ. It is clear that the optimal choice of the buyer is to buy the object if and only if $p \leq \theta_B + z(\theta_S \mid p)$. [3] That is:

$$b^*(\theta_B, p) = \begin{cases} 1 & \text{if } \theta_B \geq p - z(\theta_S \mid p) \\ 0 & \text{if } \theta_B < p - z(\theta_S \mid p). \end{cases}$$

The upper bound of the price is $\bar{p} = \bar{\theta}_S + \bar{\theta}_B$ since $\bar{\theta}_S + \bar{\theta}_B$ is the highest reservation value of the high type buyer. The lower bound of the price is $\underline{p} = \underline{\theta}_S + \underline{\theta}_B$ since this is the lowest reservation value of the buyer. The seller will offer a price in the interval $[\underline{p}, \bar{p}]$. The price $p^* = \alpha\bar{\theta}_S + (1-\alpha)\underline{\theta}_S + \bar{\theta}_B$ is the maximum price for which the high type buyer buys the object under an initial belief.

3.1 Benchmark (Neither AD nor MR)

We first analyze a benchmark case where neither AD nor MR is available. In this case, we will prove that a separating PBE exists when β is smaller than a threshold $\underline{\beta}$, and that a pooling PBE exists when β is larger than the threshold $\bar{\beta}$. Two thresholds $\underline{\beta} < \bar{\beta}$ are defined by:

$$\underline{\beta} = \frac{\underline{\theta}_B}{\Delta\theta_S + \bar{\theta}_B}, \quad \bar{\beta} = \frac{\underline{\theta}_B}{\alpha\Delta\theta_S + \bar{\theta}_B} \tag{2}$$

where $\Delta\theta_S = \bar{\theta}_S - \underline{\theta}_S$.

[3] We assume that she buys the object if buying and not buying are indifferent.

The low type seller is indifferent between offering the lowest price $\underline{p}$ and the highest $\bar{p}$ when the buyer is the high type with probability $\underline{\beta}$. If $\beta < \underline{\beta}$, the low type seller prefers to offer the lowest price $\underline{p}$. The low type seller is indifferent between offering the intermediate price p^* and the lowest price $\underline{p}$ when the buyer is the high type with probability $\bar{\beta}$. If $\beta > \bar{\beta}$, the low type seller prefers to offer the intermediate price p^*.

Proposition 1. If $\beta \leq \underline{\beta}$, there exists a unique PBE, which is separating. In equilibrium, the high type seller offers price $\bar{p} = \bar{\theta}_S + \bar{\theta}_B$ and the low type seller offers price $\underline{p} = \underline{\theta}_S + \underline{\theta}_B$. The high type buyer buys the object if and only if a price is equal to or less than $\bar{p}$ and the low type buyer buys the object if and only if a price is equal to or less than $\underline{p}$. The buyer's belief is given by:

$$\mu(\bar{\theta}_S \mid p) = \begin{cases} 1 & \text{if } p \geq \hat{p} \\ 0 & \text{if } p < \hat{p} \end{cases}$$

where $\hat{p} \in (\bar{\theta}_S + \underline{\theta}_B, \bar{p}]$.

The intuition for the proposition is as follows. When the probability that the buyer is the high type is so small that $\beta < \underline{\beta}$, the seller is likely to face to the low type buyer. Since the type of the seller is revealed to the buyer in a separating PBE, the low type seller optimally offers the lowest price $\underline{p}$ in order to trade surely with the low type buyer. The high type seller offers the highest price $\bar{p}$ since both types of the buyer form belief that the high type seller offers a very high price. If the high type seller offers lower prices, the buyer thinks that he is the low type, and she does not buy the object. If the buyer thinks that the high type seller chooses lower prices for which both types of the buyer can buy the object, the low type seller can profitably deviate to the low price. The separating PBE exhibits a standard adverse selection problem which the high type seller cannot trade with the low type buyer. When $\beta \leq \underline{\beta}$, a pooling PBE does not exist. If a pooling PBE exists, then the equilibrium price must be $\alpha\bar{\theta}_S + (1-\alpha)\underline{\theta}_S + \bar{\theta}_B$ by Assumption 2, but the low type seller can profitably deviate to the low price $\underline{p}$.

Proposition 2. If $\beta \geq \bar{\beta}$, there exists a unique PBE, which is pooling. In equilibrium, both types of the seller offer price $p^* = \alpha\bar{\theta}_S + (1-\alpha)\underline{\theta}_S + \bar{\theta}_B$. The high type buyer buys the object if and only if a price is equal to or less than p^* and the low type buyer buys the object if and only if a price is equal to or less than $\underline{\theta}_S + \underline{\theta}_B$. The buyer's belief is given by:

$$\mu(\bar{\theta}_S \mid p) = \begin{cases} \alpha & \text{if } p \geq p^* \\ 0 & \text{if } p < p^*. \end{cases}$$

The intuition for the proposition is as follows. When the probability that the buyer is the high type is so large that $\beta > \bar{\beta}$, the seller is likely to face to the high type buyer. Since the buyer cannot obtain any additional information about the seller's type in a pooling PBE, both types of the seller optimally offer the intermediate price p^* in order to trade with the high type buyer. If the low type seller chooses the lowest price $\underline{p} = \underline{\theta}_S + \underline{\theta}_B$, he can increase the probability of trade but decrease his expected payoff under $\beta > \bar{\beta}$. The high type seller has no incentive to choose the price $\underline{p}$ under Assumption 2. When $\beta \geq \bar{\beta}$, a separating PBE does not exist. If a separating PBE exists, then the low (and high) type seller's equilibrium price must be $\underline{p}$ (and $\bar{p} = \bar{\theta}_S + \bar{\theta}_B$) as in Proposition 1, but the low type seller can profitably deviate to the high price $\bar{p}$.

3.2 Advertising

In this subsection, we analyze a case where only AD is available (Case AD). We consider informative advertising as in Moraga-Gonzalez (2000). [4] If the seller uses advertising service which the advertising agent provides, the quality of the object is correctly conveyed to the buyer through various channels like TV commercials, new papers or Internet. In this case, we will prove that a separating PBE exists for every β, and that trade between the seller and the buyer is possible if and only if β is smaller than a threshold $\tilde{\beta}$. The threshold $\tilde{\beta}$ is defined by:

$$\tilde{\beta} = \frac{\underline{\theta}_B}{\bar{\theta}_B}. \tag{3}$$

When the buyer is the high type with probability $\tilde{\beta}$, both types of the seller are indifferent between choosing the highest price such that the high type buyer accepts and the highest price such that the low type buyer accepts.

[4] Moraga-Gonzalez (2000) considers a large number of potential buyers whose mass is 1. The seller decides the fraction of the buyers to be informed.

Proposition 3. For every β, there exists a unique PBE, which is separating. In equilibrium, only the high type seller uses AD. If $\beta \leq \tilde{\beta}$, the high type seller chooses $\bar{p} = \bar{\theta}_S + \underline{\theta}_B$ and the low type seller chooses $\underline{p} = \underline{\theta}_S + \underline{\theta}_B$. The equilibrium prices $\bar{p}$ and $\underline{p}$ are accepted by both types of the buyer. If $\beta > \tilde{\beta}$, the high type seller chooses $\bar{p} = \bar{\theta}_S + \bar{\theta}_B$ and the low type seller chooses $\underline{p} = \underline{\theta}_S + \bar{\theta}_B$. The equilibrium prices $\bar{p}$ and $\underline{p}$ are accepted by only the high type buyer. The advertising agent chooses $a^* = \underline{\theta}_B - \beta(\bar{\theta}_B - \Delta\theta_S)$ if $\beta \leq \tilde{\beta}$, and $a^* = \beta\Delta\theta_S$ if $\beta > \tilde{\beta}$.

The intuition for the proposition is as follows. Only the high type seller prefers to use AD for a sufficiently low service fee. The advertising agent sets the service fee just as additional value of the high type seller from using AD, and the high type seller uses AD in equilibrium. Since the buyer is fully rational, she understands that the high type seller has an incentive to use AD, and she forms a belief that the seller not using AD is the low type for sure. Therefore, a PBE is separating for every β. For $\beta > \tilde{\beta}$, since the seller is likely to face to the high type buyer, both types of the seller offer high prices $\bar{p} = \bar{\theta}_S + \bar{\theta}_B$ and $\underline{p} = \underline{\theta}_S + \bar{\theta}_B$. Therefore, both types of the seller cannot trade with the low type buyer. For $\beta \leq \tilde{\beta}$, since the seller is likely to face to the low type buyer, both types of the seller offer low prices $\bar{p} = \bar{\theta}_S + \underline{\theta}_B$ and $\underline{p} = \underline{\theta}_S + \underline{\theta}_B$. Therefore, a trade between the seller and the buyer is possible.

3.3 Market research

In this subsection, we analyze a case where only MR is available (Case MR). If the seller uses MR, the market researcher gives perfect information on the buyer's type to the seller. We assume that the buyer cannot observe whether the seller uses MR or not. [5] In this case, we will prove that a semi-separating PBE exists for every β.

Proposition 4. For every β, there exists a unique PBE, which is semi-separating. In equilibrium, only the low type seller uses MR. The high type seller chooses $p^* = \alpha\bar{\theta}_S + (1 - \alpha)\underline{\theta}_S + \bar{\theta}_B$, and the low type seller chooses p^* to the high type buyer and $\underline{p} = \underline{\theta}_S + \underline{\theta}_B$ to the low type buyer.

[5] This is a natural assumption. If the seller decides to use MR, the market researcher begins to collect data about the moderate price through Internet Research, and give the seller feedback. Though the subjects for the Internet Research know the fact that the seller uses MR, most of consumers do not know the fact.

The equilibrium price p^* is accepted by the high type buyer, and $\underline{p}$ is accepted by the low type buyer. The buyer's belief is given by:

$$\mu(\bar{\theta}_S \mid p) = \begin{cases} \alpha & \text{if } p \geq \hat{p} \\ 0 & \text{if } p < \hat{p}, \end{cases}$$

where $\hat{p} \in (\alpha\bar{\theta}_S + (1-\alpha)\underline{\theta}_S + \underline{\theta}_B, p^*]$. The market researcher chooses $m^* = \beta(\alpha\Delta\theta_S + \Delta\theta_B)$ if $\beta \leq \beta^*$, and $m^* = (1-\beta)\underline{\theta}_B$ if $\beta > \beta^*$, where

$$\beta^* \equiv \frac{\underline{\theta}_B}{\bar{\theta}_B + \alpha\Delta\theta_S}. \tag{4}$$

The intuition for the proposition is as follows. Since MR can give the seller some new information, both types of the seller seem to prefer to use MR. However, only the low type seller uses MR in equilibrium. To see that only the low type seller uses MR, suppose that the high type seller uses MR. Since the low type seller's additional value from using MR is equivalent to the high type seller's, the low type seller also uses MR. Therefore, both types of the seller use MR in equilibrium, and a PBE is pooling, where both types of the seller choose the low pooling price $\alpha\bar{\theta}_S + (1-\alpha)\underline{\theta}_S + \underline{\theta}_B$ to the low type buyer and the high pooling price $\alpha\bar{\theta}_S + (1-\alpha)\underline{\theta}_S + \bar{\theta}_B$ to the high type buyer. However, the high type seller has no incentive to choose the low pooling price by Assumption 2. Therefore the high type seller does not use MR, and a pooling PBE does not exist. In a semi-separating PBE, only the low type seller uses MR, and both types of the seller choose the same price to the high type buyer and different prices to the low type buyer, and the market researcher sets the service fee just as additional value of the low type seller from using MR. When only the low type seller uses MR, a separating PBE does not exist, either. If a separating PBE exists, there exist some types of the buyer who face to two different prices and accept both. However, since the low type seller uses MR, he can profitably deviate to choose the same price as the high type seller's, so a separating PBE does not exist. As a result, a PBE is semi-separating.

3.4 Both AD and MR

In this subsection, we analyze a case where both AD and MR are available (Case AM). In this case, we will prove that a separating PBE exists for every β.

Proposition 5. For every β, there exists a unique PBE, which is separating. In equilibrium, the only high type seller uses AD and both types of the seller use MR. The high type seller chooses $\bar{p}_* = \bar{\theta}_S + \underline{\theta}_B$ to the low type buyer and $\bar{p}^* = \bar{\theta}_S + \bar{\theta}_B$ to the high type buyer. The low type seller chooses $\underline{p}_* = \underline{\theta}_S + \underline{\theta}_B$ to the low type buyer and $\underline{p}^* = \underline{\theta}_S + \bar{\theta}_B$ to the high type buyer. The equilibrium prices are accepted with probability 1. The advertising agent chooses $a^* = \underline{\theta}_B - \beta(\bar{\theta}_B - \Delta\theta_S)$ if $\beta \leq \tilde{\beta}$, and $a^* = \beta\Delta\theta_S$ if $\beta > \tilde{\beta}$. The market researcher chooses $m^* = \beta\Delta\theta_B$ if $\beta \leq \tilde{\beta}$, and $m^* = (1-\beta)\underline{\theta}_B$ if $\beta > \tilde{\beta}$. The threshold $\tilde{\beta}$ is given by (3).

Since only the high type seller uses AD in equilibrium, a PBE is separating for every β as in Case AD. The reason why only the high type seller uses AD is similar to the intuition of proposition 3. However, both types of the seller use MR in equilibrium unlike Case MR.

The reason why only the low type seller uses MR in Case MR is that the buyer cannot distinguish between the high type seller and the low type. However, the buyer can grasp the seller's type through AD when both AD and MR are available. Therefore, MR is valuable to *both* types of the seller. In equilibrium, information asymmetry fully disappears, and both the seller and the buyer act like as under complete information. Both types of the seller choose the highest price such that the buyer buys the object, so a trade between the seller and the buyer is possible.

4 Result

In previous sections, we analyzed the four cases: *Benchmark* in which neither AD nor MR is available for a seller; *Case AD* in which only AD is available; *Case MR* in which only MR is available; and *Case AM* in which both AD and MR are available. By comparing the four cases above, we obtain the results below. We first state a proposition about market efficiency. Market efficiency can be measured by an ex ante probability of trading, so we say that *market efficiency is improved* if the probability of trading increases.

Proposition 6. MR can improve market efficiency if $\beta \geq \bar{\beta}$, while AD can improve market efficiency if $\beta \leq \underline{\beta}$.

The proposition says that the situation in which MR can improve market efficiency differs the situation in which AD can improve market efficiency. When β is small, the high type seller chooses the highest price in order to prevent the low type seller from imitating the high type seller, therefore the high type seller cannot trade with the low type buyer in Benchmark. Since the high type seller can prevent the low type seller's imitation in AD case, the high type seller can choose the low price and trade with the low type buyer. Therefore, when β is small, AD can improve market efficiency. When β is large, both types of the seller choose the high pooling price $p^* = \alpha\bar{\theta}_S + (1-\alpha)\underline{\theta}_S + \bar{\theta}_B$ and a trade between both types of the seller and the low type buyer is impossible in Benchmark. If MR is available, the low type seller can benefit from using MR and choosing the lowest price, so he can trade with the low type buyer in MR case. Therefore, when β is large, MR can improve market efficiency.

We will answer the question whether MR always increases the seller's payoff or not in the next proposition. Before stating the proposition, we represent the seller's equilibrium payoff. The high (and low) type seller's equilibrium payoff is denoted by $\bar{\Pi}$ (and $\underline{\Pi}$). [6]

In Benchmark, the seller's payoff in the separating PBE is given by:

$$\begin{cases} \bar{\Pi} = \beta\bar{\theta}_B \\ \underline{\Pi} = \underline{\theta}_B. \end{cases} \tag{5}$$

In Benchmark, the seller's payoff in the pooling PBE is given by:

$$\begin{cases} \bar{\Pi} = \beta[\bar{\theta}_B - (1-\alpha)\Delta\theta_S] \\ \underline{\Pi} = \beta(\bar{\theta}_B + \alpha\Delta\theta_S). \end{cases} \tag{6}$$

In Case AD, the seller's payoff is given by:

$$\bar{\Pi}^A = \underline{\Pi}^A = \begin{cases} \underline{\theta}_B & \text{if } \beta \leq \tilde{\beta} \\ \beta\bar{\theta}_B & \text{if } \beta > \tilde{\beta} \end{cases} \tag{7}$$

In Case MR, the seller's payoff is given by:

$$\begin{cases} \bar{\Pi}^M = \beta(\bar{\theta}_B - (1-\alpha)\Delta\theta_S) \\ \underline{\Pi}^M = (1-\beta)\underline{\theta}_B + \beta(\bar{\theta}_B + \alpha\Delta\theta_S). \end{cases} \tag{8}$$

[6]The seller's expected payoff is zero when the object remains to be unsold. In accounting terminology, payoff and marketing cost $a + m$ indicate *gross profit on sales* and *selling and general administrative expenses* (SGE) respectively.

In Case AM, the seller's payoff is given by:

$$\bar{\Pi}^{AM} = \underline{\Pi}^{AM} = \beta\bar{\theta}_B + (1-\beta)\underline{\theta}_B \tag{9}$$

Proposition 7. MR cannot increase the high type seller's payoff as compared with that in Benchmark; $\bar{\Pi}^M < \bar{\Pi}$ for $\beta \le \underline{\beta}$ (separating PBE) and $\bar{\Pi}^M = \bar{\Pi}$ for $\beta > \bar{\beta}$ (pooling PBE). MR can increase the low type seller's payoff as compared with that in Benchmark; $\underline{\Pi}^M > \underline{\Pi}$.

This proposition implies that MR is not necessarily effective. MR gives a new trade opportunity to the low type seller and increases his payoff. This change of trade opportunity has no effect on the high type seller's offer if $\beta \ge \bar{\beta}$. However, the change reduces the high type seller's offer and his payoff decreases if $\beta \le \underline{\beta}$.

Proposition 8 explains a relation between AD and MR, and shows that a synergy effect between AD and MR exists for the high type seller. We say that *a synergy effect between AD and MR exists for the high type seller* if $\bar{\Pi}^{AM} - \bar{\Pi} > (\bar{\Pi}^A - \bar{\Pi}) + (\bar{\Pi}^M - \bar{\Pi})$. This inequality means that if the increase of a payoff through both AD and MR as compared with a payoff in Benchmark is larger than the sum of the increase of a payoff through only AD and only MR.

Proposition 8. For every α and β, a synergy effect between AD and MR exists for the high type seller.

In Benchmark, the high type seller chooses the high price. Since MR gives him additional new information about the buyer's type, he can choose the low price to the low type buyer if he uses MR. However, the buyer infers that the seller is a low type when she faces to the low price, so the high type seller cannot increase his payoff by this information. The issue is that the buyer cannot distinguish the high type seller from the low type seller. AD can solve this issue. Once the high type seller can reveal his type to the buyer, this new information about the buyer's type becomes valuable. Therefore, a synergy effect between AD and MR exists for the high type seller, and we can say that AD complements MR. This synergy effect is important in the sense that parallel usage of AD and MR (or marketing-mix) can create new values for a high type seller.

Proposition 9. The low type seller's payoff in Case AM is higher than that in Benchmark if and only if $\beta < \underline{\theta}_B/(\alpha\Delta\theta_S + \underline{\theta}_B)$.

The proposition implies that AD and MR does not necessarily increase the low type seller's payoff. To see the intuition, suppose β is large. In this case, two different effects exist on the low type seller's payoff. The first is a positive effect by MR. As compared to Benchmark, he can choose different prices to each type of the buyer by MR, and he can increase his payoff. The second is a negative effect by AD. In Benchmark, he can choose the high pooling price to the high type buyer. However, since the quality of an object is public information by AD in Case AM, he must choose the relatively low price to the high type buyer. This low price reduces the low type seller's payoff. When β is large, the second negative effect dominates the first positive effect. On the other hand, when β is small, the second negative effect vanishes since PBE is separating in Benchmark. Therefore, his payoff in Case AM is higher than that in Benchmark when β is small.

5 Discussion

We assumed that both AD and MR can perfectly reveal seller's and buyer's private information. However, this assumption may not be realistic. In this section, we analyze imperfect MR and imperfect AD.

5.1 Imperfect market research

First, we consider the case in which only imperfect MR is available. We consider imperfect MR as follows: even though the seller uses MR, he cannot know the buyer's type with probability $\epsilon \in [0, 1]$, so he still keeps the initial belief about the buyer's type $(\beta, 1-\beta)$. Notice again that at least one type of seller uses MR in a PBE. As in section 3, the high type seller does not use MR for any $m \geq 0$, so only the low type seller uses imperfect MR. In equilibrium, the low type seller chooses the high pooling price $p^* = \alpha\bar{\theta}_S + (1-\alpha)\underline{\theta}_S + \bar{\theta}_B$ to the high type buyer and the lowest price $\underline{p} = \underline{\theta}_S + \underline{\theta}_B$ to the low type buyer. If he cannot know the buyer's type even though he uses MR, he chooses p^* if $\beta > \beta^*$ and $\underline{p}$ if $\beta \leq \beta^*$. We denote the low type seller's payoff by $\hat{\underline{\Pi}}^M$ and the equilibrium MR service fee by $\hat{m}^*$.

If he uses MR for $\hat{m}^*$, then he can obtain a payoff given by:

$$\hat{\underline{\Pi}}^M = \begin{cases} (1-\epsilon)\left[\beta(\bar{\theta}_B + (1-\beta)\underline{\theta}_B\right] + \alpha\Delta\theta_S) + \epsilon\underline{\theta}_B & \text{if } \beta \leq \beta^* \\ \beta(\bar{\theta}_B + \alpha\Delta\theta_S) + (1-\beta)(1-\epsilon)\underline{\theta}_B & \text{if } \beta > \beta^* \end{cases} \tag{10}$$

where the threshold $\beta^* = \underline{\theta}_B/(\bar{\theta}_B + \alpha\Delta\theta_S)$ is given by (4). Notice that $\hat{\underline{\Pi}}^M < \underline{\Pi}^M$ and $\hat{\underline{\Pi}}^M \to \underline{\Pi}^M$ as $\epsilon \to 0$, where $\underline{\Pi}^M$ is given by (8).

The equilibrium MR service fee is given by:

$$\hat{m}^* = \begin{cases} (1-\epsilon)\beta(\theta_B + \alpha\Delta\theta_S) - \beta\underline{\theta}_B & \text{if } \beta \geq \beta^* \\ (1-\epsilon)(1-\beta)\underline{\theta}_B & \text{if } \beta < \beta^* \end{cases} \tag{11}$$

Notice that $\hat{m}^* < m^*$ and $\hat{m}^* \to m^*$ as $\epsilon \to 0$. Imperfect MR is less attractive for the low type seller than perfect MR. The high type seller's strategy is the same in Proposition 4 and his payoff is given by (9).

Next, we consider the case in which both AD and MR are available but only MR is imperfect. Since AD can perfectly reveal the quality, then the high type seller uses AD in equilibrium. Again, both types of the seller use imperfect MR in equilibrium as in section 3. We denote the high (and low) type seller's payoff by $\hat{\bar{\Pi}}^{AM}$ (and $\hat{\underline{\Pi}}^{AM}$) and the equilibrium MR service fee by $\hat{m}^*$.

$$\hat{\underline{\Pi}}^{AM} = \hat{\bar{\Pi}}^{AM} = \begin{cases} [\epsilon + (1-\epsilon)(1-\beta)]\underline{\theta}_B + (1-\epsilon)\beta\bar{\theta}_B & \text{if } \beta \leq \tilde{\beta} \\ (1-\epsilon)(1-\beta)\underline{\theta}_B + \beta\bar{\theta}_B & \text{if } \beta > \tilde{\beta} \end{cases} \tag{12}$$

where the threshold $\tilde{\beta} = \underline{\theta}_B/\bar{\theta}_B$ is given by (3). Notice that $\hat{\bar{\Pi}}^{AM} < \bar{\Pi}^{AM}$ and $\hat{\underline{\Pi}}^{AM} < \underline{\Pi}^{AM}$, and $\hat{\bar{\Pi}}^{AM} \to \bar{\Pi}^{AM}$ and $\hat{\underline{\Pi}}^{AM} \to \underline{\Pi}^{AM}$ as $\epsilon \to 0$, where $\bar{\Pi}^{AM}$ and $\underline{\Pi}^{AM}$ are given by (9). Furthermore, $\hat{\bar{\Pi}}^{AM} \to \bar{\Pi}^A$ as $\epsilon \to 1$, where $\bar{\Pi}^A$ is given by (7).

This comparison of payoffs leads that a synergy effect between AD and MR for the high type seller vanishes as $\epsilon \to 1$.

The equilibrium MR fee is given by:

$$\hat{m}^* = \begin{cases} (1-\epsilon)(\beta\theta_B + \alpha\Delta\theta_S) - \beta\underline{\theta}_S & \text{if } \beta \leq \beta^* \\ (1-\epsilon)(1-\beta)\underline{\theta}_B & \text{if } \beta > \beta^* \end{cases} \tag{13}$$

Again $\hat{m}^* < m^*$ and $\hat{m}^* \to m^*$ as $\epsilon \to 0$.

5.2 Imperfect advertising

We consider the case in which only AD is available. We consider imperfect AD as follows: even though the seller uses AD, the buyer cannot know the quality with probability $\eta \in [0, 1]$, then he still keeps the initial belief about the quality $(\alpha, 1-\alpha)$. Notice that the seller cannot observe whether the buyer correctly knows the quality.

First, we consider a PBE in which only one type of seller uses imperfect AD. The buyer can perfectly distinguish the quality by the seller's advertising behavior even though she cannot judge the quality due to imperfection of AD. Therefore, only the high type seller uses imperfect AD and the same separating PBE as in Case AD realizes for any η.

Second, we consider a PBE in which both types of the seller use imperfect AD. Since the buyer cannot distinguish the quality with the probability η from the seller's advertising behavior, the equilibrium price must be pooling: $p_* = \alpha\bar{\theta}_S + (1-\alpha)\underline{\theta}_S + \underline{\theta}_B$ if β is small and $p^* = \alpha\bar{\theta}_S + (1-\alpha)\underline{\theta}_S + \bar{\theta}_B$ if β is large. However, p_* cannot be the equilibrium price by Assumption 2, so a PBE does not exist if β is small. We denote the high (and low) type seller's payoff by $\hat{\bar{\Pi}}^A$ (and $\hat{\underline{\Pi}}^A$) and the equilibrium AD service fee by $\hat{a}^*$.

$$\begin{cases} \hat{\bar{\Pi}}^A = \beta[\bar{\theta}_B - (1-\alpha)\Delta\theta_S] \\ \hat{\underline{\Pi}}^A = \beta(\bar{\theta}_B + \alpha\Delta\theta_S). \end{cases} \tag{14}$$

Notice that (14) is the same as (6). The equilibrium AD fee is given by:

$$\hat{a}^* = (1-\beta)[\alpha(1-\eta)\Delta\theta_S - \eta\bar{\theta}_B] \tag{15}$$

To prevent the high type seller from deviating from the PBE, $(1-\beta)(1-\eta)\bar{\theta}_B \leq \hat{\bar{\Pi}}^A$ or $\eta \geq (1-\alpha)\Delta\theta_S/\bar{\theta}_B$ must hold.

A PBE exists in which both types of the seller use AD if and only if both β and η are sufficiently large. This implies that a synergy effect between AD and MR for the high type seller vanishes for sufficiently large η if AD is imperfect.

5.3 Remark

Remark 1. Since the buyer can observe the seller's advertising strategy, she can perfectly know the quality even though AD is not directly informative if only one type of a seller uses AD in equilibrium. Notice that we do not consider the other functions of AD; demand stimulation, for instance.

Remark 2. If fixed cost $c > a^*$ exists for the advertising agent to provide AD, the seller cannot use AD in equilibrium. Similarly, if fixed cost $d > m^*$ exists for the market researcher to provide MR, the seller cannot use MR in equilibrium.

If marketing costs are very high for the seller and the costs exceed a payoff, then the seller has no incentive to use them. We can interpret Benchmark as the case where both c and d are sufficiently high, and the seller cannot benefit from marketing activities.

Remark 3. If we allow the non-monotone belief, so many pooling equilibria and separating equilibria exist.

We can consider the belief which does not satisfy monotonicity unlike the previous sections. Without the monotone belief, we can construct any PBE in which any prices are equilibrium prices if they are not too high to be accepted. To construct such a PBE, we can consider the belief that a buyer infers a seller of low quality with probability 1 for a price except p^*, for example, but it is not realistic.

Remark 4. We consider only pure strategy PBE, and there exists a unique PBE in Benchmark if β is in the range of $\beta \leq \underline{\beta}$ and $\beta \geq \bar{\beta}$. However, a mixed strategy PBE exists.

The seller cannot obtain higher payoff in mixed strategy PBE than in pure strategy PBE since the seller mixes the pure equilibrium prices. Therefore, propositions in section 4 almost hold.

6 Conclusion

We considered the role of market research and analyzed a relation between advertising and market research in a market under bilateral asymmetric information. Since a seller can grasp a buyer's reservation value through market research, market research may intuitively seem to be an effective marketing business method. However, market research alone cannot increase sales for a high quality seller even when the cost is zero. Furthermore, market research alone does not improve market efficiency. However, the availability of advertising and market research to a monopoly seller results in disappearance of information asymmetry and efficient trade. Advertising is needed to realize and efficient trade. We have shown that advertising complements market research and that a synergy effect between them exists. This synergy effect can increase sales for the high quality seller. This implies the significance of marketing-mix.

We considered only two types of a seller and a buyer for simplicity. There are many intermediate levels in a real world. We can know in detail how much quality of a seller uses advertising or market research by analyzing more than two types or continuous types. We considered only two marketing activities here, but we can allow other marketing methods to a seller. For example, a seller can costly invest to increase a quality of an object. We can extend our model and analyze several marketing mix. These topics will be next research issues.

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8 Appendix

Proof of Proposition 1

Proof. Given a price p offered by the seller, let $B(p)$ be a conditional probability that the buyer accepts p. Notice that there cannot exist any prices which only the low type buyer accepts, so $B(p) \in \{0, \beta, 1\}$.

First, we show that any PBE must be separating. Suppose that a pooling PBE exists. Let p^* be an equilibrium price. In equilibrium, since at least the high type buyer must accept p^*, $p^* \leq \alpha\bar{\theta}_S + (1-\alpha)\underline{\theta}_S + \bar{\theta}_B$. Furthermore, since $p^* \geq \bar{\theta}_S > \alpha\bar{\theta}_S + (1-\alpha)\underline{\theta}_S + \underline{\theta}_B$ must hold by Assumption 2, p^* must be in $[\bar{\theta}_S, \alpha\bar{\theta}_S + (1-\alpha)\underline{\theta}_S + \bar{\theta}_B]$ and $B(p^*) = \beta$. Therefore, the low type seller at most receives a payoff $B(p^*)\underline{\pi}(p^*) = \beta(\alpha\Delta\theta_S + \bar{\theta}_B)$. However, the low type seller can benefit from choosing a price $\underline{p} = \underline{\theta}_S + \underline{\theta}_B$ since the following inequality holds when $\beta \leq \underline{\beta}$:

$$B(p^*)\underline{\pi}(p^*) \leq \frac{\alpha\Delta\theta_S + \bar{\theta}_B}{\Delta\theta_S + \bar{\theta}_B} \times \underline{\theta}_B < \underline{\theta}_B = B(\underline{p})\underline{\pi}(\underline{p}).$$

The first inequality holds from (2) and the definition of $\underline{\beta}$, and the last equality holds from the fact $B(\underline{p}) = 1$. This is a contradiction.

Second, let $\bar{p}$ (and $\underline{p}$) ($\bar{p} \neq \underline{p}$) be an equilibrium price chosen by the high (and low) type seller, respectively. By Assumption 1 (monotone belief), $\bar{p} > \underline{p}$. In a separating PBE, $B(\underline{p}) \neq B(\bar{p})$ must hold. If $B(\underline{p}) = B(\bar{p})$, then the low type seller can profitably deviate to choose $\bar{p}$ since $\bar{p} > \underline{p}$. Since $\underline{\pi}(\underline{p}) = \underline{p} - \underline{\theta}_B < \bar{p} - \underline{\theta}_B = \underline{\pi}(\bar{p})$, the low type seller's incentive compatible condition, $B(\underline{p})\underline{\pi}(\underline{p}) \geq B(\bar{p})\underline{\pi}(\bar{p})$, implies $B(\underline{p}) > B(\bar{p})$. Since by Assumption 2 the high type seller can obtain a positive payoff by choosing a price $\underline{\theta}_S + \bar{\theta}_B$ which is accepted by the high type buyer, $B(\bar{p}) > 0$ must hold. Therefore, $B(\underline{p}) > B(\bar{p}) > 0$, so $B(\underline{p}) = 1$ and $B(\bar{p}) = \beta$. In equilibrium, since both types of the seller maximize payoffs, equilibrium prices are $\underline{p} = \underline{\theta}_S + \underline{\theta}_B$ and $\bar{p} = \bar{\theta}_S + \bar{\theta}_B$. A monotone belief is given by:

$$\mu(\bar{\theta}_S \mid p) = \begin{cases} 1 & \text{if } p \geq \hat{p} \\ 0 & \text{if } p < \hat{p}, \end{cases}$$

where $\hat{p} \in (\bar{\theta}_S + \underline{\theta}_B, \bar{p}]$. If $\hat{p} \leq \bar{\theta}_S + \underline{\theta}_B$, the low type seller can profitably deviate to a price $p = \bar{\theta}_S + \underline{\theta}_B$.

Finally, we show that above $\underline{p}, \bar{p}$ and μ construct a separating PBE under $\beta \leq \underline{\beta}$. The low type seller receives $B(\underline{p})\underline{\pi}(\underline{p}) = \underline{\theta}_B$ by choosing $\underline{p} = \underline{\theta}_S + \underline{\theta}_B$. Since $B(p) = \beta$ for $p \in (\underline{p}, \bar{p})$, $B(p)\underline{\pi}(p) = \beta(p - \underline{\theta}_B) < \beta(\bar{p} - \underline{\theta}_B) < \underline{\theta}_B = B(\underline{p})\underline{\pi}(\underline{p})$ when $\beta \leq \underline{\beta}$. $\underline{p}$ is optimal to the low type seller. The high type seller receives $B(\bar{p})\bar{\pi}(\bar{p}) = \beta(\bar{p} - \bar{\theta}_S) = \beta\bar{\theta}_B$ by choosing $\bar{p} = \bar{\theta}_S + \bar{\theta}_B$.

Any prices $p \in (\underline{p}, \bar{p})$ give the high type seller a payoff less than $B(\bar{p})\bar{\pi}(\bar{p})$ since $B(p) = \beta$ for $p \in (\underline{p}, \bar{p})$. By Assumption 2, $\underline{p}$ gives the high type seller a negative payoff. $\bar{p}$ is optimal to the high type seller. The belief μ is consistent with $\underline{p}$ and $\bar{p}$. $\square$

Proof of proposition 2

Proof. First, we show that any PBE must be pooling. Suppose that a PBE is separating. Let $\bar{p}$ (and $\underline{p}$) be an equilibrium price chosen by the high (and low) type seller. As shown in Proposition 1, $B(\underline{p}) = 1$ must hold and $\underline{p} = \underline{\theta}_S + \underline{\theta}_B$. However, the low type seller can benefit from choosing a price $p = \underline{\theta}_S + \bar{\theta}_B$ since $B(p) = \beta$ and $B(\underline{p})\underline{\pi}(\underline{p}) = \underline{\theta}_B < \beta\bar{\theta}_B = B(p)\underline{\pi}(p)$ when $\beta \geq \bar{\beta}$. The inequality holds from (2) and the definition of $\bar{\beta}$. This is a contradiction.

Second, let p^* be an equilibrium price. In a pooling PBE, either $B(p^*) = 1$ or $B(p^*) = \beta$ must hold. If $B(p^*) = 1$, then $p^* \leq \alpha\underline{\theta}_S + (1-\alpha)\bar{\theta}_S + \underline{\theta}_B$ and by Assumption 2, the high type seller cannot obtain a positive payoff by choosing p^*. Therefore, $B(p^*) = \beta$. In equilibrium, since both types of the seller maximize payoffs, an equilibrium price is $p^* = \alpha\underline{\theta}_S + (1-\alpha)\bar{\theta}_S + \bar{\theta}_B$. A monotone belief is given by:

$$\mu(\bar{\theta}_S \mid p) = \begin{cases} \alpha & \text{if } p \geq p^* \\ 0 & \text{if } p < p^*. \end{cases}$$

Finally, we show that above p^* and μ construct a pooling PBE under $\beta \geq \bar{\beta}$. Since $B(p) = 0$ for any prices $p > p^*$, both types of the seller have no incentive to offer such prices. Since $B(p) = 0$ for $p \in [\bar{\theta}_S, p^*)$, and since any prices $p < \bar{\theta}_S$ gives the high type seller a negative payoff by Assumption 2, p^* is optimal to the high type seller. Since $B(p) = 0$ for $p \in (\underline{\theta}_S + \bar{\theta}_B, p^*)$, and since $B(p) = \beta$ for $p \in (\underline{\theta}_S + \underline{\theta}_B, \underline{\theta}_S + \bar{\theta}_B]$, any prices $p \in (\underline{\theta}_S + \underline{\theta}_B, p^*)$ give the low type seller a payoff less than $B(p^*)\underline{\pi}(p^*)$. Since $B(\underline{p}) = 1$ for $\underline{p} = \underline{\theta}_S + \underline{\theta}_B$, $B(\underline{p})\underline{\pi}(\underline{p}) = \underline{\theta}_B < \beta(\alpha\Delta\theta_S + \bar{\theta}_S) = B(p^*)\underline{\pi}(p^*)$ when $\beta \geq \bar{\beta}$. p^* is optimal to the low type seller. The belief μ is consistent with p^*. $\square$

Proof of Proposition 3

Proof. First, we show that any PBE must be separating. Since the high type seller prefers to reveal his type to the buyer, he uses AD if a service fee is equal to of less than additional payoff of the high type seller from using AD. The advertising agent sets a service fee $a^* > 0$ as the additional payoff, otherwise he can obtain zero payoff. It is clear that the low type seller does not prefer to use AD. Since the buyer is fully rational, she understands that the high type seller has an incentive to use AD, so the buyer forms belief that the seller not using AD is the low type for sure. Therefore, a PBE is separating.

Second, let $\bar{p}$ (and $\underline{p}$) be an equilibrium price chosen by the high (and low) type seller. Both types of the seller choose the prices to maximize payoffs given β. If $\beta \leq \tilde{\beta}$, the seller is likely to face to the low type buyer, so equilibrium prices are low: $\bar{p} = \bar{\theta}_S + \underline{\theta}_B$ and $\underline{p} = \underline{\theta}_S + \underline{\theta}_B$, and $B(\bar{p})\bar{\pi}(\bar{p}) = \underline{\theta}_B$ and $B(\underline{p})\underline{\pi}(\underline{p}) = \underline{\theta}_B$. If $\beta > \tilde{\beta}$, the seller is likely to face to the high type buyer, so equilibrium prices are high: $\bar{p} = \bar{\theta}_S + \bar{\theta}_B$ and $\underline{p} = \underline{\theta}_S + \bar{\theta}_B$, and $B(\bar{p})\bar{\pi}(\bar{p}) = \beta\bar{\theta}_B$ and $B(\underline{p})\underline{\pi}^A(\underline{p}) = \beta\bar{\theta}_B$. The threshold $\tilde{\beta} = \underline{\theta}_B/\bar{\theta}_B$ is given by (3).

Third, let a^* be an equilibrium AD service fee. The advertising agent should set the equilibrium AD fee just as additional payoff of the high type seller from using AD. If the high type seller does not use AD, he can obtain maximum payoff $B(p)\bar{\pi}(p) = \beta(\bar{\theta}_B - \Delta\theta_S)$ by $p = \underline{\theta}_S + \bar{\theta}_B$ for all β since both types of the buyer infer that the seller's type is low. The additional payoff of the high type seller from using AD is given by $B(\bar{p})\bar{\pi}(\bar{p}) - B(p)\bar{\pi}(p)$, so $a^* = \beta\Delta\theta_S$ when $\beta > \tilde{\beta}$, and $a^* = \underline{\theta}_B - \beta(\bar{\theta}_B - \Delta\theta_S)$ when $\beta \leq \tilde{\beta}$.

Finally, we show that above $\bar{p}, \underline{p}$ and a^* construct a separating PBE. By above discussion, the high type seller is indifferent between choosing $\bar{p}$ with AD and choosing $p = \underline{\theta}_S + \bar{\theta}_B$ without AD. Prices $\bar{p}$ and $\underline{p}$ maximize the high type and the low type seller's payoffs respectively, and the AD fee a^* maximize the advertising agent's payoff. The buyer's belief μ is consistent with $\bar{p}$ and $\underline{p}$. □

Proof of Proposition 4

Proof. First, we show any PBE must be semi-separating. Suppose that a pooling PBE exists. In a pooling PBE, either both types of the seller use MR or no type of the seller use MR. Two pooling prices exist: a high pooling price $p^* \leq \alpha\bar{\theta}_S + (1-\alpha)\underline{\theta}_S + \bar{\theta}_B$ which only the high type buyer accepts, and a low pooling price $p_* \leq \alpha\bar{\theta}_S + (1-\alpha)\underline{\theta}_S + \underline{\theta}_B$ which both types of the buyer accept. However, by Assumption 2, the high type seller cannot obtain a positive payoff by choosing p_*, so an equilibrium pooling price must be p^* independent of the buyer's types. If both types of the seller use MR, they can benefit from stopping using MR since an equilibrium MR fee is positive. If no type of the seller use MR, the low type seller can benefit from using MR and choosing the lowest price $\underline{p} = \underline{\theta}_S + \underline{\theta}_B$ (and p^*) to the low type buyer (and the high type buyer) respectively. This is a contradiction. Suppose that a separating PBE exists. Let $\bar{p}$ (and $\underline{p}$) be an equilibrium price chosen by the high (and low) type seller. As shown in Proposition 1, both types of the buyer accept $\underline{p}$, only the high type buyer accepts $\bar{p}$ and $\underline{p}$ ($\bar{p} > \underline{p}$). However, the low type seller can benefit from using MR and choosing $\underline{p}$ (and $\bar{p}$) to the low (and high) type buyer respectively. This is a contradiction.

Second, we show that only the low type seller uses MR, and the high type buyer faces to a pooling price and the low type buyer faces to a separating price. In a semi-separating PBE, one type of the buyer faces to a pooling price and another type of the buyer faces to a separating price. Let p^* be an equilibrium pooling price and $\underline{p}$ be an equilibrium separating price. By Assumption 2, an equilibrium pooling price must be high as in Proposition 2, so $p^* = \alpha\bar{\theta}_S + (1-\alpha)\underline{\theta}_S + \bar{\theta}_B$ and only the high type seller faces to the pooling price. The low type buyer faces to a separating price $\underline{p}$. Suppose that the high type seller uses MR and offers $\underline{p}$ to the low type seller, $\underline{p} \leq \bar{\theta}_S + \underline{\theta}_B$. In this case, however, the low type seller can benefit from using MR and choosing the price $\underline{p}$ to the low type buyer since $\underline{p} - \bar{\theta}_S < \underline{p} - \underline{\theta}_S$. This is a contradiction. Therefore, only the low type seller uses MR, and chooses the lowest price $\underline{p} = \underline{\theta}_S + \underline{\theta}_B$ to the low type buyer. A monotone belief is given by:

$$\mu(\bar{\theta}_S \mid p) = \begin{cases} \alpha & \text{if } p \geq \hat{p} \\ 0 & \text{if } p < \hat{p}. \end{cases}$$

where $\hat{p} \in (\alpha\bar{\theta}_S + (1-\alpha)\underline{\theta}_S + \underline{\theta}_B, p^*]$.

Third, let m^* be an equilibrium MR service fee. The market researcher should set the equilibrium MR fee just as additional payoff of the low type seller from using MR. If the low type seller does not use MR, he can obtain maximum payoff $B(p^*)\underline{\pi}(p^*) = \beta(\bar{\theta}_B + \alpha\Delta\theta_S)$ by choosing p^* when $\beta > \beta^*$, and $B(\underline{p})\underline{\pi}(\underline{p}) = \underline{\theta}_B$ by choosing $\underline{p}$ when $\beta \leq \beta^*$. The additional payoff of the low type seller from using MR is given by $(1-\beta)\underline{\theta}_B + \beta(\alpha\Delta\theta_S + \bar{\theta}_B) - B(p^*)\underline{\pi}(p^*)$ when $\beta > \beta^*$, and $(1-\beta)\underline{\theta}_B + \beta(\alpha\Delta\theta_S + \bar{\theta}_B) - B(\underline{p})\underline{\pi}(\underline{p})$ when $\beta \leq \beta^*$. Therefore, we obtain $m^* = (1-\beta)\underline{\theta}_B$ when $\beta > \beta^*$, and $m^* = \beta(\alpha\Delta\theta_S + \Delta\theta_B)$ when $\beta \leq \beta^*$.

Finally, we show that above $p^*, \underline{p}, m^*$ and μ construct a semi-separating PBE. By the above discussion, the low type seller is indifferent between choosing p^* and $\underline{p}$ with MR and choosing p^* or $\underline{p}$ without MR. As in Proposition 2, p^* maximizes the high type seller's payoff given buyer's belief. The buyer's belief is consistent to the seller's behavior. Therefore, $p^*, \underline{p}, m^*$ and μ construct a semi-separating PBE. □

Proof of Proposition 5

Proof. First, we show that any PBE must be separating. As in the proof of Proposition 3, only the high type seller uses AD in equilibrium, so the buyer forms a belief which the seller not using AD is surely the low type for all p.

Second, we show that both types of the seller use MR in equilibrium. If the low type seller does not use MR, he chooses either the lowest price $\underline{p}_* = \underline{\theta}_S + \underline{\theta}_B$ or a high price $\underline{p}^* = \underline{\theta}_S + \bar{\theta}_B$. The former is the maximum price for which the low type buyer buys the object and the latter is the maximum price for which the high type buyer buys the object when the seller's type is public information. If the low type seller uses MR, he chooses $\underline{p}_*$ to the low type buyer and $\underline{p}^*$ to the high type buyer. The market researcher should set the equilibrium MR service fee $m^* > 0$ just as additional payoff of the low type seller from using MR, so we obtain the following equation:

$$\beta(\underline{p}^* - \underline{\theta}_S) + (1-\beta)(\underline{p}_* - \underline{\theta}_S) - m^* = \max\{\beta(\underline{p}^* - \underline{\theta}_S), (1-\beta)(\underline{p}_* - \underline{\theta}_S)\} = \begin{cases} \underline{\theta}_B & \text{if } \beta \leq \tilde{\beta} \\ \beta\bar{\theta}_B & \text{if } \beta > \tilde{\beta}. \end{cases}$$

Therefore, the low type seller uses MR only if $m^* = \beta\Delta\theta_B$ when $\beta \leq \tilde{\beta}$, and $m^* = (1-\beta)\underline{\theta}_B$ when $\beta > \tilde{\beta}$. Similar discussion holds for the high type seller. If the high type seller does not use MR, he chooses either the lowest price $\bar{p}_* = \bar{\theta}_S + \underline{\theta}_B$ or a high price $\bar{p}^* = \bar{\theta}_S + \bar{\theta}_B$. The market researcher should set the equilibrium MR service fee $m^{**} > 0$ just as additional payoff of the high type seller from using MR. Therefore, the high type seller uses MR only if $m^{**} = \beta\Delta\theta_B$ when $\beta \leq \tilde{\beta}$, and $m^{**} = (1-\beta)\underline{\theta}_B$ when $\beta > \tilde{\beta}$. Since $m^* = m^{**}$ for all β, both types of the seller use MR in equilibrium.

Third, let a^* be an equilibrium AD service fee. The advertising agent should set the equilibrium AD fee just as additional payoff of the high type seller from using AD. If the high type seller does not use AD, he can obtain maximum payoff $B(p)\bar{\pi}(p) = \beta(\bar{\theta}_B - \Delta\theta_S)$ by choosing $p = \underline{\theta}_S + \bar{\theta}_B$ for all β since both types of the buyer infer that the seller's type is low. The additional payoff of the high type seller from using AD is given by $[\beta\bar{\theta}_B+(1-\beta)\underline{\theta}_B]-B(p)\bar{\pi}(p)$, so $a^*=\beta\Delta\theta_S$ when $\beta>\tilde{\beta}$, and $a^*=\underline{\theta}_B-\beta(\bar{\theta}_B-\Delta\theta_S)$ when $\beta\leq\tilde{\beta}$.

Finally, we show that above $\bar{p}^*, \bar{p}_*, \underline{p}^*, \underline{p}_*, a^*$ and m^* construct a separating PBE. By the above discussion, the high type seller is indifferent between choosing $\bar{p}^*$ and $\bar{p}_*$ with AD and choosing p^d without AD given using MR, the low type seller is indifferent between choosing $\underline{p}^*$ and $\underline{p}_*$ with MR and choosing $\underline{p}^*$ or $\underline{p}_*$ without MR, and the high type seller is indifferent between choosing $\bar{p}^*$ and $\bar{p}_*$ with MR and choosing $\bar{p}^*$ or $\bar{p}_*$ without MR given using AD. Furthermore, $\bar{p}^*, \bar{p}_*, \underline{p}^*$ and $\underline{p}_*$ maximize the seller's payoffs if information asymmetry fully disappears, so the prices are optimal to the seller given his marketing strategy. The buyer's belief is consistent to the seller's behavior. Therefore, we must check the high type seller's deviation which he does not use AD nor MR, and the low type seller's deviation which he uses AD and does not use MR. Suppose that the high type seller does not use AD nor MR. Since he can choose only a unitary price, he can obtain maximum payoff $\beta(p^d-\bar{\theta}_S)=\beta(\bar{\theta}_B-\Delta\theta_S)$ by $p^d=\underline{\theta}_S+\bar{\theta}_B$. Therefore, he cannot profitably deviate to this marketing strategy. Next, suppose that the low type seller uses AD and does not use MR. The buyer can specify the seller's type even though the low type seller does not use AD. Therefore, since he can always save the positive AD service fee, he cannot profitably deviate to this marketing strategy. This completes the proof. □

Proof of Proposition 6

Proof. In Benchmark, the high type seller cannot trade with the low type buyer in a separating PBE (low β), and both types seller cannot trade with the low type buyer in a pooling PBE (high β). The low type seller can trade with the low type buyer in Case MR, so MR can improve market efficiency when β is high. On the other hand, trade between the both types of the seller and both types of the buyer is possible when β is low in Case AD, so AD can improve market efficiency when β is low. □

Proof of Proposition 7

Proof. $\bar{\Pi}^M<\bar{\Pi}$ holds from equations (5) and (8), $\bar{\Pi}^M=\bar{\Pi}$ from (6) and (8), and $\underline{\Pi}^M>\underline{\Pi}$ from (5), (6) and (8). □

Proof of Proposition 8

Proof. By Proposition 4.1, the seller $\bar{\theta}_S$ does not use MR for any $m \geq 0$ in Case MR. Since he cannot increase his sales even if he uses MR, then $\bar{\Pi}^M - \bar{\Pi} = 0$. Similarly, we obtain the following equations:

$$\bar{\Pi}^A - \bar{\Pi} = \begin{cases} \underline{\theta}_B - \beta\bar{\theta}_B & \text{if } \beta \leq \underline{\beta} \\ \beta(1-\alpha)\Delta\theta_S & \text{if } \beta \geq \bar{\beta} \end{cases}$$

from equations (5), (6) and (7).

$$\bar{\Pi}^{AM} - \bar{\Pi} = \begin{cases} (1-\beta)\underline{\theta}_B & \text{if } \beta \leq \underline{\beta} \\ (1-\beta)\underline{\theta}_B + \beta(1-\alpha)\Delta\theta_S & \text{if } \beta \geq \bar{\beta} \end{cases}$$

from equations (5), (6) and (9).

Therefore, we obtain $(\bar{\Pi}^M - \bar{\Pi}) + (\bar{\Pi}^A - \bar{\Pi}) < (\bar{\Pi}^{AM} - \bar{\Pi})$. □

Proof of Proposition 9

Proof. From equations (5), (6) and (9), $\underline{\Pi} < \underline{\Pi}^{AM}$ if and only if $\alpha\beta\Delta\theta_S < (1-\beta)\underline{\theta}_B$. This implies that if β is not too large and if $\Delta\theta_S$ is not too large, then the low type seller's payoff increases as compared with Benchmark. □