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Objectivist versus Subjectivist Approaches to the Marxian Theory of Exploitation

Roberto Veneziani
(Department of Economics, Queen Mary, University of London)
and
Naoki Yoshihara
(Institute of Economic Research, Hitotsubashi University)

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Institute of Economic Research
Hitotsubashi University
Kunitachi, Tokyo, 186-8603 Japan

Objectivist versus Subjectivist Approaches to the Marxian Theory of Exploitation*

Roberto Veneziani[†] and Naoki Yoshihara[‡]

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Abstract

This paper analyses two central issues in exploitation theory. First, the appropriate definition of individual and aggregate measures of exploitation is discussed. Second, the relation between profits and exploitation (the so-called Fundamental Marxian Theorem) is analysed. A general framework for the analysis of exploitation in the context of convex cone economies is proposed and various alternative equilibrium concepts are discussed. The limits of subjectivist approaches to exploitation, which crucially depend on agents' preferences, are shown. An objectivist approach to exploitation, which is related to the so-called 'New Interpretation' (Dumenil, 1980; Foley, 1982) is proposed. It is argued that it captures the core intuitions of exploitation theory and that it provides appropriate indices of individual and aggregate exploitation. Further, it is shown that it preserves the Fundamental Marxian Theorem in general economies.

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†Department of Economics, Queen Mary, University of London, Mile End Road, London E1 4NS, UK. E-mail: r.veneziani@qmul.ac.uk

‡The Institute of Economic Research, Hitotsubashi University, Naka 2-1, Kunitachi, Tokyo 186-0004, Japan. Phone: (81)-42-580-8354, Fax: (81)-42-580-8333. e-mail: yosihara@ier.hit-u.ac.jp

and other Normative Criteria and Measurement), E11 (Macro: Marxian, Sraffian, Institutional and Monetary Economics), B51 (Current Heterodox Approaches: Socialist, Marxian, Sraffian).

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1 Introduction

The theory of exploitation is arguably the cornerstone of Marxian economics, as it shows some crucial aspects of the relation between workers and capitalists in economies characterised by the private ownership of the means of production. In general, workers are said to be exploited if the labour they expend is higher than the amount of labour contained in a relevant bundle (or set of bundles) of wage goods, which measures the value of labour power. Even in stylised two-class societies, though, it has proved surprisingly difficult to provide a fully satisfactory and rigorous theory of exploitation outside the standard simplified Leontief economy. First of all, outside the Leontief economy without joint production, with homogeneous labour, and with subsistence wages, the very definition of exploitation is ambiguous. In fact, the appropriate definition of the value of labour power is not obvious, and indeed a number of definitions have been proposed (see [20] and [22], for a discussion). In turn, this implies that the definition of the appropriate *exploitation index*, measuring the amount of exploitation in the economy, is not uncontroversial. This issue seems central because a proper theory of exploitation should arguably be able to compare different societies in terms of their exploitation levels, but also to analyse an economy, and its exploitation structure, over time. Secondly, and relatedly, outside the Leontief setting, it is not trivial to prove that the core insights and propositions of exploitation theory hold. For example, the Marxian claim that labour is the source of exploitation has been challenged, because in more general economies, a number of counterexamples to the so-called Fundamental Marxian Theorem (hereafter, FMT) have been produced. In standard Marxist approaches, this is also an important issue since the FMT proves that exploitation is synonimous with positive profits (and thus with capitalist relations of production). Actually, the relevance of the FMT is such that although it is proved as a result, its epistemological status is that of a postulate: the appropriate definition of exploitation, and of an exploitation index, is considered to be one which preserves the FMT. Indeed, a number of alternative definitions of exploitation, and exploitation indices, have been proposed precisely in an attempt to generalise the FMT to economies with joint production, heterogenous labour, etc.,[1] and alternative definitions have been evaluated in terms

[1] See, among the others, [8]; [9]; [4]; [11]; [12]; [2]. For more recent debates on the FMT, see also [14], [15], and [21].

of their ability to preserve the FMT.[2]

In line with Marx's own approach, the main approaches to exploitation theory proposed in the literature have endorsed an *objectivist* perspective by defining exploitation in relation to the objective features of an economy (including data on production, consumption, labour supply, etc.), with no reference to agents' individual attitudes, beliefs, and subjective preferences. In a recent contribution, instead, [6] has proposed an original and thought-provoking *subjectivist* theory of exploitation, in which agents' preferences play a definitional role alongside the objective features (related to production and distribution) of the economy. In this approach, workers are exploited if and only if there is a bundle of goods that they weakly prefer to the bundle of wage goods they receive and that can be produced with less labour than they have expended. According to [6], this definition of exploitation is superior to alternative approches to the FMT. In particular, it can deal with counterexamples such as those constructed by Petri ([10]) and Roemer ([11], [12]) against Morishima's ([8], [9]) famous definition of exploitation, which show that in economies with joint production it is possible to have positive profits with no exploitation.

This paper thoroughly analyses the subjectivist approach and it compares it with the standard objectivist view, focusing in particular on the appropriate definition of an exploitation index and on the FMT. A general framework for the analysis of exploitation in the context of convex cone economies is proposed. First of all, the subjectivist approach to exploitation is analysed. On the one hand, the main characterisation result (the Weak System of Exploitation Theory) is significantly generalised to a convex economy with heterogeneous agents. On the other hand, it is shown that under different concepts of equilibrium, the FMT does not hold and thus, contrary to Matsuo's claim, the subjectivist approach does not solve the problems of traditional theories of exploitation. Furthermore, although no precise subjectivist exploitation index is provided, the properties of a class of subjectivist indices which satisfy two weak axioms are explored. Two rather counterintuitive implications of the subjectivist view are proved: first, even if the objective productive and distributive conditions of the economy are

[2]It is worth noting that Roemer ([13]) suggested that the Class-Exploitation Correspondence Principle - according to which agents in the lower classes are exploited and agents in the upper classes are exploiters - enjoys the same epistemological status as the FMT in Marxian economics. See also [20] and [22], which prove that under the received definitions of exploitation, the CECP does not hold in general convex cone economies.

unchanged, a subjectivist exploitation index will vary except in a very small set of economies. Second, there exist economies in which, given the same set of objective data, the exploitation index can take virtually any value in between 0 and 1, unless the set of preferences is restricted.

Secondly, a rigorous axiomatic definition of an objectivist exploitation index is provided, by requiring that if two economies differ only in the agents' individual characteristics (preference profiles), then exploitation should be the same in both of them. This is a very weak requirement to define objectivism, as it allows, for instance, workers' preferences to matter in the choice of their consumption bundle and thus, in principle, in the definition of the value of labour power. Then, an objectivist definition of exploitation is analysed, which is conceptually related to the so-called 'New Interpretation' ([1]; [3]; see [7], for a recent survey) and it is proved that it preserves the FMT in general convex economies. Although the paper focuses on Marxian exploitation theory, the discussion should be of interest for non-Marxists, too, because of the interesting positive and normative insights that the notion of exploitation as the unequal exchange of labour provides.

2 The General Model

In this section, a general framework for the analysis of exploitation is provided along the lines of Roemer ([11], [12]), which allows for a general convex cone technology, rather than the standard von Neumann framework often used in exploitation theory. This is not just for the sake of formalism: the differences between alternative approaches to exploitation and the anomalies in the relation between profits and exploitation become relevant when the linear production model is abandoned.

2.1 Production

In the economy there are n produced commodities and one non-produced good, namely labour. Let $\mathbf{0} \in \mathbb{R}^n$ be such that $\mathbf{0} = (0, ..., 0)$. Let P be the production set: P has elements of the form $\alpha = (-\alpha_0, -\underline{\alpha}, \overline{\alpha})$ where $\alpha_0 \in \mathbb{R}_+$, $\underline{\alpha} \in \mathbb{R}^n_+$, and $\overline{\alpha} \in \mathbb{R}^n_+$. Thus, elements of P are vectors in $\mathbb{R}^{2n+1}$. The first component, $-\alpha_0$, is the direct labour input of the process α; the next n components, $-\underline{\alpha}$, are the inputs of goods used in the process; and the last n components, $\overline{\alpha}$, are the outputs of the n goods from the process.

The net output vector arising from α is denoted as $\widehat{\alpha} \equiv \overline{\alpha} - \underline{\alpha}$. The set P is assumed to be a closed convex cone containing the origin in $\mathbb{R}^{2n+1}$. Moreover, it is assumed that:[3]

Assumption (A1). $\forall \alpha \in P$ s.t. $\alpha_0 \geq 0$ and $\underline{\alpha} \geqq \mathbf{0}$, $[\overline{\alpha} \geq \mathbf{0} \Rightarrow \alpha_0 > 0]$;

Assumption (A2). $\forall\, c \in \mathbb{R}^n_+$, $\exists \alpha \in P$ s.t. $\widehat{\alpha} \geqq c$;

Assumption (A3). $\forall \alpha \in P$, $\forall\, (\underline{\alpha}', \overline{\alpha}') \in \mathbb{R}^n_+ \times \mathbb{R}^n_+$,

$$[(-\underline{\alpha}', \overline{\alpha}') \leqq (-\underline{\alpha}, \overline{\alpha}) \Rightarrow (-\alpha_0, -\underline{\alpha}', \overline{\alpha}') \in P].$$

A1 implies that labour is indispensable to produce any non-negative output vector; A2 states that any non-negative commodity vector is producible as a net output; and A3 is a *free disposal* condition for the production possibility set, which states that, given any feasible production process α, any vector producing (weakly) less net output than α is also feasible using the same amount of labour as α itself.

Given P, the following notation is used:

$$\begin{aligned}
P\,(\alpha_0 = l) &\equiv \{(-\alpha_0, -\underline{\alpha}, \overline{\alpha}) \in P \mid \alpha_0 = l\}\,, \\
\widehat{P}\,(\alpha_0 = l) &\equiv \{\widehat{\alpha} \in \mathbb{R}^n \mid \exists \alpha = (-l, -\underline{\alpha}, \overline{\alpha}) \in P \text{ s.t. } \overline{\alpha} - \underline{\alpha} \geqq \widehat{\alpha}\}\,, \\
S\widehat{P}\,(\alpha_0 = l) &\equiv \left\{\widehat{\alpha} \in \widehat{P}\,(\alpha_0 = l) \mid \nexists \widehat{\alpha}' \in \widehat{P}\,(\alpha_0 = l) \text{ s.t. } \widehat{\alpha}' \geq \widehat{\alpha}\right\},
\end{aligned}$$

where $P\,(\alpha_0 = l)$ is the set of production vectors which use l units of labour as an input, $\widehat{P}\,(\alpha_0 = l)$ is the corresponding set of net outputs, and $S\widehat{P}\,(\alpha_0 = l)$ is the set of net outputs that can be produced efficiently using exactly l units of labour. Further, for any set $S \subseteq \mathbb{R}^n$, the set $\partial S \equiv \{x \in S \mid \nexists x' \in S \text{ s.t. } x' > x\}$ is the frontier of S and $\overset{\circ}{S} \equiv S \backslash \partial S$ is its interior.

The von Neumann model with joint production (analysed, among the others, by [6]) is a special case of the convex cone technology. Let A be an $n \times m$ non-negative matrix with input coefficients $a_{ij} \geqq 0$ for any $i = 1, \ldots, n$, $j = 1, \ldots, m$, and B be an $n \times m$ non-negative matrix with output coefficients $b_{ij} \geqq 0$ for any $i = 1, \ldots, n$, $j = 1, \ldots, m$. Moreover, let L be a positive $1 \times m$ vector with labour input coefficients $L_j > 0$ for any $j = 1, \ldots, m$. To

[3] For all vectors $x = (x_1, \ldots, x_n)$ and $y = (y_1, \ldots, y_n) \in \mathbb{R}^n$, $x \geqq y$ if and only if $x_i \geqq y_i$ $(i = 1, \ldots, n)$; $x \geq y$ if and only if $x \geqq y$ and $x \neq y$; $x > y$ if and only if $x_i > y_i$ $(i = 1, \ldots, n)$.

be precise, the von Neumann economy is a particular type of P, denoted as $P_{(A,B,L)}$, which can be described as follows

$$P_{(A,B,L)} \equiv \left\{(-\alpha_0, -\underline{\alpha}, \overline{\alpha}) \in \mathbb{R}_- \times \mathbb{R}^n_- \times \mathbb{R}^n_+ \mid \exists x \in \mathbb{R}^m_+ : \alpha_0 = Lx \ \& \ (-\underline{\alpha}, \overline{\alpha}) \leqq (-Ax, Bx)\right\}.$$

This $P_{(A,B,L)}$ is a closed convex cone in $\mathbb{R}_- \times \mathbb{R}^n_- \times \mathbb{R}^n_+$ with $\mathbf{0} \in P_{(A,B,L)}$. Moreover, $P_{(A,B,L)}$ is easily shown to satisfy Assumptions 1-3.

2.2 Agents

In the standard two-class model used to analyse the FMT, the economy consists of a set K of capitalists and of a set W of workers. The set of agents N, with generic element ν, is therefore given by $N = K \cup W$. To be specific, let $\omega^\nu \in \mathbb{R}^n_+$ denote the vector of initial productive endowments of agent $\nu \in N$: the working class W is the set of agents with no initial endowments, while the capitalist class K is the set of agents endowed with some non-negative and non-zero amount of inputs. Thus, $W = \{\nu \in N : \omega^\nu = \mathbf{0}\}$ and $K = \{\nu \in N : \omega^\nu \geq \mathbf{0}\}$.[4]

Each capitalist can operate any activity of the technology P and is assumed to maximise profits. For the sake of simplicity, capitalists are also assumed to save all revenues, which are invested in the next production period, and to supply no labour (e.g., they can be assumed to derive infinite disutility from labour). Each worker is endowed with one unit of labour, which is assumed to be homogenous - there is no skill heterogenity among workers.[5] For the sake of simplicity, it is also assumed that, for a given aggregate amount of (homogeneous) labour supplied, each worker works the same amount of time and gets the same amount of goods. Therefore, if b^ν denotes the wage basket of worker ν and l^ν denotes the labour performed by ν, the latter assumption implies that $b^\nu = b$ and $l^\nu = l$, all ν. It is assumed that $b \in \mathbb{R}^n_+$.

[4] In principle, one might argue that the appropriate definition of workers and capitalists relates to their financial wealth, rather than their vector of endowments. If this view is adopted, then $W = \{\nu \in N : p\omega^\nu = 0\}$ and $K = \{\nu \in N : p\omega^\nu > 0\}$. This distinction is relevant only if $p \not> \mathbf{0}$ and it does not make any significant difference for the results of this paper.

[5] The presence of heterogeneous labour does raise important issues in exploitation theory, including on the relation between exploitation and profits (for a discussion, see, e.g., [4]). Yet, this issue is not relevant in the comparison between objectivist and subjectivist approaches, which is the central theme of this paper. In his subjectivist approach to exploitation, [6] also assumes homogeneous labour (see Assumption 3).

Given the above behavioural assumptions, a complete description of an economy should be given by a list $\langle K, W; (P, b); (\omega^\nu)_{\nu \in K} \rangle$. In a subjectivist framework, such as the one proposed by [6], however, the complete description of an economy requires also the specification of the agents' (more precisely, the workers') utility functions, even if they are not essential to analyse agents' behaviour. Thus, for every agent $\nu \in W$, let $u^\nu : \mathbb{R}_+^{n+1} \to \mathbb{R}$ be the utility function representing worker ν's preferences over consumption and leisure: a convex cone economy is given by a list $E = \langle K, W; (u^\nu)_{\nu \in W}; (P, b); (\omega^\nu)_{\nu \in K} \rangle$, and the set of all such convex cone economies is denoted as $\mathcal{E}$.

Given a market economy, any price system is denoted by $p \in \mathbb{R}_+^n$, which gives one price for each of the n commodities. Moreover, the nominal wage rate is assumed to be positive and equal to unity.

3 A Subjectivist Approach to Exploitation

In a subjectivist approach, the introduction of agents' preferences in the description of an economy is not only for completeness, nor does it play a merely subsidiary role (e.g., in determining their actual consumption and leisure choices): utility functions play a central - indeed, definitional - role, in the analysis of exploitation. Consider first the definition of labour values and the value of labour power. [6] defines the labour value of a vector b referring to the notion of Minimised Labour for Equal Utility (MLEU): the labour value of bundle b corresponds to the minimum amount of labour necessary to produce another bundle c as net output, which gives at least the same utility as b. Formally, let $\mathcal{C}$ denote the set of continuous functions. For the sake of simplicity, and without loss of generality, assume that leisure does not enter the workers' utility functions, so that for every agent $\nu \in W$, $u^\nu : \mathbb{R}_+^n \to \mathbb{R}$ is the utility function representing worker ν's preferences over consumption. The equivalent of Matsuo's ([6]) Assumption 2 in a general framework with heterogeneous agents can then be written as follows.[6]

[6]Leisure is not included in utility functions for notational simplicity and conceptual clarity. First, although [6] assumes that workers have preferences over leisure, this assumption plays no role at all in his argument and indeed he imposes no restriction whatsoever concerning the effect of leisure on welfare. Second, if leisure is included in the utility function some conceptual issues arise concerning the definition of labour value (see the next footnote). Finally, the introduction of leisure in the utility functions would leave all the theoretical arguments and formal results in this paper unchanged.

Assumption 4: For every $\nu \in W$, $u^{\nu} \in \mathcal{U}$, where $\mathcal{U} = \{u^{\nu} \in \mathcal{C} | c' \geq c \Rightarrow u^{\nu}(c') > u^{\nu}(c)\}$.

In other words, each worker's utility function is continuous and strictly increasing (in the first n arguments).

Definition 1: *For a given $u^{\nu} \in \mathcal{U}$, the labour value of vector b according to the* **MLEU** *view*, relative to agent ν, *is the solution of the following problem*:

$$ML^{\nu}: \min_{\alpha=(-\alpha_0,-\underline{\alpha},\overline{\alpha})\in P} \alpha_0 \text{ s.t. } \overline{\alpha} - \underline{\alpha} \geq c \ \left(\forall c \in \mathbb{R}^n_+ : u^{\nu}(c) \geq u^{\nu}(b)\right).$$

Let us denote the solution of the above problem by $\alpha^{u^{\nu}}(b)$: the labour value of b *w.r.t.* u^{ν} is defined as $\alpha_0^{u^{\nu}}(b)$.[7] Two crucial properties of the subjectivist definition of the labour content of a bundle b are immediately apparent from Definition 1: first, the concept of labour value depends on subjective preferences, and if agents are heterogenous in principle there is no unique value of b.[8] Second, in this approach, the notion of labour value becomes more and more abstract and far from the productive conditions of the economy.

It is now possible to specify the subjectivist notion of exploitation of an agent ν.

Definition 2: *Given a utility function $u^{\nu} \in \mathcal{U}$, worker $\nu \in W$, working $l = 1$, is* exploited w.r.t. u^{ν} in the sense of Minimised labour for Equal Utility (u^{ν}-**MLEU**) *if and only if* $1 - \alpha_0^{u^{\nu}}(b) > 0$. *Further, worker $\nu \in W$, working $l = 1$, is* exploited in the sense of Minimised Labour for Equal Utility (**MLEU**) *if and only if* $1 - \alpha_0^{u^{\nu}}(b) > 0$ *holds for all $u^{\nu} \in \mathcal{U}$.*

[7] In [6] leisure is included in workers' utility functions, which are also assumed to be identical, and thus the relevant constraint in ML is written as $u(c,l) \geq u(b,l)$, where l is the amount of labour expended by workers to be able to buy b. As already noted, the inclusion of labour has no relevance for the formal results. Yet, from a theoretical viewpoint, it seems arbitrary to keep labour constant at l in the left-hand-side of the constraint. It is not at all clear why the amount of labour in workers' utility functions should remain constant even at the new allocation c.

[8] Interestingly, in the economy with $P = P_{(A,B,L)}$, Matsuo ([6], Definition 3) defines a "Narrow Effective Range of Value" as the set of strictly positive vectors t such that $t(B-A) \leqq L$. The vectors t seem the generalisation of the standard vector of embodied labour $t = L(I-A)^{-1}$ of the Leontief technology, and they only depend on the objective features of the economy relating to the conditions of production. In this framework, it would then seem natural to define the labour value of a bundle c as tc. Yet they play no essential role in Matsuo's ([6]) analysis.

An important feature of Definition 2 should be noted, in order to clarify the distinction between objectivist and subjectivist notions of exploitation. Because the condition in Definition 2 must hold *for all* functions $u^{\nu} \in \mathcal{U}$, and the existence of exploitation is proved regardless of the specific functional form of u^{ν} (subject to a proviso, to be specified below), Matsuo maintains that "This causes this condition to be objective" ([6], p.260). This claim is rather misleading: although the existence of exploitation is independent of the specific utility function (provided it belongs to $\mathcal{U}$), the actual labour value of a bundle does depend on the specific u^{ν} (thus, for instance, the value of labour power cannot be defined unless u^{ν} is known).

If workers are not homogeneous, the inherently subjective dimension of Matsuo's approach, and its implications, are particularly clear when the construction of an aggregate index of exploitation in the economy is considered. In fact, a specific problem of subjectivist approaches with heterogenous agents concerns the aggregation of the individual exploitation indices into an aggregate measure, *even if all workers consume the same bundle*, as a number of different ways of aggregating the $\alpha_0^{u^{\nu}}(b)$'s seem reasonable. One possibility might be to use the average labour value of the common bundle b.

Definition 3: *For a given* $(u^{\nu})_{\nu \in W}$*, such that* $u^{\nu} \in \mathcal{U}$ *for all* ν*, the (economy-wide) labour value of vector* b *according to the* Minimised Labour for Equal Utility *view is:* $\alpha_0 \left(b; (u^{\nu})_{\nu \in W}\right) = \sum_{\nu \in W} \frac{\alpha_0^{u^{\nu}}(b)}{|W|}$.

Yet, this is certainly not the only way of aggregating labour values, and thus some theoretical ambiguity seems inherent in the subjectivist approach. Some of these problems disappear if one assumes, as in [6], that all workers have identical preferences. If a representative agent is assumed in this economy so that $u^{\nu} = u$ for all $\nu \in W$ with $u \in \mathcal{U}$, then the minimisation problem becomes:

$$ML: \min_{\alpha = (-\alpha_0, -\underline{\alpha}, \overline{\alpha}) \in P} \alpha_0 \text{ s.t. } \overline{\alpha} - \underline{\alpha} \geq c \ \left(\forall c \in \mathbb{R}_+^n : u(c) \geq u(b)\right).$$

Given ML, the definition of individual and aggregate exploitation can be changed accordingly.

Definition 4: *Given a welfare function* $u \in \mathcal{U}$*, every worker* $\nu \in W$ *is* exploited w.r.t. u in the sense of Minimised Labour for Equal Utility (u-**MLEU**) *if and only if* $1 - \alpha_0^u(b) > 0$*. Every worker* $\nu \in W$ *is* exploited in the sense of Minimised Labour for Equal Utility (**MLEU**) *if and only if* $1-\alpha_0^u(b)>0$ *holds for all* $u\in\mathcal{U}$.

According to [6], exploitation essentially derives from the workers' lack of control over production processes, and that if workers could access all production processes, they would not be exploited and they would be able to reach a higher utility. Therefore, in addition to the minimisation programme ML^ν above, the following maximisation problem is analysed:

$$\max_{\alpha=(-\alpha_0,-\underline{\alpha},\overline{\alpha})\in P} u^\nu(\widehat{\alpha}) \text{ s.t. } \widehat{\alpha}\in\mathbb{R}_+^n \text{ and } \alpha_0\leqq 1.$$

The solution of the above problem can be denoted by $\alpha_{\max}^{u^\nu}$, and its corresponding utility value by $\overline{u}_{\max}^{u^\nu}$. If a representative agent is assumed, then the latter reduce to $\alpha_{\max}^u$ and $\overline{u}_{\max}^u$, respectively.

Given $c\in\mathbb{R}_+^n$ and $u^\nu\in\mathcal{U}$, let the upper contour set of u^ν at c be given by $U(c;u^\nu)\equiv\left\{c'\in\mathbb{R}_+^n \mid u^\nu(c')>u^\nu(c)\right\}$. The following results generalise Matsuo's ([6]) 'Weak System of the Exploitation Theory' in two important directions: first, it allows for heterogenous workers' preferences; second, all of the equivalence results are shown to hold in general convex cone economies. This generalised result is interesting in its own right, but also because - thanks to an arguably simpler and more transparent proof - it forcefully highlights some implications of a subjective approach to exploitation theory.

Theorem 1 (The Generalised Weak System of Exploitation Theory)**:** *For any economy* $E=\langle K,W;(u^\nu)_{\nu\in W};(P,b);(\omega^\nu)_{\nu\in K}\rangle\in\mathcal{E}$*, the following statements are equivalent*:

(1) $b\in\widehat{P}(\alpha_0=1)\setminus S\widehat{P}(\alpha_0=1)$;
(2) For each $\nu\in W$, $1-\alpha_0^{u^\nu}(b)>0$ holds for all $u^\nu\in\mathcal{U}$;
(3) For each $\nu\in W$, $u^\nu(b)<\overline{u}_{\max}^{u^\nu}$ holds for all $u^\nu\in\mathcal{U}$;
(4) $\nexists p\in\mathbb{R}_{++}^n$ s.t. $p[\widehat{\alpha}-b]\leqq 0$ holds for any $\widehat{\alpha}\in S\widehat{P}(\alpha_0=1)$.

Proof. 1. First, we prove that (1)⇔(2).

(⇐): Let $b\in\widehat{P}(\alpha_0=1)\setminus S\widehat{P}(\alpha_0=1)$. Then, by definition of $\widehat{P}(\alpha_0=1)$, it needs at most one unit of labour to produce b as a net output. Since $b\in\widehat{P}(\alpha_0=1)\setminus S\widehat{P}(\alpha_0=1)$, there exists $\alpha\in P$ such that $\widehat{\alpha}\in S\widehat{P}(\alpha_0=1)$ and $\widehat{\alpha}\geq b$. Then, for any $u^\nu\in\mathcal{U}$, $u^\nu(\widehat{\alpha})>u^\nu(b)$ holds. Then, we can find $c\in\widehat{P}(\alpha_0=1)\setminus S\widehat{P}(\alpha_0=1)$ such that $\widehat{\alpha}\geq c\geq b$. Then, again, for any $u^\nu\in\mathcal{U}$, $u^\nu(c)>u^\nu(b)$ holds, which implies that $c\in\cap_{u^\nu\in\mathcal{U}}U(b;u^\nu)$. Note that, since $U(b;u^\nu)$ is an open set for each u^ν, $\cap_{u^\nu \in \mathcal{U}} U(b;u^\nu)$ is also open. This implies that there is an open neighbourhood $N(c)$ of c such that $N(c) \subseteq \cap_{u^\nu \in \mathcal{U}} U(b;u^\nu)$. Thus, there is $c' \in \overset{\circ}{\widehat{P}}(\alpha_0 = 1)$ such that $c' \in \cap_{u^\nu \in \mathcal{U}} U(b;u^\nu)$. Note that $c' \in \overset{\circ}{\widehat{P}}(\alpha_0 = 1)$ implies there exists $\alpha' \in P$ such that $\overline{\alpha}' - \underline{\alpha}' \geq c'$ and $\alpha'_0 < 1$. Thus, since $u^\nu(c') > u^\nu(b)$ holds for any $u^\nu \in \mathcal{U}$, it follows from Definition 2 that $1 - \alpha_0^{u^\nu}(b) > 0$ holds for all $u^\nu \in \mathcal{U}$.

($\Rightarrow$): Suppose $b \in S\widehat{P}(\alpha_0 = 1)$. Then, there exists a suitable $u^\nu \in \mathcal{U}$ which satisfies $U(b;u^\nu) \cap \widehat{P}(\alpha_0 = 1) = \varnothing$. This implies, by the continuity of u^ν, $\alpha_0^{u^\nu}(b) = 1$. Suppose $b \notin \widehat{P}(\alpha_0 = 1)$. Then, again, there exists a suitable $u^\nu \in \mathcal{U}$ which satisfies $U(b;u^\nu) \cap \widehat{P}(\alpha_0 = 1) = \varnothing$, which implies $\alpha_0^{u^\nu}(b) \geqq 1$.

2. Next, we prove that (1) $\Leftrightarrow$ (3).

($\Leftarrow$): Let $b \in \widehat{P}(\alpha_0 = 1) \backslash S\widehat{P}(\alpha_0 = 1)$. Then, for any $u^\nu \in \mathcal{U}$, there exists $c^{u^\nu} \in S\widehat{P}(\alpha_0 = 1)$ such that $u^\nu\left(c^{u^\nu}\right) > u^\nu(b)$ holds. Note, for any $u^\nu \in \mathcal{U}$, $\overline{u}_{\max}^{u^\nu} \geqq u^\nu\left(c^{u^\nu}\right)$ holds. Thus, for any $u^\nu \in \mathcal{U}$, $u^\nu(b) < \overline{u}_{\max}^{u^\nu}$.

($\Rightarrow$): Suppose $b \in S\widehat{P}(\alpha_0 = 1)$. Then, there exists a suitable $u^\nu \in \mathcal{U}$ such that $\overline{u}_{\max}^{u^\nu} = u^\nu(b)$. Suppose $b \notin \widehat{P}(\alpha_0 = 1)$. Then, there exists a suitable $u^\nu \in \mathcal{U}$ which satisfies $U(b;u^\nu) \cap \widehat{P}(\alpha_0 = 1) = \varnothing$, which implies $u^\nu(b) \geqq \overline{u}_{\max}^{u^\nu}$.

3. Finally, we prove that (1) $\Leftrightarrow$ (4).

($\Rightarrow$): Let $b \in \widehat{P}(\alpha_0 = 1) \backslash S\widehat{P}(\alpha_0 = 1)$. Then, there exists $\alpha \in P$ such that $\widehat{\alpha} \in S\widehat{P}(\alpha_0 = 1)$ and $\widehat{\alpha} \geq b$. Then, for any $p \in \mathbb{R}^n_{++}$, $p[\widehat{\alpha} - b] > 0$ holds.

($\Leftarrow$): Suppose that for any $p \in \mathbb{R}^n_{++}$, there exists $\widehat{\alpha} \in S\widehat{P}(\alpha_0 = 1)$ such that $p[\widehat{\alpha} - b] > 0$ holds. If $b \in S\widehat{P}(\alpha_0 = 1)$, it implies there exists $p \in \mathbb{R}^n_{++}$ such that for any $\widehat{\alpha} \in S\widehat{P}(\alpha_0 = 1)$, $p[\widehat{\alpha} - b] \leqq 0$ holds, which is a contradiction. If $b \notin \widehat{P}(\alpha_0 = 1)$, then by the separating hyperplane theorem, there exists $p^* \in \mathbb{R}^n$ such that for any $\widehat{\alpha} \in \widehat{P}(\alpha_0 = 1)$, $p^*[\widehat{\alpha} - b] < 0$ holds. By A3, $\widehat{P}(\alpha_0 = 1)$ is a comprehensive set, and $\widehat{P}(\alpha_0 = 1) \cap \mathbb{R}^n_+ \neq \varnothing$ by A2. Since $b \in \mathbb{R}^n_+$, $b \notin \widehat{P}(\alpha_0 = 1)$ implies that there exists $\widehat{\alpha}' \in \partial\widehat{P}(\alpha_0 = 1) \cap \mathbb{R}^n_+$ such that $b \geq \widehat{\alpha}'$, which implies that $p^* \in \mathbb{R}^n_+$. Then, if $p^* \notin \mathbb{R}^n_{++}$, let us take another $p' \in \mathbb{R}^n_{++}$ which is sufficiently close to p^*. Then, $p'[\widehat{\alpha} - b] < 0$ still holds for all $\widehat{\alpha} \in \widehat{P}(\alpha_0 = 1)$, since $p[\widehat{\alpha} - b]$ is continuous at p^* for each $\widehat{\alpha} \in \widehat{P}(\alpha_0 = 1)$. Thus, a contradiction obtains. ■

In other words, Theorem 1 proves that, whatever the actual preferences of workers (provided they can be represented with a continuous and strictly increasing utility function), every worker in the economy is exploited in the sense of MLEU if and only if it is possible to produce her consumption bundle with less labour than she has supplied. Similarly, whatever their actual preferences, workers do not get their maximum utility if and only if it is possible to produce her consumption bundle with less labour than she has supplied. Thus, the generalisation of the Weak System of Exploitation Theory shows that the complicated formal machinery deployed by Matsuo ([6]) hides an arguably unsurprising intuition: in the subjectivist approach workers are exploited at an allocation if there is an alternative feasible allocation that gives them a higher welfare. But then, given the assumption on monotonicity, and given that capitalists play no role in the economy (and their welfare is irrelevant), workers are exploited if and only if they do not get the whole net product. This is hardly surprising.

The next results derive Matsuo's main Theorem as a Corollary of Theorem 1 above, in the special case of von Neumann technology:

Corollary 1: *For any economy* $\langle K, W; (u^\nu)_{\nu \in W}; (P, b); (\omega^\nu)_{\nu \in K} \rangle$ *with* $P = P_{(A,B,L)}$ *for some* (A, B, L)*, the following statements are equivalent*:

(1) $\nexists p \in \mathbb{R}^n_{++}$ s.t. $p[B - A - bL] \leqq 0$;

(2) $\exists x \in \mathbb{R}^m_+$ s.t. $[B - A - bL]x \geq 0$;

(3) For each $\nu \in W$, $1 - \alpha_0^{u^\nu}(b) > 0$ holds for all $u^\nu \in \mathcal{U}$;

(4) For each $\nu \in W$, $u^\nu(b) < \overline{u}^{u^\nu}_{\max}$ holds for all $u^\nu \in \mathcal{U}$.

Theorem 1, and Corollary 1, do establish some core results of the subjectivist approach to exploitation, but they do not provide fully satisfactory answers to two central issues of exploitation theory. Firstly, they provide little guidance as to the appropriate definition of an exploitation index. If agents are heterogenous, the *individual* index of exploitation of worker ν, who works l and consumes bundle b, relative to u^ν, can be defined as $e^{u^\nu}(b, l) = \frac{l - \alpha_0^{u^\nu}(b)}{l}$. As already noted, though, there is no obvious way of aggregating the different indices $e^{u^\nu}(b, l)$ into an economy-wide measure of exploitation $e\left(b, l, (u^\nu)_{\nu \in W}\right)$, unless a representative worker is assumed. But then, it is important to stress that in this context, the representative agent assumption is arguably not just an innocuous technical condition. Secondly, also Theorem 1 and Corollary 1 do not fully characterise the relation between exploitation and profits in a general way.

As a first step in the analysis of both issues, $e\left(b, l, (u^{\nu})_{\nu \in W}\right)$ is assumed to satisfy the following two reasonable properties:

Axiom 1 (Unanimity): *If* $e^{u^{\nu}}(b, l) > 0$ *for all* $\nu \in W$*, then* $e\left(b, l, (u^{\nu})_{\nu \in W}\right) > 0$.

In other words, if every worker is exploited, then the aggregate exploitation index must be positive, too.

Axiom 2 (Representative agent index): *If* $u^{\nu} = u$ *for all* $\nu \in W$*, then* $e\left(b, l, (u^{\nu})_{\nu \in W}\right) = e^{u}(b, l)$.

The second axiom states that if workers are identical, then the aggregate index coincides with the individual index of a representative worker.

Axioms 1 and 2 are by no means exhaustive, and a number of other conditions may be imposed (for example, one may argue that it is sufficient for *one* worker to be exploited for aggregate exploitation to exist). Yet, Axioms 1 and 2 are all that is necessary for the purposes of this paper, and in particular to derive the next result, which establishes that, in the subjectivist approach, if profits are positive, then the economy is exploitative.

Proposition 1: *Assume* ***Axiom 1****. For any economy* $E = \langle K, W; (u^{\nu})_{\nu \in W}; (P, b); (\omega^{\nu})_{\nu \in K} \rangle \in \mathcal{E}$ *, if* $((p, 1), \alpha)$ *is a pair of a non-negative non-zero price and a social production point such that* $\widehat{\alpha} \geqq \alpha_0 b$ *and profits are positive, then* $e\left(b, \alpha_0, (u^{\nu})_{\nu \in W}\right) > 0$ *for any* $(u^{\nu})_{\nu \in W}$ *such that* $u^{\nu} \in \mathcal{U}$ *for all* $\nu \in W$.

Proof. Let $((p, 1), \alpha)$ be a price vector and a social production point such that $\widehat{\alpha} \geqq \alpha_0 b$ and $p\widehat{\alpha} - \alpha_0 > 0$. Let $\alpha^* \equiv \alpha/\alpha_0$. Then, $p\widehat{\alpha}^* - 1 > 0$ and $\widehat{\alpha}^* \geqq b$. By definition, $\widehat{\alpha}^* \in \widehat{P}(\alpha_0 = 1)$. Since $\widehat{\alpha}^* \geqq b$, it follows that $b \in \widehat{P}(\alpha_0 = 1) \backslash S\widehat{P}(\alpha_0 = 1)$ or $\widehat{\alpha}^* = b$. Since $p\widehat{\alpha}^* - 1 > 0$ for $pb = 1$, $\widehat{\alpha}^* = b$ is impossible, so that $b \in \widehat{P}(\alpha_0 = 1) \backslash S\widehat{P}(\alpha_0 = 1)$ holds. By Theorem 1 and Axiom 1, the desired result is obtained. ■

A first problem of the Weak System of Exploitation Theory can now be noted. Corollary 1 cannot exclude the case that $p[B - A - bL] \leqq 0$ holds for some $p \geq 0$ even when condition (3) holds. Therefore, if $p \geq 0$, it may well happen that exploitation occurs without positive profits, contradicting the FMT. This is not a minor issue, as there are many cases in which market equilibrium holds with $p \geq 0$ solely, and there is no obvious explanation which permits us to focus solely on resource allocations with positive price vectors. The next section proves that this is not an abstract possibility and the FMT is indeed violated under two rather standard equilibrium definitions.

4 The subjectivist approach and the FMT

The Weak System of Exploitation Theory is only a system of equivalences and it holds whatever the equilibrium notion adopted for this economy. Indeed, Theorem 1 does not seem restricted to equilibrium allocations, whatever the concept of equilibrium adopted. In order to provide a more precise framework for the analysis of exploitation theory and the FMT, and also to illustrate the problems of the subjectivist view, though, it is opportune to define the notion of equilibrium in this economy. At least two types of equilibrium notions can be considered in this economic model.

Consider, first, von Neumann's concept of balanced growth equilibrium. Assume wages to be advanced and let $pb = 1$.

Definition 5 [[18]]**:** *A* balanced growth equilibrium (BGE) *for the economy $E \in \mathcal{E}$ with $P = P_{(A,B,L)}$ is a tuple $((p,1), x, \pi)$, where $p \in \mathbb{R}^n_+$, $x \in \mathbb{R}^m_+$, and $\pi > -1$ such that*:

(a) $pB \leqq (1+\pi)[pA + L]$;
(b) $Bx \geqq (1+\pi)[A + bL]x$;
(c) $pBx > 0$.

In Definition 5, (a) is the revenue-cost condition for each production process in equilibrium, which implies that, given competitition among production processes, in equilibrium no capitalist can gain more than the *warranted profit rate* π from operating any production process. Note that the warranted profit rate π is the minimal value of the (uniform) profit rate warranted for all production processes in equilibrium. In contrast, (b) is the demand and supply condition for each capital and/or consumption good, which implies that in the equilibrium, the demand of any capital or consumption goods used for the next production period, $(1+\pi)[A + bL]x$, does not exceed the supply of those goods, Bx, produced in this period. Here, π represents the maximum growth rate of the economy. Finally, condition (c) implies that the total market value of output should be positive, which eliminates trivial equilibria with no production.

The next result proves that the Fundamental Marxian Theorem does not hold under either Definition 3 or Definition 4, in the von Neumann balanced growth equilibrium.

Theorem 2: *There is an economy* $E = \langle K, W; (u^{\nu})_{\nu \in W}; (P, b); (\omega^{\nu})_{\nu \in K} \rangle \in \mathcal{E}$ *with* $P = P_{(A,B,L)}$ *in which for any balanced growth equilibrium, its corresponding* warranted profit rate *is* zero *while* exploitation *exists in the sense of both Definitions 3 and 4.*

Proof. Consider a von Neumann production technology (A, B, L) and a bundle of wage goods b as follows:

$$B = \begin{bmatrix} 2 & 3 \\ 2 & 2 \end{bmatrix}, A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, L = (1,1), \text{ and } b = \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$

In this case, regardless of the distribution of capital endowment $(\omega^{\nu})_{\nu \in K}$, the (normalised) set of BGEs is given by

$$BGE_{(A,B,L,b)} \equiv \{(0,1)\} \times \left\{ (x_1', x_2') \in \mathbb{R}_+^2 \mid x_1' + x_2' = 1 \right\} \times \{0\}.$$

To show this, let $p = (0,1)$. Then, since $p[B - A] = (1,1)$ and $\pi p A + (1+\pi) L = \pi (1,1) + (1+\pi)(1,1)$, the warranted profit rate at this price is $\pi = 0$. Moreover, since $[B - A] x = \begin{bmatrix} x_1 + 2x_2 \\ x_1 + x_2 \end{bmatrix}$ and $\pi A x + (1+\pi) b L x = \pi \begin{bmatrix} x_1 + x_2 \\ x_1 + x_2 \end{bmatrix} + (1+\pi) \begin{bmatrix} x_1 + x_2 \\ x_1 + x_2 \end{bmatrix}$, it follows from Definition 5(b) that $\pi = 0$ holds. Thus, if $((p,1), x, \pi) \in BGE_{(A,B,L,b)}$, then it constitutes a BGE. Let us examine whether there is another BGE. By the above argument, if $((p'',1), x'', \pi'')$ constitutes a BGE, then $\pi'' = 0$ must hold. Then, $p''[B - A] = (p_1'' + p_2'', 2p_1'' + p_2'')$ and $\pi'' p'' A + (1+\pi'') L = (1,1)$. Since $p_1'' + p_2'' = 1$ by $pb = 1$, it follows that $p''[B - A] = (1, p_1'' + 1)$. Thus, if $((p'',1), x'', \pi'')$ constitutes a BGE, $p_1'' = 0$ holds from Definition 5(a). Therefore, if $((p'',1), x'', \pi'')$ constitutes a BGE, then $((p'',1), x'', \pi'') \in BGE_{(A,B,L,b)}$.

The above argument implies that in this economy, the warranted profit rate is zero at every BGE. Hence, if either Definition 3 or Definition 4 is adopted and the FMT holds in this economy, then there should be no exploitation in the sense of Definition 3 or 4. However, recalling that $P_{(A,B,L)} \equiv \left\{ (-\alpha_0, -\underline{\alpha}, \overline{\alpha}) \in \mathbb{R}_- \times \mathbb{R}_-^n \times \mathbb{R}_+^n \mid \exists x \in \mathbb{R}_+^m : \alpha_0 = Lx \ \& \ (-\underline{\alpha}, \overline{\alpha}) \leqq (-Ax, Bx) \right\}$, it follows that

$$\widehat{P}(\alpha_0 = 1) = co\left\{ \begin{bmatrix} 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \end{bmatrix} \right\} \text{ and } S\widehat{P}(\alpha_0 = 1) = \left\{ \begin{bmatrix} 2 \\ 1 \end{bmatrix} \right\},$$

and therefore $b = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \in \widehat{P}(\alpha_0 = 1) \backslash S\widehat{P}(\alpha_0 = 1)$. Hence, by Theorem 1, for any $u^\nu \in \mathcal{U}$, $1 - \alpha_0^{u^\nu}(b) > 0$. This implies that the FMT does not hold in this economy if exploitation is given by Definition 3. Using Theorem 1, a similar argument proves that there exists exploitation according to Definition 4. ■

In other words, if the notion of BGE is adopted, the FMT does not hold as a general result if the subjectivist approach is endorsed. The problems of the subjectivist approach, however, are not specific to the BGE. Consider a different equilibrium concept, namely that of *reproducible solution* proposed by Roemer ([11];[12]). It is assumed that capitalists maximise profits, subject to the constraint that they must be able to layout the operating costs of capital in advance, whereas wages are paid out at the end of the production process. Formally, assuming *stationary expectations* on prices ([11];[12], Chapter 2), capitalist ν's program is given by:

$$\begin{aligned} \text{choose } \alpha^\nu &\in P \text{ to maximise } p\overline{\alpha}^\nu - (p\underline{\alpha}^\nu + w\alpha_0^\nu) \\ \text{s.t. } p\underline{\alpha}^\nu &\leqq p\omega^\nu. \end{aligned}$$

The set of production processes that are the optimal solutions of the above problem is denoted by $\mathcal{A}^\nu(p, w)$. Then:

Definition 6 [[11];[12], Chapter 2]**:** *A* reproducible solution (RS) *for an economy $E \in \mathcal{E}$ is a pair $\left((p, 1), \{\alpha^\nu\}_{\nu \in K}\right)$, where $p \in \mathbb{R}^n_+$ and $\alpha^\nu \in P$, such that*:

(a) $\forall \nu \in K$, $\alpha^\nu \in \mathcal{A}^\nu(p, 1)$ (profit maximisation);

(b) $\widehat{\alpha} \geqq \alpha_0 b$ (reproducibility),
where $\widehat{\alpha} \equiv \sum_{\nu \in K}(\overline{\alpha}^\nu - \underline{\alpha}^\nu)$ and $\alpha_0 \equiv \sum_{\nu \in K} \alpha_0^\nu$;

(c) $\underline{\alpha} \leqq \omega$ (availability of capital), where $\underline{\alpha} \equiv \sum_{\nu \in N} \underline{\alpha}^\nu$ and $\omega \equiv \sum_{\nu \in K} \omega^\nu$;

(d) $pb = 1$ (subsistence wage).

In other words, at a RS, (a) capitalists maximise profits; (b) aggregat output is sufficient to replace capital used up and for workers' consumption, and (c) aggregate capital is sufficient for production plans. Part (d) is the condition of labour market equilibrium. Note that feasibility requires that

$\alpha_0 \equiv \sum_{\nu\in K} \alpha_0^\nu \leqq \sum_{\nu\in W} l^\nu \leqq |W|$ (which is a standard assumption in Marxian economics).[9]

The next theorem proves that if a subjectivist definition of exploitation is adopted, the FMT does not hold at a RS of the economy.

Theorem 3: *There is an economy $E \in \mathcal{E}$ such that at a RS the* maximal profit rate *is* zero*, while* exploitation exists *in the sense of both Definitions 3 and 4.*

Proof. Consider the same von Neumann production technology (A, B, L) and the same bundle of wage goods b as in the proof of Theorem 2. Let $(\omega^\nu)_{\nu\in K}$ be given as $\omega^\nu = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, all $\nu \in K$, so that $\sum_{\nu\in K} \omega^\nu = |K| \begin{bmatrix} 1 \\ 1 \end{bmatrix}$. Assume that $|K| \leq |W|$. First, it is not difficult to prove that $p = (0, 1)$ is a competitive equilibrium price for this economy. In fact, it is immediate to show that the maximal profit rate is zero and if $x^\nu \in \left\{(x_1', x_2') \in \mathbb{R}^2_+ \mid x_1' + x_2' = 1\right\}$, then $x^\nu \in \mathcal{A}^\nu(p, 1)$, for all $\nu \in K$, with $Ax^\nu = \omega^\nu$, $(B - A)x^\nu \geqq b$, and $Lx^\nu = 1$, all $\nu \in K$. Therefore, noting that $|K| \leq |W|$, it follows that $\sum_{\nu\in K} Lx^\nu \leq |W|$, $\sum_{\nu\in K}(B - A)x^\nu \geqq \sum_{\nu\in K} Lx^\nu b$, and $\sum_{\nu\in K} Ax^\nu = \sum_{\nu\in K} \omega^\nu$. Second, because $b \in \widehat{P}(\alpha_0 = 1) \backslash S\widehat{P}(\alpha_0 = 1)$, the desired result follows from Theorem 1. ■

Theorems 2 and 3 raise serious doubts concerning the subjectivist approach to Marxian exploitation, and its relation with the FMT. Matsuo proposed the subjectivist approach precisely in order to rescue Marxian exploitation theory from the counterexample in [10], which shows that the FMT does not hold under Morishima's ([9]) definition of exploitation. In particular, Petri's counterexample shows that although Morishima's ([9]) Generalised FMT is robust in a BGE, the FMT does not hold in general if another equilibrium notion, such as RS, or if disequilibria are considered: it is possible to have positive profits without exploitation in the sense of [9]. [6] shows that Petri's counterexample can be resolved if Definition 3 is adopted instead of Morishima's ([9]). In fact, for any price vector, if profits are positive, then exploitation in the sense of Definitions 3 and 4 always exists. This is not really a solution of Petri's puzzle, however, because, as shown in Theorem 2, if Matsuo's approach is adopted, the FMT cannot hold even at a BGE.

[9] For a more detailed discussion of the notion of Reproducible Solution, see [11], [12] (Chapter 2), and [20].

From this perspective, the subjectivist approach seems to score *worse* than Morishima's definition, rather than solving its difficulties.

5 The subjectivist index of exploitation

The argument developed in the previous section provides a criticism of the subjectivist approach from an analytical point of view, and so it would apply even if Definition 3 or 4 above were deemed appropriate formulations of exploitation. There are, however, some theoretical arguments against Matsuo's notion of exploitation. In particular, it may be argued that in a subjectivist approach the concept of exploitation loses conceptual clarity and analytical strength.

First of all, even setting aside the doubts related to Theorems 2 and 3 above, the claims concerning the generality of the results are unwarranted, even if a representative worker is assumed. Although the class of utility functions in $\mathcal{U}$ is rather large, some important cases are excluded (for instance, perfect complements, lexicographic preferences, neutral goods), and this exclusion is puzzling if the motivation of the whole exercise is to provide a *general* framework that avoids counterexamples to the FMT: given the restrictions on $\mathcal{U}$, it is not difficult to build new counterexamples using utility functions outside $\mathcal{U}$. For instance, if preferences are perfect complements, then the equivalence result breaks down. So, if the approach is to be defended, this is not on the grounds of its presumed generality. One would have to argue directly that it is theoretically superior.

Matsuo defends the strict monotonicity assumption against one specific critique, namely against the claim that workers may not derive welfare from accumulation goods. However, on the one hand, this does not respond to cases such as perfect complements or lexicographic preferences. On the other hand, to postulate that "workers have some preference for accumulation goods if they - even unconsciously - have some ideas about a desirable production allocation in the society" ([6], p.263) is arguably rather ad hoc and objectionable. And it implies that an approach that aims to provide a general theory of exploitation, which is robust to counterexamples, ultimately rests on a purely empirical assumption.

A third critique of the subjectivist approach to exploitation theory is developed in the following example:

Example 1: Consider the following von Neumann production technology (A, B, L) and bundle of wage goods b:

$$B = \begin{bmatrix} 2 & 3 \\ 2 & 2 \end{bmatrix}, A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, L = (1, 1), \text{ and } b = \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$

Let $K = \{\nu\}$, $W \neq \varnothing$, and $\omega^{\nu} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$. Then, $p = (0.5, 0.5)$ and $x = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ constitute a reproducible solution: in fact, $x = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ yields a profit rate $\pi = \frac{1}{2}$, whereas $x' = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ yields a profit rate $\pi' = 0$. Moreover, $[B - A]\, x \geqq bLx$ and $Ax \leqq \omega$. Finally, $pb = 1$.

Insert Figure 1 around here.

Assume that workers are identical. Let $u \in \mathcal{U}$ be such that, for any $y' = (y'_1, y'_2) \in \mathbb{R}^2_+$, $u(y') = y'_1 \cdot y'_2$. It follows that $b = \arg\max_{py'=1} u(y')$. Thus, if u is interpreted as representing standard subjective preferences over consumption (as in the neoclassical theory of consumer behavior), the vector b can be interpreted as the worker's Marshallian demand which is purchased under the budget constraint $py' = 1$. In contrast, it follows from Definition 4 that $\alpha_0^u(b) = L\widetilde{x} = \frac{\sqrt{2}}{2}$ with $\widetilde{x} = (\widetilde{x}_1, \widetilde{x}_2) = \left(0, \frac{\sqrt{2}}{2}\right)$, and $u(b) = u(\widetilde{y})$ holds for $\widetilde{y} \equiv [B - A]\, \widetilde{x} = \left(\sqrt{2}, \frac{\sqrt{2}}{2}\right)$. Hence, since $1 - L\widetilde{x} > 0$, according to Definition 4, the worker is exploited w.r.t. u. ■

However, the socially necessary labour $L\widetilde{x}$ of the worker is given as the minimal amount of labour necessary to produce the consumption vector $\widetilde{y}$. Then, the worker can *never* purchase this vector with her income, since $p\widetilde{y} = \frac{3}{4}\sqrt{2} > 1$. This is strange, because the socially necessary labour of the worker has to be regarded as the labour hours necessary to produce some commodity vector which the worker can purchase by her wage revenue per period. However, in the case of Definition 4, the labour $L\widetilde{x}$ is necessary to produce a *non-purchasable* consumption vector $\widetilde{y}$. To purchase the vector $\widetilde{y}$ with the worker's income at p, she needs to earn $\frac{3}{4}\sqrt{2}$ per period, which is impossible because the upper bound on labour supply per period is one.

The fourth, and arguably strongest, objection to the subjectivist approach relates to the core feature of the approach, namely the definitional role played by subjective preferences. Even though, as shown by Theorem 1, the existence of exploitation may be unaffected by the specific choice of $(u^\nu)_{\nu \in W}$ within $\mathcal{U}$, both the individual and a fortiori the aggregate rate of exploitation (and thus the intensity of exploitation) in general are not. As already noted, for example, if workers are heterogeneous, there is no obvious way of defining an aggregate exploitation index. More strongly, even if a representative agent is assumed so that $u^\nu = u$ for all $\nu \in W$, the solution of the individual minimisation problem ML^ν above, and thus the actual value of b and any index on the intensity of exploitation that is based on it, will depend on the specific utility function chosen. The next Theorem provides a precise formal statement of the conditions under which this claim holds and of the potential indeterminacy of a subjectivist index of exploitation.

Theorem 4: *Assume **Axiom 2**. Consider the subset of economies $\mathcal{E}^u \subset \mathcal{E}$, such that for any $E \in \mathcal{E}^u$ and any $\nu \in W$, $u^\nu = u$. Then:*
(i) *For any $E \in \mathcal{E}^u$, $\alpha_0^u(b) = l$ for all $u \in \mathcal{U}$ if and only if $\{b\} = S\widehat{P}(\alpha_0 = l)$.*
(ii) *There exists an economy $E \in \mathcal{E}^u$, such that for all $\delta > 0$ and all $\varepsilon \in [0,1]$ there is a function $u \in \mathcal{U}$, such that $|e^u(b,l) - \varepsilon| < \delta$.*

Proof. Part (i). First of all, note that if $\{b\} = S\widehat{P}(\alpha_0 = l)$ then for all $\alpha' \in P$, such that $\alpha'_0 \leqq \alpha_0 = l$, $\widehat{\alpha}' \leqq \widehat{\alpha} = b$ and if $\alpha' \neq \alpha$, then $\widehat{\alpha}' \leq \widehat{\alpha}$. But then, it immediately follows from Assumption 4 that $\alpha_0^u(b) = l$ for all $u \in \mathcal{U}$. Conversely, suppose that there exist $b' \neq b : \{b, b'\} \subset S\widehat{P}(\alpha_0 = l)$. By definition, it follows that there exist at least two entries i, j such that $b'_i > b_i$ and $b'_j < b_j$. Furthermore, by the convex cone property of P, it follows that for all $\lambda \in [0,1]$, $\widetilde{b} = \lambda b + (1-\lambda)b' \in S\widehat{P}(\alpha_0 = l)$. Then, define the following subset of utility functions in $\mathcal{U}$: $\mathcal{U}^p = \{u \in \mathcal{U} : u(c) = \sum_{i=1}^n \delta_i c_i, \delta_i > 0, \sum_{i=1}^n \delta_i = 1\}$. It is immediate to show that there always exists $u \in \mathcal{U}^p$ such that $u(\widetilde{b}) > u(b)$, for all $\widetilde{b} = \lambda b + (1-\lambda)b'$, $\lambda \in (0,1]$, and therefore $\alpha_0^u(b) < l$.

Part (ii). Consider the following von Neumann production technology (A, B, L) and bundle of wage goods b:

$$B = \begin{bmatrix} 1 & 3 \\ 2 & 2 \end{bmatrix}, A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, L = (1,1), \text{ and } b = \begin{bmatrix} 2 \\ 0 \end{bmatrix}.$$

Let $K = \{\nu\}$, $W \neq \varnothing$, and $\omega^\nu = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$. Then, $p = (0.5, 0.5)$ and $x = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ constitute a reproducible solution associated with the maximal profit rate $\pi = \frac{1}{2}$. To see this, note that $x' = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ implies $\pi' = -\frac{1}{2}$ and thus, by the convex cone property of P the maximal profit rate is indeed $\pi = \frac{1}{2}$. Further, $Ax = \omega$, $(B-A)x \geq b$, $Lx = 1$, and $pb = 1$. Then, as in part (i), consider the subset of utility functions

$$\mathcal{U}^p = \{u \in \mathcal{U} : u(c) = \delta c_1 + (1-\delta)c_2, 1 > \delta > 0\}.$$

Define an infinite sequence of functions $\{u^t(c)\}_{t=0}^{\infty}$ in $\mathcal{U}^p$ as follows:

$$u^t(c) = \delta^t c_1 + (1-\delta^t)c_2, \text{ where } \delta^t \in (0,1)$$

for all t. Note that, by Axiom 2, $e^u(b,1) = 1 - \alpha_0^u(b)$. Therefore, let $e^{u^t}(b,1) = 1 - \alpha_0^{u^t}(b)$: it is immediate to prove that as $\delta^t \to 0$, $\alpha_0^{u^t}(b) \to 1$, and thus $e^{u^t}(b,1) \to 0$, whereas as $\delta^t \to 1$, $\alpha_0^{u^t}(b) \to 0$, and thus $e^{u^t}(b,1) \to 1$. ■

In other words, even if workers are identical, and thus no aggregation issue arises, the exploitation index will be invariant to changes in workers' preferences only in the rather special case that there exists a certain amount of labour input such that the wage basket lies on the corresponding production possibility frontier, and the latter corresponds to a single point. By focusing on a simple von Neumann technology, the following example forcefully illustrates the implications of Theorem 4, if the condition in Part (i) is violated, so that there exist two economies $E, E' \in \mathbf{E}^u$, such that $K = K'$, $L = L'$, $\left(P_{(A,B,L)}, b\right) = \left(P'_{(A,B,L)}, b'\right)$, $(\omega^\nu)_{\nu \in K} = (\omega'^\nu)_{\nu \in K}$, but $e^u(b,l) \neq e^{u'}(b,l)$.

Example 2: Consider the same economy $E = \langle K, W; (u^\nu)_{\nu \in W}; \left(P_{(A,B,L)}, b\right); (\omega^\nu)_{\nu \in K} \rangle$ as in Example 1. Let $u^\nu = u$ for all $\nu \in W$, and $u \in \mathcal{U}$ be given as in Example 1. Moreover, let $u' \in \mathcal{U}$ be given as:

$$\frac{\partial u'(b)}{\partial b_1} \Big/ \frac{\partial u'(b)}{\partial b_2} = 1 \ \& \ \forall y' (\neq b) \in \mathbb{R}_+^2 \text{ with } u(y') = u(b), \ u'(y') < u'(b).$$

Insert Figure 2 around here.

In other word, $U(b; u) \supsetneq U(b; u')$. In this case, $b = \arg\max_{py'=1} u'(y')$ holds. Thus, if we interpret u' as representing a standard subjective preference of the worker over consumption such as in the neoclassical theory of consumers behavior, the vector b can be interpreted as the worker's Marshallian demand which is purchased under the budget constraint $py' = 1$, as argued in Example

1. However, now, $\alpha_0^{u'}(b) = L\widetilde{x}' > \frac{\sqrt{2}}{2}$ holds, since by the construction of u', $u'(\widetilde{y}) < u'(b)$ for $\widetilde{y} = \left(\sqrt{2}, \frac{\sqrt{2}}{2}\right)$, so that $\widetilde{y}' \equiv [B - A]\widetilde{x}' > \widetilde{y}$. Note even in this case, $1 - L\widetilde{x}' > 0$ still holds, and so the worker is exploited w.r.t. u', since $b \in \widehat{P}_{(A,B,L)}(Lx = 1) \backslash S\widehat{P}_{(A,B,L)}(Lx = 1)$ and $u' \in \mathcal{U}$. Hence, though this worker is under the same working condition as in Example 1, because she provides one unit of labour and receives one unit of wage revenue per day, her 'exploitation rate' would *decrease* if her subjective preference is changed from u to u'. ■

To be sure, a supporter of the subjectivist approach may object that, after all, the main purpose of exploitation theory is to diagnose the existence of exploitation, whereas the construction of an exploitation index is not essential. This defence is arguably unconvincing: first, as shown in the previous section, even focusing on the issue of the existence of exploitation, in the subjectivist approach exploitation may exist even if profits are zero. Second, this defence implies that it is impossible to compare different societies based on the amount of exploitation suffered by the working class, nor is it meaningful to analyse the dynamics of exploitation of a society over time. Actually, Theorem 4(ii) has an even more puzzling implication: even assuming workers to be identical, there exists economies in which it is literally impossible in principle to say *anything* about exploitation, except whether it exists. In fact, for a given set of objective characteristics of the economy, the amount of exploitation suffered by workers can take any value if the appropriate continuous and strictly increasing utility function is chosen. In other words, by simply changing workers' subjective preferences, the economy moves from being essentially non-exploitative, to being plagued by the most extreme form of exploitation. In this kind of situation, the exploitation index is not just inaccurate, it is meaningless.[10]

[10] It is worth noting in passing that Matsuo defends his subjectivist approach by arguing that "exploitation is a matter of *alienation*" ([6], p.263), that is, it derives from the workers' "exclusion from decision making on the production allocation of the society" ([6], p.263). This argument seems false, for in Matsuo's framework, exploitation would be eliminated if capitalists continued to organise production but workers received the whole of net product. It is also arguably misleading, if not conceptually inappropriate, to conflate two distinct phenomena.

6 An objectivist approach to exploitation

In the previous sections, a thorough critical analysis of the subjectivist approach to exploitation is developed. The main criticism is not that a subjectivist approach is not consistent with Marx's own theory (although it is arguably not), but rather that it has a number of undesirable, if not counter-intuitive properties. In this section, an alternative, objectivist approach to exploitation is proposed, which provides more satisfactory answers to some of the key questions of exploitation theory, such as the construction of a robust index of exploitation, and which preserves the FMT in general economies.

There are a number of ways to define objectivism in the context of the general convex economies analysed in this paper. To be sure, it may be argued that all possible influences, direct and indirect, of subjective preferences and even *individual choices* should be excluded in the analysis of exploitation. For the purposes of this paper, however, it is unnecessary to adjudicate competing views of objectivism, and the following axiom aims to capture the minimum common denominator of all objectivist approaches, by requiring that if *all* the objective features of two economies are identical, their exploitation indices should also be identical, regardless of agents' subjective preferences.

Axiom 3 (Objectivism): *Let* $E = \langle K, W; (u^{\nu})_{\nu\in W}; (P, b); (\omega^{\nu})_{\nu\in K}\rangle$ *and* $E' = \langle K', W'; (u'^{\nu})_{\nu\in W}; (P', b'); (\omega'^{\nu})_{\nu\in K}\rangle$*, be such that* $K = K'$*,* $W = W'$*,* $(P, b) = (P', b')$ *and* $(\omega^{\nu})_{\nu\in K} = (\omega'^{\nu})_{\nu\in K}$*. Then* $e\left(b, l, (u^{\nu})_{\nu\in W}\right) = e\left(b, l, (u'^{\nu})_{\nu\in W}\right)$ *for all* $(u^{\nu})_{\nu\in W}$ *and* $(u'^{\nu})_{\nu\in W}$*.*

Note that Axiom 3 implies that the exploitation index is invariant with respect to *all* utility functions, and therefore a fortiori with respect to utility functions in $\mathcal{U}$.

Though there may be a number of definitions satisfying Axiom 3, this paper analyses a specific proposal. Given an economy $E = \langle K, W; (u^{\nu})_{\nu\in W}; (P, b); (\omega^{\nu})_{\nu\in K}\rangle$, let $\alpha \in P$ be a social production point such that $\widehat{\alpha} \in \mathbb{R}^n_+$ and let $p \in \mathbb{R}^n_+$ be the associated price vector that prevails in the economy. Let $B(p, b) \equiv \left\{c \in \mathbb{R}^n_+ \mid pc = pb\right\}$: $B(p, b)$ is the set of bundles that cost exactly as much as the wage bundle b. Then, let us take $c \in B(p, b)$ such that $c = t\widehat{\alpha}$ for some $t > 0$. Denote such $t > 0$ by $t^{(p,\alpha)}$. In this section, the following definition of exploitation is analysed.

Definition 7: *For any* $E = \langle K, W; (u^{\nu})_{\nu\in W}; (P, b); (\omega^{\nu})_{\nu\in K}\rangle$*, let* $\alpha \in P$ *be* *a social production point such that* $\widehat{\alpha} \in \mathbb{R}^n_+$ *and let* $p \in \mathbb{R}^n_+$ *be the associated price vector prevailing in this economy. Then, every worker* $\nu \in W$ *is* exploited *if and only if* $1 - t^{(p,\alpha)}\alpha_0 > 0$.

There are a number of properties of Definition 7 that are worth stressing. First, Definition 7 is conceptually related to the 'New Interpretation' developed by Duménil [1] and Foley [3]. In fact, $t^{(p,\alpha)}$ can be interpreted as the value of labour power, which coincides with the wage rate (normalised by net national product). Therefore, as in the New Interpretation, workers are exploited if and only if the share of wages in national income is less than one. Second, it is immediate to show that it satisfies Axiom 3: once prices and the aggregate social production vector are known, $t^{(p,\alpha)}$ and the exploitation rates (individual and aggregate) are identified, independently of preferences. Third, in the general convex economies considered in this paper, the definition of the aggregate index of exploitation is straightforward and no issues arise concerning the aggregation of individual indices. In fact, $e\left(b, 1, (u^\nu)_{\nu \in W}\right) = 1 - t^{(p,\alpha)}\alpha_0$. The latter index is well-defined and uniquely determined, for any set of objective characteristics of the economy, which allows meaningful comparisons across time and between countries concerning exploitation, and - more generally - fruitful empirical analysis in a Marxian context.[11] Fourth, as shown in [22], it is possible to provide a complete axiomatic characterisation of Definition 7 in the context of general convex cone subsistence economies with optimising agents: in such context, Definition 7 surprisingly emerges as the *unique* definition of exploitation that satisfies a small set of rather weak axioms.[12]

Finally, the objectivist approach in Definition 7 preserves all the essential insights of the subjectivist approach, but it also allows for the extension of the FMT to general convex cone economies. Thus, under Definition 7, the equivalent of Theorem 1 can be proved.

Theorem 5 (The General System of an Objectivist Exploitation Theory)**:** *For any economy* $E = \langle K, W; (u^\nu)_{\nu \in W}; (P, b); (\omega^\nu)_{\nu \in K} \rangle \in \mathcal{E}$*, the following statements are equivalent*:

(1) $b \in \widehat{P}(\alpha_0 = 1) \backslash S\widehat{P}(\alpha_0 = 1)$;

[11] For a detailed discussion of the empirical implications of the 'New Interpretation,' see [7].

[12] See [20] for an axiomatic analysis of Definition 7 in the context of accumulating economies.

(2) $\nexists p \in \mathbb{R}^n_{++}$ s.t. $p[\widehat{\alpha} - b] \leqq 0$ holds for any $\widehat{\alpha} \in S\widehat{P}(\alpha_0 = 1)$;

(3) For any $\alpha \in P(\alpha_0 = 1)$ such that $\widehat{\alpha} \geq b$, and for any $p \in \mathbb{R}^n_{++}$, $1 - t^{(p,\alpha)}\alpha_0 > 0$ holds.

Proof. By Theorem 1, it suffices to show (1)$\Leftrightarrow$(3). First, suppose that (1) holds. Then, there exists $\alpha \in P(\alpha_0 = 1)$ such that $\widehat{\alpha} \geq b$. Take any such α. Then, for any $p \in \mathbb{R}^n_{++}$, $p\widehat{\alpha} > pb = pc$ for any $c \in B(p, b)$. Thus, for $t^{(p,\alpha)}\widehat{\alpha} \in B(p, b)$, $p\widehat{\alpha} > p \cdot t^{(p,\alpha)}\widehat{\alpha}$, which implies $1 - t^{(p,\alpha)}\alpha_0 > 0$, since $\alpha_0 = 1$.

Next, suppose that (1) does not hold. Then, there is no $\alpha \in P(\alpha_0 = 1)$ such that $\widehat{\alpha} \geq b$, so that (3) trivially holds. ■

As in the case of Theorem 1, though, Theorem 5 does not provide fully satisfactory answers to some core issues of exploitation theory. This is because it says nothing about profit and exploitation in the case with $p \geq 0$, even though it is certainly conceivable that in a market equilibrium the price vector is such that $p \geq 0$ and there is no obvious reason to focus solely on strictly positive price vectors. Fortunately, in contrast to the subjectivist view, if Definition 7 is adopted, the FMT does hold in general convex cone economies, as shown by the next two results.

Theorem 6 (FMT in BGEs)**:** *For any economy* $E = \langle K, W; (u^\nu)_{\nu \in W}; (P, b); (\omega^\nu)_{\nu \in K} \rangle \in \mathcal{E}$ *with* $P = P_{(A,B,L)}$, *let* $((p, 1), x, \pi)$ *be a* BGE. *Then,* $\pi > 0$ *if and only if* every worker is exploited *in the sense of* Definition 7.

Proof. Let $\pi > 0$. Then, $pBx - [pA + L]x > 0$. Without loss of generality, let x be normalized so that $Lx = 1$ holds. Then, given $pb = 1$, the above inequality is reduced to $p[B - A]x > pb = t^{(p,x)}p[B - A]x$ for some $t^{(p,x)} > 0$. Since $t^{(p,x)} < 1$, $1 - t^{(p,\alpha)}Lx > 0$ holds, so that every worker is exploited in terms of Definition 7.

Let $\pi \leqq 0$. Then, $pBx - [pA + L]x \leqq 0$. Again, given $Lx = 1$ and $pb = 1$, $p[B - A]x \leqq pb = t^{(p,x)}p[B - A]x$ for some $t^{(p,x)} > 0$. Since $t^{(p,x)} \geqq 1$, $1 - t^{(p,\alpha)}Lx \leqq 0$ holds, so that every worker is not exploited in terms of Definition 7. ■

In other words, Theorem 6 proves that, unlike in the subjectivist approach, under Definition 7, the FMT holds if von Neumann's equilibrium concept is adopted. This result makes Definition 7 at least equivalent to Morishima's ([9]) classical definition, from the viewpoint of preserving the general relation between exploitation and profits. Unlike the latter approach, though, under Definition 7, the Marxian postulate that exploitation is synonimous with positive profits holds even if other equilibrium concepts are adopted, as shown by the next result.

Theorem 7 (FMT in RSs)**:** *For any economy* $E = \langle K, W; (u^\nu)_{\nu\in W}; (P,b); (\omega^\nu)_{\nu\in K}\rangle \in \mathcal{E}$*, let* $((p,1), \{\alpha^\nu\}_{\nu\in K})$ *be a* RS*. Then,* $p\left(\sum_{\nu\in K}\widehat{\alpha}^\nu\right) - \sum_{\nu\in K}\alpha_0^\nu > 0$ *if and only if* every worker is exploited *in the sense of* Definition 7*.*

Proof. Let $\alpha \equiv \sum_{\nu\in K}\alpha^\nu$ and let $\alpha' \equiv \frac{\alpha}{\alpha_0}$. Thus,

$$p\left(\sum_{\nu\in K}\widehat{\alpha}^\nu\right) - \sum_{\nu\in K}\alpha_0^\nu \lesseqqgtr 0 \Leftrightarrow p\widehat{\alpha}' - 1 \lesseqqgtr 0.$$

First, suppose that $p\widehat{\alpha}' - 1 > 0$. Then, since $pb = 1$, it follows that $p\widehat{\alpha}' > pb = t^{(p,\alpha')}p\widehat{\alpha}'$, for some $t^{(p,\alpha')} > 0$. Because $t^{(p,\alpha')} < 1$, it follows that $1 - t^{(p,\alpha')}\alpha_0' > 0$, so that every worker is exploited in terms of Definition 7.

Next, if $p\widehat{\alpha}' - 1 = 0$, then in a similar way, it can be proved that $1 - t^{(p,\alpha')}\alpha_0' = 0$, so that no worker is exploited in the sense of Definition 7. ■

As argued above, Definition 7 has a number of attractive features that warrant its adoption in the context of exploitation theory. Theorems 6 and 7 arguably provide further independent support to its adoption as the appropriate definition of exploitation. In fact, *if* the epistemological role of the FMT is indeed as a postulate, as assumed in much of the discussion on the Marxian theory of exploitation (see the Introduction above), Theorems 6 and 7 show that Definition 7 is preferable to the main received definitions, and to the subjectivist approach, because it allows to derive a general, robust relation between exploitation and profits.

7 Conclusions

This paper analyses two central issues in exploitation theory. First, the appropriate definition of individual and aggregate measures of exploitation is discussed. Second, the relation between profits and exploitation (the so-called Fundamental Marxian Theorem) is formally analysed. A general framework for the analysis of exploitation in the context of convex cone economies is proposed and various alternative equilibrium concepts - such as von Neumann's Balanced Growth Equilibrium, and Roemer's Reproducible Solution - are discussed.

The limits of subjectivist approaches to exploitation, which crucially depend on agents' preferences, are shown. It is argued that in a subjectivist approach, it may be possible to accurately identify the existence of exploitation, but it is in general impossible to construct a reliable measure of its intensity, which makes it difficult to develop any meaningful empirical analysis (both cross-section and time series) of existing economies from the viewpoint of exploitation theory. Further, it is shown that in general the subjectivist approach does not preserve the relation between profits and exploitation that characterises the Marxian theory of exploitation, for positive exploitation can well coexist with zero profits in equilibrium.

A novel axiomatic definition of objectivist approaches to exploitation theory is rigorously formulated. Then, a specific objectivist definition of exploitation is proposed, which is related to the so-called 'New Interpretation' ([1], [3]). A number of desirable properties of this definition are discussed, which suggest that it captures the core intuitions of exploitation theory and that it provides appropriate, and empirically meaningful, indices of individual and aggregate exploitation. Further, it is shown that under the definition of exploitation proposed in this paper, the Fundamental Marxian Theorem holds in general convex economies. Actually, as proved in [19] and [22], if the approach presented in this paper is adopted, the FMT can be extended to subsistence economies (in which agents minimise labour, instead of maximising revenues) and, in less polarised economies with a more complex class structure, the Class-Exploitation Correspondence Principle ([13]) holds, which states that agents in the lower classes are exploited and agents in the higher classes are exploiters. Indeed, as shown in [17], the definition proposed in this paper can be meaningfully applied to the analysis of the global economy and to fully dynamic economic models.

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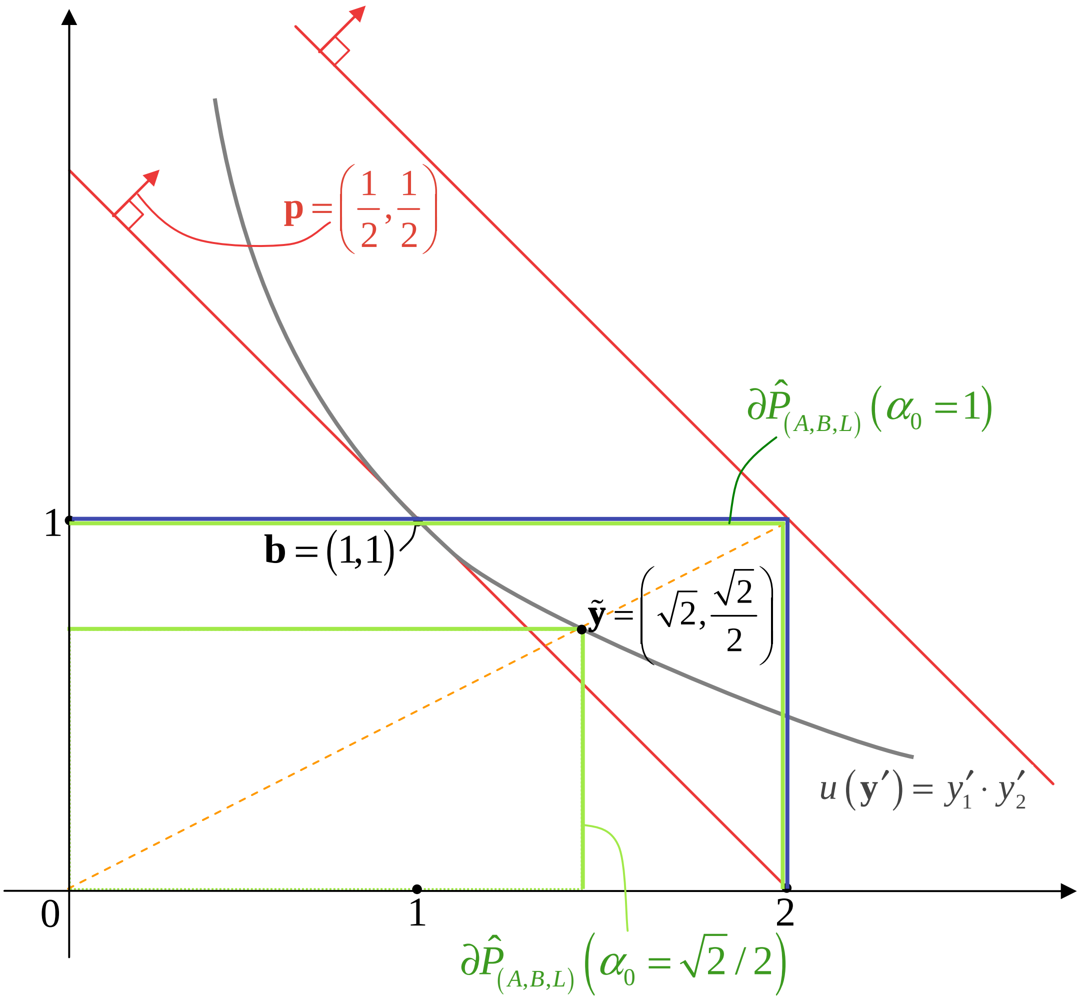
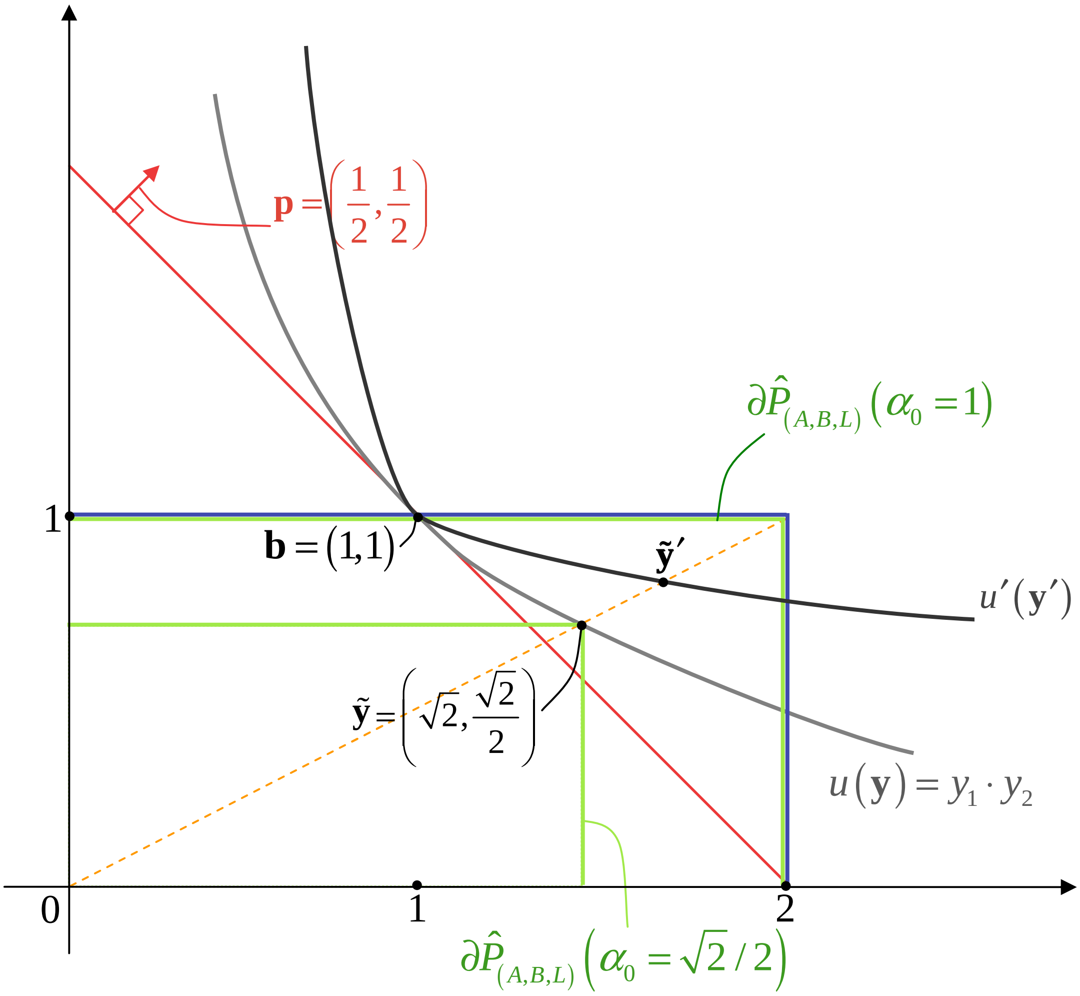


$\mathbf{p} = \left(\frac{1}{2}, \frac{1}{2}\right)$
$\partial\hat{P}_{(A,B,L)}\left(\alpha_0 = 1\right)$
1
$\mathbf{b} = (1,1)$
$\tilde{\mathbf{y}} = \left(\sqrt{2}, \frac{\sqrt{2}}{2}\right)$
$u\left(\mathbf{y}'\right) = y_1' \cdot y_2'$
0
1
2
$\partial\hat{P}_{(A,B,L)}\left(\alpha_0 = \sqrt{2}/2\right)$

Figure 1



$\mathbf{p} = \left(\frac{1}{2}, \frac{1}{2}\right)$
$\partial\hat{P}_{(A,B,L)}(\alpha_0 = 1)$
1
$\mathbf{b} = (1,1)$
$\tilde{\mathbf{y}}'$
$u'(\mathbf{y}')$
$\tilde{\mathbf{y}} = \left(\sqrt{2}, \frac{\sqrt{2}}{2}\right)$
$u(\mathbf{y}) = y_1 \cdot y_2$
0
1
2
$\partial\hat{P}_{(A,B,L)}\left(\alpha_0 = \sqrt{2}/2\right)$

Figure 2