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Matching, Quality Upgrading, and Trade between Heterogeneous Firms*

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Abstract

This paper analyzes trade between firms that are heterogeneous in product quality in a simple general equilibrium model. The multi-sided heterogeneity of exporters and importers creates a new source of gains from trade. The opening of trade raises the quality of final goods by improving matching of firms. The quality upgrading is decomposed as the short run effect of a reduction in the quality gap among parts and components and the long run effect of intensified competition among suppliers. Under the existence of fixed trade costs, firms' trade pattern is consistent with a variety of stylized facts that have not been explained in the conventional love of variety model. Firms selectively trade with those with similar sizes at similar quality levels. Both exporting and importing are concentrated into large and high quality firms, though not all large and high quality firms engage in trade. Trade in intermediate goods improves the quality of even firms that do not import intermediate goods.

Key Words: matching, heterogeneous firms, quality, vertical differentiation, trade in intermediate goods, offshoring.

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1 Introduction

Trade in intermediate goods has been becoming an important source of welfare gains from the globalization. A significant amount of traded products are intermediate goods and their share in the total volume is rapidly growing.[1] In addition to traditional consumer's gains from trade such as the availability of more varieties in cheaper prices, trade in intermediate goods improves the quality/productivity of final good producers.[2]

A new feature of the recent trade in intermediate goods is that firms often form international production teams. Thanks to the development in communication and transportation technologies, firms in distant countries cooperatively develop a product and trade specially designed parts and components within a team. Boeing 787 dreamliner is a good example. For this new midsize jet, Boeing sets up a team that consists of 43 suppliers mostly selected from developed countries.[3] The suppliers, which Boeing proudly calls "the world's most capable top-tier supplier partners", produce cutting-edge components newly designed for this airplane. Similar team production of high-tech firms in developed countries is observed in other quality-differentiated products such as automobiles and electronics.[4]

The international team production suggests a new channel though which international trade affects the quality of final products. It is well-known that there is a considerable degree of heterogeneity in firms' performances within industry. Furthermore, empirical works on price data suggest the observed firm-heterogeneity reflects the difference in product quality.[5] Given this

[1] For instance, according to Bernard, Jensen, and Schott (2005), imports by good/service producers, most of which are likely to be intermediate goods for domestic production, account for 70% of total imports in the US in 2000, while imports by wholesalers/retailers account for only 27%. Feenstra (1998) surveys evidence for a growing importance for trade in intermediate goods.

[2] Broda, Greenfield, and Weinstein (2006) report the aggregate gains in total factor productivity (TFP) from trade in intermediate goods account for 15% of TFP growth in a typical country.

[3] For the list of suppliers, see http://www.boeing.com/commercial/787family/background.html.

[4] Another famous example is Apple ipod. Though products are assembled and exported from China, more than 90% of the gross profits of Apple's iPod are taken by US and Japanese firms producing cutting-edge components (Linden, Kraemer, and Dedrick, 2007).

[5] Schott (2004) and Hummels and Klenow (2005) are two early studies on the heterogeneity of unit prices of traded goods within product categories. Hummels and Skiba (2004), Baldwin and Harrigan (2007), Johnson (2008), and Bernard et al. (2007) observe fob price of traded goods increases in the distance and trade costs, which implies firms producing higher quality products are more likely to be exported. From plant data, Kugler and Verhoogen

prevalence in the heterogeneity of firms in product quality, matching of team members determines the combination of the quality of parts and components and the quality of finished products.

Although recent empirical studies emphasize the prevalence of heterogeneity of exporting firms, importing firms, and product quality, the matching of heterogeneous firms is understudied in the literature. Existing models of trade by heterogeneous firms abstract away from matching of firms. Most studies (e.g. Melitz, 2003; Kasahara and Lapham, 2008) employ the love of variety (Krugman, 1980; Ethier, 1982) as a source of gains from trade, which automatically implies that all importers trade with all exporters.[6] Existing models of matching of firms focus on a random matching between symmetric firms rather than heterogeneous firms (Casella and Rauch, 2001; Rauch and Casella, 2003; Rauch and Trindade, 2003; Grossman and Helpman, 2005). However, casual observations and some empirical study suggest that the matching of firms is assortative in quality rather than purely random. For instance, parts and components for luxury cars are usually higher quality than those for standard cars. Furthermore, from the price data of Colombian plants, Kugler and Verhoogen (2009) found plants using expensive imported inputs tend to use expensive domestic inputs. The conventional love of variety model and random matching model fail to analyze these systematic patterns in transactions of intermediate goods.

To fill this gap in the literature, this paper develops a tractable general equilibrium model of international team production of quality differentiated firms. Ex ante symmetric firms become heterogeneous in product quality as a result of R&D investment as a quality-version of Melitz (2003) model developed by Baldwin and Harrigan (2007), Johnson (2008), and Kugler and Verhoogen (2008). Firms form production teams in a competitive matching market as in Becker (1973) and Sattinger (1979). Team members complement with each other in developing the quality of final products. Consumers prefer a moderate combination of the quality of parts and components. As the simplest model of trade between developed countries, I consider two symmetric countries that differ in their R&D technologies.

The model presents a new mechanism of firm-level gains from trade. International trade in intermediate goods, i.e. international matching of firms, raises the quality of final goods by improving matching of firms within a team. In the autarky, matching patterns differ across countries

(2007) find exporting plants tend to have a higher index of output prices.

[6] Bernard et al. (2003) use the perfectly competitive model instead of the love of variety model. However, firms do not care about their trading partners under the perfect competition.

reflecting the difference in their technologies. In the short run after the opening of trade, the matching pattern converges across countries. This reduces the difference in the quality among parts and components to improve the quality of final goods. In the long run, countries' specialization in low entry cost sectors increases competition among suppliers to raise the quality of suppliers available for final producers.

Combined with fixed trade costs, the pattern of firm-level trade based on this new gain is consistent with a variety of stylized facts, some of which are difficult to explain in the conventional love of variety model. First of all, firms selectively trade with those with similar characteristics instead of trading with all firms. Second, both exporting and importing are concentrated into a small share of large and high quality firms within industries.[7] Furthermore, the assortative matching explains why exporting firms and importing firms are larger in common characteristics than non-trading firms as observed by Bernard, Jensen, and Schott (2005) and Kasahara and Lapham (2008). Third, not all large and high quality firms necessarily trade. While in the love of variety model, the most productive firms always choose to trade, in the current model, some portion of high quality firms always choose not to trade.[8] Finally, trade upgrades the quality of final producers that do not import intermediate goods. In the conventional model of trade in intermediate goods such as the love of variety model and the quality-ladder model, firms must import foreign intermediate goods in order to raise the productivity/quality. This last prediction is consistent with a recent finding by Amiti and Konings (2007) that a reduction in tariffs on intermediate goods improves the total factor productivity (TFP) of even firms that do not use imported intermediate goods.

The paper contributes to the literature of so-called heterogeneous firm trade theories. Many theories have been developed to analyze exporters heterogeneous in productivity (Bernard, Jensen, Eaton, and Kortum, 2003; Melitz, 2003), exporters heterogeneous in product quality (Baldwin and Harrigan, 2007; Johnson, 2008; Kugler and Verhoogen, 2008; Verhoogen, 2008), and heterogeneous

[7] Bernard, Jensen, Redding, and Schott (2007) survey empirical and theoretical studies on firm-level trade. See Bernard and Jensen (1999) and Clerides, Lach, and Tybout (1998) and the papers cited in Bernard et al. (2007) for the concentration of exporting. See Bernard, Jensen, Redding, and Schott (2007), Bernard, Jensen, and Schott (2005), Biscourp and Kramarz (2007), and Kasahara and Lapham (2007) for the concentrantion of importing.

[8] In the Ricardian model by Bernard et al. (2003), it is possible for the most productive firm to choose not to export because the high productivity does not assure the comparative advantage. However, their model has no heterogeneous importers.

importers (Antràs and Helpman, 2003; Kasahara and Lapham, 2008). However, these studies treat heterogeneous exporters and importers in separate frameworks. The paper offers the first model of trade between firms that are heterogeneous in product quality.

The paper relies on the long history of the matching literature developed by Gale and Shapley (1962), Becker (1973), and other many studies. Especially, my model applies Sattinger (1979)'s model of a continuum of agents. My innovation is to let the distribution of firms at each side of matching endogenously determined, which allows me to analyze the effect of trade liberalization on the distribution of firms across industries in a general equilibrium framework.

The paper is closely related with recent studies on international matching of heterogeneous agents.[9] Kremer and Maskin (2006) and Antràs, Garicano, and Rossi-Hansberg (2006) study North-South matching of workers based on a hierarchical order in the skill intensity of production stages. My paper considers matching of firms between developed countries and does not assume any hierarchical order in the characteristics of production stages. Furthermore, their models allow workers to move across production stages, but my model prohibits firms from moving across production stages. This last point leads to very different predictions on the distribution of gains from trade across agents. In Antràs, Garicano, and Rossi-Hansberg (2006), the most skilled managers always reduce team's size and output by being matched with less skilled workers after the opening of trade, while in my model, the highest quality final producers improve the output from better matching. Nocke and Yeaple (2008) analyze two-sided matching between a corporate asset and a manager to model international M&A. The current paper analyzes three-sided matching and introduces a richer structure of firms' entry and exit to derive systematic predictions on the pattern of international matching. Furthermore, none of the above three studies analyzes costs of international matching.

The theoretical literature of trade in vertically-differentiated goods has focused on North-South trade in final goods, e.g. Flam and Helpman (1987). The literature has also been interested in whether low wage allows developing countries to export low quality products, e.g. Murphy and Shleifer (1997) and Sutton (2007). The current paper complements the literature by offering a

[9] A matching model is also becoming a popular tool to study trade between countries with different distributions of workers' skill. Grossman and Maggi (2000) model domestic matching between heterogeneous workers. Ohnsornge and Trefler (2007), Costinot (2008), and Costinot and Vogel (2008) study domestic matching between heterogeneous workers and different industries.

model of trade in vertically differentiated intermediate goods between countries at a similar income level.

The rest of the paper proceeds as follows. Section 2 sets up the model of a closed economy. Section 3 analyzes trade between symmetric countries. Section 4 concludes the paper and remarks on future extensions.

2 Closed Economy

This section introduces a general equilibrium model of team production in a closed economy setting. I explain the basic structure of the model and then demonstrate that the distribution of firms and matching patterns of firms reflect the technology of the economy.

2.1 Basic Structure

Consider a closed economy endowed with one production factor, labor. Final goods are both vertically and horizontally differentiated. A representative consumer maximizes a CES utility function,

$$U = \left[\int_{\omega \in \Omega} q\left(\omega\right) c\left(\omega\right)^{\rho} d\omega \right]^{1/\rho},$$

where Ω is the set of available varieties of final goods, ω is a particular variety, $c(\omega)$ is consumption of variety ω, $q(\omega)$ is product quality of ω, and $\rho \in (0,1)$ is a parameter. Let $p(\omega)$ be a price of ω and I be an aggregate income of this economy. The demand function for ω is derived as

$$c\left(\omega\right) = \frac{I q\left(\omega\right)^{\sigma} p(\omega)^{-\sigma}}{P^{1-\sigma}},$$

where $\sigma \equiv 1/(1-\rho) > 1$ is an elasticity of substitution, and $P \equiv \left[\int_{\omega \in \Omega} p\left(\omega\right)^{1-\sigma} q(\omega)^{\sigma} d\omega \right]^{1/(1-\sigma)}$ is a quality-adjusted price index. The product quality $q\left(\omega\right)$ is a demand shifter: a higher q implies a larger demand at a given price.

There exist three types of firms: final producers in final goods sector, suppliers of intermediate good $Z1$ ($Z1$-suppliers) in $Z1$-sector, and suppliers of intermediate good $Z2$ ($Z2$-suppliers) in $Z2$-sector. A final producer, a $Z1$-supplier, and a $Z2$-supplier form a production team to produce one variety of final good. Intermediate goods are specially designed for a particular variety of final good, e.g. engines and bodies for a particular model of car; therefore, firms transact them only within a team. In the following, I use subscripts i and j to denote variables and functions of $Z1$-suppliers and $Z2$-suppliers. They always mean that $i, j \in \{1, 2\}$ and $i \neq j$ when i and j are used together.

Firms are continuum and heterogeneous in production quality. Let x, z_1, and z_2 be the quality parameters of final producers, $Z1$-suppliers, and $Z2$-suppliers, respectively.[10] Quality parameters can also be interpreted as the quality of components, or product characteristics in a terminology used in the industrial organization literature.

The quality of a final good depends on the quality of team members in the following simple way,

$$q = xz_1z_2. \tag{1}$$

The function (1) exhibits three properties. First, q is increasing as is normally expected. Second, q is supermodular. A smooth twice-differentiable function is called (strictly) supermodular if all of its partial cross-derivatives are positive. The supermodularity expresses complementarity among the quality of team members. Suppose a car production team replaces an engine with the one with higher quality. The supermodularity just requires the improvement in the overall car performance to be increasing in the quality of the other parts and components such as transmission, body, tires, etc. Finally, q is quasi-concave. Consumers prefer a moderate combination of the quality of parts and components to an extreme combination. For instance, consumers might prefer a standard-class car with normal equipment to a luxury-class car with a poor air conditioner.

The labor market is perfectly competitive and wage is normalized as one. When a team produces X unit of a final good of quality q, the final producer requires $L_X(q, X)$ unit of labor, X unit of intermediate goods $Z1$, and X unit of intermediate goods $Z2$. To produce X unit of intermediate goods Zi designed for final goods with quality q, each Zi-supplier requires $L_{Zi}(q, X)$ unit of labor. The labor requirement is symmetric across team members, consists of fixed and variable components, and increases in q,

$$L_h(q, X) = \frac{qX + f}{3} \quad \text{for} \quad h = X, Z1, \text{and } Z2. \tag{2}$$

The assumption in (2) that variable costs increase in the quality of final goods reflects costs of quality control in team production. Since even one defect component can destroy the whole

[10] Readers may call x, z_1,and z_2 "productivity" of quality production.

product, as emphasized by Kremer (1993), production of high quality final goods requires extra costs of quality control for all team members. The fixed cost f includes costs of participating in the matching market as well as physical production costs.

The firm heterogeneity arises from firms' entry and exit. There exist infinitely many potential entrants of final producers and Zi-suppliers. These firms are ex ante symmetric, but become heterogeneous as a result of uncertain R&D investment. When firms enter, each firm independently draws its quality parameter from a common Pareto distribution.[11] The distribution function is $G(s) \equiv 1 - s^{-k}$ for $s \in [1, \infty)$ where $k > 3$ is assumed to assure the existence of finite GDP. Entry requires f_{Xe} unit of labor for final producers and f_{Zie} unit of labor for Zi-suppliers. These entry costs include not only setup costs but also R&D investment for blueprint of products. Firms are risk neutral so that they enter until their expected profits are zero.

After knowing quality parameters, firms form production teams. The matching of firms is frictionless in the following two senses: (i) firms have all information about the other firms and (ii) firms can write a complete contract on the distribution of team's joint profit.[12] For mathematical tractability, I assume one-to-one matching, i.e. each firm can join at most one team.[13]

The model consists of four stages. (i) Pre-entry Stage: a Walrasian auctioneer announces wage to clear the labor market. (ii) Entry Stage: firms enter and draw quality parameters by paying fixed entry costs. (iii) Matching Stage: firms form production teams. (iv) Production Stage: teams compete in a final good market under the monopolistic competition and distribute the joint profit.

[11] The Pareto distribution is commonly used to characterize empirical distributions of firm size. See Axtell (2001) and Helpman, Melitz, and Yeaple (2004), for example. In the current model, the sales of final goods follow the Pareto distributions both in autarky and free trade.

[12] The latter assumption abstracts away from a problem whether a team is formed within or across the boundaries of firms.

[13] The assumption of "one-to-one" is not necessary here. The main results are derived from the assorative matching of firms. A crucial assumption for the assortative matching is that each Zi-supplier has a capacity constraint on the number of teams that they can join. There are also several theoretical reasons for this capacity constraint: search costs convex in the number of buyers, convex marketing costs (Arkolakis, 2008), and several reasons for vertical foreclosure discussed in the industrial organization literature. I leave incorporating one of these reasons in the model for future research.

2.2 Equilibrium

I derive an equilibrium allocation by backward induction. Although the model has a trivial equilibrium where no firm enters, I focus on an equilibrium with entry.

Production Stage Consider a team producing a final good with quality q. Since team's marginal cost is q, it follows that the optimal price $p(q)$ of final goods, the sales $r(q)$ of final goods, and team's joint profit $\Pi(q)$ are

$$p(q) = \frac{q}{\rho},\ r(q) = I\left(P\rho\right)^{\sigma-1} q, \text{ and } \Pi(q) = Aq - f. \tag{3}$$

Parameter $A \equiv \sigma^{-1} I \left(\rho P\right)^{\sigma-1}$ expresses the market condition exogenous to individual teams. The optimal output, $\bar{c} = \rho^{\sigma} I P^{\sigma-1}$, is independent of q. This is because both consumer's demand and marginal costs increase in q and the two effects are balanced under the current specification. Since the price increases in q, both revenue and profit increase in q. From (1) and (3), team's joint profit is increasing and supermodular in the quality of team members,

$$\Pi(x, z_1, z_2) = Axz_1z_2 - f. \tag{4}$$

Matching Stage Firms choose their partners and decide the distribution of team's joint profit, taking A as given. Two types of functions, profit schedules, $\pi_X(x)$ and $\pi_{Zi}(z_i)$, and assignment functions, $m_{Zi}(x)$, characterize equilibrium matching. A final producer with quality x chooses Zi-suppliers with quality $m_{Zi}(x)$ and receives a residual profit $\pi_X(x)$ after paying profits $\pi_{Zi}(z_i)$ for Zi-suppliers. Firms can also choose not to join any team and exit. Following the matching literature, I focus on stable matching satisfying two conditions: (i) no individual firm is willing to deviate from the current team; (ii) no trio of a final producer, a $Z1$-supplier, and a $Z2$-supplier are willing to deviate from the current teams to form a new team.[14] The two conditions require the following two conditions, respectively: (i) all firms earn non-negative profit, $\pi_X(x) \geq 0$ and

[14] The first condition is often called *individual rationality* and the second condition is *pair-wise stability*. Roth and Sotomayer (1990) is an excellent textbook on the literature.

$\pi_{Zi}(z_i) \geq 0$ for all x and z_i; (ii) each firm in a team is the optimal partner for the other members,

$$\begin{aligned}
\pi_X(x) &= \Pi(x, m_{Z1}(x), m_{Z2}(x)) - \pi_{Z1}(m_{Z1}(x)) - \pi_{Z2}(m_{Z2}(x)) \\
&= \max_{z_1, z_2} \Pi(x, z_1, z_2) - \pi_{Z1}(z_1) - \pi_{Z2}(z_2) \text{ and} \qquad (5) \\
\pi_{Zi}(m_{Zi}(x)) &= \Pi(x, m_{Zi}(x), m_{Zj}(x)) - \pi_X(x) - \pi_{Zj}(m_{Zj}(x)) \\
&= \max_{x', z_j} \Pi\left(x', m_{Zi}(x), z_j\right) - \pi_X(x') - \pi_{Zj}(z_j). \qquad (6)
\end{aligned}$$

The first order conditions for maximization (5) and (6),

$$\pi_X'(x) = A m_{Z1}(x) m_{Z2}(x) \text{ and } \pi_{Zi}'(m_{Zi}(x)) = A x m_{Zj}(x), \qquad (7)$$

prove that profit schedules increase in quality parameters.

From the supermodularity of joint profit in (4), firms are assortatively matched according to their quality as in Becker (1973) and Sattinger (1979). The logic is straightforward. Since a high quality firm has a higher willingness to pay for extra quality of partners, a high quality firm is matched with a high quality firm in a stable matching.

Lemma 1 *(Assortative matching)* $m_{Zi}(x) \geq m_{Zi}(x')$ *if only if* $x \geq x'$.

Proof. In Appendix. ■

The assortative matching in Lemma 1 predicts a systematic pattern on the combination of the quality of parts and components. It implies that more expensive final goods have higher quality in all parts and components than less expensive final goods. This complementarity of quality of components is quite common for many quality-differentiated goods, such as cars, electronics, and clothing. In the literature of hedonic regression, this complementarity is often observed as multi-collinearity (see e.g. Triplet, 2004). Furthermore, Kugler and Verhoogen (2009) found Colombian manufacturing plants using expensive foreign inputs tend to use expensive domestic inputs, while plants using cheap foreign inputs tend to use cheap domestic inputs. If observed prices are proxy for product quality, which will be shown to hold in this model, the assortative matching by product quality is consistent with their finding.

Fixed costs f allow only teams producing high quality final goods to survive. Under assortative matching, teams producing the lowest quality, which consist of the lowest quality firms, must be break even

$$\Pi(x_L, z_{1L}, z_{2L}) = A x_L z_{1L} z_{2L} - f = 0, \qquad (8)$$

where x_L and z_{iL} are the lowest quality thresholds of final producers and $Z1$-suppliers, respectively, and satisfy

$$\pi_X(x_L) = \pi_{Z1}(z_{1L}) = \pi_{Z2}(z_{2L}) = 0. \tag{9}$$

Firms with lower quality than the lowest quality thresholds choose not to join teams and exit.

Assignment functions must clear the demand and supply for firms in the matching market. Let M_{Xe}, M_{Z1e}, and M_{Z2e} be the mass of entrants of final producers, $Z1$-suppliers, and $Z2$-suppliers, respectively. Under assortative matching, the market clearing condition is written as

$$M_{Xe}\left[1 - G\left(x\right)\right] = M_{Zie}\left[1 - G\left(m_{Zi}\left(x\right)\right)\right] \text{ for all } x \geq x_L. \tag{10}$$

The left hand side of (10) is the mass of final producers with higher quality than x. The right hand side is the mass of Zi-suppliers matched with those final producers. They must be equal for all x of survival final producers. Figure 1 describes the market clearing conditions (10) in the matching market. Bars in Figures 1 express the distributions of final producers, $Z1$-suppliers, and $Z2$-suppliers. The vertical axis draws values of the distribution $G(x)$, $G(z_1)$, and $G(z_2)$ in a range of $[0, 1]$. The areas of rectangles surrounded by solid lines are equal to the mass of survival firms, $M_{Xe}\left[1 - G\left(x_L\right)\right]$ and $M_{Zie}\left[1 - G\left(z_{iL}\right)\right]$, respectively, each of which must have the same area under one-to-one matching. Grey areas, which are equal to $M_{Xe}\left[1 - G\left(x\right)\right]$ and $M_{Zie}\left[1 - G\left(m_{Zi}\left(x\right)\right)\right]$, respectively, must have the same area from (10).

The relative mass of entrants across sectors determines the matching pattern. Under the Pareto distribution, assignment functions are solved from (10),

$$m_{Zi}\left(x\right) = x\left(\frac{M_{Zie}}{M_{Xe}}\right)^{1/k} \text{ for all } x \geq x_L. \tag{11}$$

Figure 2 draws $m_{Zi}\left(x\right)$ for a given x as an increasing and concave curve in the relative mass of entrants. Figure 3 explains the intuition. As more firms enter Zi-sector, a final producer becomes matched with a Zi-supplier with better quality. Suppose new Zi-suppliers with mass dM'_{Zie} enter. A final producer with quality x can be matched with better Zi-suppliers only if new entrants have higher quality than the current partner. Under the Pareto distribution, the mass of those high quality Zi-suppliers $\left[1 - G\left(m_{Zi}\left(x\right)\right)\right] dM_{Zie}$ falls as $m_{Zi}\left(x\right)$ rises further by dM''_{Zie}; therefore, the marginal improvement is diminishing.[15]

[15] This concave relationship holds under a wide class of distributions including those exhibiting the non-decreasing hazard rate $g(x)/(1 - G(x))$, which includes uniform, normal, exponential, and other frequently used distributions.

The profit schedules are obtained by integrating the first order conditions (7) with initial conditions (9),

$$\pi_X(x) = A\int_{x_L}^{x} m_{Z1}(t)\, m_{Z2}(t)\, dt \text{ and } \pi_{Zi}(m_{Zi}(x)) = A\int_{x_L}^{x} t m_{Zj}(t)\, m'_{Zi}(t)\, dt, \tag{12}$$

for all $x \geq x_L$ and $z_i \geq z_{iL}$. Profits are increasing in the market size A, the quality parameters of the partners, and the degree of its advantage in quality over the lowest quality firm. The cutoff condition (9) and the assignment functions (11) further simplify the profit schedules as

$$\pi_X(x) = \frac{f}{3}\left[\left(\frac{x}{x_L}\right)^3 - 1\right] \text{ and } \pi_{Zi}(z_i) = \frac{f}{3}\left[\left(\frac{z_i}{z_{iL}}\right)^3 - 1\right], \tag{13}$$

for all $x \geq x_L$ and $z_i \geq z_{iL}$.[16] The profit schedule is decreasing in the lowest quality threshold because of two negative effects. When the threshold increases, firms in that sector become assigned with lower quality partners (see (11)) and the market size A shrinks (see (8)).[17]

Entry Stage Since firms are ex ante identical and risk neutral, their expected profits must be equal to entry costs,

$$[1 - G(x_L)]\,\bar{\pi}_X = f_{Xe} \text{ and } [1 - G(z_{iL})]\,\bar{\pi}_{Zi} = f_{Zie}, \tag{14}$$

where $\bar{\pi}_X$ and $\bar{\pi}_{Zi}$ are the average profits of firms in the market, $\bar{\pi}_X = [1 - G(x_L)]^{-1}\int_{x_L}^{\infty}\pi_X(t)\, g(t)\, dt$ and $\bar{\pi}_{Zi} = [1 - G(z_{iL})]^{-1}\int_{z_{iL}}^{\infty}\pi_{Zi}(t)\, g(t)\, dt$. A straightforward manipulation from (13) proves that the average profits turn are constant,

$$\bar{\pi}_X = \bar{\pi}_{Zi} = \frac{f}{k-3}. \tag{15}$$

This constant average profit is consistent with a well-known property of the Melitz-type model (2003) that the average profit of firms becomes constant when firms' productivity follows the Pareto distribution. In the current model, teams' quality q follows the Pareto distribution.[18] To

[16] To derive (13), I use $m_{Zi}(x) = x\,(z_{iL}/x_L)$ derived from (10).

[17] The profit schedules in (13) are independent of the quality thresholds in the other sectors because of the consequence of two opposite effects. When the threshold increases in the other sectors, the firm becomes assigned with better partners, but the other rival firms in the same sector enjoy that improvement, which reduces the market size A for individual firms. These two effects are cancelled under the Pareto distribution.

[18] The distribution of q is $\Pr(q \leq s) = 1 - (q_L/s)^{k/3}$, where $q_L \equiv x_L z_{1L} z_{2L}$ is the lowest quality of final goods in the market.

assure the positive mass of entry, I assume $f/(k-3) \geq \max\{f_{Xe}, f_{Zie}\}$. Then, the lowest quality thresholds are solved from (14) and (15) as

$$x_L = \left[\frac{f}{f_{Xe}(k-3)}\right]^{1/k} \text{ and } z_{iL} = \left[\frac{f}{f_{Zie}(k-3)}\right]^{1/k}. \tag{16}$$

The lowest thresholds decrease in entry costs and increases in production fixed costs. The intuition will be clear below after I solve the mass of consumption varieties, M, and the mass of entrants in each sector.

Since firms earn zero expected profits, the aggregate revenue from final goods must be equal to the aggregate labor income, $M\bar{r} = \bar{L}$, where $\bar{r}$ is the average revenue of survival teams. From $\bar{r} = \sigma(\bar{\pi}_X + \bar{\pi}_{Z1} + \bar{\pi}_{Z2} + f)$ and (15), the mass of consumption varieties is proportional to the ratio of labor endowment to production fixed costs,

$$M = \frac{(k-3)}{k\sigma}\left(\frac{\bar{L}}{f}\right). \tag{17}$$

Under one-to-one matching, the mass of teams is equal to the mass of survival firms in each sector. Therefore, the mass of entrants are solved as

$$M_{Xe} = \frac{M}{1-G(x_L)} = \frac{\bar{L}}{f_{Xe}k\sigma} \text{ and } M_{Zie} = \frac{M}{1-G(z_{iL})} = \frac{\bar{L}}{f_{Zie}k\sigma}.$$

While the relative magnitude of labor endowment to production fixed costs determine the mass of varieties consumed, the relative size of entry costs determines the relative size of the mass of entrants.

The lowest quality thresholds in (16) link the mass of consumption varieties and the mass of entrants. Lower entry costs attract relatively more entrants, but the total mass of survival firms (17) is independent of the size of entry costs. Therefore, in the sector with low entry costs, firms have to be higher quality to survive than in the other sectors.

From (11) and (16), the distribution of product characteristics reflects the distribution of entry costs across sectors,

$$\frac{m_{Zi}(x)}{x} = \left(\frac{f_{Xe}}{f_{Zie}}\right)^{1/a} \text{ and } \frac{m_{Z1}(x)}{m_{Z2}(x)} = \left(\frac{f_{Z2e}}{f_{Z1e}}\right)^{1/a} \text{ for all } x \geq x_L.$$

A component produced by a sector with relatively lower entry costs is relatively higher quality than the other components. Although our main interest is in an open economy, the closed economy model predicts horizontal differences in the quality of parts and components. National product differentiation is often loosely stated in terms of horizontal differences in the quality of components, e.g. "European cars have safety bodies" or "Japanese cars are energy efficient", even though the most expensive Japanese cars will be safer than the cheapest European cars. In this model, this kind of comparison is valid when final goods with the same price are compared.

3 Open Economy

In this section, I extend the model in a two-country framework and analyze the effect of trade intermediate goods, i.e. international matching, on the matching patterns and the quality of final goods. Throughout this paper, I assume that final goods are non-tradable.[19]

3.1 Symmetric Two Countries

I consider two symmetric countries, Home and Foreign, which are identical except entry costs in the Zi-sectors. Foreign is a mirror image of Home: Foreign $Z1$-sector is identical to Home $Z2$-sector, while Foreign $Z2$-sector is identical to Home $Z1$-sector. Without loss of generality, Home $Z1$-sector and Foreign $Z2$-sector require smaller entry costs than the other sectors,

$$f_{Z1e} = f^*_{Z2e} < f_{Z2e} = f^*_{Z1e}, \tag{18}$$

where foreign variables and functions are labeled by asterisks. I call Home $Z1$-sector and Foreign $Z2$-sector *Low Entry Cost (LEC) sectors* and Home $Z1$-sector and Foreign $Z2$-sector *High Entry Cost (HEC) sectors*.

The symmetric model tries to capture trade in quality-differentiated intermediate goods between developed countries at similar income level. Furthermore, the mirror-image structure greatly simplifies the analysis. Wage is equalized across countries and normalized as one. Equilibrium values of functions and variables of Home Zi-sector are the same as those of Foreign Zj-sector. The other aspects are identical between Home and Foreign.

The horizontal difference in entry costs creates a horizontal difference in the autarky matching patterns. Final producers with quality x produce the same quality of final goods in both countries, but in the teams, Zi-suppliers from LEC-sectors are higher quality than Zj-suppliers from HEC-

[19]Since the structure of the final good market is similar to the Melitz (2003) model, introducing trade in final goods will add few new insight.

sectors. Point A and Point A* in Figure 4 represent the quality of Zi-suppliers in a Home team $(m^a_{Z1}(x), m^a_{Z2}(x))$ and in a Foreign team $(m^{*a}_{Z1}(x), m^{*a}_{Z2}(x))$, respectively, for given x. The curve in Figure 4 is an "iso-$q(x)$ curve," which depicts a combination of the quality of Zi-suppliers in that final producers with quality x need to produce final goods with quality $q(x)$. In the following, endogenous variables and functions in the autarky equilibrium are labeled by "a".

3.2 Trade Costs and Specialization

The opening of trade in intermediate goods allows international matching of firms between the two countries. Forming international teams requires fixed trade costs, which include transportation costs, communication costs, and costs of adopting foreign standards and regulations, etc. If an international team has n foreign suppliers, then the team must hire nf_I unit of labor as fixed trade costs. Each firm in an international team equally shares nf_I by hiring $nf_I/3$ unit of labor. From the symmetric structure, I will show that $n \leq 1$ holds in equilibrium. For simplicity, I assume that trade does not require any variable cost.

Three regimes arise depending on the level of f_I: autarky, incomplete specialization, and complete specialization.

Lemma 2 *There exists a threshold level of trade costs $\tilde{f}_I$: (i) (autarky) if $f_I = \infty$, then no international matching occurs; (ii) (incomplete specialization) if $\tilde{f}_I < f_I < \infty$, then both countries have positive mass of entrants in all three sectors; (iii) (complete specialization) if $f_I < \tilde{f}_I$, then both countries have positive mass of entrants in final goods sector and LEC sector, but no entrant in HEC sector.*

Proof. In Appendix. ■

As I will show below, in the incomplete specialization equilibrium, international teams and local teams coexist, i.e. some firms trade but some firms do not in line with stylized facts on firm-level trade, while in the specialization equilibrium, all firms engage in international trade. In the following, I first analyze the incomplete specialization equilibrium and then, the complete specialization equilibrium.

3.3 Incomplete Specialization Equilibrium

The incomplete specialization equilibrium is more complicated than the autarky equilibrium since trade costs prevent some firms from engaging in international trade. To distinguish suppliers who can export from those who cannot, I introduce the concept of exportability.

Definition 1 *Home Zi-suppliers with quality z_i are called exportable if $\pi_{Zi}(z_i) + f_I = \pi^*_{Zi}(z_i)$ and non-exportable if $\pi_{Zi}(z_i) + f_I > \pi^*_{Zi}(z_i)$. Similarly, Foreign Zi-suppliers with quality z_i are called exportable if $\pi^*_{Zi}(z_i) + f_I = \pi_{Zi}(z_i)$ and non-exportable if $\pi^*_{Zi}(z_i) + f_I > \pi_{Zi}(z_i)$.*

When Home Zi-suppliers with quality z_i are exportable, Foreign final producers are indifferent between Home Zi-suppliers with quality z_i and Foreign Zi-suppliers with the same quality. When Home Zi-suppliers with quality z_i are non-exportable, Foreign final producers strictly prefer Foreign Zi-suppliers with z_i to Home Zi-suppliers with the same quality. Notice that non-exportable Zi-suppliers never export, but exportable Zi-suppliers do not necessarily export. All Home Zi-suppliers are classified as either exportable or non-exportable because $\pi_{Zi}(z_i) + f_I < \pi^*_{Zi}(z_i)$ never holds; otherwise, no final producer would choose Foreign Zi-suppliers with z_i.

I prove two lemmas to derive the market clearing conditions in the matching market. I assume that as in the autarky equilibrium, countries have more entrants in LEC sectors than HEC sectors,

$$M_{Z1e} = M^*_{Z2e} > M_{Z2e} = M^*_{Z1e}, \tag{19}$$

and prove this inequality later at the end of this section. The first lemma shows that the quality of exportable Zi-suppliers are equalized within a team.

Lemma 3 *A $Z1$-supplier is matched with a $Z2$-supplier with the same quality if either one of them is exportable.*

Proof. In Appendix. ■

Lemma 3 is derived from the definition of exportability and the mirror-image structure of Home and Foreign. Suppose an exportable Home $Z1$-supplier with quality z is matched with a Foreign $Z2$-supplier. Consider the case of international matching first. Since the profit schedules of Foreign $Z2$-suppliers is identical to that of Home $Z1$-suppliers and the joint profit is symmetric in the quality parameters of $Z1$-supplier and $Z2$-supplier, the partner Foreign $Z2$-supplier also has the same quality z. A similar logic is applied for the case of local matching.

Although Lemma 3 depends on the mirror-image assumption, it greatly saves notations. In more general setting, a complete description of possible matching patterns requires eight functions. However, from Lemma 3, four functions $m_{Zi}(x)$ and $m^*_{Zi}(x)$ can summarize an equilibrium matching pattern. A Home final producer with quality x is matched with Zi-suppliers with quality $m_{Zi}(x)$ regardless of the nationality of Zi-suppliers. Similarly, a Foreign final producer with quality x is matched with Zi-suppliers with quality $m^*_{Zi}(x)$. Notice that the assortative matching continues to hold, i.e. $m_{Zi}(x) \geq m_{Zi}(x')$ and $m^*_{Zi}(x) \geq m^*_{Zi}(x')$ if and only if $x \geq x'$, since team's joint profit remains supermodular even with fixed trade costs.

The second lemma shows only high quality Zi-suppliers in LEC sectors are exportable.

Lemma 4 *There exists a threshold quality z_T such that only Home $Z1$-suppliers and Foreign $Z2$-suppliers with higher quality than z_T are exportable. The other suppliers are non-exportable.*

Proof. In Appendix. ■

The intuition for Lemma 4 is simple. From inequality (19), international matching occurs only between Home $Z1$-suppliers and Foreign $Z2$-suppliers, i.e firms that are abundant in the two countries. However, not all firms can trade. Exportable suppliers must be high quality since low quality teams cannot afford trade costs.[20]

Lemmas 3 and 4 derive three conditions for matching of firms. First, since final producers are indifferent between domestic Zi-suppliers and foreign Zi-suppliers if they are higher quality than z_T, these high quality firms in Home and Foreign are pooled together. The market clearing condition for high quality firm can be written as

$$(M_{Xe} + M^*_{Xe})[1 - G(x)] = (M_{Zie} + M^*_{Zie})[1 - G(m_{Zi}(x_T))] \text{ for all } x \geq x_T, \tag{20}$$

where x_T is defined by $z_T = m_{Zi}(x_T)$. Notice that $m_{Z1}(x) = m_{Z2}(x)$ for all $x \geq x_T$ from (19). After the opening of trade, the cross-country difference in matching patterns that exists in autarky disappears for high quality teams. For low quality teams that cannot form international teams, the market clearing condition holds for local firms,

$$M_{Xe}[G(x_T) - G(x)] = M_{Zie}[G(z_T) - G(m_{Zi}(x))] \text{ for all } x \in [x_L, x_T]. \tag{21}$$

[20] Lemma 4 crucially depends on the assumption that the distribution of firm size G is similar across sectors and across countries. Once the model allows asymmetric distributions of firms, it will be possible for the highest quality suppliers to be non-exportable. I do not pursue the case of asymmetric distributions here since the shape of the firm-size distribution is similar across industries and across countries in data.

The pattern of international matching is derived from conditions (20), (21), and Lemma 4. Since the mass of entrants of Zi-suppliers differs across countries, international matching must occur between high quality firms. Following a tradition in the literature, I focus on an equilibrium that minimizes the amount of international matching.

Lemma 5 *(i) Among final producers with given quality $x \geq x_T$, the share of final producers that import intermediate goods is $s_X = M_{Xe}\left(M_{Z1e} - M_{Z2e}\right) / \left(M_{Z1e} + M_{Z2e}\right) \in (0,1)$. (ii)Among Zi-suppliers in LEC sectors with given quality $z \geq z_T$, the share of Zi-suppliers exporting intermediate goods is $s_Z = (M_{Z1e} - M_{Z2e})/2 \in (0,1)$.*

Lemma 5 shows that only high quality final producers import and only high quality Zi-suppliers in LEC sectors export, though not all of them trade. Figure 5 describes the market clearing conditions (20) and (21). The area of each of six rectangles surrounded by solid lines is the mass of survival firms in each sector. Trade costs divide firms into three groups and firms are assortatively matched among each group. High quality firms in grey area are matched together and low quality firms in each stripe area are matched together. Firms that trade are expressed by the shaded area in Figure 6. Home final producers in area A are matched with Foreign $Z2$-suppliers in area A'; Foreign final producers in area B are matched with Home $Z1$-suppliers in area B'.

3.3.1 New Predictions on Firm-level Trade

The model provides new predications on the observable characteristics of trading firms, some of which are observed in empirical studies. Before stating them, I explain how quality parameters link several observable characteristics of firms. Unit price is a proxy for product quality. This is clear for final goods from (3). A unit price of an intermediate good Zi, $p_{Zi}(z_i)$, is obtained by dividing the revenue by output,

$$p_{Zi}(z_i) \equiv \frac{\pi_{Zi}(z_i) + L_{Zi}(q(z_i), \bar{c})}{\bar{c}}, \text{ where } q(z_i) = m_{Zi}^{-1}(z_i)\, m_{Zj}\left(m_{Zi}^{-1}(z_i)\right) z_i.$$

Since the output $\bar{c}$ is common for all teams and q is increasing in z_i, a unit price is positively correlated with product quality. From (3), the assortative matching implies that product quality is positively correlated with revenue, employment, profit, and unit prices in each sector.

Trade patterns in the incomplete specialization equilibrium offer three predictions on characteristics of trading firms.

Proposition 1 *(1) Firms trade with those with similar characteristics such as revenue, employment, profit, and unit prices. (2) When the type of sectors and the nationality of firms are controlled, the average exporter is larger than the average non-exporter and the average importer is larger than the average non-importer in such common variables as employment, revenue, profit, and unit prices. (3) However, there exist firms that are larger in employment, revenue, profit, and unit prices than other trading firms but choose not to trade.*

Three predictions in Proposition 1 have not been presented in the previous models of heterogeneous firm trade theories. First, in contrast to standard models of heterogeneous firms based on the love of variety such as Melitz (2003) and Kasahara and Lapham (2007), which predicts all exporters trade with all importers, in the current model, high quality exporters selectively trade only with high quality importers, while low quality exporters selectively trade only with low quality importers. This international assortative matching is consistent with many anecdotal stories on the production of high quality goods, e.g. Boeing 787 and Apple ipod, although to my knowledge, no systematic econometric study has tested whether the assortative matching is a chief trading pattern in data yet.

Second, Proposition 1 is the first demonstration of the concentration of exporting and importing into large and high quality firms in a single framework. Recent empirical studies report that exports and imports are concentrated into a small share of firms within an industry and that on average, exporting firms and importing firms are larger than non-trading firms in common characteristics such as employment, sales, productivity, wage, capital intensity, and skill intensity.[21] Although many papers have been written on these styled facts, none of them treated heterogeneous exporters and heterogeneous importers within one framework. The assortative matching in this model naturally explains the similarity of characteristics of exporting firms and importing firms.

Finally, the large size and the high quality are necessary conditions for trade, but not sufficient conditions. While the standard love of variety models predicts firms that are larger than a certain threshold always choose to trade, in data, the correlation between firm size (or measured productivity) and firm's trading status is obviously not perfect (see e.g. Bernard et al. 2003), i.e. there are many large firms that do not trade.[22] The current model predicts the existence of non-trading

[21] Bernard and Jensen (1999) and Clerides, Lach, and Tybout (1998) observed that better performances of exporting firms reflect their intrinsic ability.

[22] In Bernard et al. (2003), the most productive firms do not necessarily export since high productivity does

large firms without relying on any idiosyncratic factor.

Before moving to the analysis on the change of the quality in the next subsection, I list equilibrium conditions for obtaining endogenous variables and prove the inequality (19). From Lemma 4, $f_I = \pi_{Z2}(m_{Z2}(x)) - \pi_{Z1}(m_{Z1}(x))$ holds if and only if $x \geq x_T$. Therefore, the threshold x_T is determined by

$$\begin{aligned} f_I &= \pi_{Z2}(m_{Z2}(x_T)) - \pi_{Z1}(m_{Z1}(x_T)) \\ &= A \int_{x_L}^{x_T} t\left[m_{Z1}(t)\, m'_{Z2}(t) - m'_{Z1}(t)\, m_{Z2}(t)\right] dt, \end{aligned}$$

where profit schedules are solved from (9), (12), and (21). Finally, the mass of entrants are solved from the free entry conditions (14) and the average revenue $M\bar{r} = \bar{L}$. The inequality (19) follows from the free entry conditions and the inequality of fixed entry costs (18).

Lemma 6 $M_{Z1e} = M^*_{Z2e} > M_{Z2e} = M^*_{Z1e}$.

Proof. In Appendix. ■

3.4 Quality Upgrading of Final Goods

Trade liberalization affects the quality of final goods by changing matching patterns. I analyze the change of final good quality by comparing the autarky equilibrium and the trade equilibrium. Following Proposition 5, I divide final producers by quality at threshold x_T. I first consider the quality change of high quality final producers and, then, that of low quality final producers.

Quality Upgrading of High Quality Firms The opening of trade affects the matching market in two ways. First, high quality firms in Home and Foreign are pooled together. Second, firms enter and exit under free entry. To separate the former effect from the latter one, I decompose trade liberalization into short run and long run: in the short run, international matching is allowed, with the mass of entrants kept at the autarky level; in the long run, the mass of entrants adjusts to satisfy the free entry conditions.

not necessarily implies the comparative advantage as in a traditional Ricardian model. In their model, there is no heterogeneity in importers.

Short Run Effect Trade improves the quality of final goods even in the short run. All propositions and lemmas in the last section holds in the short run equilibrium, though the mass of entrants should be at the autarky level there, since they are derived from the inequality (19) and do not depend on the levels of the mass of entrants. I obtain assignment functions $m^s_{Zi}(x)$ from the market clearing condition (20),

$$m^s_{Z1}(x) = m^s_{Z2}(x) = x\left(\frac{M^a_{Z1e} + M^{a*}_{Z1e}}{2M^a_{Xe}}\right)^{1/k} \text{for } x \geq x_T. \tag{22}$$

The assignment functions (22) are comparable with those in the autarky equilibrium (11) since they are commonly expressed in terms of the relative mass of entrants into the Zi-sector and the final goods sector. Figure 7, which replicates Figure 2, draws assignment functions (11) and (22) for a given x. Since the relative mass of entrants of $Z1$-supplier and final producers, $\left(M^a_{Z1e} + M^{*a}_{Z1e}\right)/2M^a_{Xe}$, is the average of those in the autarky, M^a_{Z1e}/M^a_{Xe} and M^{*a}_{Z1e}/M^{*a}_{Xe}, the concave curve implies that $m^s_{Zi}(x)$ is higher than the average of $m^a_{Z1}(x)$ and $m^{*a}_{Z1}(x)$. By the quasi-concavity of q, a final producer with quality $x \geq \hat{x}$ raises the product quality. This is shown in Figure 8, which draws $m^s_Z(x)$ (Point B) and $m^a_{Zi}(x)$ and $m^{*a}_{Zi}(x)$ (Point A and Point A*) with iso-$q(x)$ curves.

The source of this short run quality upgrading is the reduction in the difference in the quality of Zi-suppliers within a team. The competition with foreign final producers forces a final producer to be matched with a lower quality Zi-supplier in LEC sector. However, trade also allows it to be matched with a higher quality Zj-supplier in HEC sector than in the autarky. Since consumers prefer a moderate combination of the quality of components, the latter positive effect compensates for the former negative effect and improves the overall quality.

Long Run Effect In the long run, the mass of entrants are adjusted to satisfy the free entry conditions.

Lemma 7 *(1)The mass of entrants of final producers remains at the autarky level:* $M_{Xe} = M^a_{Xe}$*.(2) The mass of entrants of* Zi*-suppliers in LEC sector rises while that of* Zi*-suppliers in HEC sector falls:*

$$M_{Z1e} > M^a_{Z1e} > M^a_{Z2e} > M_{Z2e}. \tag{23}$$

(3)The relative mass of entrants of Zi-suppliers to final producers in the world increases but is bounded by the relative mass in Home autarky:

$$\frac{M^a_{Zie}}{M^a_{Xe}} > \frac{M_{Zie} + M^*_{Zie}}{2M_{Xe}} > \frac{M^a_{Zie} + M^{*a}_{Zie}}{2M^a_{Xe}}. \tag{24}$$

Proof. In Appendix. ■

Trade liberalization invites more entrants in LEC sectors and reduces entrants in HEC sectors. This specialization of entry into sectors with low entry costs results in the increase in the mass of entrants of Zi-suppliers in the world. This specialization of R&D investment in more efficient sectors reminiscent a classical Ricardian comparative advantage.

The long run adjustment of firms' entry and exit further improves the quality of final goods. From (20), the assignment function in the long run is

$$m_{Z1}(x) = m_{Z2}(x) = \left(\frac{M_{Zie} + M^*_{Zie}}{2M_{Xe}}\right)^{1/k} x \text{ for } x \geq x_T. \tag{25}$$

From Figure 7, final producers become matched with higher quality Zi-suppliers than in the short run equilibrium, i.e. $m_{Zi}(x) > m^s_{Zi}(x)$. Point C in Figure 8 expresses $m_{Zi}(x)$ on a iso-quality curve $q(x) = q^l(x)$, where $q^l(x)$ is the quality of final good produced by a final producer with quality x in the trade equilibrium. From (24) and (25), the upper bound of $m_{Zi}(x)$ is $m^a_{Z1}(x) = m^{*a}_{Z2}(x)$. Therefore, Point C is located somewhere between Point B and Point D.

The source of the long run quality upgrading could be interpreted as increased competition among Zi-suppliers. Countries' specialization in LEC sectors increases the mass of entrants of Zi-suppliers in the world. The intensified competition among Zi-suppliers allows final producers to be matched with higher quality Zi-suppliers.

In sum, trade liberalization improves the quality of high quality final goods in two steps. While trade eliminates the quality gap between Zi-suppliers in the short-run, trade increases the quality level of Zi-suppliers matched with a given final producer in the long-run.

Let $\Theta(x) \equiv q^l(x)/q^a(x)$ be the degree of quality-change of a final producer with x. It is straight forward to show the quality upgrading is at a constant rate.

Proposition 2 $\Theta(x) = K > 1$ *for* $x \geq x_T$, *where* $K = (M_{Z1e} + M_{Z2e})^{2k} / (M_{Z1e}M_{Z2e})^k$.

Finally, I should remark the quality upgrading does not require the use of imported intermediate goods. High quality final producers equally gain from the opening of trade whether they are matched with foreign suppliers or not. I will show this property distinguishes the current model from the conventional models later below.

Quality Upgrading of Low Quality Firms Final producers with lower quality than x_T cannot form international teams. From the market clearing condition, the assignment function is

$$m_{Zi}(x) = \left(\frac{M_{Z1e} + M_{Z2e}}{2M_{Xe}}\right)^{1/k} x \left[1 + \left(1 - \left(\frac{x}{x_T}\right)^k\right)\left(\frac{M_{Zje} - M_{Zie}}{2M_{Zie}}\right)\right]^{-1/k} \tag{26}$$

for $x \in [x_L, x_T)$. If x is close to x_T, then $m_{Zi}(x)$ in (26) is close to the value predicted by (20). As x becomes smaller, the difference between (26) and (20) becomes wider. Therefore, the degree of quality-upgrading is increasing in x.

Proposition 3 *(i)* $\Theta(x_T) < K$ *and* $\Theta'(x) > 0$ *for* $x \in [x_L, x_T)$. *(ii)* $\Theta(x_L^a) > 1$ *if* M_{Zie}/M_{Xe} *are sufficiently close to* M_{Zie}^a/M_{Xe}^a.

Proof. In Appendix. ■

Surprisingly, even final producers whose quality is too low to be matched with foreign suppliers can upgrade the product quality. The intuition is simple. After the opening of trade, the inflow of high quality $Z2$-suppliers from Foreign makes high quality final producers to release high quality $Z2$-suppliers for low quality final producers in Home. Furthermore, it is possible for all final producers with $x \geq x_L^a$ to upgrade product quality if the mass of entrants is close to the autarky level. However, another opposing effect arises in the long run. The specialization into $Z1$-sectors from $Z2$-sectors reduces the mass of entrants of $Z2$-suppliers in Home. Whether all final producers upgrade the product quality is generally ambiguous.

3.4.1 Testable predictions of the Model against the Conventional Models

The model presents the new mechanism of gains from trade in intermediate goods. However, it is not the only model demonstrating the positive effect of trade in intermediate goods on the performance of final producers. Trade in intermediate goods improves the productivity/quality in the conventional models such as the love of variety model, e.g. Ethier (1982), and the quality ladder model, e.g. Grossman and Helpman (1991). Of course, an ideal empirical test on the current model against these conventional models is to check the assortative matching of trading firms. However, at this moment, it is very difficult to link transaction data with firms' characteristics data in multiple countries to see characteristics of exporters and importers at transaction level. Therefore, in this section, I propose an alternative form of empirical test of the current model against the conventional models that is relatively easier to implement.

The prediction on the relationship between the degree of quality-upgrading and the importing status of final producers provides a basis distinguishing the current model from the conventional models.

Remark 1 *After trade liberalization of intermediate goods, (i) the average degree of quality-upgrading of firms that use imported intermediate goods (importing firms) is larger than that of firms that do not (non-importing firms). (ii) The average degree of quality-upgrading of non-importing firms can be positive.*

To test the current model against these models, it is sufficient to check whether Remark 1 hold in data. In the conventional models, a necessary condition for improving the productivity/quality is to import intermediate goods. Therefore, the prediction (ii) of Remark 1 should not be observed in these models.

To my knowledge, there is no econometric study that investigates the effect of trade liberalization of intermediate goods on the quality of final goods.[23] To my knowledge, a recent study by Amiti and Konings (2007) on Indonesian plants is the closest. The authors estimate the effect of a reduction in tariffs on intermediate goods on the total factor productivity (TFP) of importing firms and non-importing firms instead of product quality.

Their findings are consistent with Remark 1. Non-importing firms improve TFPs though importing firms experience a larger improvement more than non-importing firms. Their finding was puzzling in the conventional love of variety model or the quality ladder model unless there exists some externality between importing firms and non-importing firms as the authors suggest. However, it is totally plausible in the current model. There are at least two remarks on using Amiti and Konings (2007) as an test for Remark 1. First, Indonesia is not a developed country which the current model mainly considers. Indonesian manufacturing sector can be regarded as a part of regional production chains among East and Southeast Asian countries, many of which are developing countries. Therefore, it is likely that trade liberalization may have increased trade

[23] Verhoogen (2008) investigates the effect of trade liberalization with respect to final goods on the quality of final goods in Mexico.

in intermediate goods with those developing countries. Second, instead of product quality, the authors estimate TFP without controlling product quality. Since it is known that the measured TFP may reflect the quality of output as well as true TFP, the change in measured TFP might reflect the change in product quality (Katayama, Lu, and Tybout, 2006; Foster, Haltiwanger, and Syverson, 2008).[24]

3.5 The Lowest Quality Thresholds and the Mass of Consumption Varieties

Finally, I analyze the effect of trade liberalization on the levels of the lowest quality thresholds and the mass of consumption varieties. Consistent with Melitz (2003) and Kasahara and Lapham (2008), trade liberalization raises the lowest quality thresholds both in exporting sectors and in importing sectors and reduces the mass of survival teams, i.e. the mass of consumption varieties.

Proposition 4 *(i)The lowest quality thresholds of final producers and Zi-suppliers in LEC sectors rise, i.e.* $x_L \in (x_L^a, x_T)$ *and* $z_{1L} = z_{2L}^* \in (z_{1L}^a, z_T)$. *The lowest quality thresholds of Zi-suppliers in HEC sectors fall. (ii) The mass of consumption varieties falls.*

Proof. In Appendix. ■

Since high quality final producers upgrade the quality of final goods at a higher rate than low quality final producers, low quality final producers must exit from the market, even though they might upgrade the product quality. Therefore, the mass of consumption varieties falls. On the other hand, the mass of Zi-suppliers increases in LEC sectors and decreases in HEC sectors from Lemma 7. Therefore, the lowest quality threshold of Zi-suppliers rises in LEC sectors and falls in HEC sectors.[25]

[24] It is possible to show Remark 1 can be applied for a coarse measure of team's productivity, revenue per worker (revenue TFP) $RTFP(x) \equiv r\left(q\left(x\right)\right) / \left[\bar{c}q\left(x\right) + f\right]$. Alternatively, the quality parameters x, z_1, and z_2 can be interpreted as productivity in such a model where teams produce symmetric goods in a Cobb-Douglass production technology.

[25] From Proposition 3, it is possible that even final producers that upgrade the final good must exit since the degree of their quality upgrading is smaller than that of high quality firms. Therefore, some might think that the quality upgrading of firms that cannot import is not important if they cannot survive. However, notice that Proposition 4 heavily depends on the assumption that the total expenditure on final goods is constant. If the quality upgrading expands the total expenditure, e.g. in a multi-industry setting, it would be possible that all final producers upgrading the product quality can survive after trade liberalization.

3.6 Complete Specialization Equilibrium

When f_I is sufficiently small, countries specialize in the final good sector and the LEC sector. All teams are international teams. Because of the symmetry of Home and Foreign, Home final producers are matched with a half of Home $Z1$-suppliers and a half of Foreign $Z2$-suppliers. The world economy is equivalent with a closed economy with $2\bar{L}$ of labor endowment, common entry costs f_{Z1e} both in $Z1$-sector and in $Z2$-sector, and production fixed costs $f_I + f$ instead of f. The assignment functions are

$$m_{Z1}(x) = m_{Z2}(x) = m_{Z1}^{a}(x) = \left(\frac{f_{Xe}}{f_{Z1}}\right) x \text{ for } x \geq x_L. \tag{27}$$

Point D in Figure 9 expresses $m_{Zi}(x)$ with iso-quality curve of $\bar{q}(x)$. Points A, A*, and C are those for the Home autarky equilibrium, the Foreign autarky equilibrium, and the incomplete specialization equilibrium in Figure 8. The quality of final goods produced by a final producer with given quality is higher than in an incomplete specialization equilibrium. Since (27) is independent of f_I, trade liberalization does not affect the quality of final goods. The mass of entrants of final producers and Zi-suppliers are the same as the autarky, which are also independent of f_I. The lowest quality thresholds in final goods sector and in LEC sectors are

$$x_L = \left[\frac{f + f_I}{f_{Xe}(k-3)}\right]^{1/k}, \; z_{iL} = \left[\frac{f + f_I}{f_{Zie}(k-3)}\right]^{1/k} \text{ and } M = \frac{(k-3)}{k\sigma}\left(\frac{\bar{L}}{f + f_I}\right).$$

Therefore, trade liberalization increases the mass of consumption varieties, by lowering the lowest quality thresholds.

4 Concluding Remarks

This paper presents a new mechanism of quality-upgrading in a tractable general equilibrium model of matching of firms heterogeneous in product quality. Trade in intermediate goods between developed countries raises the quality of final goods by improving matching of firms in a production process. The quality upgrading arises both from the short-run effect of convergence in the matching of firms and from the long-run competition effect of specialization. The model provides a number of plausible predictions on firm-level trade in contrast to the previous model of heterogeneous firm models. Firms selectively trade with those with similar characteristics. Trade costs concentrate both exporting and importing into a small portion of large firms producing high quality products, though some portion of large and high quality firms always choose not to trade. Trade upgrades the quality of final producers that do not use imported intermediate goods, which is supported by an empirical study by Amiti and Konings (2007).

The model presented in the paper is highly simplified. I remark on some extensions. The current model abstracts away from several frictions in matching, especially search frictions and incompleteness of contracts. One way to incorporate search frictions is to introduce a dynamic search process into multi-sided matching a la Shimer and Smith (2000). In such setting, the authors confirm the assortative matching holds on average but with some frictional deviations. Since the main predictions in the current paper are derived from the assortative matching result, an introduction of search frictions will make the model quantitatively more realistic with maintaining the qualitative predictions. Second, the hold up problem due to incomplete contracts affects team's choice on the organizational form, i.e. FDI or arm's length. An introduction of contract costs will allow us to examine the interaction between matching of heterogeneous firms and firm's boundaries.

Throughout this paper, firms trade with relationship-specific inputs under nonlinear pricing. An alternative model of quality-differentiated goods might be a market-based model with linear pricing, in which suppliers announce the price and quality of intermediate goods and wait for final producers to come and buy them. The market-based model may be realistic for standardized intermediate goods, e.g. steal. Under the imperfect competition, even the best supplier does not usually take all of the market in order to raise a price; therefore, such model will again see a matching problem between suppliers and final producers. Whether the mechanism of international matching found in this paper continue to exist is an interesting question for future research.

Finally, empirical studies on international matching are necessary. To construct an ideal data set to directly test assortative matching, one needs to match customs transaction data with data on characteristics of exporters and importers in at least two countries. Although international matching of firm-level data is currently very difficult, I believe that it will greatly improve our understanding of firms' trade.

References

Antràs, Pol, Luis Garicano, and Estaben Rossi-Hansberg. 2006. "Offshoring in a Knowledge Economy." *Quarterly Journal of Economics*, 121(1): 31-77.

Antràs, Pol, and Elhanan Helpman. 2004. "Global Sourcing." *Journal of Political Economy*, 112(3): 552-80.

Arkolakis, Costas. 2008. "Market Penetration Costs and the New Consumers Margin in International Trade." NBER working paper 14214.

Amiti, Mary, and Jozef Konings. 2007. "Trade Liberalization, Intermediate Inputs, and Productivity: Evidence from Indonesia." *American Economic Review*, 97(5): 1611-1638.

Axtell, Robert L. 2001. "Zipf Distribution of U.S. Firm Sizes." *Science*, 293(5536): 1818-20.

Baldwin, Richard E., and James Harrigan. 2007. "Zeros, Quality and Space: Trade Theory and Trade Evidence." NBER Working Paper No.13214.

Becker, Gary S. 1973. "A Theory of Marriage: Part I." *Journal of Political Economy*, 81(4): 813-46.

Bernard, Andrew B., and J. Bradford Jensen. 1999. "Exceptional Exporter Performance: Cause, Effect, or Both?" *Journal of International Economics*, 47(1): 1-25.

Bernard, Andrew B., Jonathan Eaton, J. Bradford Jensen, and Samuel Kortum. 2003. "Plants and Productivity in International Trade." *American Economic Review*, 93(4): 1268-90.

Bernard, Andrew B., J. Bradford Jensen, Stephen J. Redding, and Peter K. Schott. 2007. "Firms in International Trade." *Journal of Economic Perspectives*, 21(3): 105-30.

Bernard, Andrew B., J. Bradford Jensen, and Peter K. Schott. 2005. "Importers, Exporters and Multinationals: A Portrait of Firms in the U.S. That Trade Goods." NBER working paper No.11404.

Biscourp, Pierre, and Francis Kramarz. 2007. "Employment, Skill Structure and International Trade: Firm-Level Evidence for France." *Journal of International Economics*, 72(1): 22-51.

Broda, Christian, Joshua Greenfield, and David E. Weinstein. 2006. "From Groundnuts to Globalization: A Structural Estimate of Trade and Growth." NBER Working Paper No.12512.

Casella, Alessandra, and James E. Rauch. 2002. "Anonymous Market and Group Ties in International Trade." *Journal of International Economics*, 58(1): 19-47.

Clerides, Sofronis K., Saul Lach, and James R. Tybout. 1998. "Is Learning by Exporting Important? Micro-Dynamic Evidence from Colombia, Mexico, and Morocco." *Quarterly Journal of Economics*, 113(3): 903-947.

Costinot, Arnaud. 2008. "An Elementary Theory of Comparative Advantage." forthcoming in *Econometrica*.

Costinot, Arnaud, and Jonathan Vogel. 2008. "Matching and Inequality in the World Economy." http://econ-www.mit.edu/faculty/costinot/papers.

Ethier, Wilfred J. 1982. "National and International Returns to Scale in the Modern Theory of International Trade." *American Economic Review*, 72(3): 389-405.

Feenstra, Robert C. 1998. "Integration of Trade and Disintegration of Production in the Global Economy." *Journal of Economic Perspectives*, 12(4): 31-50.

Flam, Harry, and Elhanan Helpman. 1987. "Vertical Product Differentiation and North-South Trade." *American Economic Review*, 77(5): 810-22.

Foster, Lucia, John Haltiwanger, and Chad Syverson. 2008 "Reallocation, Firm Turnover, and Efficiency: Selection on Productivity or Profitability?" *American Economic Review*, 98(1): 394-425

Gale, David, and Lloyd Shapley. 1962. "College Admissions and the Stability of Marriage." *American Mathematical Monthly*, 69(1): 9-15.

Grossman, Gene M., and Elhanan Helpman. 1991. *Innovation and Growth in the Global Economy*. Cambridge: MIT Press.

Grossman, Gene M., and Elhanan Helpman. 2005. "Outsourcing in a Global Economy." *Review of Economic Studies*, 72(1): 135-59.

Grossman, Gene M., and Giovanni Maggi. 2000. “Diversity and Trade.” *American Economic Review*, 90(5): 1255-75.

Helpman, Elhanan, Marc J. Melitz, and Stephen R. Yeaple. 2004. “Export Versus FDI with Heterogeneous Firms.” *American Economic Review*, 94(1): 300-16.

Hummels, David, and Peter J. Klenow. 2005. “The Variety and Quality of a Nation’s Exports.” *American Economic Review*, 95(3): 704-23.

Hummels, David, and Alexander Skiba. 2004. “Shipping the Good Apples Out: An Empirical Confirmation of the Alchian-Allen Conjecture.” *Journal of Political Economy*, 112(6): 1384-402.

Johnson, Robert C. 2008 “Trade and Prices with Heterogeneous Firms.” http://robjohnson41.googlepages.com/.

Katayama, Hajime, Shihua Lu, and James R. Tybout. 2006. “Firm-level Productivity Studies: Illusions and a Solution.” http://econ.la.psu.edu/~jtybout/tyboutj.html

Kasahara, Hiroyuki, and Beverly Lapham. 2007. “Productivity and the Decision to Import and Export: Theory and Evidence.” http://economics.uwo.ca/faculty/kasahara/.

Kremer, Michael. 1993. “The O-Ring Theory of Economic Development.” *Quarterly Journal of Economics*, 108(3): 551-75.

Kremer, Michael, and Eric Maskin. 2006. “Globalization and Inequality.” http://www.economics.harvard.edu/faculty/kremer/.

Krugman, Paul. 1980. “Scale Economies, Product Differentiation, and the Pattern of Trade.” *American Economic Review*, 70(5): 950-59.

Kugler, Maurice, and Eric A. Verhoogen. 2008. “The Quality-Complementarity Hypothesis: Theory and Evidence from Colombia” NBER Working Paper No. 14418.

Kugler, Maurice, and Eric A. Verhoogen. 2009. “Plants and Imported Inputs: New Facts and an Interpretation.” forthcoming in *American Economic Review Papers and Proceedings.*.

Linden, Greg, Kenneth L. Kraemer, Jason Dedrick. 2007. “Who Captures Value in a Global Innovation Network? The Case of Apple’s iPod.” http://pcic.merage.uci.edu/papers.asp.

Melitz, Marc J. 2003. "The Impact of Trade on Intra-Industry Reallocations and Aggregate Industry Productivity." *Econometrica*, 71(6): 1695-725.

Milgrom, Paul, and John Roberts. 1990. "The Economics of Modern Manufacturing: Technology, Strategy, and Organization." *American Economic Review*, 80(3): 511-28.

Murphy, Kevin M., and Andrei Shleifer. 1997. "Quality and Trade." *Journal of Development Economics,* 53(1): 1-15.

Nocke, Volker, and Stephen R. Yeaple. 2008. "An Assignment Theory of Foreign Direct Investment." *Review of Economic Studies*, 75: 529-557.

Ohnsorge, Franziska and Daniel Trefler. 2007. "Sorting It Out: International Trade with Heterogeneous Workers." *Journal of Political Economy*,115(5): 868-92.

Rauch, James E., and Alessandra Casella. 2003. "Overcoming Informational Barriers to International Resource Allocation: Prices and Ties." *Economic Journal*, 113(484): 21-42.

Rauch, James E., and Vitor Trindade. 2003. "Information, International Substitutability, and Globalization." *American Economic Review*, 93(3): 775-91.

Roth, Alvin E., and Oliveira Sotomayor. 1990. *Two-sided Matching: A Study on Game-Theoretic Modeling and Analysis,* Cambridge University Press, Cambridge.

Sattinger, Michael. 1979. "Differential Rents and the Distribution of Earnings." *Oxford Economic Papers*, 31(1): 60-71.

Sattinger, Michael. 1993. "Assignment Models of the Distribution of Earnings." *Journal of Economic Literature*, 31(2): 831-80.

Schott, Peter K. 2004. "Across-Product Versus within-Product Specialization in International Trade." *Quarterly Journal of Economics*, 119(2): 647-78.

Shimer, Robert, and Lones Smith. 2000. "Assortative Matching and Search." *Econometrica*, 68(2): 343-70.

Sutton, John. 2007. "Quality, Trade and The "Moving Window": The Globalization Process.", *Economic Journal*, 117(November): F469-F498.

Triplet, Jack E. 2006. *Handbook on Hedonic Indexes and Quality Adjustments in Price Indexes.* OECD publishing.

Verhoogen, Eric A. 2008. "Trade, Quality Upgrading and Wage Inequality in the Mexican Manufacturing Sector." *Quarterly Journal of Economics*, 123(2): 489-530.

5 Appendix

Proof for Lemma 1 Without loss of generality, it is sufficient to show that neither of the following two cases holds: (i) there exist x and x' such that $x > x'$, $m_{Z1}(x) > m_{Z1}(x')$, and $m_{Z2}(x') > m_{Z2}(x)$; (ii) there exist x and x' such that $x > x'$, $m_{Z1}(x') > m_{Z1}(x)$, and $m_{Z2}(x') > m_{Z2}(x)$.

(i) Suppose $x > x'$ and $m_{Z1}(x) > m_{Z1}(x')$. By definition of $m_{Zi}(x)$ and $\pi_X(x)$, it follows that

$$\begin{aligned} \pi_X(x) &= Axm_{Z1}(x)\,m_{Z2}(x) - f - \pi_{Z1}(m_{Z1}(x)) - \pi_{Z2}(m_{Z2}(x)) \\ &\geq Axm_{Z1}(x)\,m_{Z2}(x') - f - \pi_{Z1}(m_{Z1}(x)) - \pi_{Z2}(m_{Z2}(x')), \qquad (28) \\ \text{and } \pi_X(x') &= Ax'm_{Z1}(x')\,m_{Z2}(x') - f - \pi_{Z1}(m_{Z1}(x')) - \pi_{Z2}(m_{Z2}(x')) \\ &\geq Ax'm_{Z1}(x')\,m_{Z2}(x) - \pi_{Z1}(m_{Z1}(x')) - \pi_{Z2}(m_{Z2}(x)). \qquad (29) \end{aligned}$$

By adding (28) to (29), I obtain

$$\begin{aligned} & Axm_{Z1}(x)\,m_{Z2}(x) + Ax'm_{Z1}(x')\,m_{Z2}(x') - Axm_{Z1}(x)\,m_{Z2}(x') - Ax'm_{Z1}(x')\,m_{Z2}(x) \\ = \;& A\left[xm_{Z1}(x) - x'm_{Z1}(x')\right]\left[m_{Z2}(x) - m_{Z2}(x')\right] \geq 0. \qquad (30) \end{aligned}$$

The inequality (30) implies $m_{Z2}(x) \geq m_{Z2}(x')$.

(ii) Suppose $x > x'$ and $m_{Z1}(x') > m_{Z1}(x)$. By definition of $m_{Zi}(x)$ and $\pi_X(x)$, it follows that

$$\begin{aligned} \pi_X(x) &= Axm_{Z1}(x)\,m_{Z2}(x) - f - \pi_{Z1}(m_{Z1}(x)) - \pi_{Z2}(m_{Z2}(x)) \\ &\geq Axm_{Z1}(x')\,m_{Z2}(x') - f - \pi_{Z1}(m_{Z1}(x')) - \pi_{Z2}(m_{Z2}(x')), \qquad (31) \\ \text{and } \pi_X(x') &= Ax'm_{Z1}(x')\,m_{Z2}(x') - f - \pi_{Z1}(m_{Z1}(x')) - \pi_{Z2}(m_{Z2}(x')) \\ &\geq Ax'm_{Z1}(x)\,m_{Z2}(x) - f - \pi_{Z1}(m_{Z1}(x)) - \pi_{Z2}(m_{Z2}(x)) \qquad (32) \end{aligned}$$

By adding (31) to (32), I obtain

$$\begin{aligned} & Axm_{Z1}(x)\,m_{Z2}(x) + Ax'm_{Z1}(x')\,m_{Z2}(x') - Axm_{Z1}(x')\,m_{Z2}(x') - Ax'm_{Z1}(x')\,m_{Z2}(x') \\ = \; & A\left(x - x'\right)\left[m_{Z1}(x)\,m_{Z2}(x) - m_{Z1}(x')\,m_{Z2}(x')\right] \geq 0 \end{aligned} \tag{33}$$

The inequality (30) implies $m_{Z2}(x) \geq m_{Z2}(x')$. Q.E.D.

Proof for Lemma 2 (i) Suppose the two countries are in autarky for some finite f_I. Since $\pi^{*a}_{Z1}(z) - \pi^{a}_{Z1}(z) = z^3 f\left[\left(z^{*a}_{1L}\right)^{-3} - \left(z^{a}_{1L}\right)^{-3}\right]/3$ and $z^{*a}_{1L} > z^{a}_{1L}$ hold from (13) and (16), there must be some z' such that $\pi^{*}_{Z1}(z') > \pi_{Z1}(z') + f_I$. Then, final producers and $Z2$-suppliers in Foreign prefer Home $Z1$-suppliers with quality z' to Foreign $Z1$-suppliers with the same quality. Therefore, the autarky matching is unstable for any finite f_I.

(ii) Suppose the two countries have positive mass of entrants in final good sectors and in LEC sectors, but no entrant in HEC sectors. I show that when very small mass of Home $Z2$-suppliers enter, their expected profits must exceed entry costs when f_I is sufficiently large. Suppose that the mass of the new entrants is so small that it does not change matching pattern. If the new Home $Z2$-suppliers draw $z_2 > z^{*}_{2L} (= z_{1L})$, then trade costs allow them to earns $\pi_{Z2}(z_2) = \pi^{*}_{Z2}(z_2) + f_I = \pi_{Z1}(z_2) + f_I$.

Notice that the allocation in a complete specialization equilibrium with f_I is equivalent with the one in the free trade equilibrium in which fixed cost f is replaced with $f + f_I$ since all teams must pay f_I. From (13) and (16), it follows that

$$x_L = \left[\frac{f + f_I}{f_{Xe}(k-3)}\right]^{1/k}, \; z_{1L} = \left[\frac{f + f_I}{f_{Z1e}(k-3)}\right]^{1/k}, \text{ and } \pi_{Z1}(z) = \frac{(f + f_I)}{3}\left[\left(\frac{z}{z_{1L}}\right)^3 - 1\right]. \tag{34}$$

Then, I obtain

$$\pi_{Z2}(z_2) = \pi_{Z1}(z_2) + f_I = \frac{z_2^3 (f + f_I)^{(k-3)/k}\left[(k-3)\, f_{Z1e}\right]^{3/k}}{3} + \frac{2f_I - f}{3} \text{ if } z_2 \in [z_{1L}, \infty). \tag{35}$$

On the other hand, when the new Home $Z2$-suppliers draw $z'_2 < z_{1L}$, they receive approximately all of the joint profit by forming teams with final producers with $x_L - \epsilon_x$ and $Z1$-suppliers with $z_{1L} - \epsilon_{z1}$ for some very small $\epsilon_x > 0$ and $\epsilon_{z1} > 0$ and by paying them only approximately zero profit, i.e.

$$\pi_{Z2}(z'_2) \simeq Ax_L z_{1L} z'_2 - f = f\left(\frac{z'_2}{z_{2L}} - 1\right). \tag{36}$$

I used the cutoff condition (8) to obtain (36). From $Ax_L z_{1L} z_{2L}^* = f + f_I$, $z_{2L}^* = z_{1L}$, and (34), z_{2L} is obtained as

$$z_{2L} = \frac{f}{Ax_L z_{1L}} = \frac{f}{(f + f_I)^{(k-1)/k} \left[f_{Z1e}(k-3)\right]^{1/k}}. \tag{37}$$

Since z_{2L} decreases in f_I from (37) and $\pi_{Z2}(z_2)$ increases in f_I for all $z_2 \geq z_{2L}$ from (35) and (36), $[1 - G(z_{2L})]\bar{\pi}_{Z2} = \int_{z_{2L}}^{\infty} \pi_{Z2}(z) g(z) dz$ is increasing in f_I. Therefore, for sufficiently high f_I, the expected profit of $Z2$-suppliers must be strictly positive so that countries must produce both intermediate goods. Q.E.D.

Proof for Lemma 3 It is sufficient to consider two cases. Case (i): an exportable Home Zi-supplier with quality z_i is matched with a Foreign final producer with x and a Foreign Zj-supplier with z_j. Suppose $z_i \neq z_j$. From the mirror-image structure of Home and Foreign, $\pi_{Zi}(z) = \pi_{Zj}^*(z)$ for all z. Therefore, the Foreign final producer earns

$$\begin{aligned}
\pi_X^*(x) &= Axz_i z_j - \pi_{Zi}(z_i) - \pi_{Zj}^*(z_j) - f_I - f \\
&= Axz_i z_j - \pi_{Zj}^*(z_i) - \pi_{Zj}^*(z_j) - f_I - f \\
&\geq \max_{z_i', z_j'} Axz_i' z_j' - \pi_{Zj}^*(z_i') - \pi_{Zj}^*(z_j') - f_I - f.
\end{aligned}$$

Since the second order condition for maximization requires $\pi_{Zj}^{*\prime\prime}(z) > 0$, $\bar{z} \equiv (z_i + z_j)/2$ satisfies

$$\begin{aligned}
Axz_i z_j - \pi_{Zj}^*(z_i) - \pi_{Zj}^*(z_j) - f_I &< Ax(\bar{z})^2 - 2\pi_{Zj}^*(\bar{z}) - f - f_I \\
&= Ax(\bar{z})^2 - \pi_{Zi}(\bar{z}) - \pi_{Zj}^*(\bar{z}) - f - f_I.
\end{aligned}$$

The inequality implies that the Home final producer with x forms a new team with a Home Zi-supplier with $\bar{z}$ and Foreign Zj-supplier with $\bar{z}$, which contradicts with stable matching.

Case (ii): an exportable Home Zi-supplier with quality z_i is matched with a Home final producer with x and a Home Zj-supplier with z_j. Suppose $z_i \neq z_j$. From the mirror-image structure, Foreign Zj-suppliers with quality z_i are also exportable. Since $\pi_{Zi}(z) = \pi_{Zj}^*(z)$ holds for all z, the Home final producer earns

$$\begin{aligned}
\pi_X(x) &= Axz_i z_j - \pi_{Zi}(z_i) - \pi_{Zj}(z_j) - f \\
&= Axz_i z_j - \pi_{Zi}(z_i) - \pi_{Zj}^*(z_j) - f - f_I \\
&= Axz_i z_j - \pi_{Zj}^*(z_i) - \pi_{Zj}^*(z_j) - f - f_I \\
&\geq \max_{z_i', z_j'} Axz_i' z_j' - \pi_{Zj}^*(z_i') - \pi_{Zj}^*(z_j') - f_I - f.
\end{aligned}$$

From the second order condition $\pi^{*\prime\prime}_{Zj}(z) > 0$, $\bar{z} \equiv (z_i + z_j)/2$ satisfies

$$\begin{aligned} Axz_iz_j - \pi^*_{Zj}(z_i) - \pi^*_{Zj}(z_j) - f_I &< Ax(\bar{z})^2 - 2\pi^*_{Zj}(\bar{z}) - f - f_I \\ &= Ax(\bar{z})^2 - \pi_{Zi}(\bar{z}) - \pi^*_{Zj}(\bar{z}) - f - f_I. \end{aligned}$$

The inequality implies that the Home final producer with x forms a new team with a Home Zi-supplier with $\bar{z}$ and Foreign Zj-supplier with $\bar{z}$, which contradicts with stable matching. Q.E.D.

Proof for Lemma 4 The proof consists of Claims 1 to 3.

Claim 1 *Home $Z2$-suppliers and Foreign $Z1$-suppliers are non-exportable.*

Proof. Suppose Home Zi-suppliers with quality z are exportable. Under the assortative matching, the market clearing condition for matching between Home Zi-suppliers is expressed

$$M_{Z1e} \int_z^\infty \theta^D_{Z1}(t)\, g(t)dt = M_{Z2e} \int_z^\infty \theta^D_{Z2}(t)\, g(t)dt, \tag{38}$$

where $\theta^D_{Zi}(z_i)$ is the share of Home Zi-suppliers with quality z_i choosing domestic partners. By definition, $\theta^D_{Zi}(z_i) < 1$ holds only when Home Zi-suppliers with quality z_i are exportable. A differentiation of (38) with respect to z leads to

$$M_{Z1e}\theta^D_{Z1}(z) = M_{Z2e}\theta^D_{Z2}(z). \tag{39}$$

Notice that if Home Zi-suppliers with quality z are exportable, $\pi^*_{Zi}(z) = \pi_{Zi}(z) - f_I$, then Home Zj-suppliers with quality z are non-exportable since under the mirror-image structure, it follows that $\pi_{Zj}(z) = \pi^*_{Zj}(z) - f_I$. Therefore, only one of $\theta^D_{Z1}(z)$ or $\theta^D_{Z2}(z)$ can be smaller than unity. From $M_{Z1e} > M_{Z2e}$, only a combination of $\theta^D_{Z2}(z) = 1$ and $\theta^D_{Z1}(z) = M_{Z2e}/M_{Z1e} < 1$ satisfies condition (39). Therefore, Home $Z2$-suppliers are all non-exportable. Q.E.D. ■

Claim 2 *$m_{Z1}(x) \geq m_{Z2}(x)$ for all $x \geq x_L$.*

Proof. Since $\theta^D_{Z2}(z) = 1$ for all $z \geq z_{2L}$ from Claim 1, the market clearing condition (38) becomes

$$M_{Z1e} \int_{m_{Z1}(x)}^\infty \theta^D_{Z1}(t)\, g(t)dt = M_{Z2e}[1 - G(m_{Z2}(x))] \text{ for all } x \geq x_L.$$

A straightforward manipulation yields

$$\frac{M_{Z1e}}{M_{Z2e}} \int_{m_{Z1}(x)}^\infty \left(\theta^D_{Z1}(t) - \frac{M_{Z2e}}{M_{Z1e}}\right) g(t)dt = G(m_{Z1}(x)) - G(m_{Z2}(x)) \text{ for all } x \geq x_L. \tag{40}$$

Since $\theta^D_{Z1}(z) \geq M_{Z2e}/M_{Z1e}$ for all $x \geq x_L$ from the proof for Claim 1, the left hand side of (40) is non-negative for all $x \geq x_L$. Q.E.D. ■

Claim 3 *(i)* $\pi^*_{Z1}(z) - \pi_{Z1}(z) = \pi_{Z2}(z) - \pi^*_{Z2}(z) = f_I$ *for all* $z \geq z_T$*. (ii)* $0 \leq \pi^*_{Z1}(z) - \pi_{Z1}(z) = \pi_{Z2}(z) - \pi^*_{Z2}(z) < f_I$ *for all* $z \in [z^*_{1L}, z_T)$*.*

Proof. (i) Consider two teams with bundles of quality parameters, (x, z_1, z_2) and (x', z'_1, z'_2), respectively. Suppose $z_2 = z'_1 (\equiv \hat{z})$. Claim 2 implies that $x \geq x'$ and $z_1 \geq z_2 = z'_1 \geq z'_2$. Therefore, from the first order condition, we obtain

$$\pi'_{Z2}(\hat{z}) = Axz_1 \geq \pi'_{Z1}(\hat{z}) = Ax'z'_2. \tag{41}$$

In a complete specialization equilibrium, there exists some exportable Home $Z1$-supplier with quality $\tilde{z} \geq z_L$ such that $\pi^*_{Z1}(\tilde{z}) - \pi_{Z1}(\tilde{z}) = f_I$. Suppose there exists $z > \tilde{z}$ such that $\pi^*_{Z1}(z) - \pi_{Z1}(z) < f_I$ on the contrary. Since $\pi^*_{Z1}(z) = \pi_{Z2}(z)$ holds in equilibrium, the difference in the profit schedules satisfies

$$\pi^*_{Z1}(z) - \pi_{Z1}(z) = f_I + \int_{z_T}^{z} \left[\pi'_{Z2}(u) - \pi'_{Z1}(u)\right] du.$$

The second term in the right hand side must be non-negative from (41), which it contradicts with $\pi^*_{Z1}(z) - \pi_{Z1}(z) < f_I$. Therefore, if $\pi^*_{Z1}(z') - \pi_{Z1}(z') = f_I$ holds for some z', then $\pi^*_{Z1}(z) - \pi_{Z1}(z) = f_I$ holds for all $z \geq z'$. (ii) Since $z_{2L} = z^*_{1L} < z_{1L} = z^*_{2L}$ from $M_{Z2e} > M_{Z1e}$, the difference in the profit schedules is

$$\pi^*_{Z1}(z) - \pi_{Z1}(z) = \pi^*_{Z1}(z_{1L}) + \int_{z^*_{1L}}^{z} \left[\pi'_{Z2}(u) - \pi'_{Z1}(u)\right] du \text{ for all } z \in [z_{1L}, z_T). \tag{42}$$

From (41), $\pi^*_{Z1}(z) - \pi_{Z1}(z) \in [0, f_I)$ for $z \in [z^*_{1L}, z_T)$. Q.E.D. ■

Finally, I prove LEC sectors have more entrants than HEC sectors. Q.E.D.

Proof for Lemma 6 Suppose $M_{Z1e} = M^*_{Z2e} < M_{Z2e} = M_{Z1e}$, on the contrary. Then, from similar arguments in Claims 1 to 3, it is possible to show $m_{Z2}(x) \geq m_{Z1}(x)$ for $x \geq x_L$, $\pi'_{Z1}(z) = \pi'_{Z2}(z)$ for $z \geq z_T$ and $\pi'_{Z1}(z) \geq \pi'_{Z2}(z)$ for $z < z_T$.

From integration by parts, the free entry condition becomes

$$f_{Zie} = [1 - G(z_{iL})]\,\bar{\pi}_{Zi} = \int_{z_{iL}}^{z_T} \pi'_{Zi}(t)\,[1 - G(t)]\,dt + \int_{z_T}^{\infty} \pi'_{Zi}(t)\,[1 - G(t)]du.$$

Since $\pi'_{Z1}(z) = \pi'_{Z2}(z)$ for $z \geq z_T$, the difference in the free entry conditions is

$$f_{Z2e} - f_{Z1e} = \int_{z_{2L}}^{z_T} \pi'_{Z2}(t)\,[1 - G(t)]\,dt - \int_{z_{1L}}^{z_T} \pi'_{Z1}(t)\,[1 - G(t)]\,dt > 0.$$

Since $\pi'_{Z1}(z) \geq \pi'_{Z2}(z)$ for $z < z_T$, it requires $z_{1L} > z_{2L}$, which contradicts with $m_{Z2}(x) \geq m_{Z1}(x)$ for $x \geq x_L$. Q.E.D.

Proof for Lemma 7 (1) From $k\left[1-G\left(x\right)\right]=xg(x)$ and the integration by parts, the free entry condition can be rewritten as

$$\begin{aligned}\frac{f_{Xe}}{1-G\left(x_L\right)} &= \frac{A}{1-G\left(x_L\right)}\int_{x_L}^{\infty} m_{Z1}\left(t\right)m_{Z1}\left(t\right)\left[1-G\left(t\right)\right]dt \\ &= \frac{A}{k}\bar{q}.\end{aligned}$$

Since $A=\bar{L}/\left(\sigma M\bar{q}\right)$ from the aggregate zero profit, it follows that

$$M_{Xe}=\frac{M}{1-G\left(x_L\right)}=\frac{\bar{L}}{f_{Xe}k\sigma}=M_{Xe}^{a}.$$

(2)(3)The proof for (2) and (3) consists of Claims 4 to 6.

Claim 4 *Let $\eta_{Zi}\left(x\right)\equiv xm'_{Zi}\left(x\right)/m_{Zi}\left(x\right)$. Then, it follows that*

$$\eta_{Zi}\left(x\right)=1 \text{ for } x>x_T \text{ and } \eta_{Zi}\left(x\right)=\frac{M_{Xe}\left[1-G\left(x\right)\right]}{M_{Zie}\left[1-G\left(m_{Zi}\left(x\right)\right)\right]}.$$

Proof. Since $m_{Zi}\left(x\right)$ is linear in x for $x>x_T$, $\eta_{Zi}\left(x\right)=1$ for $x>x_T$. From (21), $m_{Zi}\left(x\right)$ satisfies

$$\left(\frac{1}{m_{Zi}\left(x\right)}\right)^k=\frac{M_{Xe}}{M_{Zie}}\left(\frac{1}{x}\right)^k+\left(\frac{1}{z_T}\right)^k-\frac{M_{Xe}}{M_{Zie}}\left(\frac{1}{x_T}\right)^k \text{ for } x\in\left[x_L,x_T\right].$$

From the Implicit Function theorem, $\eta_{Zi}\left(x\right)$ is solved as

$$\eta_{Zi}\left(x\right)=\frac{M_{Xe}}{M_{Zie}}\left(\frac{m_{Zi}\left(x\right)}{x}\right)^k=\frac{M_{Xe}\left[1-G\left(x\right)\right]}{M_{Zie}\left[1-G\left(m_{Zi}\left(x\right)\right)\right]} \text{ for } x\in\left[x_L,x_T\right]. \tag{43}$$

■

Claim 5 *The mass of entrants satisfies*

$$\frac{M_{Z1e}}{M_{Xe}}\frac{f_{Z1e}}{f_{Xe}}+\frac{M_{Z2e}}{M_{Xe}}\frac{f_{Z2e}}{f_{Xe}}=2. \tag{44}$$

Proof. From the integration by parts and the first order condition, the free entry condition for final producers is

$$\begin{aligned}f_{Xe} &= \int_{x_L}^{\infty}\pi_X\left(t\right)g(t)dt \\ &= \int_{x_L}^{\infty}\pi'_X\left(t\right)\left[1-G(t)\right]dt \\ &= A\int_{x_L}^{\infty}m_{Z1}\left(t\right)m_{Z2}\left(t\right)\left[1-G(t)\right]dt. \end{aligned} \tag{45}$$

From Claim 4, the free entry condition for Zi-suppliers is

$$
\begin{aligned}
f_{Zie} &= A\int_{x_L}^{\infty} m_{Z1}(t)\, m_{Z2}(t)\, \eta_{Zi}(t)\left[1-G(m_{Zi}(t))\right] dt \\
&= A\int_{x_T}^{\infty} m_{Z1}(t)\, m_{Z2}(t)\left[1-G(m_{Zi}(t))\right] dt \\
&\quad +\frac{M_{Xe}}{M_{Zie}} A\int_{x_L}^{x_T} m_{Z1}(t)\, m_{Z2}(t)\left[1-G(t)\right] dt. \qquad (46)
\end{aligned}
$$

Since $2M_{Xe}\left[1-G(x)\right]=\sum_{i=1,2} M_{Zie}\left[1-G\left(m_{Zi}(x)\right)\right]$ for $x\geq x_L$, it follows that

$$
\frac{M_{Z1e}}{M_{Xe}} f_{Z1e}+\frac{M_{Z2e}}{M_{Xe}} f_{Z2e}=2A\int_{x_L}^{\infty} m_{Z1}(t)\, m_{Z2}(t)\left[1-G(t)\right] dt=2f_{Xe}.
$$

■

Claim 6

$$
\frac{M_{Z1e}}{M_{Xe}}>\frac{M_{Z1e}^{a}}{M_{Xe}^{a}}>\frac{M_{Z2e}^{a}}{M_{Xe}^{a}}>\frac{M_{Z2e}}{M_{Xe}}.
$$

Proof. From $f_{Z1e}/f_{Xe}=M_{Xe}^{a}/M_{Z1e}^{a}$ and (11), condition (45) is rewritten as

$$
\begin{aligned}
f_{Z1e}=f_{Xe}\frac{f_{Z1e}}{f_{Xe}}= \quad & A\int_{x_T}^{\infty} m_{Z1}(t)\, m_{Z2}(t)\left[1-G(m_{Z1}^{a}(t))\right] dt \\
&+\frac{M_{Xe}^{a}}{M_{Z1e}^{a}} A\int_{x_L}^{x_T} m_{Z1}(t)\, m_{Z2}(t)\left[1-G(t)\right] dt \qquad (47)
\end{aligned}
$$

Since $m_{Z1}^{a}(x)>m_{Z1}(x)$ for $x\geq x_T$, the comparison of (46) and (47) proves that $M_{Z1e}/M_{Xe}>M_{Z1e}^{a}/M_{Xe}^{a}$. From (44), we also obtain $M_{Z2e}^{a}/M_{Xe}^{a}>M_{Z2e}/M_{Xe}$. ■

Under the constraint of (44), it follows that

$$
\frac{M_{Z1e}+M_{Z2e}}{2M_{Xe}}\geq\frac{M_{Z1e}^{a}+M_{Z2e}^{a}}{2M_{Xe}^{a}} \text{ if and only if } \frac{M_{Z1e}}{M_{Xe}}>\frac{M_{Z1e}^{a}}{M_{Xe}^{a}}.
$$

Since $\left(M_{Z1e}^{a}+M_{Z2e}^{a}\right)/2M_{Xe}^{a}>\sqrt{M_{Z1e}^{a}M_{Z2e}^{a}}/M_{Xe}^{a}$, $K>1$ holds. Q.E.D.

Proof for Proposition 3 (i) The degree of quality upgrading is

$$
\Theta(x)=K\left[1-\left[1-\left(\frac{x}{x_T}\right)^{k}\right]\left[3-\left(\frac{x}{x_T}\right)^{k}\right]\frac{\left(M_{Z1e}-M_{Z2e}\right)^{2}}{4M_{Z1e}M_{Z2e}}\right]^{-1/k} \qquad (48)
$$

for $x\in[x_L,x_T)$. From (48), $\Theta(x)$ is increasing in x. (ii) From (20) and (21), $m_{Zi}(x)$ satisfies

$$
\left(\frac{1}{m_{Zi}(x)}\right)^{k}=\frac{M_{Xe}}{M_{Zie}}\left(\frac{1}{x}\right)^{k}+\left(\frac{1}{z_T}\right)^{k}\left(\frac{M_{Zie}-M_{Zje}}{2M_{Zie}}\right) \text{ if } x\leq x_T.
$$

From $M_{Z1e} > M_{Z2e}$, if M_{Zi}/M_{Xe} are close to M^a_{Zi}/M^a_{Xe}, then

$$\begin{aligned}\left(\frac{1}{m_{Z1}\left(x^a_L\right)m_{Z2}\left(x^a_L\right)}\right)^k &= \frac{\left(M_{Xe}\right)^2}{M_{Z1e}M_{Z2e}}\left(\frac{1}{x^a_L}\right)^{2k} - \left(\frac{1}{z_T}\right)^{2k}\left(\frac{\left(M_{Z1e}-M_{Z2e}\right)^2}{4M_{Z1e}M_{Z2e}}\right)\\ &> \frac{\left(M^a_{Xe}\right)^2}{M^a_{Z1e}M^a_{Z2e}}\left(\frac{1}{x^a_L}\right)^{2k} = \left(\frac{1}{z^a_{1L}z^a_{2L}}\right)^k.\end{aligned}$$

Therefore, $q^a\left(x^a_L\right) = x^a_L z^a_{1L} z^a_{2L} > x^a_L m_{Z1}\left(x^a_L\right)m_{Z2}\left(x^a_L\right) = q^t\left(x^a_L\right)$. Q.E.D.

Proof for Proposition 4 Let $\bar{\pi}_X\left(x_L\right)$ be the average profit of final producers when the lowest quality threshold is x_L. From the integration by parts and the first order condition, $\bar{\pi}_X\left(x_L\right)$ is

$$\begin{aligned}\bar{\pi}_X &= \int_{x_L}^{\infty}\pi_X\left(t\right)\left(\frac{g\left(t\right)}{1-G\left(x_L\right)}\right)dt\\ &= \int_{x_L}^{\infty}\pi'_X\left(t\right)\left(\frac{1-G\left(t\right)}{1-G\left(x_L\right)}\right)dt\\ &= A\int_{x_L}^{\infty}m_{Z1}\left(t\right)m_{Z2}\left(t\right)\left(\frac{1-G\left(t\right)}{1-G\left(x_L\right)}\right)dt.\end{aligned}$$

From the cutoff condition (31), it follows that

$$\begin{aligned}\bar{\pi}_X &= A\int_{x_L}^{\infty}\frac{z_{1L}z_{2L}t^2}{\left(x_L\right)^2}\left(\frac{1-G\left(t\right)}{1-G\left(x_L\right)}\right)dt + A\int_{x_L}^{\infty}\left[m_{Z1}\left(t\right)m_{Z2}\left(t\right)-\frac{z_{1L}z_{2L}t^2}{\left(x_L\right)^2}\right]\left(\frac{1-G\left(t\right)}{1-G\left(x_L\right)}\right)dt\\ &= \frac{f}{k-3} + A\int_{x_L}^{\infty}\left[m_{Z1}\left(t\right)m_{Z2}\left(t\right)-\frac{z_{1L}z_{2L}t^2}{\left(x_L\right)^2}\right]\left(\frac{1-G\left(t\right)}{1-G\left(x_L\right)}\right)dt.\end{aligned}$$

From assignment functions (25) and (26), if $x_L < x_T$, then $m_{Z1}\left(x\right)m_{Z2}\left(x\right) > \left(z_{1L}z_{2L}x^2\right)/\left(x_L\right)^2$ for all $x \geq x_L$, while if $x_L \geq x_T$, then $m_{Z1}\left(x\right)m_{Z2}\left(x\right) = \left(z_{1L}z_{2L}x^2\right)/\left(x_L\right)^2$ for all $x \geq x_L$. Therefore, we have

$$\bar{\pi}_X\left(x^a_L\right) > \frac{f}{k-3} = \frac{f_{Xe}}{1-G\left(x^a_L\right)} \text{ and } \bar{\pi}_X\left(x_T\right) = \frac{f}{k-3} < \frac{f_{Xe}}{1-G\left(x_T\right)}.$$

Therefore, there exists $x_L \in \left(x^a_L, x_T\right)$ such that $\bar{\pi}_X\left(x_L\right) = f_{Xe}\left[1-G\left(x_L\right)\right]^{-1}$.

It takes similar steps to prove $z_{1L} \in \left(z^a_{1L}, z_T\right)$. The average profit of Home $Z1$-suppliers when the lowest quality threshold is z_{1L}, $\bar{\pi}_{Z1}\left(z_{1L}\right)$, is

$$\begin{aligned}\bar{\pi}_{Z1}\left(z_{1L}\right) &= A\int_{z_{1L}}^{\infty}\frac{x_L z_{2L}t^2}{\left(z_{1L}\right)^2}\left(\frac{1-G\left(t\right)}{1-G\left(z_{1L}\right)}\right)dt\\ &\quad + A\int_{z_{1L}}^{\infty}\left[m^{-1}_{Z1}\left(t\right)m_{Z2}\left(m^{-1}_{Z1}\left(t\right)\right)-\frac{x_L z_{2L}t^2}{\left(z_{1L}\right)^2}\right]\left(\frac{1-G\left(t\right)}{1-G\left(z_{1L}\right)}\right)dt\\ &= \frac{f}{k-3} + A\int_{z_{1L}}^{\infty}\left[m^{-1}_{Z1}\left(t\right)m_{Z2}\left(m^{-1}_{Z1}\left(t\right)\right)-\frac{x_L z_{2L}t^2}{\left(z_{1L}\right)^2}\right]\left(\frac{1-G\left(t\right)}{1-G\left(z_{1L}\right)}\right)dt,\end{aligned}$$

where $m^{-1}_{Z1}\left(\cdot\right)$ is an inverse function of $m_{Z1}\left(\cdot\right)$. From assignment functions (25) and (26), it follows that if $z_{1L} < z_T$, $m^{-1}_{Z1}\left(z_1\right)m_{Z2}\left(m^{-1}_{Z1}\left(z_1\right)\right) > \left(z_{1L}z_{2L}z_1^2\right)/\left(z_{1L}\right)^2$ for all $z_1 \geq z_{1L}$, while if $z_{1L} \geq z_T$,

$m_{Z1}^{-1}(z_1)\, m_{Z2}\left(m_{Z1}^{-1}(z_1)\right) = \left(z_{1L} z_{2L} z_1^2\right) / \left(z_{1L}\right)^2$ for all $z_1 \geq z_{1L}$. Therefore, we have

$$\bar{\pi}_{Z1}\left(z_{1L}^a\right) > \frac{f}{k-3} = \frac{f_{Z1e}}{1 - G\left(z_{1L}^a\right)} \text{ and } \bar{\pi}_{Z1}\left(z_T\right) = \frac{f}{k-3} < \frac{f_{Z1e}}{1 - G\left(z_T\right)}.$$

Therefore, there exists $z_{1L} \in \left(z_{1L}^a, z_T\right)$ such that $\bar{\pi}_{Z1}\left(z_{1L}\right) = f_{Z1e}\left[1 - G\left(z_{1L}\right)\right]^{-1}$.

Finally,

$$\begin{aligned} \bar{\pi}_{Z2}\left(z_{2L}\right) &= A \int_{z_{2L}}^{\infty} \frac{x_L z_{1L} t^2}{\left(z_{2L}\right)^2} \left(\frac{1 - G(t)}{1 - G\left(z_{2L}\right)}\right) dt \\ &\quad + A \int_{z_{2L}}^{\infty} \left[m_{Z2}^{-1}(t)\, m_{Z1}\left(m_{Z2}^{-1}(t)\right) - \frac{x_L z_{1L} t^2}{\left(z_{2L}\right)^2}\right] \left(\frac{1 - G(t)}{1 - G\left(z_{2L}\right)}\right) dt \\ &= \frac{f}{k-3} + A \int_{z_{2L}}^{\infty} \left[m_{Z2}^{-1}(t)\, m_{Z1}\left(m_{Z2}^{-1}(t)\right) - \frac{x_L z_{1L} t^2}{\left(z_{2L}\right)^2}\right] \left(\frac{1 - G(t)}{1 - G\left(z_{2L}\right)}\right) dt. \end{aligned}$$

From assignment functions (25) and (26), it follows that if $z_{1L} < z_T$, $m_{Z2}^{-1}(z_2)\, m_{Z1}\left(m_{Z2}^{-1}(z_2)\right) < \left(x_L z_{1L} z_2^2\right) / \left(z_{2L}\right)^2$ for all $z_2 \geq z_{2L}$, while if $z_{1L} \geq z_T$, $m_{Z2}^{-1}(z_2)\, m_{Z1}\left(m_{Z2}^{-1}(z_2)\right) = \left(x_L z_{1L} z_2^2\right) / \left(z_{2L}\right)^2$ for all $z_2 \geq z_{2L}$. Therefore, we have

$$\bar{\pi}_{Z2}\left(z_{2L}^a\right) \leq \frac{f}{k-3} = \frac{f_{Z2e}}{1 - G\left(z_{2L}^a\right)} \leq \frac{f_{Z2e}}{1 - G\left(z_{2L}\right)} \text{ if } z_{2L} \geq z_{2L}^a.$$

Therefore, $z_{2L} < z_{2L}^a$ must hold.

(ii) The average revenue of teams is $\bar{r} = \sigma\left(\bar{\pi}_X + \bar{\pi}_{Z1} + \bar{\pi}_{Z2} + f + f_I(M_I/M)\right)$, where M_I is the mass of international teams. Since $\bar{\pi}_{Z1} + \bar{\pi}_{Z2} = 2\bar{\pi}_X$ holds both in the autarky and in the trade equilibrium, the mass of consumption varieties is

$$M = \frac{L}{\bar{r}} = \frac{L}{\sigma\left(3\bar{\pi}_X + f + f_I(M_I/M)\right)}.$$

From the proof for (i) of this Lemma, $\bar{\pi}_X$ is higher than the autarky level. Therefore, $M < M^a$. Q.E.D.

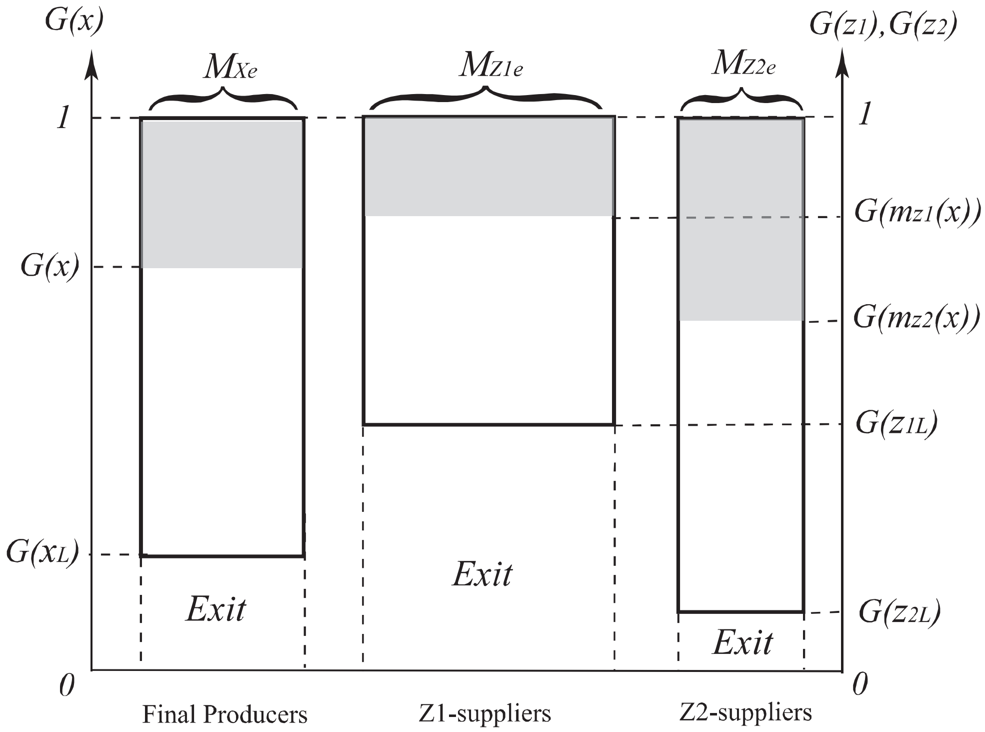

Figure 1: Matching market equilibrium. The vertical axis expresses the share of firms whose quality is lower than a certatin level. The mass of the survival firms $M_{Xe}\left[1-G\left(x_L\right)\right]$ and $M_{Zie}\left[1-G\left(z_{iL}\right)\right]$, which are the areas of rectangles surounded by solid lines must be equalized. Firms with lower quality than the thresholds x_L, z_{1L}, and z_{2L} exit from the market. The assortative matching implies for all $x \geq x_L$, the mass of final producers with higher quality than x, which is the area of the left grey rectangle, must be equal to the mass of Zi-suppliers with higher quality than $m_{Zi}\left(x\right)$, which are the areas of the center and right grey rectangles.

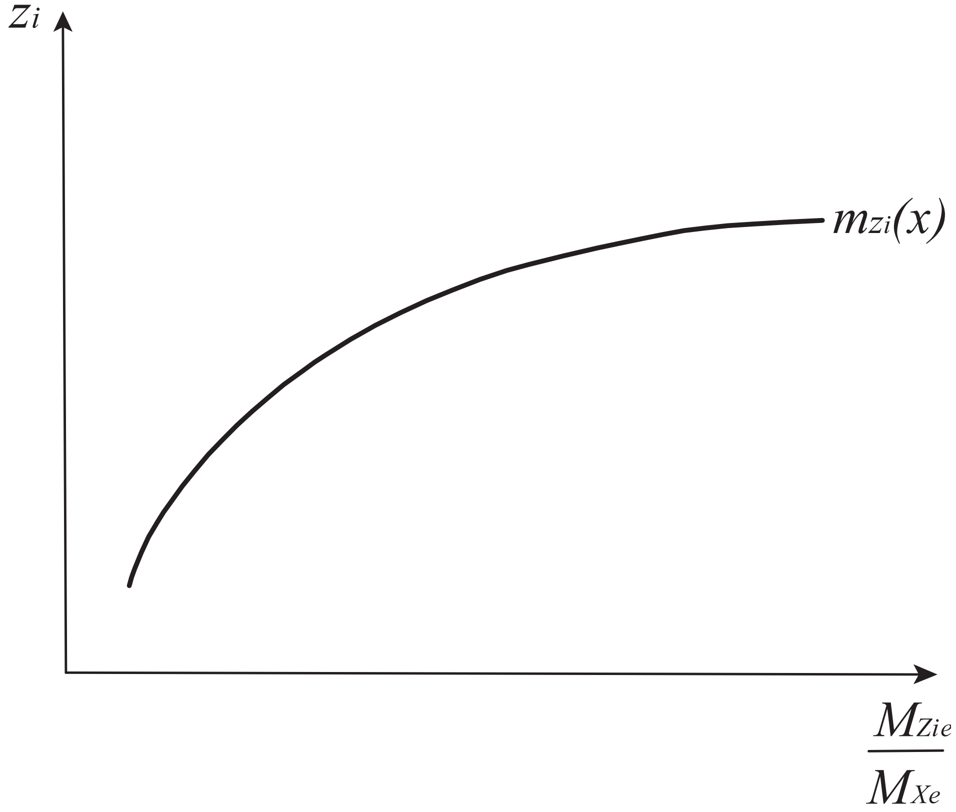

Figure 2: The quality of Zi-supplier matched with a final producer with quality x, $m_{Zi}(x)$, is increasing and concave in the relative mass of entrants into Zi-sector and final goods sector, M_{Zie}/M_{Xe}.

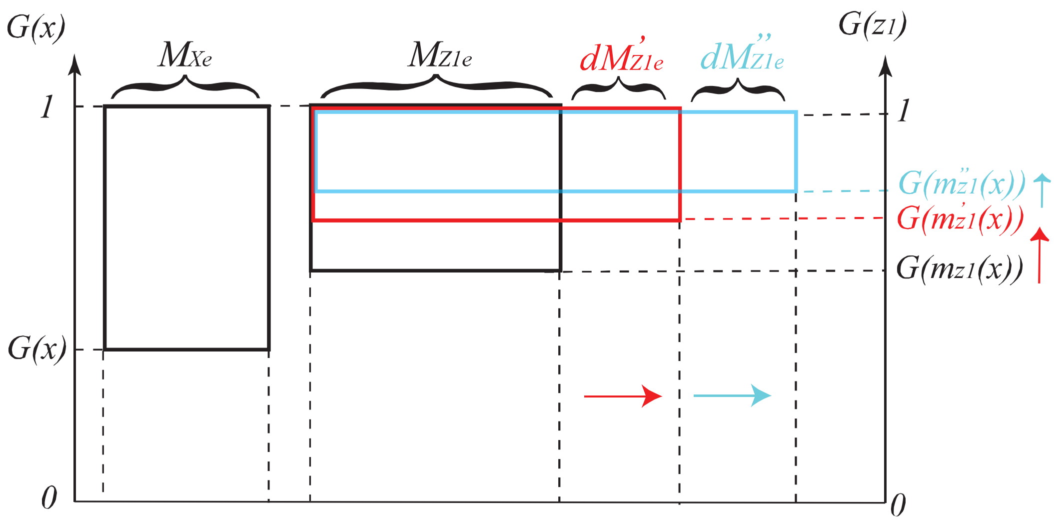

Figure 3: As the mass of entrants into Zi-sector and final goods sector, M_{Zie}, increases, a final producer with quality x becomes matched with higher quality of Zi-supplier with $m_{Zi}(x)$ though the marginal improvement is diminishing.

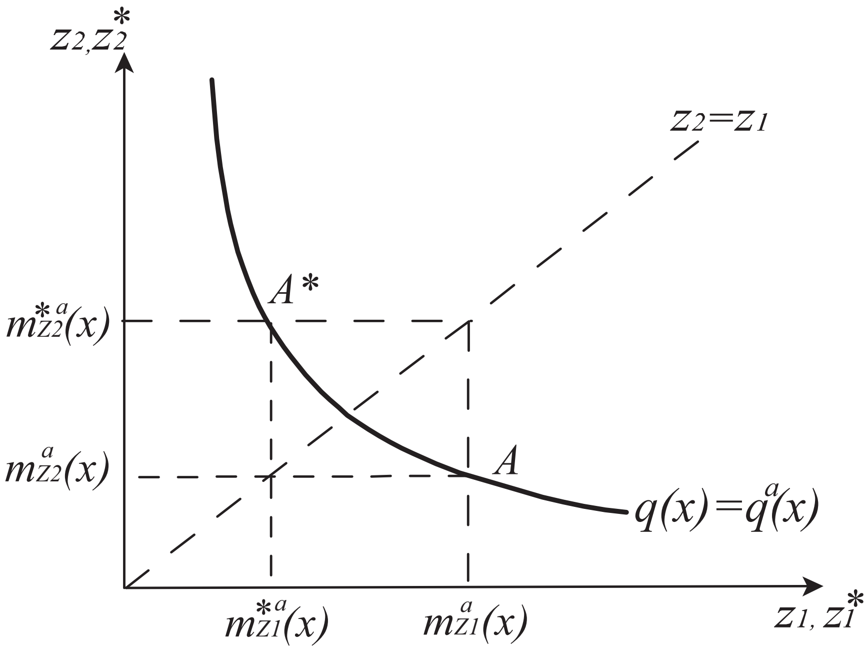

Figure 4: Autarky matching: The curve depicts a combination of the quality of Zi-suppliers in teams that produces final goods of quality $q^a(x)$ with a final producer with quality x. Point A and Point A* expresse the quality of Zi-suppliers in Home autarky teams $(m_{Z1}^{A}(x), m_{Z2}^{A}(x))$ and in Foreign autarky teams $(m_{Z1}^{A*}(x), m_{Z2}^{A*}(x))$, respectively. In each team, a Zi-supplier in low entry cost sector has higher quality than the other Zj-supplier within the same team.

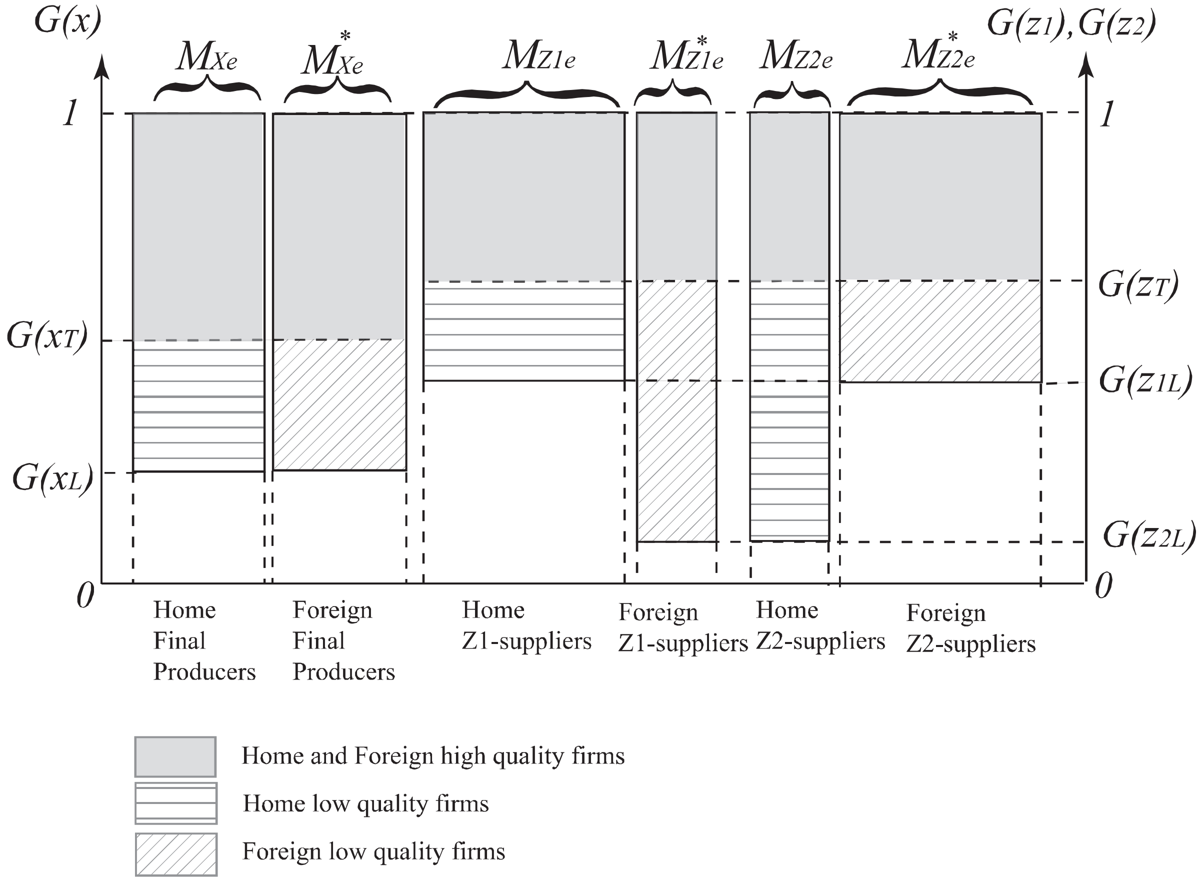

Figure 5: Fixed trade costs separate firms into three groups: high quality firms in Home and Foreign, which are expressed in grey areas, are matched together; low quality firms in each country, which are expressed in the same stripe areas, are matched locally.

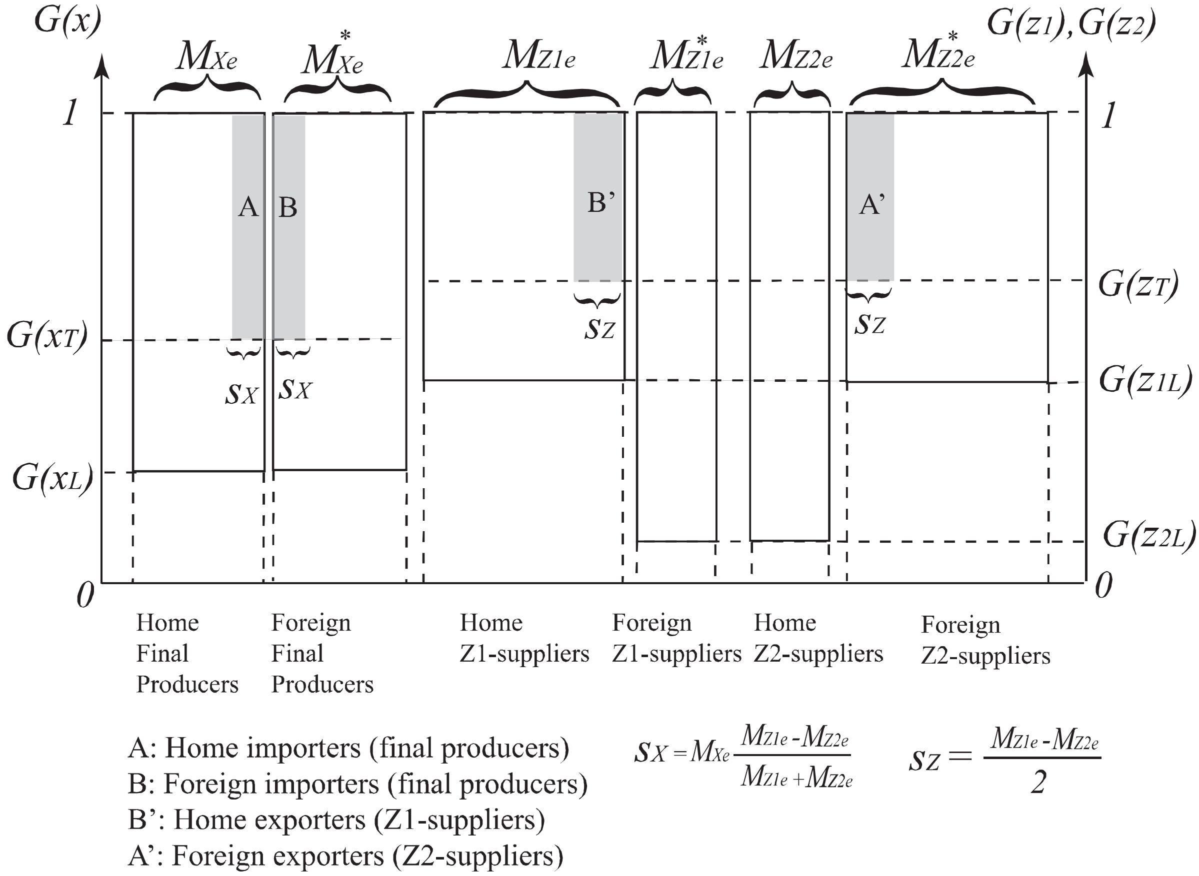

Figure 6: Equilibrium trade patterns. Home final producers in area A import from Foreign $Z2$-suppliers in Area A'. Foreign final producers in area B import from Home $Z1$-suppliers in Area B'.

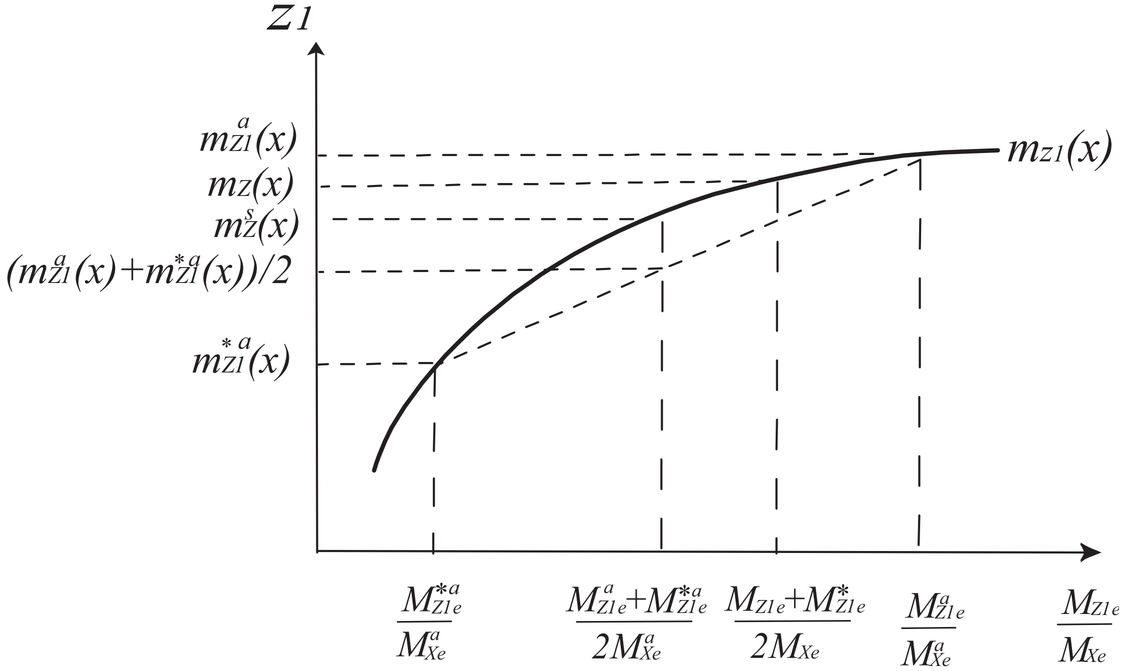

Figure 7: The quality of $Z1$-suppliers matched with high quality final producers with $x \geq x_T$ is higher than the average of those in two countries in the autarky. In the short run equilibrium, the relative mass of entrants into $Z1$-sector to final producers, $(M^a_{Z1e} + M^{*a}_{Z1e})/2M^a_{Xe}$, is the average of those in the autarky, M^a_{Z1e}/M^a_{Xe} and M^{*a}_{Z1e}/M^{*a}_{Xe}. The quality of $Z1$-suppliers matched with final producers with quality x, $m^s_{Z1}(x)$, is higher than the average of the two autarky levels, $m^a_{Z1}(x)$ and $m^{a*}_{Z1}(x)$. In the long run, the relative mass of $Z1$-suppliers to final producer, $(M_{Z1e} + M^*_{Z1e})/2M_{Xe}$, increases so that $m_{Z1}(x)$ rises further.

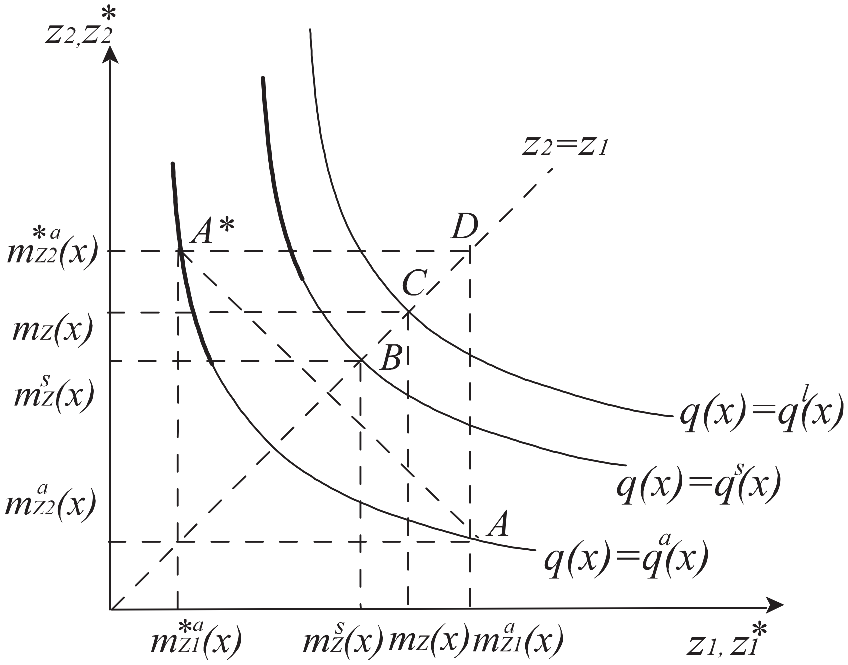

Figure 8: After the opening of trade, high quality final producers with quality $x \geq x_T$ upgrade the quality of final goods both in the short run, from $q^a(x)$ to $q^s(x)$, and in the long run, from $q^s(x)$ to $q^l(x)$. In the short run, final producers change Zi-suppliers with $m_{Zi}^a(x)$ to those with $m_Z^s(x)$; in the long run, final producers change Zi-suppliers with $m_{Zi}^s(x)$ to those with $m_Z(x)$.

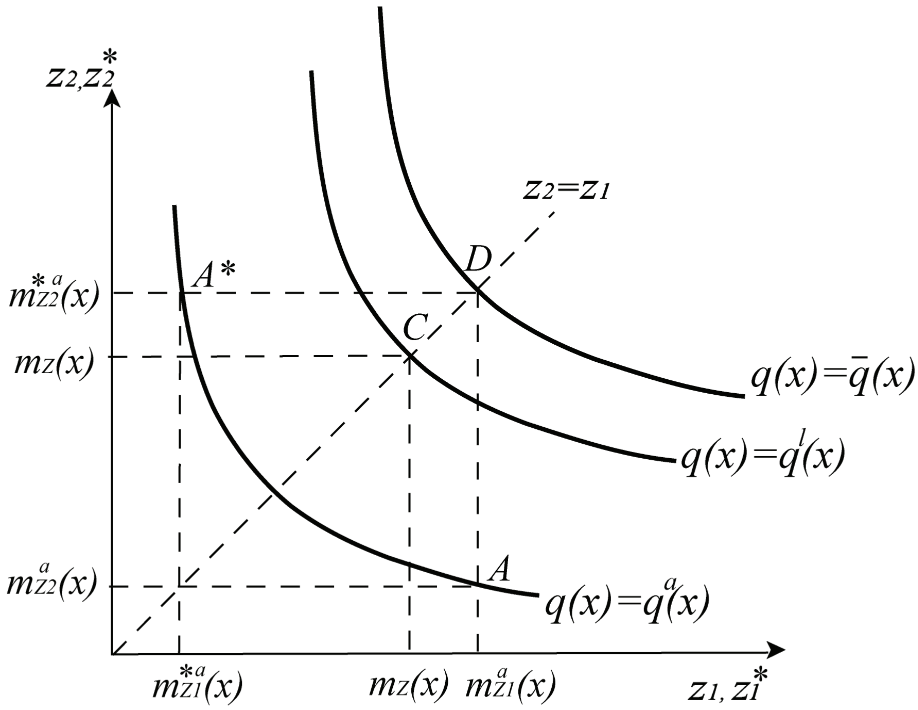

Figure 9: In a complete specialization equilibrium, a final producer with quality x is matched with Zi-suppliers with quality $m^a_{Z1}(x) = m^a_{Z2}(x)$ expressed by Point D. The quality of final good $\bar{q}(x)$ is higher than the level in the incomplete specialization equilibrium.