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REAL INTEREST RATES AS A MEASURE OF WELFARE IN AN ECONOMY WITH INCENTIVE CONSTRAINTS*

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Abstract

In this paper, we study the relationship between welfare and real interest rates in Aiyagari and Williamson's (2000) setting. They developed a dynamic insurance contract model with an incomplete market monetary economy and demonstrated that the optimal inflation rate is positive. This paper investigates the relationship between real interest rates and welfare in their model with more precise measures of inflation around the optimal inflation rate. Our numerical results demonstrate that real interest rates and welfare have a positive correlation and suggest that real interest rates can serve as a measure of welfare.

JEL classification: E4, G2
Keywords: Incomplete markets, dynamic insurance contract, measure of welfare

I. *Introduction*

It is well known that asset prices contain useful information about macroeconomic performance. In this paper, we investigate whether asset prices can serve as a measure of welfare in a dynamic contract economy. Exploring the idea of Lucas (1987) within a representative agent framework, Alvarez and Jermann (2004) advanced a method for measuring the cost of business cycles using asset prices. Among models with exogenous market incompleteness, Bewley (1983), Huggett (1993), and Aiyagari (1994) analyzed a situation where agents cannot insure their idiosyncratic income fluctuations because of borrowing constraints. Then, they demonstrated that real interest rates tend to be low because of strong precautionary demand for assets. Therefore, their models reveal a positive relationship between real interest rates and welfare (measured in terms of the expected utility of ex ante identical agents).

More recent models consider endogenous market incompleteness and pay serious attention

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to contractual arrangements among agents. Aiyagari and Williamson (2000) and Saito and Takeda (2006) incorporated a dynamic insurance contract into a monetary economy with incomplete markets. They introduced fiat money into an economy where agents cannot borrow from financial markets, but can contract with an intermediary to insure themselves against idiosyncratic shocks to their endowments.

In particular, Aiyagari and Williamson (2000) explored the welfare effect of inflation. In general, inflation lowers economic welfare because of an increase in the cost of holding money. However, in their model, because an insurance contract is a substitute for money, a rise in the cost of holding money endogenously increases the efficiency of insurance contracts; consequently, the expected utility of ex ante identical agents is maximized at a moderate rate of inflation. In addition, the volume of transfers and the level of real interest rates change according to the level of inflation. That is, in their setting, monetary policy has an impact on real interest rates as well as on welfare. However, they did not discuss the relationship between real interest rates and economic welfare.

In this paper, we examine whether the positive relationship between real interest rates and welfare is applicable to the dynamic insurance contract model constructed by Aiyagari and Williamson (2000). Using an intensive numerical calculation, we demonstrate that the positive relationship between real interest rates and welfare still holds in this environment. Our findings imply that we can use real interest rates as a measure of welfare in an economy with incentive constraints.

This paper is organized as follows. Section II summarizes the basic elements of the Aiyagari and Williamson (2000) model. Section III presents the numerical computation procedures and results. In Section IV, we discuss the relationship between economic welfare and real interest rates. In Section V, we provide a conclusion.

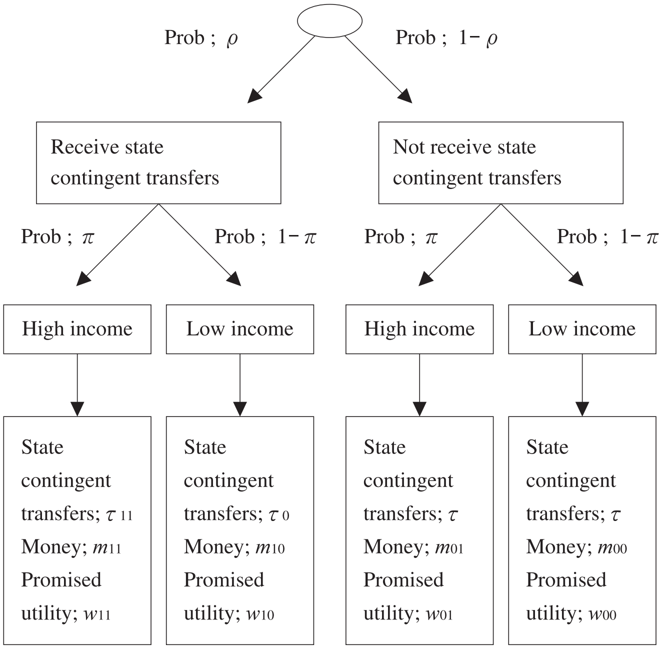
II. *Aiyagari and Williamson (2000) Model*

In this section, we briefly sketch the model, while a detailed description of the model is provided in Aiyagari and Williamson (2000). There are ex ante identical consumers, who face unobservable idiosyncratic income shocks. These consumers cannot hold interest-bearing bonds. There are two ways to smooth their consumption: to hold money for self-insurance, and to make a long-term insurance contract with a financial intermediary. It is assumed that consumers are allowed to cancel the long-term contract with the intermediary. Once they defect, they must permanently live in a Bewley-type monetary economy, where only money can serve as self-insurance instruments. We also assume that agents occasionally fail to receive the insurance transfers, even if they participated in the long-term contract. In preparation for such an event, agents have to hold money.

Figure 1 summarizes the process of events.

In this economy, the intermediary accesses the competitive bond market and delivers the transfer payments depending on reports by consumers. When the intermediary fails to identify the state of some consumers, the intermediary delivers the same payments for high-endowment consumers as for low-endowment consumers. The insurance contract must be self-enforcing and incentive-compatible. The contract must satisfy incentive constraints and enforcement constraints. An incentive-compatible insurance contract requires that the value of payments based

FIG 1. PROCESS OF EVENTS



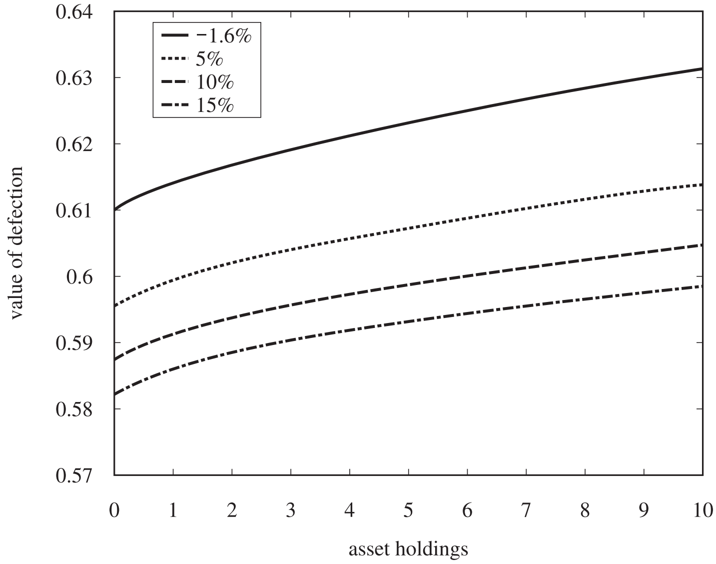
on true reports is higher than the value of payments based on false reports. A self-enforcing contract requires that the value of honoring the long-term contract is higher than the value of defecting from the long-term contract, which is the value of living in a Bewley-type self-insurance economy. Of central importance are the enforcement constraints:

$$(1-\beta)u[y_i+m+\tau_i(w,m)-\gamma m_{1i}(w,m)]+\beta w_{1i}(w,m)\geq\delta[y_i+m+\max_{i\in\{h,l\}}\tau_i(w,m)],(i=h,l)$$

$$(1-\beta)u[y_i+m+\tau(w,m)-\gamma m_{0i}(w,m)]+\beta w_{0i}(w,m)\geq\delta[y_i+m+\max_{i\in\{h,l\}}\tau(w,m)],(i=h,l)$$

$i=h, l$ denotes the state of endowments and $j=0, 1$ denotes the states of contingent transfers. When $j=0$, the intermediary fails to deliver the state-contingent transfers. β denotes the time discount factor, $y_i=1\pm\varepsilon$ denotes endowments, and γ denotes the gross inflation rate. w_{ji} denotes the promised utility. τ_i and m_{ji} denote insurance and monetary transfers. w_{ji}, τ_{ji}, and m_{ji} depend on the agent's report and the current state variable, w and m respectively. u denotes the periodic utility function, and δ denotes the defection value function. Considering the incentive and enforcement constraints, the intermediary delivers the insurance payments in a cost-minimizing way.

Fig 2. Defection Value Functions



III. *Computation and Results*

1. The Computational Procedures

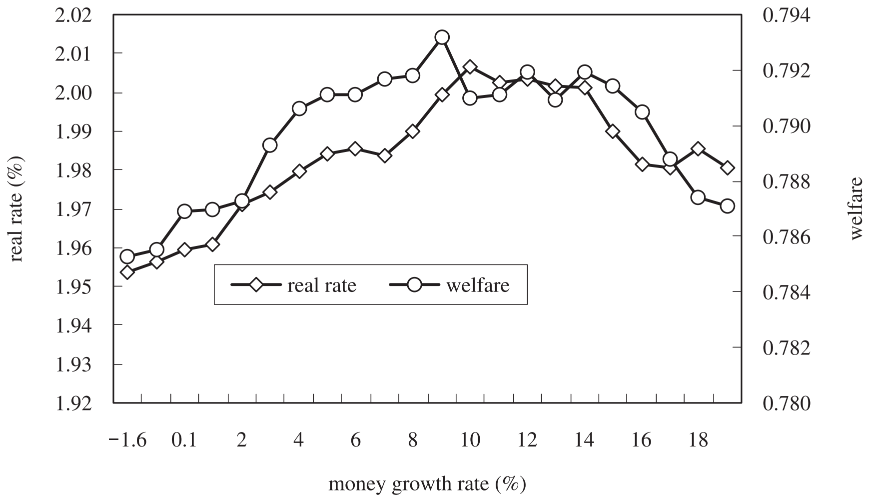
Aiyagari and Williamson (2000) provided results in the case where inflation rates equal the Friedman rule, 10%, 15%, 100%, and 1500%. We cannot find a precise relationship between real interest rates and welfare around the optimal inflation rates. Therefore, we investigate the relationship between real interest rates and welfare with more precise measures of inflation.

The first step in solving the model is to compute a defection value function, which is the value of living in the Bewley-type monetary economy for a given inflation rate. The second step is to choose an interest rate. The third step is to solve the intermediary's cost minimization problem and compute allocation under the interest rate. The final step is to examine whether the computed allocation satisfies the market clearing condition. If the market is not cleared, we return to the second step and update the real interest rate. The interest rate, which is consistent with the market clearing condition, is the equilibrium real interest rate.

The specification of exogenous parameters is as follows. Following Aiyagari and Williamson, we assume CARA utility, $u(c)=1-\exp(-c)$, and set the probability of receiving the state-contingent transfers, ρ, at 0.81 and the probability of receiving the high endowment, π, at 0.5. The subjective discount factor, β, and the shock size, ε, are specified as 0.99 and 0.6 as in Aiyagari and Williamson. In addition, we compute the cases where β equals 0.99 and ε equals 0.4, β equals 0.98 and ε equals 0.6, and β equals 0.98 and ε equals 0.4.

We then fix the money growth rate and compute the value function of a Bewley-type

FIG 3. INFLATION, WELFARE, AND REAL INTEREST RATES; $\beta=0.99$ and $\varepsilon=0.6$



monetary economy, δ, which is used as the outside opportunity in enforcement constraints. In computing defection value functions, there is a subtle difference between Aiyagari and Williamson and ours. Aiyagari and Williamson employed the Chebyshev polynomial and computed defection value functions through value iteration. However, they did not refer to the order of the Chebyshev polynomial that they used in approximating defection value functions. In addition, when we compute defection value functions through value iteration with relatively low order (third to fifth) Chebyshev polynomials, approximated value functions are slightly convex around the upper bound of the state variables, while value functions must be concave.

In order to avoid such convexity in defection value functions, we follow Imrohorouglu (1992) and compute defection value functions through state space discretization. Figure 2, which presents value functions under several money growth rates, shows that the outside value functions are concave and decrease as the money growth rate increases. We approximate computed defection value functions by a fifth-order Chebyshev polynomial.

Finally, we choose a real interest rate and compute the cost function of the intermediary to obtain optimal decision rules. To compute the cost function, we use the same function approximation method as in Aiyagari and Williamson, a third-order Chebychev polynomial.

2. Results

Figure 3 plots welfare and real interest rates under money growth rates ranging from -1.6%, to 19% with increments of 1.0% in the case of $\beta=0.99$ and $\varepsilon=0.6$. For money growth rates ranging from -1.6% to about 10%, welfare rises as money growth rates increase. However, above 10% inflation, welfare tends to fall. This effect of inflation on welfare is similar to that in Aiyagari and Williamson (2000). Figure 3 shows the positive correlation between real interest rates and welfare.

Real interest rates rise until inflation rates reach 9%, and fall above this inflation rate. This

FIG 4. WELFARE AND REAL INTEREST RATES; $\beta = 0.99$ and $\varepsilon = 0.6$

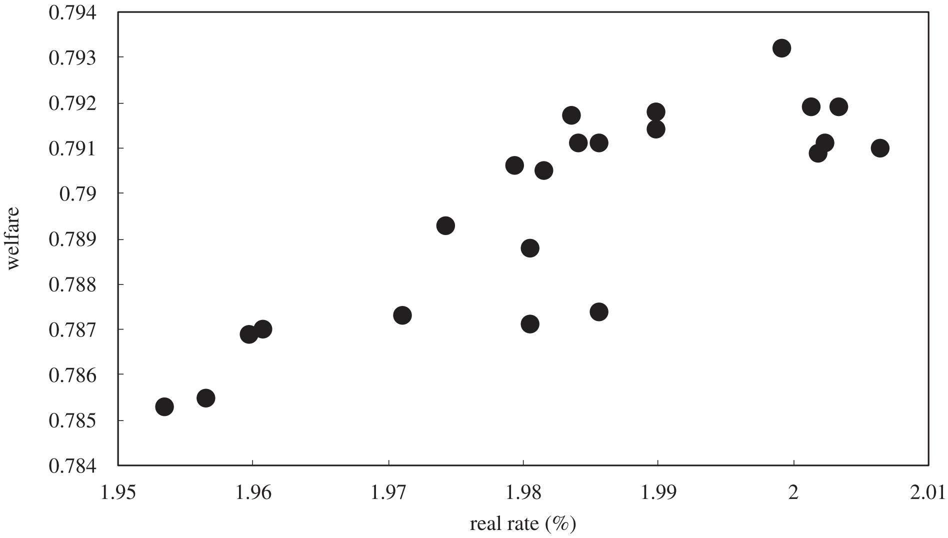
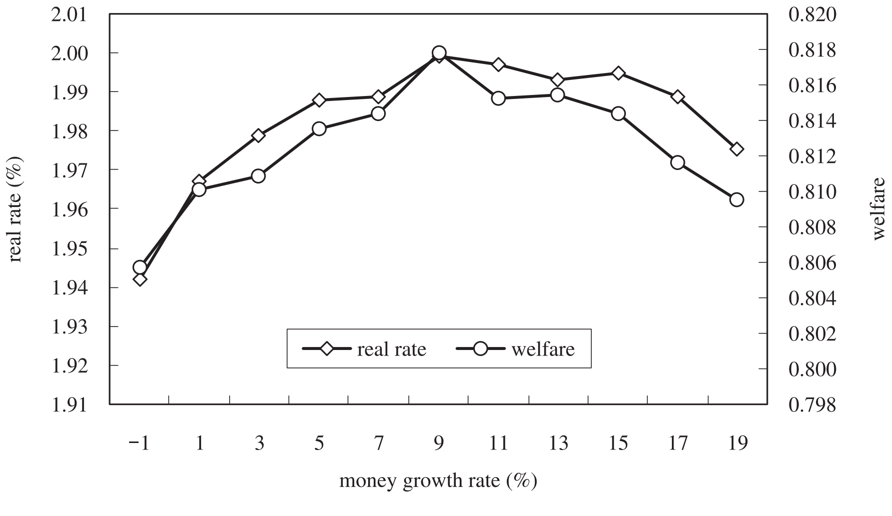


FIG 5. INFLATION, WELFARE, AND REAL INTEREST RATES; $\beta = 0.99$ and $\varepsilon = 0.4$



locus is similar to welfare. In addition, such a relationship can be found at higher inflation rates. For example, when the money growth rate is 100%, the level of welfare equals 0.775 and the real interest rate equals 1.923%. In addition, when the money growth rate is 200%, the level of welfare equals 0.772 and the real interest rate equals 1.921%. These numbers are lower than those under Friedman's rule.

Therefore, we find that the positive correlation between real interest rates and welfare

FIG 6. INFLATION, WELFARE, AND REAL INTEREST RATES; $\beta = 0.98$ and $\varepsilon = 0.6$

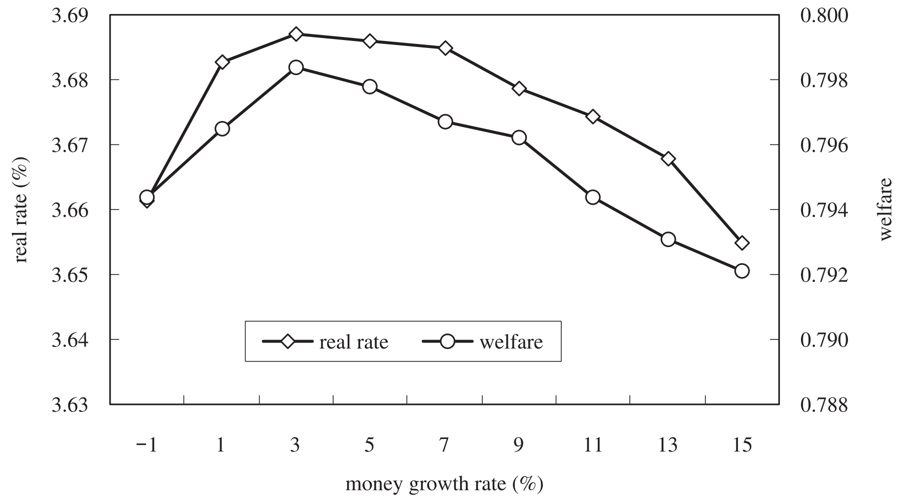
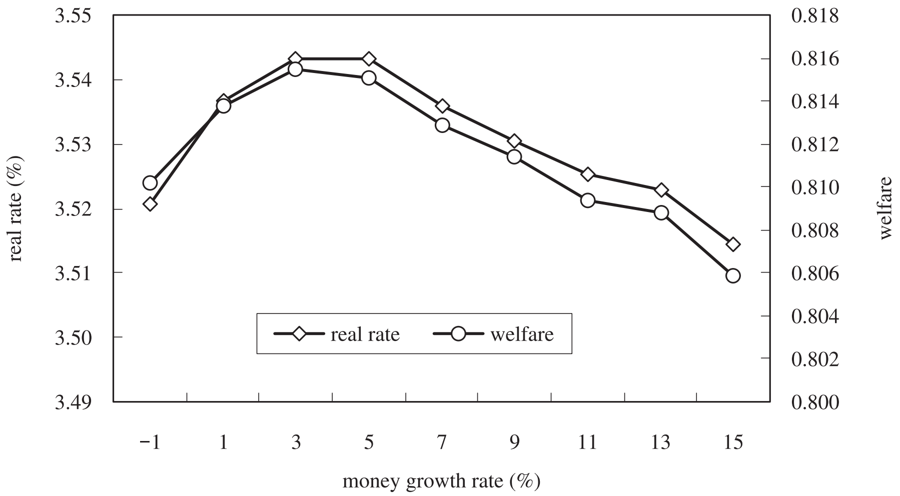


FIG 7. INFLATION, WELFARE, AND REAL INTEREST RATES; $\beta = 0.98$ and $\varepsilon = 0.4$



tends to hold at higher inflation rates. Figure 4 plots real interest rates and welfare. The figure shows that there is a near-positive correlation between welfare and real interest rates.

Figure 5 plots welfare and real interest rates under money growth rates ranging from -1% to 19% with increments of 2.0% in the case of β=0.99 and ε=0.4. For money growth rates ranging from Friedman's rule to about 9%, welfare rises as money growth rates increase. However, above 9% inflation, welfare tends to fall. In addition, real interest rates rise until inflation rates reach 9%, and fall above this inflation rate. Therefore, as in the case of β=0.98

and $\varepsilon = 0.6$, there is a near-positive correlation between welfare and real interest rates.

Figure 6 plots welfare and real interest rates under money growth rates ranging from -1% to 15% with increments of 2.0% in the case of $\beta = 0.98$ and $\varepsilon = 0.6$. We find that welfare and real interest rates are maximized when money growth rates are about 3-5%. Figure 7 plots welfare and real interest rates under money growth rates ranging from -1% to 15% with increments of 2.0% in the case of $\beta = 0.98$ and $\varepsilon = 0.4$. We find that welfare and real interest rates are also maximized when money growth rates are about 3-5%. In these cases, while optimal inflation rates are lower than the case of $\beta = 0.99$, there is a near-positive correlation between welfare and real interest rates.

Therefore, the numerical results show that there is a near-positive correlation between welfare and real interest rates. Our numerical results of a positive correlation between real interest rates and welfare differ from those of Aiyagari and Williamson. The difference between the two papers stems from the difference in computing the defection value functions.

Besides the relationship between welfare and real interest rates, we find that a decrease in variation of endowments lowers real interest rates and that optimal inflation rates are lower in the case of a low time discount factor.

IV. *Discussion*

In this section, we briefly discuss the intuition behind the relationship between real interest rates and welfare in this economy.

Given that endowments are private information, high-endowment agents pretend to be low-endowment agents to receive current transfers and raise their future consumption. Because of incentive compatibility constraints, the intermediary lowers the real interest rate so that agents reduce their future consumption. In this economy, because agents can hold money in preparation for future consumption, a rise in the costs of holding money mitigates the incentive compatibility constraints. Therefore, inflation enables the intermediary to offer more efficient contracts and raises real interest rates.

However, given that agents can cancel long-term contracts, increases in money growth rates imply a decrease in the value of canceling the long-term contract with the intermediary. Then, because inflation decreases agents' incentive to cancel long-term insurance contracts, the relaxed enforcement constraints enable the intermediary to offer more efficient insurance contracts. As a result, inflation reduces the variation of consumption. In general, such a decrease in the variation of consumption can improve welfare and raise real interest rates because of mitigating precautionary demand for assets. However, such a decrease in the variation of consumption implies that high-endowment agents have an incentive to pretend to be low-endowment agents. Then, because of incentive compatibility constraints, such a decrease in the variation of consumption reduces the efficiency of insurance contracts offered by the intermediary and lowers real interest rates.

In addition, an increase in money growth rates decreases the lower bound of expected utilities because of a decrease in the value of canceling the long-term contract. Then, inflation tends to increase the variation of expected utilities. An increase in the variation of expected utilities has two effects on real interest rates. On the one hand, an increase in the variation of expected utilities implies that high-endowment agents have less incentive to pretend to be low-endowment agents. Thus, such an increase in the variation of expected utilities raises the efficiency of insurance contracts offered by the intermediary and raises real interest rates. On the other hand, an increase in the variation of expected utilities raises asset demands, because of precautionary motives, and lowers real interest rates.

As discussed above, the relationship among inflation, welfare, and real interest rates is quite complex, and the effects of inflation on welfare and real interest rates can be positive or negative. However, our numerical results suggest that real interest rates can serve as a measure of welfare in the model economy.

Besides the relationship between welfare and real interest rates, we find that a decrease in variation of endowments lowers real interest rates. When variation of endowments is small, high-endowment agents have an incentive to pretend to be low-endowment agents. Then, the intermediary has to raise real interest rates and future consumption of high-endowment agents so that high-endowment agents have an incentive to report truthfully. In addition, we find that optimal inflation rates are lower in the case of a low time discount factor. An increase in inflation rates has two effects on welfare. On the one hand, relaxed enforcement constraints enable the intermediary to offer more efficient insurance contracts. On the other hand, a rise in the costs of holding money is not preferable in terms of self-insurance in preparation for failure in receiving state contingent transfers from the intermediary. Because a decrease in the time discount factor weakens the former effect, which is the merit of continuing to make a long-term insurance contract with the intermediary, optimal inflation rates are low in the case of a low time discount factor.

V. *Conclusion*

In this paper, we investigated the relationship between real interest rates and welfare in Aiyagari and Williamson's (2000) setting. For this purpose, we computed real interest rates and welfare with more precise measures of inflation around the optimal inflation rate. As a result, we found that there is a near positive correlation between welfare and real interest rates. These results differ from those of Aiyagari and Williamson. The major reason for the difference between the two papers is the difference in computing defection value functions.

In this monetary economy with incentive constraints, the relationship among inflation, welfare, and real interest rates is quite complex, and the effects of inflation on welfare and real interest rates can be positive or negative. However, as in the standard incomplete market economy, our numerical results suggest that real interest rates can serve as a measure of welfare in the model economy.

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